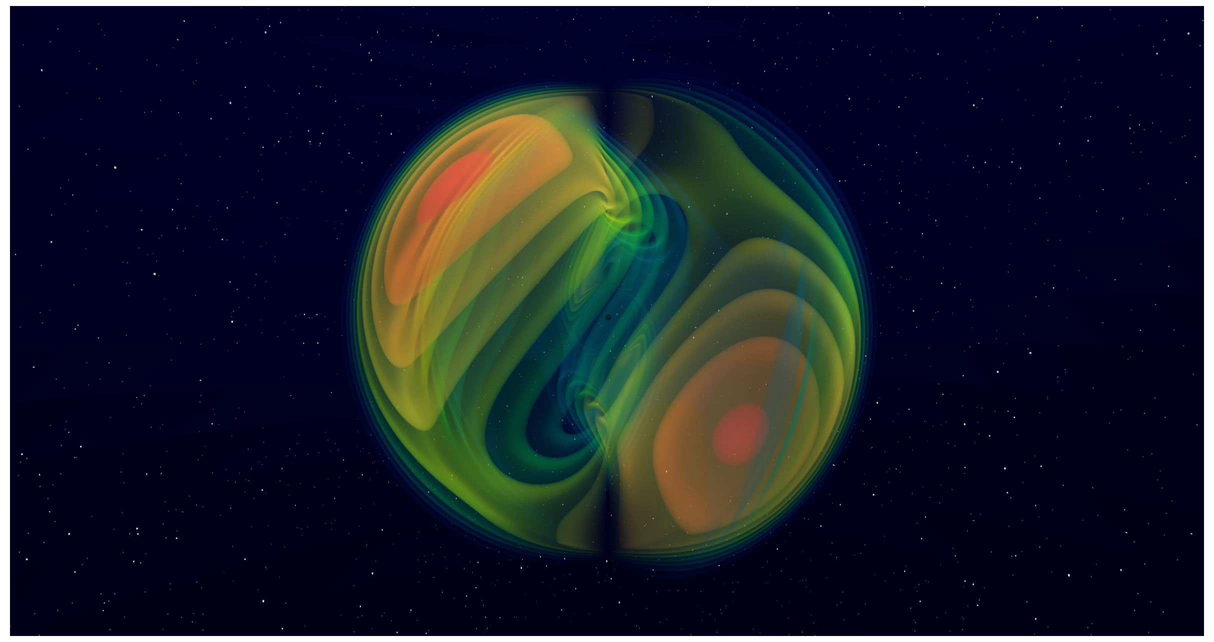
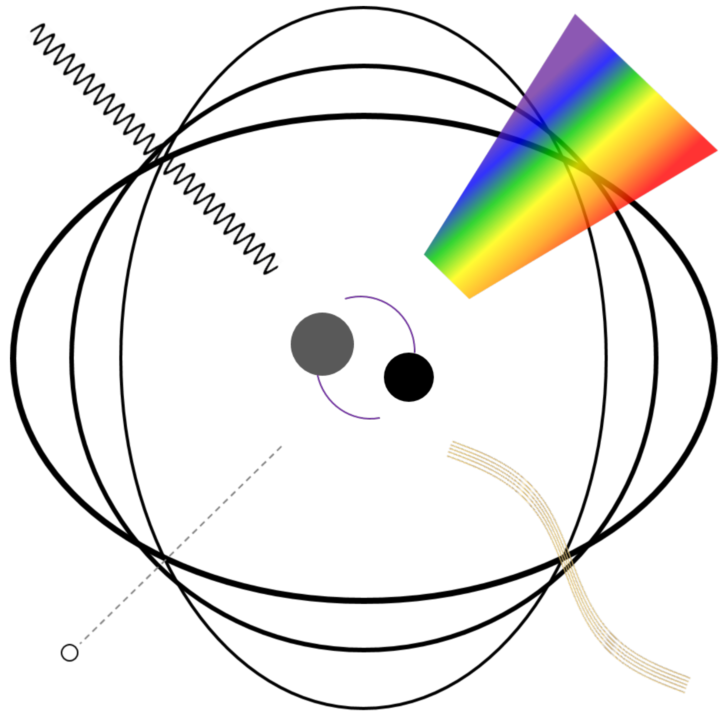
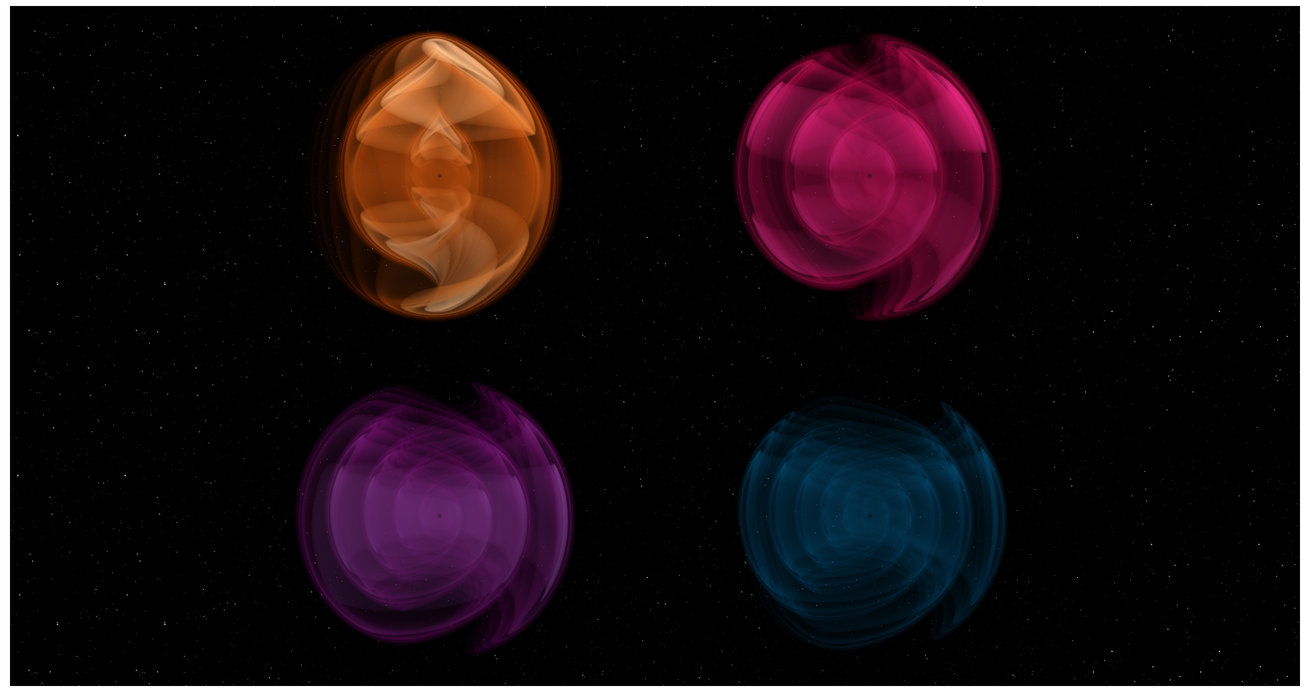
GW190412





GW190814



Developing High-Precision Gravitational-Wave Models

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

(Albert Einstein Institute), Potsdam

Deutsche Forschungsgemeinschaft
German Research Foundation

International Balzan Foundation

"From Amplitudes to Gravitational Waves", Stockholm

July 25, 2023

Motivations/Outline

- **Gravitational waves** have become a **groundbreaking tool to explore** the Universe.

- Inferring **astrophysical and cosmological information** from GW observations, detecting **possible deviations from GR** and **discriminating** them **from astrophysical environmental** and **cosmological effects, rely on accurate predictions** of **two-body dynamics** and **gravitational radiation.**

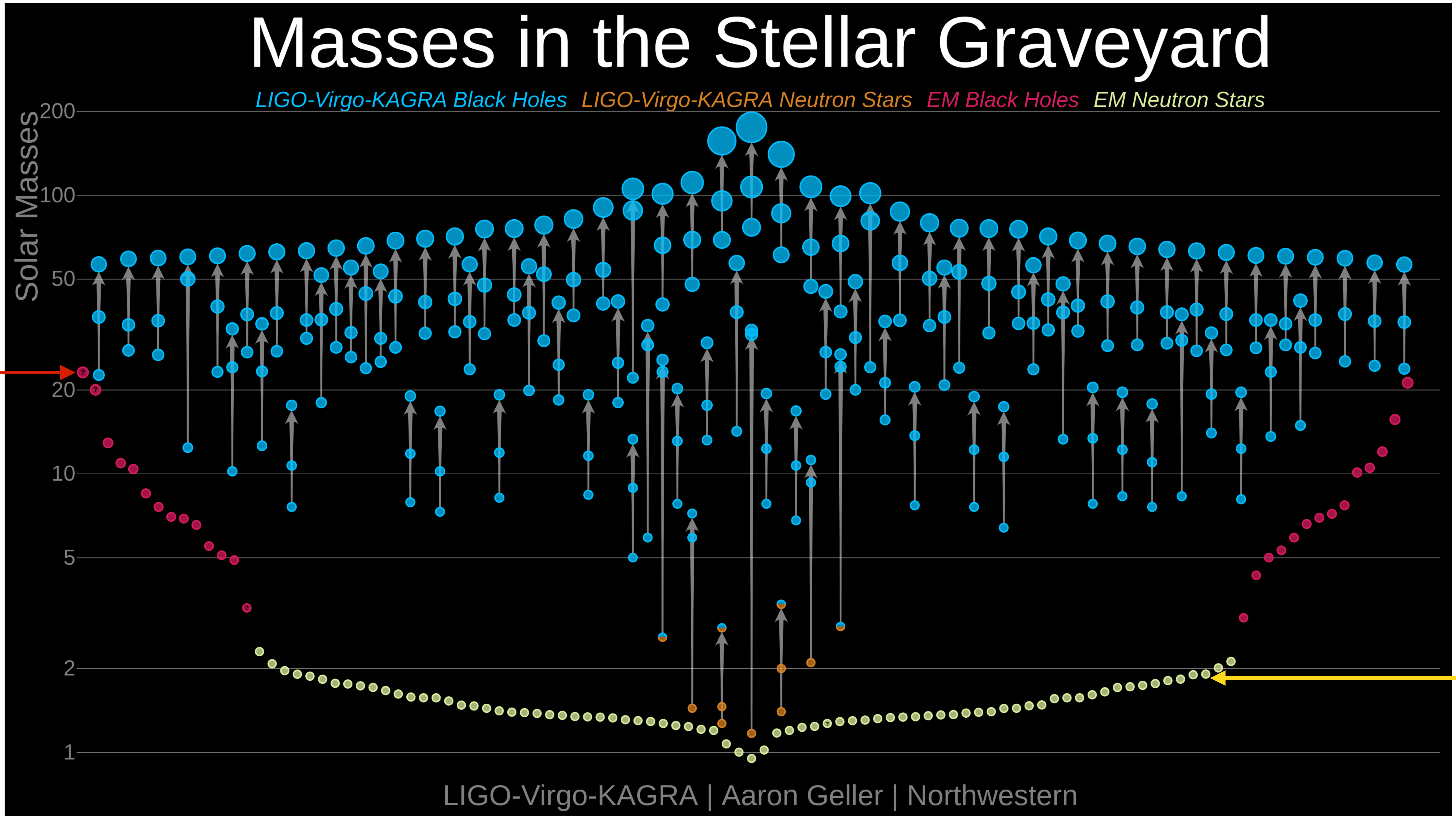
- **Upcoming runs with LIGO-Virgo-KAGRA and future detectors** in space and on the ground, **require ever more accurate and precise** waveform models, which **include *all* physical effects** (spins, tides, eccentricity, beyond-GR effects, non-vacuum GR's effects, etc.).

- What does it take to **build faithful waveform models** for the entire coalescence **combining the different analytical** methods with **numerical** relativity, and **how perturbative results from scattering-amplitude/ EFT/QFT (PM) and GSF** calculations could **be employed to improve waveforms?**

Discovering/Characterizing Black Holes & Neutron stars in the Universe

- As today, gravitational waves were observed by **LIGO-Virgo detectors** from about **100 coalescences.**

Cygnus X-1

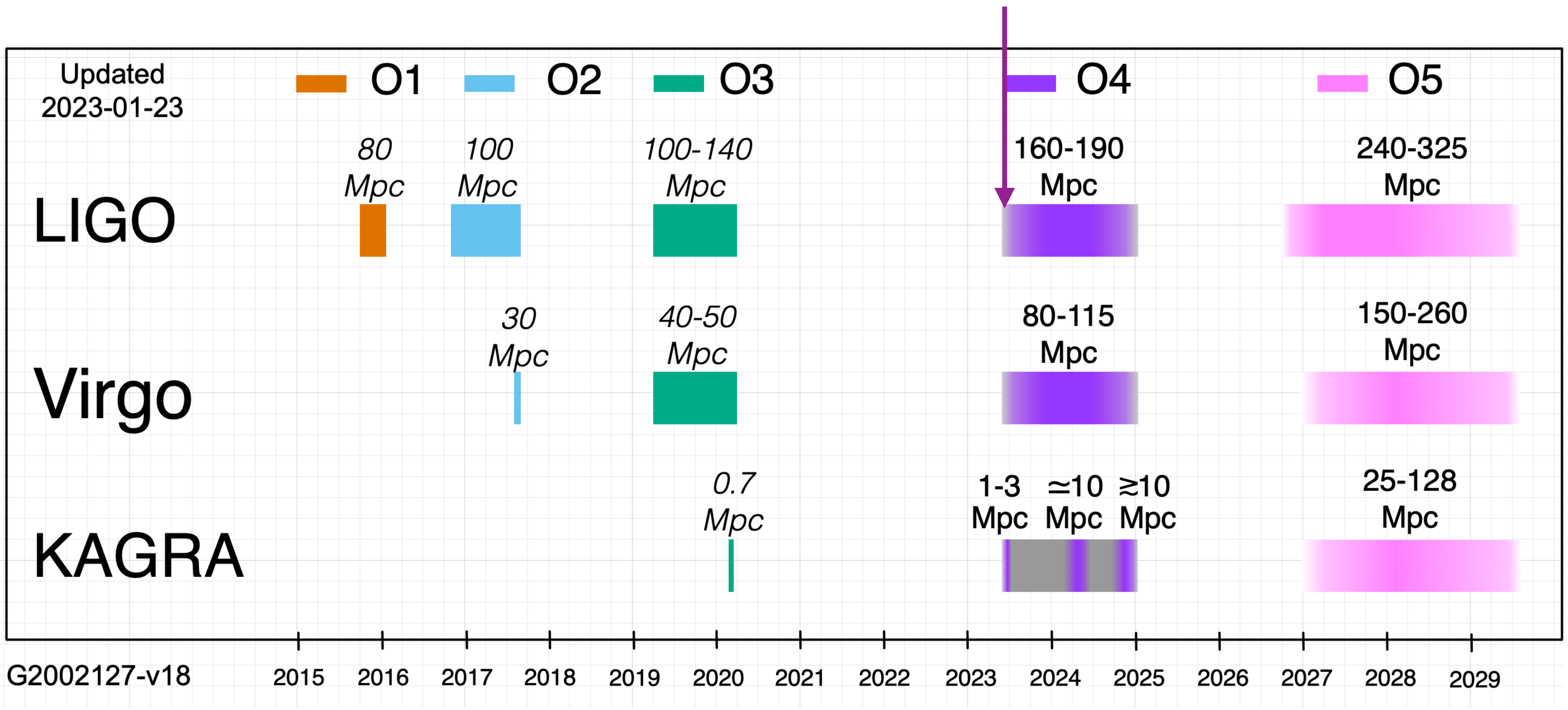


Pulsar

GW Astronomy on the Ground until 2030 in the hectoHz

Fourth observational (O4) run started on May 24 for LIGO detectors!

20 GW signals have been already observed!



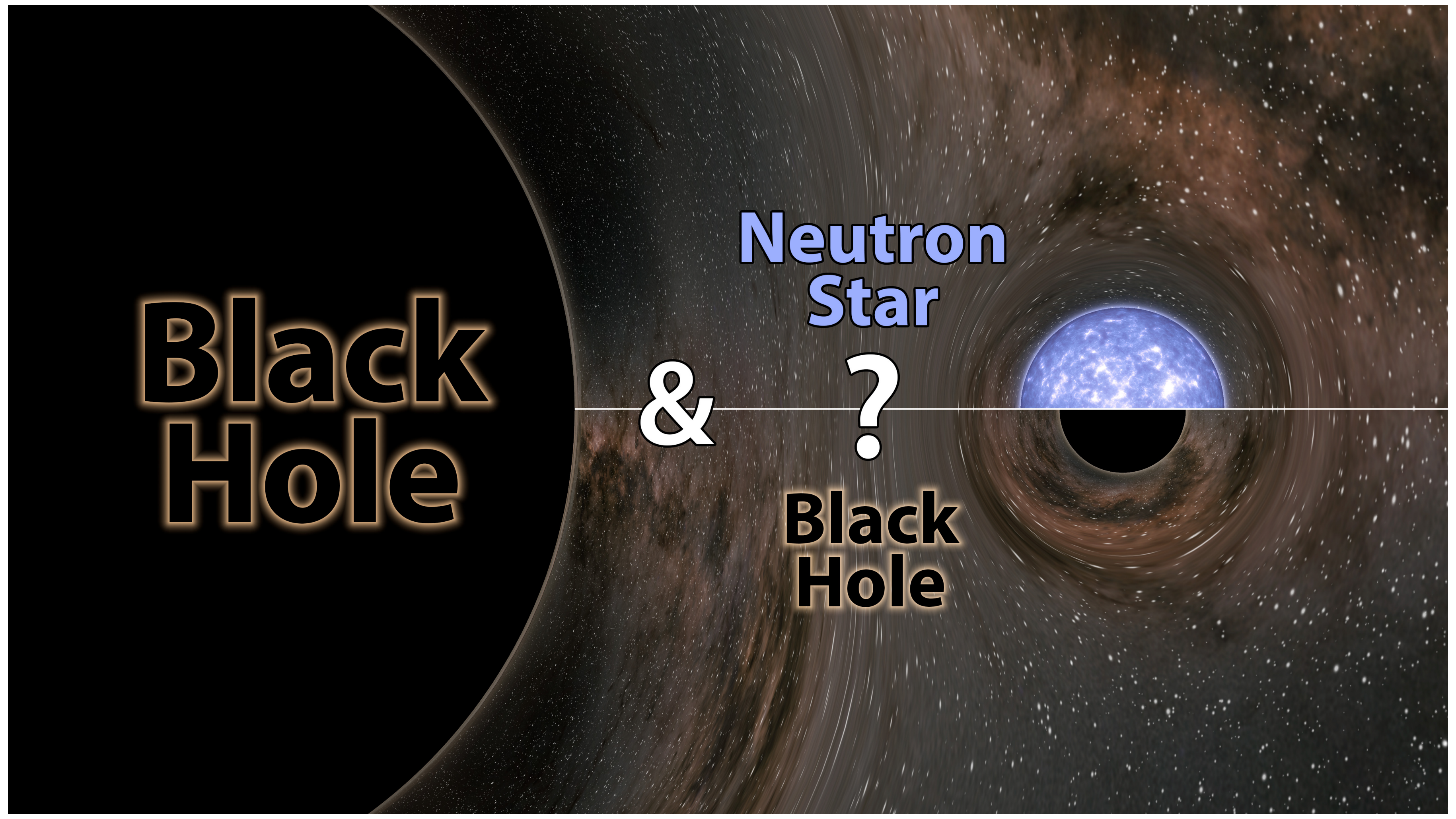
- From **several tens to thousand** of binary detections per year.
- Inference of **astrophysical properties** of BBHs, NSBHs and BNSs **in local Universe ($z \lesssim 1-2$).**

Some highlights on the science of the last observing run (O3).

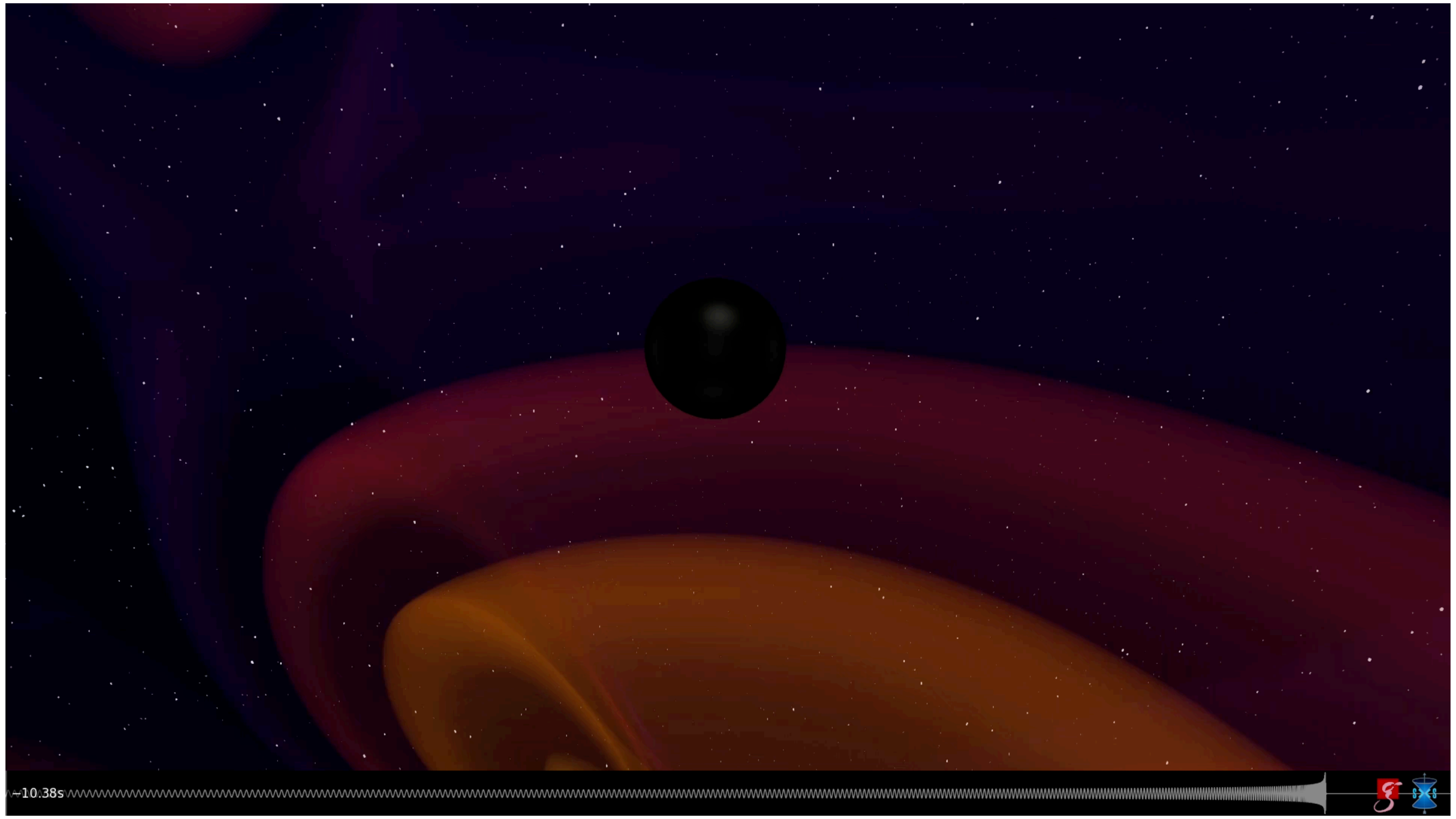
GW190814: a Binary with a Puzzling Companion

GW190814: a binary with a puzzling companion

- A black hole 23 times the mass of our Sun merging with an *object* just 2.6 times the mass of the Sun.



- The more substructure and complexity the binary has (e.g., masses or spins of black holes are different) the richer is the spectrum of radiation emitted: higher harmonics.

