Holographic Correlators for all Λs

Charlotte Sleight

Durham U., Naples U. and INFN Naples

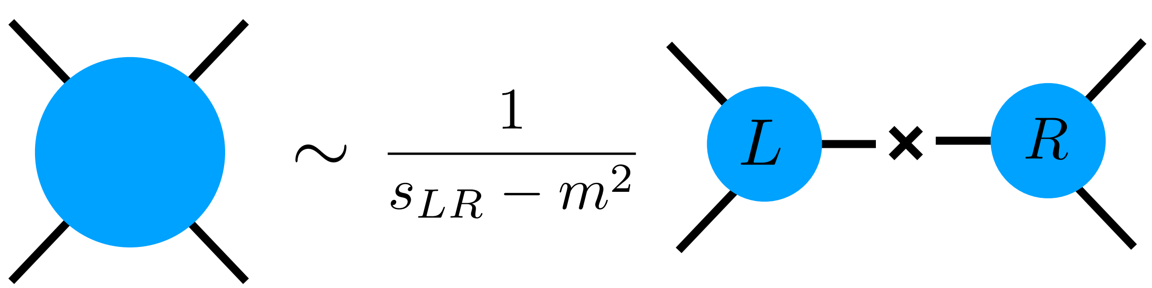
Bootstrap approach to QFT

"Nature is as it is because this is the only possible Nature consistent with itself " *Geoffrey Chew*

> Constrain observables using symmetries and consistency conditions

Basic physical criteria for consistent scattering amplitudes:

- Lorentz invariance
- Unitarity $SS^\dagger = 1$
- Locality



?? $\vdots$

Bootstrap approach to CFT

"Nature is as it is because this is the only possible Nature consistent with itself " *Geoffrey Chew*

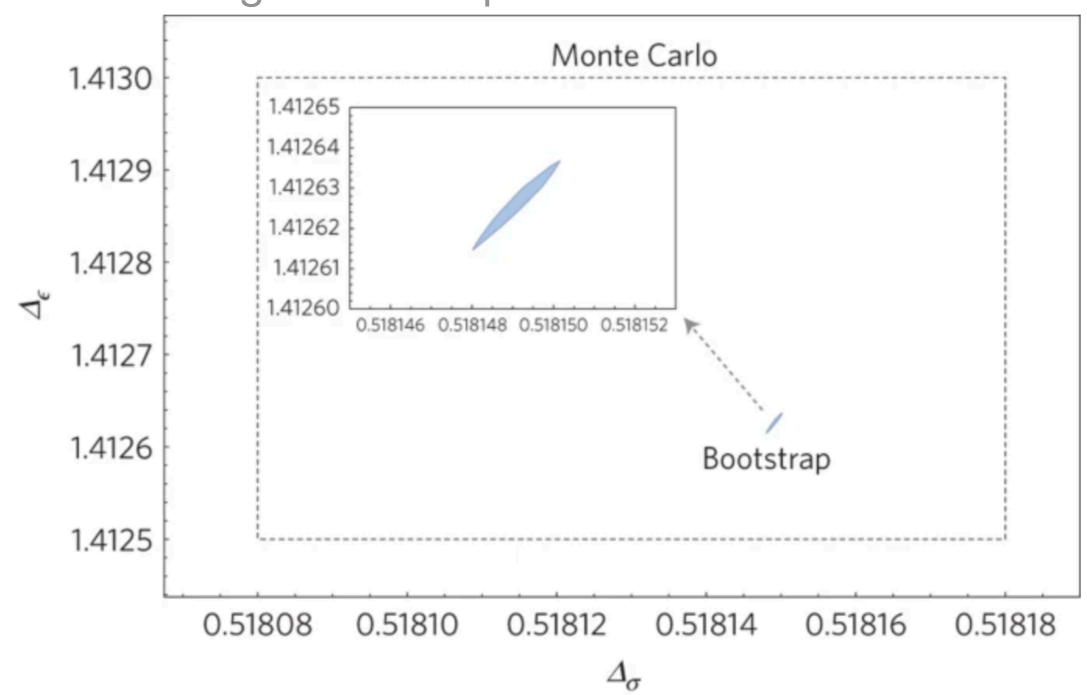
Constrain observables using
symmetries and consistency conditions

Correlation functions in CFTs are constrained non-perturbatively by:

- Conformal Symmetry
- Unitarity
- Associative Operator Product Expansion

[Belavin, Polyakov, Zamolodchikov 1984;
Rattazzi, Rychkov, Tonni, Vichi 2008]

Ising Critical Exponents

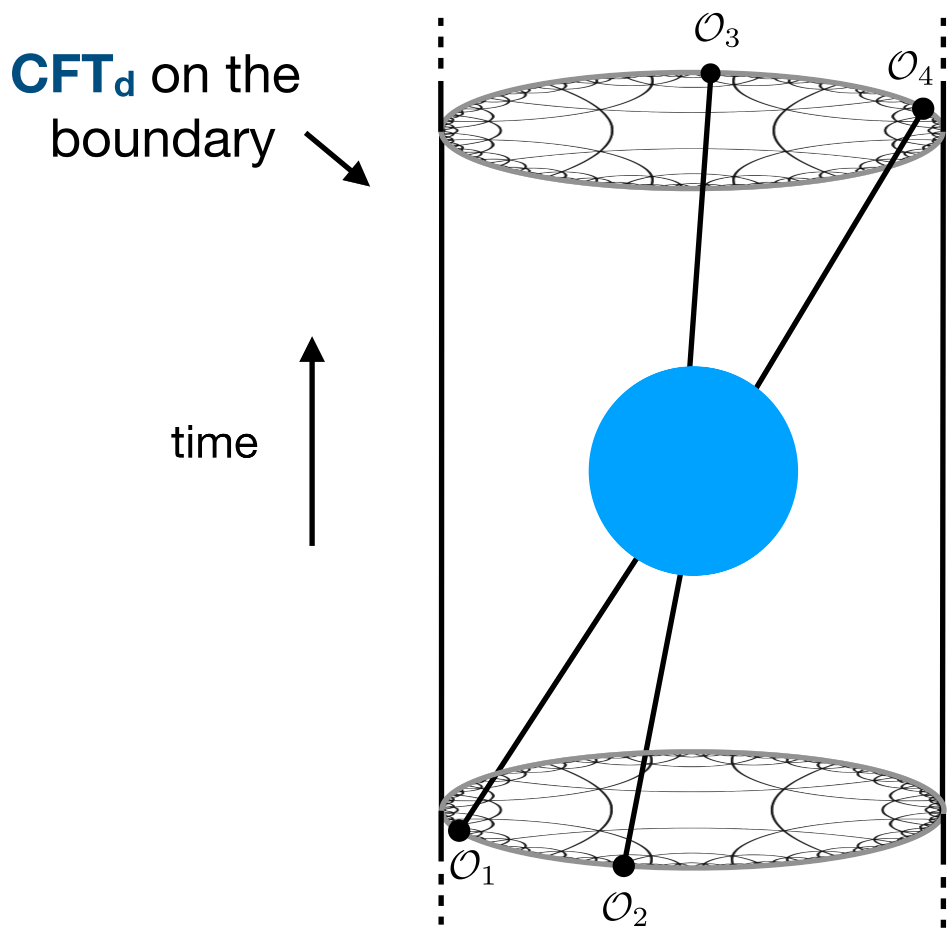


[Kos, Poland, Simmons-Duffin, Vichi 2016]

QFT applied to Gravity: AdS/CFT

Quantum Gravity in AdS_{d+1}	=	(non-gravitational) CFT in $\mathbb{M}^d$

Observables ?! ⟷ **Correlation functions**



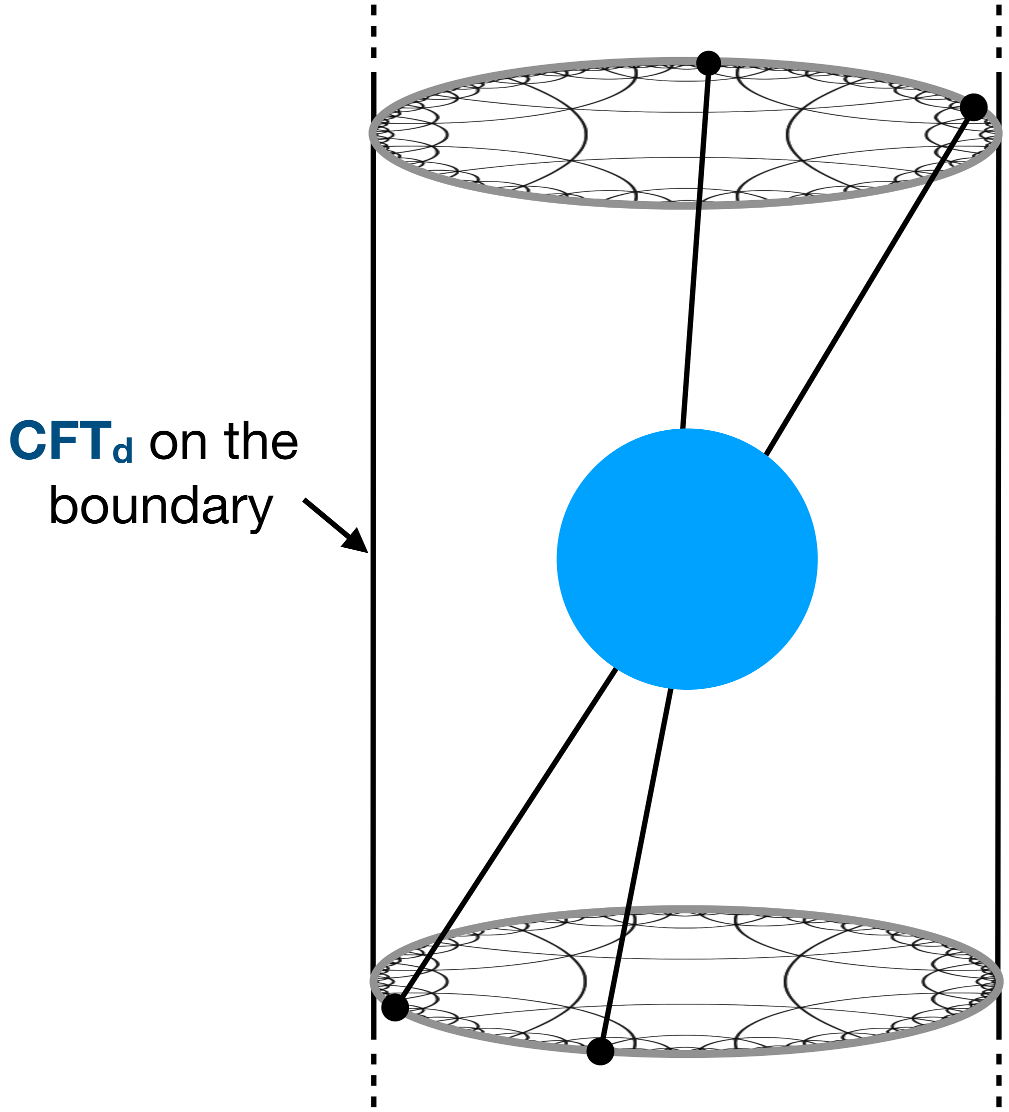
Constrained non-perturbatively by the Conformal Bootstrap:

- Conformal symmetry
- Unitarity
- Associative OPE

$$(\mathcal{O}_1\mathcal{O}_2)\,\mathcal{O}_3 = \mathcal{O}_1\,(\mathcal{O}_2\mathcal{O}_3)$$

