Progress in Flat Space Holography

Ana-Maria Raclariu
University of Amsterdam

From Amplitudes to Gravitational Waves - Nordita, July 2023

Motivation

Holography (most basic test)

- Asymptotic/large gauge symmetries of bulk (gravity) theory ↔ global symmetries of boundary (quantum) theory

[Brown, Henneaux ’86]

Gravity in 4D Asymptotically Flat Spacetimes

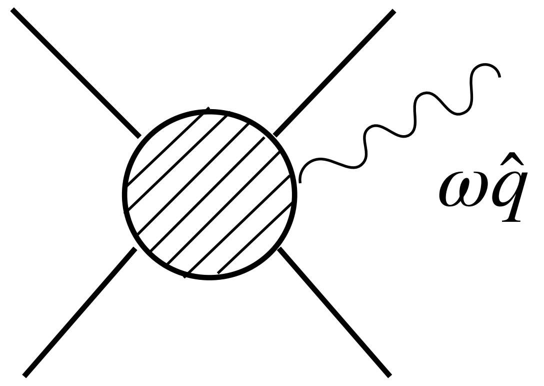
- Lorentz symmetries act like global conformal symmetries of the 2D sphere at infinity superrotations ↔ Virasoro

[Barnich, Troessaert ’09; Kapec, Lysov, Pasterski, Strominger ’14]

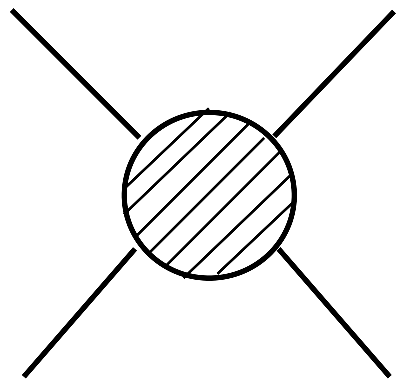
- Subleading soft graviton mode ↔ generator of Virasoro!

[Cachazo, Strominger ’14; Kapec, Mitra, A.R., Strominger ’16]

$$\lim_{\omega \to 0}$$



$$= \left[\frac{1}{\omega} S^{(0)}(\hat{q}; p_i) + S^{(1)}(\hat{q}; p_i, J_i) + \mathcal{O}(\omega) \right] \times$$



Goal: Look for 2D conformal field theory dual to gravity in 4D AFS: $\Lambda = 0$ holography

Outline

Celestial amplitudes from
AdS Witten diagrams

Top-down celestial holography

Conformally soft sector of CFT_3

Soft gluon and graviton algebras

Operator product expansions
and conformally soft symmetries

Generic features of celestial
2, 3, 4-point functions

Massless scattering
The boost basis and celestial amplitudes

Spinor helicity variables

Massless Poincare representations: particle states $|p_\mu, \sigma\rangle$ labelled by momentum p_μ and helicity σ

Lorentz group $SO^+(1,3) \simeq SL(2,\mathbb{C})/\mathbb{Z}_2$:

$$p_\mu \to P \equiv p_\mu \sigma^\mu, \qquad \sigma_\mu \quad \text{Pauli matrices}$$

$$\Lambda_{\mu\nu} p^\nu \to XPX^\dagger, \qquad X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C})$$

Spinor helicity variables: $p^2 = 0 \iff \det P = 0 \implies P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$

Let $\lambda = \sqrt{2\omega}\begin{pmatrix} z \\ 1 \end{pmatrix}$, $\tilde{\lambda} = \eta\lambda^*$ with $\eta = \pm$ depending on whether particle is incoming or outgoing. The Lorentz group then acts as:

$$\lambda \to \lambda' = e^{i\theta}\sqrt{2\omega'}\begin{pmatrix} z' \\ 1 \end{pmatrix} \qquad \text{where} \qquad \omega' = |cz+d|^2\omega, \quad z' = \frac{az+b}{cz+d}, \quad e^{2i\theta} = \frac{cz+d}{\bar{c}\bar{z}+\bar{d}}$$

Massless scattering amplitudes

Lorentz transformations: $\lambda \to \lambda' = e^{i\theta}\sqrt{2\omega'}\begin{pmatrix} z' \\ 1 \end{pmatrix}$ where $\omega' = |cz+d|^2\omega, \quad z' = \dfrac{az+b}{cz+d}, \quad e^{2i\theta} = \dfrac{cz+d}{\bar{c}\bar{z}+\bar{d}}$

- Little group = Lorentz subgroup that preserves momenta: $\lambda' = e^{i\theta}\lambda \implies p' = p$
- Under Lorentz transformations: $X(\Lambda)|p,\sigma\rangle = \sum_{\sigma'} D_{\sigma\sigma'}|\Lambda p, \sigma'\rangle$, where D is some representation of the little group

in 4D little group is $SO(2)$ → reps. labelled by helicity σ

Scattering amplitudes of n massless particles labelled by n pairs (p_a, σ_a):

- Translation invariance $\implies$ $\mathscr{M}(p_a,\sigma_a) = M(p_a,\sigma_a)\delta^4\left(\sum_{a=1}^{n} p_a\right)$
- Lorentz covariance $\implies$ $M^\Lambda(p_a,\sigma_a) = \prod_a D_{\sigma_a\sigma_a'} M((\Lambda p)_a; \sigma_a')$

[for short review see eg. Arkani-Hamed, Huang, Huang ’21]

Massless scattering amplitudes

- Little group = Lorentz subgroup that preserves momenta: $\lambda' = e^{i\theta}\lambda \implies p' = p$

In terms of the spinor-helicity variables, the (stripped) amplitudes M are homogeneous functions of $(\lambda_a, \tilde{\lambda}_a)$ with helicity-weights given by the little group:

$$M(e^{i\theta}\lambda, e^{-i\theta}\tilde{\lambda}) = e^{-2i\theta h} M(\lambda, \tilde{\lambda})$$

- This strongly restricts the scattering amplitudes. For example, for massless 3-point scattering:

$$\text{Momentum conservation + Lorentz invariance} \implies \begin{cases} \lambda_1 \sim \lambda_2 \sim \lambda_3 \implies M \sim [12]^a[23]^b[31]^c & [ij] \equiv \tilde{\lambda}_{i,\dot{\alpha}}\tilde{\lambda}_j^{\dot{\alpha}} \\ \tilde{\lambda}_1 \sim \tilde{\lambda}_2 \sim \tilde{\lambda}_3 \implies M \sim \langle 12\rangle^a\langle 23\rangle^b\langle 31\rangle^c & \langle ij\rangle \equiv \lambda_{i,\alpha}\lambda_j^{\alpha} \end{cases}$$

a, b, c fixed in terms of helicities by little group scaling!

Boost eigenstates

Lorentz transformations: $\lambda \to \lambda' = e^{i\theta}\sqrt{2\omega'}\begin{pmatrix} z' \\ 1 \end{pmatrix}$ where $\omega' = |cz+d|^2\omega$, $\boxed{z' = \frac{az+b}{cz+d}}$, $e^{2i\theta} = \frac{cz+d}{\bar{c}\bar{z}+\bar{d}}$

Lorentz group ~ global conformal group in 2D

Apply 2D CFT methods to scattering amplitudes in 4D?

Obstacle: asymptotic states are not in highest weight representations of $SL(2,\mathbb{C})$

Under $p(\omega, z, \bar{z}) \to \lambda p(\omega, z, \bar{z})$ (4D boosts towards $(z, \bar{z})$ = 2D dilatations), $|\omega, z, \bar{z}\rangle \to |\lambda\omega, z, \bar{z}\rangle \neq \lambda|\omega, z, \bar{z}\rangle$ (since eg. $[K, P] \neq 0$)

Diagonalize boosts: $|\Delta, z, \bar{z}\rangle \equiv \int_0^\infty d\omega\omega^{\Delta-1}|\omega, z, \bar{z}\rangle \to \int_0^\infty d\omega\omega^{\Delta-1}|\lambda\omega, z, \bar{z}\rangle = \lambda^{-\Delta}|\Delta, z, \bar{z}\rangle$

Celestial amplitudes

Diagonalize boosts:

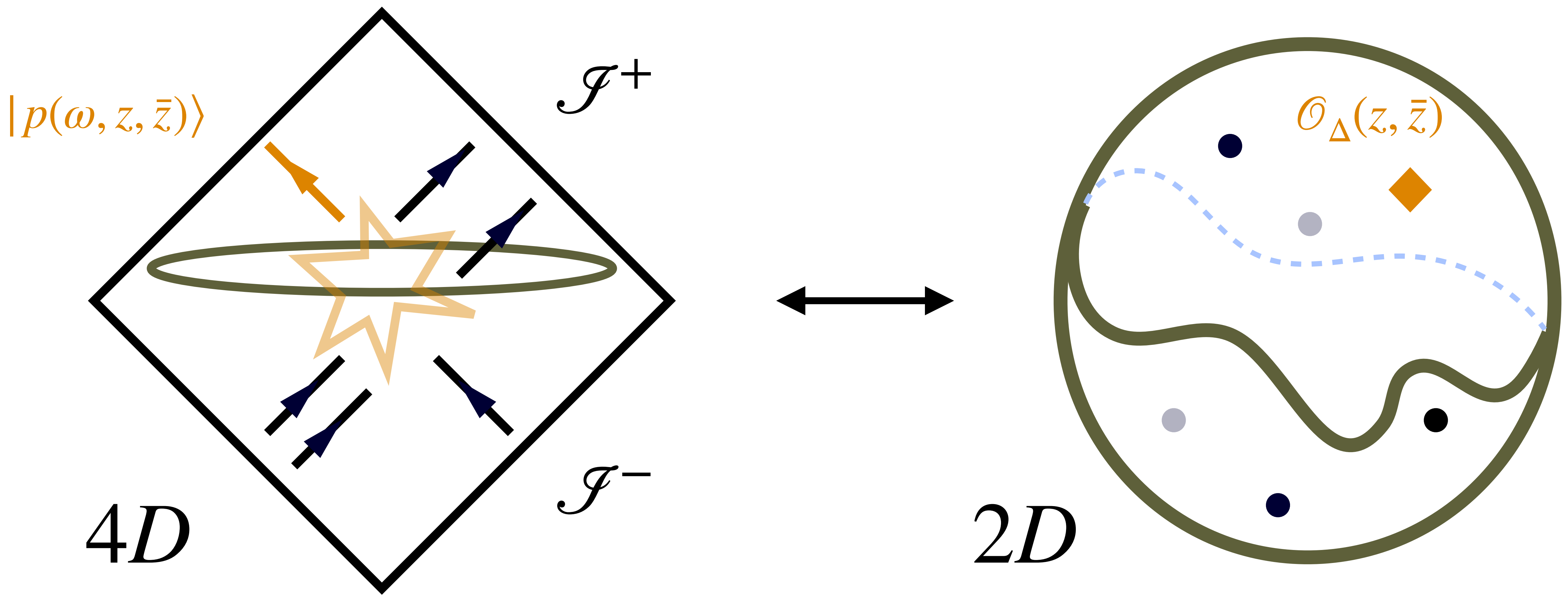
$$|\Delta, z, \bar{z}\rangle \equiv \int_0^\infty d\omega \omega^{\Delta-1} |\omega, z, \bar{z}\rangle \rightarrow \int_0^\infty d\omega \omega^{\Delta-1} |\lambda\omega, z, \bar{z}\rangle = \lambda^{-\Delta} |\Delta, z, \bar{z}\rangle$$

Scatter boost eigenstates $|\Delta, z, \bar{z}\rangle$ instead of energy-momentum-eigenstates $|\omega, z, \bar{z}\rangle$

Conformal primary basis:

$$\mathscr{M}(p_a, \sigma_a) \longrightarrow \widetilde{\mathscr{M}}(\Delta_a, z_a, \bar{z}_a) = \prod_{i=1}^{n} \left(\int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} \right) M(p_a, \sigma_a) \delta^4 \left(\sum_{a=1}^{n} p_a \right)$$

Scattering amplitude $\mathscr{M}(p_a, \sigma_a)$



Celestial amplitude $\langle \mathscr{O}_{\Delta_1}(z_1, \bar{z}_1) \cdots \mathscr{O}_{\Delta_n}(z_n, \bar{z}_n) \rangle$

Examples of celestial amplitudes

Two-point functions: $\widetilde{\mathcal{M}}(z_1, \bar{z}_1; z_2, \bar{z}_2) \propto \delta(\Delta_1 + \Delta_2 - 2)\delta^{(2)}(z_1 - z_2)$ [Pasterski, Shao, Strominger '17; Pasterski, Shao '17; Stieberger, Taylor '18]

Three-point functions: $\widetilde{\mathcal{M}} \propto z_{21}^{-h_1-h_2+h_3} z_{23}^{-h_2-h_3+h_1} z_{13}^{h_2-h_1-h_3} \delta(\bar{z}_{12})\delta(\bar{z}_{23}) \int_0^\infty d\omega \omega^{\Delta_1+\Delta_2+\Delta_3-3-s_1-s_2-s_3-2}, \quad h = \frac{\Delta + s}{2}$

Four-point functions: $\widetilde{\mathcal{M}}(z_i, \bar{z}_i; \beta) = \underbrace{K(z_i, \bar{z}_i)X(z, \beta)}_{\text{kinematics}} \underbrace{\int_0^\infty d\omega \omega^{\beta-1} \mathcal{M}(\omega^2, -z\omega^2)}_{\equiv \mathcal{A}(\beta, z),\ \text{dynamics}}, \quad \beta \equiv \sum_{i=1}^{4} \Delta_i - 4,$ $X \propto \delta(z - \bar{z})$

[momentum conservation]

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[momentum conservation]

Analytic properties

Poor UV behavior $\mathcal{M} \propto \omega^p \implies \widetilde{\mathcal{M}} \propto \int_0^\infty d\omega\, \omega^{\beta+p-1} \propto \delta(\beta + p),\ \beta + p \in i\mathbb{R}$

Good UV behavior $\mathcal{M} = \lambda \frac{M^2}{\omega^2 - M^2} \implies \widetilde{\mathcal{M}} \propto \frac{\lambda M^\beta}{\sin \pi\beta/2}$ ``low-" and ``high-energy" poles in β

Properties of celestial amplitudes

Two-point functions: $\widetilde{\mathcal{M}}(z_1, \bar{z}_1; z_2, \bar{z}_2) \propto \delta(\Delta_1 + \Delta_2 - 2)\delta^{(2)}(z_1 - z_2)$

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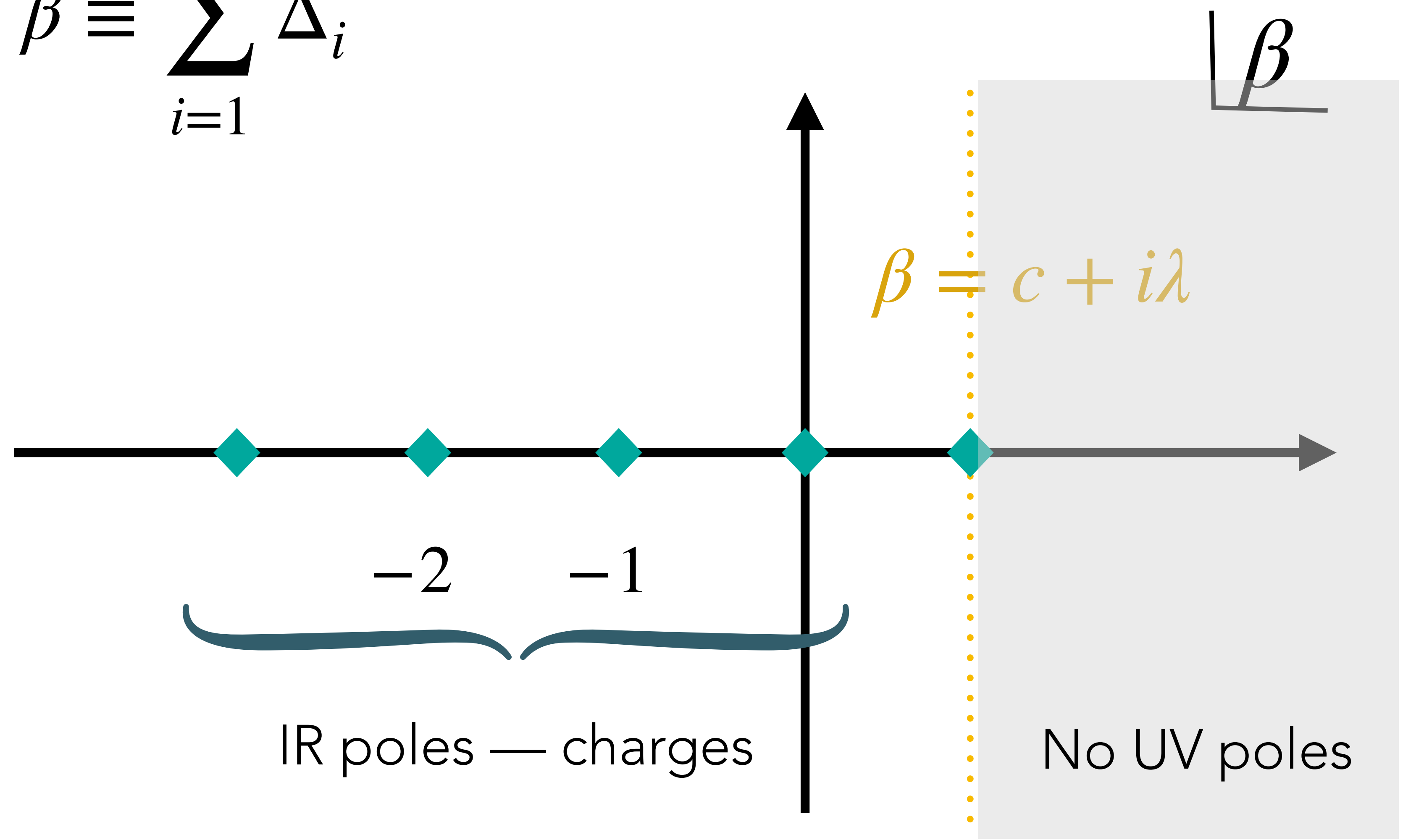
Analytic properties Very good UV behavior