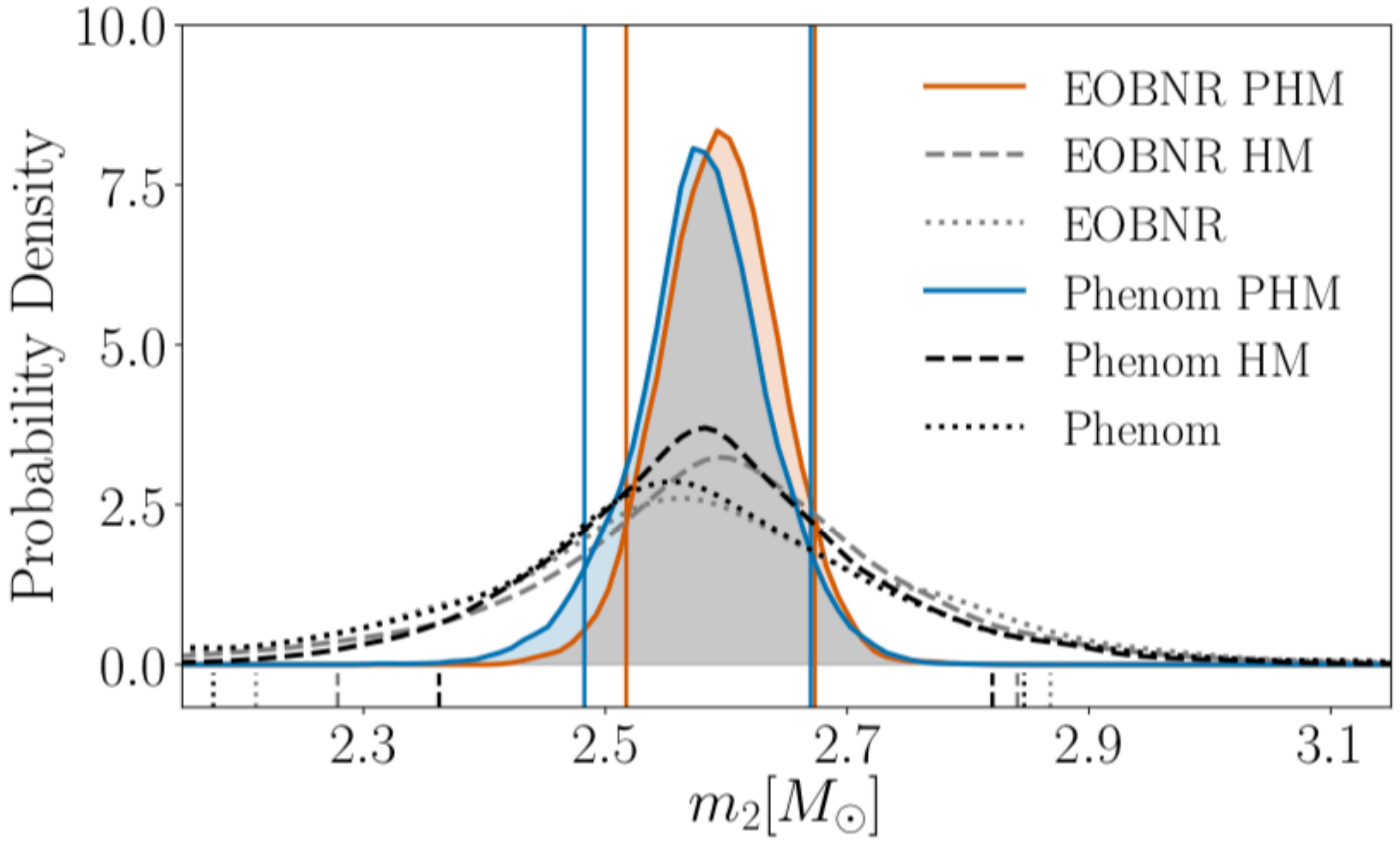


(credit: Fischer, Pfeiffer, Ossokine & AB; SXS project)

GW190814: a Binary with a Puzzling Companion (contd.)

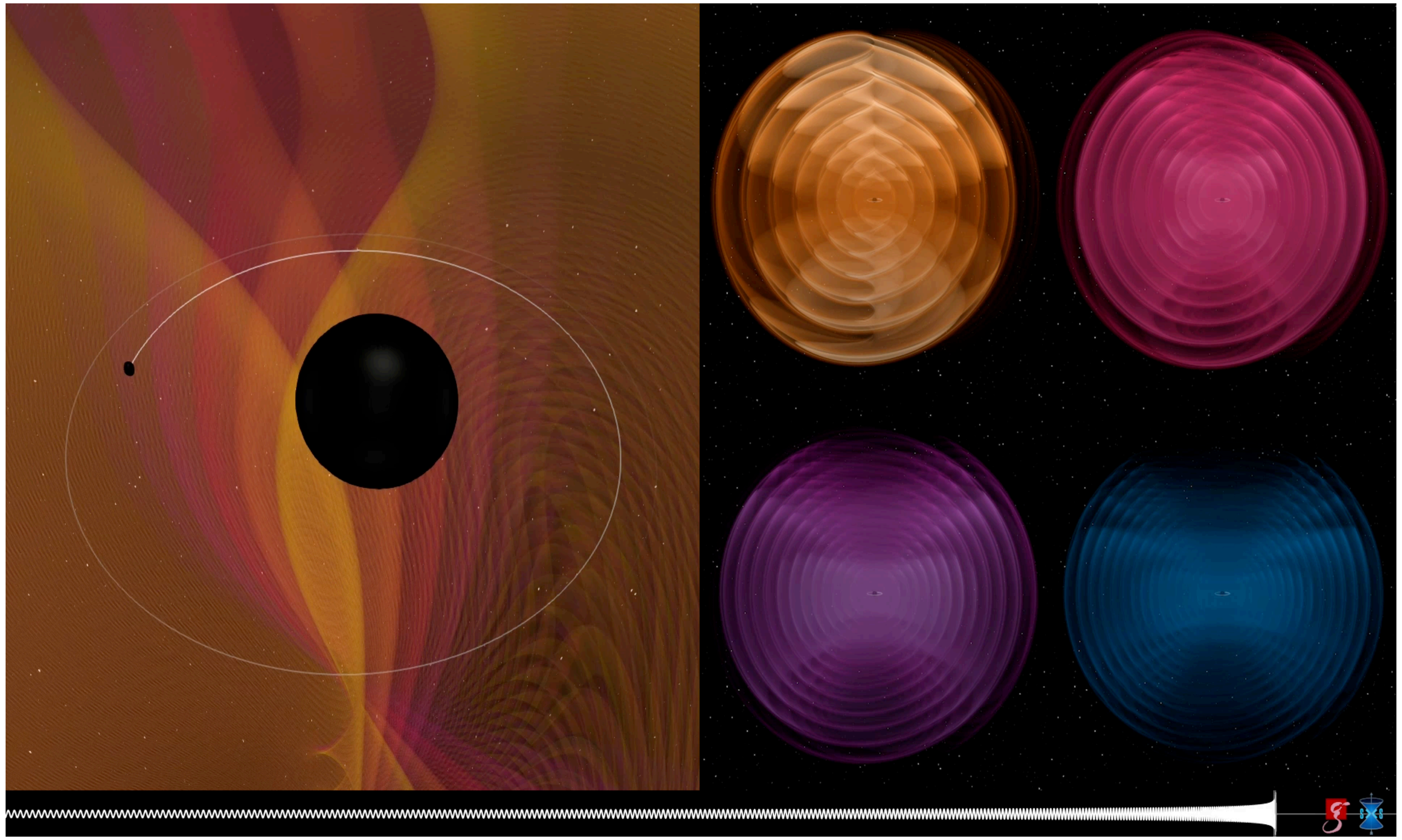
- Either the **largest neutron star** or the **smallest black hole.**

$$m_1 = 23.2^{+1.1}_{-1.0}\,M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\,M_\odot$$



- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

- The **more substructure and complexity** the binary has (e.g., masses or spins of black holes are different) **the richer is the spectrum of radiation** emitted: **higher harmonics**.



(credit: Fischer, Pfeiffer, Ossokine & AB; SXS project)

GW190521: a Signal Produced by the Largest BHs

(Abbott et al. PRL 125 (2020) 10, ApJ Lett 900 (2020) L13)

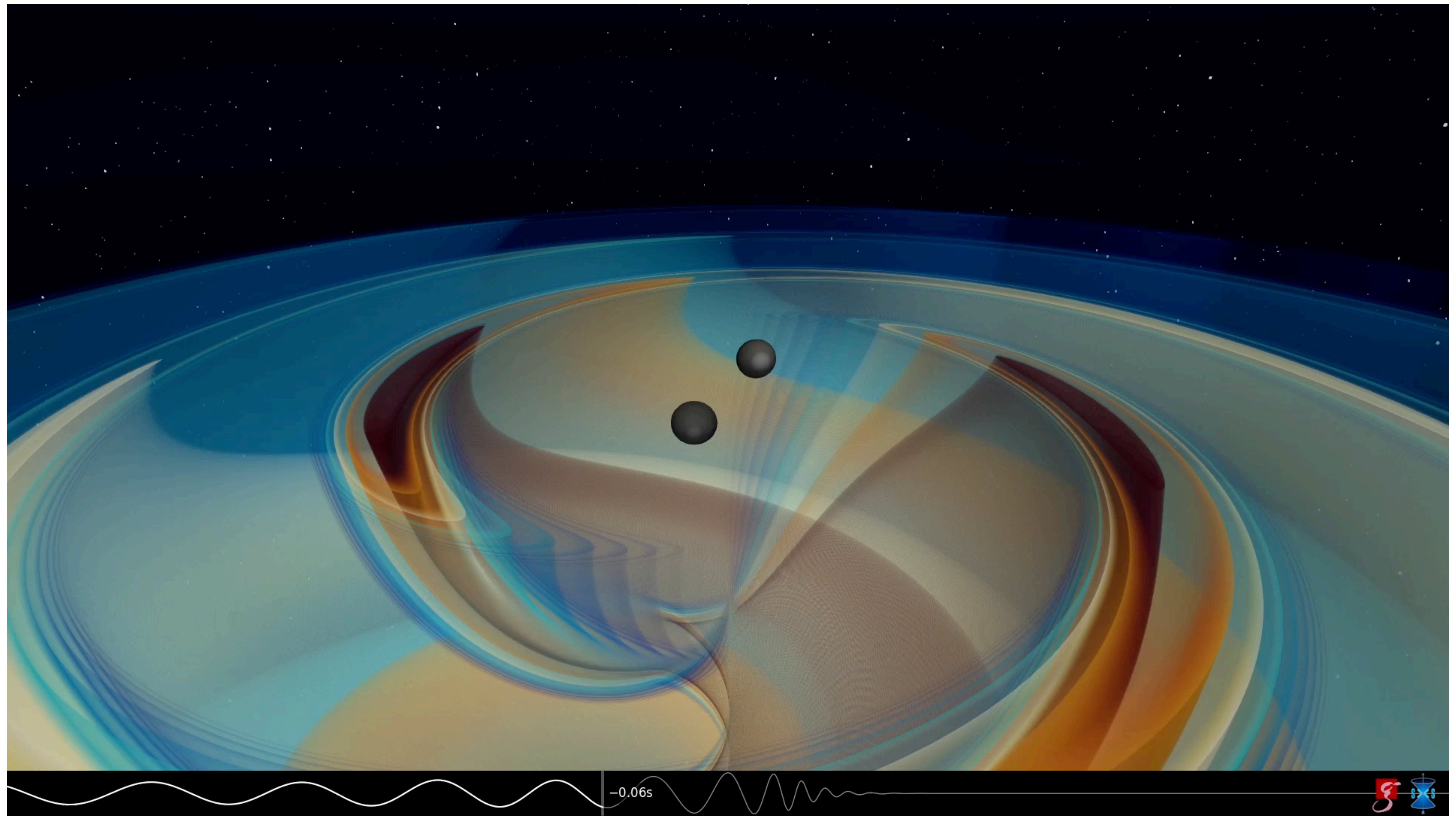
- Likely, BHs **too massive** to have been formed **from a collapsed star, because of Pair-Instability SN (high mass gap).**

$$q = m_1/m_2 \qquad \chi_1 = S_1/m_1^2 \qquad \chi_2 = S_2/m_2^2$$

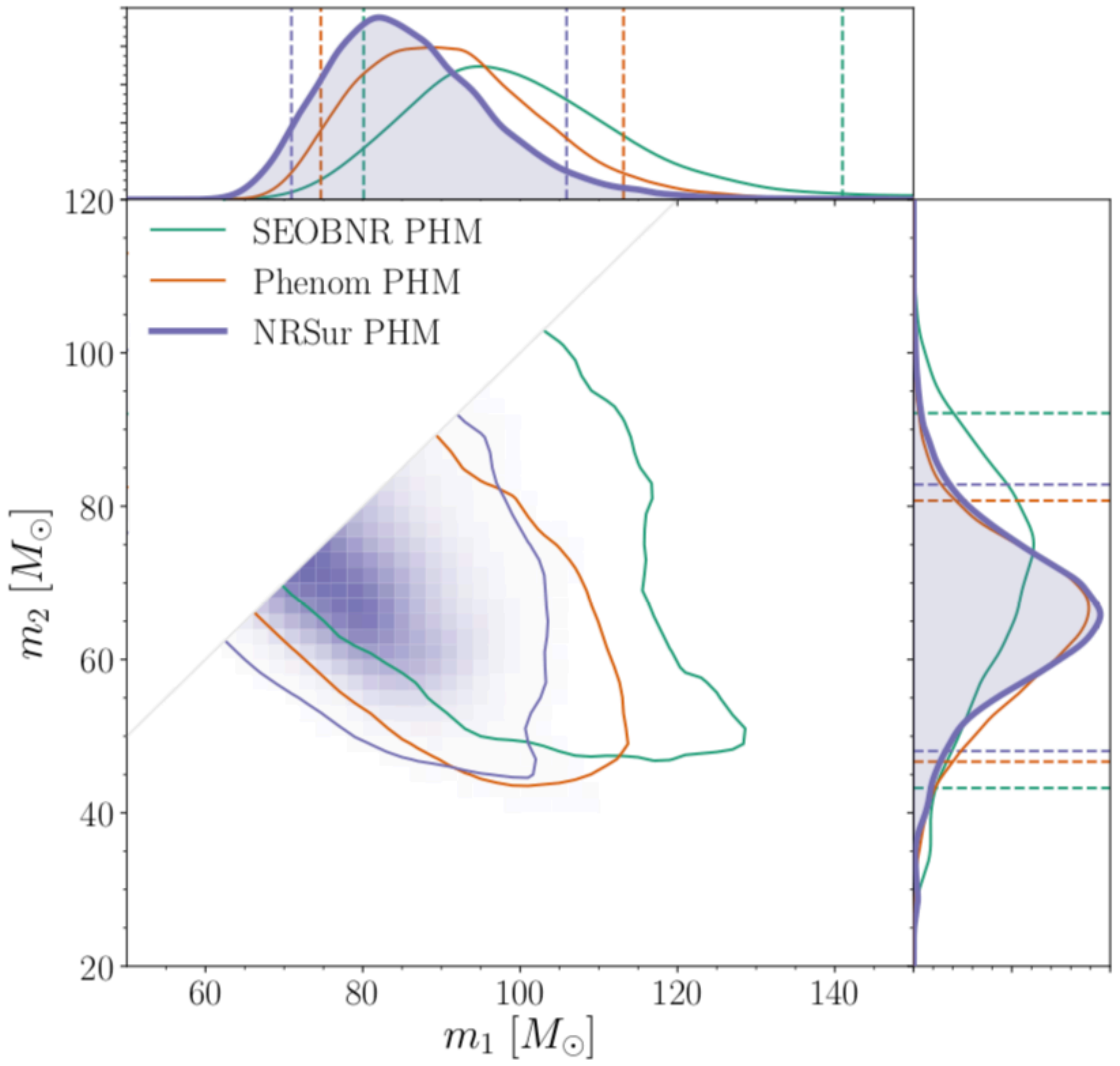
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot \hat{\mathbf{L}}$$

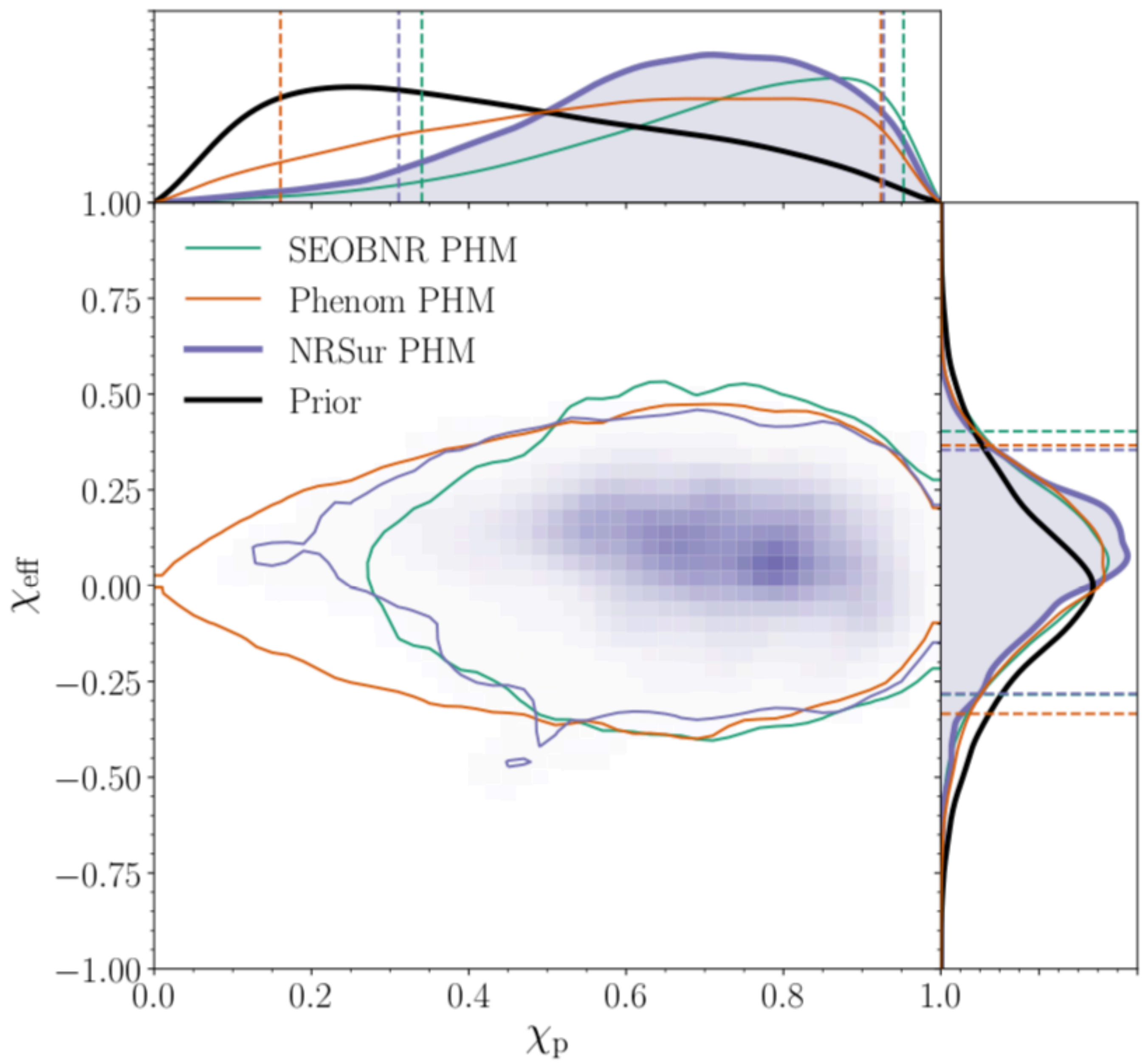
χ_p measures the spin components on the orbital plane

$$m_1 = 91.4^{+29.3}_{-17.5}\,M_\odot \qquad m_2 = 66.8^{+20.7}_{-20.7}\,M_\odot$$



(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

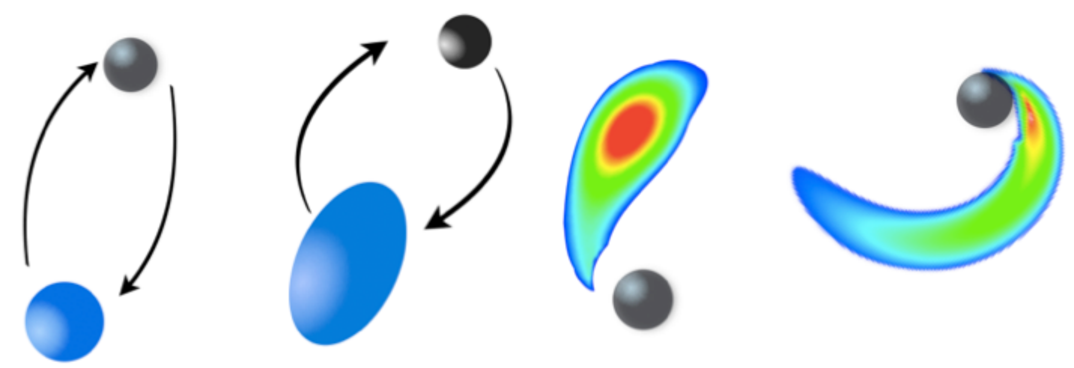




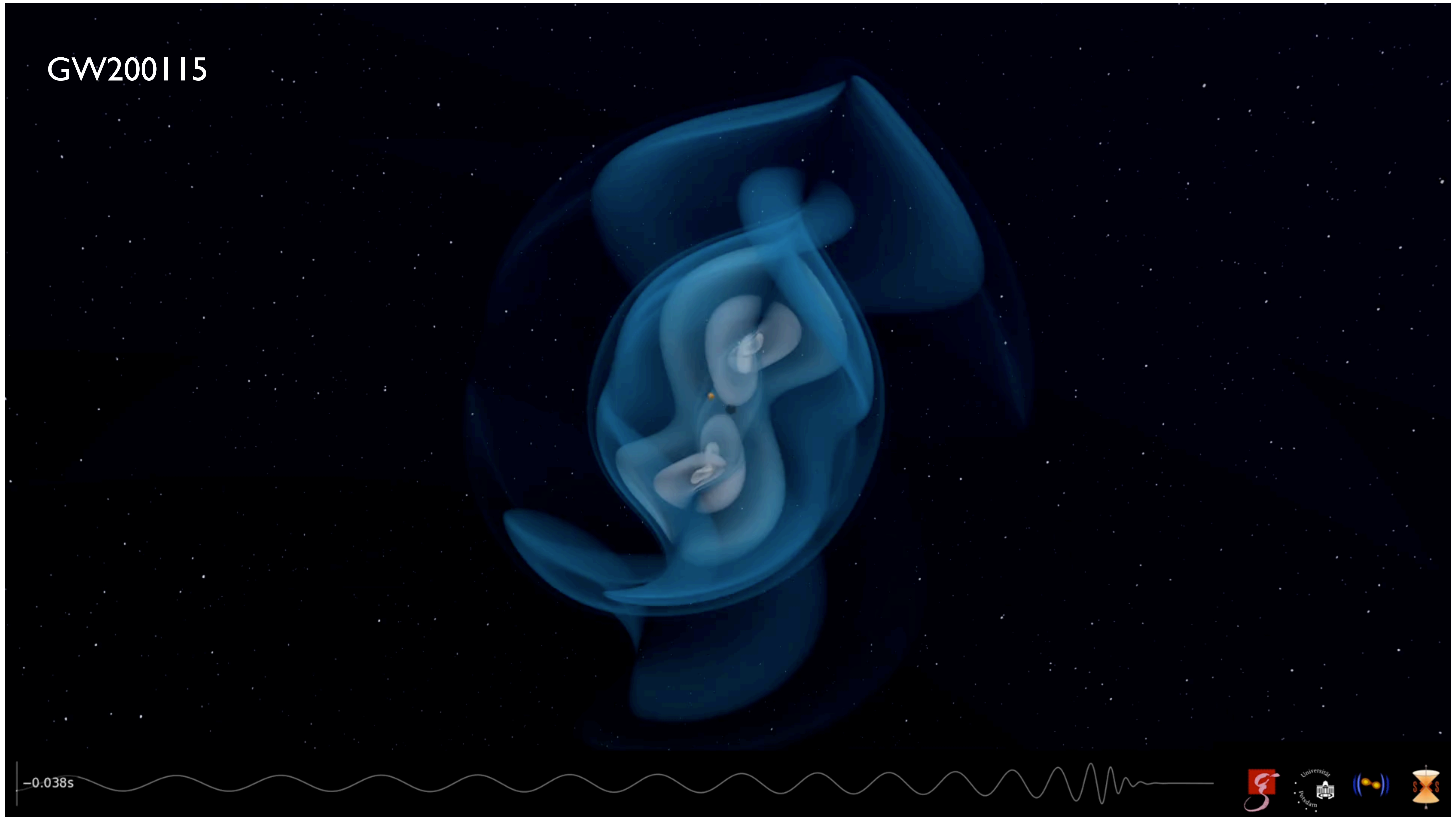
- **Systematics** due to waveform modeling **are not negligible when spin precession** and **higher modes are relevant,** but they are still **subdominant with respect to statistical uncertainty.**

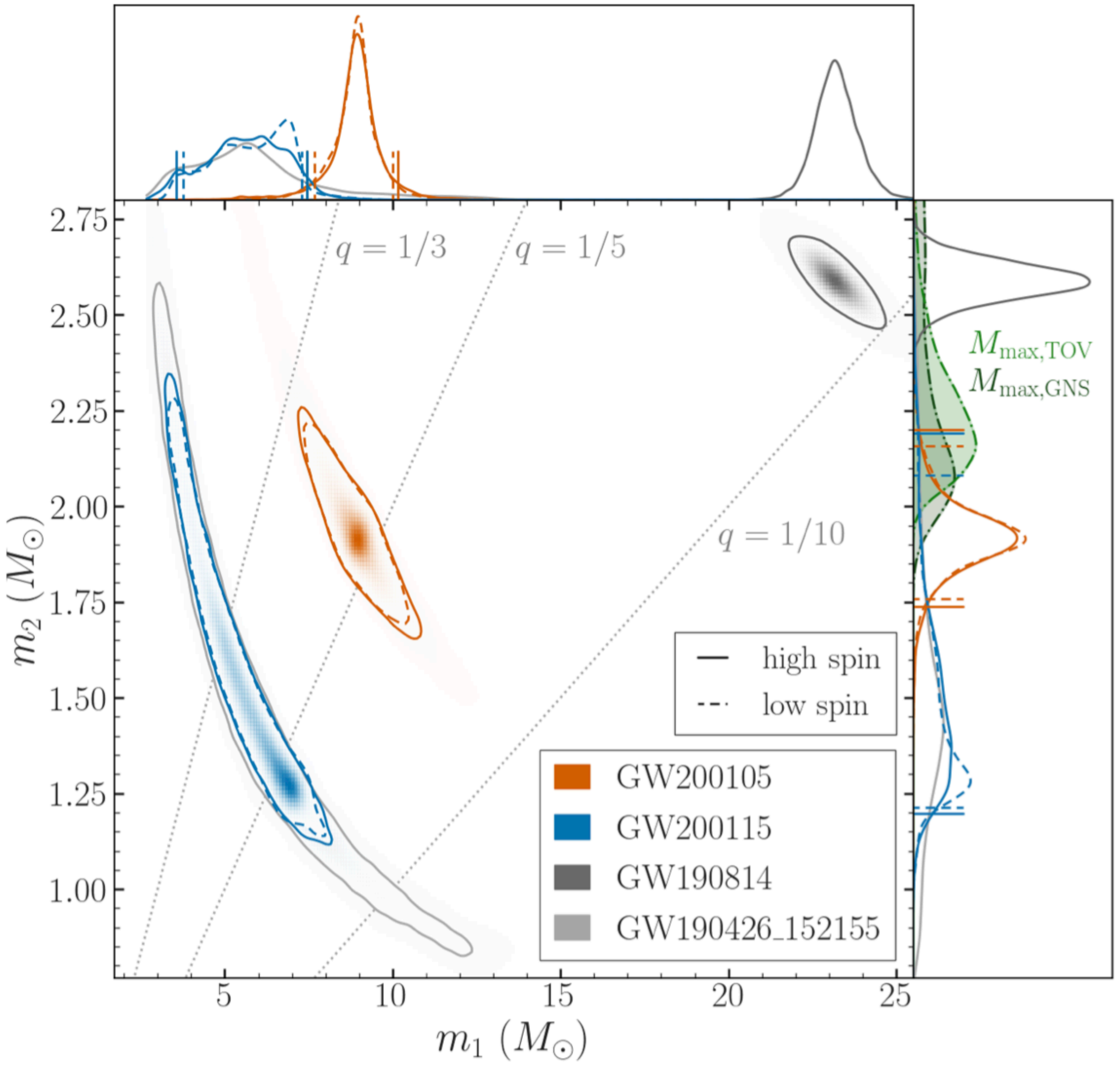
GW200115: a BH swallowing the NS whole

- First **robust detection** of a mixed binary.



