Associativity of celestial OPEs and supersymmetry

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Based on upcoming work with Adam Ball, Marcus Spradlin and Anastasia Volovich and on
2206.08322, 2208.14416 with Rishabh Bhardwaj, Luke Lippstreu, Lecheng Ren, Marcus Spradlin and Anastasia Volovich

Introduction

- Scattering amplitudes are incredibly constrained quantities
- Many different ways of computing them
- An entirely new formulation as a 2D CFT seems tantalizing
- Currently no first principles definition of CCFT
- What happens when we try to force amplitude to be CFT correlators?

Scattering amplitudes

A convenient parametrization for the momentum

$$p_i^\mu = \omega_i \, \epsilon_i \left\{1 + z_i \, \bar{z}_i, \; z_i + \bar{z}_i, \; i \left(z_i - \bar{z}_i\right), 1 - z_i \, \bar{z}_i\right\}$$

Stereographic co-ordinates on the celestial sphere

S-Matrix in a basis $\mathbf{P}_i, \mathbf{J}_z$ eigenstates

$$\mathcal{A}_n \equiv {}_{\text{out}}\left\langle p_1, s_1; \ldots p_m, s_m \right| \underbrace{p_{m+1}, s_{m+1}}_{\text{Momentum Eigenstates}}, \ldots p_n, s_n \Big\rangle_{\text{in}}$$

Momentum Helicity

$$\omega \to (cz + d)^2 \, \omega, \qquad z \to \frac{az + b}{cz + d}, \qquad \bar{z} \to \frac{a\bar{z} + b}{c\bar{z} + d}$$

Conformal symmetry is not manifest.

Celestial Amplitudes

Instead define

$$|\Delta, z, \bar{z}, \sigma\rangle = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta |\omega, z, \bar{z}, \sigma\rangle$$

S-Matrix in a basis of $\mathbf{J}_z, \mathbf{K}_z$ eigenstates.

$$\tilde{\mathcal{A}}_n = \prod_{i=1}^{n} \int_0^\infty d\omega_i \, \omega_i^{\Delta_i - 1} \, \mathcal{A}_n$$

Now, under a Loretnz transformation

$$\tilde{\mathcal{A}}_n \left(\left\{ \Delta_i, \frac{az_i + b}{cz_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}}, s_i \right\} \right) = \prod_{i=1}^{n} \left[(cz_i + d)^{2h_i} \left(\bar{c}\bar{z}_i + \bar{d} \right)^{2\bar{h}_i} \right] \tilde{\mathcal{A}}_n \left(\{ \Delta_i, z_i, \bar{z}_i, s_i \} \right)$$

Pasterski, Shao, Strominger

$$h_i = \frac{\Delta_i + s_i}{2}, \qquad \bar{h}_i = \frac{\Delta_i - s_i}{2}, \qquad s_i \longrightarrow \text{helicity of particle } i$$

Loretnz covariance guarantees *global* conformal covariance

From Celestial Amplitudes to CCFT

Soft graviton theorems

$$\mathcal{A}_n\left(1^{+2}, 2^{s_2}, \ldots\right) \xrightarrow{\omega_1 \to 0} \Bigg[\underbrace{\frac{1}{\omega_1}\mathcal{S}^{(-1)}\left(z_1, \bar{z}_1\right)}_{\text{Supertranslations}} + \underbrace{\mathcal{S}^{(0)}\left(z_1, \bar{z}_1\right)}_{\text{Superrotations}}\Bigg] \mathcal{A}_{n-1}\left(2^{s_2}, \ldots\right)$$

[Weinberg, Cachazo, Strominger]

This guarantees *local* conformal covariance

$$\tilde{\mathcal{A}}_n = \langle \mathcal{O}^{s_1}_{\Delta_1}\left(z_1, \bar{z}_1\right) \ldots \mathcal{O}^{s_n}_{\Delta_n}\left(z_n, \bar{z}_n\right)\rangle$$

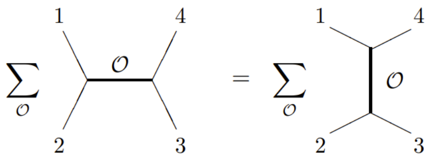
Celestial amplitudes are Celestial Conformal Field Theory (CCFT) correlators

CFT with "extra" translation symmetry which imposes exotic constraints on CFT

[Talks by Raclariu, Taylor]

Conformal bootstrap

Does the CFT structure impose strange constraints on amplitudes?