[Belavin, Polyakov, Zamolodchikov 1984; Rattazzi, Rychkov, Tonni, Vichi 2008]

QFT applied to Gravity: AdS/CFT

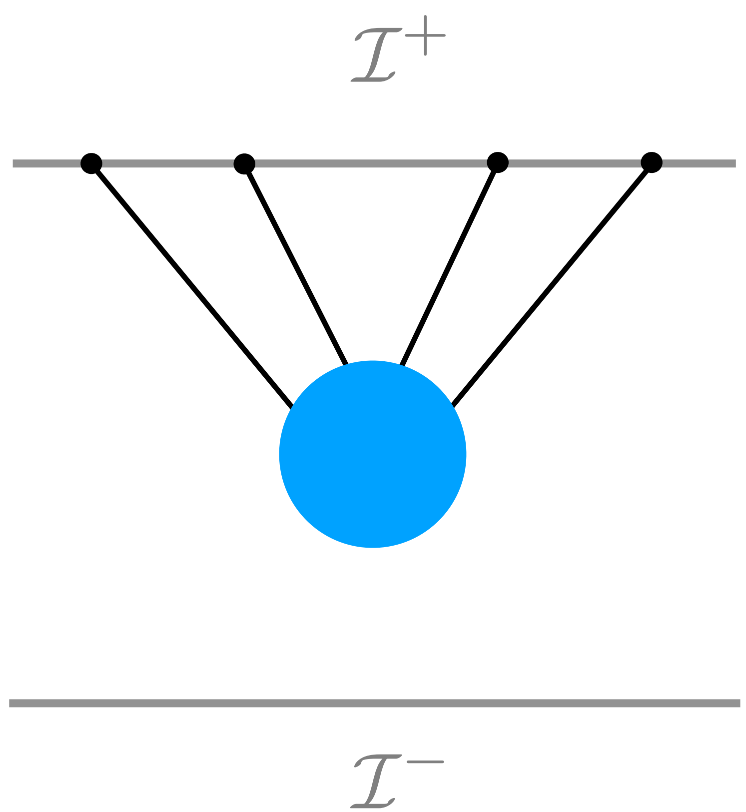


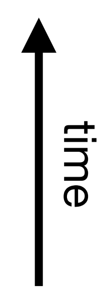
Can we extend this understanding to our own universe?

Amplifying Gravity at all scales

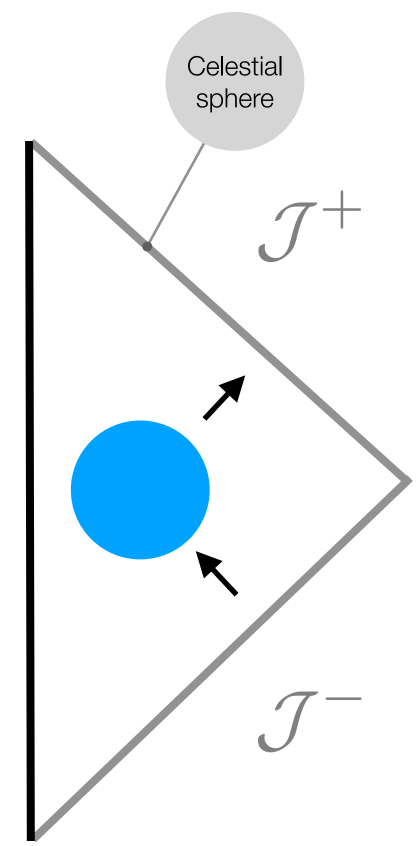
The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter





$\Lambda = 0$ Minkowski



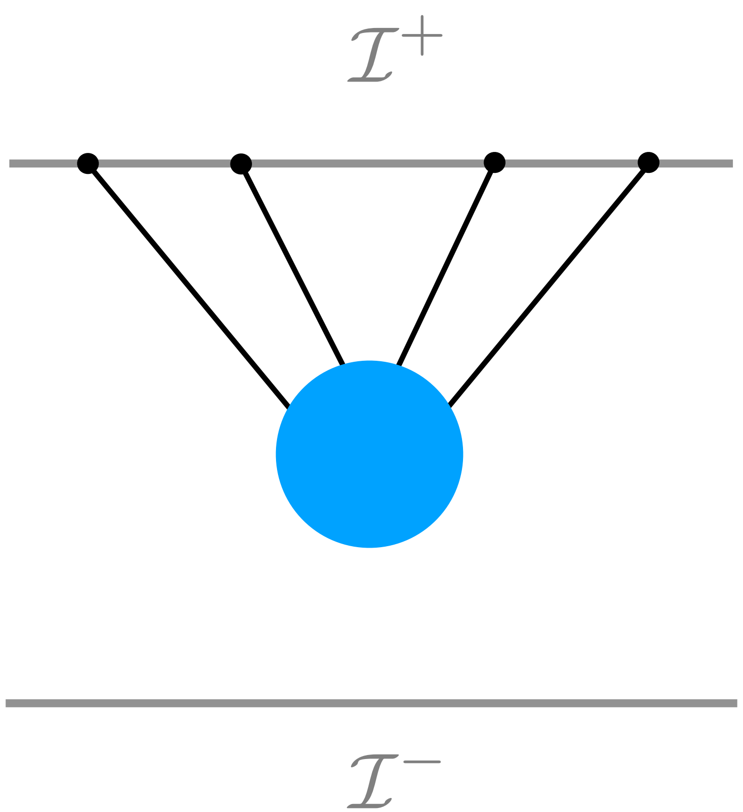
- Cosmological scales
- Primordial inflation

- intermediate scales

Amplifying Gravity at all scales

The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter



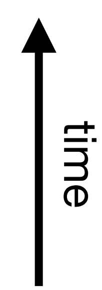
Cosmological Bootstrap

[Arkani-Hamed and Maldacena ’15]

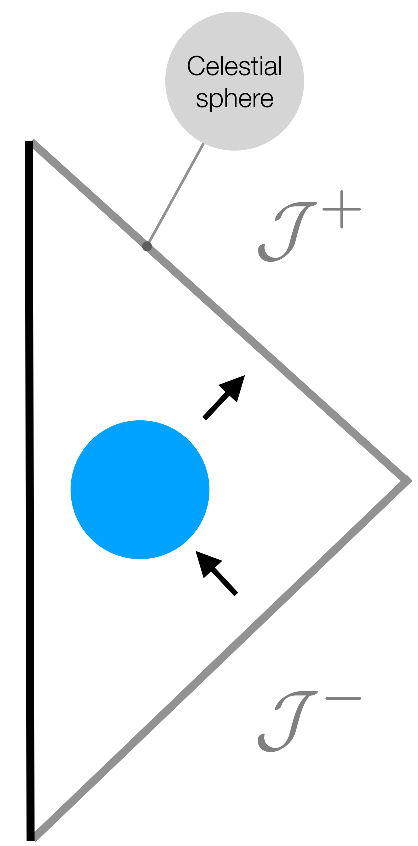
[Arkani-Hamed and Benincasa ’17]

[Arkani-Hamed, Baumann, Lee and Pimentel ’18]

[Sleight and Taronna ’19] [Pajer et al ’20] […]



$\Lambda = 0$ Minkowski



Celestial holography

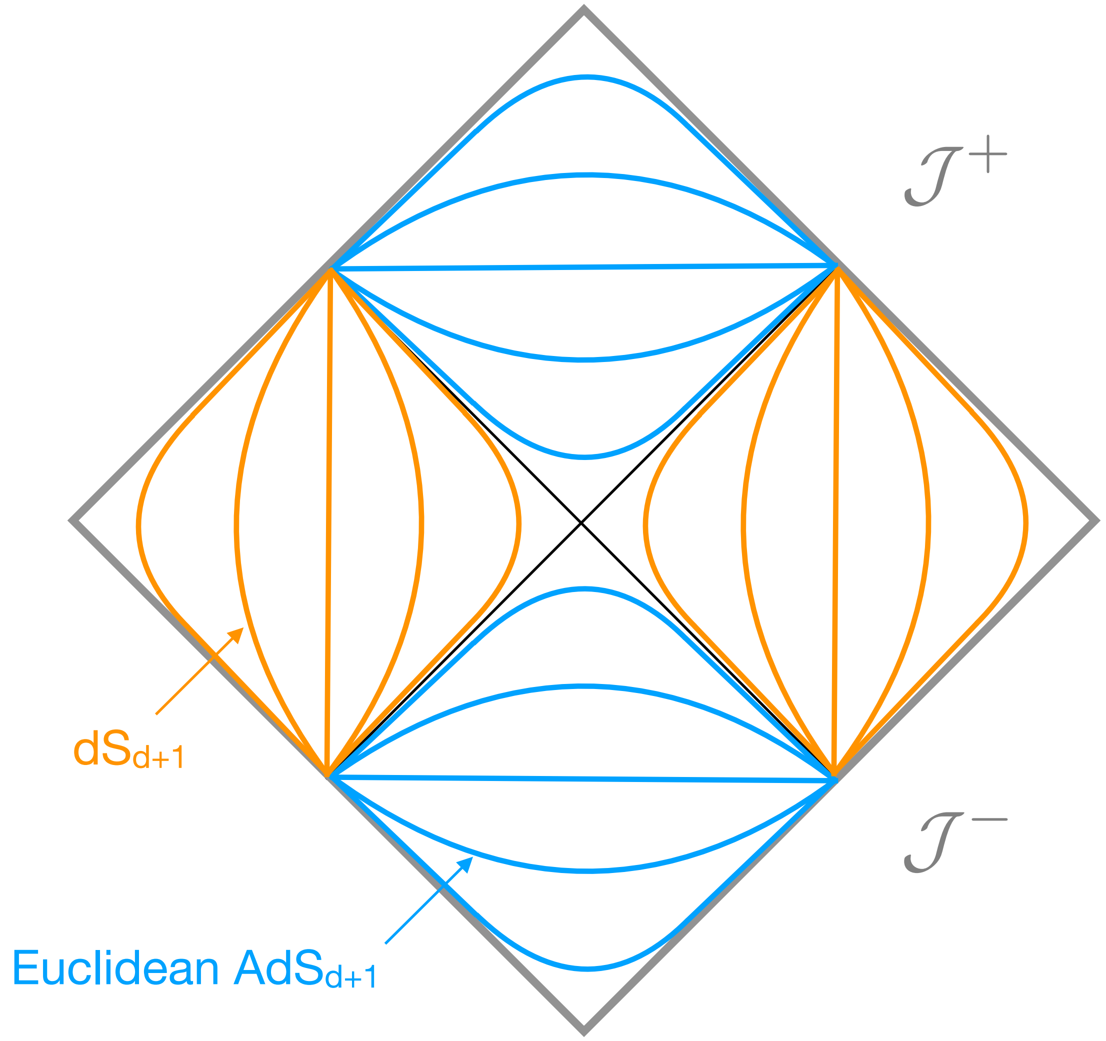
[de Boer and Solodukhin '03]

[Strominger ’17] [Pasterski, Shao, Strominger ’17]

[Pasterski, Shao ’17] […]

