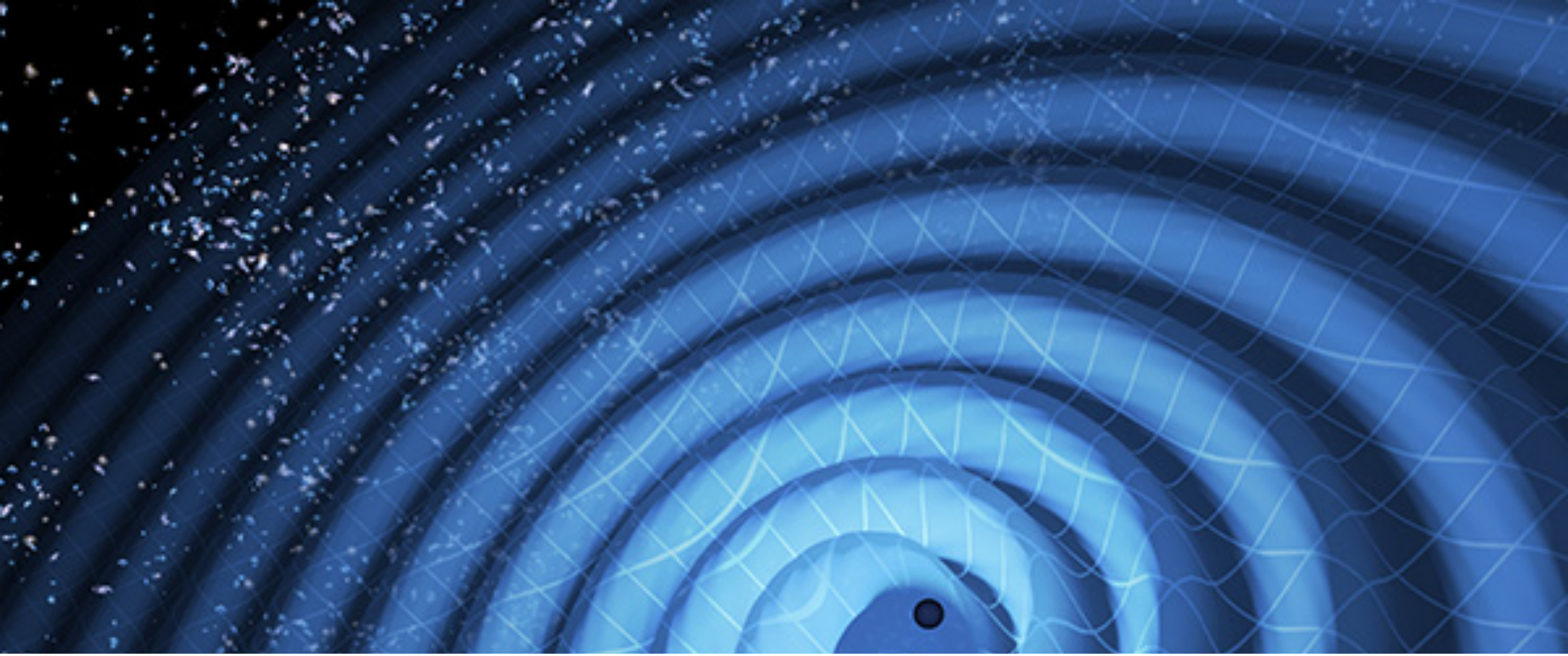


FROM AMPLITUDES TO GRAVITATIONAL WAVES

SCATTERING OF SPINNING BLACK HOLES AT 4PM ORDER

RTG 2575:
Rethinking Quantum Field Theory

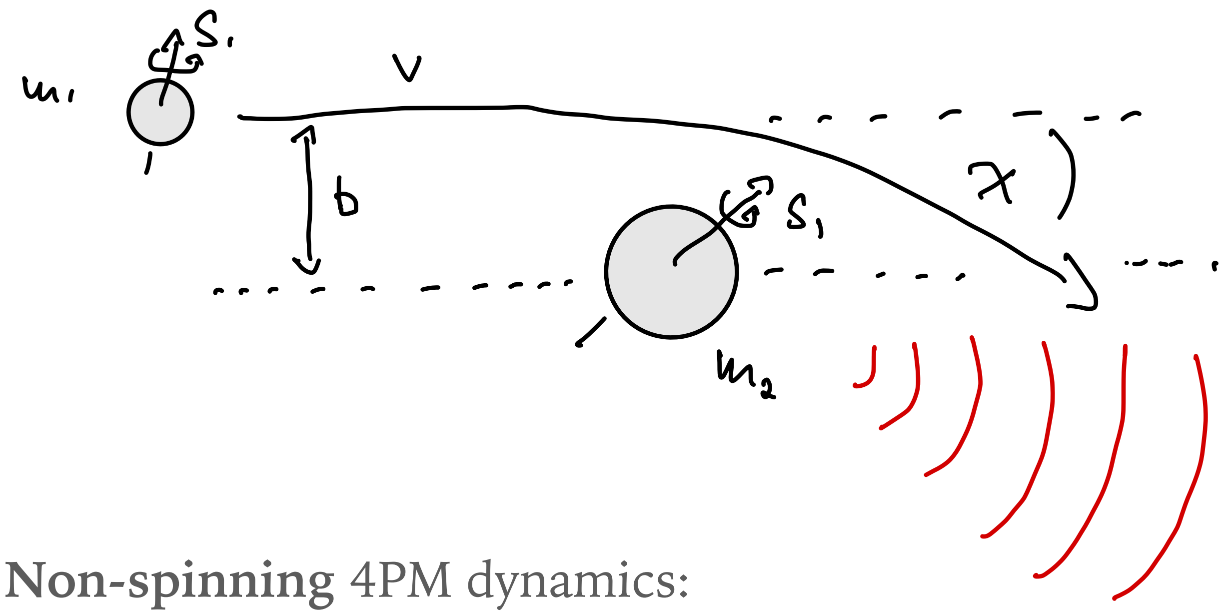
Gustav Mogull

4PM SCATTERING WITH SPIN

*Jakobsen, **GM**, Plefka, Sauer, Xu* ***2306.07104***

*Jakobsen, **GM**, Plefka, Sauer* ***2307.xxxxx***

- Scattering of two **spinning particles** (black holes) at 4PM (G^4):



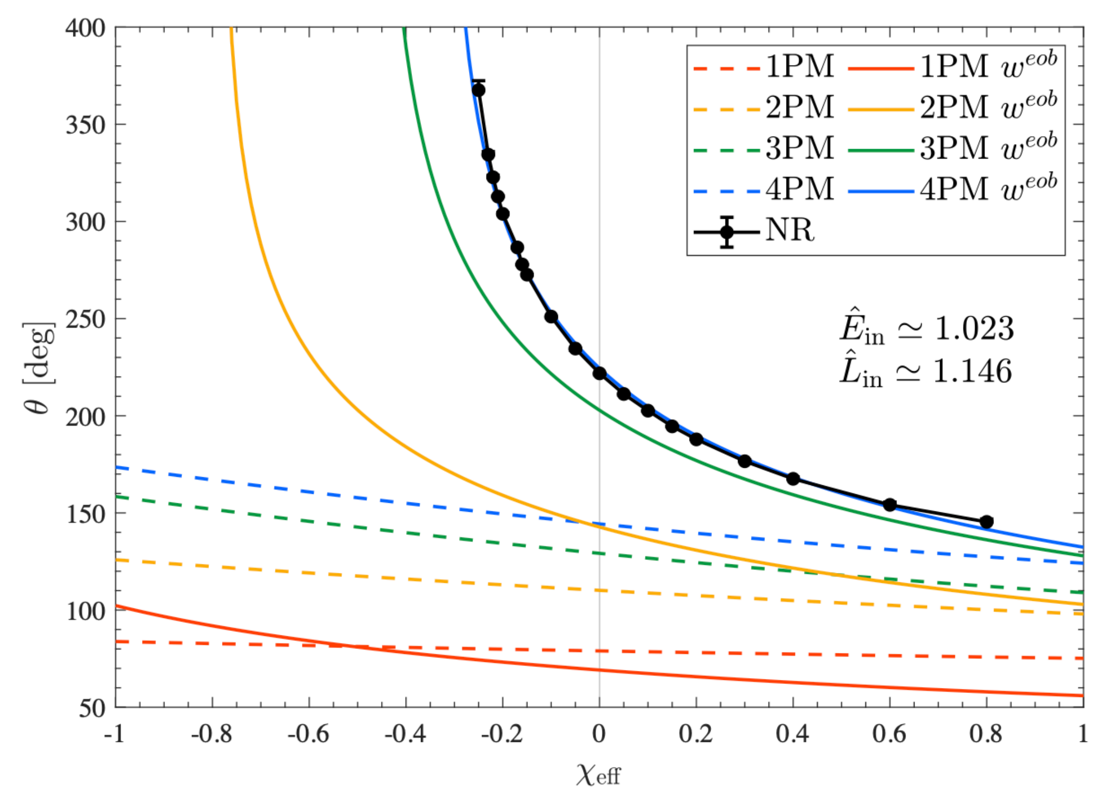
$$\gamma = v_1 \cdot v_2$$

$$\Delta p_1^\mu = p_1(\tau = +\infty) - p_1(\tau = -\infty)$$

$$\Delta S_1^\mu = S_1(\tau = +\infty) - S_1(\tau = -\infty)$$

$$\Delta p_1^\mu = G\Delta p_1^{(1)\mu} + G^2\Delta p_1^{(2)\mu} + G^3\Delta p_1^{(3)\mu} + G^4\Delta p_1^{(4)\mu}$$

