Exact Integrated Correlators in N=4 Super Yang-Mills

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From Amplitudes to Gravitational Waves
Nordita, 24 - 28 July

References

With Daniele Dorigoni & Michael Green, + Haitian Xie

- arXiv:2102.08305, arXiv:2102.09537, arXiv:2210.14038

With Shun-Qing Zhang

- arXiv:2203.01890

With Augustus Brown & Haitian Xie, + Paul Heslop

- arXiv:2303.13195, arXiv:2303.17570, + in progress

Also See the talk by Michael Green at the workshop

Outline

- Review on correlators of superconformal primaries in N=4 SYM Hansen's talk
- Introduction to integrated correlators and connections with periods of Feynman integrals (planar limit)
- Integrated correlators at finite YM coupling and supersymmetric localisation
- Exact expressions for integrated correlators and their generating functions
- Large-N expansion and connections with α' expansion of string amplitudes
- Conclusion and outlook

Four-point correlators

- Superconformal primaries in N=4 Super Yang-Mills (SYM)

$$T_{p_1\dots p_n}(x,Y) = \frac{\prod_i p_i}{\sum_i p_i} T_{p_1}(x,Y)\dots T_{p_n}(x,Y)$$

$$T_p(x,Y) = \frac{1}{p}\mathrm{tr}(\phi_{I_1}(x)\dots\phi_{I_p}(x))Y^{I_1}\dots Y^{I_p}$$

$\phi_I(x)$ are scalars, and Y_I are SO(6) R-symmetry null vectors.

- We denote linear combinations of $T_{p_1\dots p_n}$ with charge-p as $\mathcal{O}_p^{(i)}$; They are 1/2 BPS operators.

Four-point correlators

Two and three-point functions are protected; we will consider four-point functions: Eden, Petkou, Schubert, Sokatchev; Nirschl, Osborn

$$\langle \mathcal{O}_{p_1}^{(i_1)} \mathcal{O}_{p_2}^{(i_2)} \mathcal{O}_{p_3}^{(i_3)} \mathcal{O}_{p_4}^{(i_4)} \rangle = \text{free part} + \mathcal{I}_4(x_i, Y_i)\, \mathcal{H}(U, V; \alpha, \beta; \tau, \bar{\tau})$$

Fixed by symmetry $\delta^{16}(Q)$ Our focus

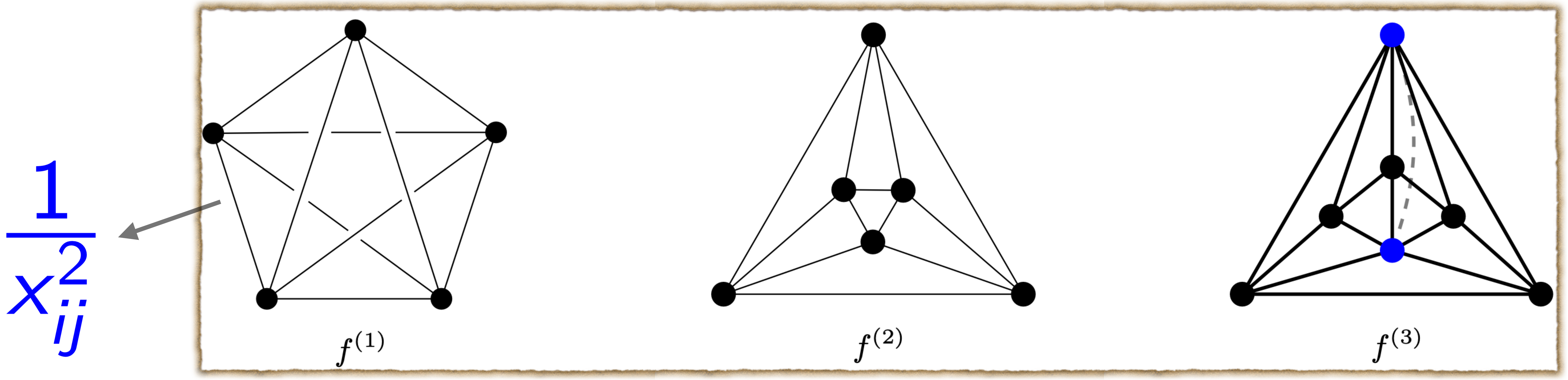
U, V and α, β are cross ratios of x_i and Y_i, respectively.

