A Field Theory View on
Spin-Magnitude Change in Orbital Evolution

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Based on work with

Zvi Bern, Dimitrios Kosmopoulos, Andres Luna,
Trevor Scheopner, Fei Teng, Justin Vines

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The detection of gravitational waves opened a new window on our Universe

- Probe aspects of dynamics in General Relativity in strong field regime
- Probe properties of black holes
- Probe/discriminate extensions of General Relativity
- Probe certain astrophysical environments, including dark matter
- Probe properties of (ultra-) dense nuclear matter
- Probe BH origin, formation mechanisms, population, etc
- …

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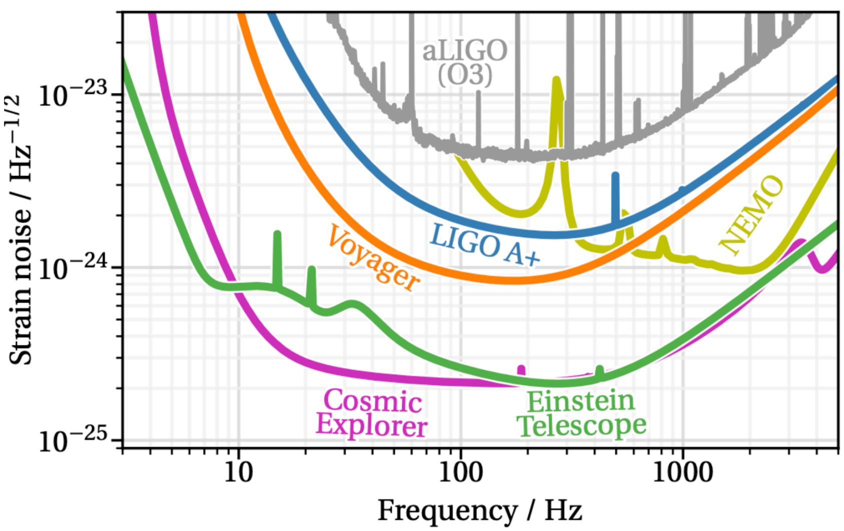
and gave new impetus towards new theoretical tools and structures

- Search for new symmetries
- Exploration of the structure of perturbation theory
- Resummation of perturbation theory
- Analytic continuations
- …

Future ground-based observatories

https://cosmicexplorer.org/sensitivity.html

Advanced LIGO, Einstein Telescope, Cosmic Explorer

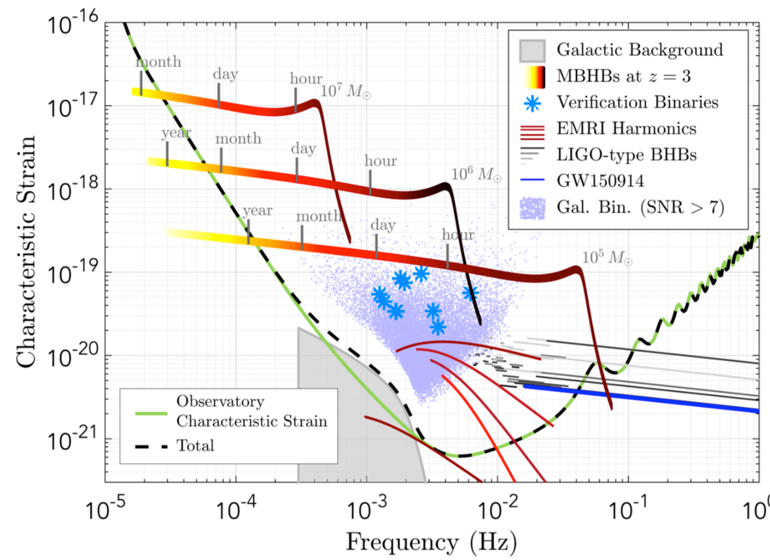


Sensitivity improvement up to 100 depending on parameters

Future space-based observatories

From Danzmann et al 1702.00786; LISA proposal

LISA (2035+), TianQin (2035+)

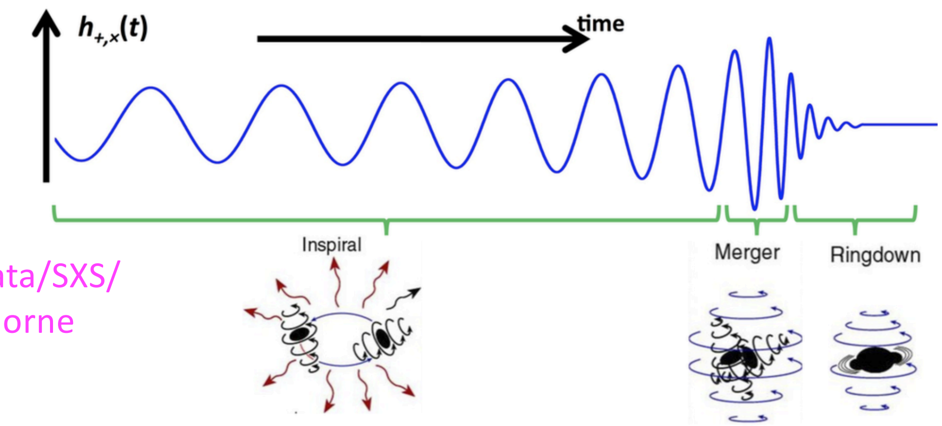


long accurate waveforms are required buildup of theoretical error over long-time evolution must be avoided

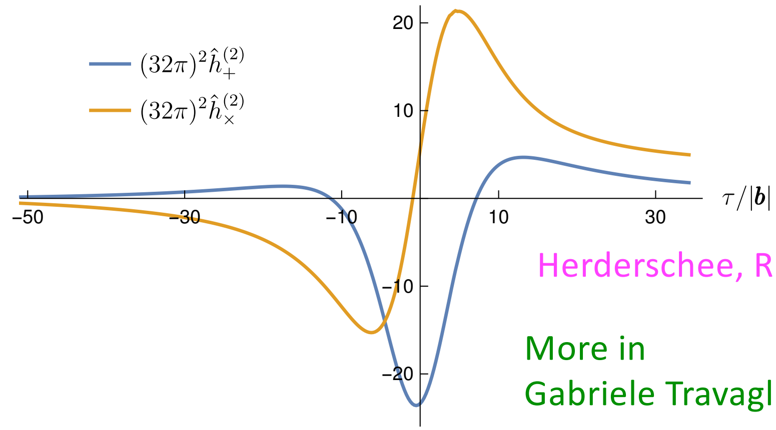
Interplay of the various available approaches will be important to maximize theoretical output

More in Alessandra Buonanno's talk

Anatomy of an idealized binary merger



Favata/SXS/
K.Thorne



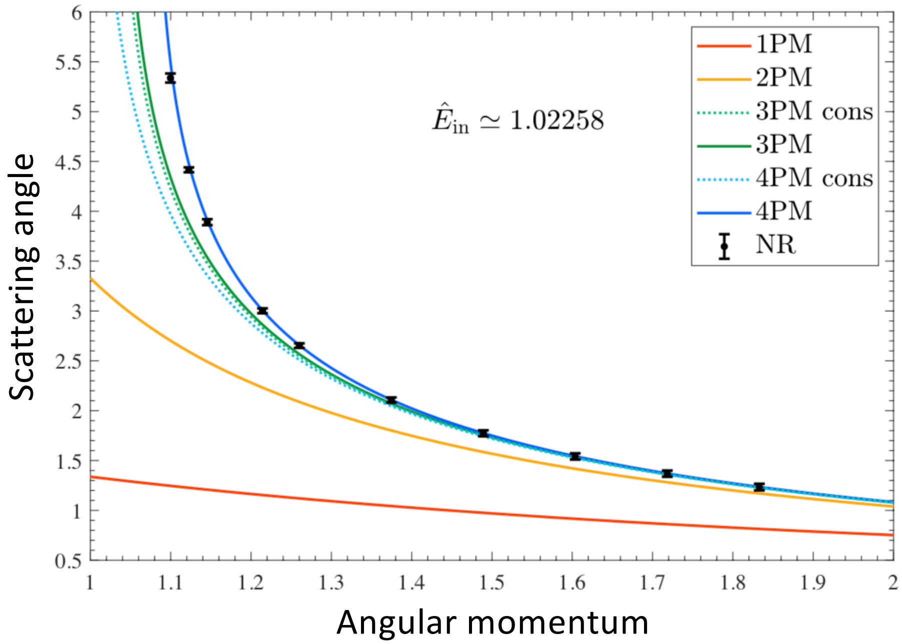
Herderschee, RR, Teng

More in
Gabriele Travaglini's talk

- (Numerical) relativity – "the truth", but expensive
- Post-Newtonian expansion (weak field, nonrelativistic): $\boldsymbol{v}^2 \sim \frac{GM}{|\boldsymbol{r}|} \ll 1$
- Post-Minkowskian expansion (weak-field, relativistic): $\frac{GM}{|\boldsymbol{r}|} \ll \boldsymbol{v}^2 \sim 1$
- Small mass ratio expansion/gravitational self-force $\boldsymbol{v}^2 \sim GM/|\boldsymbol{r}| \sim 1$ See Adam Pound's talk
- Ringdown: black hole perturbation theory