- **Non-spinning** 4PM dynamics:
  - **[Dlapa, Kälin, Liu, Neef, Porto]** (complete, using PM EFT)
  - **[Damgaard, Hansen, Planté, Vanhove]** (complete, using KMOC + exponential S-matrix)
  - **[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng]** (conservative only, KMOC)
- Scattering observables **encode 2-body problem:** inform **EOB models**, c.f. **[Damour, Rettegno]**



We now have full spin-orbit (S^1) observables including radiation-reaction!

WORLDLINE QUANTUM FIELD THEORY

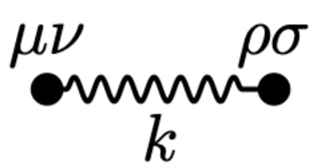
GM, Plefka, Steinhoff ***JHEP* 02 (2021)**

$$S = -\frac{2}{\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R - \sum_{i=1}^{2}\int \mathrm{d}\tau_i \frac{m_i}{2} g_{\mu\nu}\dot{x}_i^\mu \dot{x}_i^\nu$$

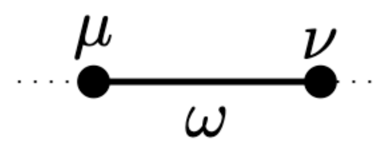
$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$$

$$x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + z_i^\mu(\tau_i)$$

- Promote gravitons, deflections to propagating d.o.f's… with **retarded** i0 prescription!



$$= i\frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i\epsilon)^2 - \mathbf{k}^2}$$

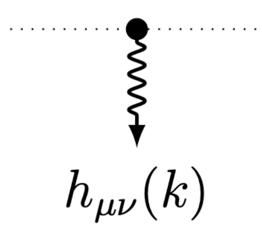


$$= -i\frac{\eta^{\mu\nu}}{m\,(\omega + i\epsilon)^2}$$

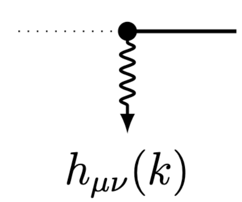
$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

← Theory trivially lifts to D=4-2ε dimensions

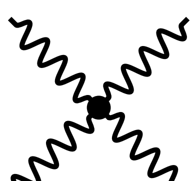
- **Gravitons** live in the bulk, carry **momentum.**
- **Deflections** live on the worldline, carry **energy**.



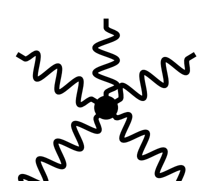
$$= -i\frac{m}{2m_{\rm Pl}}e^{ik\cdot b}\bar{\delta}(k\cdot v)v^\mu v^\nu\,,$$



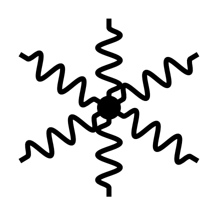
$$= \frac{m}{2m_{\rm Pl}}e^{ik\cdot b}\bar{\delta}(k\cdot v + \omega)\left(2\omega v^{(\mu}\delta_\rho^{\nu)} + v^\mu v^\nu k_\rho\right)$$



$$\sim \sqrt{G}^2 k^2\,,$$



$$\sim \sqrt{G}^3 k^2$$



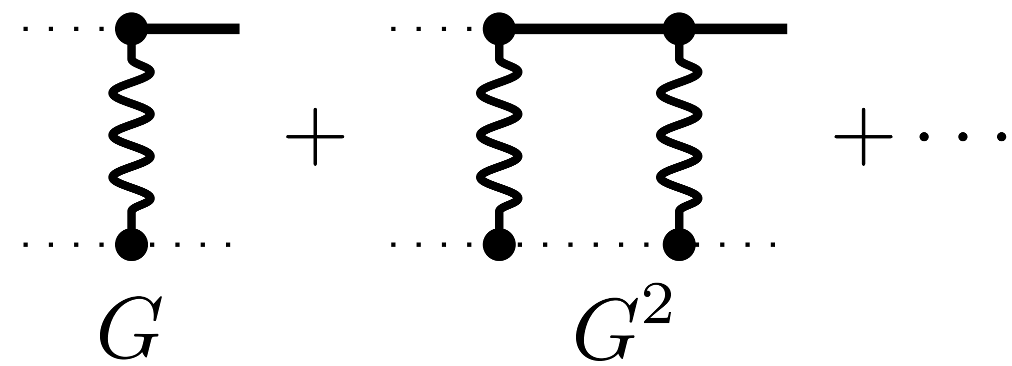
$$\sim \sqrt{G}^4 k^2\,,\ldots$$

Tree-level one-point functions = Solutions to classical equations of motion

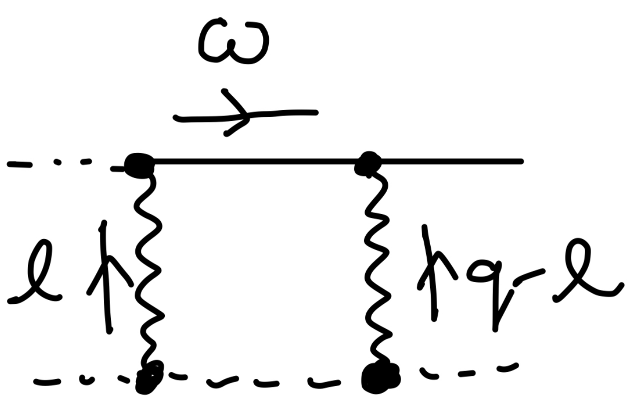
SCATTERING OBSERVABLES

- For momentum impulse draw tree diagrams with 1 outgoing line:

$$\Delta p_1^\mu = -m_1 \omega^2 \, \langle z_1^\mu(\omega)\rangle\big|_{\omega=0} =$$



- **All graphs are trees.** Loop integrals arise from **lack of momentum conservation:**

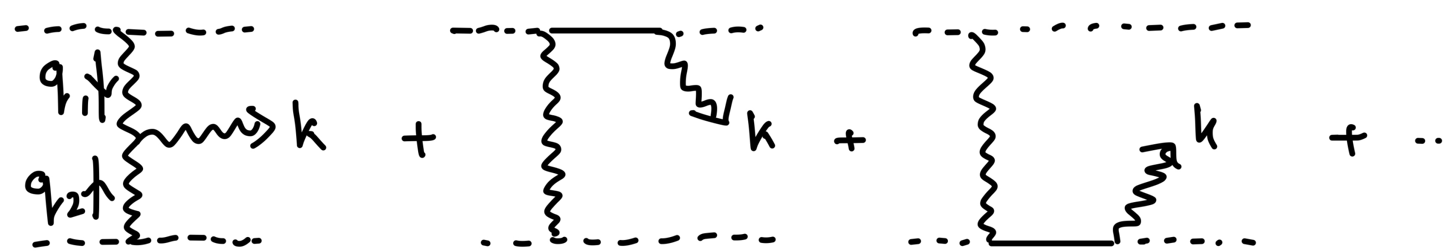


$$= \int_{q,\ell,\omega} \frac{\delta(\omega - \ell\cdot v_1)\delta(\omega + (q-\ell)\cdot v_1)\delta(\ell\cdot v_2)\delta((q-\ell)\cdot v_2)}{(\omega + i0)^2 \ell^2 (\ell - q)^2} e^{iq\cdot b}$$

$$= \int_q \delta(q\cdot v_1)\delta(q\cdot v_2) e^{iq\cdot b} \int_\ell \frac{\delta(\ell\cdot v_2)}{(\ell\cdot v_1 + i0)^2 \ell^2 (\ell - q)^2}$$