$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{\rm YM}^2}$ is complexified YM coupling, transforms under $SL(2, Z)$.

Correlators in the planar limit

- Perturbative integrands for $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ can be represented by $S_{4+\ell}$ permutation invariant f-graphs

Eden, Heslop, Korchemsky, Sokatchev



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- The correlator @ 3 loops

Drummond, Duhr, Eden, Heslop, Pennington, Smirnov

$$\begin{aligned}
\mathcal{H}(x,\bar{x}) = {} & -128G^+_{4,\bar{2}} - 512G^+_{5,\bar{1}} - 64G^+_{3,1,\bar{2}} + 64G^+_{3,\bar{1},2} - 64G^+_{3,\bar{1},\bar{2}} - 128G^+_{3,\bar{2},\bar{1}} \\
& + 64G^+_{4,1,\bar{1}} - 64G^+_{4,\bar{1},1} - 448G^+_{4,\bar{1},\bar{1}} + 64G^+_{2,\bar{1},2,1} + 64G^+_{2,\bar{1},\bar{2},\bar{1}} + 64G^+_{2,2,1,\bar{1}} + 64G^+_{2,2,\bar{1},1} \\
& - 64G^+_{2,2,\bar{1},\bar{1}} + 128G^+_{2,\bar{2},1,1} + 128G^+_{2,\bar{2},\bar{1},\bar{1}} + 256G^+_{3,1,1,\bar{1}} + 128G^+_{3,1,\bar{1},1} - 128G^+_{3,1,\bar{1},\bar{1}} \\
& + 192G^+_{3,\bar{1},1,1} - 64G^+_{3,\bar{1},1,\bar{1}} - 64G^+_{3,\bar{1},\bar{1},1} + 192G^+_{3,\bar{1},\bar{1},\bar{1}} + 128H^+_{2,4} - 128H^+_{4,2} \\
& + \frac{640}{3}H^+_{2,1,3} - \frac{64}{3}H^+_{2,3,1} - \frac{256}{3}H^+_{3,1,2} + 64H^+_{2,1,1,2} - 64H^+_{2,2,1,1} + 64L_0G^+_{3,\bar{2}} \qquad (5
\end{aligned}$$