OPE associativity imposes constraints on the spectrum

We will implement a version of this

What does this mean in terms of scattering amplitudes?

Outline

1. Collinear limits and OPEs in CCFT
2. OPE associativity and scattering amplitudes
3. OPE associativity at tree-level
4. OPE associativity at loop-level
5. The role of SUSY

Collinear limits and OPEs

OPEs in CCFT are controlled by the collinear limits of scattering amplitudes

$$p_i^\mu = \omega_i\,\epsilon_i\,\{1 + z_i\,\bar{z}_i,\, z_i + \bar{z}_i,\, i\,(z_i - \bar{z}_i)\,, 1 - z_i\,\bar{z}_i\}$$

In this parametrization

$$\lambda_i = \epsilon_i\sqrt{\omega_i}\begin{pmatrix}1\\ z_i\end{pmatrix}, \qquad \tilde{\lambda}_i = \sqrt{\omega_i}\begin{pmatrix}1\\ \bar{z}_i\end{pmatrix}$$

$$2p_i\cdot p_j = -\langle ij\rangle\,[ij] = 2\epsilon_i\epsilon_j\omega_i\omega_j\,z_{ij}\,\bar{z}_{ij}$$

As $p_i \cdot p_j \to 0$, the operators $\mathcal{O}_{\Delta_i}\,(z_i, \bar{z}_i)$ and $\mathcal{O}_{\Delta_j}\,(z_j, \bar{z}_j)$ approach each other.

We are interested in *holomorphic* OPEs, i.e. $z_{ij} \to 0$

OPEs from collinear limits

Amplitudes have a particularly simple behaviour in the collinear limit

$$\mathcal{A}_n\left(1^{s_1}, 2^{s_2} \ldots n^{s_n}\right) \xrightarrow{1||2} \sum_{s,\text{loops}} \underbrace{\text{Split}_s\left(1^{s_1}, 2^{s_2}\right)}_{\text{Universal functions}} \mathcal{A}_{n-1}\left(P^s, \ldots n^{s_n}\right)$$

$$\text{Split} = \text{Split}^{\text{tree}} + \text{Split}^{\text{massive loops}} + \text{Split}^{\text{massless loops}}$$

Bern, Chalmers, Dixon, Kosower, Dunbar, Perelstein, Rozowsky ...

From this, one can obtain the OPE

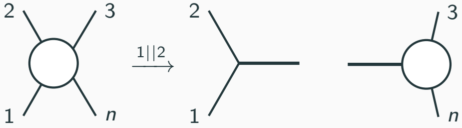
$$\mathcal{O}^{s_1}_{\Delta_1}\left(z_1, \bar{z}_1\right) \mathcal{O}^{s_2}_{\Delta_2}\left(z_2, \bar{z}_2\right) \sim \frac{1}{z_{12}} \sum_s C^s\left(\Delta_1, \Delta_2\right) \mathcal{O}^s_{\Delta}\left(z_2, \bar{z}_2\right) + \ldots$$

Depends on $\Delta_1, \Delta_2, s_1, s_2$.

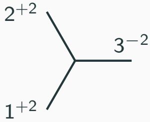
Computed from the splitting function

Fan, Fotopoulos, Taylor

Collinear limits : Trees



Determined by the three point amplitudes.