- Similar derivation of the **gravitational waveform**:

$$\langle h_{\mu\nu}(k)\rangle =$$



SUSY IN THE SKY WITH GRAVITONS

*Jakobsen, **GM**, Plefka, Steinhoff* ***JHEP 01 (2022)***

- First-quantized theory of a **spin-N/2 particle** in a **flat background (D=4+1)**:

$$S = -\int \mathrm{d}\tau \left[p_M \dot{x}^M + \frac{i}{2}\psi_\alpha^A \dot{\psi}_\alpha^B \eta_{AB} - e\,H - i\chi^\alpha Q_\alpha - \frac{1}{2} f_{\alpha\beta} M^{\alpha\beta} \right]$$

$$H = \frac{1}{2}p^2\,, \quad Q_\alpha = p\cdot\psi_\alpha\,, \quad M_{\alpha\beta} = i\psi_\alpha\cdot\psi_\beta\,. \quad \{x^M, p_N\} = \delta_N^M\,, \quad \{\psi_\alpha^A, \psi_\beta^B\} = -i\delta_{\alpha\beta}\,\eta^{AB}\,,$$

- Theory enjoys an **N-SUSY algebra** (α=1,2,…,N):

$$\{Q_\alpha, Q_\beta\} = -2i\delta_{\alpha\beta}H\,, \quad \{H, Q_\beta\} = \{H, M_{\alpha\beta}\} = 0\,,$$
$$\{M_{\alpha\beta}, Q_\gamma\} = -2\delta_{\gamma[\alpha}Q_{\beta]}\,, \quad \{M_{\alpha\beta}, M^{\gamma\delta}\} = -4\delta_{[\alpha}{}^{[\gamma}\, M_{\beta]}{}^{\delta]}\,.$$

- Coupling to a **curved background** possible up to **N=2** (spin-1) [Bastianelli, Benincasa, Giombi ’05]:

$$Q = \psi^a\, e_a^\mu(x)\,\pi_\mu \qquad\qquad \pi_\mu = p_\mu - i\omega_{\mu ab}\bar{\psi}^a\psi^b$$

$$\{Q, \bar{Q}\} = -2i\underbrace{\left[\frac{1}{2}(g^{\mu\nu}\pi_\mu\pi_\nu - m^2 - R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d)\right]}_{H}$$

GLOBAL SUSY ACTION

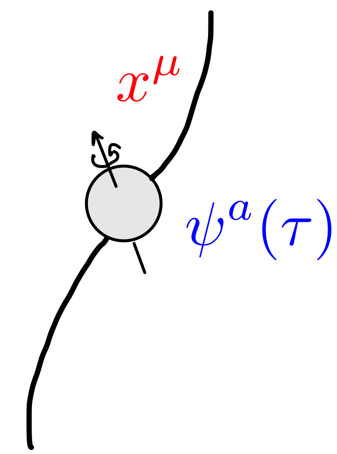
*Jakobsen, **GM**, Plefka, Steinhoff* ***Phys. Rev. Lett. 128 (2022)***

- Gauge fix action by setting **e=1**, Lagrange multipliers to zero:

$$S_{\mathrm{BH/NS}} = -m\int d\tau\Big[\tfrac{1}{2}g_{\mu\nu}\dot{x}^{\mu}\,\dot{x}^{\nu} + i\bar{\psi}D_{\tau}\psi + \tfrac{1}{2}R_{abcd}\bar{\psi}^{a}\psi^{b}\bar{\psi}^{c}\psi^{d} + C_E R_{a\mu b\nu}\dot{x}^{\mu}\dot{x}^{\nu}\bar{\psi}^{a}\psi^{b}\,\bar{\psi}\cdot\psi\Big]$$

spin degrees of freedom *neutron star term*

- Theory now enjoys a **global SUSY:**



$$\delta x^{\mu} = i e^{\mu}_{a}(\bar{\epsilon}\psi^{a} + \epsilon\bar{\psi}^{a})\,,$$

$$\delta\psi^{a} = -\epsilon e^{a}_{\mu}\,\dot{x}^{\mu} - \delta x^{\mu}\omega_{\mu}{}^{a}{}_{b}\psi^{b}$$

- Symmetries imply **conserved charges**:

$$S^{\mu\nu} = -2i\bar{\psi}^{[\mu}\psi^{\nu]} = \epsilon^{\mu\nu\rho\sigma}p_{\rho}a_{\sigma}$$

$$\dot{x}^2 = 1 + R_{abcd}\bar{\psi}^{a}\psi^{b}\bar{\psi}^{c}\psi^{d} \qquad \bar{\psi}\cdot\psi = s$$

Conserved spin length

$$p\cdot\psi = p\cdot\bar{\psi} = 0 \quad\Longrightarrow\quad p_{\mu}S^{\mu\nu} = 0$$

Covariant SSC

- Neutron star term **preserves SUSY up to O(S²)**.

SPINNING WQFT FEYNMAN RULES

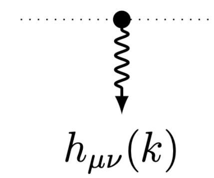
- Inclusion of spin requires **extended Feynman rules**:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$$

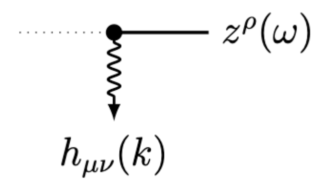
$$x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + z_i^\mu(\tau_i)$$

$$\psi_i^\mu(\tau_i) = \Psi_i^\mu + \psi_i'^\mu(\tau_i)$$

$$\mathcal{S}^{\mu\nu} = -2i\bar\Psi^{[\mu}\Psi^{\nu]}$$



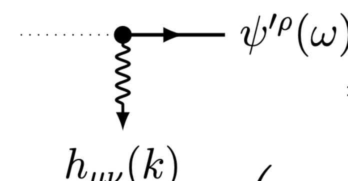
$$= -i\frac{m\kappa}{2}e^{ik\cdot b}\hat\delta(k\cdot v) \times \left(v^\mu v^\nu + i(k\cdot\mathcal{S})^{(\mu}v^{\nu)} - \frac{1}{2}(k\cdot\mathcal{S})^\mu(k\cdot\mathcal{S})^\nu + \frac{C_E}{2}v^\mu v^\nu(k\cdot\mathcal{S}\cdot\mathcal{S}\cdot k)\right),$$



$$= \frac{m\kappa}{2}e^{ik\cdot b}\hat\delta(k\cdot v+\omega)$$

$$\times\Big(2\omega v^{(\mu}\delta^{\nu)}_\rho + v^\mu v^\nu k_\rho + i(k\cdot\mathcal{S})^{(\mu}(k_\rho v^{\nu)} + \omega\delta^{\nu)}_\rho) + \frac{1}{2}k_\rho(k\cdot\mathcal{S})^\mu(\mathcal{S}\cdot k)^\nu$$

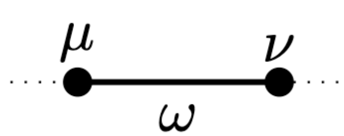
$$+\frac{C_E}{2}\Big(\big(2\omega v^{(\mu}\delta^{\nu)}_\rho + v^\mu v^\nu k_\rho\big)(k\cdot\mathcal{S}\cdot\mathcal{S}\cdot k) - \omega^2 k_\rho(\mathcal{S}\cdot\mathcal{S})^{\mu\nu} + 2\omega^2(k\cdot\mathcal{S}\cdot\mathcal{S})^{(\mu}\delta^{\nu)}_\rho\Big)\Big)$$



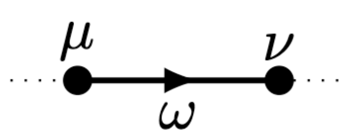
$$= -im\kappa e^{ik\cdot b}\hat\delta(k\cdot v+\omega)$$

$$\times\Big(k_{[\rho}\delta^{(\mu}_{\sigma]}\big(v^{\nu)} - i(\mathcal{S}\cdot k)^{\nu)}\big) + iC_E\big(v^{(\mu}k_\lambda + \omega\delta^{(\mu}_\lambda\big)\big(v^{\nu)}k_{[\rho} + \omega\delta^{\nu)}_{[\rho}\big)\mathcal{S}^\lambda{}_{\sigma]}\Big)\bar\Psi^\sigma.$$

- Propagators:



$$= -i\frac{\eta^{\mu\nu}}{m(\omega+i\epsilon)^2},$$



$$= -i\frac{\eta^{\mu\nu}}{m(\omega+i\epsilon)},$$

- Equivalent to solving **Mattison-Papapetrou-Dixon (MPD)** EoMs.
- Combine worldline modes into a "superfield": $Z_i = \{z_i, \psi_i'\}$