→ Effective one-body theory (EOB) and phenomenological models consolidate available results

Damour, Retegno



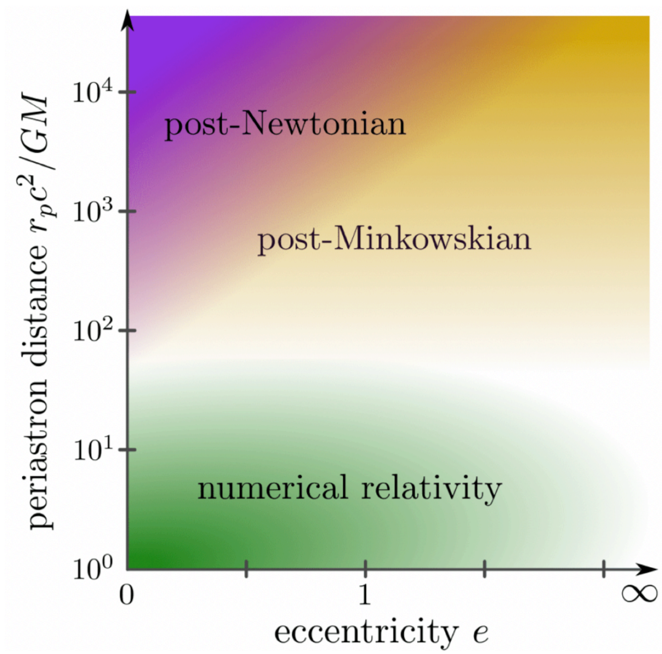
4PM conservative: Bern, Parra-Martinez, RR, Ruf, Shen, Solon, Zeng

4PM dissipative: Manohar, Ridgway, Shen; Dlapa, Kälin, Neef, Porto

Improved by EOB resummation

Numerical: Damour, Guercilena, Hinder, Hopper, Nagar, Rezzolla

Khalil, Buonanno, Steinhoff, Vines



Post-Minkowskian expansion is the relevant expansion for certain eccentric bounded and for hyperbolic motion

Importance of 5PM contributions in regions of parameter space Rettegno, Pratten, Thomas, Schmidt, Damour

Where do amplitudes and amplitude methods fit in?

1. An efficient way to integrate out off-shell gravitons — Cheung, Rothstein, Solon; Bern, Cheung, RR, Shen, Solon, Zeng

GR + (spinning) matter $\xrightarrow{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}^{\text{off shell}} + h_{\mu\nu}^{\text{radiation}}}$ $S_{eff} = S_{eff}(\text{matter}, h_{\mu\nu}^{\text{radiation}})$

Subtlety: radiation can also be off shell

2. An efficient path to scattering observables — Kosower, Maybee, O'Connell; + Cristofoli

Including scattering waveform, see Travaglini's talk

3. Provide technical backing for worldline approaches — Kälin, Porto; Mogull, Plefka, Steinhoff;...

3.5 Connection between 2. & worldline methods in Hansen's talk — Damgaard, Hansen, Planté, Vanhove

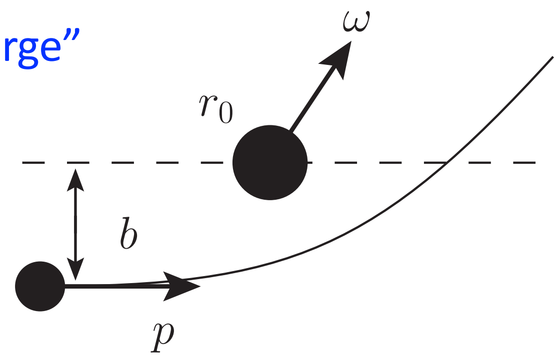
An important open problem: bound dynamics from unbound dynamics

Boundary-to-bound (B2B) (covers local-in-time effects) — Kälin, Porto

General? (i.e. including nonlocal-in-time effects)

(Semi) classical limit – correspondence pp: "all conserved charges are large"

momenta, angular momenta, electric/color charges, etc



- Momentum transfer q much smaller that external momenta

- Spin is $S \sim \mathcal{O}(1/q)$

Structure of two-body effective action: $\mathcal{F}_{\boldsymbol{r}}\Big[V(\boldsymbol{p},\boldsymbol{r},\boldsymbol{S}),\boldsymbol{q}\Big] \sim \frac{c_{ijk}(\boldsymbol{p})}{|\boldsymbol{q}|^3}\,(Gm|\boldsymbol{q}|)^i\left(\frac{\boldsymbol{q}\cdot\boldsymbol{S}}{m}\right)^j(R\,|\boldsymbol{q}|)^k$

A field theory approach to classical dynamics requires higher-spin particles/fields

- What are the fields (or field configurations) that describe a general spinning body?
- What are its/their interactions?
- HS field theories are interpreted as classical FTs; sequence of finite-spin calculations may be used to identify the leading large-spin dependence

We will hear a number of talks reviewing and casting new light on various aspects of classical gravity: Ruf, Kalin, Mogull, Heissenberg, van de Meent, Buonanno, Vines, Cangemi, Travaglini, Cristofoli, Pound

This said, gravity is complicated: nonlinearities, high-derivative couplings, etc

- Interesting effects (perhaps also the kind we did not yet foresee) appear at high orders
- "it ain't over till it's over": feasibility of higher order calculations is always on one's mind
- High-order calculations in both GR/standard techniques and with amplitudes-based techniques are complicated $\longrightarrow$ a direct trial-and-error approach to merging information is cumbersome

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Simpler/toy models capturing relevant aspects of gravity are important testing grounds

- Scalar model for gravity ⟷ compare/combine PM & SF data (Leor Barack's talk last week) — Barack, Bern, Herrmann, Long, Parra-Martinez, RR, Shen, Solon, Teng, Zeng
- Scalar QED through 5PL ⟷ feasibility of 5PM calculations (may also be relevant for aspects of heavy ion collisions) — Bern, Herrmann, RR, Ruf, Smirnov, Smirnov. See Michael Ruf's talk
- Earlier results on scalar QED — Bern, Gatica, Herrmann, Luna, Zeng; Saketh, Vines, Steinhoff, Buonanno; Kosower, Maybee, O'Connell; +Cristofoli; Elkhidir, O'Connell, Sergola, Vazquez-Holm;…
- $\mathcal{N}$=8 supergravity ⟷ effects of radiation gravitons — Di Vecchia, Heissenberg, Russo, Veneziano, …

As for the next 30 min or so --

Use QED w/ higher-spin fields to explore the classical dynamics of spinning compact bodies

The puzzle:

- HS + gravity: “extra” Wilson coefficients compared to WL — Bern, Kosmopoulos, Luna, RR, Teng

$$\mathcal{L} = \frac{1}{2}(-1)^s \phi_s(-\nabla^2 - m^2)\phi_s + \frac{H_2}{8} R_{abcd}\phi_s M^{ab} M^{cd}\phi_s - \frac{C_{ES^2}}{2m} R_{af_1bf_2}\nabla^a\phi_s \mathbb{S}^{(f_1}\mathbb{S}^{f_2)}\nabla^b\phi_s \ldots \qquad \mathbb{S}^\mu \equiv \frac{-i}{2m}\varepsilon^{\mu\nu\rho\sigma}M_{\rho\sigma}\nabla_\nu$$

- Is there a worldline theory?