$$\begin{aligned}
& + 256L_0G^+_{4,\bar{1}} + 32L_0G^+_{2,1,\bar{2}} + 64L_0G^+_{2,2,\bar{1}} + 96L_0G^+_{3,1,\bar{1}} + 32L_0G^+_{3,\bar{1},1} + 96L_0G^+_{3,\bar{1},\bar{1}} \\
& - 64L_0G^+_{2,1,1,\bar{1}} + 64L_0G^+_{2,\bar{1},\bar{1},\bar{1}} - 32L_1G^+_{3,\bar{2}} - 128L_1G^+_{4,\bar{1}} - 16L_1G^+_{2,1,\bar{2}} \\
& - 32L_1G^+_{2,2,\bar{1}} - 80L_1G^+_{3,1,\bar{1}} - 16L_1G^+_{3,\bar{1},1} - 16L_1G^+_{3,\bar{1},\bar{1}} - 64L_2G^-_{2,\bar{1},\bar{1}} + 64L_4G^-_{1,\bar{1}} \\
& + 32L_{2,2}G^-_{1,\bar{1}} - \frac{32}{3}H^+_2H^+_{2,2} - 64H^+_2H^+_{2,1,1} - 128H^+_2H^+_4 - 64H^-_1L_0G^-_{2,\bar{1},\bar{1}} \\
& - 32L_0^2G^+_{3,\bar{1}} - 32L_0^2G^+_{2,\bar{1},\bar{1}} + 32L_0^2G^+_{1,1,1,\bar{1}} + 32L_1L_0G^+_{3,\bar{1}} + 16L_1L_0G^+_{2,1,\bar{1}} \\
& + 16L_1L_0G^+_{2,\bar{1},\bar{1}} - \frac{80}{3}H^-_1L_0L_{2,2} - 48H^-_1L_0L_{2,1,1} + 12H^-_1L_1L_{2,2} + 16L_0^2H^+_{2,2} \\
& + 32L_0^2H^+_{2,1,1} - 64H^-_1L_4L_0 + 16H^-_1L_1L_4 + 64L_3G^+_{1,1,\bar{1}} - \frac{640}{3}H^-_3H^-_{2,1} \\
& + 64(H^-_{2,1})^2 + 128(H^-_3)^2 + 32L_0L_2G^-_{2,\bar{1}} - 32L_0L_2G^-_{1,1,\bar{1}} - 16L_1L_2G^-_{2,\bar{1}} \\
& + \frac{16}{3}L_0L_2H^-_{2,1} + 16H^-_1L_2L_{2,1} - \frac{112}{3}H^+_2L_0L_{2,1} - 8H^+_2L_1L_{2,1} - 32H^-_3L_0L_2 \\
& - 48H^-_1L_3L_2 + 32H^+_2L_0L_3 + 16H^+_2L_1L_3 + 32H^-_1L_0^2G^-_{2,\bar{1}} - 16H^-_1L_1L_0G^-_{2,\bar{1}} \\
& + \frac{16}{3}L_0^3G^+_{2,\bar{1}} + \frac{16}{3}L_0^3G^+_{1,1,\bar{1}} - 8L_1L_0^2G^+_{2,\bar{1}} - 8L_1L_0^2G^+_{1,1,\bar{1}} + \frac{16}{3}H^-_1L_0^2H^-_{2,1} \\
& - 16(H^-_1)^2L_0L_{2,1} - 32H^-_1H^-_3L_0^2 + \frac{8}{3}(H^-_1)^2L_3L_0 - 12(H^-_1)^2L_1L_3 + 28H^+_2L_2^2 \\
& + \frac{368(H^+_2)^3}{9} - 16L_0^2L_2G^-_{1,\bar{1}} - 8L_0L_1L_2G^-_{1,\bar{1}} + \frac{56}{3}H^-_1H^+_2L_0L_2 - 8H^-_1H^+_2L_1L_2 \\
& + 8(H^-_1)^2L_2^2 + 8(H^+_2)^2L_0^2 + 8(H^+_2)^2L_0L_1 + \frac{28}{3}(H^-_1)^2H^+_2L_0^2 - 4(H^-_1)^2H^+_2L_0L_1 \\
& - 96H^-_2(H^-_1)^3L_0 + \frac{160}{3}(H^-_1)^3L_0L_2 + \frac{52}{3}H^-_1L_0^3L_2 + 4H^-_1L_0L_1^2L_2 \\
& + 4H^-_1L_0^2L_1L_2 + H^+_2L_0L_1^3 + \frac{2}{3}H^+_2L_0^2L_1^2 - 8H^+_2L_0^3L_1 + \frac{148}{3}(H^-_1)^4L_0^2 \\
& + \frac{10}{3}(H^-_1)^2L_0^4 + 5(H^-_1)^2L_0^2L_1^2 - \frac{10}{3}(H^-_1)^2L_0^3L_1 - 128\zeta_3G^+_{2,\bar{1}} - 128\zeta_3G^+_{1,1,\bar{1}} \\
& + \frac{16}{3}\zeta_3(H^-_1)^2L_0 + 24\zeta_3(H^-_1)^2L_1 + \frac{64}{3}\zeta_3H^-_1L_2\,,
\end{aligned}$$

Correlators in the planar limit

- Ten-dim hidden symmetry in planar limit: Caron-Huot, Coronado; Caron-Huot, Trinh

Single trace

Integrands of $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$ can be obtained from $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ via

$$x_{ij}^2 \to x_{ij}^2 - Y_i \cdot Y_j$$ → Ten-dim'l distance

Expansion to $(Y_i)^{p_i-2}$ leads to correlators involving operator $\mathcal{O}_{p_i}$.