$$\lim_{\omega\to\infty} \mathcal{M}(\omega^2, -z\omega^2) \propto e^{-\omega^2/M^2} \longrightarrow \lim_{\beta\to\infty} \widetilde{\mathcal{M}}(\beta, z) \to \frac{M^\beta}{2}\Gamma(\beta/2)$$



[Arkani-Hamed, Pate, A.R., Strominger '20; Chang, Huang, Huang, Li '22; Garcia-Sepulveda, Guevara, Kulp, Wu '22]

Other examples

Loop corrections $\widetilde{\mathcal{M}}(\beta, z) \supset \int_0^{\omega_*} d\omega \omega^{\beta-1} \log^r \omega \propto \frac{\partial^r}{\partial \beta^r}\frac{1}{\beta} \propto \frac{1}{\beta^{r+1}}$ (loop order ~ higher order pole in β)

All loop formula in planar $\mathcal{N} = 4$ SYM: UV finite, IR divergences exponentiate

$$M = \exp\left[\sum_{\ell=1}^{\infty} a^{\ell}\left(f_{\epsilon}^{(\ell)} M_{\ell\epsilon}^{(1)} + C_{\epsilon}^{(\ell)} + E_{\epsilon}^{(\ell)}\right)\right] M_{\text{tree}} \longrightarrow \widetilde{\mathcal{M}} = \exp\left[\sum_{L=1}^{\infty} a^{L}\left(f_{\epsilon}^{(L)} \mathcal{F}_1(z, L\epsilon) + C_{\epsilon}^{(L)} + \mathcal{E}^{(L)}(z, L\epsilon)\right) \hat{P}^{L\epsilon}\right] \widetilde{M}_{\text{tree}}$$

$$a = \frac{g^2 N}{8\pi^2}(4\pi e^{-\gamma_E})^{\epsilon}$$

- $\hat{P}$ is a conformally invariant operator $\propto e^{\frac{i}{2}\sum_{i=1}^{4}\partial_{\Delta_i}}$
- z is a conformally invariant cross-ratio
- Recent work celestial open string amplitudes

[Bern, Dixon, Smirnov '05]
[Gonzales, Puhm, Rojas '20]
[above + Donnay, Giribet '23]

- IR divergences in QED and gravity may be removed in a basis of eigenstates of large gauge charge

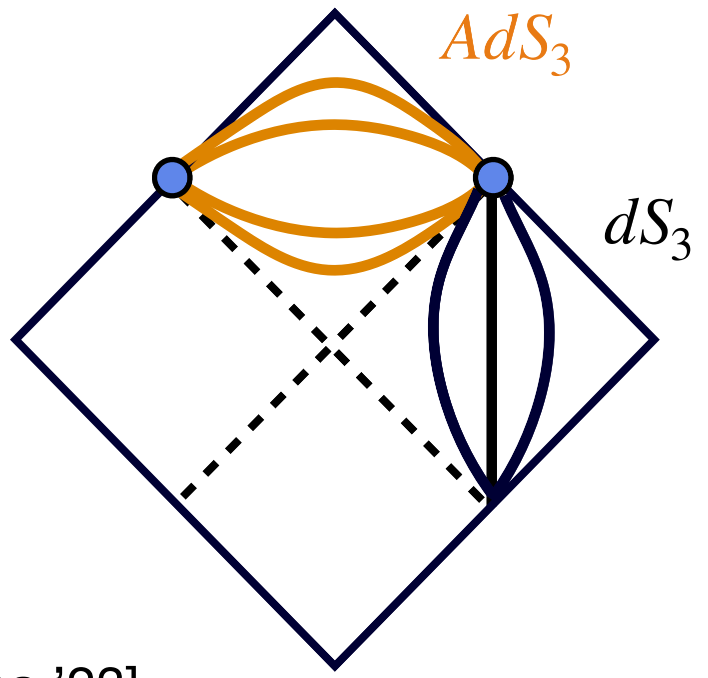
[Kapec, Perry, A.R., Strominger '17; Arkani-Hamed, Pate, A.R., Strominger '20]

Non-perturbative backgrounds

Translation breaking backgrounds appear to smoothen out singularities

$$\widetilde{\mathcal{M}}_B(1^-,2^-,3^+,\cdots n^+) \sim \frac{z_{12}^3}{z_{23}z_{34}\cdots z_{n1}} \int \widetilde{d^3Q}\, g(Q) \int d\omega_1 \omega_1^{\Delta_1} \int d\omega_2 \omega_2^{\Delta_2} \int \prod_{j=3}^{n} d\omega_j \omega_j^{\Delta_j - 2} \delta^{(4)}\left(Q + \sum_i \eta_i \omega_i \hat{q}_i\right)$$

conformal primary massive scalar ($\Delta = 2$)



- 3-point function ∝ standard CFT 3-point function [Casali, Melton, Strominger ’22; Stieberger, Taylor, Zhu ’22; Sleight, Taronna ’23]
- Celestial two-point functions in various different backgrounds recently computed to leading order in the coupling [Gonzo, McLaughlin, Puhm ’22]

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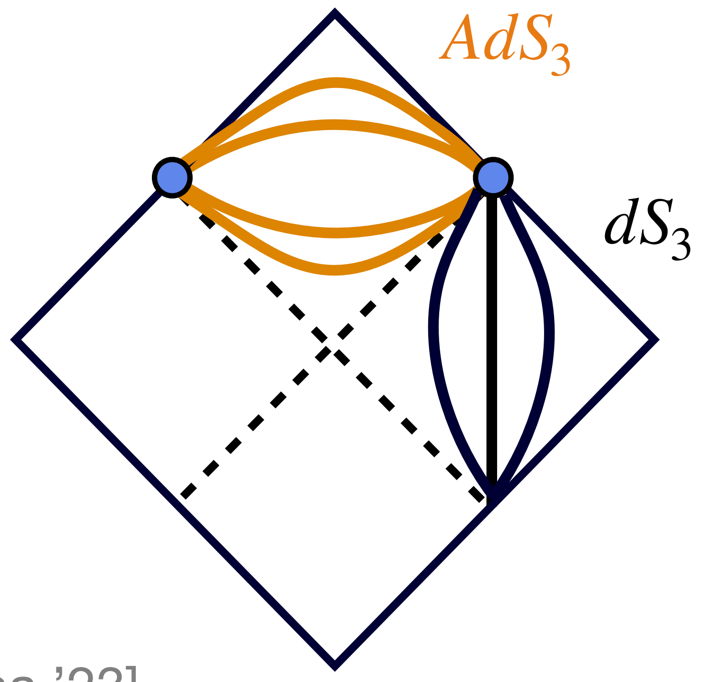