- In QED + massive spin-1 field there are no extra Wilson coefficients — Kim, Steinhoff

Same from BCFW — Haddad

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More Wilson coefficients ⟷ more possible degrees of freedom

Consider a Hamiltonian that depends on the rest-frame spin vector

$$H = H(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{S}) \xrightarrow{\{S^i, S^j\}_{\rm PB} \sim \epsilon^{ijk}S^k} \dot{\boldsymbol{S}} = \frac{\partial H}{\partial \boldsymbol{S}} \times \boldsymbol{S}$$

The magnitude of the spin vector is conserved

$$\frac{d}{dt}\boldsymbol{S}^2 = 2\boldsymbol{S}\cdot\dot{\boldsymbol{S}} = 2\boldsymbol{S}\cdot\left(\frac{\partial H}{\partial \boldsymbol{S}} \times \boldsymbol{S}\right) = 0$$

Emerging physical picture:

Bern, Kosmopoulos, Luna, RR, Scheopner, Vines, Teng

Rotational invariance implies conservation of total angular momentum

$$J = S_1 + S_2 + L + J_{\rm field}$$

Physically, dynamical rearrangement between spin and orbital angular momentum is allowed; extra Wilson coefficients capture this effect

Dynamical change of the magnitude of $\boldsymbol{S}$ requires $H = H(\ldots, \boldsymbol{K})$ such that $\{S^i, K^j\}_{\rm PB} \neq 0$

Hamiltonian: more operators $\longrightarrow$ bigger Hilbert space $\longrightarrow$ more d.o.f.-s $\longrightarrow$ more QFT d.o.f.-s

QFT options: 1. relax the SSC $\longrightarrow$ 6 d.o.f.-s instead of the standard 3
2. more QFT fields $\longrightarrow$ any # of d.o.f. but limited # in classical limit } not unrelated
3. others?

Amplitudes: $\mathcal{O}(q)$ change in spin; resumed by equations of motion to a finite spin kick ΔS and ΔS^2
close analogy with momentum transfer q vs. impulse Δp

Three (classical, almost-free) field theories: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm HS}$

$$D_\mu\phi_s = \partial_\mu\phi_s - iQA_\mu\phi_s$$
$$D_\mu\bar\phi_s = \partial_\mu\bar\phi_s + iQA_\mu\bar\phi_s$$

FT1: $\mathcal{L}_{\rm HS} = \mathcal{L}_{\rm min} \equiv -(-1)^s\phi_s(D^2+m^2)\bar\phi_s$

FT2: $\mathcal{L}_{\rm HS} = \mathcal{L}_s \equiv -(-1)^s\left[\phi_s(D^2+m^2)\bar\phi_s + s(D\phi_s)(D\bar\phi_s) + \text{auxiliary lower-spin fields}\right]$

Singh, Hagen

e.g.
$$\begin{aligned}\mathcal{L}_{s=3} =\ & \phi^{\mu_1\mu_2\mu_3}(D^2+m^2)\bar\phi_{\mu_1\mu_2\mu_3} + 3(D_\mu\phi^{\mu\mu_2\mu_3})(D^\nu\bar\phi_{\nu\mu_2\mu_3})\\ & - 3\phi_\mu{}^{\mu\mu_3}(D^2+m^2)\bar\phi^\nu{}_{\nu\mu_3} + 3\phi_\mu{}^{\mu\mu_3}D^\rho D^\lambda\bar\phi_{\rho\lambda\mu_3} + 3\bar\phi^\mu{}_{\mu\mu_3}D_\rho D_\lambda\phi^{\rho\lambda\mu_3}\\ & + \frac{3}{2}(D_\mu\phi^{\mu\rho}{}_\rho)(D_\nu\bar\phi^{\nu\lambda}{}_\lambda) + 2\varphi(D^2+4m^2)\bar\varphi + m(\varphi D_\mu\bar\phi^{\mu\lambda}{}_\lambda + \bar\varphi D_\mu\phi^{\mu\lambda}{}_\lambda)\end{aligned}$$

Chang

FT3: $\mathcal{L}_{\rm HS} = \mathcal{L}_s + \mathcal{L}_{s-1}(+\text{lower spins})$

The fields:

FT1: (l,r) reps of Lorentz group $(l+r=2s)$ $\quad \phi_s \equiv \phi^{\dot\beta_1\dot\beta_2\ldots\dot\beta_r}_{\alpha_1\alpha_2\ldots\alpha_l}$ $\quad \bar\phi_s \equiv \bar\phi^{\alpha_1\alpha_2\ldots\alpha_l}_{\dot\beta_1\dot\beta_2\ldots\dot\beta_r}$

FT2 & FT3: (s,s) reps of Lorentz group $\quad \phi_s \equiv \phi^{\dot\beta_1\dot\beta_2\ldots\dot\beta_s}_{\alpha_1\alpha_2\ldots\alpha_s} \propto \phi^{(\mu_1\mu_2\ldots\mu_s)}(\sigma_{\mu_1})_{(\alpha_1}{}^{(\dot\beta_1}\ldots(\sigma_{\mu_s})_{\alpha_s)}{}^{\dot\beta_s)}$

Classical asymptotic states: coherent states to minimize dispersion of observables

For (l,r) reps of Lorentz group:

$$\mathcal{E}(p)_{\alpha(l)\dot\beta(r)} = \xi(p)_{\alpha_1}\ldots\xi(p)_{\alpha_l}\chi(p)_{\dot\beta_1}\ldots\chi(p)_{\dot\beta_r}$$

$$\bar{\mathcal{E}}(p)^{\alpha(l)\dot\beta(r)} = \tilde\xi(p)^{\alpha_1}\ldots\tilde\xi(p)^{\alpha_l}\tilde\chi(p)^{\dot\beta_1}\ldots\tilde\chi(p)^{\dot\beta_r}$$

factorization is convenient but not necessary

Momentum dependence via boost…

$$\xi(p)_\alpha = \exp(i\eta\hat p^k\hat K^k_L)_\alpha{}^\beta\xi_{0\beta} \quad \cdots \quad \tilde\chi(p)^{\dot\alpha} = \exp(i\eta\hat p^k\hat K^k_R)^{\dot\alpha}{}_{\dot\beta}\tilde\chi_0^{\dot\beta}$$

… of rest-frame coherent-state spinors:

$$\xi_{0\alpha} = \exp(z_L\hat N^L_+ - z_L^*\hat N^L_-)_\alpha{}^\beta\xi^+_{0\beta} \quad \cdots \quad \tilde\chi_0^{\dot\alpha} = \exp(z_R\hat N^R_+ - z_R^*\hat N^R_-)^{\dot\alpha}{}_{\dot\beta}\chi_0^{-,\dot\beta}$$

$$z_{L,R} \equiv -(\theta_{L,R}/2)e^{-i\phi_{L,R}}$$

$$n^i_L = \xi_0\sigma^i\tilde\xi_0 \qquad \boldsymbol{n}_L = (\sin\theta_L\cos\phi_L, \sin\theta_L\sin\phi_L, \cos\theta_L)$$

$$n^i_R = \chi_0\sigma^i\tilde\chi_0 \qquad \boldsymbol{n}_R = (\sin\theta_R\cos\phi_R, \sin\theta_R\sin\phi_R, \cos\theta_R)$$

Rest-frame Lorentz sandwiches vs. covariant SSC:

$$\mathcal{E}_0\cdot\hat{\boldsymbol{S}}\cdot\bar{\mathcal{E}}_0 = \frac{1}{2}(l\,\boldsymbol{n}_L + r\,\boldsymbol{n}_R) \equiv \boldsymbol{S} \qquad \mathcal{E}_0\cdot\hat{\boldsymbol{K}}\cdot\bar{\mathcal{E}}_0 = \frac{i}{2}(l\,\boldsymbol{n}_L - r\,\boldsymbol{n}_R) \equiv i\boldsymbol{K}$$

$$\mathcal{E}_0\cdot M^{\mu\nu}\cdot\bar{\mathcal{E}}_0 = S_0^{\mu\nu} + iK_0^{\mu\nu} \equiv \mathcal{S}_0^{\mu\nu}$$

$$p_{0\mu}S_0^{\mu\nu} = 0 \qquad \frac{1}{m}(p_0^\mu K_0^\nu - p_0^\nu K_0^\mu)$$