Integrated four-point correlators

$$\mathcal{C}_{p_i}^{(i)}(\tau,\bar\tau) = -\frac{2}{\pi}\int_0^\infty dr r^3 \int_0^\pi d\theta \sin^2(\theta) \frac{\mathcal{H}_{p_i}^{(i)}(U,V;\alpha,\beta;\tau,\bar\tau)|_{Y_a\cdot Y_b=x_{ab}^2}}{U\,V}$$

with $U = 1 + r^2 - 2r\cos(\theta)$, $V = r^2$.

- Originally for $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p^{(i)} \mathcal{O}_p^{(j)} \rangle$, for which H is independent of α, β .

  Binder, Chester, Pufu, Wang

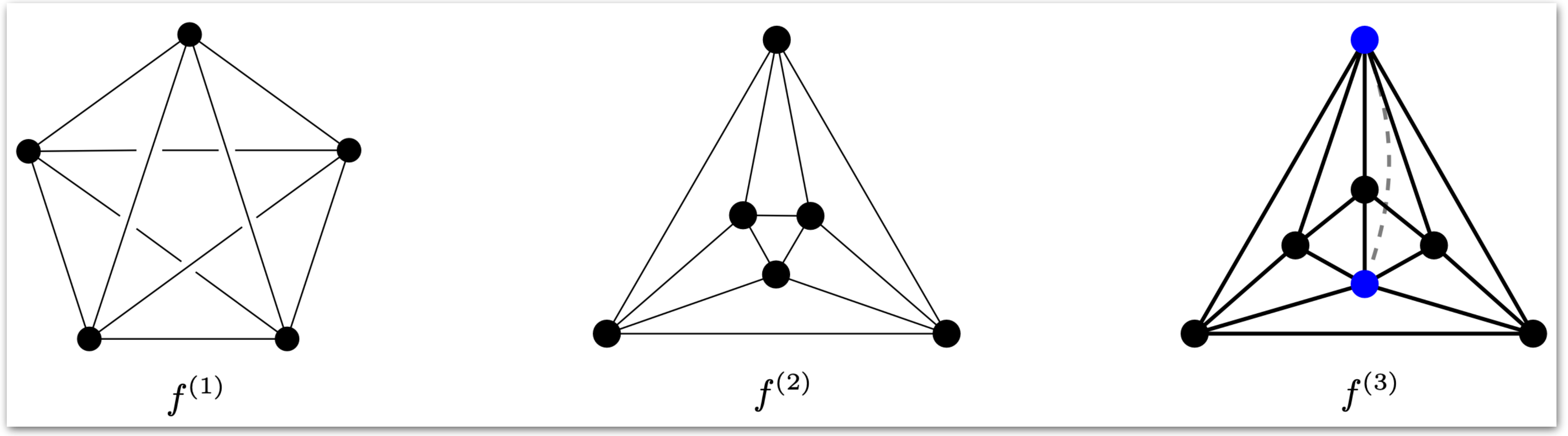
- We generalised it for $\langle \mathcal{O}_{p_1}^{(i_1)} \mathcal{O}_{p_2}^{(i_2)} \mathcal{O}_{p_3}^{(i_3)} \mathcal{O}_{p_4}^{(i_4)} \rangle$ with $Y_i \cdot Y_j = x_{ij}^2$.

  (ten-dim light-like limit)

Integrated correlators as Feynman integral periods

- For the perturbative contribution, integrated correlators are sum of periods of the f-graphs Brown; Panzer; Schnetz;....

$$-\frac{1}{\pi^2 L!(-4\pi^2)^L}\int \frac{d^4x_i}{\mathrm{vol\,(conf.)}} f^{(L)}(x_i)$$



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- Each graph (with L>4) generally contains MZVs, which cancel out in the integrated correlator; the cancellation happens if $Y_i \cdot Y_j = x_{ij}^2$.