Amplifying Gravity at all scales

Hyperbolic slicing of $\mathbb{M}^{d+2}$

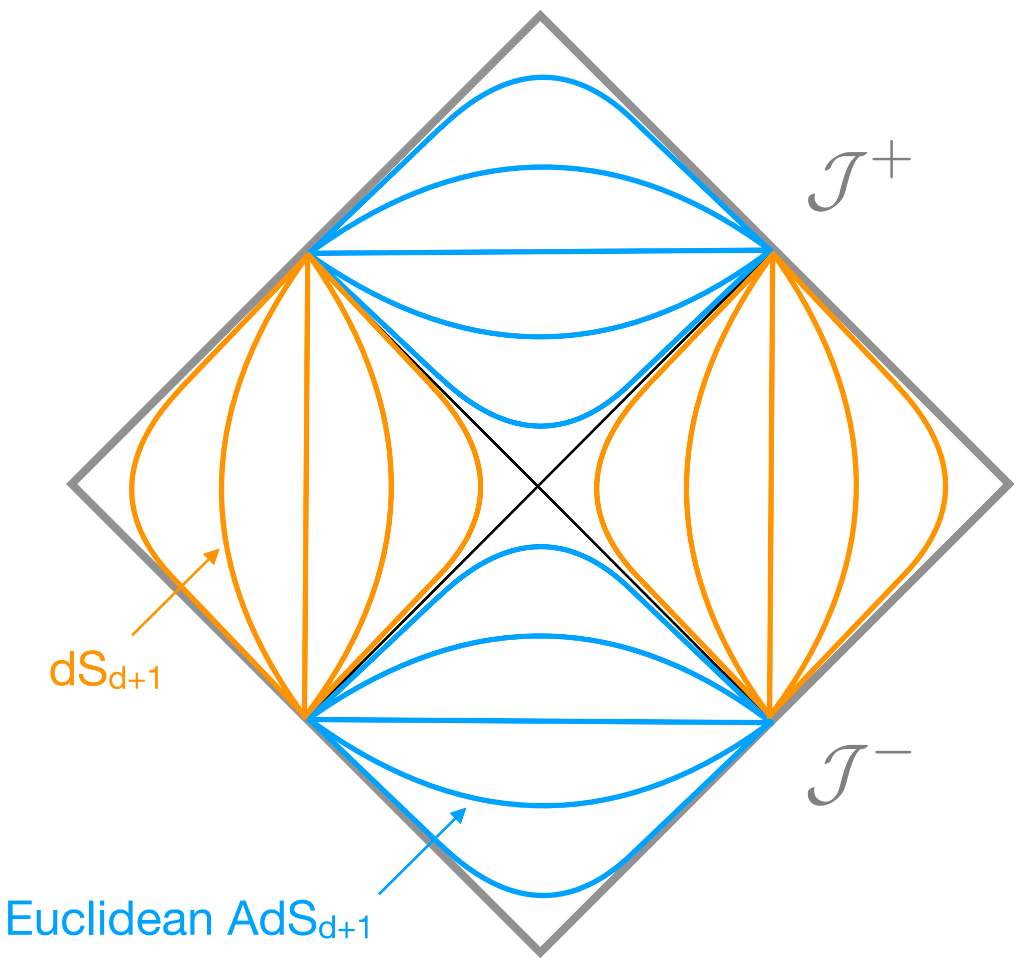


Idea: Apply holography to each slice!

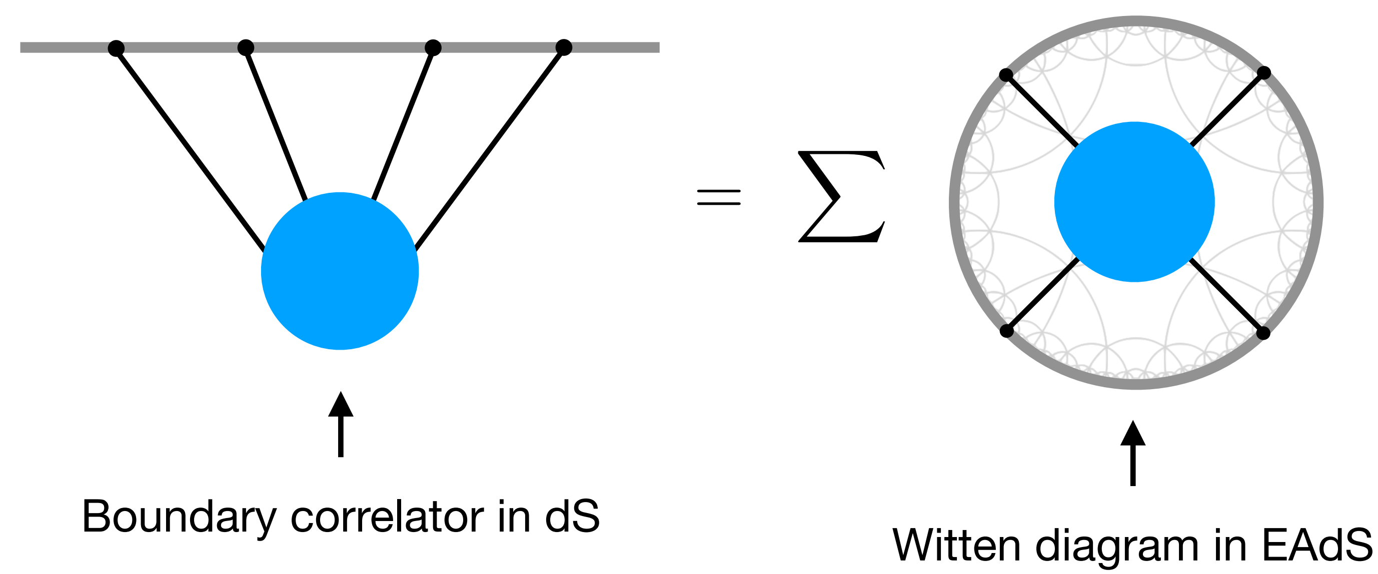
[de Boer and Solodukhin '03]

Amplifying Gravity at all scales

Holography for all Λs on the same footing:



- In de Sitter [Sleight and Taronna ’19, ’20, ’21]



- Hyperbolic slicing of $\mathbb{M}^{d+2}$

⟶ Decomposition of celestial correlators into EAdS Witten diagrams

[Iacobacci, Sleight and Taronna ’22, Sleight and Taronna ’23]

dS and Celestial correlators therefore have a similar analytic structure to their EAdS counterparts! On a practical level, can use such identities to import techniques and understanding from AdS.

Outline

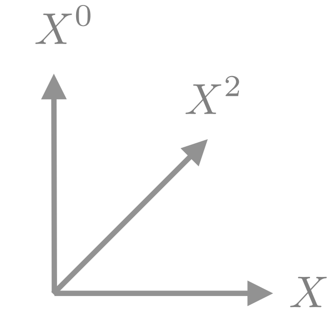
I. $\Lambda < 0$

II. $\Lambda > 0$

III. $\Lambda = 0$

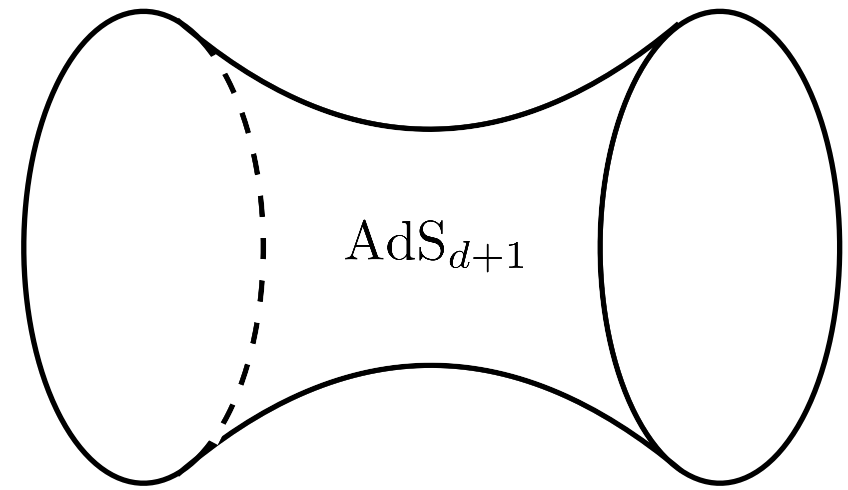
$$\Lambda < 0$$

Anti-de Sitter space-time



$\mathrm{AdS}_{d+1} \subset \mathbb{R}^{d,2}$:

$$-\left(X^0\right)^2 - \left(X^{d+1}\right)^2 + \sum_{i=1}^{d} \left(X^i\right)^2 = -R_{\mathrm{AdS}}^2$$

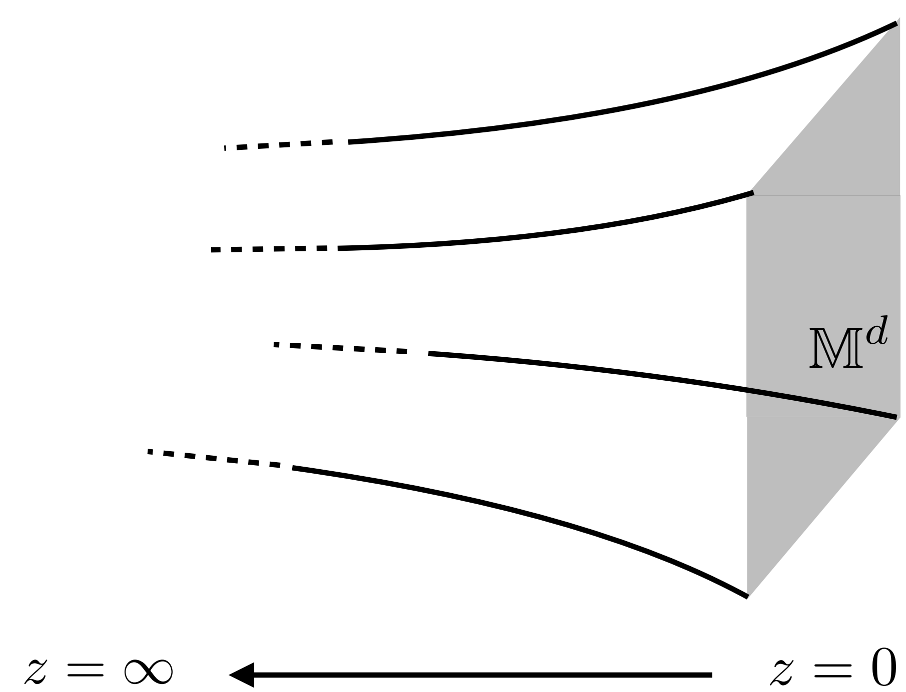


It is manifest that

Isometry group: $SO\left(d,2\right)$ = conformal group in $\mathbb{M}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