Classical asymptotic states: coherent states to minimize dispersion of observables

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$$\bar{\mathcal{E}}(p)^{\alpha(l)\dot\beta(r)} = \tilde\xi(p)^{\alpha_1}\ldots\tilde\xi(p)^{\alpha_l}\tilde\chi(p)^{\dot\beta_1}\ldots\tilde\chi(p)^{\dot\beta_r}$$

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$$\mathcal{E}_0\cdot M^{\mu\nu}\cdot\bar{\mathcal{E}}_0 = S_0^{\mu\nu} + iK_0^{\mu\nu} \equiv \mathcal{S}_0^{\mu\nu}$$

$$p_{0\mu}\mathcal{E}_0^{\mu\mu_2\ldots\mu_s} = 0 \qquad 0 = (p_{0\mu}\sigma^\mu)^\alpha{}_{\dot\beta}(\mathcal{E}_0)^{\dot\beta\dot\beta_2\ldots\dot\beta_s}_{\alpha\alpha_2\ldots\alpha_s} \propto \xi_{0\alpha}\epsilon^\alpha{}_{\dot\alpha}\chi_0^{\dot\alpha}$$

$$\xi_{0\alpha} = \chi_0^{\dot\alpha} \text{ as column vectors} \quad\Longrightarrow\quad \boldsymbol{n}_L = \boldsymbol{n}_R \longrightarrow \boldsymbol{K} = 0$$

Transverse polarization tensors lead to spin tensors obeying covariant SSC: $\mathcal{S}^{\mu\nu} = S^{\mu\nu}$

Momentum-dependent Lorentz sandwiches:

$$\mathcal{E}_1\cdot\{M^{\mu_1\nu_1},\ldots,M^{\mu_n\nu_n}\}\cdot\bar{\mathcal{E}}_2 = \mathcal{S}(p_1)^{\mu_1\nu_1}\ldots\mathcal{S}(p_n)^{\mu_n\nu_n}\mathcal{E}_1\cdot\bar{\mathcal{E}}_2 + \mathcal{O}(q^{1-n})$$

$$(-1)^r\mathcal{E}_1\cdot\bar{\mathcal{E}}_2 = \exp\left[-\frac{1}{m}\boldsymbol{q}\cdot\boldsymbol{K}\right]\exp\left[-i\frac{\epsilon_{rsk}u_1^r q^s S^k}{m(1+\sqrt{1+\boldsymbol{u}_1^2})} + \mathcal{O}(q^2)\right] + \mathcal{O}(q)$$

Non-minimal interactions:

QED analogs of the "standard" GR Wilson coefficients

Porto, Rothstein; … ; generalized to all orders Levi, Steinhoff

… of a single fields:

$$(-1)^s\mathcal{L}_{\text{non-min}} = C_1 F_{\mu\nu}\phi_s M^{\mu\nu}\bar\phi_s + \frac{D_1}{m^2}F_{\mu\nu}(D_\rho\phi_s M^{\rho\mu}D^\nu\bar\phi_s + \text{cc})$$
$$- \frac{iC_2}{2m^2}\partial_{(\mu}F_{\nu)\rho}(D^\rho\phi_s\mathbb{S}^\mu\mathbb{S}^\nu\bar\phi_s - \text{cc}) - \frac{iD_2}{2m^2}\partial_\mu F_{\nu\rho}(D_\alpha\phi_s M^{\alpha\mu}M^{\nu\rho}\bar\phi_s - \text{cc})$$

QED analog of the "extra" Wilson coefficients

… of two fields of different spins:

$$\mathbb{S}^\mu \equiv \frac{-i}{2m}\varepsilon^{\mu\nu\rho\sigma}M_{\rho\sigma}D_\nu$$

$$\mathcal{L}^{s,s-1}_{\text{non-min}} = C_1 F_{\mu\nu}\phi_s M^{\mu\nu}\bar\phi_s - \frac{2i\widetilde{C}_1\sqrt{s}}{m}F_{\mu\nu}\Big[(\phi_s)^\mu{}_{\alpha_2\ldots\alpha_s}D^\nu\bar\phi_{s-1}^{\alpha_2\ldots\alpha_s} - \text{cc}\Big]$$
$$- \frac{iC_2}{2m^2}\partial_{(\mu}F_{\nu)\rho}(D^\rho\phi_s\mathbb{S}^\mu\mathbb{S}^\nu\bar\phi_s - \text{cc}) - \frac{2i\widetilde{C}_2\sqrt{s}}{m}F_{\mu\nu}\Big[(\phi_s)^\mu{}_{\alpha_2\ldots\alpha_s}D^{\alpha_2}\bar\phi_{s-1}^{\nu\alpha_3\ldots\alpha_s} - \text{cc}\Big]$$

Name	Lagrangian	External state
FT1	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{min}} + \mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\text{EM}} + \mathcal{L}_s + \mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{s,s-1} + \mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

- "Extra" Wilson coefficients appear at $\mathcal{O}(S^1)$
- "Tidal" operators appear at $\mathcal{O}(S^2)$

$$(-1)^s\mathcal{L}_{F^2} = \frac{E_1}{m^2}F_{\mu\nu}F_{\rho\sigma}\phi_s M^{\mu\nu}M^{\rho\sigma}\bar\phi_s + \frac{E_2}{m^2}F_{\mu\nu}F_\rho{}^\mu\phi_s M^{\nu\lambda}M_\lambda{}^\rho\bar\phi_s$$
$$+ \frac{E_3}{m^4}F_{\mu\nu}F_{\rho\sigma}D^\mu\phi_s M^{\nu\lambda}M_\lambda{}^\rho D^\sigma\bar\phi_s + \mathcal{O}(M^3)$$

A digression – standard scattering theory and amplitudes in FT3

$$\mathcal{A}(k_1^{\text{in}}, \dots k_{m+1}^{\text{out}}, \dots) = {}_{\text{out}}\langle p_{m+1} \dots | p_1 \dots \rangle_{\text{in}} \qquad |p, \lambda\rangle = a_\lambda^\dagger(p)|0\rangle$$

little-group label suppressed in $\mathcal{A}$

LSZ reduction: quantum amplitudes ⟷ correlation functions of $(p_i^2 - m^2)\mathcal{E}_\lambda(p_i) \cdot \tilde{\phi}_*(p_i)$

Classically, there is no penalty to consider, e.g. $(p_i^2 - m^2) \sum_j A_j \, \mathcal{E}_j(p_i) \cdot \tilde{\phi}_j(p_i)$

A_j: constrained by normalization

for FT3: $(p_i^2 - m^2)(A_s \, \mathcal{E}_s(p_i) \cdot \tilde{\phi}_s(p_i) + A_{s-1} \, \mathcal{E}_{s-1}(p_i) \cdot \tilde{\phi}_{s-1}(p_i))$

we will consider:

1. spin-*s* $\quad A_s = 1 \, , \quad A_{s-1} = 0$
2. indefinite $\quad A_s = 1/\sqrt{2} \, , \quad A_{s-1} = 1/\sqrt{2}$

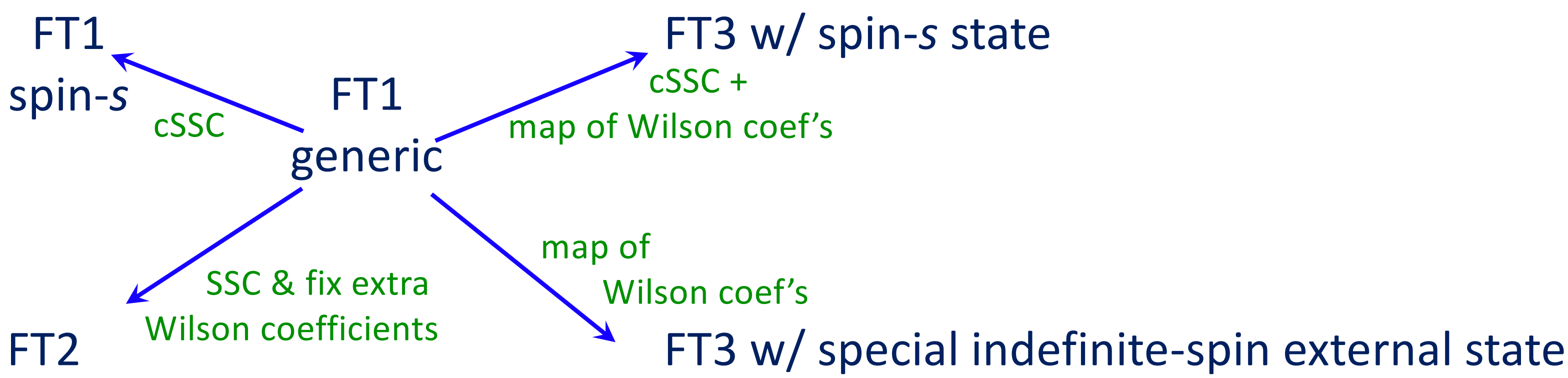
Why these field theories and not others?