IN-IN FORMALISM

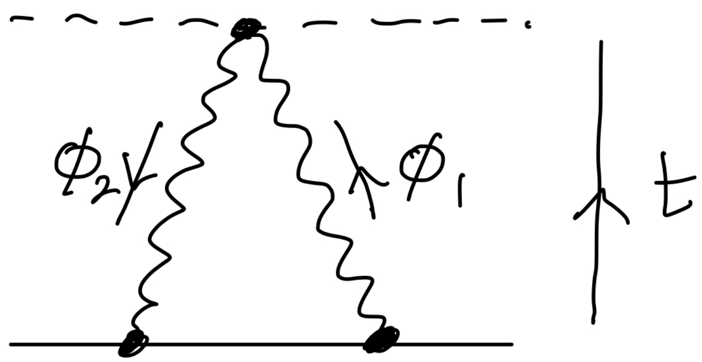
*Jakobsen, **GM**, Plefka, Sauer JHEP 10 (2022)*

- Formal path-integral description using **Schwinger-Keldysh in-in (CTP)** formalism:

$$\langle \mathcal{O}(t,\mathbf{x})\rangle_{\text{in}-\text{in}} := {}_{\text{in}}\langle 0|U(-\infty,t)\mathcal{O}(t,\mathbf{x})U(t,-\infty)|0\rangle_{\text{in}}$$

- Path integral involves **2 copies of the theory**:

$$Z[J_1,J_2] = \int \mathrm{D}[\phi_1,\phi_2]\exp\left\{\frac{i}{\hbar}\left[(S[\phi_1]-S[\phi_2]) + \int \mathrm{d}^4x\big(J_1(x)\phi_1(x) - J_2(x)\phi_2(x)\big)\right]\right\}$$



$$\phi_1(t=+\infty,\mathbf{x}) = \phi_2(t=+\infty,\mathbf{x})$$
$$\phi_1(t=-\infty,\mathbf{x}) = \phi_2(t=-\infty,\mathbf{x}) = 0$$

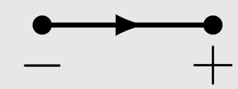
$$\langle \phi_A(x)\,\phi_B(y)\rangle = \begin{pmatrix} \langle 0|\mathcal{T}\phi(x)\phi(y)|0\rangle & \langle 0|\phi(y)\phi(x)|0\rangle \\ \langle 0|\phi(x)\phi(y)|0\rangle & \langle 0|\mathcal{T}^*\phi(x)\phi(y)|0\rangle \end{pmatrix} = \begin{pmatrix} D_F(x,y) & D_-(x,y) \\ D_+(x,y) & D_D(x,y) \end{pmatrix}$$

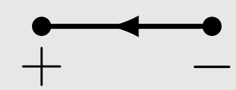
KELDYSH BASIS

- **Huge simplification** in the **Keldysh basis**, with **advanced & retarded propagators:**

$$\langle \phi_a(x)\,\phi_b(y)\rangle = \begin{pmatrix} \frac{1}{2}D_H(x,y) & D_{\rm ret}(x,y) \\ -D_{\rm adv}(x,y) & 0 \end{pmatrix}$$

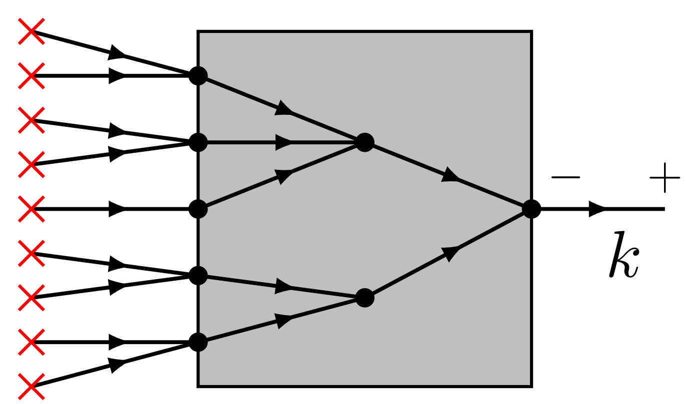
$$\phi_+ = \frac{1}{2}(\phi_1+\phi_2) \qquad \phi_- = \phi_1 - \phi_2$$

$$\widetilde{D}_{\rm ret}(k) =$$ 
$$= \frac{-i}{(k^0+i0)^2 - \mathbf{k}^2}\,,$$

$$\widetilde{D}_{\rm adv}(k) =$$ 
$$= \frac{-i}{(k^0-i0)^2 - \mathbf{k}^2}\,,$$

- Diagrams conspire to ensure **forward-in-time flow of causality:**

$$\langle \phi(k)\rangle_{\text{in-in}} =$$

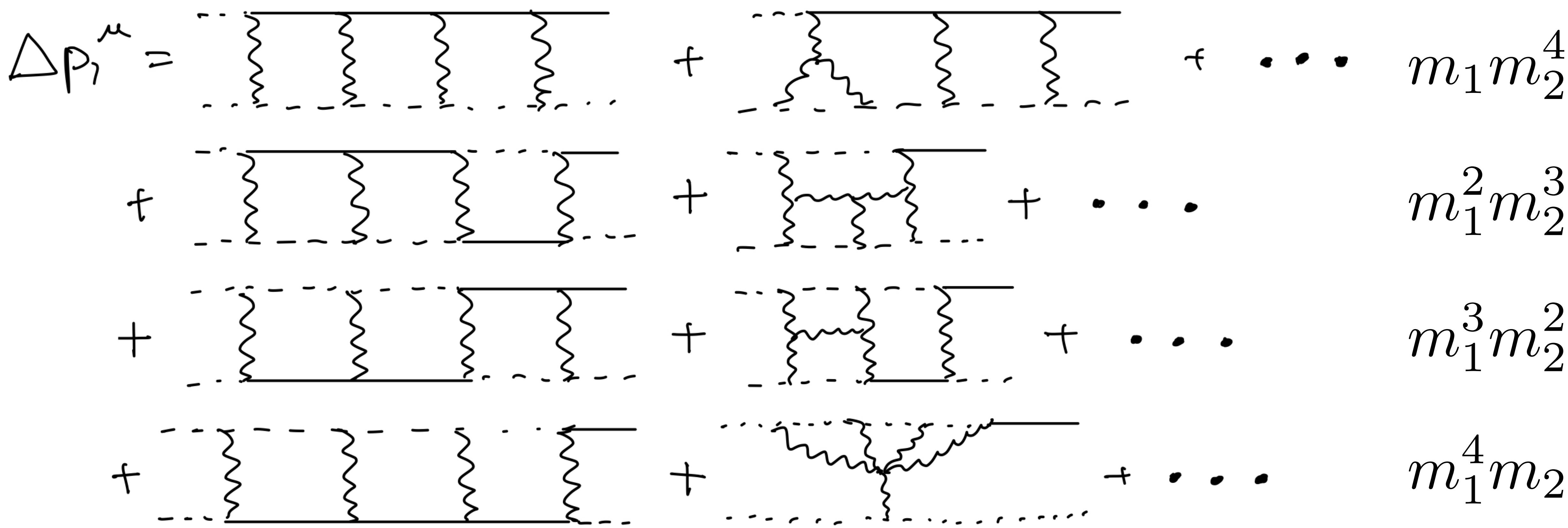


Absorbs the cuts present in e.g. KMOC formalism

$$\frac{1}{k^2+\mathrm{sgn}(k^0)i0} = \frac{1}{k^2+i0} + 2i\pi\theta(-k^0)\delta(k^2)$$

- Upshot: calculate tree-level 1-point functions using **in-out Feynman rules + retarded propagators**.
- But: we must now handle loop integrals with a **retarded i0 prescription!**

4PM MOMENTUM IMPULSE



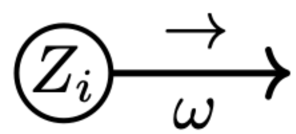
- With the retarded prescription, **causality flows towards the outgoing line.**

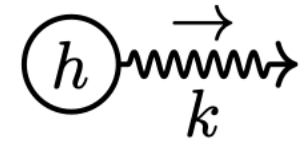
$$\int_q e^{-iq\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{\ell_1,\ell_2,\ell_3}\frac{\mathrm{num}[\ell_i]}{D_1\cdots D_{12}}\delta(\ell_1\cdot v_{i_1})\delta(\ell_2\cdot v_{i_2})\delta(\ell_3\cdot v_{i_3})$$

Passarino-Veltman reduction yields scalar integrals — no irreducible numerators

$$\ell_i^\mu \to \ell_i\cdot v_1\,\hat{v}_1^\mu + \ell_i\cdot v_2\,\hat{v}_2^\mu + \frac{\ell_i\cdot q}{q^2}q^\mu \qquad v_i\cdot\hat{v}_j = \delta_{ij}$$