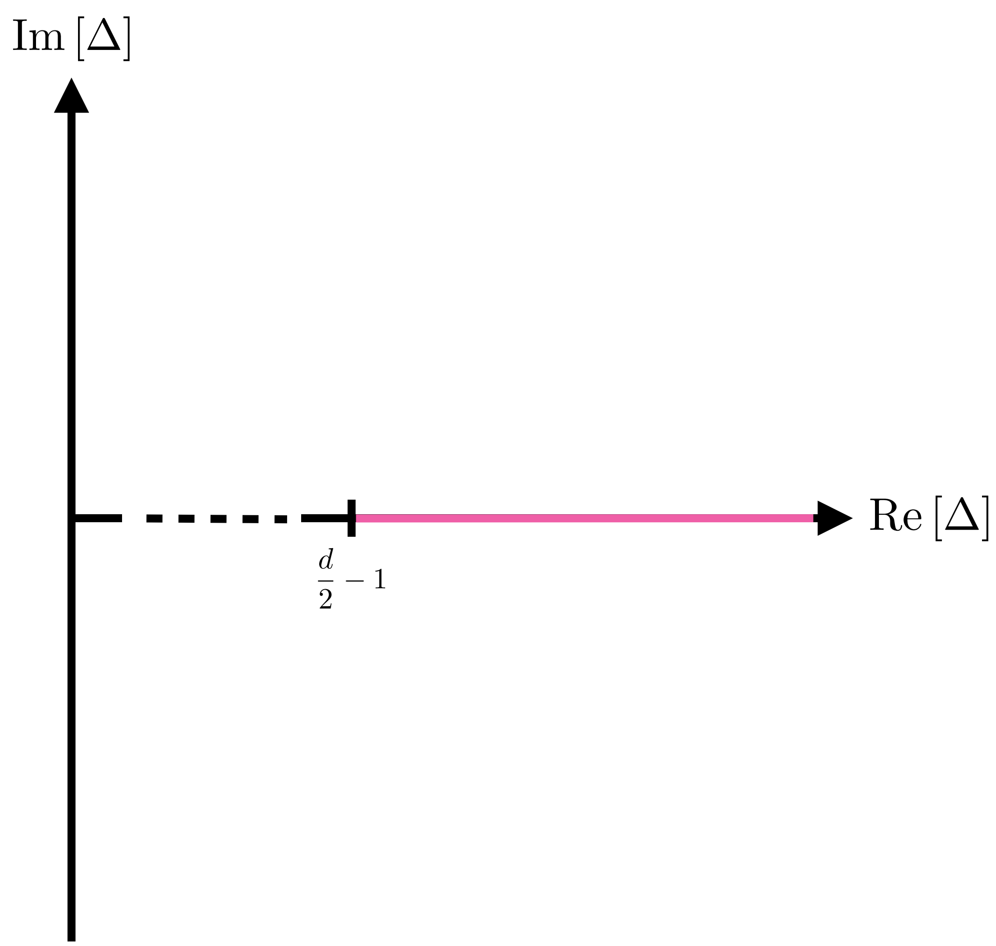


Particles in AdS

Particles in AdS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO(d,2)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

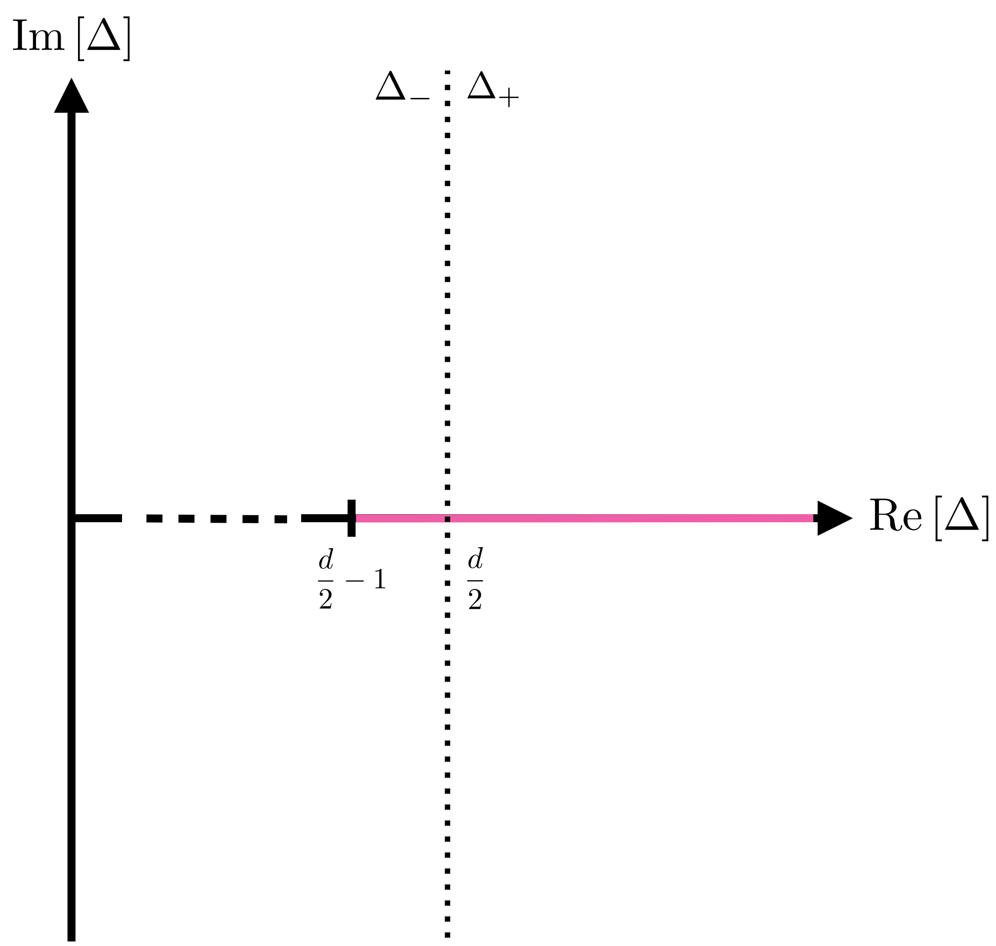
- $\Delta \in \mathbb{R}$
- Bounded from below $\Delta \geq \dfrac{d}{2} - 1$

Particles in AdS

Particles in AdS_{d+1} ⟷ unitary irreducible representations of $SO(d,2)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in AdS_{d+1} :

E.g. Spin J=0 representations



Quadric Casimir equation

$\langle \mathcal{C}_2 \rangle = \Delta(\Delta - d)$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle \mathcal{C}_2 \rangle\right)\varphi = 0$$

$$m^2 R_{\text{AdS}}^2 = \Delta(\Delta - d)$$

Boundary behaviour ($\Delta_- = d - \Delta_+$):

$$\lim_{z\to 0} \varphi(z,x) = O_{\Delta_+}(x)\, z^{\Delta_+} + O_{\Delta_-}(x)\, z^{\Delta_-}$$

Dirichlet boundary condition

Neuman boundary condition

N.B. Δ_- may be ruled out by unitarity

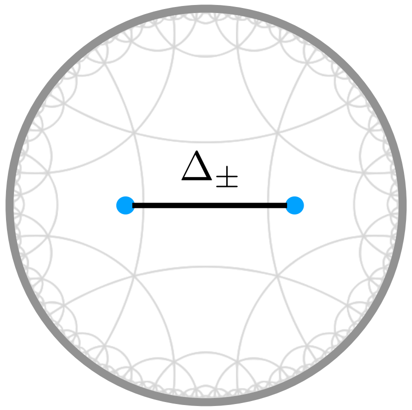
$O_{\Delta_\pm}(x)$ transform as primary fields with scaling dimension $\Delta_\pm$ in Minkowski CFT_d

AdS boundary correlators

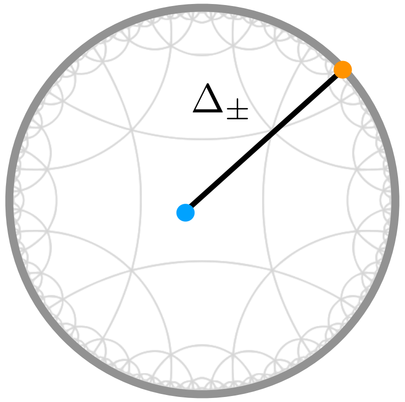
$$\lim_{z\to 0} z^{-(\Delta_1+\ldots+\Delta_n)}\langle\varphi_1(x_1,z)\ldots\varphi_n(x_n,z)\rangle \overset{!}{=} \langle\mathcal{O}_{\Delta_1}(x_1)\ldots\mathcal{O}_{\Delta_n}(x_n)\rangle$$

Feynman rules:

Bulk-to-bulk propagator, $\Delta_\pm$ boundary condition:



Bulk-to-boundary propagator, $\Delta_\pm$ boundary condition:

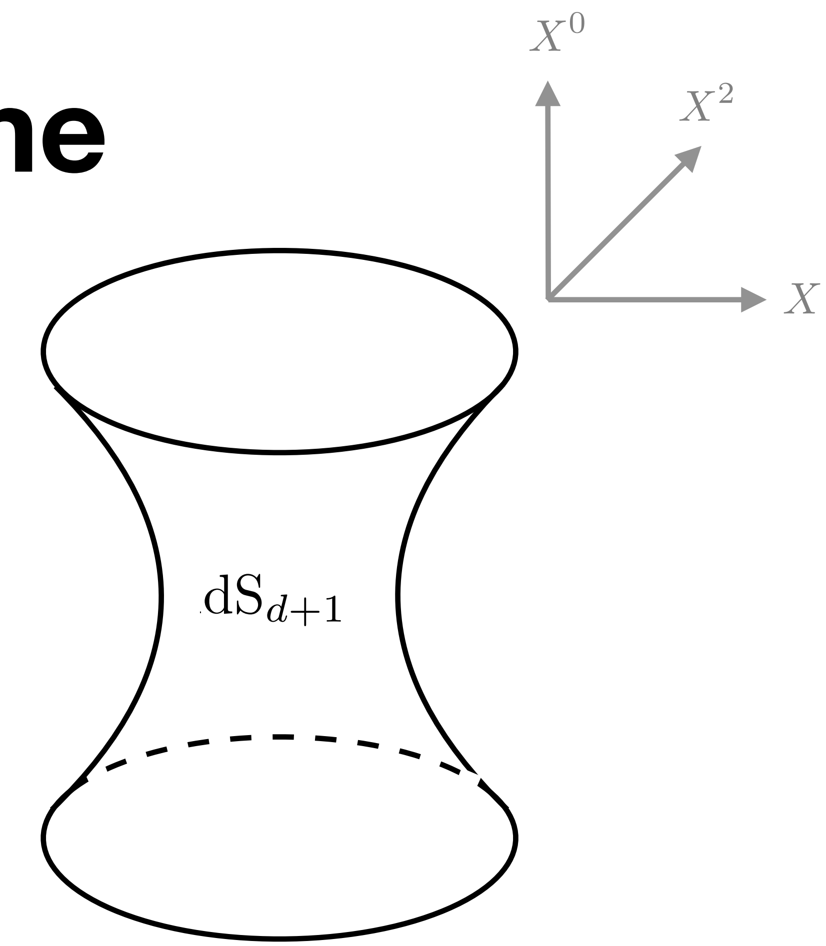


$$\Lambda > 0$$

de Sitter space-time

$\mathrm{dS}_{d+1} \subset \mathbb{M}^{d+2}$:

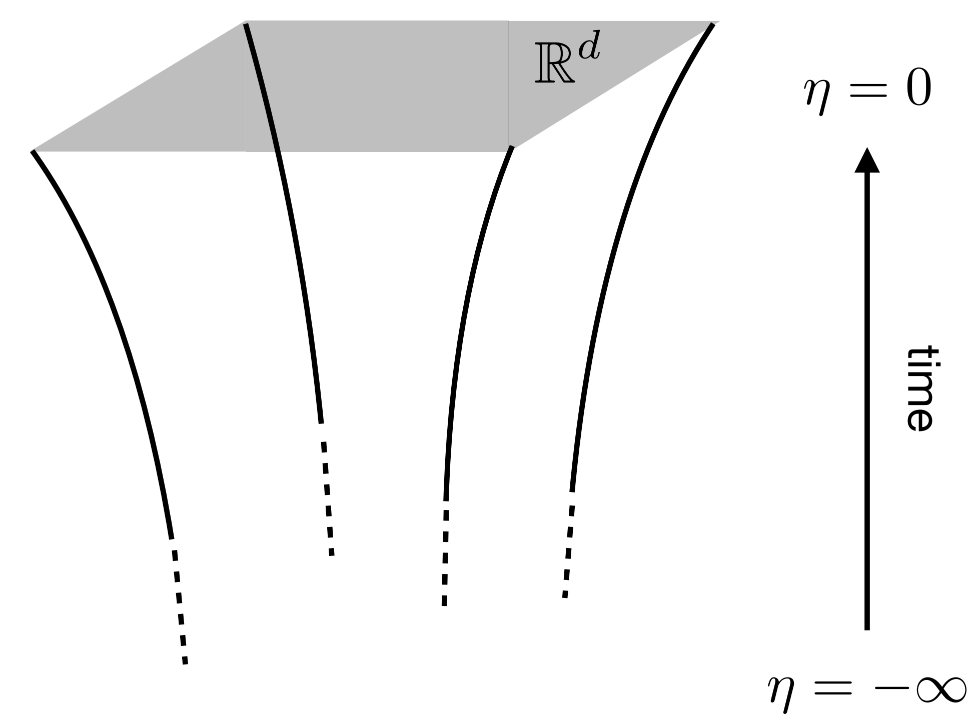
$$-\left(X^0\right)^2 + \sum_{i=1}^{d+1} \left(X^i\right)^2 = R_{\mathrm{dS}}^2$$