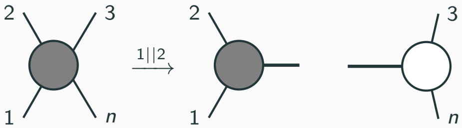


$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2}$$

$$\mathcal{O}^{+2}_{\Delta_1}(z_1, \bar{z}_1)\, \mathcal{O}^{+2}_{\Delta_2}(z_2, \bar{z}_2) \sim \kappa_{2,2,-2} \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 - 1\right) \mathcal{O}^{+2}_{\Delta_1 + \Delta_2}(z_2, \bar{z}_2) + \ldots$$

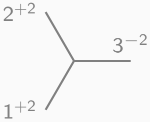
Other massless bosons can contribute

Fermions cannot contribute

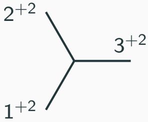


We can integrate out massive particles to generate new effective vertices.

Integrating out a massive fermion/ scalar



$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2}$$



$$= \kappa_{2,2,2} [12]^2[23]^2[13]^2$$

$$\mathcal{O}^{+2}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+2}_{\Delta_2}(z_2,\bar{z}_2) \sim \kappa_{2,2,-2}\frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 - 1, \Delta_2 - 1\right)\mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2,\bar{z}_2)$$
$$+\kappa_{2,2,2}\frac{\bar{z}^5_{12}}{z_{12}} B\left(\Delta_1 + 3, \Delta_2 + 3\right)\mathcal{O}^{-2}_{\Delta_1+\Delta_2+4}(z_2,\bar{z}_2) + \dots$$

Himwich, Pate, Singh

Collinear limits : Trees & massive loops

We can account for massive particles by simply including new three point amplitudes

Operators which do not generate new three point amplitudes will not contribute to the singular term in the OPE

Excluding massless higher spins, there are only a finite number of new three point amplitudes

These correspond to operators like $R^3, F^3, R^2\phi$

OPE associativity

Are these OPEs associative?

$$\mathcal{O}_{\Delta}(z,\bar{z}) = \sum_m \mathcal{O}_{\Delta,m}(z)\,\bar{z}^m$$

$$\forall m_1, m_2, m_3 \left[\left[\mathcal{O}_{\Delta_1,m_1}, \mathcal{O}_{\Delta_2,m_2}\right], \mathcal{O}_{\Delta_3,m_3}\right] + cyclic \stackrel{!}{=} 0$$

Translate to a statement about scattering amplitudes

$$\left[\mathcal{O}_{\Delta_1,m_1}, \mathcal{O}_{\Delta_2,m_2}\right] = \mathcal{P}_{m_1,m_2} \oint_{z_{12}=0} \mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}_{\Delta_2}(z_2,\bar{z}_2)$$

The Jacobi identity is equivalent to

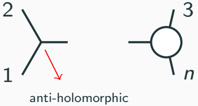
$$\underbrace{\left[\operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_2} - \operatorname*{Res}_{z_1\to z_3}\operatorname*{Res}_{z_2\to z_3} + \operatorname*{Res}_{z_2\to z_3}\operatorname*{Res}_{z_1\to z_3}\right]} \left\langle \mathcal{O}^{\sigma_1}_{\Delta_1}(z_1,\bar{z}_1)\;\mathcal{O}^{\sigma_2}_{\Delta_2}(z_2,\bar{z}_2)\ldots\right\rangle \stackrel{!}{=} 0$$

Independent of Δ_1, Δ_2.