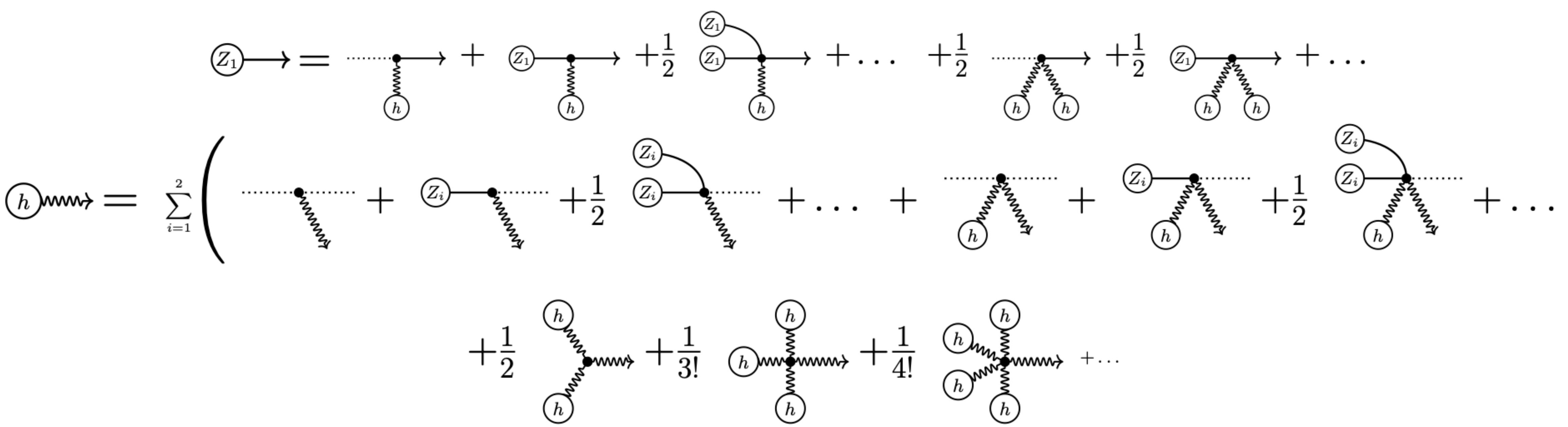
INTEGRAND CONSTRUCTION

$\langle Z_i(\omega)\rangle =$ 

, $\quad \langle h_{\mu\nu}(k)\rangle =$ 

.

- All graphs are **nested trees**, so we can use **Berends-Giele recursion** to conveniently obtain all contributions. We get **529 distinct graphs!**
- Fast implementation with **FORM**, which is also good for handling **anti-commuting spin variables**.



- All contributions will reduce to scalar integrals, so having a messy loop integrand **doesn't matter**!

METHOD OF REGIONS: A FIRST GLIMPSE

- Loop momenta are characterised by **2 possible scalings**:

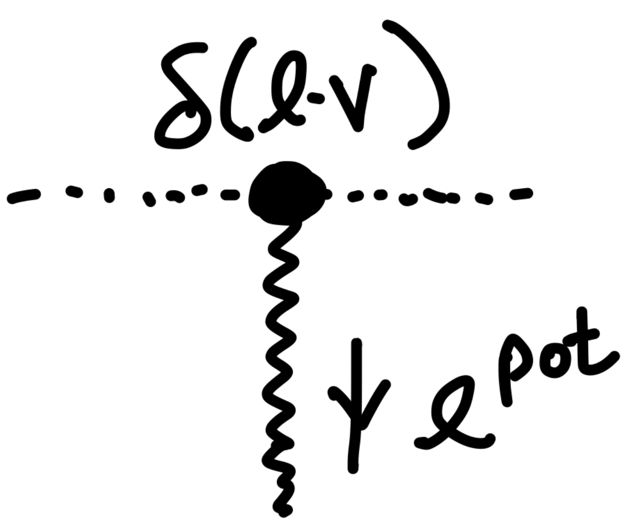
$$\ell_i^{\rm pot} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, 1)\,, \qquad \ell_i^{\rm rad} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, v)$$

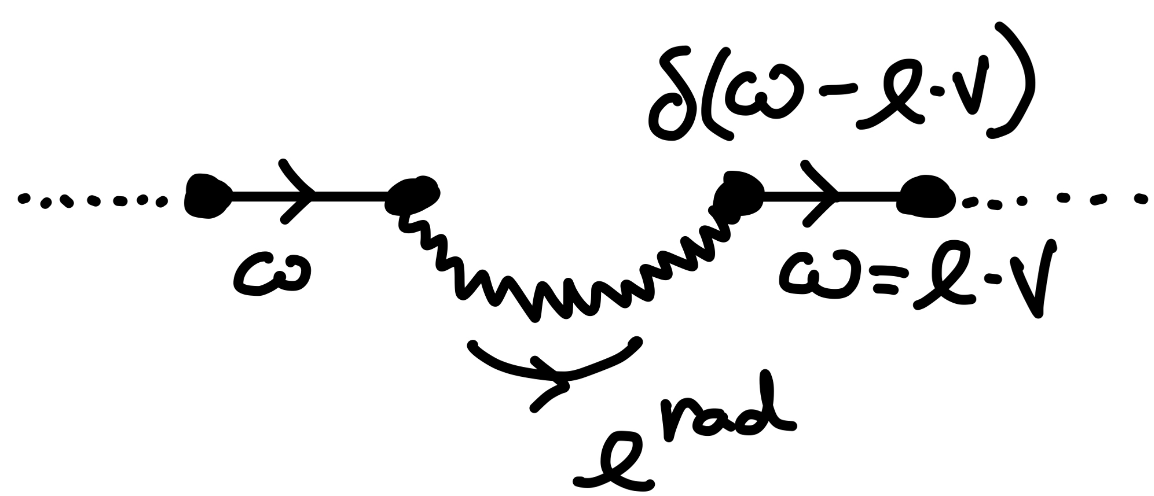
- We distinguish between **active & passive** propagators. These **can & can't** go on-shell over the domain of integration:

$$\ell_i^2 = (\ell_i^0)^2 - \boldsymbol{\ell}_i^2 = 0 \qquad \Longrightarrow \qquad \ell_i \sim \ell_i^{\rm rad}$$

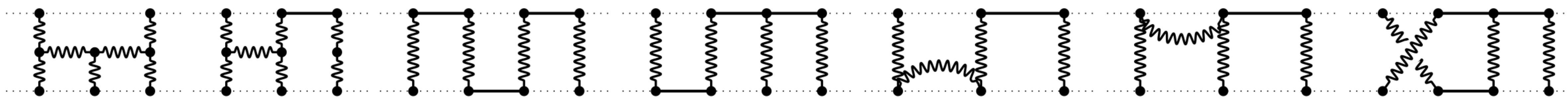
$$\ell_i \cdot v = \gamma(\ell_i^0 - \mathbf{v}\cdot\boldsymbol{\ell}_i) = 0 \qquad \Longrightarrow \qquad \ell_i \sim \ell_i^{\rm pot}$$

If a propagator is passive, then we don't care about its i0 prescription!





INTEGRAL FAMILIES



- At 4PM we need **2 integral bases** to handle all contributions:

$$J^{(\sigma_1,\sigma_2,\ldots,\sigma_5)}_{n_1,n_2,\ldots,n_{12}} = \int_{\ell_1,\ell_2,\ell_3} \frac{\delta(\ell_1\cdot v_1)\delta(\ell_2\cdot v_1)\delta(\ell_3\cdot v_2)}{D_1^{n_1} D_2^{n_2} \ldots D_{12}^{n_{12}}} \qquad I^{(\sigma_1,\sigma_2,\ldots,\sigma_6)}_{n_1,n_2,\ldots,n_{12}} = \int_{\ell_1,\ell_2,\ell_3} \frac{\delta(\ell_1\cdot v_2)\delta(\ell_2\cdot v_1)\delta(\ell_3\cdot v_1)}{\tilde{D}_1^{n_1} \tilde{D}_2^{n_2} \ldots \tilde{D}_{12}^{n_{12}}}$$

$$D_1 = \ell_1\cdot v_2 + \sigma_1 i0^+\,,\ D_2 = \ell_2\cdot v_1 + \sigma_2 i0^+\,,\ D_3 = \ell_3\cdot v_1 + \sigma_3 i0^+\,,$$
$$D_4 = (\ell_1-\ell_3)^2 + \sigma_4 \mathrm{sgn}(\ell_1^0-\ell_3^0) i0^+\,,\ D_5 = (\ell_2-\ell_3)^2 + \sigma_5 \mathrm{sgn}(\ell_2^0-\ell_3^0) i0^+\,,$$
$$D_6 = (\ell_1-\ell_2)^2\,,\ D_7 = \ell_1^2\,,\ D_8 = \ell_2^2\,,\ D_9 = \ell_3^2\,,$$
$$D_{10} = (\ell_1+q)^2\,,\ D_{11} = (\ell_2+q)^2\,,\ D_{12} = (\ell_3+q)^2\,,$$