Isometry group: $SO\left(d+1,1\right)$ = conformal group in $\mathbb{R}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

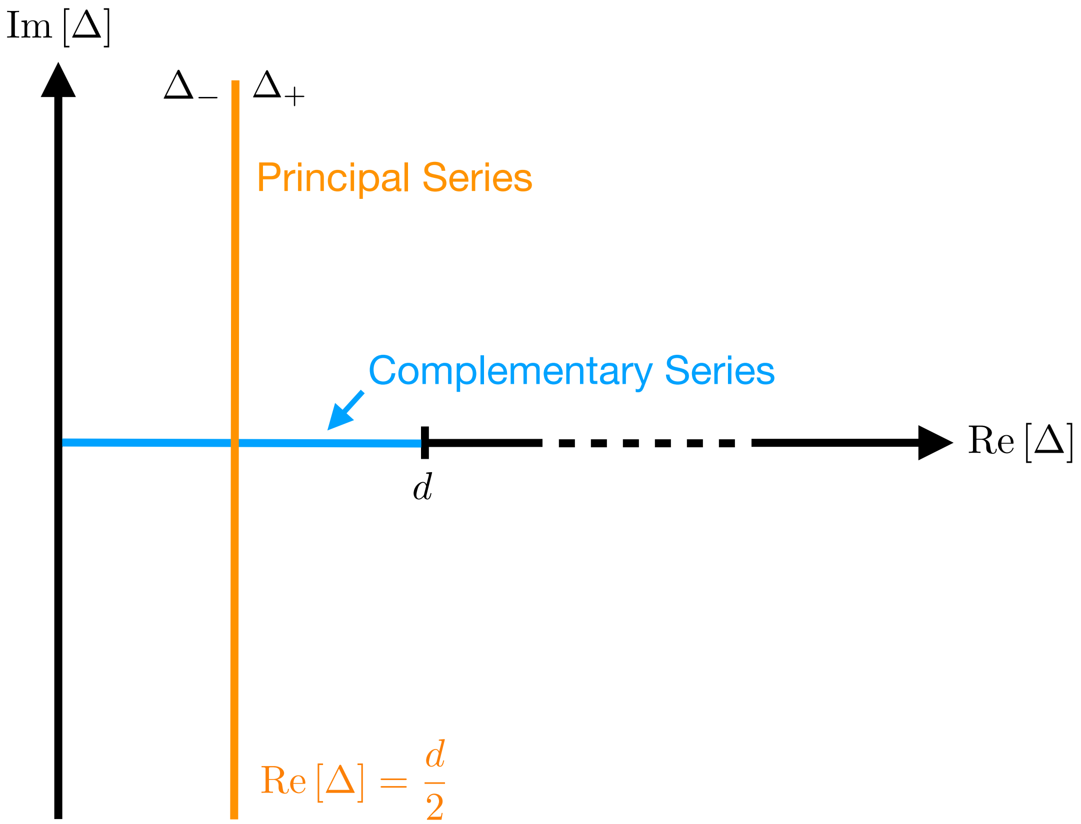


Particles in dS

Particles in dS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO(d+1,1)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

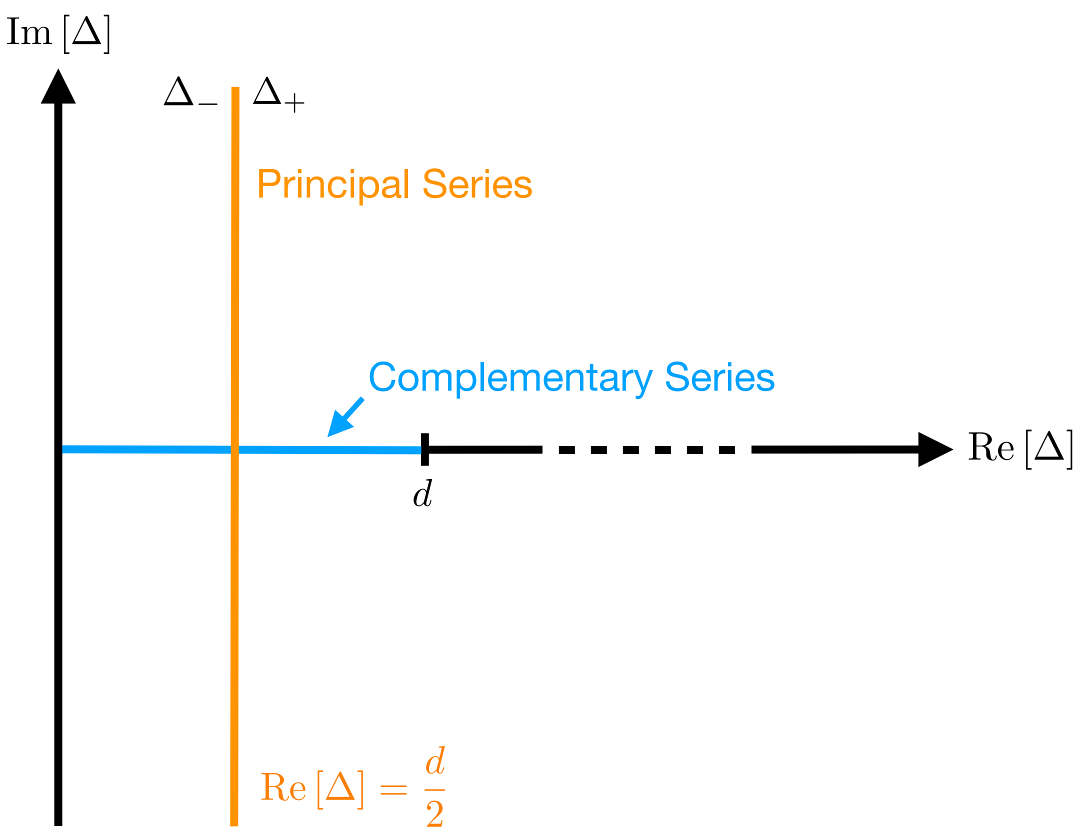
- Both Δ_+ and Δ_- are unitary
- Δ can be complex - Principal Series

Particles in dS

Particles in dS$_{d+1}$ $\longleftrightarrow$ unitary irreducible representations of $SO(d+1,1)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in dS$_{d+1}$.

E.g. Spin J=0 representations



Quadric Casimir equation

$\langle \mathcal{C}_2 \rangle = \Delta(d-\Delta)$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle \mathcal{C}_2 \rangle\right)\varphi = 0$$

$$m^2 R_{\mathrm{dS}}^2 = \Delta(d-\Delta)$$

Boundary behaviour:

$$\lim_{\eta\to 0} \varphi(\eta, x) = O_{\Delta_+}(\mathbf{x})\,\eta^{\Delta_+} + O_{\Delta_-}(\mathbf{x})\,\eta^{\Delta_-}$$

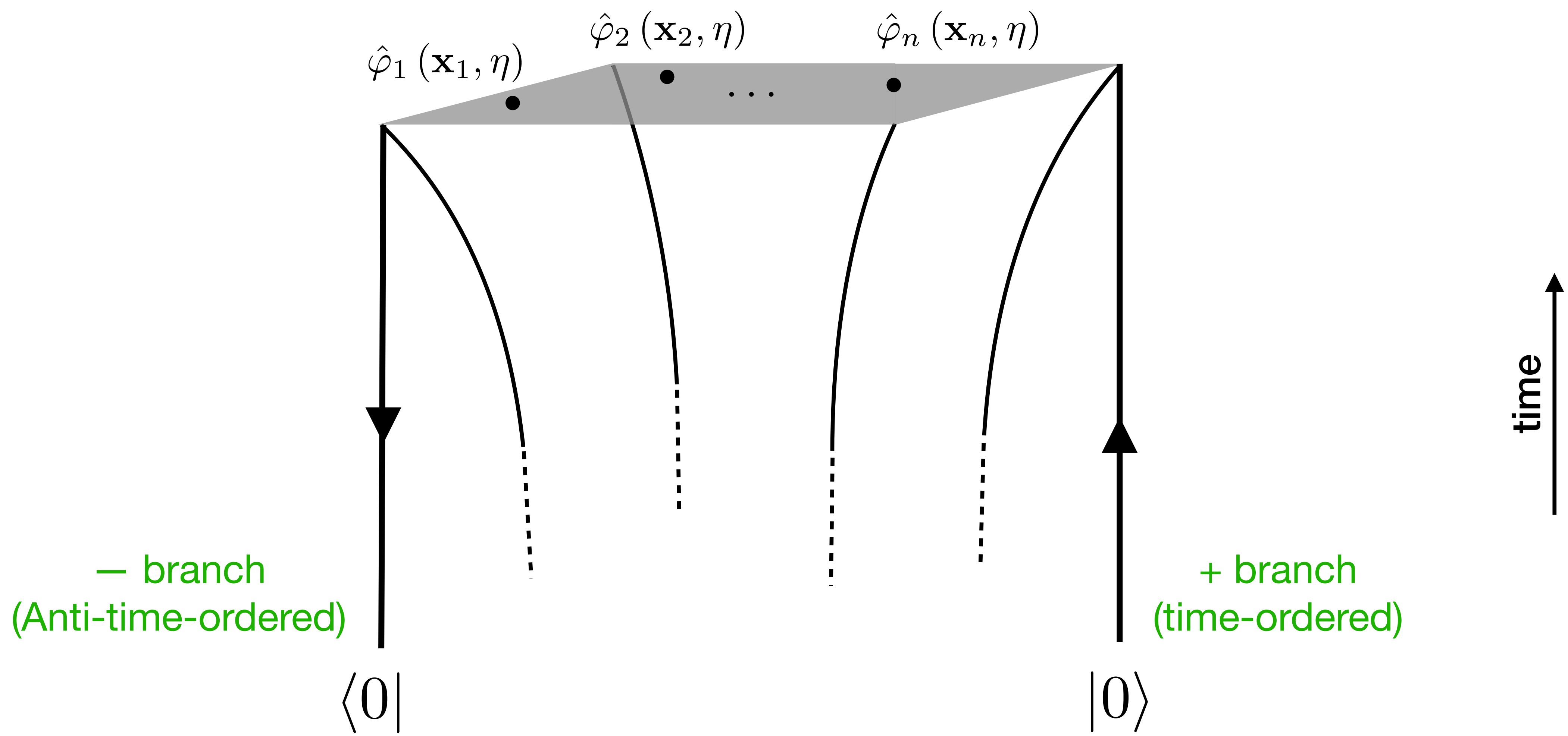
Determined by the initial state

$O_{\Delta_\pm}(\mathbf{x})$ transform as primary fields with scaling dimension $\Delta_\pm$ in Euclidean CFT$_d$

dS Boundary Correlators

in-in formalism [Maldacena ’02, Weinberg '05]

$$\lim_{\eta\to 0}\langle 0|\hat{\varphi}_1(\mathbf{x}_1,\eta)\dots\hat{\varphi}_n(\mathbf{x}_n,\eta)|0\rangle$$



Take $|0\rangle$ to be the de Sitter vacuum which reduces to the Minkowski vacuum at early times.