OPE associativity

$$\left[\underset{z_2\to z_3}{\mathrm{Res}}\;\underset{z_1\to z_2}{\mathrm{Res}} - \underset{z_1\to z_3}{\mathrm{Res}}\;\underset{z_2\to z_3}{\mathrm{Res}} + \underset{z_2\to z_3}{\mathrm{Res}}\;\underset{z_1\to z_3}{\mathrm{Res}}\right]\mathcal{A}_n \overset{!}{=} 0$$

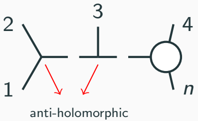
$$\underset{z_1\to z_2}{\mathrm{Res}}\;\mathcal{A}_n =$$



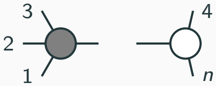
$$\mathcal{A}_3\left(1^{s_1},2^{s_2},3^{s_3}\right) = \begin{cases} \kappa_{s_1,s_2,s_3}\,[12]^{s_1+s_2-s_3}[23]^{s_2+s_3-s_1}[31]^{s_3+s_1-s_2} & s_1+s_2+s_3>0 \\ \\ \kappa_{s_1,s_2,s_3}\langle 12\rangle^{s_3-s_1-s_2}\langle 23\rangle^{s_1-s_2-s_3}\langle 31\rangle^{s_2-s_1-s_3} & s_1+s_2+s_3<0 \end{cases}$$

Associativity of the hard OPE

$$\operatorname*{Res}_{z_2 \to z_3} \operatorname*{Res}_{z_1 \to z_2} \mathcal{A}_n =$$



$$\left[\operatorname*{Res}_{z_2 \to z_3} \operatorname*{Res}_{z_1 \to z_2} - \operatorname*{Res}_{z_1 \to z_3} \operatorname*{Res}_{z_2 \to z_3} + \operatorname*{Res}_{z_2 \to z_3} \operatorname*{Res}_{z_1 \to z_3} \right] \mathcal{A}_n =$$



The part of amplitudes that can be constructed *solely* out of anti-holomorphic vertices vanish

Sufficient to apply this to four point amplitudes.

Ren, Spradlin, AY, Volovich

All line shift recursion

These amplitudes are precisely the ones constructible by holomorphic all line shift recursion relations

$$\hat{\lambda}_i = \lambda_i - w\, c_i\, X, \qquad i = 1, \ldots n$$

For large w, $\mathcal{A}_n \sim w^a \mathcal{A}_n$. Constructibility condition

$$2a = -\sum_{i=1}^{4} s_i - c < 0$$

exponent of shift parameter

helicities

Dimension of couplings

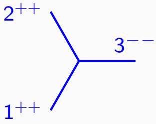
Cohen, Elvang, Kiermaier

At 4 points, the single minus and MHV amplitudes are not all line shift constructible.

$$\mathcal{A}\left(1^{+2}, 2^{+2}, 3^{+2}, 4^{-2}\right) = \kappa_{2,2,2}\kappa_{-2,-2,2}(\langle 14\rangle[13]\langle 34\rangle)^2 \frac{[12][23][31]}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,,$$

$$\mathcal{A}\left(1^{+2}, 2^{-2}, 3^{-2}, 4^{+2}\right) = \frac{\left(\langle 23\rangle\,[14]\right)^4}{s_{14}} \left(\kappa^2_{2,2,2} s_{12}\, s_{13} - \kappa^2_{2,2,0} + \kappa^2_{2,2,-2} \frac{1}{s_{12}\, s_{13}}\right)\,,$$

Pure gravity



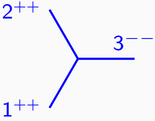
$$= \kappa_{2,2,-2}\,\frac{[12]^6}{[23]^2[13]^2}$$

Only anti-holomorphic vertex

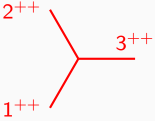
No amplitudes can be constructed with just this

All OPEs are associative. Similarly in EYM.