- **Two active gravitons** per graph define regions: **(PP), (PR), (RP), (RR)**
- **Integration-by-parts relations (IBPs) [FIRE,LiteRed] don't care about the i0 prescription; symmetries care!**
- All integrals are **strictly real/imaginary (b-type/v-type)**

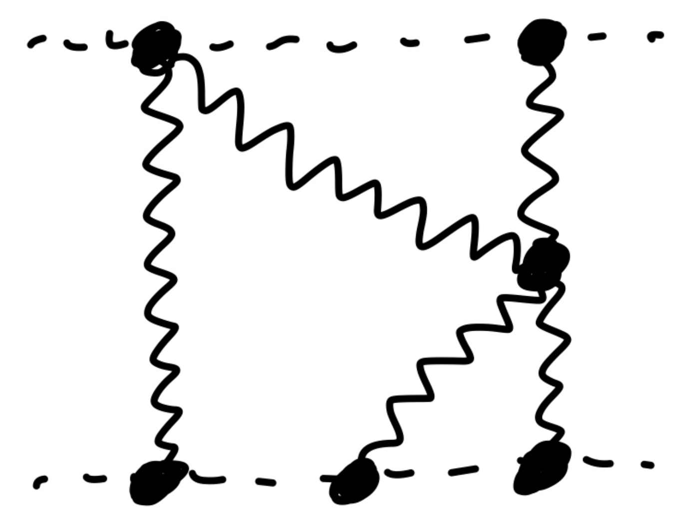
Number of masters integrals: J: 64+66, I: 23+23

DIFFERENTIAL EQUATIONS

- We seek a **canonical basis of integrals** c.f. [Dlapa, Kälin, Neef, Liu, Porto]:

$$\frac{\mathrm{d}\vec{I}(x)}{\mathrm{d}x} = \epsilon A(x)\,\vec{I}(x) \qquad x = \gamma - \sqrt{\gamma^2 - 1}$$

- This is a **highly non-trivial task**! A few hints…
  1. Organise into **block diagonals**, using **top-level sectors**
  2. Most useful algorithms: **INITIAL, CANONICA** (we also used **Fuchsia, Libra, Epsilon**)
  3. Elliptic K/E 3x3 block in the **canonical transformation** — solves a **degree-3 Picard-Fuchs equation**



- Canonical basis enables an order-by-order solution to DEs.

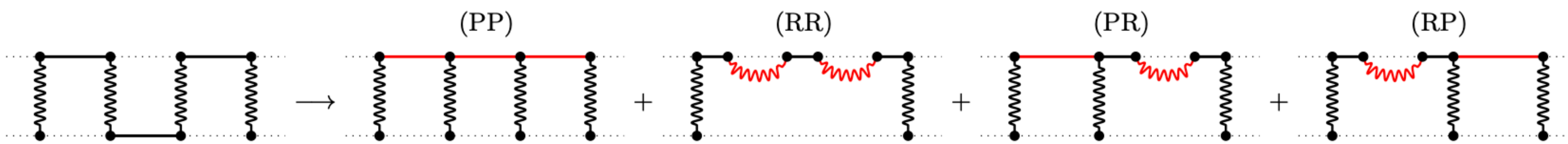
$$\vec{I}(x) = \vec{I}_0 + \epsilon\left(\vec{I}_1 + \int \mathrm{d}x\, A(x)\vec{I}_0\right) + \mathcal{O}(\epsilon^2)$$

- Remaining task, to fix the boundary constants. Here **the i0 matters**!

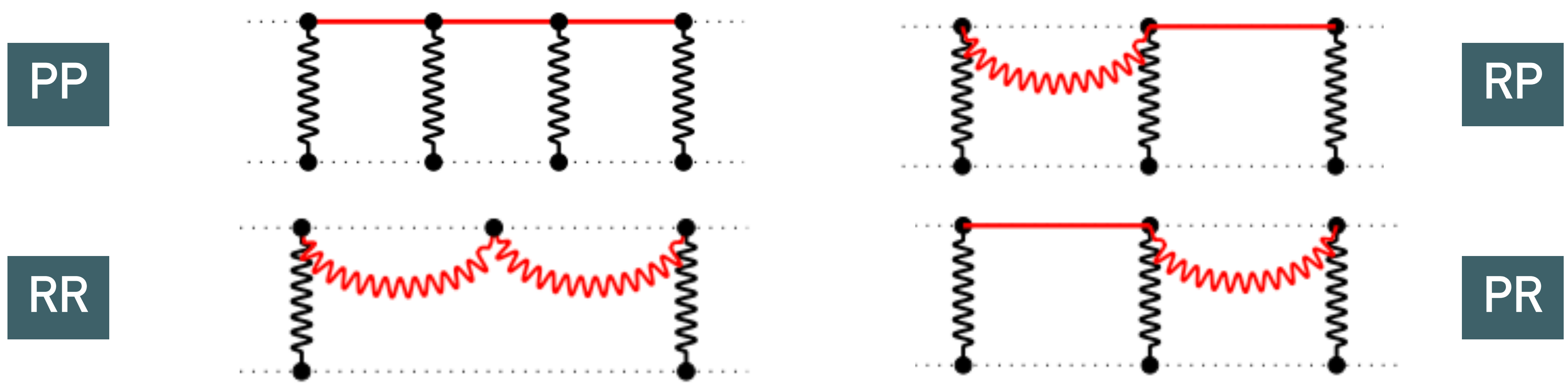
BOUNDARY FIXING

$$\ell_i^{\text{pot}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, 1)\,, \qquad \ell_i^{\text{rad}} = (\ell_i^0, \boldsymbol{\ell}_i) \sim (v, v)$$

- In the slow-velocity limit, re-express master integrals in terms of those with a **simpler velocity dependence**:



- Simpler boundary integrals are handled using IBPs, with **top-level sectors**:



- Passive worldline propagators **don't appear in the masters!**
- Graviton bubbles are integrated out yielding more worldline propagators:

$$\int_k \frac{\delta(\omega - k\cdot v_1)}{((k^0+i0)^2 - \mathbf{k}^2)^n} = \int_{\mathbf{k}} \frac{1}{((\omega+i0)^2 - \mathbf{k}^2)^n} = \left(\frac{e^{-i\pi}}{4\pi}\right)^{\frac{D-1}{2}} \frac{\Gamma(n - \frac{D-1}{2})}{\Gamma(n)} (\omega + i0)^{D-1-2n}\,.$$

ASSEMBLING RESULTS, CONSISTENCY CHECKS

- Two N=1 **supercharges are conserved**:

$$p_1^2 = (p_1 + \Delta p_1)^2 \implies p_1 \cdot \Delta p_1^{(4)} = -\Delta p_1^{(1)} \cdot \Delta p_1^{(3)} - \Delta p_1^{(2)} \cdot \Delta p_1^{(2)}$$

$$p_1 \cdot \psi_1 = (p_1 + \Delta p_1) \cdot (\psi_1 + \Delta \psi_1) \implies \psi_1 \cdot \Delta p_1^{(4)} + p_1 \cdot \Delta \psi_1^{(4)} = -\Delta p_1^{(1)} \cdot \Delta \psi_1^{(3)} - \Delta p_1^{(2)} \cdot \Delta \psi_1^{(2)} - \Delta p_1^{(3)} \cdot \Delta \psi_1^{(1)}$$

- Dimensional regularisation 1/ε poles **cancel between PP/RR**, implying a **logarithmic velocity dependence (non-localities)**:

$$\Delta p_1^{(PP)} = \frac{P(\gamma)}{2\epsilon} + \dots, \quad \Delta p_1^{(RR)} = -v^{-4\epsilon} \frac{P(\gamma)}{2\epsilon} + \dots$$

$$\implies \Delta p_1^{(PP+RR)} = \Delta p_1^{(\mathrm{PP})} + \Delta p_1^{(\mathrm{RR})} = P(\gamma) \log \frac{\gamma - 1}{2} + \dots$$

- Change in the **spin tensor, spin vector** given by:

$$S_1^{\mu\nu} = -2i\bar{\psi}_1^{[\mu}\psi_1^{\nu]} \implies \Delta S_1^{\mu\nu} = -2i(\Delta\bar{\psi}_1^{[\mu}\psi_1^{\nu]} + \bar{\psi}_1^{[\mu}\Delta\psi_1^{\nu]} + \Delta\bar{\psi}_1^{[\mu}\Delta\psi_1^{\nu]})$$

$$S_1^{\mu} = \frac{1}{2}\epsilon^{\mu}{}_{\nu\rho\sigma} p_1^{\nu} S_1^{\rho\sigma} \implies \Delta S_1^{\mu} = \frac{1}{2}\epsilon^{\mu}{}_{\nu\rho\sigma}(\Delta p_1^{\nu} S_1^{\rho\sigma} + p_1^{\nu}\Delta S_1^{\rho\sigma} + \Delta p_1^{\nu}\Delta S_1^{\rho\sigma})$$