Name	Lagrangian	External state
FT1	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{min}} + \mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{s} + \mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{s,s-1} + \mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

- Explore the effects of extra states (FT2 vs. others)

- Explore consequences of the unphysical nature of the extra states of FT1 (FT1 vs. FT3)

- Explore consequences of choice of external state (various choices of states in FT3)



Will find that the extra Wilson coeff's characterize change in $|S|$ during dynamical evolution

FT1 w/ generic external state is related with a worldline theory with no SSC See Justin Vines's talk

Amplitudes and relations en route to the Hamiltonian

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FT3	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{s,s-1} + \mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

Three-point amplitudes:

$$\mathcal{A}_3^{\text{FT1}}\Big|_{S^1}^{\text{generic}} = (-1)^s \mathcal{E}_1 \cdot \bar{\mathcal{E}}_2 \left[2\varepsilon_3 \cdot p_1 - 2iC_1 \mathcal{S}_{\mu\nu} q_3^\mu \varepsilon_3^\nu - \frac{2iD_1}{m^2} \varepsilon_3 \cdot p_1 \mathcal{S}_{\mu\nu} p_1^\mu q_3^\nu\right]$$

The other FTs:

$$\mathcal{A}_3^{\text{FT1}}\Big|_{S^1}^{\text{spin-}s} = \mathcal{A}_3^{\text{FT1}}\Big|_{S^1}^{\text{generic}}\Big|_{\mathcal{S}\to S} = \mathcal{A}_3^{\text{FT2}}\Big|_{S^1} = \mathcal{A}_3^{\text{FT3}}\Big|_{S^1}^{\text{spin-}s}$$

$$\mathcal{S}_{\mu\nu} = S_{\mu\nu} + iK_{\mu\nu}$$

FT3 with special indef.-spin external states:

$$\mathcal{A}_3^{\text{FT3}}\Big|_{S^1}^{\text{indef.}} = 2(-1)^s \mathcal{E}_1 \cdot \bar{\mathcal{E}}_2 \left[\varepsilon_3 \cdot p_1 - iC_1 S_{\mu\nu} q_3^\mu \varepsilon_3^\nu + (i\tilde{C}_1 - 1)\varepsilon_3 \cdot p_1 \frac{q \cdot K}{m}\right]$$

$$\mathcal{A}_3^{\text{FT3}}\Big|_{S^1}^{\text{indef.}}\Big|_{i\tilde{C}_1 = 1 - C_1 + D_1} = \mathcal{A}_3^{\text{FT1}}\Big|_{S^1}^{\text{generic}}$$

i reflects the unphysical nature of spin-(s-1) state in FT1

Extra Wilson coefficients $\not\leftrightarrow$ unphysical nature of lower-spin states in FT1

Compton amplitudes in the three field theories:

Name	Lagrangian	External state
FT1	$\mathcal{L}_{\rm EM} + \mathcal{L}_{\rm min} + \mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\rm EM} + \mathcal{L}_{s} + \mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\rm EM} + \mathcal{L}_{s,s-1} + \mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

FT1: Feynman rules straight out of Lagrangian

$$\mathcal{A}^{\rm FT1}_{4,\,\rm cl}\Big|^{\text{spin-}s}_{S^1} = (-)^s \mathcal{E}_1 \cdot \bar{\mathcal{E}}_4 S(p_1)_{\mu\nu}\left[\frac{iC_1}{(p_1\cdot q_2)^2}(f_2^{\mu\nu} q_{2\rho} f_3^{\rho\lambda} + f_3^{\mu\nu} q_{3\rho} f_2^{\rho\lambda})p_{1\lambda} + \frac{2iC_1^2}{p_1\cdot q_2} f_2^{\nu\rho} f_{3\rho}{}^{\mu} + \frac{2iD_1(2C_1 - D_1 - 2)}{(p_1\cdot q_2)m^2} p_{1\rho} f_2^{\rho\mu} f_3^{\nu\lambda} p_{1\lambda}\right]$$

$$\mathcal{A}^{\rm FT1}_{4,\,\rm cl}\Big|^{\text{generic}}_{S^1} = \mathcal{A}^{\rm FT1}_{4,\rm cl}\Big|^{\text{spin-}s}_{S^1\, S\to\mathcal{S}} + (-)^s \mathcal{E}_1 \cdot \bar{\mathcal{E}}_4\, \mathcal{S}(p_1)_{\mu\nu} p_1^{\nu}\left[\frac{2iD_1(C_1+1)}{(p_1\cdot q_2)m^2} p_{1\rho}(f_3^{\rho\lambda} f_{2\lambda}{}^{\mu} - f_2^{\rho\lambda} f_{3\lambda}{}^{\mu}) + \frac{2iD_1}{(p_1\cdot q_2)^2 m^2} p_{1\rho} p_{1\lambda}(f_3^{\rho\mu} q_{3\sigma} f_2^{\sigma\lambda} + f_2^{\rho\mu} q_{2\sigma} f_3^{\sigma\lambda})\right]$$

killed if SSC is imposed

Compton amplitude of FT1 with generic external state agrees to $\mathcal{O}(S^2)$ (up to contact terms) with a worldline theory with no SSC

See Justin Vines talk

Name	Lagrangian	External state
FT1	$\mathcal{L}_{\rm EM}+\mathcal{L}_{\rm min}+\mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\rm EM}+\mathcal{L}_{s}+\mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\rm EM}+\mathcal{L}_{s,s-1}+\mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

Compton amplitudes in the three field theories:

FT1: Feynman rules straight out of Lagrangian

$$\mathcal{A}^{\rm FT1}_{4,\,\rm cl}\Big|^{\text{spin-}s}_{S^1} = (-)^s\mathcal{E}_1\cdot\bar{\mathcal{E}}_4 S(p_1)_{\mu\nu}\left[\frac{iC_1}{(p_1\cdot q_2)^2}(f_2^{\mu\nu}q_{2\rho}f_3^{\rho\lambda}+f_3^{\mu\nu}q_{3\rho}f_2^{\rho\lambda})p_{1\lambda}+\frac{2iC_1^2}{p_1\cdot q_2}f_2^{\nu\rho}f_{3\rho}{}^{\mu}+\frac{2iD_1(2C_1-D_1-2)}{(p_1\cdot q_2)m^2}p_{1\rho}f_2^{\rho\mu}f_3^{\nu\lambda}p_{1\lambda}\right]$$

$$\mathcal{A}^{\rm FT1}_{4,\,\rm cl}\Big|^{\rm generic}_{S^1} = \mathcal{A}^{\rm FT1}_{4,\rm cl}\Big|^{\text{spin-}s}_{S^1,\,S\to\mathcal{S}} + (-)^s\mathcal{E}_1\cdot\bar{\mathcal{E}}_4\,\mathcal{S}(p_1)_{\mu\nu}p_1^{\nu}\left[\frac{2iD_1(C_1+1)}{(p_1\cdot q_2)m^2}p_{1\rho}(f_3^{\rho\lambda}f_{2\lambda}{}^{\mu}-f_2^{\rho\lambda}f_{3\lambda}{}^{\mu})+\frac{2iD_1}{(p_1\cdot q_2)^2m^2}p_{1\rho}p_{1\lambda}(f_3^{\rho\mu}q_{3\sigma}f_2^{\sigma\lambda}+f_2^{\rho\mu}q_{2\sigma}f_3^{\sigma\lambda})\right]$$