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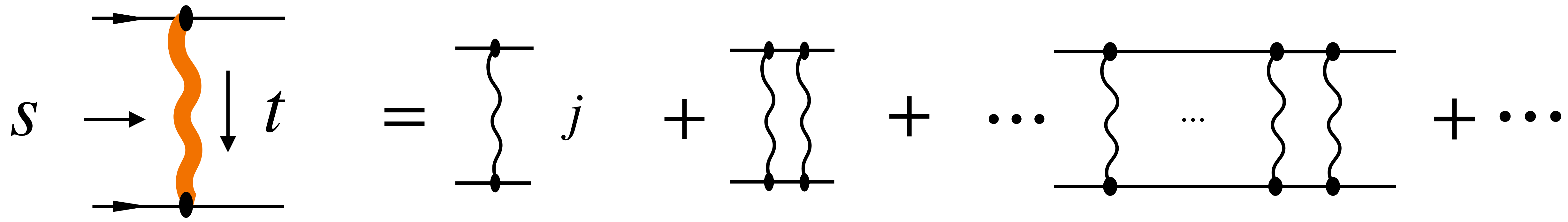
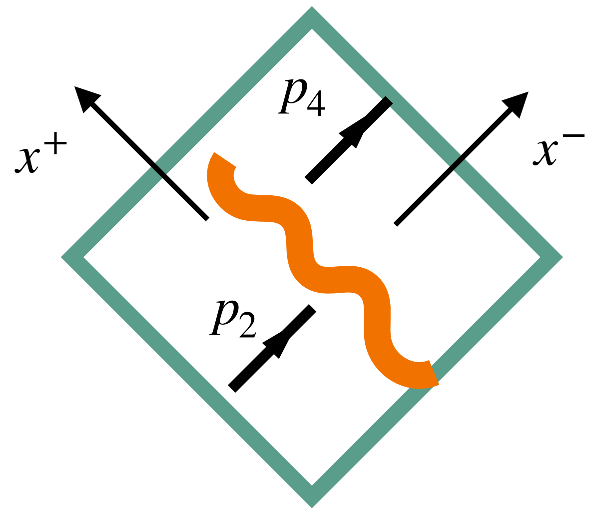


Propagation through shockwave ($j = 2$)



- Celestial 2-point function

$$\widetilde{A}_{\text{shock}}(\Delta_2, z_2, \bar{z}_2; \Delta_4, z_4, \bar{z}_4) = 4\pi \int d^2x_\perp \frac{i^{\Delta_2+\Delta_4}\Gamma(\Delta_2+\Delta_4)}{\left[-q_{24,\perp}\cdot x_\perp - h(x_\perp) + i\epsilon\right]^{\Delta_2+\Delta_4}}$$

[de Gioia, A.R. '22]

Chiral algebras

- 2D QFT with $SL(2,\mathbb{C})$ symmetry generated by $\begin{Bmatrix} L_{-1} = -\partial_z, & L_0 = -z\partial_z, & L_{-1} = -z^2\partial_z \\ \bar{L}_{-1} = -\partial_{\bar{z}}, & \bar{L}_0 = -\bar{z}\partial_{\bar{z}}, & \bar{L}_{-1} = -\bar{z}^2\partial_{\bar{z}} \end{Bmatrix}$

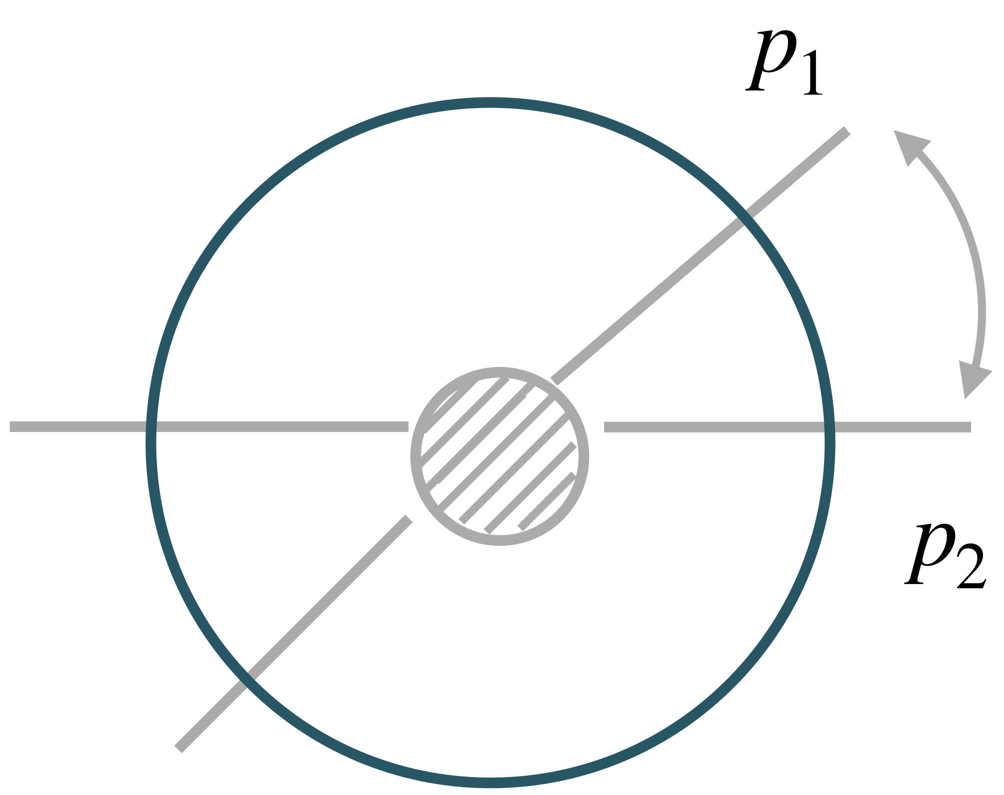
$sl(2)_L \times sl(2)_R$ commutation relations $[L_m, L_n] = (m-n)L_{m+n}, \quad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}$

- Meromorphicity condition $\bar{\partial}\mathcal{O}^s_\Delta(z,\bar{z}) = 0 \implies \mathcal{O}^s_\Delta(z,\bar{z}) = \mathcal{O}_h(z)$ of dimension/weight $\Delta = h = s \in \mathbb{N}/2$

$\implies$ infinity of conserved charges $O_n \equiv \oint dz z^{n+h-1}\mathcal{O}(z)$ $\qquad \mathcal{O}_h(z) = \sum_n \frac{O_n}{z^{n+h}}$

- Global subalgebra = modes that annihilate the vacuum at both 0 and ∞ $\implies 1-h \leq n \leq h-1$

Celestial operator products



Leading OPE in CCFT = collinear factorization in 4D

Gluons in Yang-Mills theory:

Global conformal invariance in (Lorentzian) 2D CCFT $\Longrightarrow$ $\mathcal{O}^a_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^b_{\Delta_2}(z_2,\bar{z}_2) \sim if^{ab}{}_c \dfrac{C(\Delta_1,\Delta_2)}{z_{12}}\mathcal{O}^c_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2) + \ldots$

Subleading soft gluon theorem ~ symmetry action in 2D CCFT

- Invariance of OPE under these transformations $\Longrightarrow$ $C(\Delta_1,\Delta_1) = B(\Delta_1 - 1, \Delta_2 - 1)$, $\quad B(x,y) = \int_0^1 dt t^{x-1}(1-t)^{y-1}$
- Also follows from associativity or Poincare symmetry upon including SL$(2,\mathbb{R})$ descendants

[Pate, A.R., Strominger, Yuan ’19; Himwich, Pate, Singh ’21]

Conformally soft gluon algebras

- Conformally soft gluons of **positive helicity** are operators with $s = 1, \quad \Delta = k \in \mathbb{Z}, \quad k \leq 1 \iff h = \frac{k+1}{2}, \quad \bar{h} = \frac{k-1}{2}$

(Similar story can be told for negative helicity gluons $s = -1$)

- Note: $\bar{h} \leq 0 \Longrightarrow$ finite dimensional $sl(2)_R$ representations: $[\bar{L}_1, \bar{\partial}^m \mathcal{O}(z,\bar{z})] = m(2\bar{h} + m - 1)\bar{\partial}^{m-1}\mathcal{O}(z,\bar{z})$

$$\bar{\partial}^{2-k}\mathcal{O}(z,\bar{z}) = 0 \implies \mathcal{O}^a_k(z,\bar{z}) = \sum_n \frac{O^a_{k,n}(z)}{\bar{z}^{n+\frac{k-1}{2}}}, \qquad \frac{k-1}{2} \leq n \leq \frac{1-k}{2}$$

- Similar to global symmetry algebras with respect to $sl(2)_R$ upon taking light transform!