Higher derivative Gravity

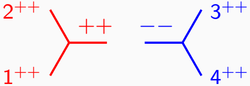


$= \kappa_{2,2,-2} \dfrac{[12]^6}{[23]^2[13]^2}$



$= \kappa_{2,2,2} [12]^2[23]^2[13]^2$

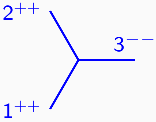
$\mathcal{A}_4\left(1^{++}, 2^{++}, 3^{++}, 4^{++}\right) =$



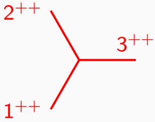
$+$ perms

$$= 10\kappa_{-2,2,2}\kappa_{2,2,2}\, s\, t\, u\, \frac{[12][23][34][41]}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

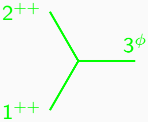
Only way to make this vanish is to set $\kappa_{2,2,2} = 0$



$= \kappa_{2,2,-2} \, \dfrac{[12]^6}{[23]^2[13]^2}$

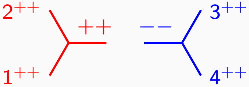


$= \kappa_{2,2,2} \, [12]^2[23]^2[13]^2$



$= \kappa_{0,2,2} \, [12]^4$

$\mathcal{A}_4\left(1^{++}, 2^{++}, 3^{++}, 4^{++}\right) =$



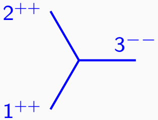
$+$

$$= \left(10\kappa_{-2,2,2}\kappa_{2,2,2} - 3\kappa_{0,2,2}^2\right) \, s \, t \, u \, \frac{[12][23][34][41]}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

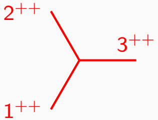
Vanishes if $10\kappa_{-2,2,2}\kappa_{2,2,2} - 3\kappa_{0,2,2}^2 = 0$

Ren, Spradlin, AY, Volovich

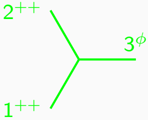
Higher derivative gravity with one scalar



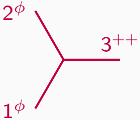
$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2}$$



$$= \kappa_{2,2,2}\, [12]^2[23]^2[13]^2$$

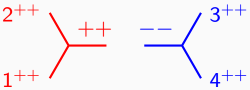


$$= \kappa_{0,2,2}\, [12]^4$$

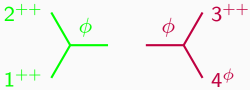


$$= \kappa_{0,0,2} \frac{[13]^2\,[23]^2}{[12]^2}$$

$$\mathcal{A}_4\left(1^{++}, 2^{++}, 3^{++}, 4^{\phi}\right) =$$



$$+$$



$$= \kappa_{2,2,0}\left[\kappa_{-2,2,2} \frac{[12]\,[34]^4}{\langle 12\rangle} \frac{\langle 4X\rangle^4}{\langle 1X\rangle^2\langle 2X\rangle^2} + \kappa_{0,0,2} \frac{[12]^3\,[34]^2}{\langle 12\rangle} \frac{\langle 4X\rangle^2}{\langle 3X\rangle^2}\right]$$

Vanishes if $\kappa_{-2,2,2} = \kappa_{2,0,0}$

Ren, Spradlin, AY, Volovich

Analysis of constraints

If R^3 like interactions are present, there *must* be atleast one scalar

Furthermore, the couplings of the theory must be tuned

$$(\kappa_{-2,2,2} - \kappa_{0,0,2})\,\kappa_{0,2,2} = 0, \qquad 3\kappa_{0,2,2}^2 = 10\,\kappa_{-2,2,2}\,\kappa_{2,2,2}\,.$$

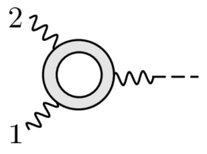
These constraints change as the spectrum changes

$$\mathcal{A}_n\left(p_1^{s_1}, p_2^{\sigma_2} \dots p_n^{s_n}\right) \xrightarrow{1||2} \sum_{s,\text{loops}} \underbrace{\text{Split}_s\left(p_1^{s_1}, p_2^{s_2}\right)}_{\text{Universal functions}} \mathcal{A}_{n-1}\left(P^s, \dots p_n^{s_n}\right)$$

$$\text{Split} = \underset{\checkmark}{\text{Split}^{\text{tree}}} + \underset{\checkmark}{\text{Split}^{\text{massive loops}}} + \text{Split}^{\text{massless loops}}$$

$$\text{Split}^{\text{massless loops}} = \text{Split}^{\text{factorizing}} + \text{Split}^{\text{non - factorizing}}$$