Integrated correlators in the planar limit

This leads to an all-order expression (conjectured) for planar integrated correlators of $\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle$ with $Y_i \cdot Y_j = x_{ij}^2$

't Hooft coupling

$$\sum_{\ell=1}^{\infty} \zeta(2\ell+1)\lambda^{\ell} \sum_{p=2}^{\ell+1} \frac{4(-1)^{p+\ell+1}\Gamma\left(\ell+\frac{3}{2}\right)^2}{\pi^{2\ell+1}\Gamma(\ell+2-p)\Gamma(p+\ell+1)} \frac{\mathcal{S}_{p-2,p-2,0,0}(g_i) - \mathcal{S}_{p-2,p-2,1,1}(g_i)}{\prod_{i<j\leq 4}(g_i g_j - 1)}\Big|_{g_i^{p_i-2}}$$

where S are Schur polynomials.

For $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_p\mathcal{O}_p\rangle$, the result was known. Binder, Chester, Pufu, Wang

Integrated correlators in the planar limit

- One can re-sum the perturbative results, and obtain strong coupling expansion:

$$\Big(\sum_{p=2}^{\infty}\Big[\frac{1}{2(p-1)p}+\sum_{n=1}^{\infty}\frac{4n(-1)^n\zeta(2n+1)\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(n+p-\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\sqrt{\pi}\Gamma(n)\Gamma\left(p+\frac{1}{2}-n\right)}+O(e^{-2\sqrt{\lambda}})\Big] \times \frac{\mathcal{S}_{p-2,p-2,0,0}(g_i)-\mathcal{S}_{p-2,p-2,1,1}(g_i)}{\prod_{i<j\leq 4}(g_i g_j-1)}\Big)\Big|_{g_1^{p_1-2}g_2^{p_2-2}g_3^{p_3-2}g_4^{p_4-2}}$$

- Strong-coupling result agrees known type IIB string amplitudes on AdS (known up to $\lambda^{-5/2}$), and provide constraints for the unknowns.

Abl, Heslop, Lipstein; Aprile, Drummond, Paul, Santagata

Exact results of Integrated correlators

- Having seen the simplicity in the planar limit, we now consider integrated correlators at finite $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{\rm YM}^2}$.
- This is necessary for studying the S-duality of N=4 SYM. Finite coupling is difficult!
- Localisation to rescue: the integrated correlator associated with

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p^{(i)} \mathcal{O}_p^{(j)} \rangle$$

is related to derivatives acting on the partition function of N=2* SYM (mass deformation of N=4 SYM) that can be computed by localisation.

Integrated correlators and Susic localisation

$$\widehat{\mathcal{C}}^{(i,j)}_{N,p}(\tau,\bar\tau) = \frac{v_p^{i,\mu}\,\bar v_p^{j,\nu}\,\partial_{\tau'_\mu}\partial_{\bar\tau'_\nu}\partial_m^2 \log \mathcal{Z}_N(\tau,\tau'_A;m)\Big|_{m=\tau'_A=0}}{v_p^{i,\mu}\,\bar v_p^{j,\nu}\,\partial_{\tau'_\mu}\partial_{\bar\tau'_\nu} \log \mathcal{Z}_N(\tau,\tau'_A;0)\Big|_{\tau'_A=0}}$$

Binder, Chester, Pufu, Wang

the partition function of N=2* SYM (deformed by higher-dim operators) on S^4 from supersymmetric localisation

Pestun; Nekrasov

$$\mathcal{Z}_N(\tau,\tau'_A;m) = \int d^{N-1}a \left|\exp\left(i\pi\tau\sum_{i=1}^N a_i^2 + i\sum_{p>2}\pi^{p/2}\tau'_p\sum_{i=1}^N a_i^p\right)\right|^2 Z_{1-\mathrm{loop}}(a;m)\left|Z_{\mathrm{inst}}(\tau,\tau',a;m)\right|^2 ,$$

a finite-dim integral, rather than a Path Integral !