killed if SSC is imposed

FT2: spin dependence extrapolated from *s*=1,2,3 calculations. Large-spin limit yields

$$\mathcal{A}^{\rm FT2}_{4,\,\rm cl}\Big|_{S^1} = (-\varepsilon_1\cdot\varepsilon_4)^s S(p_1)_{\mu\nu}\left[\frac{iC_1}{(p_1\cdot q_2)^2}(f_2^{\mu\nu}q_{2\rho}f_3^{\rho\lambda}+f_3^{\mu\nu}q_{3\rho}f_2^{\rho\lambda})p_{1\lambda}+\frac{2iC_1^2}{p_1\cdot q_2}f_2^{\nu\rho}f_{3\rho}{}^{\mu}+\frac{2i(C_1-1)^2}{(p_1\cdot q_2)m^2}p_{1\rho}f_2^{\rho\mu}f_3^{\nu\lambda}p_{1\lambda}\right]$$

FT1 generic ⟶ FT2 $\quad D_1\longrightarrow C_1-1$

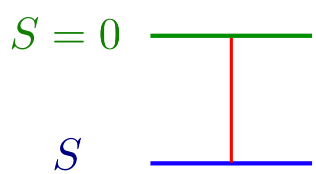
Similar map at $\mathcal{O}(S^2)$; $\quad D_1\longrightarrow C_1-1$ also removes D_2

FT3: we verified that for *both* spin-*s* and indefinite-spin external states, the Compton amplitude is related to FT1 by $i\widetilde{C}_1=1-C_1+D_1$ and also $i\tilde{C}_2=D_2-C_1$ at $\mathcal{O}(S^2)$

Two-body amplitudes; focus on spin-0 on spin-*s* to first order in spin -- $\mathcal{O}(\alpha)$/1PL

relations between three-point amplitudes ⟶ relations between two-body amplitudes

$$i\mathcal{M}^{(0)} = \mathcal{E}_1 \cdot \bar{\mathcal{E}}_4 \left(\frac{d_{\rm T}}{q^2}\right)$$

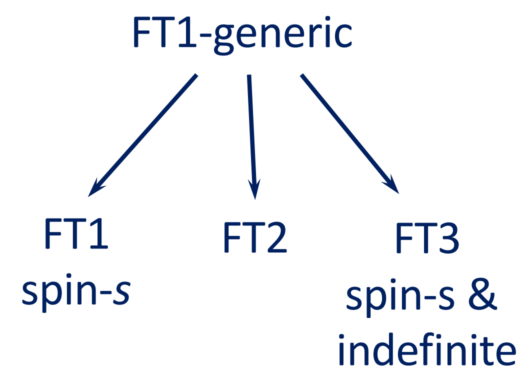


Name	Lagrangian	External state
FT1	$\mathcal{L}_{\rm EM} + \mathcal{L}_{\rm min} + \mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\rm EM} + \mathcal{L}_{s} + \mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\rm EM} + \mathcal{L}_{s,s-1} + \mathcal{L}_{\text{non-min}}^{s,s-1}$	spin-s and indefinite

$$d_{\rm T}\Big|_{S_1^0 S_2^0} = 4iy\bar{m}_1\bar{m}_2 \qquad d_{\rm T}^{\rm FT1}\Big|_{S_1^1 S_2^0}^{\text{generic}} = -4\bar{m}_2\mathcal{S}_{1\,\mu\nu}\left(C_1\bar{u}_2^{\mu}q^{\nu} - D_1 y\bar{u}_1^{\mu}q^{\nu}\right)$$

$$d_{\rm T}^{\rm FT1}\Big|_{S_1^1 S_2^0}^{\text{spin-}s} = d_{\rm T}\Big|_{S_1^1 S_2^0}^{\rm FT2} = d_{\rm T}^{\rm FT3}\Big|_{S_1^1 S_2^0}^{\text{spin-}s} = d_{\rm T}^{\rm FT1}\Big|_{S_1^1 S_2^0}^{\text{generic}}\Big|_{S_{1\,\mu\nu}\bar{u}_1^{\nu}=0}$$

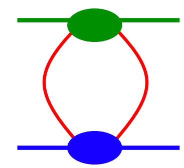
$$d_{\rm T}^{\rm FT3}\Big|_{S_1^1 S_2^0}^{\text{indef.}} = d_{\rm T}^{\rm FT1}\Big|_{S_1^1 S_2^0}^{\text{generic}}\Big|_{D_1 = i\tilde{C}_1 + C_1 - 1}$$



Extra Wilson coefficients in FT1-generic and FT3-indefinite tree-level two-body amplitudes

Two-body amplitudes; focus on spin-0 on spin-*s* to first order in spin -- $\mathcal{O}(\alpha^2)$/2PL

One-loop two-body amplitudes via generalized unitarity



Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng

Rel's between Compton amp's ⟶ rel's between two-body amp's

To classical order: $i\mathcal{M}_4^{(1)} = C_{\rm box}(I_{\rm B} + I_{\bar{\rm B}}) + i\mathcal{M}_{\triangle+\triangledown}$ Classically-relevant part

Classically singular; in the right variables contains no classical terms

$$S^{\mu\nu} = S^{\mu\nu}(\bar{p})\,, \quad \bar{p}_1 = p_1 + q/2\,, \quad \bar{p}_2 = p_2 - q/2$$

$C_{\rm box} \sim d_{\rm T} \times d_{\rm T}$ expected factorized structure; straightforward match with Hamiltonian

The classically-relevant part:

$$i\mathcal{M}_{\triangle+\triangledown}\Big|_{S_1^{n_1} S_2^{n_2}} = \frac{\mathcal{E}_1 \cdot \bar{\mathcal{E}}_4 \mathcal{E}_2 \cdot \bar{\mathcal{E}}_3}{4\sqrt{-q^2}} \sum_i \alpha^{(n_1,n_2,i)} \mathcal{O}^{(n_1,n_2,i)}$$

Linear in spin: $\mathcal{O}^{(1,0,1)} = \mathcal{S}_1^{\mu\nu} \bar{u}_{2\mu} q_\nu$ $\qquad \mathcal{O}^{(1,0,2)} = \mathcal{S}_1^{\mu\nu} \bar{u}_{1\mu} q_\nu$

Bi-linear & quadratic in spin: $\mathcal{O}^{(1,1,i)} \quad i = 1, \dots, 11$ $\qquad \mathcal{O}^{(2,0,i)} \quad i = 1, \dots, 9$

Two-body amplitudes; focus on spin-0 on spin-*s* to first order in spin -- $\mathcal{O}(\alpha^2)$/2PL

Name	Lagrangian	External state
FT1	$\mathcal{L}_{\rm EM}+\mathcal{L}_{\rm min}+\mathcal{L}_{\text{non-min}}$	spin-s and generic
FT2	$\mathcal{L}_{\rm EM}+\mathcal{L}_{s}+\mathcal{L}_{\text{non-min}}$	spin-s
FT3	$\mathcal{L}_{\rm EM}+\mathcal{L}_{s,s-1}+\mathcal{L}^{s,s-1}_{\text{non-min}}$	spin-s and indefinite

To classical order: $i\mathcal{M}_4^{(1)} = C_{\rm box}(I_{\rm B}+I_{\bar{\rm B}}) + i\mathcal{M}_{\triangle+\triangledown}$

The classically-relevant part: $i\mathcal{M}_{\triangle+\triangledown}\Big|_{S_1^{n_1}S_2^{n_2}} = \frac{\mathcal{E}_1\cdot\bar{\mathcal{E}}_4\mathcal{E}_2\cdot\bar{\mathcal{E}}_3}{4\sqrt{-q^2}}\sum_i \alpha^{(n_1,n_2,i)}\mathcal{O}^{(n_1,n_2,i)}$