Collinear limits of scattering amplitudes: Massless loops



Contribution from one boson

$$\mathrm{Split}_{-2}^{\mathrm{factorizing}}\left(p_1^{+2}, p_2^{+2}\right) \propto \frac{[12]^4}{\langle 12\rangle^2}$$

This leads to an OPE

$$\begin{aligned}\mathcal{O}_{\Delta_1}^{+2}(z_1,\bar{z}_1)\,\mathcal{O}_{\Delta_2}^{+2}(z_2,\bar{z}_2) \sim & c_1(\Delta_1,\Delta_2)\frac{\bar{z}_{12}^4}{z_{12}^2}\mathcal{O}_{\Delta}^{-2}(z_2,\bar{z}_2) \\ & + c_2(\Delta_1,\Delta_2)\frac{\bar{z}_{12}^4}{2z_{12}}\underbrace{\frac{\partial}{\partial z_2}\mathcal{O}_{\Delta}^{-2}(z_2,\bar{z}_2)}_{\text{Non universal corrections}}\end{aligned}$$

Collinear limits of scattering amplitudes: Massless loops

$$\mathcal{O}^{+2}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+2}_{\Delta_2}(z_2,\bar{z}_2) \sim c_1(\Delta_1,\Delta_2)\,\frac{\bar{z}_{12}^4}{z_{12}^2}\mathcal{O}^{-2}_{\Delta}(z_2,\bar{z}_2) + c_2(\Delta_1,\Delta_2)\,\frac{\bar{z}_{12}^4}{2z_{12}}\frac{\partial}{\partial z_2}\mathcal{O}^{-2}_{\Delta}(z_2,\bar{z}_2)$$

This contribution fails to be associative

Fails for form factors in self-dual gravity [Bittleston]

Fails for form factors in self-dual YM [Costello, Paquette]

Associativity restored by introducing a scalar with a quartic kinetic term

Massive particles

Loop correction involving massive particles generically make the OPE non associative.

This obstruction can be characterized by holomorphic all line shift constructible four point amplitudes.

For gravity, these are $\mathcal{A}_4^{+2,+2,+2,+2}, \mathcal{A}_4^{+2,+2,+2,\phi}$.

Massless particles

Massless particles can generate non universal terms in the OPE

OPE associativity and SUSY

SUSY imposes constraints on amplitudes via SUSY Ward identities.

$$\bar{\mathcal{Q}}\mathcal{A}_4\left(1^{+2},2^{+2},3^{+2},4^{+3/2}\right)=0$$
$$\implies \mathcal{A}_4\left(1^{+2},2^{+2},3^{+2},4^{+2}\right)=0$$

This is a non-perturbative result

In our specific example, it cures all problems

More generally, in a theory of bosons and fermions with spins ≤ 2, with $\mathcal{N}=1$ SUGRA, Ward identities suffice to guarantee the vanishing of all "bad" amplitudes.

They also ensure that the non universal contributions to the OPE vanish

To appear: Ball, Spradlin, AYS, Volovich

OPE associativity and string theory

Do field theories which arise from compactifying string theory have an associative OPE?

$$\mathcal{A}_{bos}\left(1^{+2}, 2^{+2}, 3^{+2}, 4^{+2}\right) = \left(\alpha'\right)^2 \frac{\Gamma\left(1-\alpha' s\right)\Gamma\left(1-\alpha' t\right)}{\Gamma\left(1-\alpha' s-\alpha' t\right)} \times (\dots) \neq 0$$

This amplitude (in 26D) is compactification independent.