Integrated correlators and Susic localisation

Schematically:

- ∂_m^2 brings down two $\int \mathcal{O}_2$; $v_p^{i,\mu}\partial_{\tau'_\mu}$ and $\bar{v}_p^{j,\nu}\partial_{\bar{\tau}'_\nu}$ lead to $\mathcal{O}_p^{(i)}$ and $\mathcal{O}_p^{(j)}$ at north and south poles of S^4.
- $v_p^{i,\mu}$ are introduced due to operators with different dims on S^4 can mix; they are determined by a Gram-Schmidt procedure.

Gerchkovitz,Gomis,Ishtiaque,Karasik, Komargodski,Pufu

Exact results of Integrated correlators

- Localisation formula is NOT easy: $Z_{\rm inst}(\tau, \tau', a; m)$ is not well-understood; many properties (especially $SL(2, \mathbb{Z})$) are not manifest.
- We hope to do better! Indeed, we found integrated correlators can be written as a 2-dimensional lattice sum

$$\widehat{\mathcal{C}}^{(i,j)}_{N,p}(\tau, \bar{\tau}) = \sum_{(m,n)\in\mathbb{Z}^2} \int_0^\infty e^{-tY_{m,n}(\tau,\bar{\tau})} \widehat{B}^{(i,j)}_{N,p}(t)\, dt \,,$$

with $Y_{m,n}(\tau, \bar{\tau}) := \pi \frac{|m+n\tau|^2}{\tau_2}$; the formula is manifestly $SL(2, \mathbb{Z})$ invariant; all the information is contained in $\widehat{B}^{(i,j)}_{N,p}(t)$.

Exact results of Integrated correlators

- One can further introduce generating functions by resuming N or charge p

$$\widehat{B}_{p}^{(i,j)}(z;t)=\sum_{N=2}^{\infty}\widehat{B}_{N,p}^{(i,j)}(t)z^{N}\,,\qquad \widehat{B}_{N}^{(i,j)}(w;t)=\sum_{p=2}^{\infty}\widehat{B}_{N,p}^{(i,j)}(t)w^{p}\,.$$

- The generating functions contain all the information, and are extremely useful for large-N or large-p expansions.

Exact results of Integrated correlators

Examples:

- $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle$ of SU(N), after resuming N:

$$\widehat{B}_{p=2}(z;t) = \frac{3tz^2\left[(t-3)(3t-1)(t+1)^2 - z(t+3)(3t+1)(t-1)^2\right]}{2(1-z)^{\frac{3}{2}}\left[(t+1)^2-(t-1)^2 z\right]^{\frac{7}{2}}}.$$

- $\langle \mathcal{O}_2\mathcal{O}_2(\mathcal{O}_2)^{p/2}(\mathcal{O}_2)^{p/2}\rangle$ of SU(2), after resuming p:

Unique Independent operators for SU(2)

$$\widehat{B}_{N=2}(w;t) = 4t\frac{(t-1)^2\left(3t^2-2t+3\right)w-(t+1)^2\left(3t^2-10t+3\right)}{(t+1)^3(w-1)\left[(t+1)^2-(t-1)^2 w\right]^2}$$

Laplace-difference equations

Generating functions obey differential equations, which imply Laplace-difference equations for integrated correlators. Examples:

- In N-space for $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle$

$$(\Delta_\tau - 2)\,\mathcal{C}_{N,2} = N(N-1)(\mathcal{C}_{N+1,2} - \mathcal{C}_{N,2}) + N(N+1)(\mathcal{C}_{N,2} - \mathcal{C}_{N-1,2})$$

with $\Delta_\tau = 4\tau_2^2\partial_\tau\partial_{\bar\tau}$

- In p-space for $\langle \mathcal{O}_2\mathcal{O}_2(\mathcal{O}_2)^{p/2}(\mathcal{O}_2)^{p/2}\rangle$

see also: Paul, Perlmutter, Raj

$$\Delta_\tau\,\widehat{\mathcal{C}}_{2,p} = (p+1)\left(p + N^2/2 - 1/2\right)(\widehat{\mathcal{C}}_{2,p+1} - \widehat{\mathcal{C}}_{2,p}) + p\left(p + N^2/2 - 3/2\right)(\widehat{\mathcal{C}}_{2,p} - \widehat{\mathcal{C}}_{2,p-1}) - 4\,\mathcal{C}_{N,2}$$