(Abbott et al. ApJ 915 (2021))

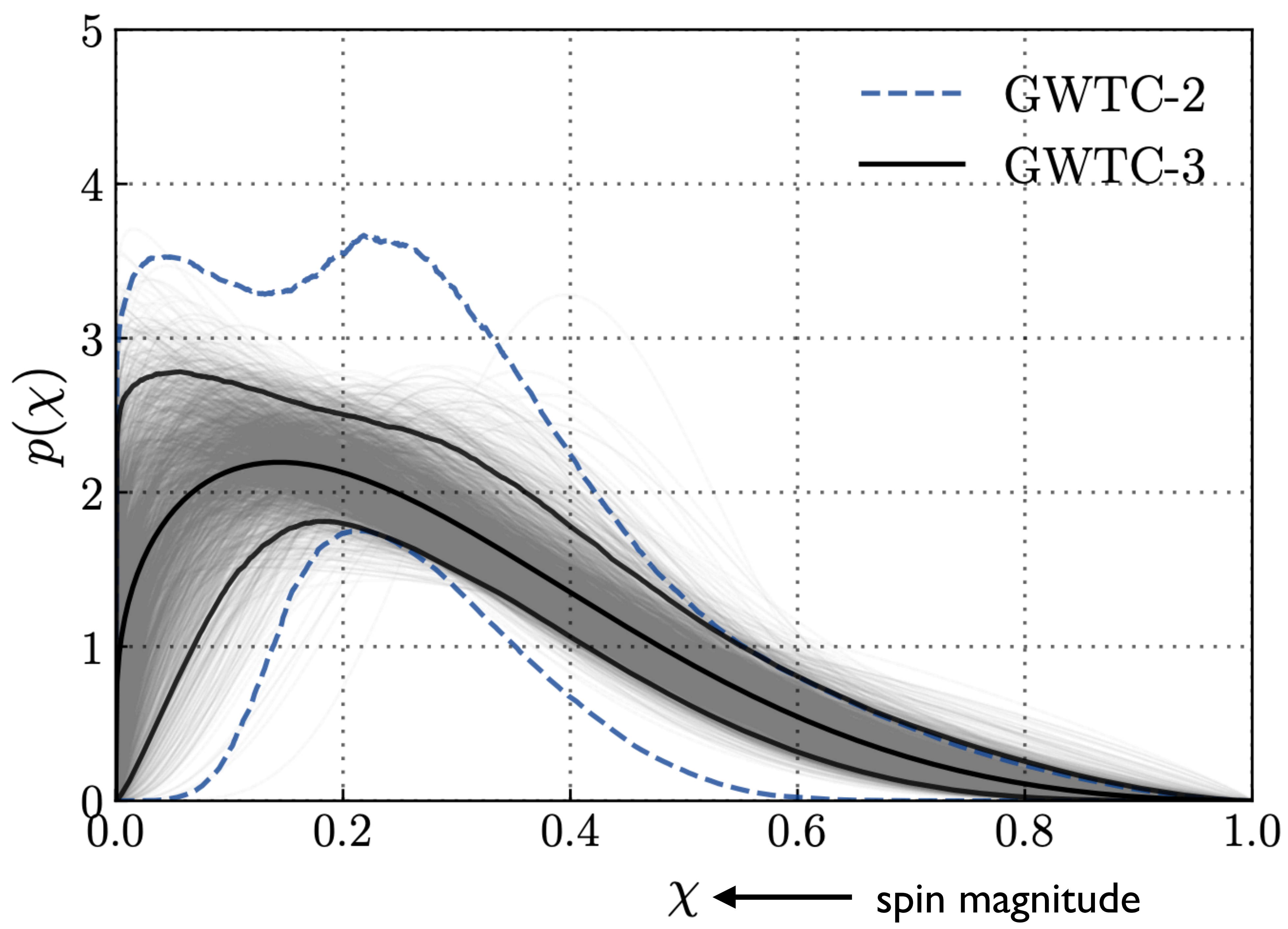


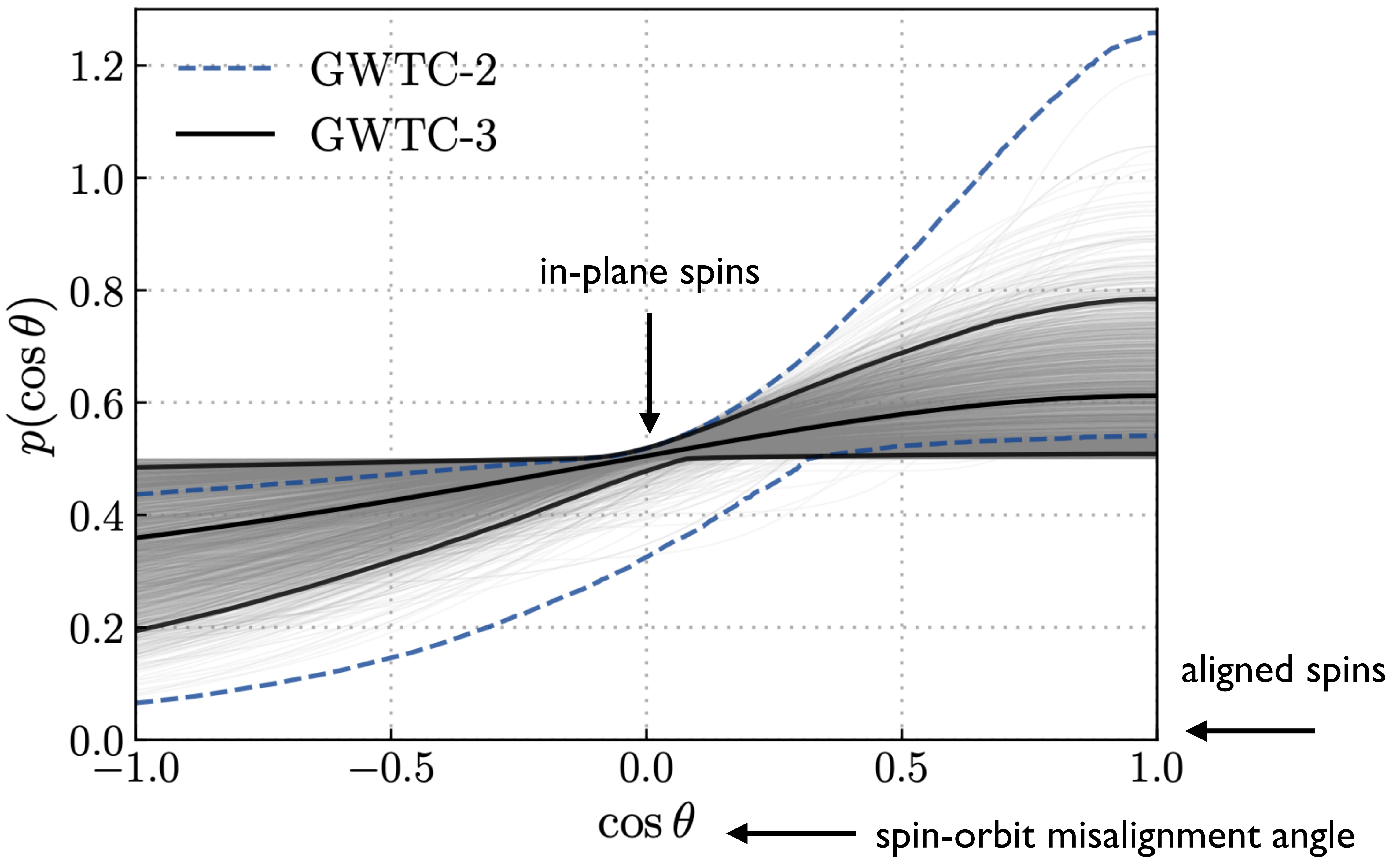


(credit: Chaurasia, Dietrich, Fischer, Ossokine & Pfeiffer)

Gravitational-Wave Transient Catalog 3: Spin Properties

(Abbott et al. Phys. Rev. X13 (2023) 1, 011048)



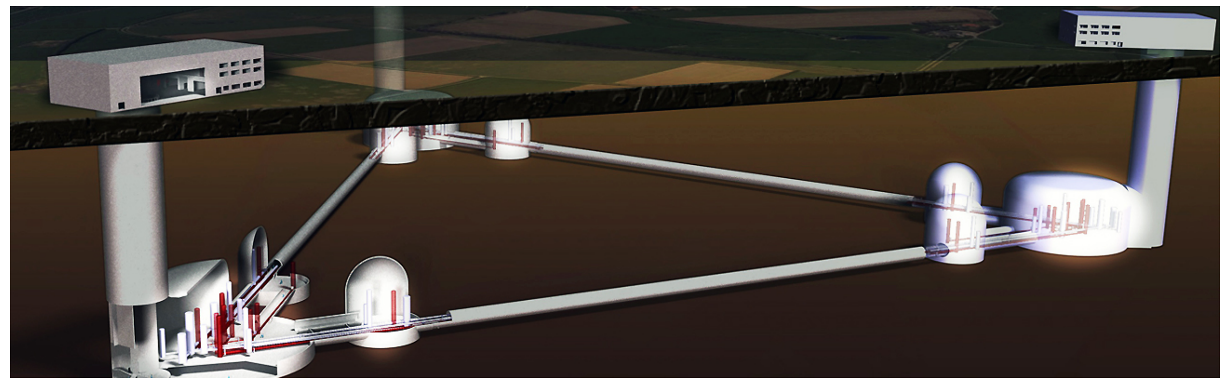


- Observed BH's **spins are small**, but **tail extends to large or maximal values**.
- Evidence of **misalignment of spins** relative to the orbital angular momentum.
- Evidence of **negative aligned spins**, and an **increase in spin magnitude** for systems with **more unequal mass ratio.**

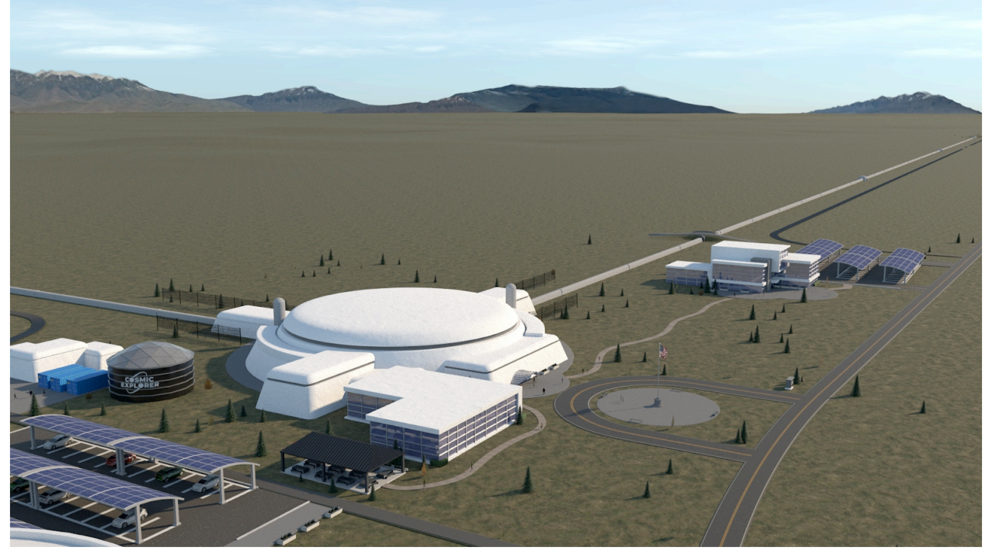
Ever more sensitive detectors in the next decade.

GW Astronomy on the Ground in 2030: from hectoHz to 1Hz

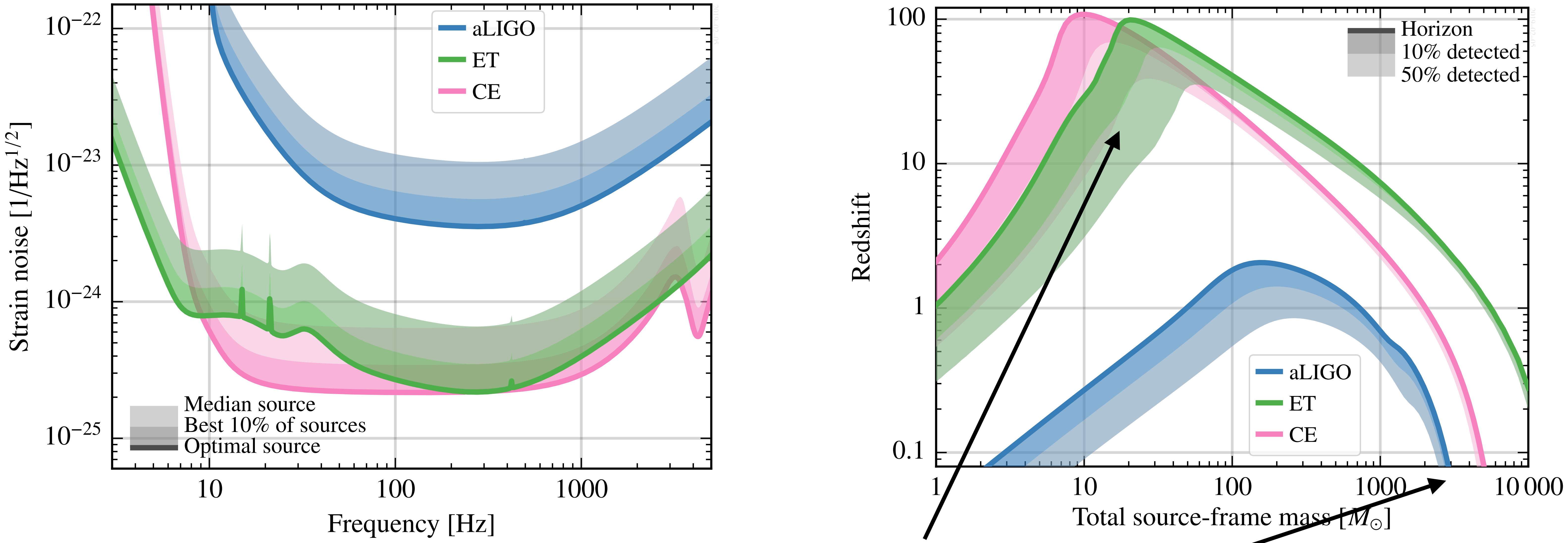
Einstein Telescope (ET)



Cosmic Explorer (CE)



(3G Science-Case Report 21)

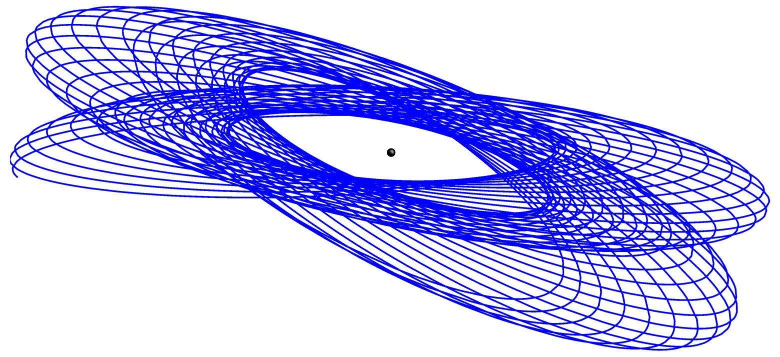


Observe BHs at much larger distance, when first stars formed, and more massive.

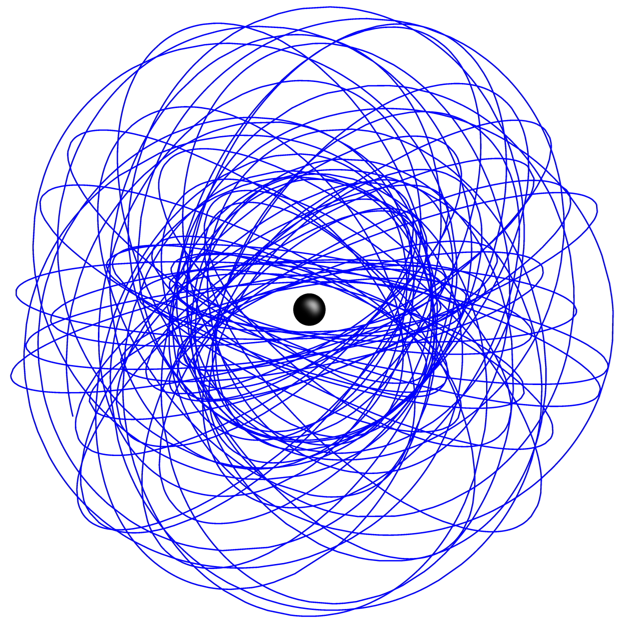
- **Stellar-mass binaries:**
  - Observe each year $\sim 20{,}000$ **BBH signals** with **SNRs** > 100.
  - Observe each year ~ 780 **BNS signals** with **SNRs** > 100.

(Borhanian & Sathyaprakash 22)

- **Intermediate Mass-Ratio Inspirals (IMRIs),** with mass ratio 10^3



at GW frequency ~1Hz

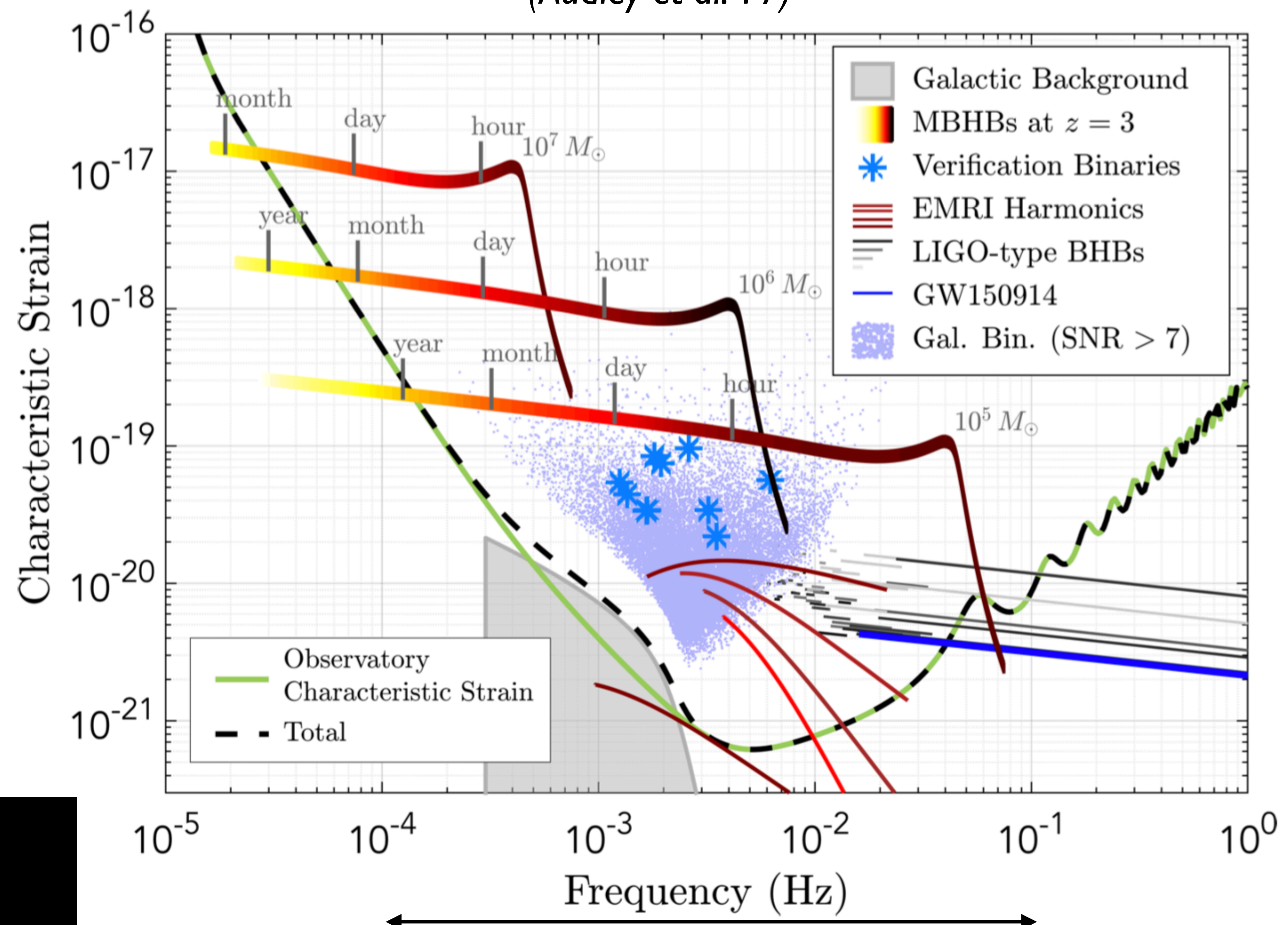


at GW frequency ~10 Hz

(credit: van de Meent)