[Gelfand; Banerjee; Pasterski, Puhm, Trevisani; Guevara, Himwich, Pate, Strominger]

Conformally soft gluon algebras

$$\mathcal{O}^a_k(z,\bar{z}) = \sum_n \frac{O^a_{k,n}(z)}{\bar{z}^{n+\frac{k-1}{2}}}, \qquad \frac{k-1}{2} \leq n \leq \frac{1-k}{2}$$

$k \leq 1, \quad k \in \mathbb{Z}$

$s \equiv 1-k \geq 0$

- **Light transform** $L[\mathcal{O}_{h,\bar{h}}](z,\bar{z}) \equiv \int_{\mathbb{R}} \frac{d\bar{w}}{2\pi i} \frac{\mathcal{O}_{h,\bar{h}}(z,\bar{w})}{(\bar{z}-\bar{w})^{2-2\bar{h}}}$ $\qquad (h,\bar{h}) \to (h, 1-\bar{h})$

- Take $\mathcal{O}_{h,\bar{h}} = \mathcal{O}^a_k(z,\bar{z}) \implies L[\mathcal{O}^a_k](z,\bar{z}) = \sum_n O^a_{k,n}(z) \int \underbrace{\frac{d\bar{w}}{2\pi i} \frac{1}{\bar{w}^{n+\frac{k-1}{2}}} \frac{1}{(\bar{z}-\bar{w})^{2-(k-1)}}}$

$\propto \bar{z}^{\frac{k-1}{2}-n-1}$ upon changing variables $\bar{w} \to \bar{w}\bar{z}$

$\implies$ The light-transformed operator has $\bar{h}_L = \dfrac{3-k}{2}$ and $1-\bar{h}_L \leq n \leq \bar{h}_L - 1$ cf. global modes of $sl(2)_R$ chiral algebra

Celestial symmetry algebras

- The conformally soft gluon modes (as well as their light-transforms) form an algebra

$k \leq 1, \quad k \in \mathbb{Z}$

$s \equiv 1 - k \geq 0$

Strategy to compute it

$$\mathcal{O}^a_{\Delta_1}(z_1, \bar{z}_1)\mathcal{O}^b_{\Delta_2}(z_2, \bar{z}_2) \sim if^{ab}{}_c \frac{B(\Delta_1 - 1, \Delta_2 - 1)}{z_{12}} \mathcal{O}^c_{\Delta_1+\Delta_2-1}(z_2, \bar{z}_2) + \ldots$$

Include $sl(2)_R$ descendants

$$\sim if^{ab}{}_c \frac{1}{z_{12}} \sum_{n=0}^{\infty} B(\Delta_1 - 1 + n, \Delta_2 - 1) \frac{\bar{z}_{12}^n}{n!} \bar{\partial}^n \mathcal{O}^c_{\Delta_1+\Delta_1-1}(z_j, \bar{z}_j) + O(z_{ij}^0)$$

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Include $sl(2)_R$ descendants

$$\sim if^{ab}_{\ \ c}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1 - 1 + n, \Delta_2 - 1)\frac{\bar{z}^n_{12}}{n!}\bar{\partial}^n\mathcal{O}^c_{\Delta_1+\Delta_1-1}(z_j, \bar{z}_j) + O(z^0_{ij})$$

Key observation: only a finite number of terms have poles at $\Delta_1 = 1 - s,\ s \in \mathbb{Z}_+$

$$R^a_s(z_1, \bar{z}_1)\mathcal{O}^b_{\Delta_2}(z_2, \bar{z}_2) \sim if^{ab}_{\ \ c}\frac{1}{z_{12}}\sum_{n=0}^{s}\binom{s+1-\Delta_2-n}{s-n}\frac{\bar{z}^n_{12}}{n!}\bar{\partial}^n\mathcal{O}_{\Delta_2-s}(z_2, \bar{z}_2) + O(z^0_{12})$$

residue of conformal primary gluon at $\Delta_i = 1 - s$

[Guevara, Himwich, Pate, Strominger ’21; Jiang ’21; Guevara ’21]

Celestial symmetry algebras

- The conformally soft gluon modes (as well as their light-transforms) form an algebra

Taking $\mathcal{O}^b_{\Delta_2}$ soft too (ie. take residue at $\Delta_2 = 1 - s'$)

$$R^a_s(z_1,\bar{z}_1)R^b_{s'}(z_2,\bar{z}_2) \sim if^{ab}{}_c \frac{1}{z_{12}} \sum_{n=0}^{s} \binom{s+s'-n}{s'} \frac{\bar{z}_{12}^n}{n!} \bar{\partial}^n R^c_{s+s'}(z_2,\bar{z}_2) + O(z_{12}^0)$$

The $sl(2)_R$ modes $R^{s,a}_n(z) \equiv \oint \frac{d\bar{z}}{2\pi i} \bar{z}^{n+\frac{k-3}{2}} R^a_s(z,\bar{z})$ form an algebra: $[R^{s,a}_n, R^{s',b}_m] = C(s,n;s',m) if^{ab}{}_c R^{s+s',c}_{m+n}$

Much simpler algebra of modes of the (right) light transforms of $R^a_s(z)$: $[S^{s,a}_m, S^{s',b}_n] = if^{ab}{}_c S^{s+s',c}_{m+n}$

S^a_s has weights $(h,\bar{h}) = \left(\frac{2-s}{2}, \frac{2+s}{2}\right)$

[Guevara, Himwich, Pate, Strominger ’21; Jiang ’21; Guevara ’21]

Celestial symmetry algebras

Light transforms of (conformally soft gluons) $R^a_s(z)$: $[S^{s,a}_m, S^{s',b}_n] = if^{ab}{}_c S^{s+s',c}_{m+n}$

Similar story in gravity: light transforms of (conformally) soft gravitons form a w-infinity algebra

$$[w^s_m, w^{s'}_n] = \big(m(s+1) - n(s'+1)\big)\, w^{s+s'}_{m+n}$$

Spacetime interpretation: algebra also follow from hierarchy of differential equations extracted from the Einstein equations

$$\dot{\mathcal{Q}}_s = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2}, \quad s \in \mathbb{Z}_+$$

tower of celestial Ward identities ⟷ tower of soft theorems

$\mathcal{Q}_s$ encode multipole moments of the gravitational field

[Guevara, Himwich, Pate, Strominger ’21; Jiang ’21; Guevara ’21; Freidel, Pranzetti, AR ’21, Compere, Oliveri, Seraj ’22; Freidel, Pranzetti, A.R. ’23]

Chiral algebras from higher dimensions

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13]

- Chiral algebras are special, rigid structures; theories that possess them are vastly constrained

- Unexpected to encounter beyond 2D CFT

- Nevertheless they may appear as ``sectors” of higher-dimensional SCFT, eg. $4D\ \mathcal{N}=2$ SYM!

Consider $\mathbb{R}^2 \subset \mathbb{R}^4$ preserving $sl(2)_L \times sl(2)_R \subset so(6)$ & look for operators that transform trivially under an $sl(2)$ copy

Naive obstruction: Trivial under $sl(2) \implies$ trivial under full $so(6)$

Bypass by looking for $\widehat{sl}(2)$ that is exact with respect to some operator $\mathbb{Q}$ such that $\mathbb{Q}^2=0$ and take cohomology wrt. $\mathbb{Q}$

Chiral algebras from higher dimensions

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees ’13]

Taking cohomology wrt. $\mathbb{Q}$ = twisting

- Schematically $\mathbb{Q} \sim \mathcal{Q} + \mathcal{S} \implies [\mathcal{O}]_{\mathbb{Q}}(z)$ where $[\mathcal{O}]_{\mathbb{Q}} = \left\{ \mathcal{O} \,\middle|\, \{\mathbb{Q}, \mathcal{O}] = 0, \;\; \mathcal{O} \neq \{\mathbb{Q}, \mathcal{O}'] \right\}$

Examples of chiral algebras for $\mathcal{N} = 2$ SCFT:

Free hypermultiplet → free symplectic boson algebra $\quad q_I(z) q_J(w) \sim \dfrac{\varepsilon_{IJ}}{z - w}$

Free vector multiplet → (b, c) system $\quad b(z)\partial c(w) \sim \dfrac{1}{(z-w)^2}, \quad \partial c(z) b(w) \sim -\dfrac{1}{(z-w)^2}$

Top down construction of flat space holography

Celestial gluon and graviton algebras related to 2D chiral algebras arising from SCFT/twisted holography/twistor theory

[Gaiotto, Costello ’18; Adamo, Bu, Costello, Mason, Paquette, Sharma ’21, ’22, ’23]

Goal: Look for 4D asymptotically flat bulk theory dual to celestial 2D chiral algebras

Top down construction of flat space holography

[Costello, Paquette, Sharma ’22, ’23]

Consider $\mathbb{R}^4 \simeq \mathbb{C}^2$ with coordinates $x^\mu \to x = x_\mu \sigma^\mu$

$$u^{\dot\alpha} \equiv x^{1\dot\alpha}, \quad \hat{u}^{\dot\alpha} \equiv x^{2\dot\alpha}$$