Seeing superstring amplitudes

- Large-N expansion (with fixed YM coupling): $1/\sqrt{N} \sim \alpha'$

Supergravity

α' corrections

$$\mathcal{C}_{N,2}(\tau,\bar\tau) = \frac{N^2}{4} + \sum_{r=0}^{\infty} N^{\frac{1}{2}-r} \sum_{m=0}^{\lfloor r/2 \rfloor} b_{r,m} E(\tfrac{3}{2}+\delta_r+2m;\tau,\bar\tau)$$

$$\pm i \sum_{r=0}^{\infty} N^{2-\frac{r}{2}} \sum_{m=0}^{r} d_{r,m} D_N\left(2m-\frac{3r}{2};\tau,\bar\tau\right).$$

(p,q)-string world-sheet instantons

See also: Chester, Green, Pufu, Wang, C.W.

Seeing superstring amplitudes

- Non-holomorphic Eisenstein series

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2_{\rm YM}} \to \chi + \frac{i}{g_s} = \tau_1 + i\tau_2$$

$$E(s;\tau,\bar\tau) = \sum_{(p,q)\neq(0,0)} \frac{\tau_2^s}{\pi^s|p+q\tau|^{2s}}$$

$$= \frac{2\zeta(2s)}{\pi^s}\tau_2^s + \frac{2\zeta(2s-1)\Gamma(s-\frac{1}{2})}{\pi^{s-\frac{1}{2}}\Gamma(s)}\tau_2^{1-s} + \sum_{k\neq 0}\mathcal{F}_k(s;\tau_2)e^{2\pi i k\tau_1}$$

String genus expansion — D-instantons

Green, Gutperle + Vanhove; ....

- $E(3/2;\tau,\bar\tau)$ is the coefficient of R^4, $E(5/2;\tau,\bar\tau)$ coefficient of d^4R^4.
Precision AdS/CFT: beyond supergravity & beyond perturbation.

Earlier work: Bianchi, Green, Kovacs, Rossi; Dorey, Hollowood, Khoze, Mattis, Vandoren;...

Seeing superstring amplitudes

- New non-holomorphic modular function

exponentially decay in large-N

1/N expansion is not Borel summable

$$D_N(s;\tau,\bar{\tau}) = \sum_{\ell=1}^{\infty} \sum_{\gcd(p,q)=1} \exp\left(-4\sqrt{N\pi}\,\ell\frac{|p+q\tau|}{\sqrt{\tau_2}}\right)\frac{1}{\pi^s}\frac{\tau_2^s}{\ell^{2s}|p+q\tau|^{2s}}$$

$$\sim \sum_{\ell=1}^{\infty} \sum_{\gcd(p,q)=1} \exp\left(-4\pi L^2 \ell \frac{|p+q\tau|}{2\pi\alpha'}\right).$$

(p, q)-string tension

- 't Hooft limit: it behaves as $\exp(-2n\sqrt{\lambda})$, while Eisenstein series $\sim \sum \lambda^{-1/2-\ell}$

  they are related via resurgence.

See also: Hatsuda, Okuyama; Collier, Perlmutter;....