(Bunch Davies vacuum)

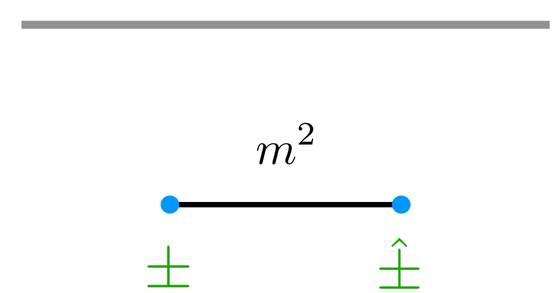
dS Boundary Correlators

in-in formalism [Maldacena ’02, Weinberg '05]

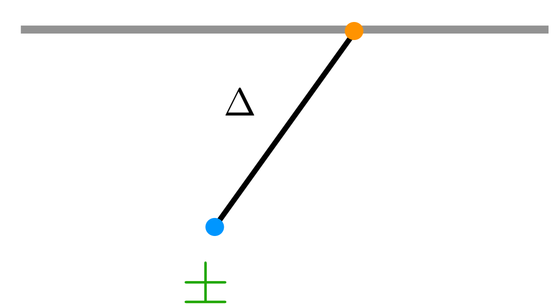
$$\lim_{\eta \to 0} \langle 0 | \hat{\varphi}_1 \left(\mathbf{x}_1 , \eta \right) \ldots \hat{\varphi}_n \left(\mathbf{x}_n , \eta \right) | 0 \rangle$$

Feynman rules:

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:



$\pm$ bulk-to-boundary propagator:

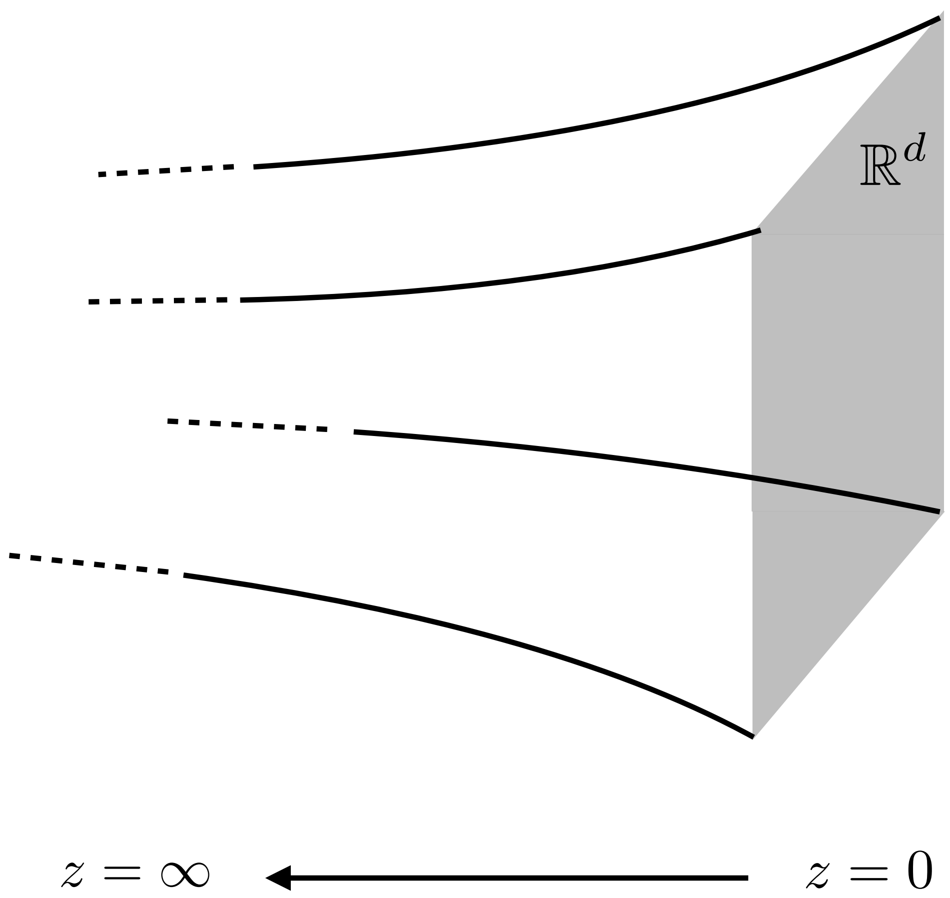


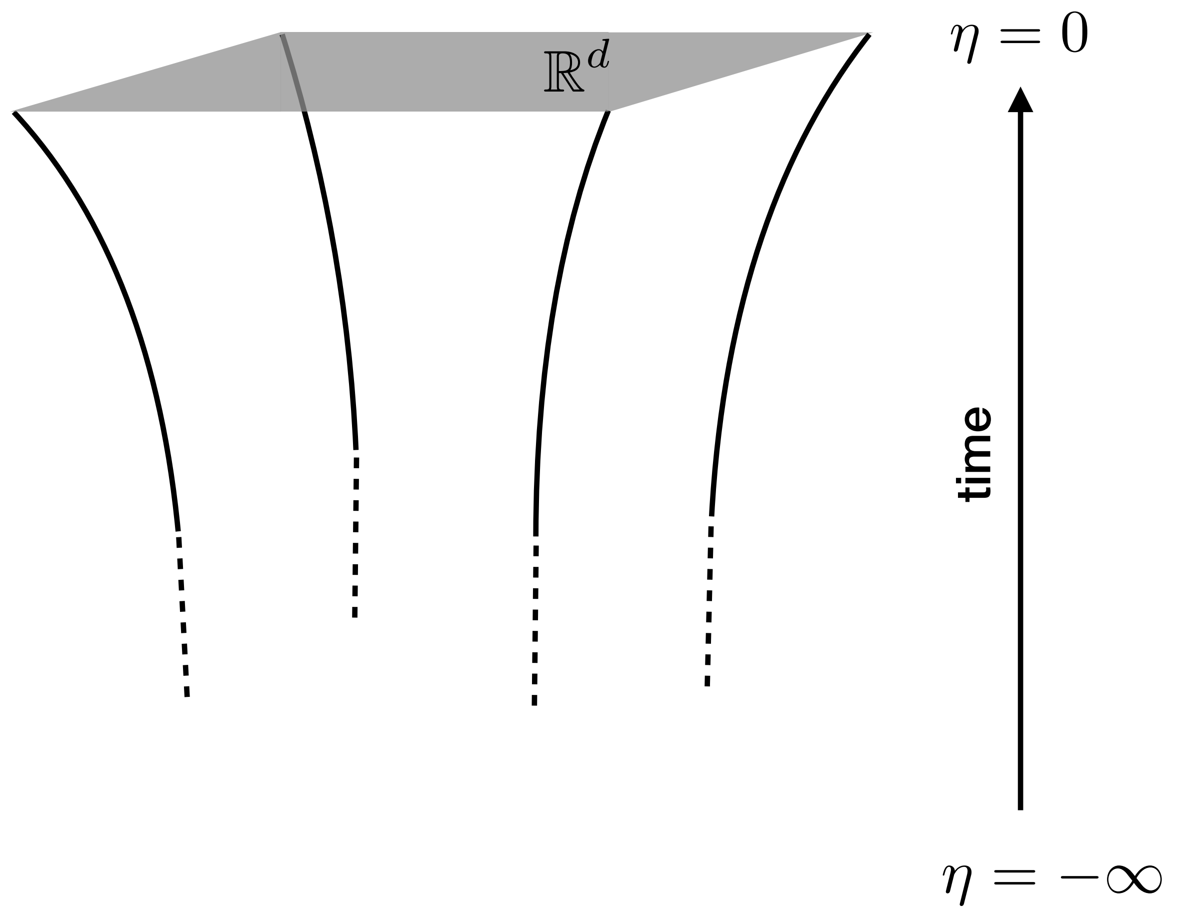
Sum contributions from each branch (±) of the time (in-in) contour!

From dS to Euclidean AdS

Euclidean AdS

dS





$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \mathrm{d}\mathbf{x}^2}{z^2} \quad \longleftrightarrow \quad \mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

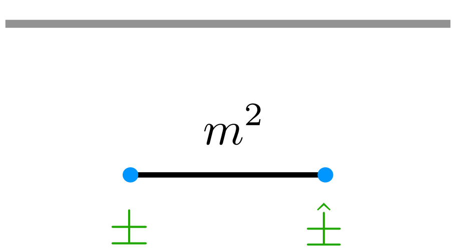
EAdS and dS are identified under:

$$R_{\mathrm{AdS}} = i R_{\mathrm{dS}} \qquad z = i\,(-\eta)$$

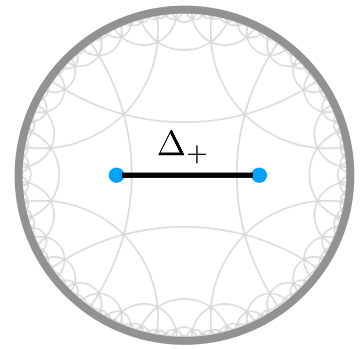
From dS to Euclidean AdS

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:

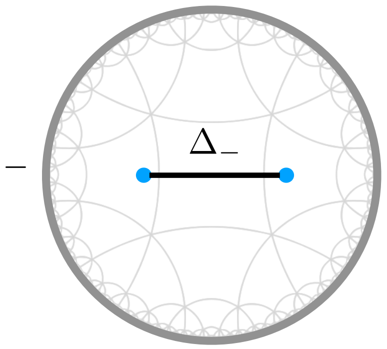
[C.S. and M. Taronna ’19, ’20, ’21]