Linear in spin: $\mathcal{O}^{(1,0,1)} = \mathcal{S}_1^{\mu\nu}\bar{u}_{2\mu}q_\nu$ $\qquad \mathcal{O}^{(1,0,2)} = \mathcal{S}_1^{\mu\nu}\bar{u}_{1\mu}q_\nu$

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

FT1-generic $\alpha^{(1,0,1)} = -\frac{y}{(y^2-1)\bar{m}_1}\Big[2C_1\bar{m}_1 + (C_1^2 - 2C_1D_1 + D_1^2 + 2D_1)\bar{m}_2\Big]$

$$\alpha^{(1,0,2)} = \frac{1}{(y^2-1)\bar{m}_1}\Big\{\Big[(y^2+1)C_1 + (y^2-1)D_1\Big]\bar{m}_1 + \Big[C_1^2 - (y^2+1)C_1D_1 + y^2D_1^2 + (3y^2-1)D_1\Big]\bar{m}_2\Big\}$$

FT1-spin-*s*: $\mathcal{S}\to S,\quad \mathcal{O}^{(1,0,2)} = 0$ $\qquad$ FT2: $\mathcal{S}\to S,\quad \mathcal{O}^{(1,0,2)} = 0,\quad D_1 = C_1 - 1$

FT3: from FT1 via the maps identified at tree level/1PL $i\widetilde{C}_1 = 1 - C_1 + D_1$

Quadratic in spin: Similar structure, just more involved

Extra Wilson coefficients are present in all FTs with more states than spin-*s*

Two-body Hamiltonians to one power of spin

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

Two Hamiltonians, one without *K* … (for FT1 spin-*s*, FT2 and FT3 spin-*s*)

$$\mathcal{H}_1 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\mathbb{1} + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\hat{\boldsymbol{S}}}{\boldsymbol{r}^2}$$

rest-frame spin and boost operators

… and one with *K* (for FT1 generic and FT3 indefinite)

$$\mathcal{H}_2 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\mathbb{1} + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\hat{\boldsymbol{S}}}{\boldsymbol{r}^2} + V^{(2)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{r}\cdot\hat{\boldsymbol{K}}}{\boldsymbol{r}^2}$$

"Magnetic moment" coupling

"Electric dipole" coupling

The structure of the potentials: $V^{(a)}(\boldsymbol{r}^2, \boldsymbol{p}^2) = \frac{\alpha}{|\boldsymbol{r}|}c_1^{(a)}(\boldsymbol{p}^2) + \left(\frac{\alpha}{|\boldsymbol{r}|}\right)^2 c_2^{(a)}(\boldsymbol{p}^2) + \mathcal{O}(\alpha^3)$

Two-body Hamiltonians to one power of spin

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

Two Hamiltonians, one without *K* … (for FT1 spin-*s*, FT2 and FT3 spin-*s*)

$$\mathcal{H}_1 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\mathbb{1} + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\hat{\boldsymbol{S}}}{\boldsymbol{r}^2}$$

rest-frame spin and boost operators

… and one with *K* (for FT1 generic and FT3 indefinite)

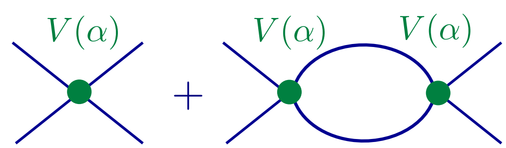
$$\mathcal{H}_2 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\mathbb{1} + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\hat{\boldsymbol{S}}}{\boldsymbol{r}^2} + V^{(2)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{r}\cdot\hat{\boldsymbol{K}}}{\boldsymbol{r}^2}$$

The structure of the potentials: $V^{(a)}(\boldsymbol{r}^2, \boldsymbol{p}^2) = \frac{\alpha}{|\boldsymbol{r}|}c_1^{(a)}(\boldsymbol{p}^2) + \left(\frac{\alpha}{|\boldsymbol{r}|}\right)^2 c_2^{(a)}(\boldsymbol{p}^2) + \mathcal{O}(\alpha^3)$

Coefficients from amplitude matching:

Cheung, Rothstein, Solon; Vaidya; Chung, Huang, Kim, Lee; Bern, Luna, RR, Shen, Zeng; Kosmopoulos, Luna

$$\alpha\mathcal{M}_4^{(0)} + \alpha^2\mathcal{M}_4^{(1)} + \mathcal{O}(\alpha^3) =$$



$+\mathcal{O}(\alpha^3)$

$$[S_i, S_j] = i\epsilon_{ijk}S_k$$
$$[K_i, K_j] = -i\epsilon_{ijk}S_k$$
$$[S_i, K_j] = i\epsilon_{ijk}K_k$$

External states -- rest-frame coherent state $|\Psi\rangle$: $\langle\Psi|\hat{\boldsymbol{S}}|\Psi\rangle = \boldsymbol{S}$ $\quad\langle\Psi|\hat{\boldsymbol{K}}|\Psi\rangle = \boldsymbol{K}$

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

E.g. 1PL Hamiltonian coefficients matching FT1 w/generic ext. state

$$c_1^{(0)} = \frac{m_1 m_2 \sigma}{4E_1E_2}\,, \quad c_1^{(1)} = \frac{m_1 m_2 \sigma - E C_1 (m_1 + E_1)}{4E_1E_2 m_1 (m_1+E_1)}\,, \quad c_1^{(2)} = \frac{m_2\sigma(-C_1 + D_1 + 1)}{4E_1E_2} \qquad \sigma = \frac{\boldsymbol{p}^2 + E_1E_2}{m_1m_2}$$

2PL coefficients are somewhat more complicated

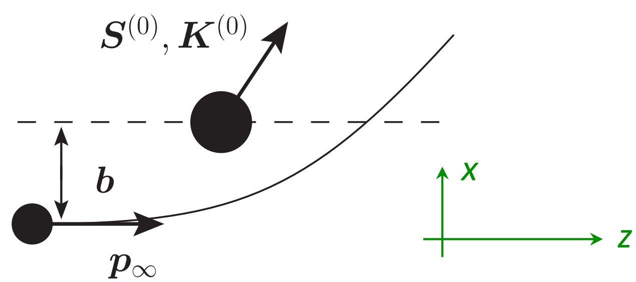
Classical Hamiltonian: $\mathcal{H}_i^{\text{classical}} := \langle\Psi|\mathcal{H}_i|\Psi\rangle$ $\quad |\Psi\rangle =$ rest-frame coherent state

equations of motion:

$$\dot{\boldsymbol{r}} = \frac{\partial\mathcal{H}_*}{\partial\boldsymbol{p}} \qquad \dot{\boldsymbol{p}} = -\frac{\partial\mathcal{H}_*}{\partial\boldsymbol{r}} \qquad \dot{\boldsymbol{S}} = \frac{\partial\mathcal{H}_*}{\partial\boldsymbol{S}}\times\boldsymbol{S} + \frac{\partial\mathcal{H}_*}{\partial\boldsymbol{K}}\times\boldsymbol{K} \qquad \dot{\boldsymbol{K}} = \frac{\partial\mathcal{H}_*}{\partial\boldsymbol{S}}\times\boldsymbol{K} - \frac{\partial\mathcal{H}_*}{\partial\boldsymbol{K}}\times\boldsymbol{S}$$

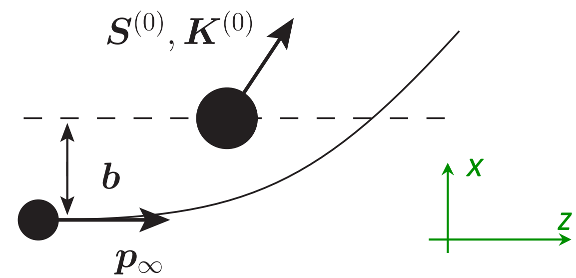
$$\frac{d}{dt}\boldsymbol{S}^2 = \frac{d}{dt}\boldsymbol{K}^2 \neq 0$$

Construct perturbative (scattering) solution, with initial conditions:



Observables -- spin-magnitude change

$$\mathcal{H}_2 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\mathbb{1} + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\hat{\boldsymbol{S}}}{\boldsymbol{r}^2} + V^{(2)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{r}\cdot\hat{\boldsymbol{K}}}{\boldsymbol{r}^2}$$