Equip $\mathbb{C}^2 \backslash \{0\}$ with metric associated to Kähler form $\omega = \partial\bar\partial K$,

$$K = \|u\|^2 + \log\|u\|^2$$

$$\partial \equiv du^{\dot\alpha}\partial_{u^{\dot\alpha}}, \quad \bar\partial \equiv d\hat{u}^{\dot\alpha}\partial_{\hat{u}^{\dot\alpha}}, \quad \|u\|^2 = u^{\dot\alpha}\hat{u}_{\dot\alpha}$$

- also known as the Burns metric
- self-dual, $R = 0, \;\; R_{\mu\nu} \neq 0,$ asymptotically flat:

$$g_{\mu\nu} = \delta_{\mu\nu} + \mathcal{O}(\|u\|^{-2}), \quad \|u\|^2 \propto \delta_{\mu\nu}x^\mu x^\nu \to \infty$$

- bulk theory: WZW_4 on Burns space

Top down construction of flat space holography

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- bulk theory: WZW_4 on Burns space

$$\widetilde{\mathbb{C}}^2 = \mathbb{C}^2 \text{ with origin replaced by } \mathbb{CP}^1$$

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar\partial K \wedge tr\left(g\partial g^{-1} \wedge g\bar\partial g^{-1}\right) - \frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar\partial K \wedge tr\left(\tilde{g}d\tilde{g}^{-1}\right)^3$$

$$g : \widetilde{\mathbb{C}}^2 \to SO(8), \quad \frac{iN}{2\pi}\int_{\mathbb{CP}^1} \partial\bar\partial K = N \in \mathbb{Z}_+$$

Top down construction of flat space holography

[Costello, Paquette, Sharma '22]

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$$-\frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}K \wedge tr\left(\widetilde{g}d\widetilde{g}^{-1}\right)^3$$

$$\to \frac{N}{8\pi^2}\int_{\mathbb{C}^2} \partial\bar{\partial}K \wedge tr\left(\partial\phi \wedge \bar{\partial}\phi - \frac{1}{3}\phi[\partial\phi, \bar{\partial}\phi]\right) + \mathcal{O}(\phi^4)$$

- look for perturbative solutions by setting $g = e^{\phi}, \quad \widetilde{g} = e^{t\phi}$
- ``asymptotic" states obey the wave equation on Burns space admitting a family of solutions:

$$\phi_a(z, \widetilde{\lambda}) = \sum_{k,\ell} \frac{1}{k!\ell!}\widetilde{\lambda}_1^k\widetilde{\lambda}_2^\ell\phi_a[k,\ell](z)$$

Top down construction of flat space holography

[Costello, Paquette, Sharma '22]

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Holographic dictionary

$$\phi_a[k,\ell](z) \leftrightarrow J_a[k,\ell](z)$$

$J_a[k,\ell](z)$ 2D chiral algebra generators

Top down construction of flat space holography

[Costello, Paquette, Sharma '22]

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar{\partial}K \wedge tr\left(g\partial g^{-1} \wedge g\bar{\partial}g^{-1}\right)$$
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$ij \to a$

4D perturbative gluon amplitudes on Burns space ⟷ 2D OPE of $J_{ij}[\widetilde{\lambda}](z)$

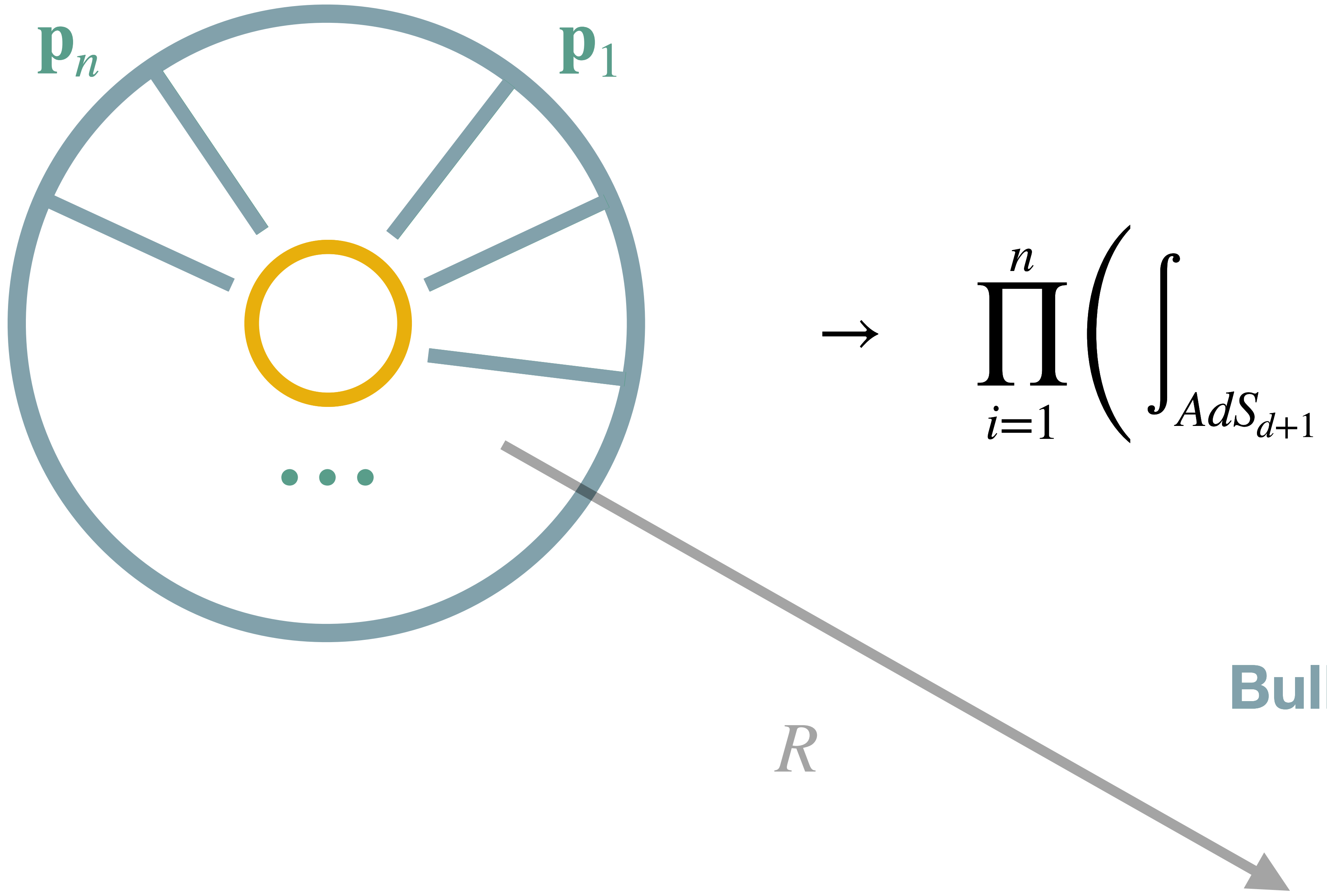
- two-point $\quad A(1,2) = -\frac{N}{z_{12}^2}J_0\left(2\sqrt{\frac{[12]}{z_{12}}}\right) tr\left(T_{a_1}T_{a_2}\right)$ $\quad$ matches identity contribution to OPE
- Matching also established for the three-point amplitudes/correlators

AdS/CFT in the flat space limit

$\sum$ Amplitudes in AdS$_{d+1}$ (Witten diagrams) $\leftrightarrow$ Correlation functions in CFT$_d$

- CFT (Mellin) correlators related to flat space scattering amplitudes at **large AdS radius**

[Polchinski ‘99; Susskind ’99; Giddings ’99; Penedones ’10;…; Hijano, Neuenfeld ’20]