Conclusion

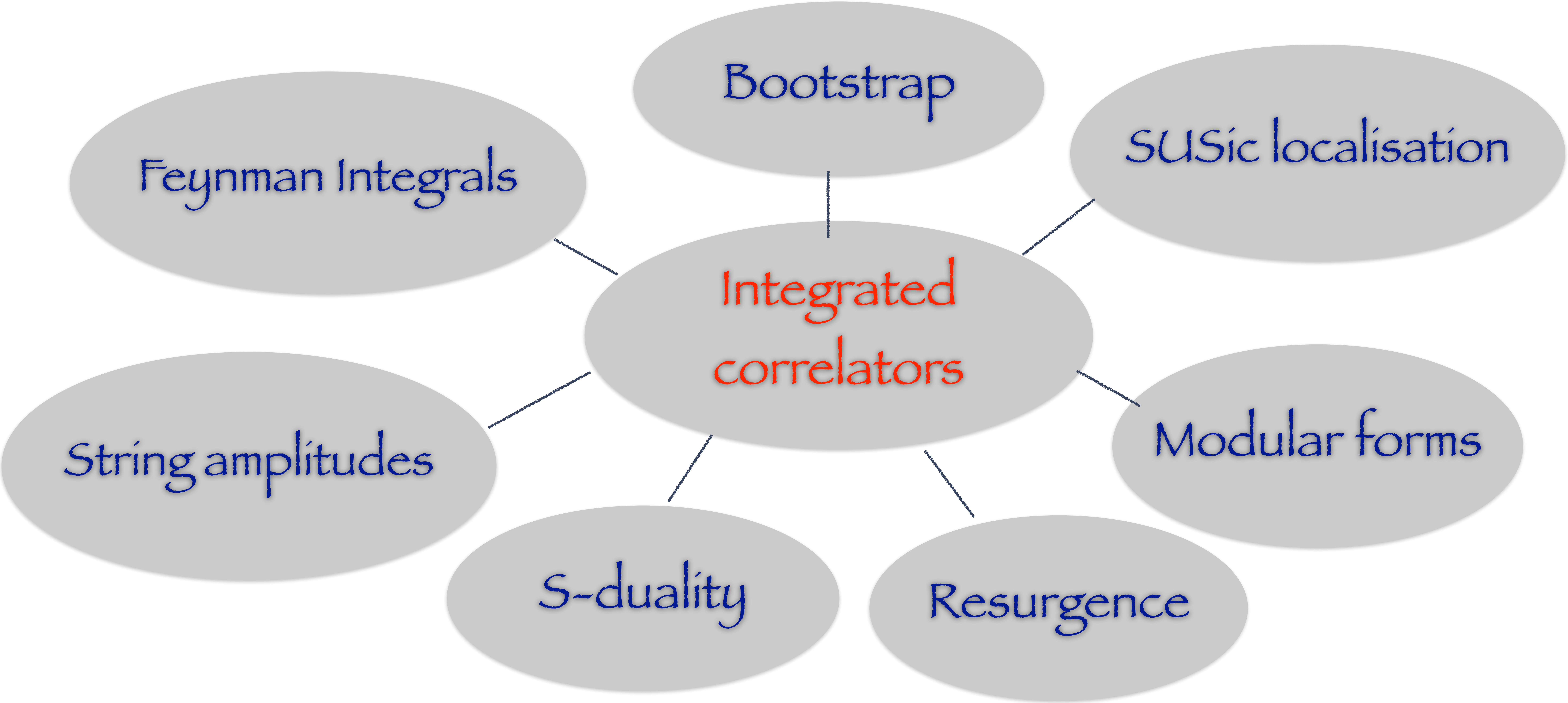
Integrated correlators may be viewed as the "simplest" (yet highly non-trivial) observable in N=4 SYM.

- Exact expression for $\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle$ in planar limit with $Y_i \cdot Y_j = x_{ij}^2$ This is beyond localisation!
- Manifestly SL(2,Z)-invariant expression for $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_p^{(i)}\mathcal{O}_p^{(j)}\rangle$; generating functions are obtained by resuming N or charge p.
- Large-N expansion was studied, and made contact with α' expansion of string amplitudes.

Outlook

- Integrated correlators for $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$ beyond the planar limit?
- Integrated correlators with other gauge groups, relevant for Goddard-Nuyts-Olive duality. For $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$, see Alday, Chester, Hansen; Dorigoni, Green, C.W.
- Integrated correlators with a different measure ($\partial_m^4 \log Z\big|_{m=0}$) ?

  Chester, Pufu; Chester, Green, Pufu, Wang, C.W.; Collier, Perlmutter;
  Alday, Chester, Dorigoni, Green, C.W., in progress
- Other observables? lower SUSY? ……


Bootstrap
Feynman Integrals
SUSic localisation
Integrated correlators
String amplitudes
Modular forms
S-duality
Resurgence