FINAL RESULTS

- **Momentum impulse:**

$$\Delta p_1^{(4)\mu} = \frac{m_1^2 m_2^2}{|b|^4} \sum_{l,\sigma=b,v} \rho_l^{(\sigma)\mu} \left[\left(\frac{m_2^2}{m_1} c_l^{(\sigma)}(\gamma) + \frac{m_1^2}{m_2} \bar{c}_l^{(\sigma)}(\gamma) \right) + \sum_\alpha F_\alpha(\gamma) \left(m_2 d_{\alpha,l}^{(\sigma)}(\gamma) + m_1 \bar{d}_{\alpha,l}^{(\sigma)}(\gamma) \right) \right],$$

- **Spin kick:**

$$\Delta S_1^{(4)\mu} = \frac{m_1^2 m_2^2}{|b|^4} \sum_{l,\sigma} \tilde{\rho}_l^{(\sigma)\mu} \left[\left(\frac{m_2^2}{m_1} e_l^{(\sigma)}(\gamma) + \frac{m_1^2}{m_2} \bar{e}_l^{(\sigma)}(\gamma) \right) + \sum_\alpha F_\alpha(\gamma) \left(m_2 f_{\alpha,l}^{(\sigma)}(\gamma) + m_1 \bar{f}_{\alpha,l}^{(\sigma)}(\gamma) \right) \right].$$

Probe Limit

$$\rho_l^{(b)\mu} = \left\{ \hat{b}^\mu, \frac{a_i \cdot \hat{L}}{|b|} \hat{b}^\mu, \frac{a_i \cdot \hat{b}}{|b|} \hat{L}^\mu \right\},$$

$$\rho_l^{(v)\mu} = \left\{ v_j^\mu, \frac{a_i \cdot \hat{L}}{|b|} v_j^\mu, \frac{a_i \cdot v_{\bar{i}}}{|b|} \hat{L}^\mu \right\}.$$

Comparable Mass

$$\tilde{\rho}_l^{(b)\mu} = \left\{ \hat{b}^\mu a_1 \cdot v_2, \hat{b} \cdot a_1 v_1^\mu, \hat{b} \cdot a_1 v_2^\mu \right\},$$

$$\tilde{\rho}_l^{(v)\mu} = \left\{ \hat{b} \cdot a_1 \hat{b}^\mu, a_1 \cdot v_1 v_2^\mu, a_1 \cdot v_2 v_2^\mu \right\}$$

- Expanded on basis of **weight-0,1,2 functions**, and **elliptic E/K:**

$$F_\alpha^{(b)}(\gamma) = \{1, \text{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_\pm}{2}\right], \text{arccosh}^2[\gamma], \text{arccosh}[\gamma] \log\left[\frac{\gamma_\pm}{2}\right], \log\left[\frac{\gamma_+}{2}\right] \log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right], \text{Li}_2\left[\pm\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], K^2\left[\frac{\gamma_-}{\gamma_+}\right], E^2\left[\frac{\gamma_-}{\gamma_+}\right], K\left[\frac{\gamma_-}{\gamma_+}\right] E\left[\frac{\gamma_-}{\gamma_+}\right]\}$$

$$F_\alpha^{(v)}(\gamma) = \{1, \text{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_+}{2}\right], \text{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \text{Li}_2\left[-x^2\right], \text{Li}_2\left[-x\right]\}$$

$$\gamma_\pm = \gamma \pm 1$$

$$x = \gamma - \sqrt{\gamma^2 - 1}$$

CONCLUSIONS & OUTLOOK

State-of-the-art 4PM spinning observables demonstrates **power of worldline-based methodology:**

- **Avoids quantum corrections: classical observables** from **tree-level diagrams**
- **Supersymmetric** encoding of **spin degrees of freedom**
- **Flow of causality** assured by use of **retarded propagators, in-in formalism**
- **Energy conservation** on the worldline controls **regions of integration**

- **Spin^2**: calculations on the way!
- **Spin$^{>2}$**: must first overcome theoretical limitations
- **5PM**: work begun by [Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]
- **EOB Resummation** for large scattering angles, mapping to bound orbits

THANKS FOR LISTENING!

SCATTERING ANGLE

- For **aligned spins**, introduce a **scattering angle**:

$$\cos\theta = \frac{\mathbf{p}_i \cdot \mathbf{p}_f}{|\mathbf{p}_i||\mathbf{p}_f|}$$

$$p_1^\mu = (E_1, \mathbf{p}_i)$$
$$p_2^\mu = (E_2, -\mathbf{p}_i)$$

- Defined with respect to the **centre-of-mass frame** at t=-∞

$$\theta^{(4)} = \theta_0^{(4)} + \nu\theta_1^{(4)} + \frac{\nu}{\Gamma^2}\left(\theta_2^{(4)} + \nu\theta_3^{(4)}\right)$$

Probe limit ($\theta_0^{(4)}$) | Cons PP+RR ($\theta_1^{(4)}$) | Rad PR ($\theta_2^{(4)}$) | Rad PR+RR ($\theta_3^{(4)}$)

$$\nu = m_1 m_2/M^2$$
$$\Gamma = \sqrt{1+2\nu(\gamma-1)}$$

$$\theta_m^{(4)} = \theta_m^{(4,0)} + \theta_m^{(4,+)} s_+ + \theta_m^{(4,-)}\delta s_-$$

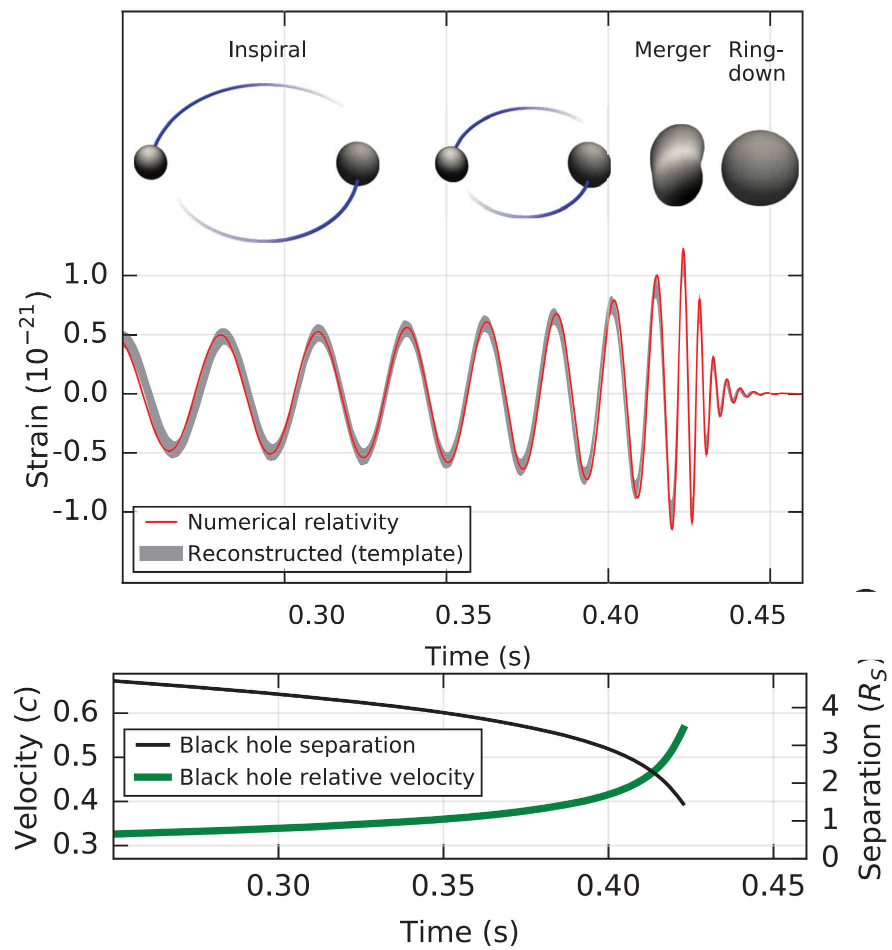
$$s_\pm = -(a_1 \pm a_2)\cdot\hat{L}$$
$$\delta = (m_2 - m_1)/M$$