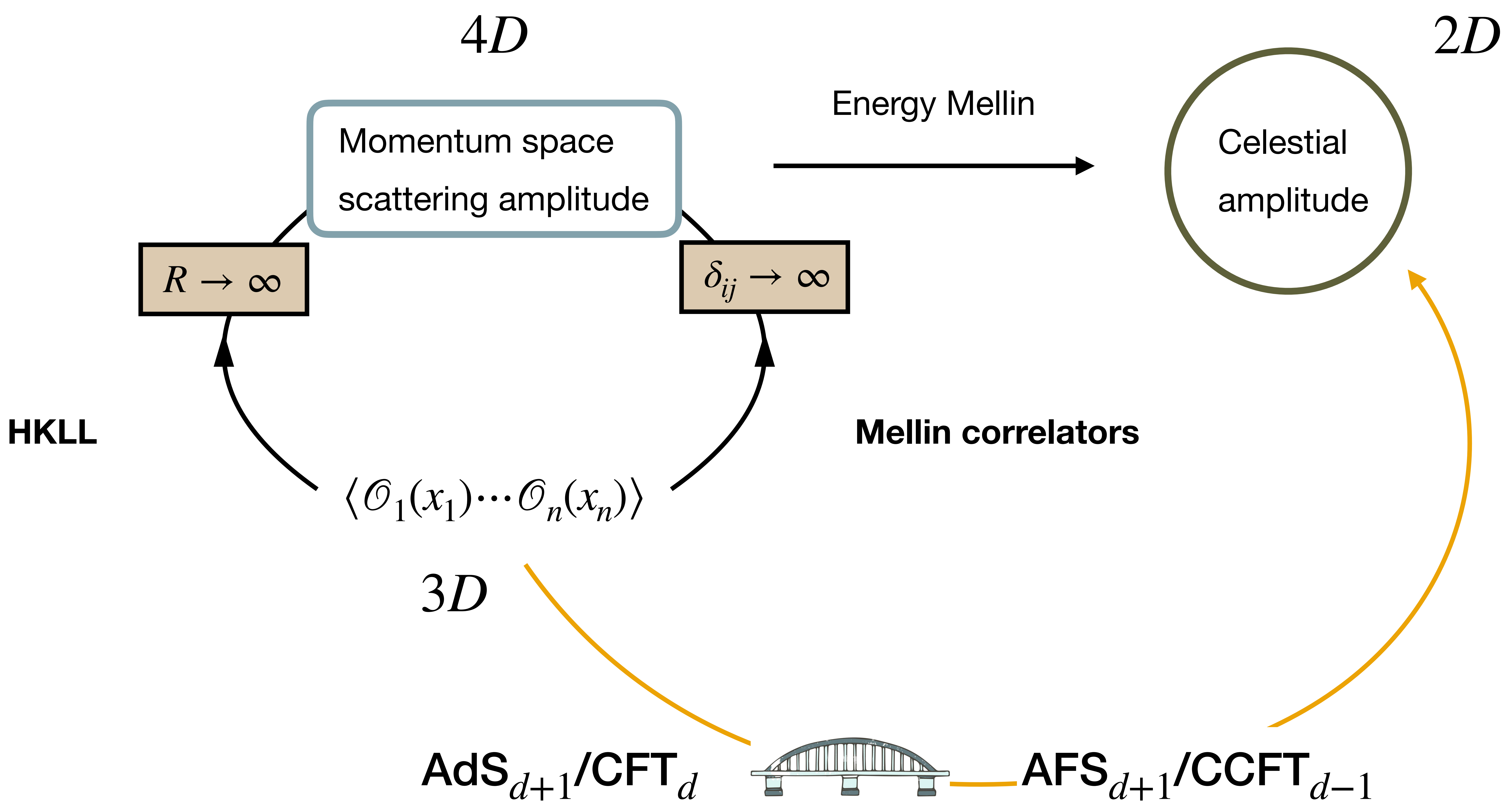


$$\rightarrow \prod_{i=1}^{n}\left(\int_{AdS_{d+1}} d^{d+1}\mathbf{x}_i K_{\Delta_i}(\mathbf{p}_i, \mathbf{x}_i)\right)\mathbf{\Pi}_\Delta(\mathbf{x}_i) \xrightarrow{R\to\infty} \langle p_{\text{out}} | \mathcal{S} | p_{\text{in}}\rangle$$

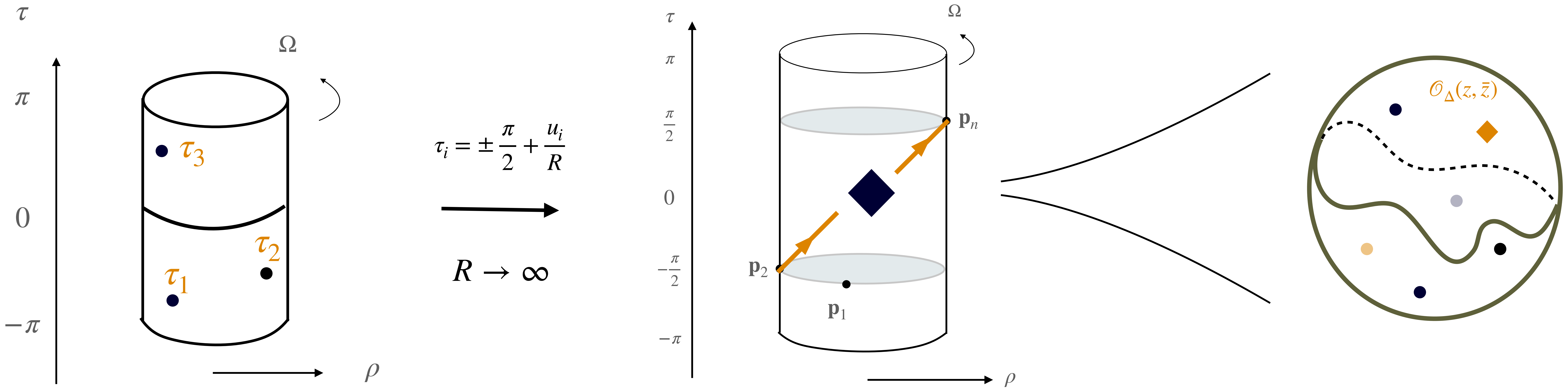
Bulk-to-boundary propagator

Bulk-to-bulk propagators

Celestial amplitudes from AdS/CFT



Celestial amplitudes from AdS/CFT

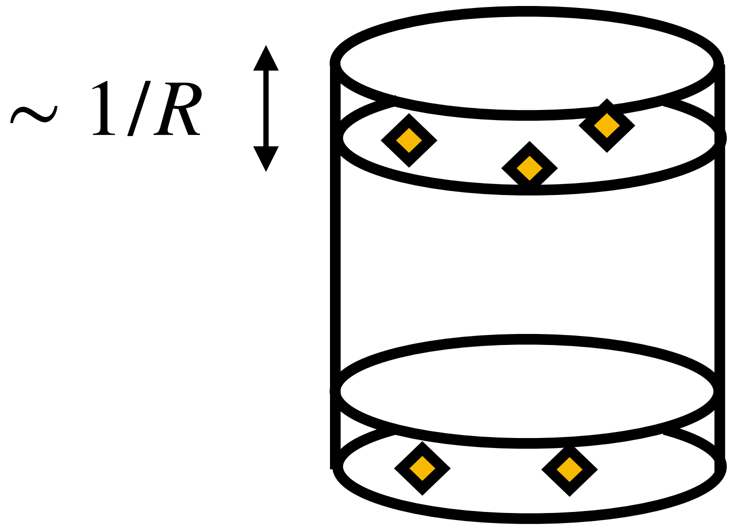


AdS$_4$ boundary observables → 3+1D flat space celestial observables

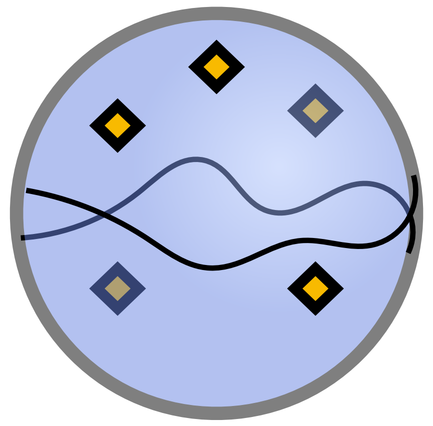
[de Gioia, A.R. '22]