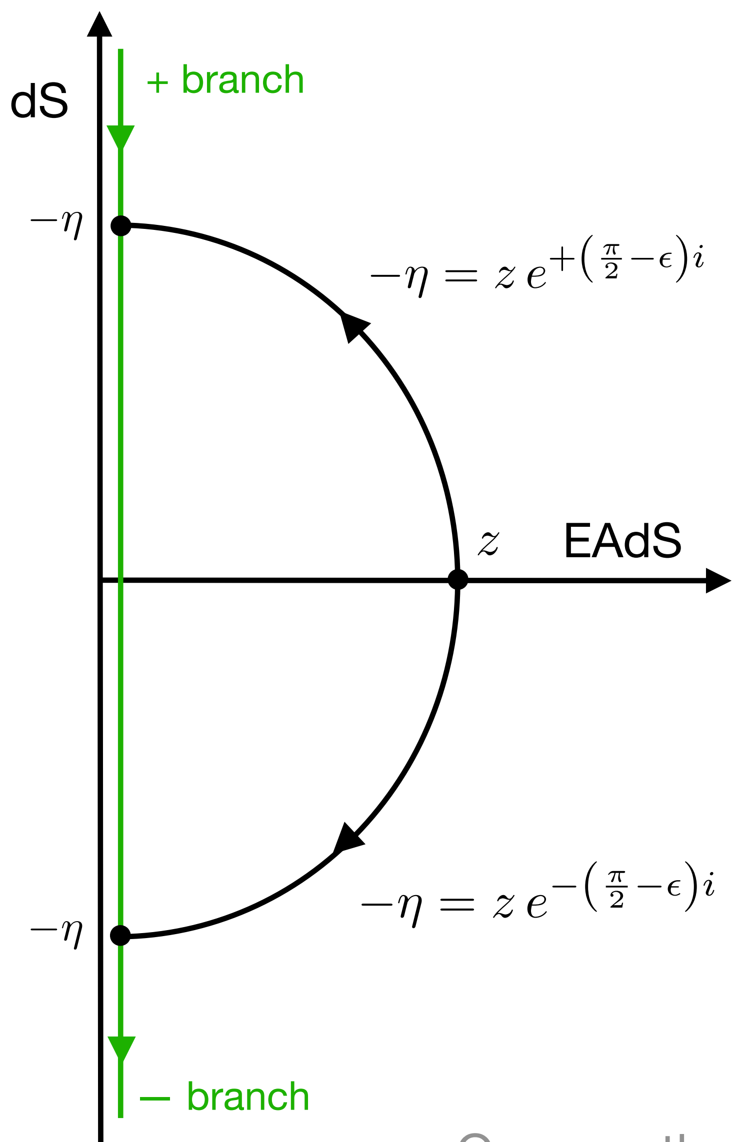


$= e^{\mp\frac{i\pi}{2}\Delta_+} e^{\hat{\mp}\frac{i\pi}{2}\Delta_+}$

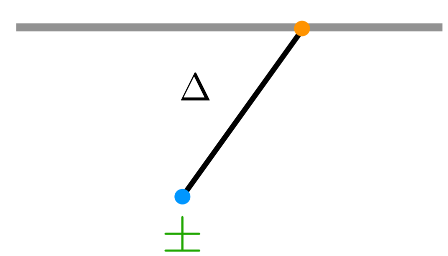


$+ e^{\mp\frac{i\pi}{2}\Delta_-} e^{\hat{\mp}\frac{i\pi}{2}\Delta_-}$

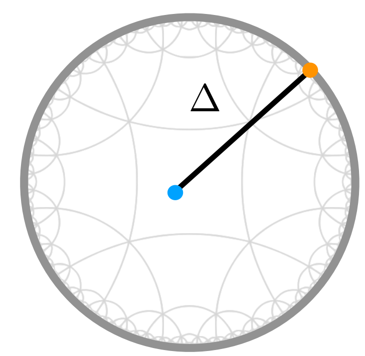




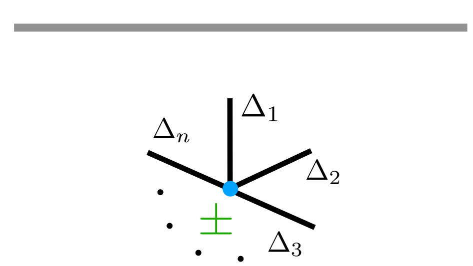
$\pm$ bulk-to-boundary propagator:



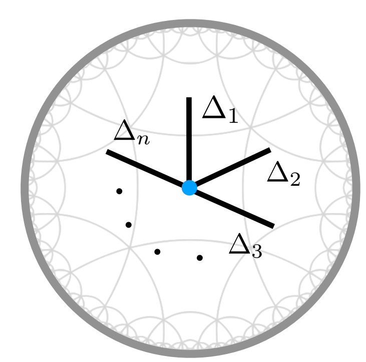
$= e^{\mp\frac{i\pi}{2}\Delta}$



$\pm$ bulk integrals:



$= e^{\pm(d-1)\frac{\pi i}{2}}$



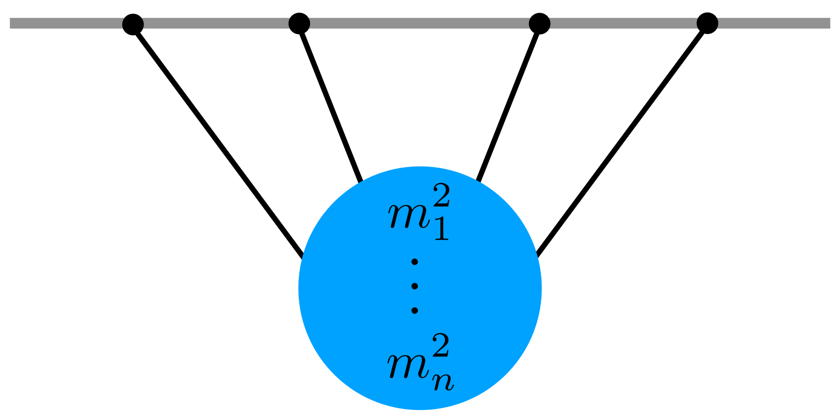
One can then write an EAdS Lagrangian for dS correlators [di Pietro, Gorbenko and Komatsu ’21]

From dS to EAdS, and back

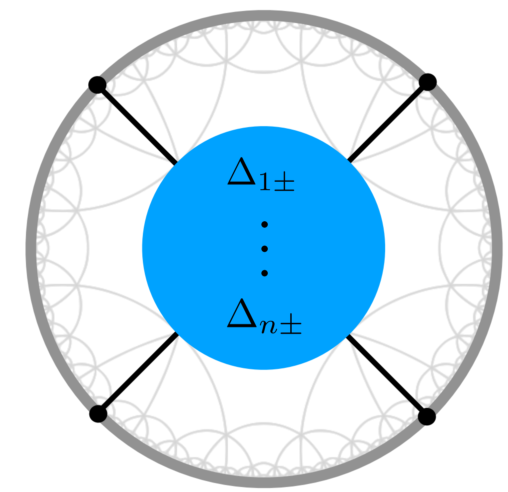
dS boundry correlators are perturbatively recast as Witten diagrams in EAdS:

e.g. four-points

Contributions from
each branch of the in-in contour



$$= \sum_{\Delta_{1\pm}\dots\Delta_{n\pm}} C_{\Delta_{1\pm}\dots\Delta_{n\pm}}$$



Sum over boundary conditions
for exchanged particles

Notes:

- Contributions from both $\Delta_{\pm}$ modes
- $\Delta_{i\,\pm} \in$ Unitary Irreducible Representation of **dS** isometry

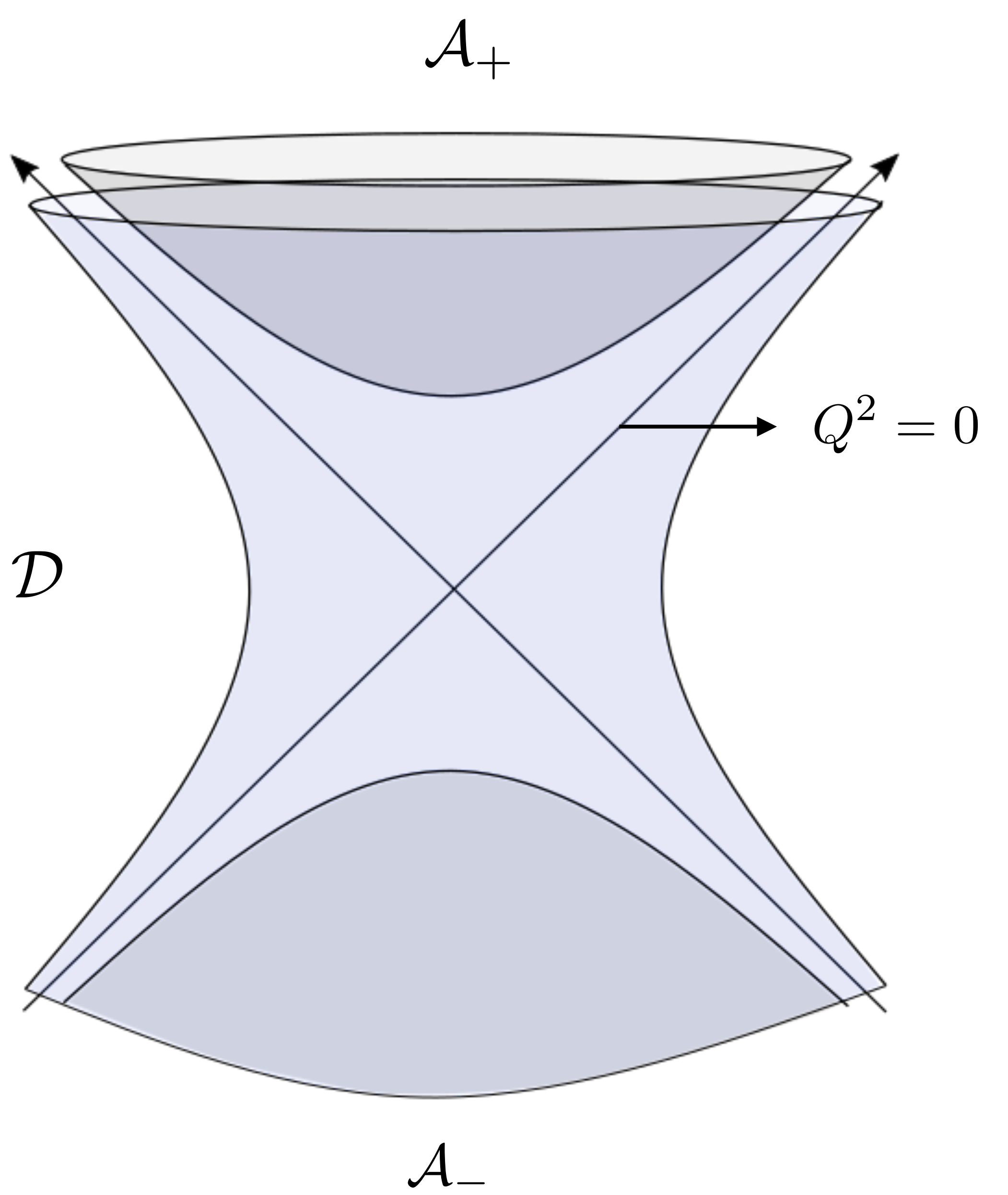
Can use to import techniques, results and understanding from AdS to dS!

$$\Lambda = 0$$

Hyperbolic slicing of Minkowski space

[de Boer and Solodukhin '03]

(d+2)-dimensional Minkowski space $\mathbb{M}^{d+2}$, coordinates $X^A, \quad A = 0, \ldots d+1$



$\mathcal{A}_\pm : \quad X^2 = -t^2$ (EAdS$_{d+1}$, radius t)

$\mathcal{D} : \quad X^2 = R^2$ (dS$_{d+1}$, radius R)

Conformal boundary:

$$Q^2 = 0, \quad Q \equiv \lambda Q, \quad \lambda \in \mathbb{R}^+$$

Introduce projective coordinates:

$$\xi_i = Q^i / Q^0, \quad i = 1, \ldots, d+1$$

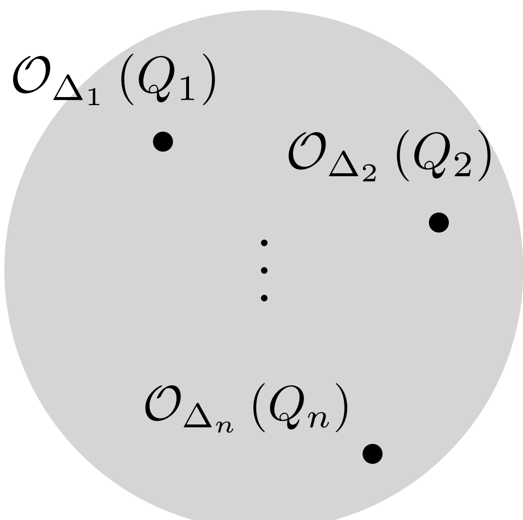
$$\xi_1^2 + \ldots + \xi_{d+1}^2 = 1$$ d-dimensional unit sphere

$SO\,(d+1,1)$ acts on the celestial sphere as the Euclidean conformal group!