GW Astronomy in Space in 2030s: from hectoHz to milliHz

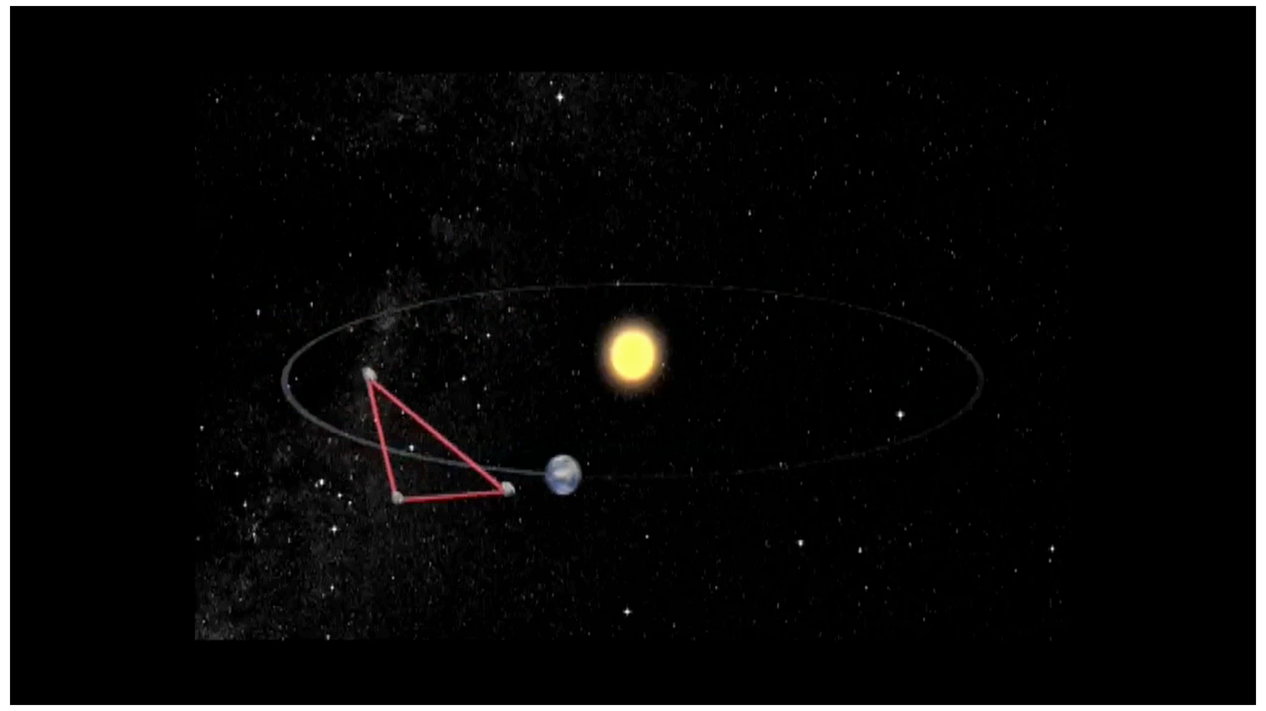
(Audley et al. 17)



opening **three decades** of GW spectrum

- **New data-analysis challenges with LISA** (and in part also with ET/CE).
- GW signals are **much louder,** they have **long-duration and overlap.**

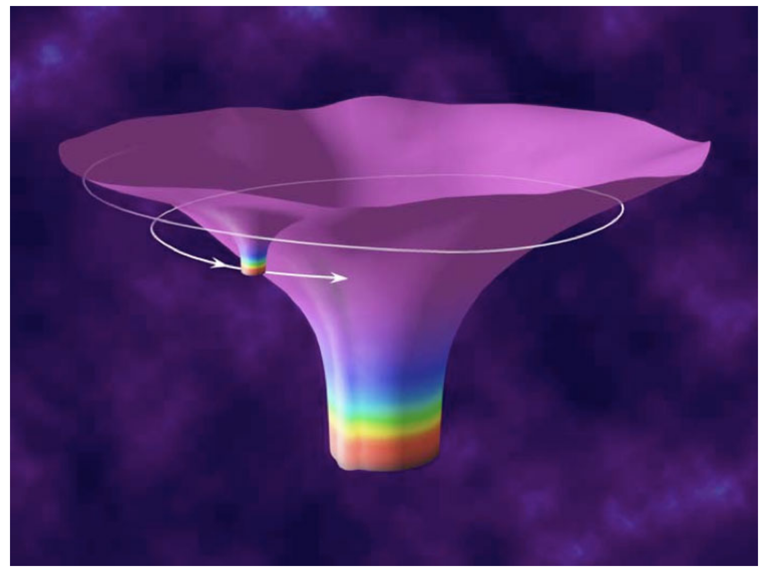
LISA in ~ 2035



(credit: AEI/Milde Marketing)

ESA leading mission with NASA junior partner

- **New** GW **sources**:
  - extreme mass-ratio inspirals (**EMRIs**)
  - massive BHs (**MBHs**)
  - White-Dwarf binaries in our galaxy



EMRI

- **Probing black-hole** properties and gravity **with exquisite precision**.

With ever more sensitive GW detectors, we need ever more accurate waveform models to avoid systematics.

Systematics in Waveform Models with Future Detectors: BBHs

- **Massive BH binary with LISA**

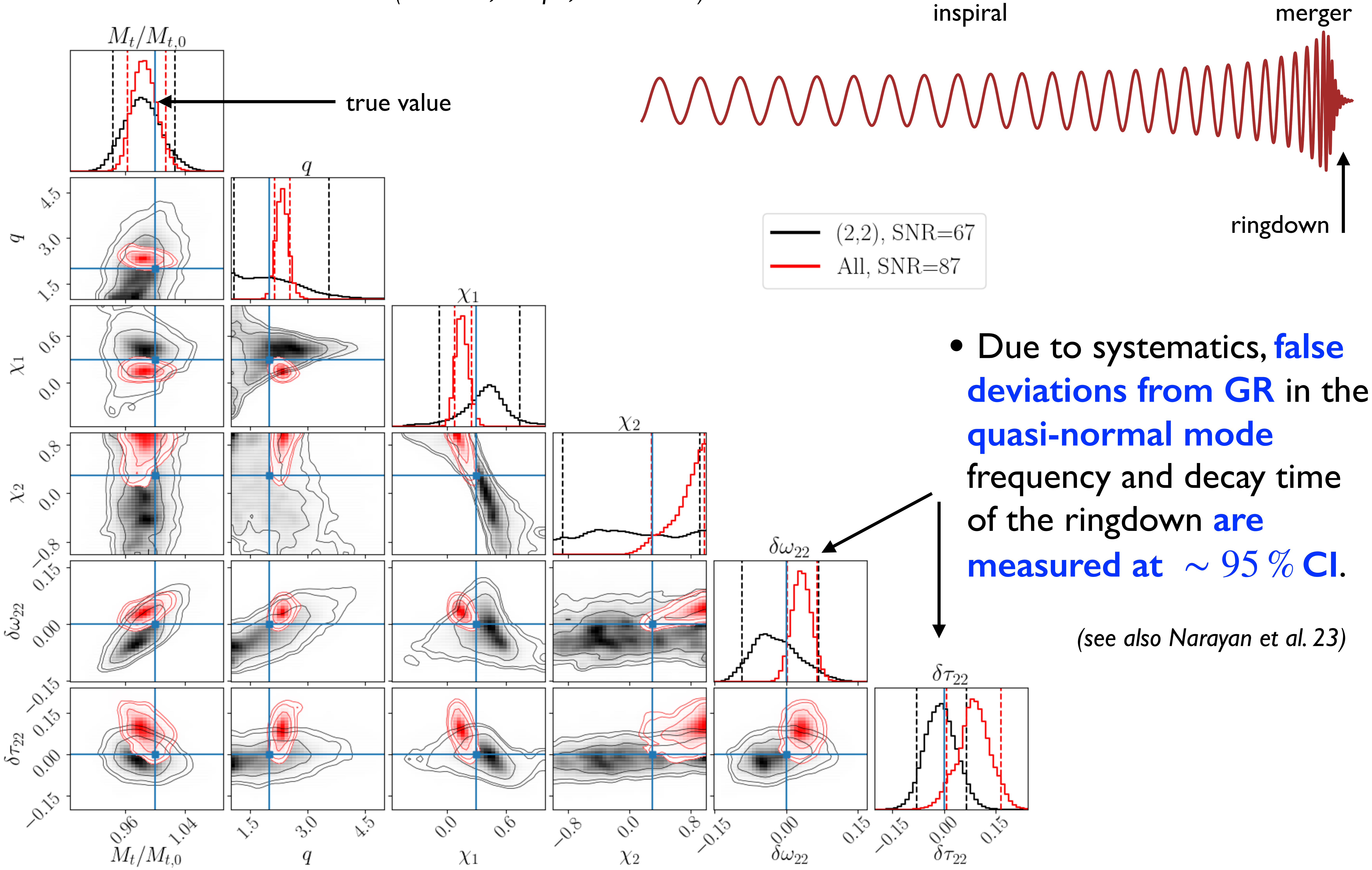
- **Parameters** of synthetic **NR signal in GR** that is **injected**:

$$M = 10^8 M_\odot, \chi_1 = \chi_2 = 0.3$$

$$q = m_1/m_2 = 2, \, z = 5$$

- **Signal is recovered with** (a parameterized) waveform model **pSEOBNRv5HM** using Bayesian analysis.

(Toubiana, Pompili, AB et al. 23)



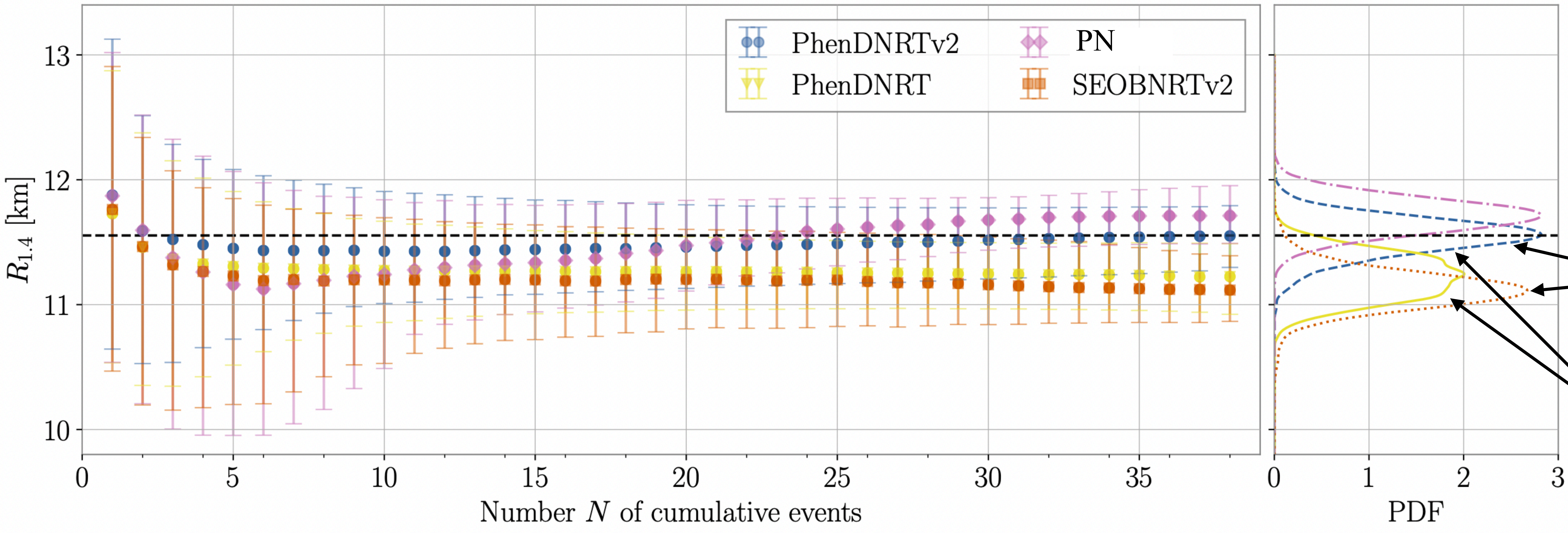
- Due to systematics, **false deviations from GR** in the **quasi-normal mode** frequency and decay time of the ringdown **are measured at** $\sim 95\,\%$ **CI**.

(see also Narayan et al. 23)

Systematics in Waveform Models with Future Detectors: BNS

- **"Stacking"** events **reduces statistical errors,** but **systematic biases can show up**.

(Kunert, Pang, Tews, Coughlin & Dietrich 22)



LIGO-Virgo **O5 run** (~2026)

Synthetic **signal** injected is **PhenDNRTv2**.

Differences in models only **due to BBH baseline model**, i.e., tidal terms are identical.

Differences in models only **due to tidal corrections**.

- With 38 NS detections, **statistical uncertainties in NS radius decrease to $\pm 250\,\mathrm{m}$** ($2\,\%$ at $90\,\%$ CI) but **systematic differences** between current waveform models **can be twice as large**.

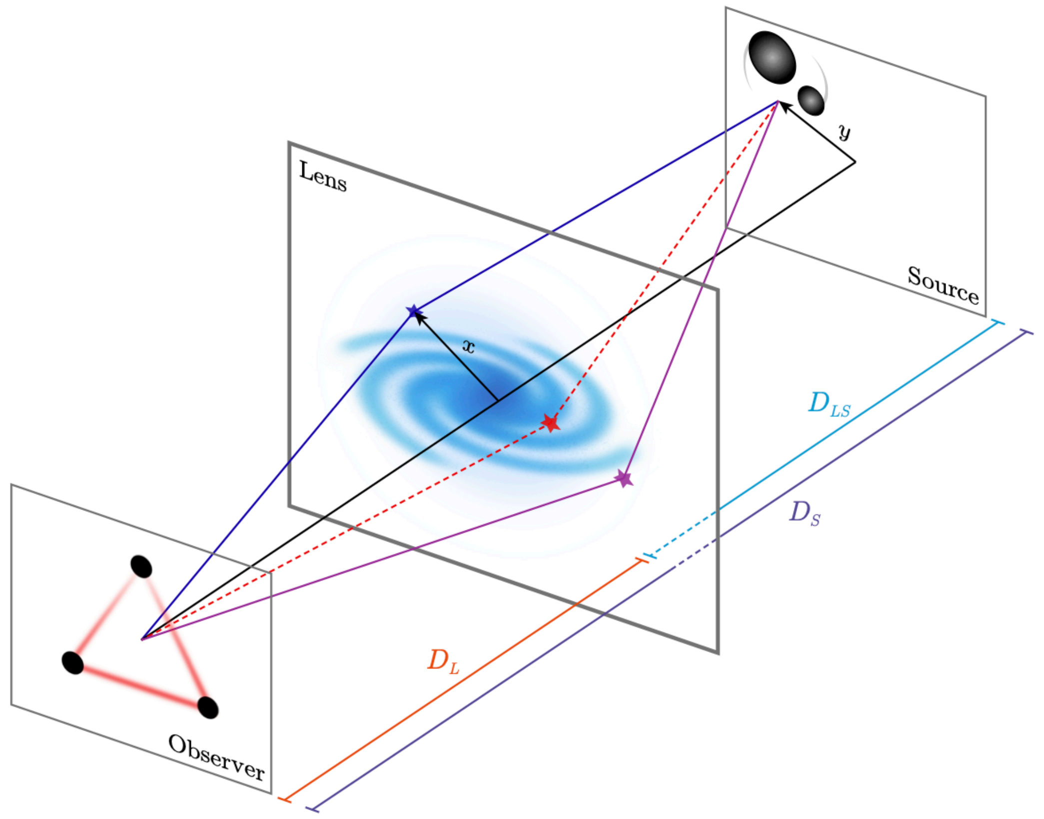
(see also Purrer & Halster 19, Huang et al. 20, Gamba et al. 21)

- Crucial to make **BBH model more accurate. Tidal corrections** also need **to be improved**.

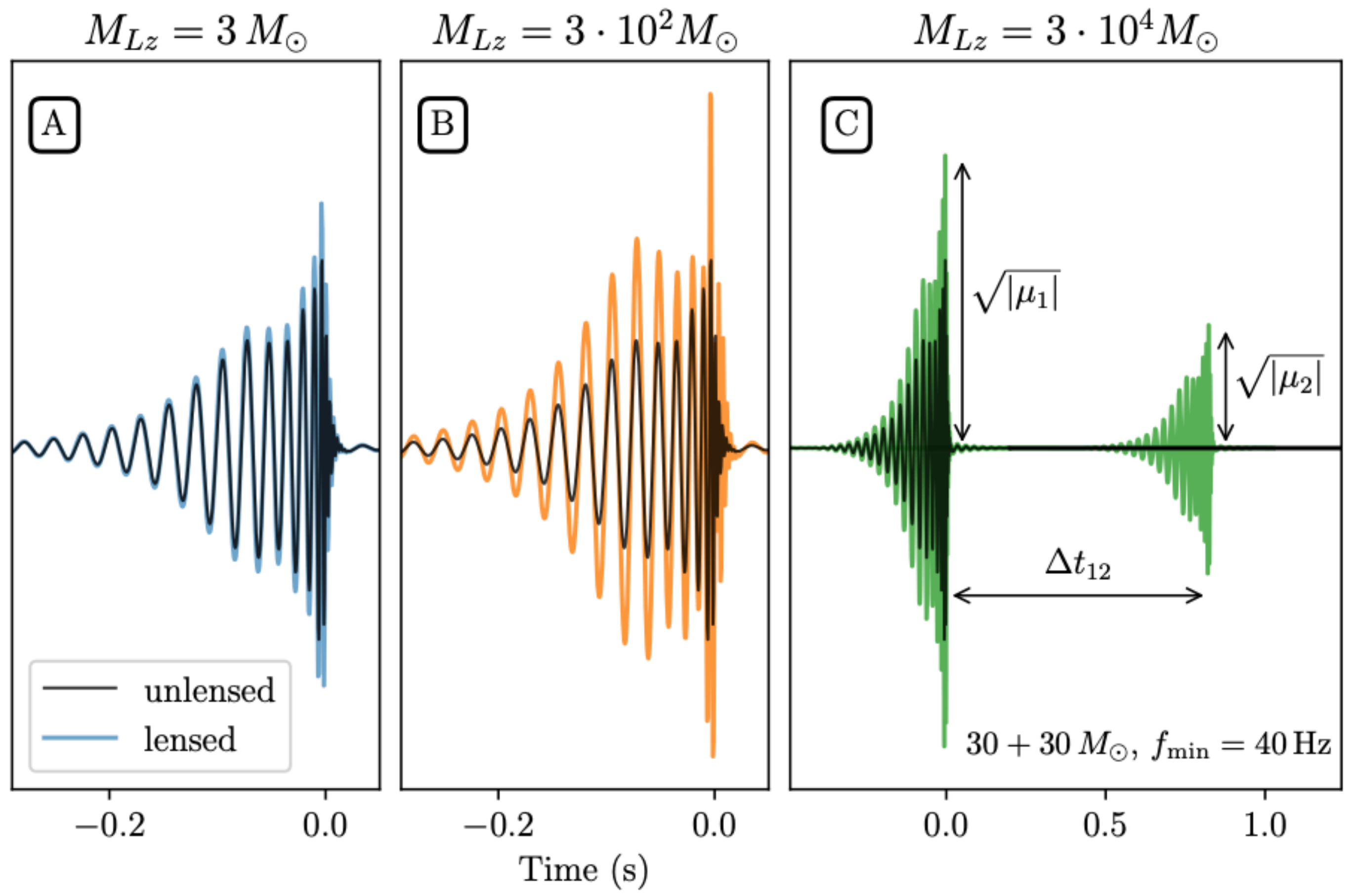
Toward High-Precision Gravitational Waves

- **Accuracy of current waveform models** would need **to be improved by 1-2 orders of magnitude**. **Numerical-relativity simulations** would also need to become **more accurate (by 1 order of magnitude)**.

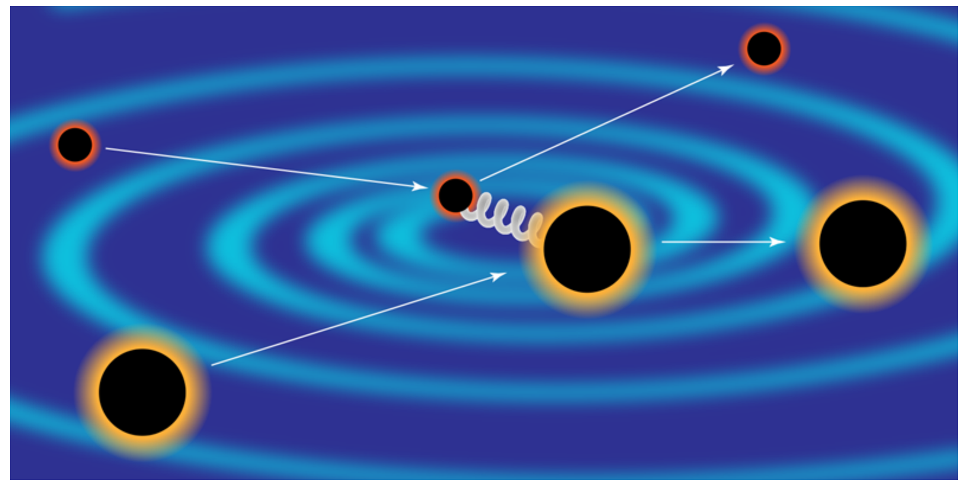
- **All physical effects** would **need to be included in waveform models** (generic orbits, beyond-GR deviations, gravitational lensing, astrophysical environmental effects, etc.) **to avoid wrong scientific** conclusions.



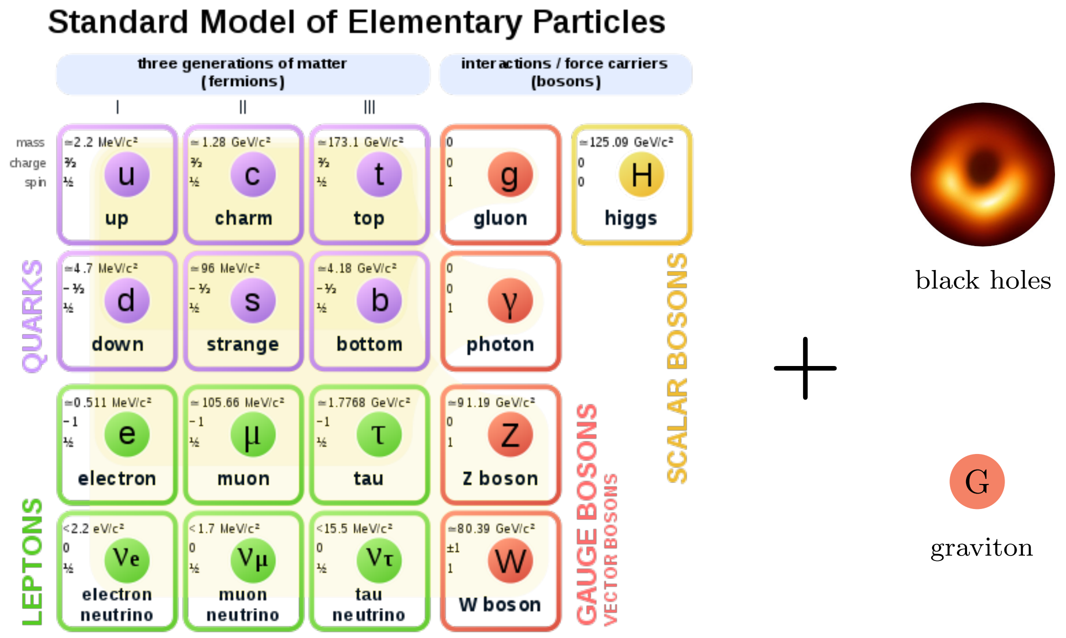
(Tambalo et al. 22)



- **Scattering-amplitude/EFT/QFT** methods from high-energy physics **have brought new tools to solve two-body problem in classical gravity**.



(APS/Stonebraker)



(credit:Steinhoff)

Methods to build accurate waveform models.