1PL: $\Delta \boldsymbol{S}^2\Big|_{\rm 1PL} = \Delta \boldsymbol{K}^2\Big|_{\rm 1PL} \propto \frac{\alpha}{b\, p_\infty}\left(K_z^{(0)} S_y^{(0)} - K_y^{(0)} S_z^{(0)}\right) c_1^{(2)}(p_\infty^2)$

$c_1^{(2)} \propto D_1 - (C_1 - 1)$

2PL: $\Delta \boldsymbol{S}^2\Big|_{\rm 2PL} = \Delta \boldsymbol{K}^2\Big|_{\rm 2PL} \propto \frac{\alpha^2}{b^2\, p_\infty}\left(K_z^{(0)} S_y^{(0)} - K_y^{(0)} S_z^{(0)}\right) c_2^{(2)}(p_\infty^2)$

$$+\frac{\alpha^2}{b^2\, p_\infty^2}\left(\left(S_y^{(0)}\right)^2 + \left(S_z^{(0)}\right)^2\right)\left(c_1^{(2)}(p_\infty^2)\right)^2 + \frac{\alpha^2}{b^2\, p_\infty^2}\left(\left(K_y^{(0)}\right)^2 + \left(K_z^{(0)}\right)^2\right)\left(c_1^{(2)}(p_\infty^2)\right)^2$$

$$+\frac{\alpha^2}{b^2}\left((K^{(0)}S^{(0)})c_1^{(2)}c_1^{(1)} + (K^{(0)}S^{(0)})c_1^{(2)}c_1^{(0)} + (K^{(0)}S^{(0)})c_1^{(2)}(c_1^{(0)})' + (K^{(0)}S^{(0)})c_1^{(0)}(c_1^{(2)})'\right)$$

Change of spin magnitude is governed by $V^{(2)}(\boldsymbol{r}^2, \boldsymbol{p}^2)$

- Spin-magnitude is conserved in FTs with fixed-*s* states, and in all FTs for special value $D_1 = C_1 - 1$
- For $K^{(0)} = 0$: magnitude of spin can change, but spinless bodies don't spin up

All observables of FT1 agree to $\mathcal{O}(S^1)$ with a WL theory *effectively* with no SSC see Justin Vines talk

Worldline preview --

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

Standard worldline theory, with dynamical mass function $\mathcal{M}(z,\hat{p},S)$ and covariant SSC:

$$S[\mathsf{e},\xi,\chi,z,p,e,S] = \int_{-\infty}^{\infty}\left(-(p_\mu - QA_\mu)\dot{z}^\mu + \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \mathsf{e}(|p| - \mathcal{M}(z,\hat{p},S)) + \xi_\mu S^{\mu\nu}p_\nu\right)d\lambda$$

$$\Omega^{\mu\nu} = \eta^{AB}e^\mu{}_A\frac{De^\nu{}_B}{D\lambda} \qquad g^{\mu\nu} = e^\mu{}_A e^\nu{}_B\eta^{AB}$$

Extend with additional degrees of freedom such that the Compton and thus all observables reproduce those from QFT/Hamiltonian (FT1):

$$S[\ldots,K] = \int_{-\infty}^{\infty}\left(\left(-(p_\mu - QA_\mu)\dot{z}^\mu + \tfrac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \mathsf{e}(|p| - \mathcal{M}(z,\hat{p},S))\right)\Big|_{S_{\mu\nu}\to S_{\mu\nu}+p_\mu K_\nu - p_\nu K_\mu} + \xi_\mu S^{\mu\nu}p_\nu\right)d\lambda$$

$$\mathcal{M}(z,\hat{p},S) = m - \frac{QC_1}{2m}S^{\mu\nu}\mathcal{F}_{\mu\nu} - \frac{QD_1}{m}\hat{p}_\mu S^{\mu\nu}\mathcal{F}_{\nu\rho}\hat{p}^\rho$$

Introduction of K *effectively* relaxes the SSC

Observables from amplitudes: the boost-modified eikonal formula

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

- Schematic form of 1PL and classical part of 2PL two-body amplitude:

$$\mathcal{M}^{1\mathrm{PL}} = \frac{4\pi\alpha}{\boldsymbol{q}^2}\left[a_1^{(0)} + a_1^{(1)}\boldsymbol{L_q}\cdot\boldsymbol{S} + a_1^{(2)} i\boldsymbol{q}\cdot\boldsymbol{K}\right] \qquad \mathcal{M}^{2\mathrm{PL}}_{\triangle+\triangledown} = \frac{2\pi^2\alpha^2}{|\boldsymbol{q}|}\left[a_2^{(0)} + a_2^{(1)}\boldsymbol{L_q}\cdot\boldsymbol{S} + a_2^{(2)} i\boldsymbol{q}\cdot\boldsymbol{K}\right]$$

- The eikonal to 2PL order: $\chi = \frac{1}{4E|\boldsymbol{p}|}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}(\mathcal{M}^{1\mathrm{PL}} + \mathcal{M}^{2\mathrm{PL}}_{\triangle+\triangledown} + \mathcal{O}(\alpha^3))$

- Observables from eikonal: $\mathbb{O} \in \{\boldsymbol{p}_\perp, \boldsymbol{S}, \boldsymbol{K}\}$ $\qquad \mathcal{D}_L(f,g) = -\epsilon_{ijk}\left(S_i\frac{\partial f}{\partial S_j} + K_i\frac{\partial f}{\partial K_j}\right)\frac{\partial g}{\partial L_k}$

$$\Delta\mathbb{O} \equiv \mathbb{O}(t=+\infty) - \mathbb{O}(t=-\infty) = \{\chi,\mathbb{O}\} + \frac{1}{2}\{\chi,\{\chi,\mathbb{O}\}\} + \mathcal{D}_L(\chi,\{\chi,\mathbb{O}\}) - \frac{1}{2}\{\mathcal{D}_L(\chi,\chi),\mathbb{O}\} + \mathcal{O}\left(\chi^3\right)$$

Including longitudinal impulse: $\Delta p = \Delta p_\perp - \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^2}\left(\frac{\partial\chi}{\partial\boldsymbol{b}}\right)^2 + \mathcal{O}(\chi^3)$

- Identical in structure with the original $\mathcal{D}_{SL}$ formula of Bern, Luna, RR, Shen, Zeng

may expect analogous resummation: $\Delta\mathbb{O} = e^{-\chi\mathcal{D}}[\mathbb{O}, e^{\chi\mathcal{D}}] \qquad \chi\mathcal{D}g \equiv \chi g + \mathcal{D}_L(\chi,g)$

Summary and outlook

- Studied QED coupled to higher-spin fields as a means to understand puzzling aspects of spinning bodies interacting with gravity
  - extra Wilson coefficients describe additional degrees of freedom
  - they govern a physical effect: the change in the magnitude of the rest-frame spin vector
  - if FT has states of only one spin, the extra Wilson coefficients drop out
  - probed the additional d.o.f. by (1) releasing SSC (2) introducing more transverse fields
  - constructed Hamiltonian and observables; they also follow an improved eikonal formula
  - there exists a worldline theory w/o SSC whose two-body observables and Compton amplitudes and agree with FT expressions to $\mathcal{O}(S^2)$ -- see Justin Vines talk tomorrow
  - the effect is present in both field theories with and without ghosts
  - extra d.o.f.s have the interpretation of electric dipole; spin-up requires a dipole
  - Incorporate $|S|$-change in other formalisms, e.g. KMOC? Eikonal to higher orders?

Summary and outlook

- Studied QED coupled to higher-spin fields as a means to understand puzzling aspects of spinning bodies interacting with gravity

  Extra Wilson coefficients govern the change in the magnitude of the spin vector

- What about the original problem -- gravity?

  - Physically, one may expect that the magnitude of the rest-frame spin can change
  - First effect may appear at $\mathcal{O}(S^2)$; consequence of 2-derivative nature of gravity
  - Physical meaning of the gravitational version of K ?
  - Should there exist a gravitational story that parallels QED, we might also expect that there also exist an improved eikonal formula for observables

Expect renewed progress and understanding in the future