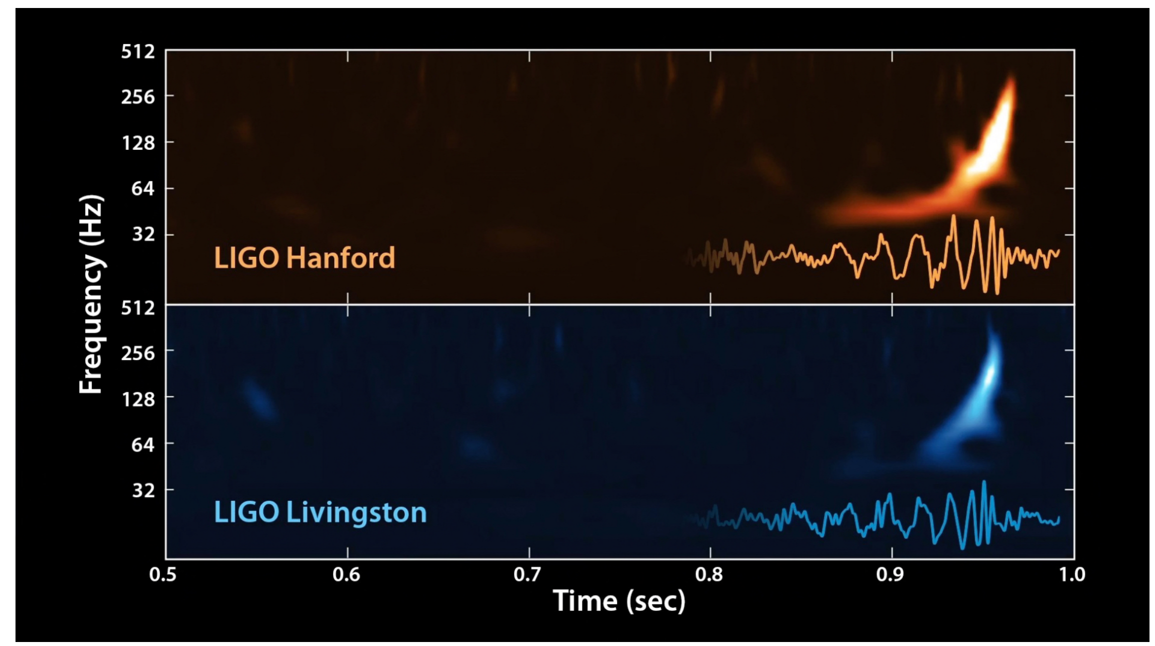
- From **linear response**, learn the **radiated angular momentum**:

$$\theta_{\rm diss}^{(4,{\rm PR})} = \frac{1}{2}\frac{\partial\theta}{\partial J}\Delta J + \frac{1}{2}\frac{\partial\theta}{\partial E}\Delta E\bigg|_{G^4}$$

AMPLITUDES MEETS GRAVITATIONAL WAVES

- In the meantime, amplitudes was shifting its focus to **black holes**… the most perfect *macroscopic* objects in the Universe.





First detection of a binary merger event by GW emission ***GW150914***

- Can ideas and techniques from *quantum* particle scattering be used to describe the *classical* 2-body problem?
- Can we make **high-precision GW predictions**? Needed for upcoming LIGO/Virgo/KAGRA runs, 3rd generation detectors (Cosmic Explorer, LISA, Einstein Telescope)

STATE-OF-THE-ART PM CALCULATIONS

WQFT [Plefka, Steinhoff, Jakobsen, GM, Sauer]

WEFT Worldline effective theory [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

Amps Scattering amplitudes [Bern,Roiban,Shen,Parra-Martinez,Ruf,..] [Bjerrum-Bohr,Damgaard,Vanhove,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..][Huang,..][Guevera,Ochirov,Vines,…] [Johansson,Pichini[Kosower,O'Connell,Maybee,Cristofoli, Gonzo…]

HEFT Heavy BH effective theory [Aoude,Haddad,Helset] [Brandhuber,Travaglini,Chen]

	Impulse & spin kick				waveform			
	plain	spin²	spin>2	tidal	plain	spin²	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	WQFT WEFT Amps	WQFT WEFT	WQFT WEFT	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT HEFT	Amps	WQFT WEFT Amps	Amps			~ 1-loop
3PM (Cons)	WQFT WEFT Amps HEFT	WQFT (Amps)		WQFT WEFT				~ 2-loop
3PM (Rad)	WQFT WEFT Amps HEFT	WQFT (WEFT)		WQFT WEFT				~ 2-loop
4PM w/o r-r	WEFT Amps							~ 3-loop

EFFECTIVE-ONE-BODY (EOB)

- Ongoing work with **Alessandra Buonanno, Raj Patil (AEI)**.
- Achieve better agreement with NR close to merger (strong-field regime) by **resumming 2-body motion**.
- Map real 2-body motion to a test particle in a **deformed Kerr background.**

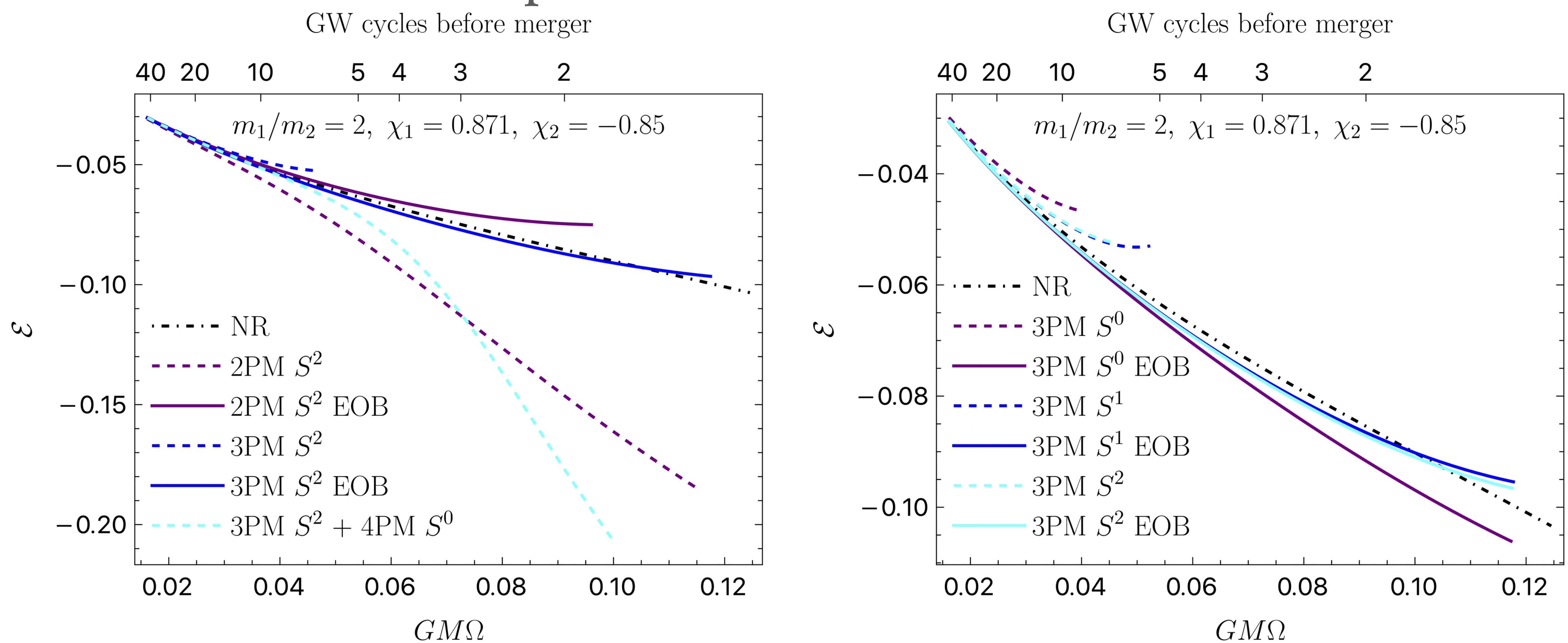
$$H_{\text{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_{\text{eff}}}{\mu} - 1\right)} \qquad \mu = \frac{m_1}{m_2}M, \quad \nu = \frac{\mu}{M}, \quad M = m_1 + m_2$$

- Use a **deformed Kerr Hamiltonian** (aligned spins, same setup as SEOBNRv5), match by comparison with the scattering angle

$$H_{\text{eff}} = \frac{J(g_{a_+}a_+ + g_{a_-}\delta a_-)}{r^3 + a_+^2(r+2)} + \sqrt{A\left(1 + Bp^2 + C\frac{J^2a_+^2}{r^2} + Q\right)}$$

EOB BINDING ENERGIES

- Binding energy plots show a **big improvement** over non-resummed counterparts!



- Work still to be done: include 4PM in the EOB model, and higher-PN information.
- My ultimate goal is to complement PN waveform models