Solving Two-Body Problem in General Relativity

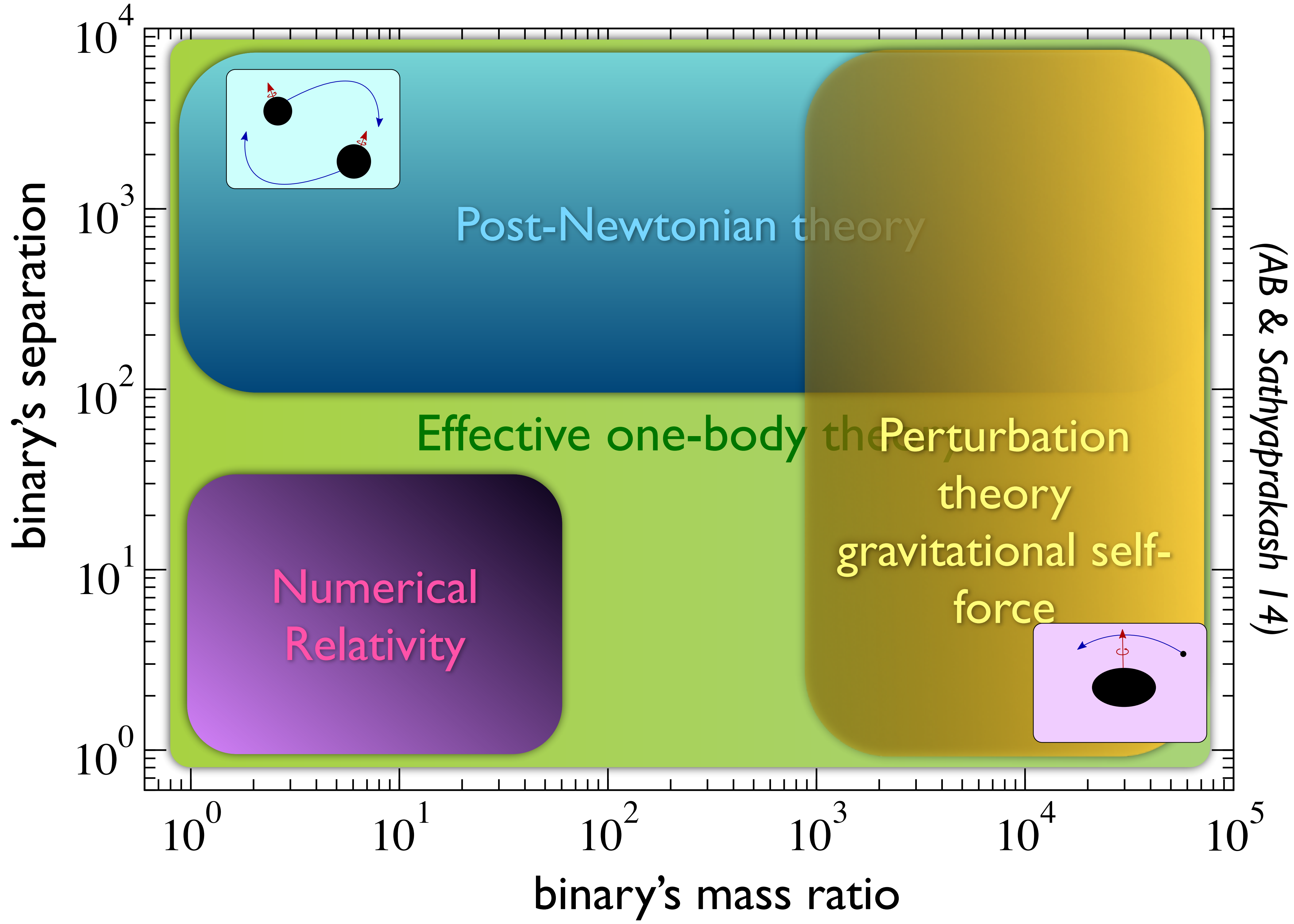
- **GR** is **non-linear theory**. $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

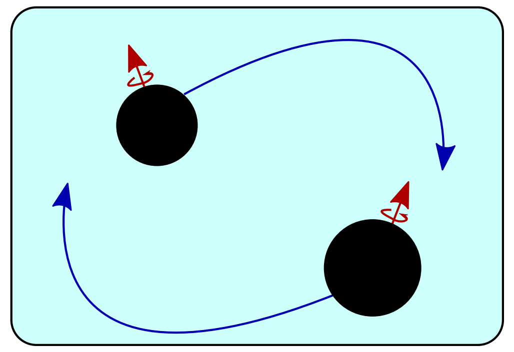
-**accurately**, but **numerically** on supercomputers (**slow** way)

- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.



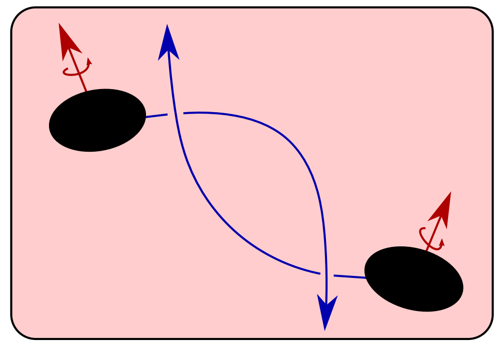
- **Post-Newtonian** (large separation, and slow motion)

expansion in $v^2/c^2 \sim GM/rc^2$



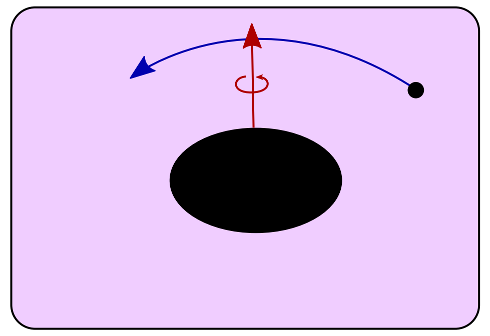
- **Post-Minkowskian** (large separation, and fast motion)

expansion in G



- **Gravitational self-force**

expansion in m_2/m_1



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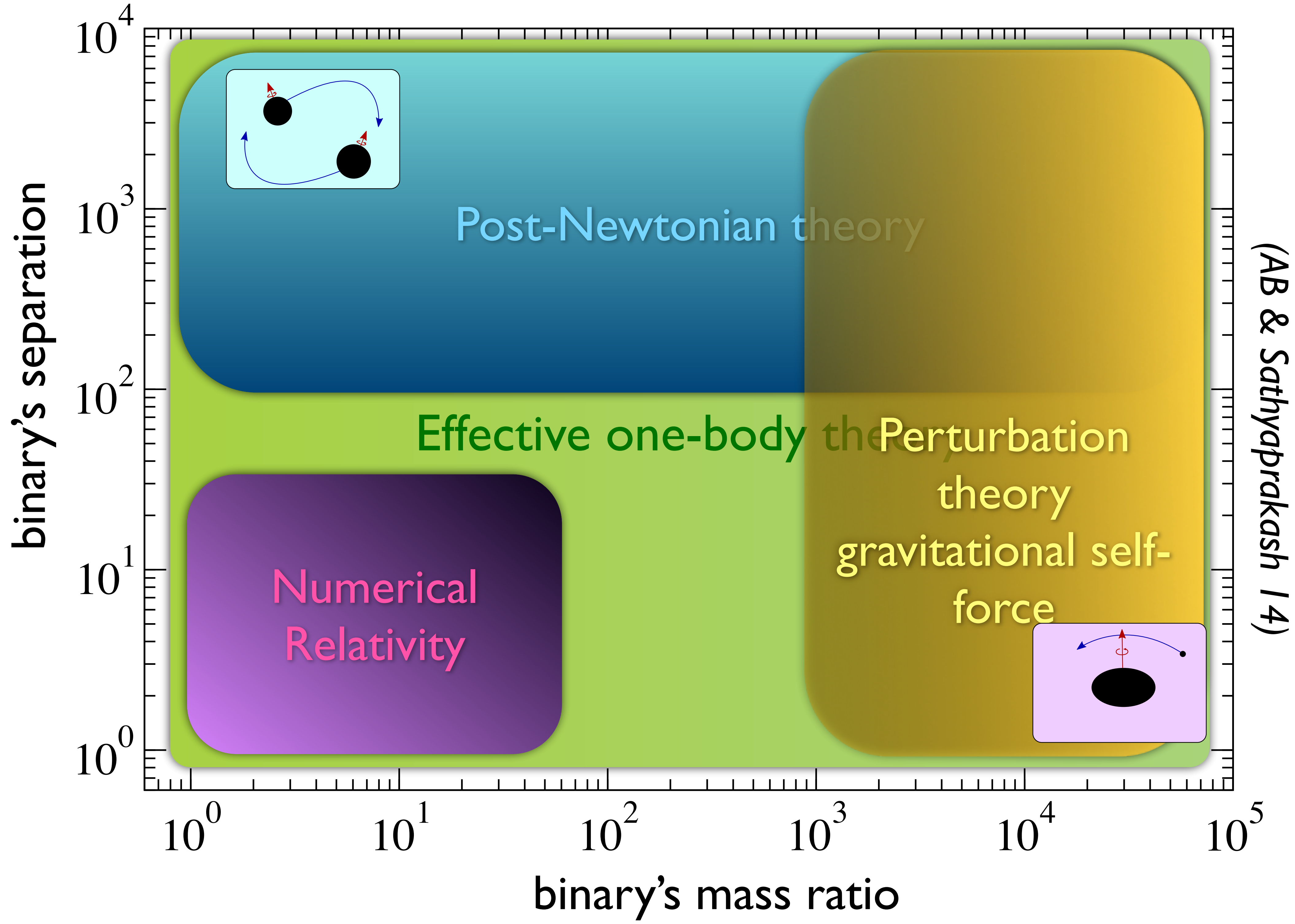
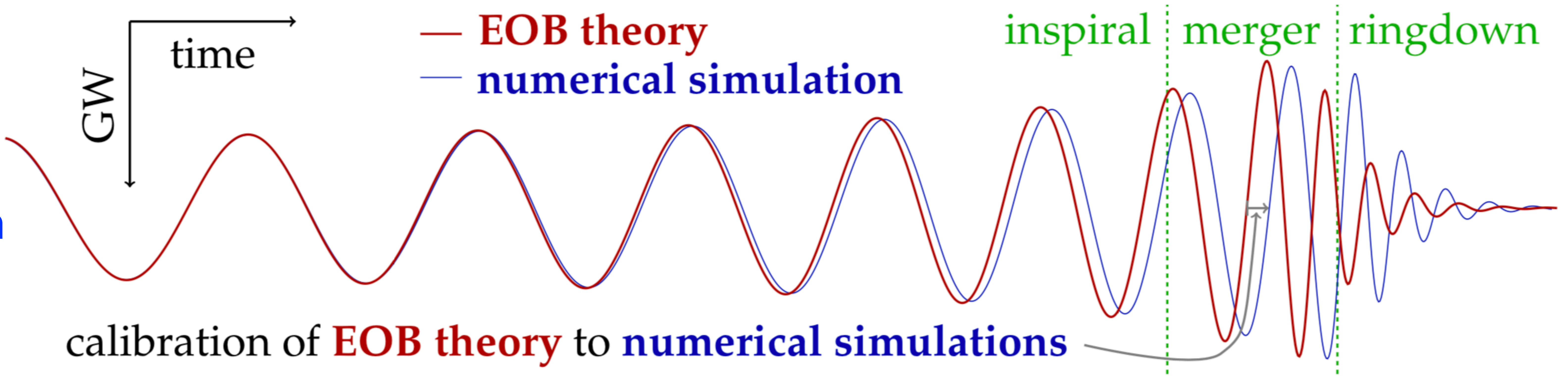


- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Effective-one-body** (EOB) **theory** (combines results from all methods, i.e., for **entire coalescence**)

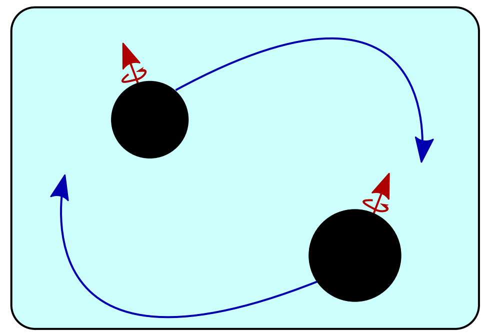
- **Phenomenological frequency-domain** waveforms (Phenom) hybridizing EOB and NR waveforms, and fitting.



Toward High-Precision Gravitational Waves

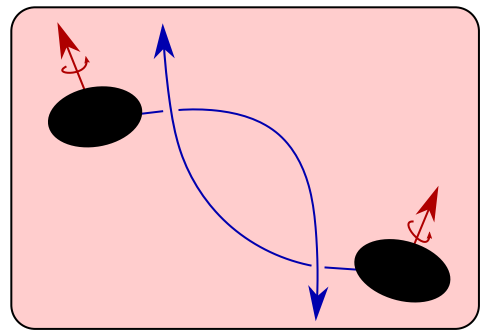
- **Post-Newtonian, PN** (large separation, and slow motion)

  expansion in
  $$v^2/c^2 \sim GM/rc^2$$



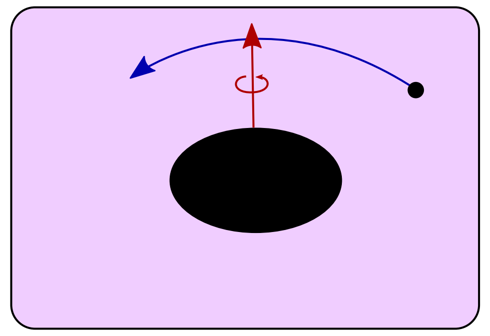
- **Post-Minkowskian, PM** (large separation, and fast motion)

  expansion in
  $$G$$

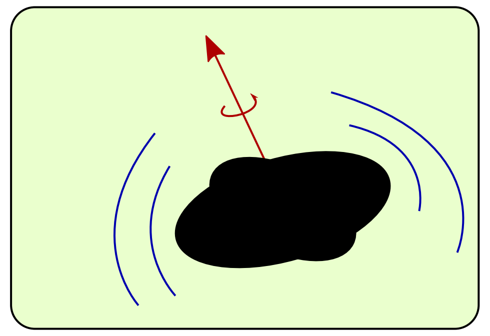


- **Small mass-ratio (SMR)/ gravitational-self force, GSF**

  expansion in
  $$m_2/m_1$$



- **Perturbation theory** (e.g., **ringdown** of final object)



- **Numerical relativity**

- **Waveform accuracy** would need to be **improved by two or more orders of magnitude** depending on the parameter space.
  (e.g., Pürrer & Halster 19)

- **Toward Improving Waveform Accuracy: PN**

- **GW phasing completed through 4.5PN** order.
  (Blanchet, Faye, Henry, Larrouturou & Trestini 23)

PN order	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5
no spin	N	1PN		2PN		3PN		4PN		5PN		6PN	
spin-orbit			LO SO		NLO SO		N2LO SO		N3LO SO		N4O SO		
spin^2				LO S2		NLO S2		N2LO S2		N3LO S2			
spin^3							LO S3		NLO S3		NNLO S3		
spin^4								LO S4		NLO S4		NNLO S4	
spin^5											LO S5		NLO S5
spin^6												LO S6	

+ need radiation

for bound orbits

(credit: Justin Vines)

Toward Improving Waveform Accuracy: PM

- Nonspinning **conservative dynamics derived through 3PM**, it is local and valid for generic orbits.

  (Cheung, Rothstein & Solon 19; Bern et al. 19; Blümlein et al. 20; Kälin, Liu & Porto 20; Cheung & Solon 20; Brandhuber, Chen, Travaglini & Wen 21)

- Nonspinning **conservative dynamics derived at 4PM** with non-local part for *hyperbolic* orbits.

  (Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, & Zeng 21; Dlapa, Kälin, Liu & Porto 21)

- **Total impulse in nonspinning BH scattering derived at 3PM, and then at 4PM** including linear, nonlinear and hereditary RR effects.

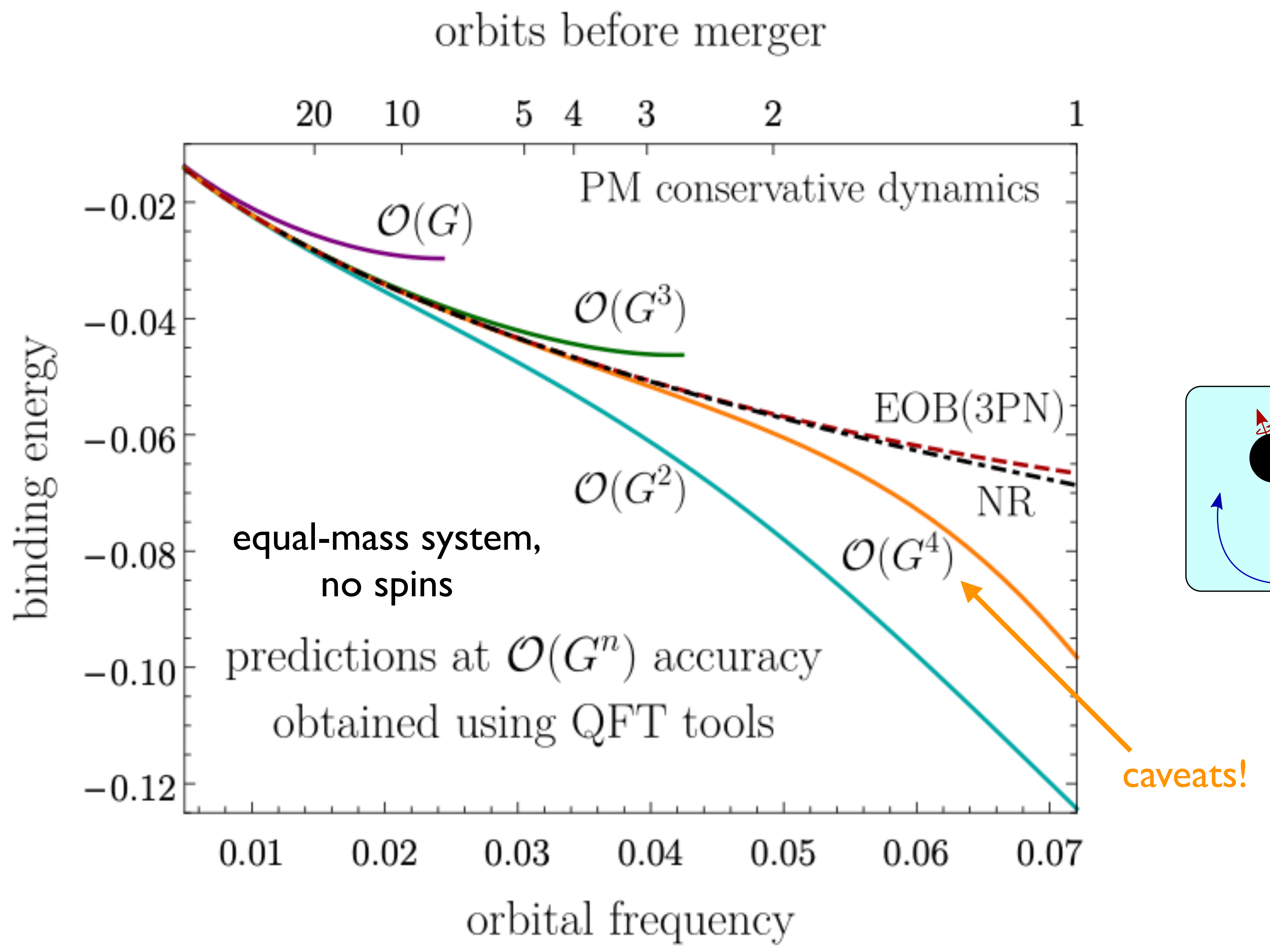
  (Di Vecchia, Heissenberg, Russo & Veneziano 21; Hermann, Parra-Martinez, Ruf & Zeng 21; Manohar, Ridgway & Shen 22; Dlapa, Kälin, Liu, Neef & Porto 22; Damgaard, Hansen, Planté & Vanhove 23)

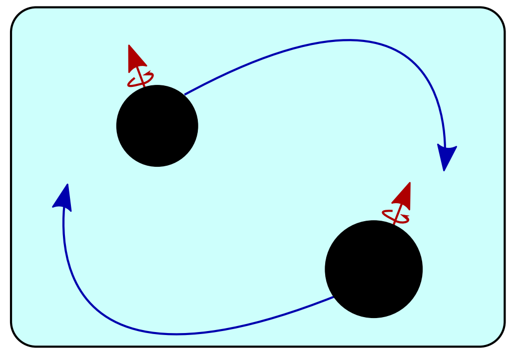
- Nonspinning **waveform derived at next-to-leading order.**

  (Kovacs & Thorne 1975; Jakobsen et al. 21; Brandhuber et al. 23; Georgoudis et al. 23; Herdershee et al. 23; Elkhidir et al. 23)

- Spinning **conservative dynamics derived through 4PM**, for generic orbits.

  (Bern, Luna, Roiban, Shen & Zeng 20; Liu, Porto & Yang 21; Jakobsen, Mogull, Steinhoff & Plefka 22; Jakobsen & Mogull 22; Riva, Vernizzi & Wang 22; Bern, Kosmopoulos, Lusa, Roiban & Teng 23; Jakobsen, Mogull, Plefka, Sauer and Xu 23)





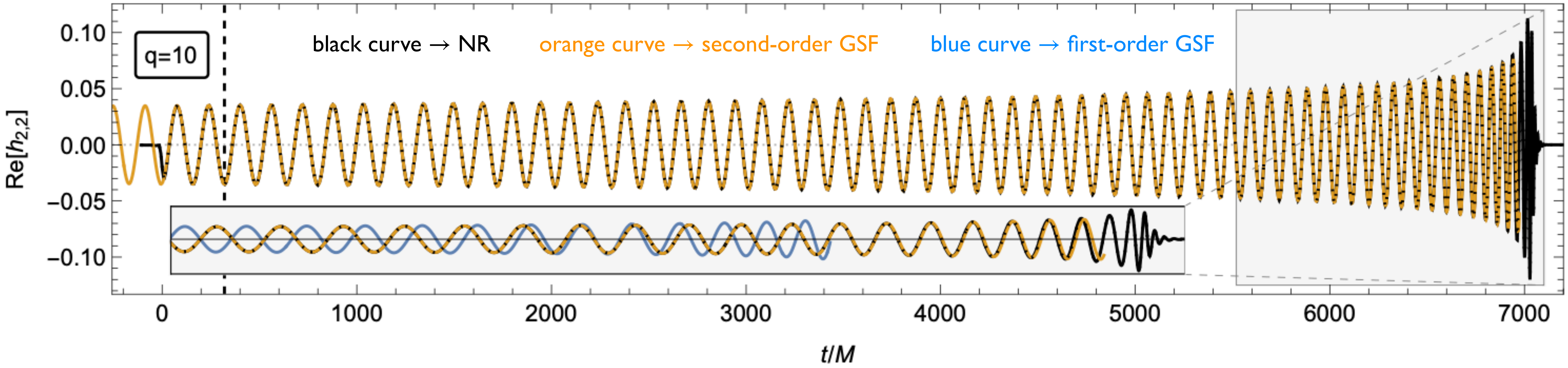
(Khalil, AB, Steinhoff & Vines 22; AB, Khalil, O'Connell, Roiban, Solon & Zeng 22)

Toward Improving Waveform Accuracy: GSF

- For **nonspinning binaries** in **quasi-circular orbits**, GSF effects **at second order in mass ratio** (all order in velocities, strong field) have been computed.

(Pound, Wardell, Warburton & Miller 20; Warburton, Pound, Wardell, Miller & Durkan 21; Wardell, Pound, Warburton, Miller & Durkan 21)

- Although **GSF approximation** is **designed for** cases in which **mass ratio** is extreme, it also **performs remarkably well for more comparable mass ratios** including $1:10$.