Minkowski boundary correlators

[C.S. and M. Taronna '23]

Radial Mellin transform of Minkowski correlators implements a radial reduction onto the hyperbolic slicing:



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i}\, t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \ldots \phi_n(t_n \hat{X}_n) \right\rangle$$

Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

Mellin transform

$$\int_0^\infty \frac{dt}{t} t^\Delta (\ldots)$$

Inverse Mellin transform

$$\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i}\, t^{-\Delta} (\ldots)$$

Unitary Principal Series representations of SO(d+1,1)

Minkowski boundary correlators

[C.S. and M. Taronna '23]

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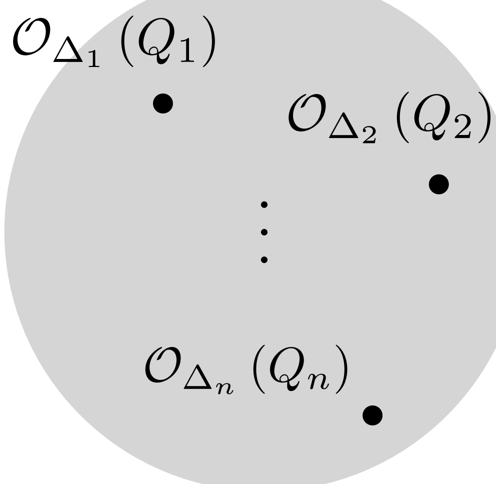

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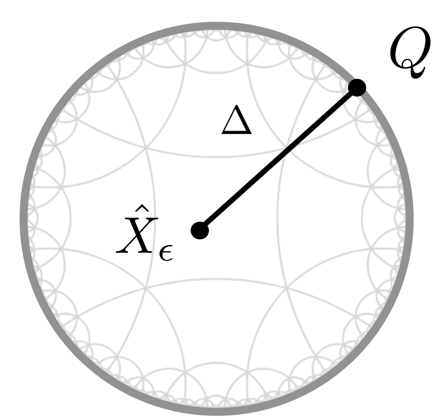
Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

"Celestial" bulk-to-boundary propagator:

$$G_\Delta^{\text{flat}}(X, Q) = \lim_{\hat{Y} \to Q} \int_0^\infty \frac{dt}{t} t^\Delta\, G_F\left(X, t\hat{Y}\right) =$$



Kernel of the radial reduction (Bessel function)

$$\times\ \mathcal{K}^{(m)}_{i\left(\frac{d}{2}-\Delta\right)}\left(\sqrt{X^2 + i\epsilon}\right)$$

Factorises into (analytically cont'd) **EAdS bulk-boundary propagator** + **radial component!**

From the Celestial Sphere to EAdS

[C.S. and M. Taronna '23]

Examples.

Free theory Celestial two point function:

$$\langle \mathcal{O}_{\Delta_1}(Q_1)\,\mathcal{O}_{\Delta_2}(Q_2)\rangle = \lim_{\hat{X}\to Q_2}\int_0^\infty \frac{dt}{t}\, t^{\Delta_2} G^{\text{flat}}_{\Delta_1}(t\hat{X}, Q_1)$$

Q_i can be null separated

$$= \frac{C^{\text{flat}}_{\Delta_1}(m)}{(-2Q_1\cdot Q_2 + i\epsilon)^{\Delta_1}}(2\pi)\delta(i(\Delta_1-\Delta_2))$$

Consequence of continuous spectrum

Form required by Conformal Symmetry

From the Celestial Sphere to EAdS

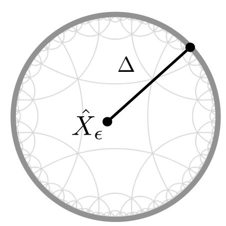
[C.S. and M. Taronna ’23]

Examples.

Non-derivative vertex of scalars fields $\mathcal{V}(X) = g\phi_1(X)\ldots\phi_n(X)$

Contact diagram:

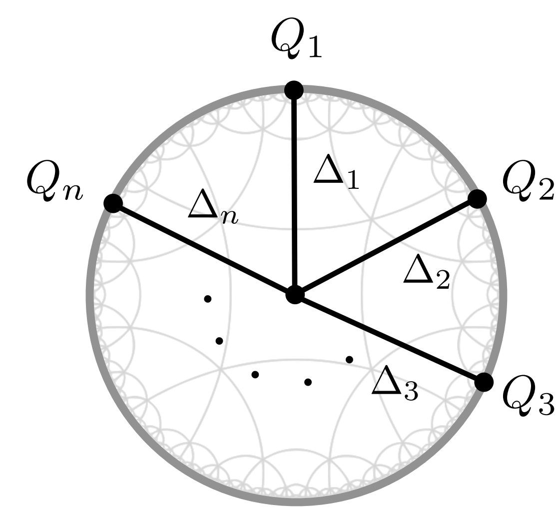
$$\langle \mathcal{O}_{\Delta_1}(Q_1)\ldots\mathcal{O}_{\Delta_n}(Q_n)\rangle = -ig\int d^{d+2}X\, G^{\text{flat}}_{\Delta_1}(X,Q_1)\cdots G^{\text{flat}}_{\Delta_n}(X,Q_n)\,.$$

$G^{\text{flat}}_{\Delta}(X,Q) =$  $\times\ \mathcal{K}^{(m)}_{i(\frac{d}{2}-\Delta)}\left(\sqrt{X^2+i\epsilon}\right)$

$$= R_{\Delta_1\ldots\Delta_n}(m_1,\ldots,m_n)\ \times$$

Radial integral. Encodes all mass dependence
(Can be evaluated as a Mellin-Barnes integral)

(analytically cont'd)
EAdS contact diagram

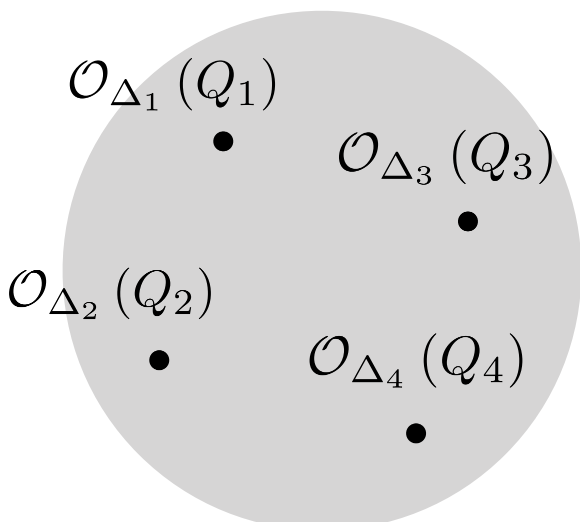


$\longrightarrow$ Celestial contact diagrams are proportional to their EAdS counterparts (like in dS)

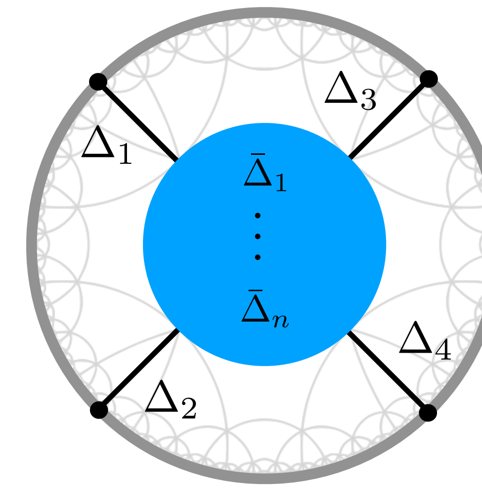
From the Celestial Sphere to EAdS

[C.S. and M. Taronna '23]

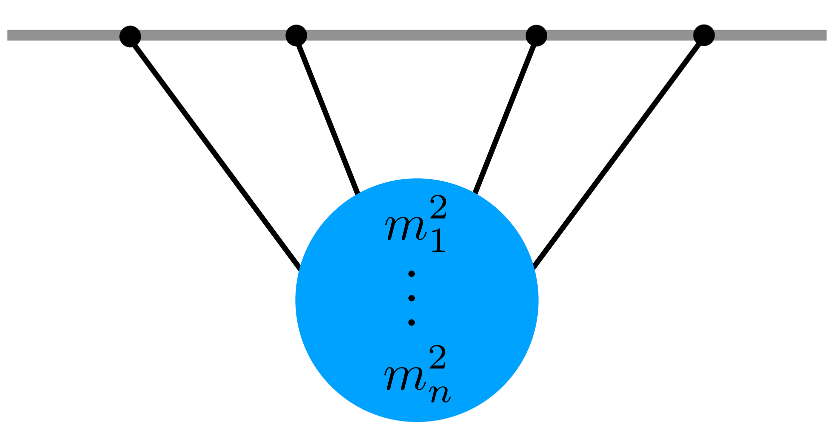
In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$



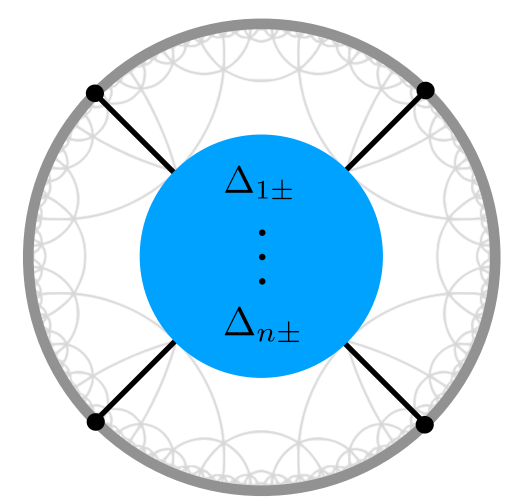
$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i} C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}(m_1, \ldots, m_n)$$



Compare with de Sitter:



$$= \sum_{\Delta_{1\pm} \ldots \Delta_{n\pm}} C_{\Delta_{1\pm} \ldots \Delta_{n\pm}}$$



Outlook

- Relation to definition [Pasterski, Shao, Strominger '17] of celestial correlators as scattering amplitudes in a conformal basis?

  [Pasterski, Shao, Strominger '17] = LSZ ([Sleight, Taronna '23]) ?

- Celestial correlators defined as an extrapolation of bulk Minkowski correlators give a definition of celestial correlators for theories without an S-matrix.

  What lessons can we draw from Minkowski CFT?

- dS and celestial correlators have a similar analytic structure to those in AdS. What about non-perturbatively?

Analytic structure ⟶ Conformal partial wave expansion [Sleight, Taronna '20]:

$$\langle \mathcal{O}(\mathbf{x}_1)\,\mathcal{O}(\mathbf{x}_2)\,\mathcal{O}(\mathbf{x}_3)\,\mathcal{O}(\mathbf{x}_4)\rangle = \sum_{J=0}^{\infty}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}\frac{d\Delta}{2\pi i}\rho_J(\Delta)\,\mathcal{F}_{\Delta,J}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)$$

Spectral density: $\rho_J(\Delta)$

Conformal Partial Wave: $\mathcal{F}_{\Delta,J}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)$

Unitarity: $\rho_J(\Delta) \geq 0$

[Hogervorst, Penedones, Vaziri '21, di Pietro, Komatsu, Gorbenko 21' Iacobacci, Sleight, Taronna '23]

Non-perturbative Bootstrap of Euclidean CFTs dual to physics in Minkowski/de Sitter?