The Bosonic string can never give theories with associative OPEs

$$\mathcal{A}_{het}\left(1^{+2}, 2^{+2}, 3^{+2}, 4^{+2}\right) = 0$$

There could be compactifications to 4D of the heterotic string that lead to non associative OPEs

Compatifications which preserves at least $\mathcal{N} = 1$ SUSY

Collinear limits of scattering amplitudes

$$\mathcal{A}_n\left(p_1^{s_1}, p_2^{\sigma_2} \ldots p_n^{s_n}\right) \xrightarrow{1||2} \sum_{s,\text{loops}} \underbrace{\text{Split}_s\left(p_1^{s_1}, p_2^{s_2}\right)}_{\text{Universal functions}} \mathcal{A}_{n-1}\left(P^s, \ldots p_n^{s_n}\right)$$

$$\text{Split} = \underset{\checkmark}{\text{Split}^{\text{tree}}} + \underset{\checkmark}{\text{Split}^{\text{massive loops}}} + \text{Split}^{\text{massless loops}}$$

$$\text{Split}^{\text{massless loops}} = \underset{\checkmark}{\text{Split}^{\text{factorizing}}} + \text{Split}^{\text{non - factorizing}}$$

Non factorizing contributions to the OPE

These are better understood for gluons which will be the focus here

Non factorizing contributions arise from IR divergences

A one-loop gluon amplitude has IR divergences

$$\mathcal{A}_n^{1-\text{loop}}|_{\text{singular}} = -c_\Gamma \mathcal{A}_n^{\text{tree}} \frac{1}{\epsilon^2} \sum_{i=1}^{n} \left(\frac{\mu^2}{-s_{ii+1}} \right)^{\epsilon}$$

If $z_{12} \to 0, \omega_1 = t\omega, \omega_2 = (1-t)\omega$

$$s_{12}^{-\epsilon} \sim \frac{1}{\epsilon} \log(t(1-t)), \quad s_{n1}^{-\epsilon} \sim \frac{1}{\epsilon} \log t, \quad s_{23} \sim \frac{1}{\epsilon} \log(1-t)$$

These contributions can't be attributed to a left or right amplitude

Non factorizing contributions

$$\text{Split}^{\text{non-factorizing}} = \frac{1}{z_{12}}\left[f_0 + f_1 \log\left(-\omega^2 z_{12}\bar{z}_{12} t(1-t)\right) + f_2 \log^2\left(-\omega^2 z_{12}\bar{z}_{12} t(1-t)\right)\right]$$

f_0, f_1, f_2 are independent of z_{ij} and ω.

$$\mathcal{O}^a_{\Delta_1,+}(z_1,\bar{z}_1)\,\mathcal{O}^b_{\Delta_2,+}(z_2,\bar{z}_2) \sim \frac{if^{abc}}{z_{12}}\left[1 + \frac{a}{2}\left(C^{(1)}_{0,+} + C^{(1)}_{1,+}\hat{\mathcal{D}}_{12} + C^{(1)}_{2,+}\hat{\mathcal{D}}^2_{12}\right)\right] C^{(0)}_{+}\,\mathcal{O}^c_{\Delta,+}(z_2,\bar{z}_2)$$

$$\hat{\mathcal{D}}_{12} = \frac{\partial}{\partial\Delta_1} + \frac{\partial}{\partial\Delta_2} + \frac{\partial}{\partial\Delta} + \frac{1}{2}\log\left(-z_{12}\bar{z}_{12}\right)$$

Unavoidable logarithms in the OPE

Bhardwaj, Lippstreu, Ren, Spradlin, AYS, Volovich

Summary and outlook

- Associativity of the celestial OPE is *very* restrictive
- The role of SUSY in associativity
- At what scale is SUSY required?
- IR divergences introduce logarithms into the OPE. Does this point to a logarithmic CFT?

Thank you for your attention!