(Wardell, Pound, Warburton, Miller, Durkan & Le Tiec 21)

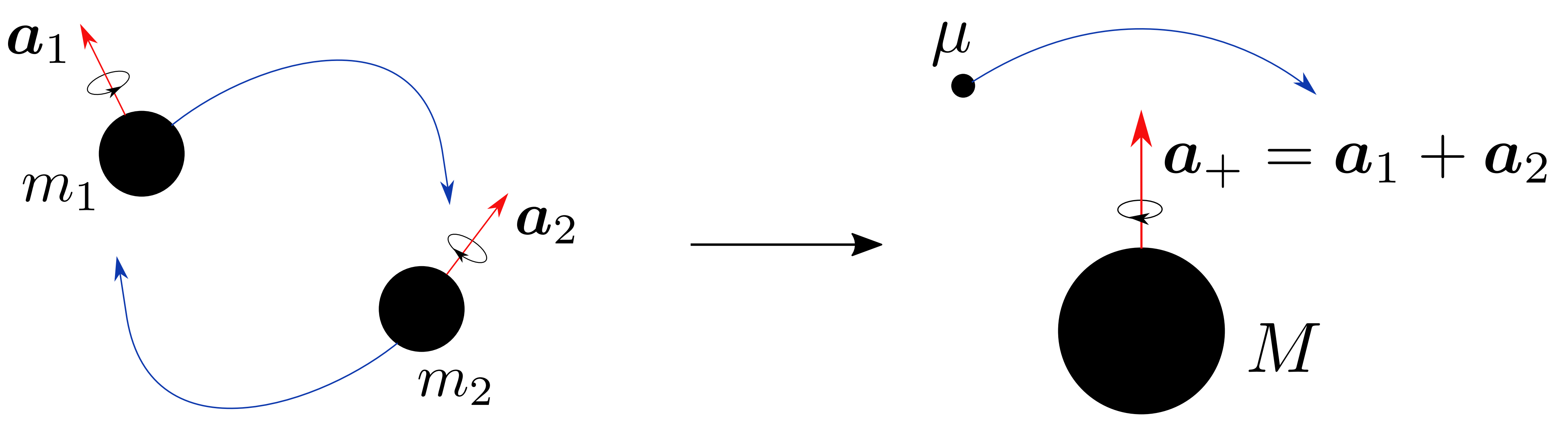
How to take advantage of new results in PN, GSF, PM, …

EOB Hamiltonian: Non-Spinning Bodies

$\mu = m_1 m_2/M$

$M = m_1 + m_2$

$\nu = \mu/M \qquad 0 \le \nu \le 1/4$



(credit: Khalil)

$\mathbf{a}_i = m_i \chi_i \quad i = 1,2$

$0 \le \chi_i \le 1$

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20; Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)

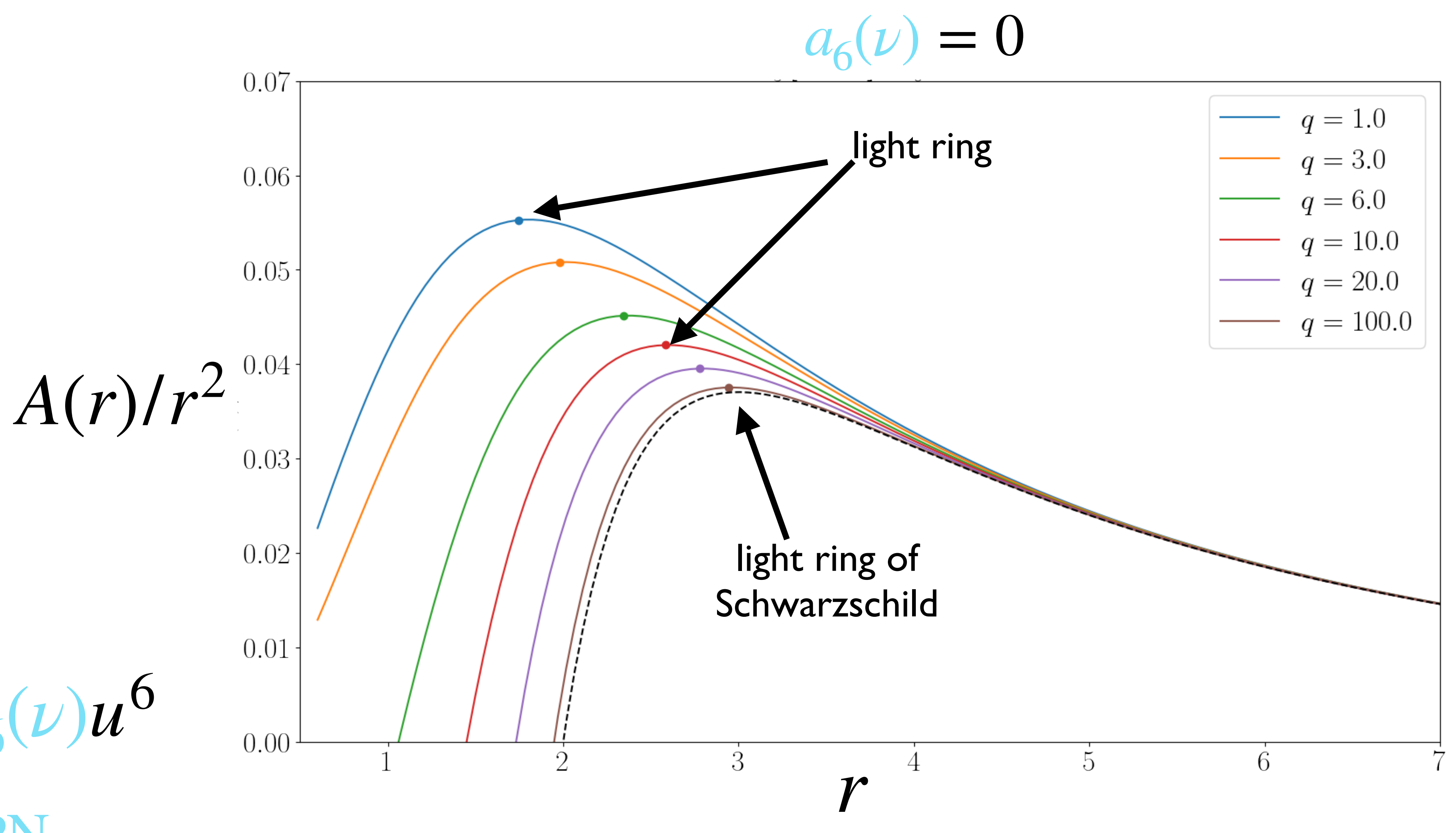
$\mathbf{a}_i = 0 \quad i = 1,2 \qquad g_{\rm eff}^{\mu\nu}\, p_\mu\, p_\nu + \mu^2 + \cdots = 0$

$$H_{\rm eff} = \sqrt{A(r; a_6)\left[\mu^2 + p_r^2\, B_{np}(r) + \frac{L^2}{r^2} + Q(r, p_r)\right]}$$

$$A(u, a_6) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + [a_5(\nu) + a_5^{\log}(\nu)\,\log(u)]u^5 + a_6(\nu)u^6$$

4PN 5PN

$u = M/r$



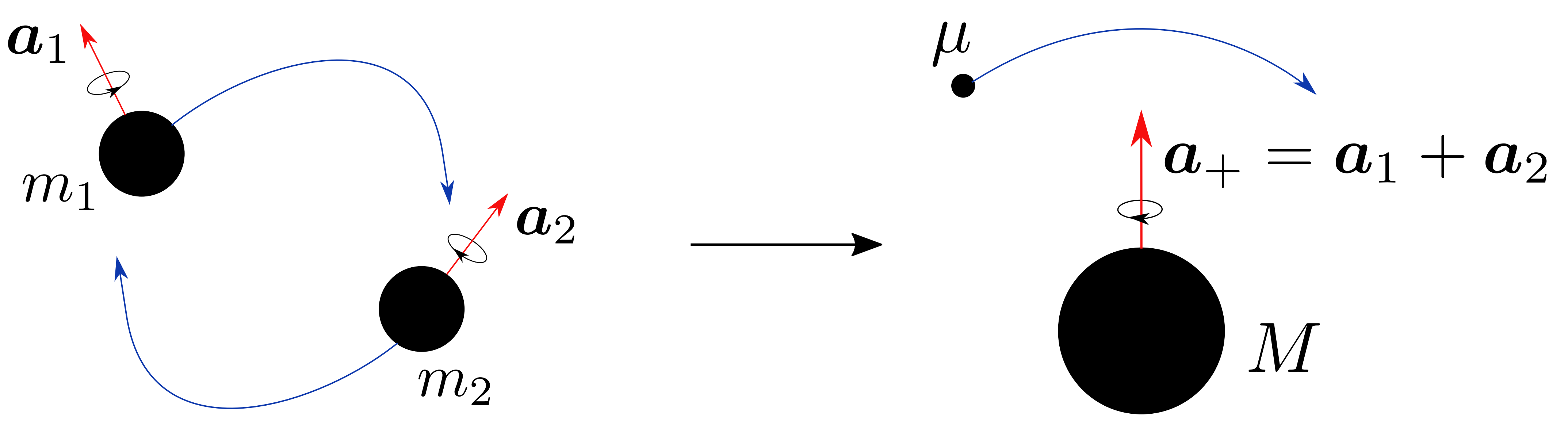
resummation of $A(r)$ potential

EOB Hamiltonians: Spinning Bodies

$\mu = m_1 m_2/M$

$M = m_1 + m_2$

$\nu = \mu/M \qquad 0 \leq \nu \leq 1/4$



(credit: Khalil)

$\mathbf{a}_i = m_i\, \chi_i \quad i = 1{,}2$

$0 \leq \chi_i \leq 1$

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20; Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)

$$H^{\rm eff} = H_{\rm odd}^{\rm eff} + H_{\rm even}^{\rm eff}$$

odd (even) powers in BH's spin

restricted to aligned-spins, equatorial orbits *(Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)* **@4PN order**

$a_+ = a_1 + a_2 \qquad a_- = a_1 - a_2 \qquad \delta = \sqrt{1 - 4\nu}$

$$H_{\rm even}^{\rm eff} = \sqrt{A(a_6)\left[\mu^2 + p_r^2\,(1 + B_{np}) + \frac{L^2}{r^2}(1 + a_+^2\, B_{npa}) + Q\right]}$$

$$H_{\rm odd}^{\rm eff} = \frac{M\,L\left[g_{a_+}(d_{\rm SO})\, a_+ + g_{a_-}\,\delta\, a_- - a_+^2/(4r^2)\,\left(a_+ - a_-\delta\right)\right]}{a_+^2\,(r + 2M) + r^3}$$

gyro-gravitomagnetic functions

resummation of Hamiltonian

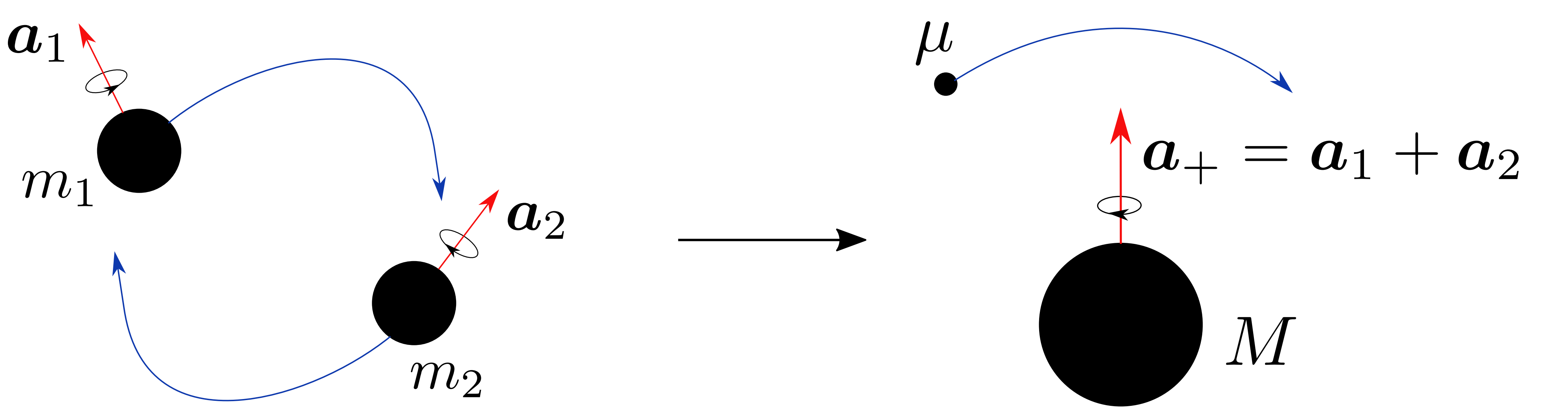
- **Non-spinning 5PN terms** are known except two coefficients, which can be fixed by second-order GSF. *(Bini, Damour & Geralico 20; Blümlein et al. 21)*
- **5.5PN SO terms** are known except for one coefficient, which could be fixed by second-order GSF. *(Khalil 22)*
- **5PN SS terms** are known for quasi-circular orbits. *(Kim, Levi & Yin 22)*

EOB EOM and RR Force for Spinning Bodies

$\mu = m_1 m_2/M$

$M = m_1 + m_2$

$\nu = \mu/M \qquad 0 \le \nu \le 1/4$



$\mathbf{a}_i = m_i\,\chi_i \quad i = 1{,}2$

$0 \le \chi_i \le 1$

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20; Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)

- **EOB equations of motion:**

(AB & Damour 00; AB, Chen & Damour 05; Damour et al. 09)

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{p}} \qquad \dot{\mathbf{a}}_{\rm i} = \left\{\mathbf{a}_i, H_{\rm real}^{\rm EOB}\right\}$$

$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r},\mathbf{p},\mathbf{a}_i)$$

- **Radiation-reaction force** and gravitational **modes**:

(AB & Damour 00; Damour et al. 09; Pan, AB et al. 11)

$$F_\phi \propto \frac{dE}{dt} \propto \sum_{\ell m} (m\,\Omega)^2\,|\,h_{\ell m}^{\rm insp}(r,\Omega)\,|^2 \longleftarrow \text{quasicircular orbits}$$

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\phi}\, S_{\ell m}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

resummation of PN results

non-quasicircular (NQC) corrections

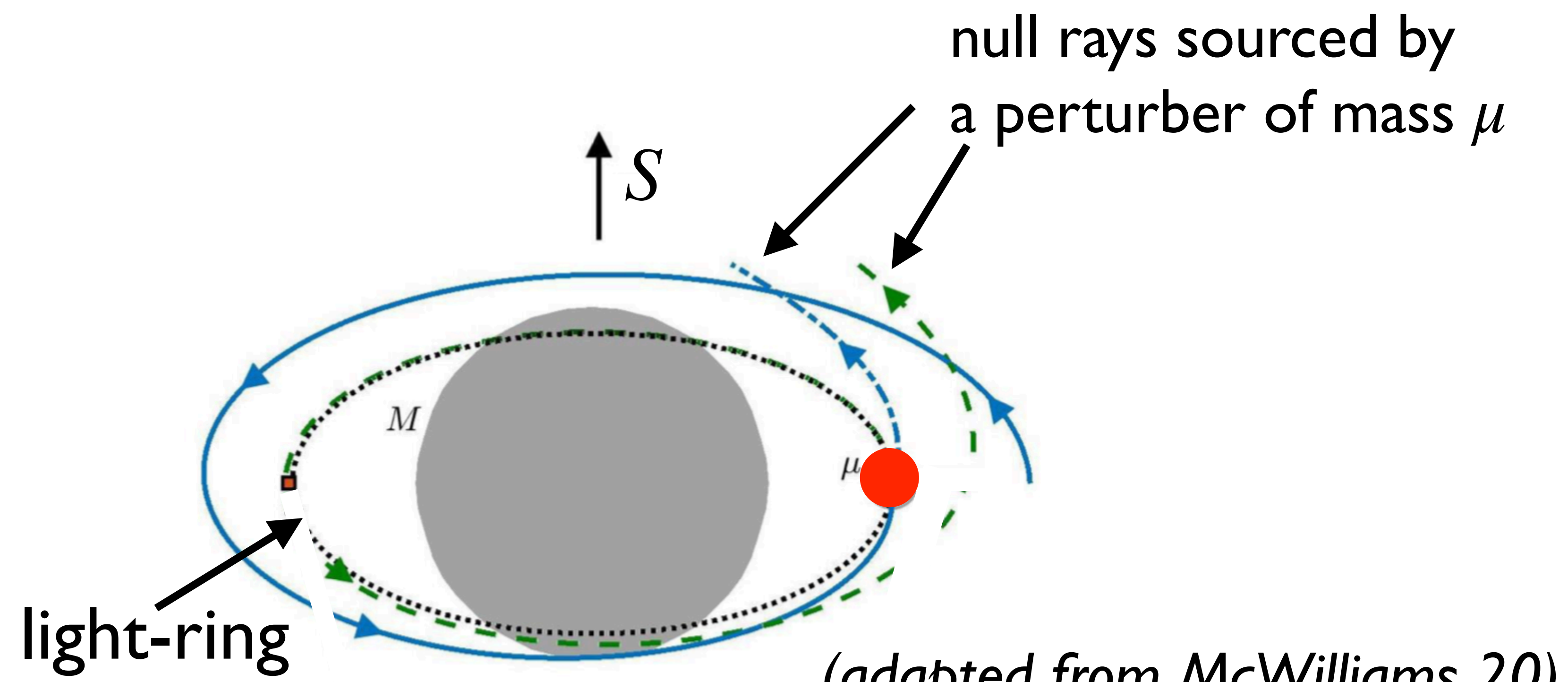
Inspiral-Plunge EOB Waveform & Frequency

- **EOB equations of motion**

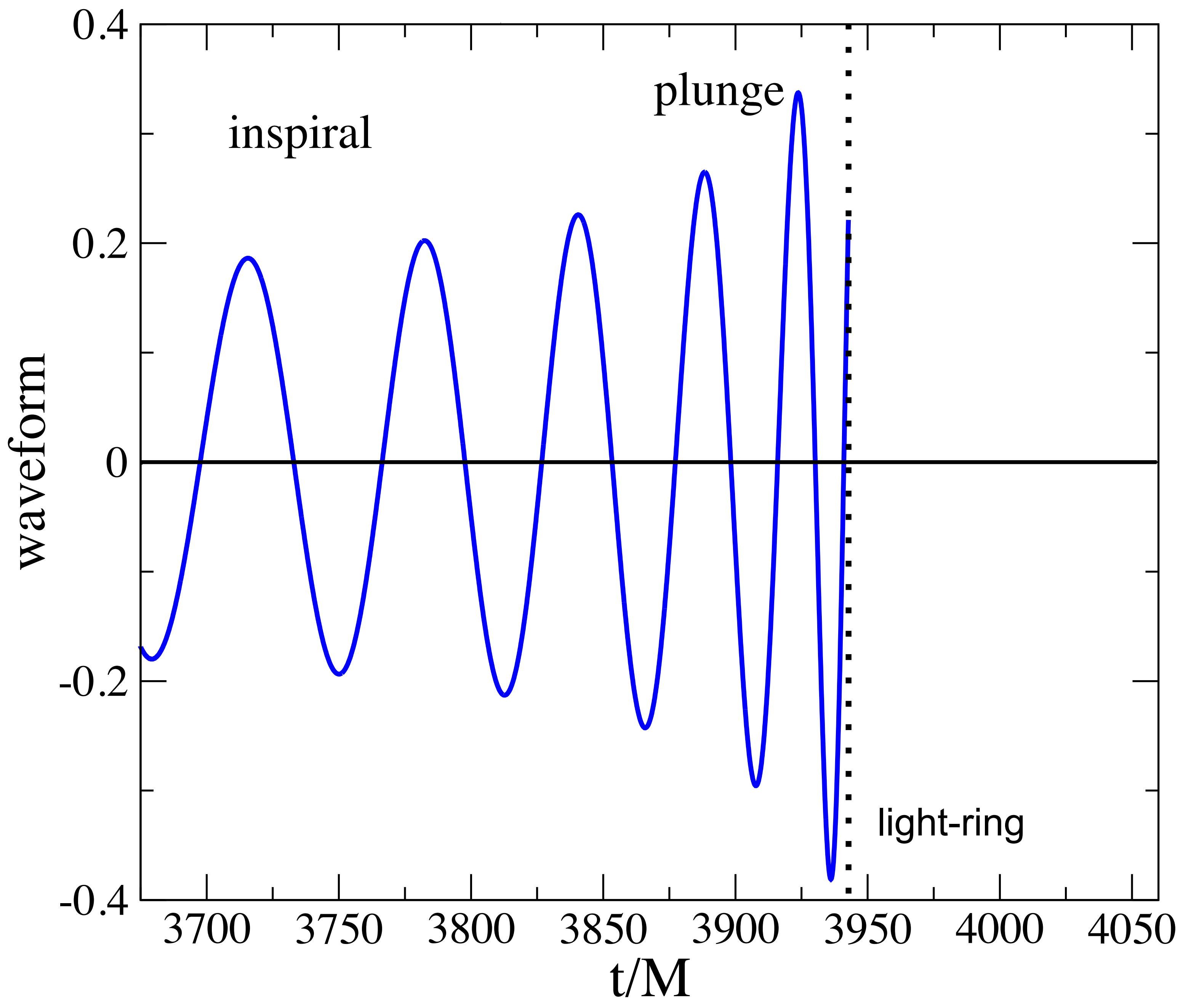
$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}_{\rm real}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{p}}$$

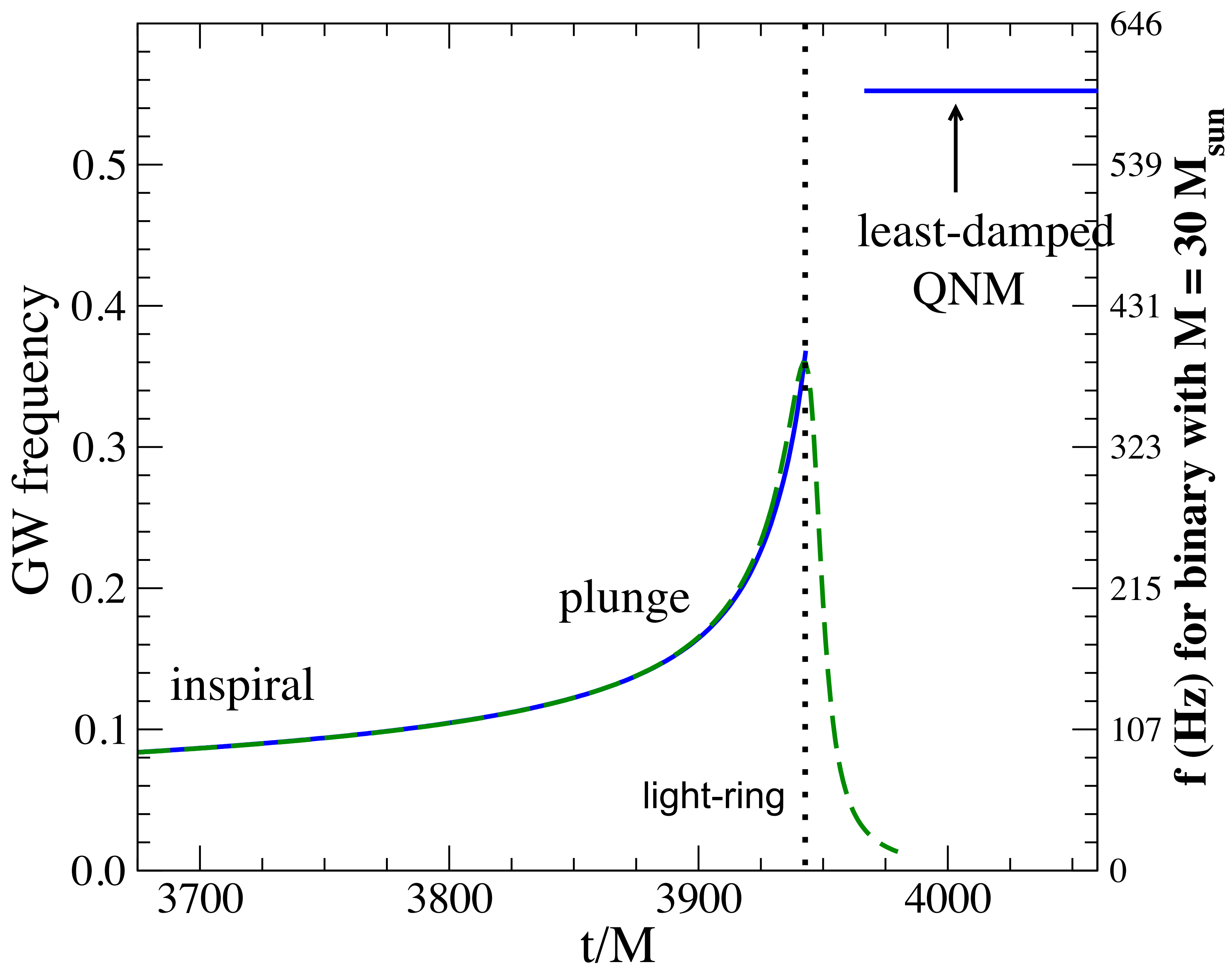
$$\dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}_{\rm real}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)$$

- Evolve **two-body dynamics up to light ring** (or photon orbit) and then …



(adapted from McWilliams 20)





- **Quasi-normal modes** excited at **light-ring crossing.** *(Goebel 1972; Davis, Ruffini & Tiomno 1972; Ferrari et al. 1984; Price and Pullin 1994)*

Inspiral-Merger-Ringdown EOB Waveform & Frequency

- **EOB equations of motion**

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r},\mathbf{p},\mathbf{a}_i)$$

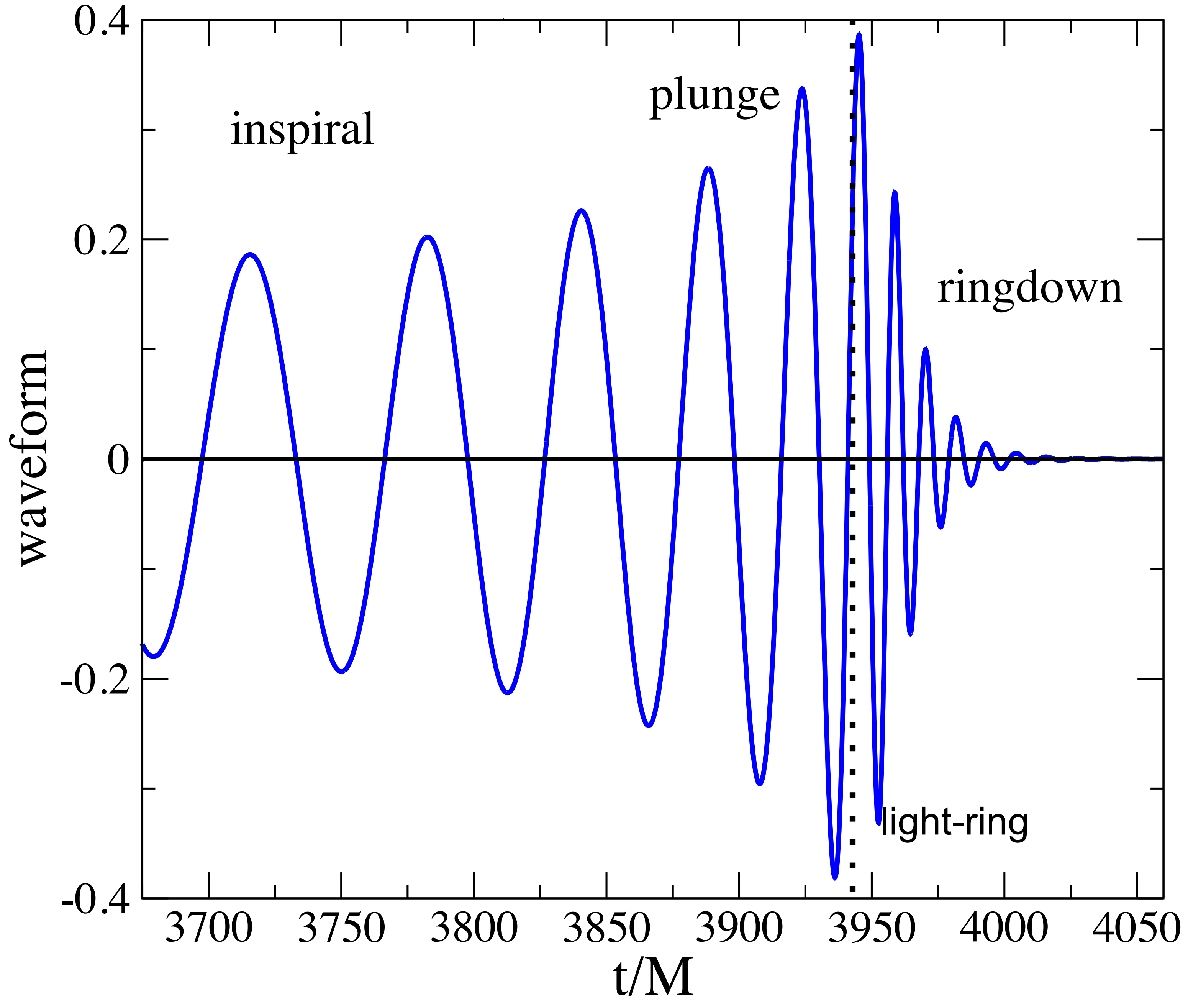
- … attach a function representing **quasi-normal mode ringing** of remnant BH.