Celestial sector in CFT

Infinity of symmetries from CFT_d



CFT_3 $\xrightarrow{R \to \infty}$



CCFT_2

$\cup$ $\cup$

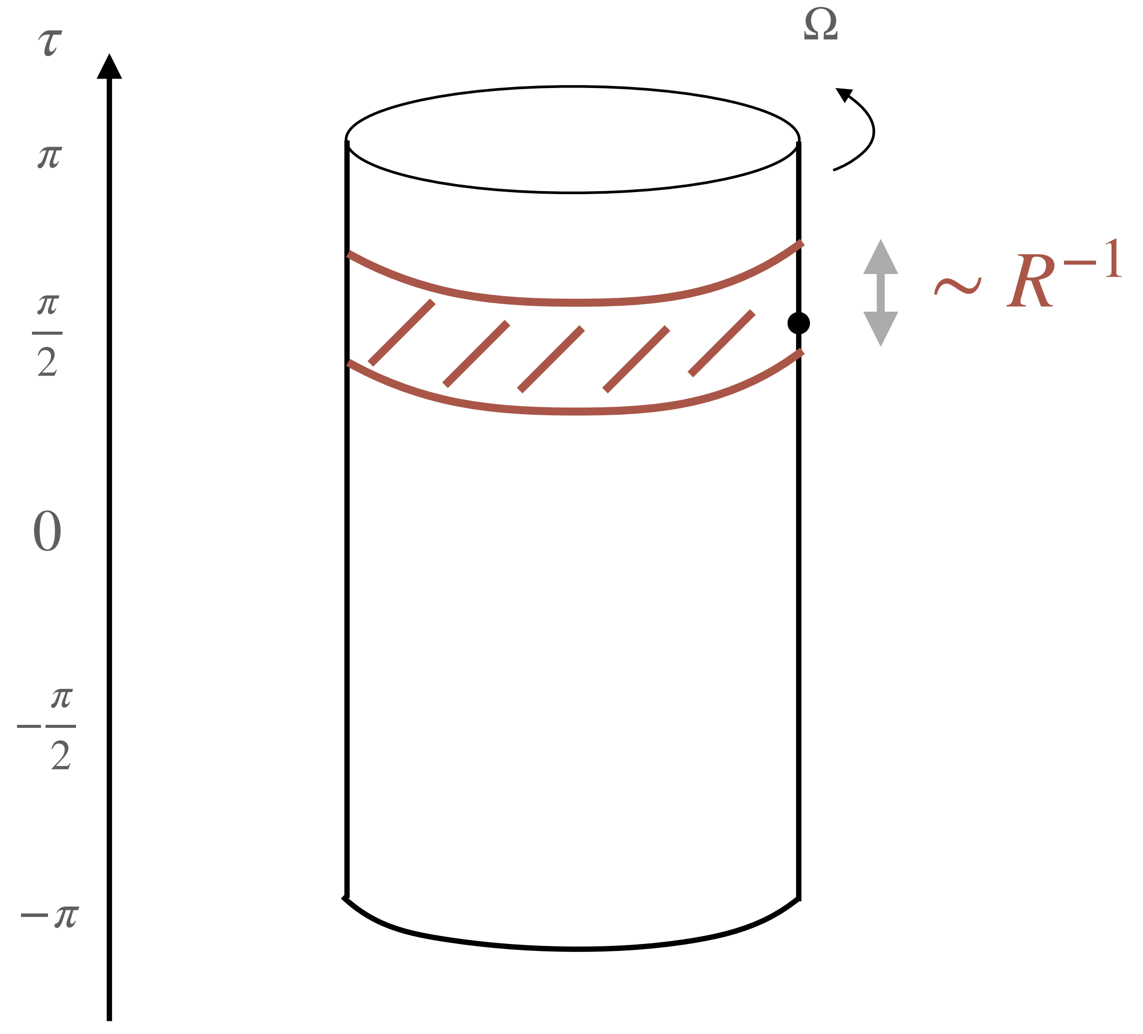
$so(3,2)$ —— ? ——→ $\left\{ \begin{array}{l} \text{Supertraslations} \\ \text{Virasoro} \times \text{Virasoro} \\ \dots\, w_\infty\,? \end{array} \right.$

(Supertraslations, Virasoro × Virasoro) from [shadow of] T_{CFT_3}!

[de Gioia,. A.R. '23]

Symmetries of infinitesimal time intervals in 3D CFT

Analyze conformal symmetries in infinitesimal interval on the Lorentzian cylinder:



$$ds^2 = -d\tau^2 + 2\gamma_{z\bar{z}}dzd\bar{z}, \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2} \quad \xrightarrow{\tau = \tau_0 + \frac{u}{R}} \quad ds^2 = -\boxed{R^{-2}du^2} + 2\gamma_{z\bar{z}}dzd\bar{z}$$

$\rightarrow 0$ as $R \rightarrow \infty$

Conformal Killing vectors in the interval: $\nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu = \frac{2}{d}\nabla \cdot \epsilon(x) g_{\mu\nu}$

As $R \rightarrow \infty$, solutions parameterized by a function f and a vector field Y^A on the sphere:

$$\epsilon^\pm = \left[\mp \frac{iR}{2} F_\pm(u) D \cdot Y(z,\bar{z}) + f(z,\bar{z})\right] \partial_u + F_\pm(u) Y^A(z,\bar{z}) \partial_A$$

[de Gioia, A.R. '23]

BMS_4 algebra in the strip

- For constant f and Y global CKV, $\epsilon^{\pm}$ reorganize into generators of $so(3,2)$ - Lorentz generators $M^{\mu\nu}$ in 5d embedding space

- Inonu-Wigner contraction $\mathscr{P}^{\mu} = \frac{1}{R} M^{4\mu}, \quad \mu = 0,\cdots,3$ with $\mathscr{P}^{\mu}, \quad M_{\mu\nu}$ fixed as $R \to \infty$ yields Poincare algebra

- For $Y^A(z,\bar{z})$ arbitrary CKV, contraction yields

$$\begin{cases} L_Y = iY^A\partial_A + i\frac{u}{2}D\cdot Y\partial_u + O(R^{-2}) \\ T_f \equiv i\epsilon_f = if(z,\bar{z})\partial_u + O(R^{-2}) \end{cases}$$

which generate $\mathfrak{ebms}_4$ $\quad [T_{f_1}, T_{f_2}] = O(R^{-2}), \quad [L_{Y_1}, L_{Y_2}] = iL_{[Y_1,Y_2]} + O(R^{-2}), \quad [T_f, L_Y] = iT_{f'=\frac{1}{2}(D\cdot Y)f - Y(f)} + O(R^{-2})$

$\rightarrow$ asymptotic symmetry algebra of 4D AFS!

CCFT operators from 3D CFT operators

Conformal transformations in the strip ~ celestial symmetries in the ``flat space” limit

$$\delta_\epsilon \mathcal{O}_\Delta(x) = -\left[(\nabla\cdot\epsilon)\frac{\Delta}{3} + \epsilon^\mu \nabla_\mu + \frac{i}{2}\nabla_\mu \epsilon_\nu S^{\mu\nu}\right]\mathcal{O}_\Delta(x) \longrightarrow$$

transformation of CCFT_2 primary operator

$$\mathfrak{h} \equiv \frac{\hat{\Delta}+s}{2}, \quad \bar{\mathfrak{h}} \equiv \frac{\hat{\Delta}-s}{2}, \quad \hat{\Delta} \equiv \Delta + u\partial_u$$

- Diagonalize weights via $\widehat{\mathcal{O}}_\Delta(z,\bar{z};\Delta_0) \equiv N(\Delta,\Delta_0)\int_{-\infty}^{\infty} du\ u^{-\Delta_0}\mathcal{O}_\Delta(u,z,\bar{z})$

- Same as transform relating Carrollian and celestial conformal field theories

[Donnay, Fiorucci, Herfray, Ruzziconi ‘22]

- Shadow stress tensor Ward identity in 3D CFT lead to leading and subleading conformally soft graviton theorems in 2D CCFT

[Kapec, Mitra ’18; de Gioia, A.R. ’23]

Flat space holography summary

- top down realization of flat space holography
- some of the novelties recovered in flat limit of (bottom up) AdS/CFT
- high- and low-energy behavior of 4D scattering amplitudes reflected in analytic structure of celestial amplitudes in Δ
- from OPE to conformally soft currents
- low point amplitudes are distributions
- singularities disappear in special backgrounds

Boost eigenbasis $|\Delta, z, \bar{z}\rangle \equiv \int_0^\infty d\omega \omega^{\Delta-1} |\omega, z, \bar{z}\rangle \to \int_0^\infty d\omega \omega^{\Delta-1} |\lambda\omega, z, \bar{z}\rangle = \lambda^{-\Delta} |\Delta, z, \bar{z}\rangle$

- Bulk/geometric interpretation of in flat space limit
- AdS boundary conditions - TTbar deformations?
- Central extensions
- Entanglement entropy; black holes?
- Spectrum of CCFT; conformal block decompositions
- Connections to all Λ via AdS/dS slicing
- Top down constructions

$\ldots$

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. . .

Connections to scattering amplitudes

- OPE from EFT
- Higher-derivative & loop corrections to OPE

[He, Jiang, Ren, Spradlin, Taylor, Volovich, Zhu….]

- Double copy constructions [Casali, Puhm, Sharma,…]
- IR divergences + Dan’s talk
- Self-dual amplitudes and black holes
- Discrete basis

Asymptotic symmetries and Carrollian FT

String theory, BFFS, …

[Adamo, Ball, Cotler, Crawley, Donnay, Fiorucci, Guevara, He, Kapec, Mason, Mitra, Narayana, Ruzziconi, Salzer, Strominger, Sharma, Tropper, Wang…]

Thank you!