BH quasi-normal modes

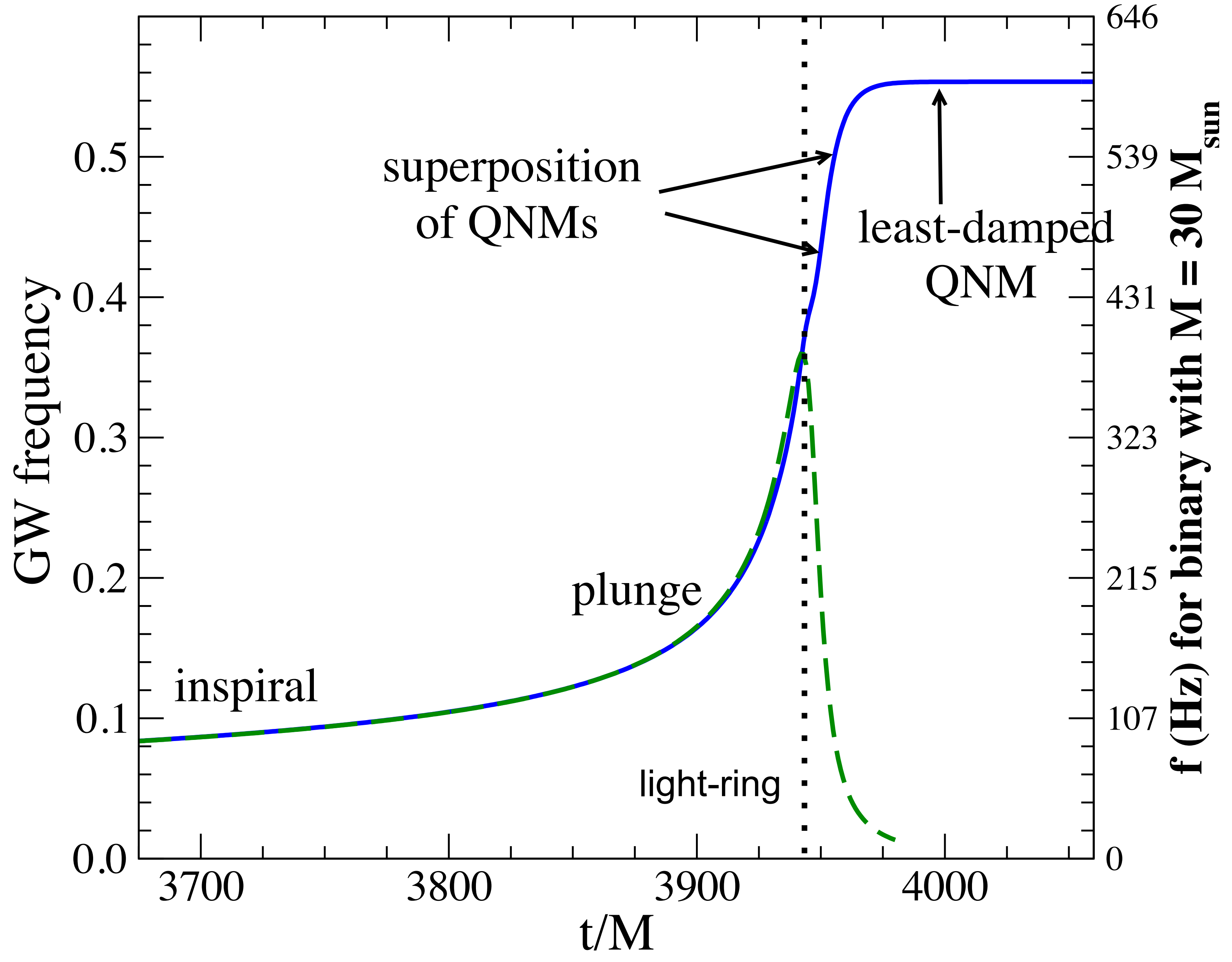
$$h_{\ell m}^{\rm merger-RD}(t) = \nu\, \tilde{A}_{\ell m}(t)\, e^{i\,\tilde{\phi}_{\ell m}(t)}\, e^{-i\sigma_{\ell m0}(t-t_{\rm match}^{\ell m})}$$

(Baker et al. 08; Damour & Nagar14; London et al. 14; Bohé, … AB et al. 17; Cotesta, AB et al. 19; Pompili, AB et al. 23)

$$t_{\rm match}^{\ell m} = t_{\rm ISCO} + \Delta t^{\ell m}$$



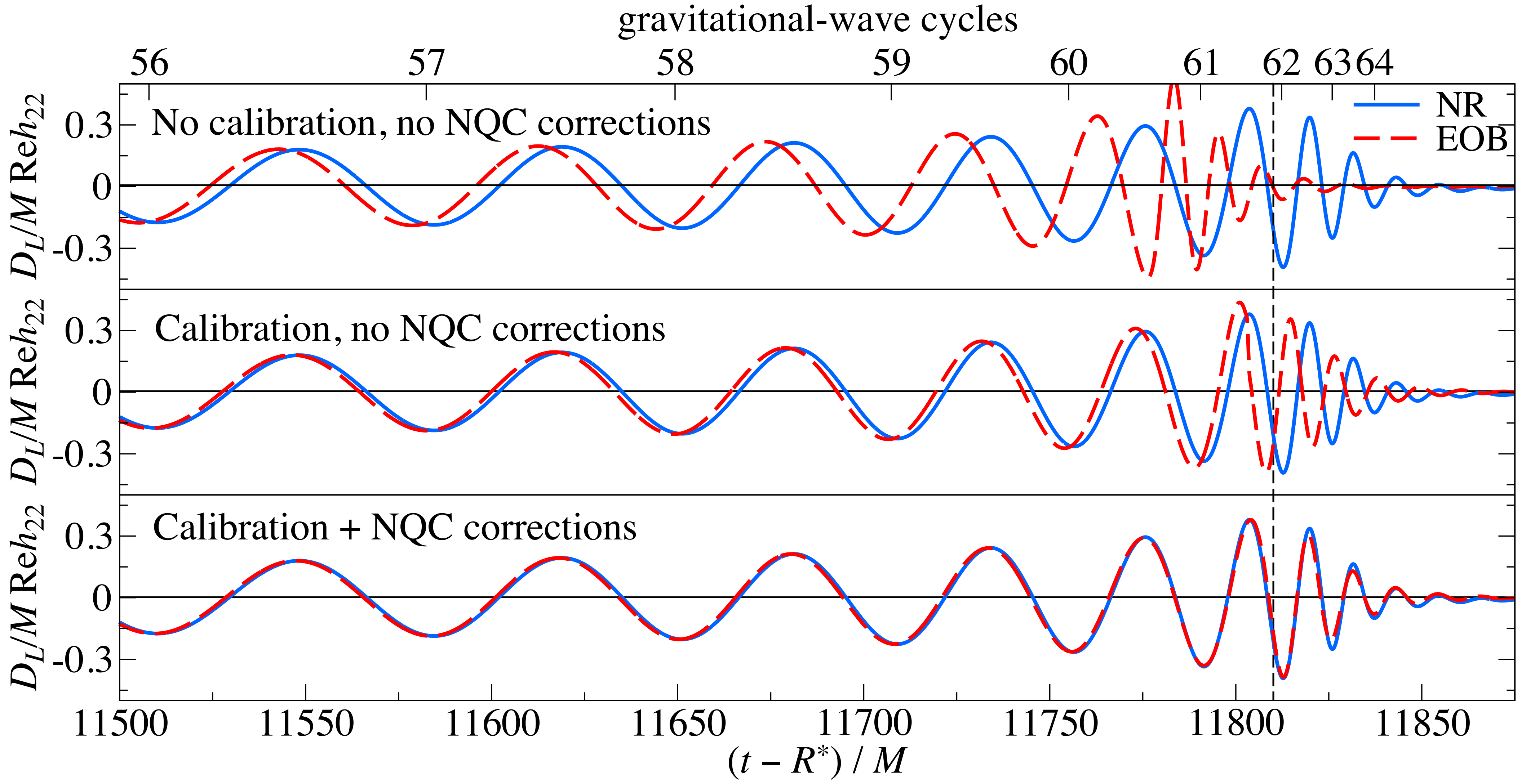
(AB & Damour 00; AB, Chen & Damour 05;AB, Cook & Pretorius 07)



$$h_{22}(t) = h_{22}^{\rm insp-plunge}(t)\,\theta(t_{\rm match}^{22}-t) + h_{22}^{\rm merger-RD}(t)\,\theta(t-t_{\rm match}^{22})$$

Completing EOB Waveforms with NR Information & Template Bank

- We calibrate models to **inspiral-merger-ringdown NR** waveforms.



(credit: Taracchini)

(NQC: non-quasi-circular corrections)

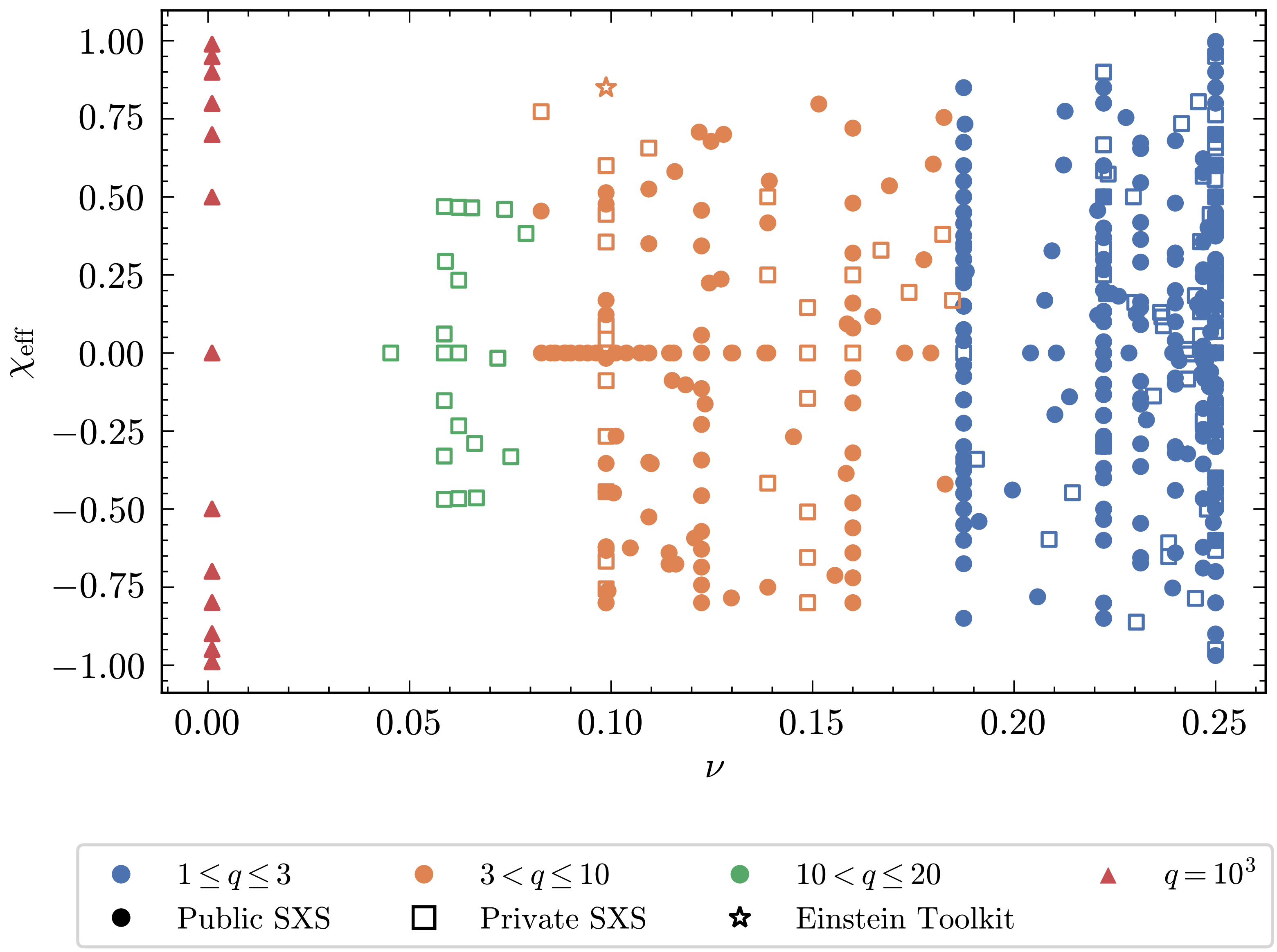
$$\chi_1 = S_1/m_1^2$$

$$\chi_2 = S_2/m_2^2$$

$$\chi_{\rm eff} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2$$

$$\nu = \frac{m_1 m_2}{M^2} \qquad q = \frac{m_1}{m_2}$$

Calibration of **SEOBNRv5** using about **440 NR waveforms**

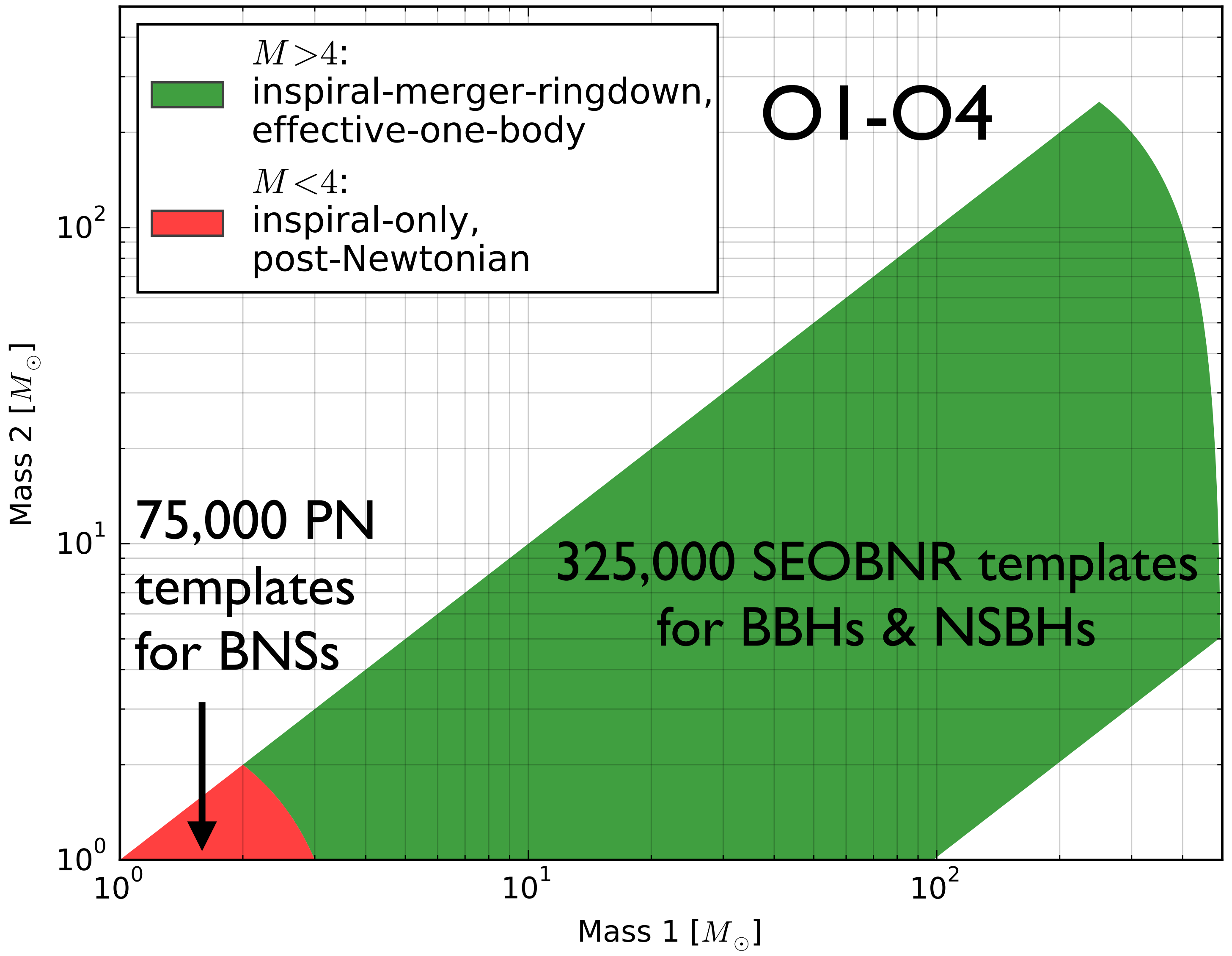


(Pompili et al. 23)

(SXS: Simulating eXtreme Spacetime)

(Khalil, AB et al. 23, Pompili, AB et al. 23, van de Meent, AB et al. 23, Ramos-Buades, AB et al. 23, Mihaylov, Ossokine, AB et al. 23; **SEOBNR***)*

- **Matched filtering** employed in LIGO/Virgo searches.



(Dal Canton & Harry 17)

(García-Quíros et al. 20, Pratten et al. 20; **IMRPhenom***)*

(Gamba et al. 21; **TEOBResumS***)*

(Varma et al. 19; **NRSur***)*

Accuracy of SEOBNR & IMRPhenomX Models

quasi-circular, spin-precessing case

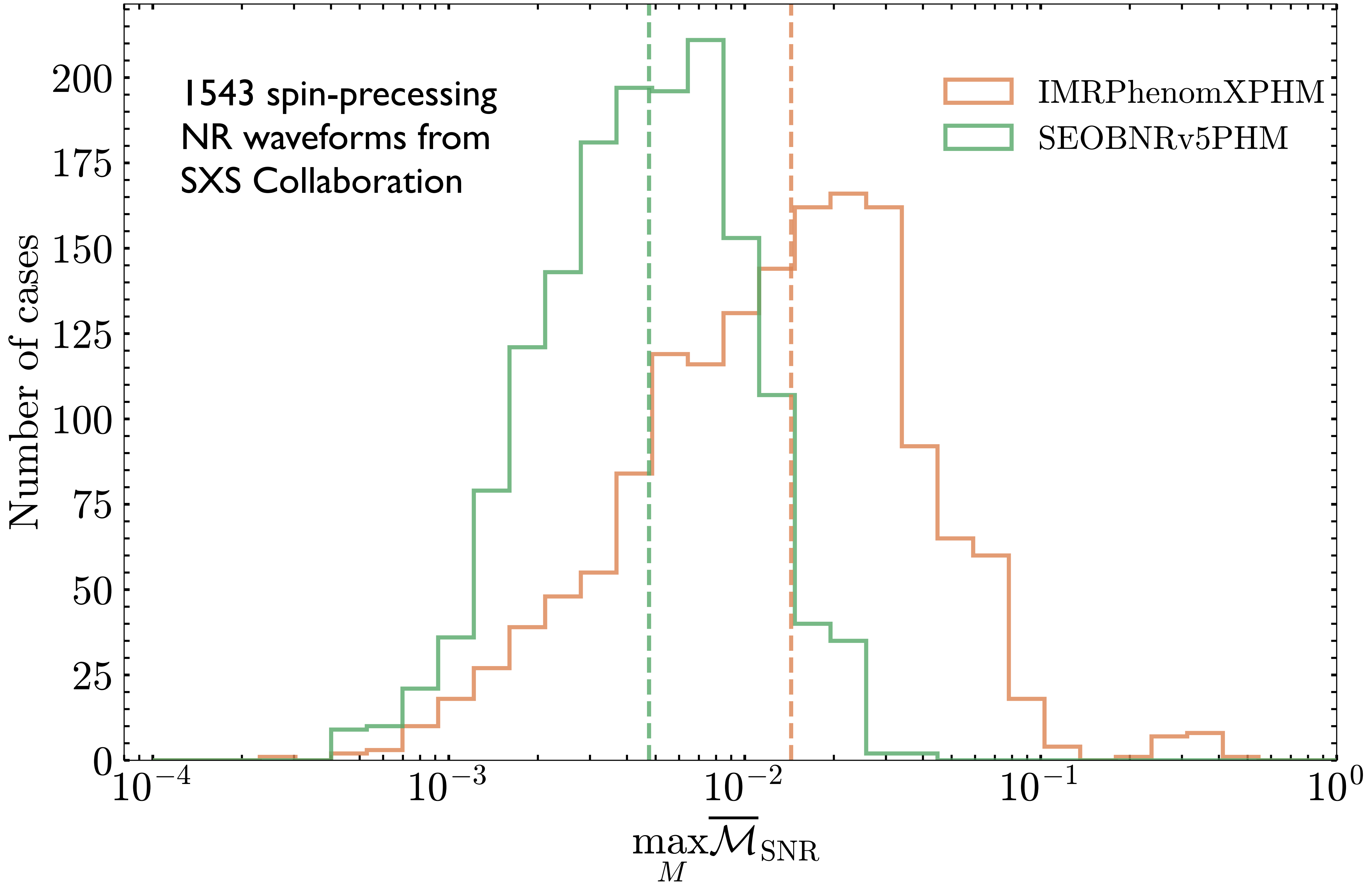
(Ramos-Buades, AB, Khalil, Estelles, Pompili & Ossokine 23)

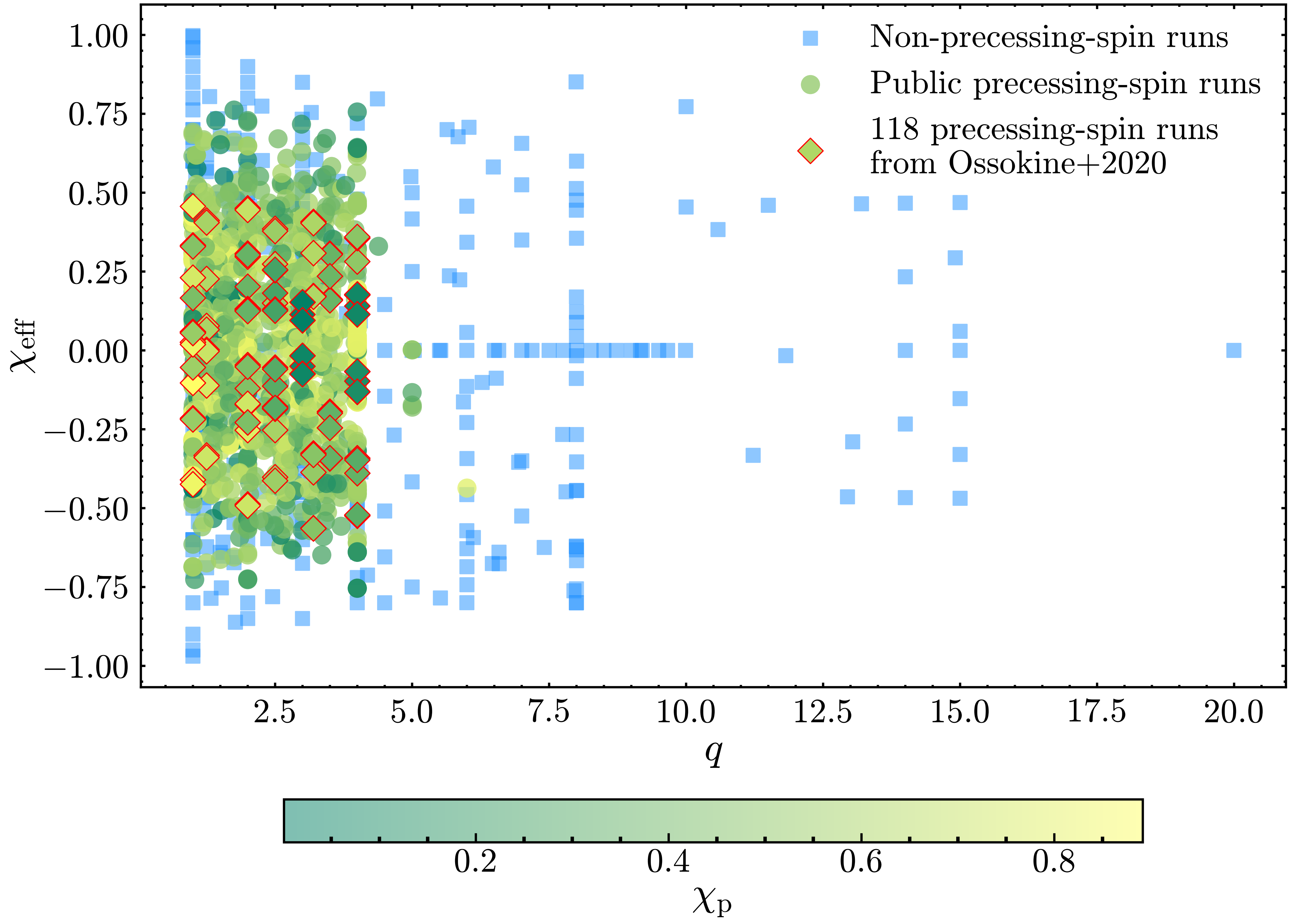
$$\mathcal{M} = 1 - \max_{t_0,\phi_0} \frac{(h_{\rm model}, h_{\rm NR})}{\sqrt{(h_{\rm model}, h_{\rm model})\,(h_{\rm NR}, h_{\rm NR})}} \qquad (h,g) = 4\mathrm{Re}\left[\int_{f_{\min}}^{f_{\max}} \frac{h(f)\,g^*(f)\,df}{S_n(f)}\right]$$

$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$

- **Mismatch $\mathcal{M} = 0$** implies models & NR **match perfectly**

χ_p measures the spin components on the orbital plane





Systematics in the Spin-Precessing sector

quasi-circular, spin-precessing case

(Ramos-Buades, AB, Khalil, Estelles, Pompili & Ossokine 23)

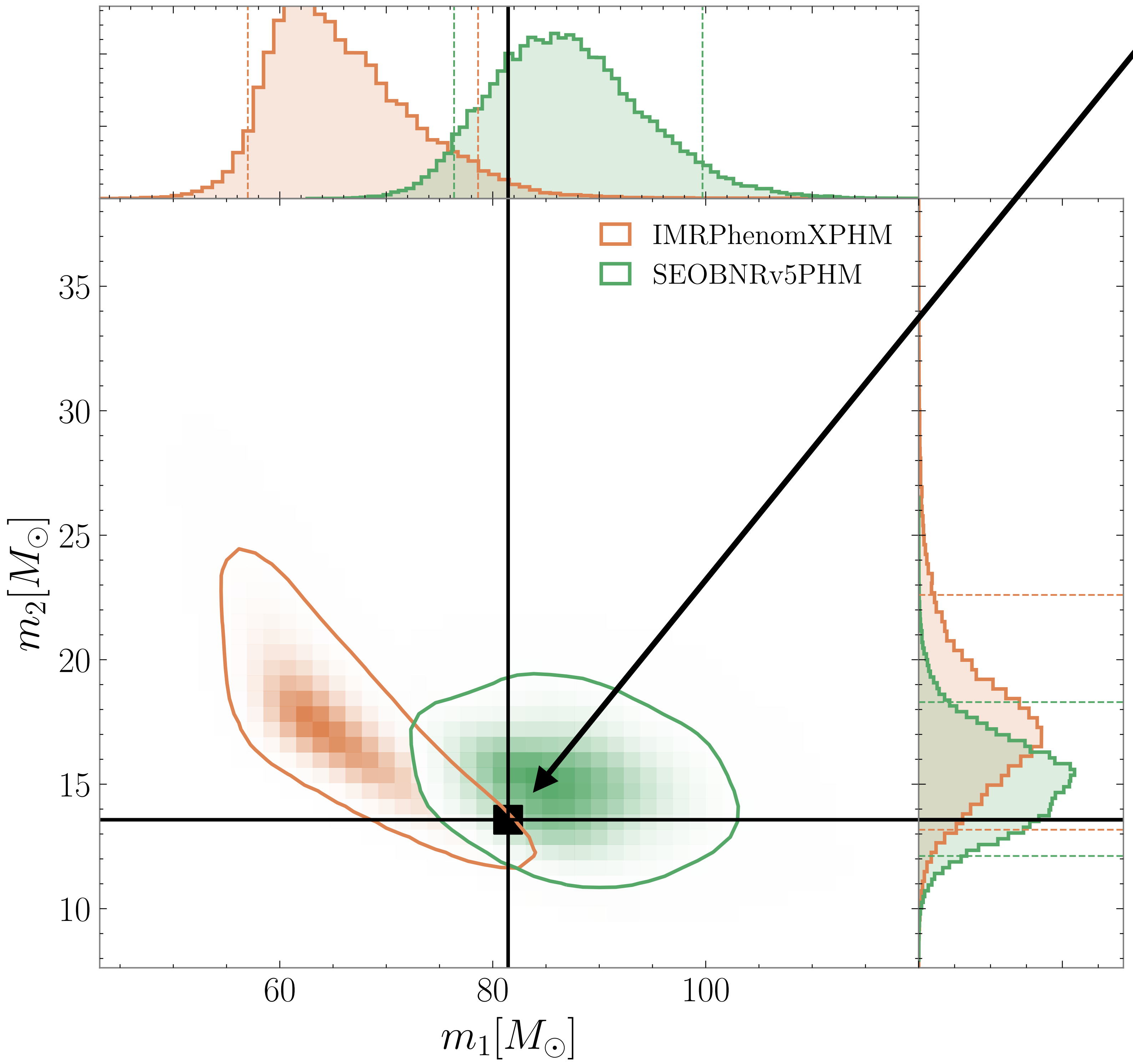
$$\mathcal{M} = 1 - \max_{t_0,\phi_0} \frac{(h_1, h_2)}{\sqrt{(h_1, h_1)\,(h_2, h_2)}} \qquad (h_1, h_2) = 4\mathrm{Re}\left[\int_{f_{\min}}^{f_{\max}} \frac{h_1(f)\, h_2^*(f)\, df}{S_n(f)}\right]$$

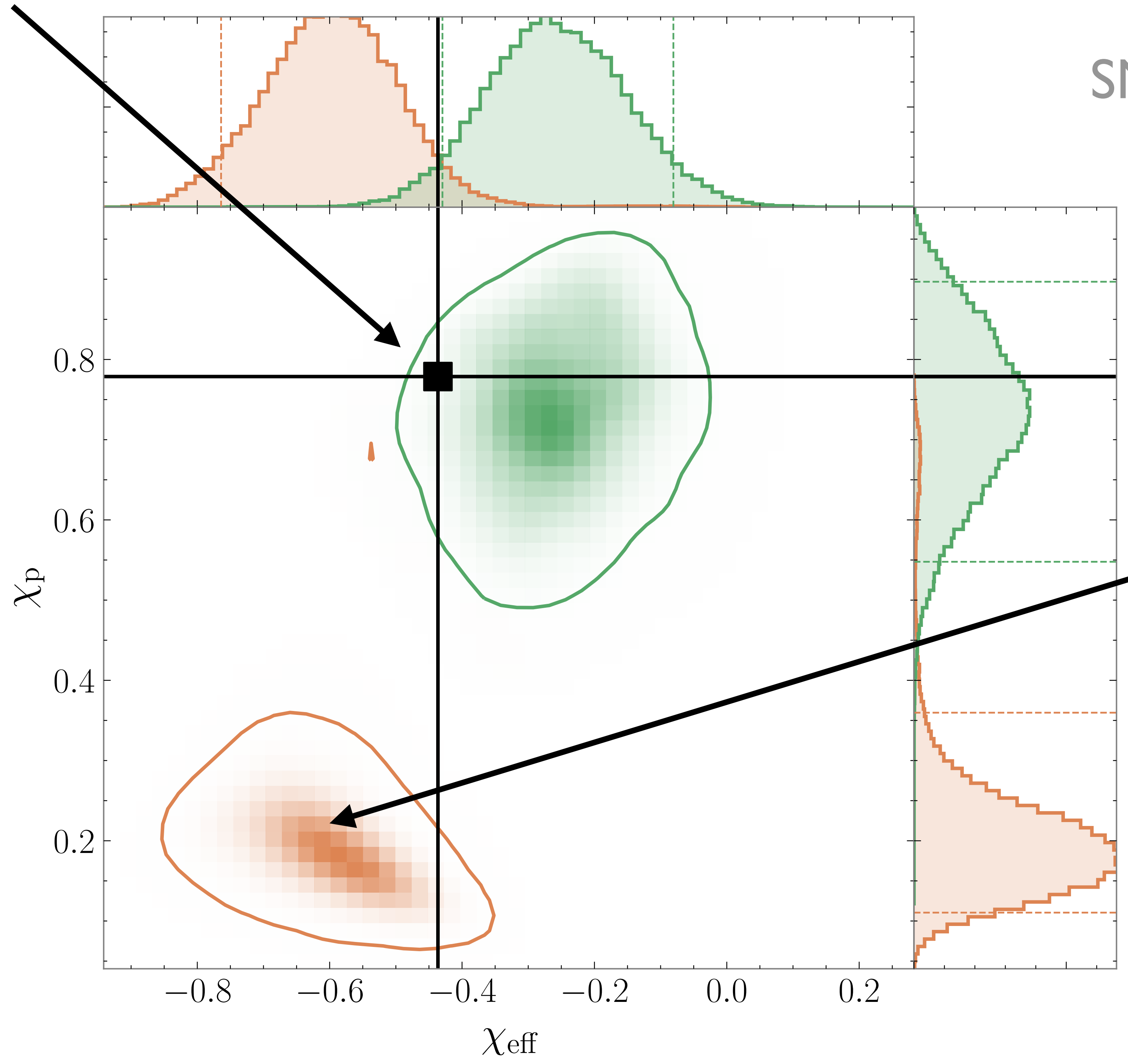
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right) \cdot \hat{\mathbf{L}}$$

$\mathcal{M}(\text{IMRPhenomXPHM}\,|\,NR) = 12\,\%$ $\quad \mathcal{M}(\text{SEOBNRv5PHM}\,|\,\text{NR}) = 2\,\%$

χ_p measures the spin components on the orbital plane

- Synthetic **NR signal is injected, and recovered with both models**





SNR = 20 with O5

- Due to larger systematics **IMRPhenomXPHM erroneously measures small spin-precession.**

Toward Improving Waveform Accuracy: GSF/EOB & Fluxes

- The **second-order GSF** (2GSF) correction to the **energy flux,** and corresponding first post-adiabatic (1PA) **waveforms are available** (when central BH is nonspinning).

  (Pound, Wardell, Warburton & Miller 20; Warburton, Pound, Wardell, Miller & Durkan 21; Wardell, Pound, Warburton, Miller & Durkan 21)

- 2GSF energy flux corrections can be **incorporated in EOB GW mode amplitudes and RR force.**

  (van de Meent, AB, Pompili, Pound, Warburton, Wardell, Durkan & Miller 23)

- For the inspiral, **EOB GW modes/flux** are obtained **resumming the PN-expanded modes/flux** in factorized form:

$$h_{\ell m}^{\rm insp} = h_{\ell m}^{\rm Newt}\, e^{-im\phi}\, S_{\ell m}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}$$

$$\mathcal{F} = \sum_{\ell m} \mathcal{F}_{\ell m} \propto \sum_{\ell m} (m\, M\Omega)^2\, |\, h_{\ell m}^{\rm insp}\, |^2$$

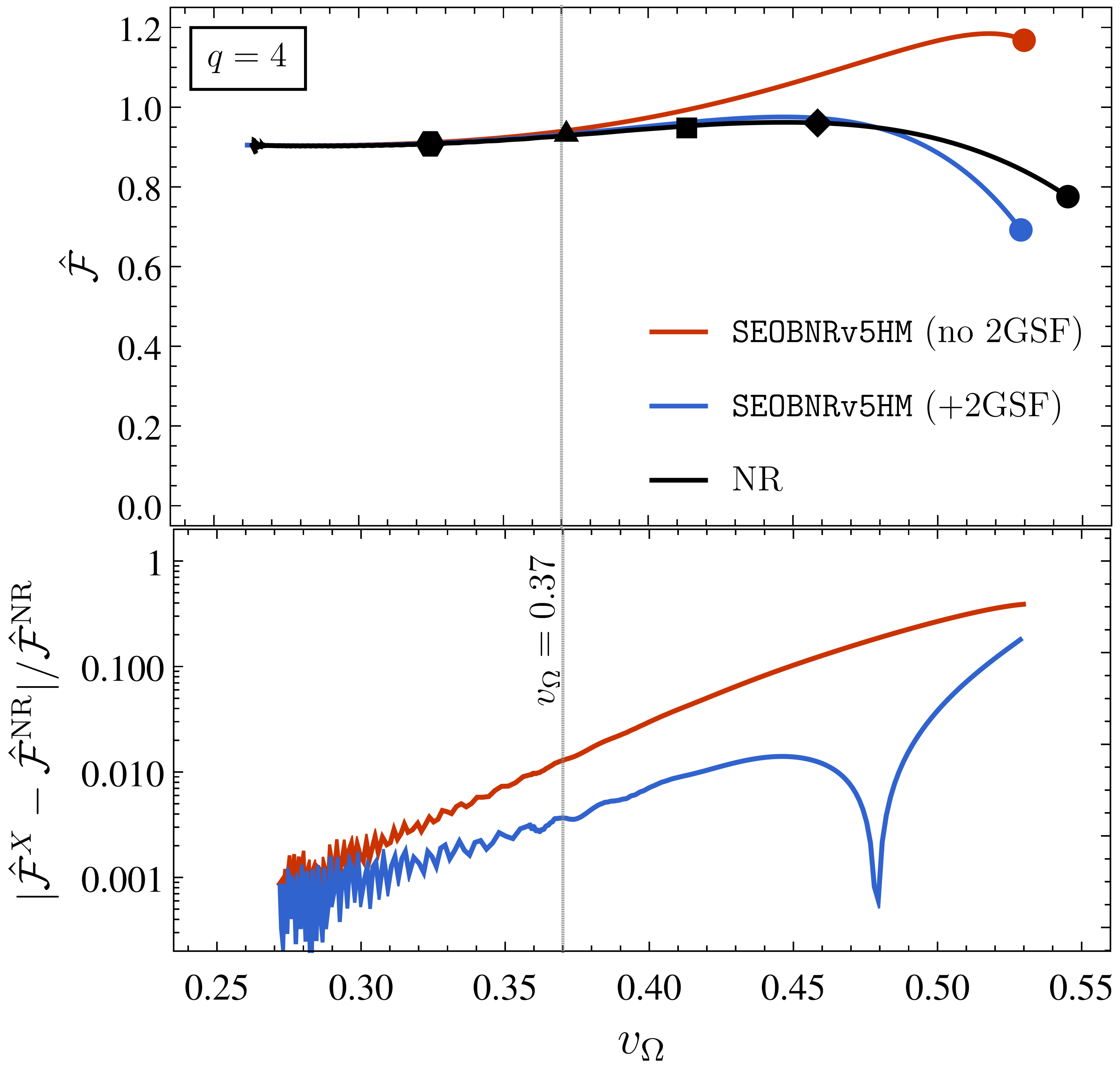
- For the inspiral, **GSF energy-flux modes** are:

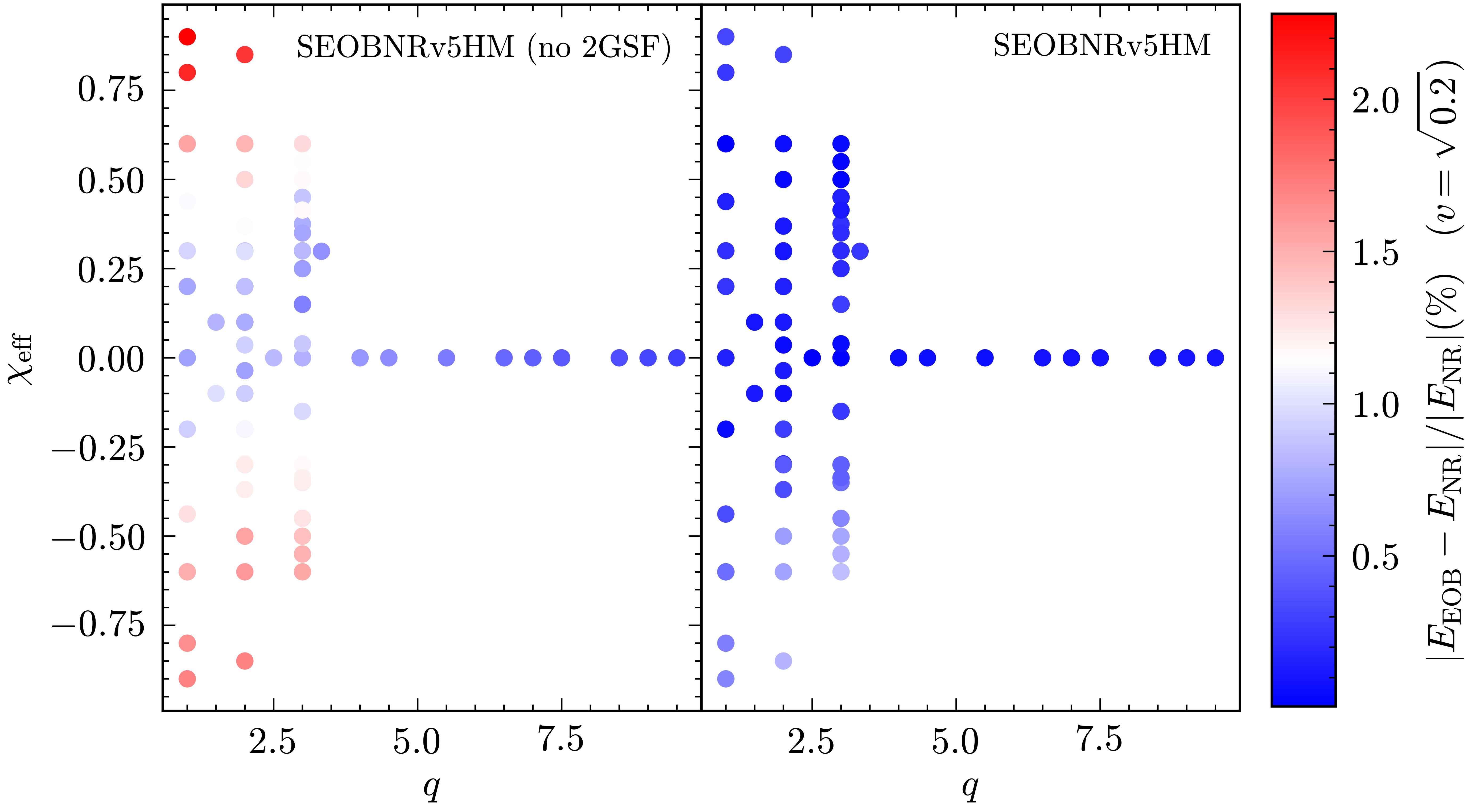
$$\mathcal{F}_{\ell m}^{\rm GSF} = \nu^2\, \mathcal{F}_{\ell m}^{\rm 1GSF} + \nu^3\, \mathcal{F}_{\ell m}^{\rm 2GSF} + \mathcal{O}(\nu^4)$$

- The **1GSF and 2GSF information** is included **in** $\rho_{\ell m}, S_{\ell m}, T_{\ell m}$.

Toward Improving Waveform Accuracy: GSF/EOB & Fluxes (contd.)

(van de Meent, AB, Pompili, Pound, Warburton, Wardell, Durkan & Miller 23)





- **Better agreement** with NR **for all mass ratios**.
- **After calibration to NR**, the binding energy of the waveform model **with 2GSF information is more accurate**.

Toward Improving Waveform Accuracy: GSF/EOB & Hamiltonian

- **EOB Hamiltonian with 1GSF terms was derived**, but in standard EOB resummation/gauge it has a **pole at the light-ring**.

  (Barausse, AB & Le Tiec 12; Le Tiec, Barausse & AB 12; Ackay, Barack, Damour & Sago 12)

- **Alternative resummation/gauge was introduced to avoid the pole** and describe plunging dynamics.

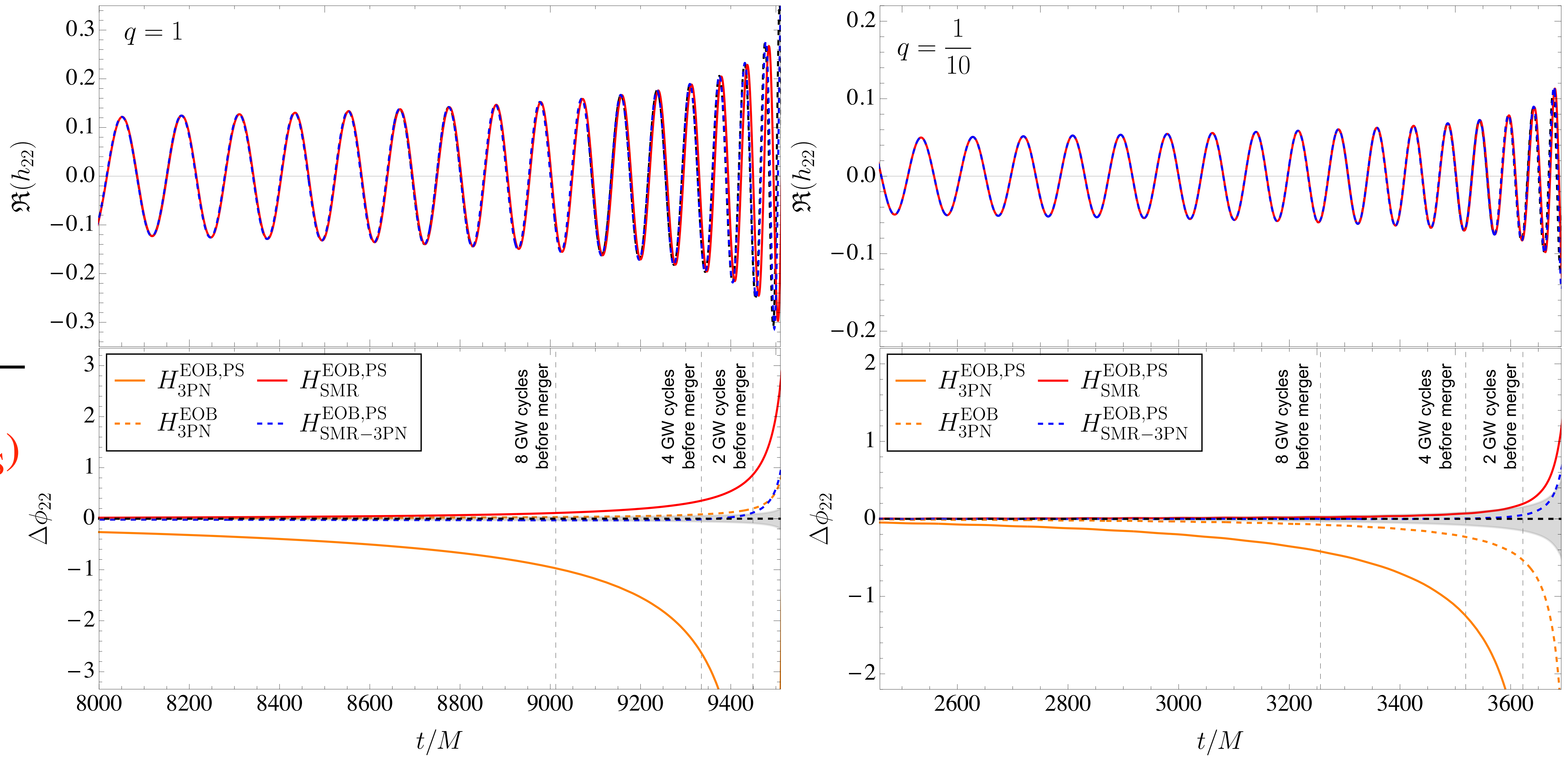
  (Antonelli, van den Meent, AB, Steinhoff & Vines 19)

$$H^{\rm EOB}_{\rm real} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

nonspinning case

$$H_{\rm eff} = \sqrt{\underbrace{(1-2u)\left[\mu^2 + p_r^2\, B_{np}(r) + \frac{L^2}{r^2}\right]}_{H_S^2} + (1-2u)\,\mu^2\, Q(u,\nu,H_S)}$$

$$u = M/r$$

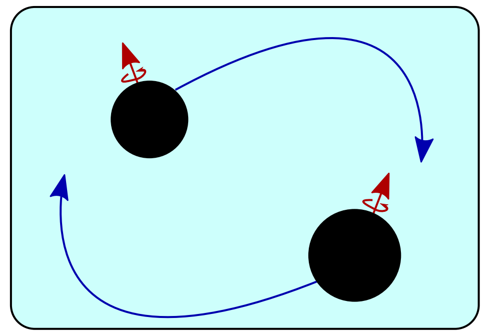


- **EOB/GSF Hamiltonian improves accuracy against NR**, for mass ratios larger than one, **when including GSF & PN** information.

  (see Nagar & Albanesi 22; Albertini, Nagar, Pound, Warburton, Wardell, Durkan & Miller 22)

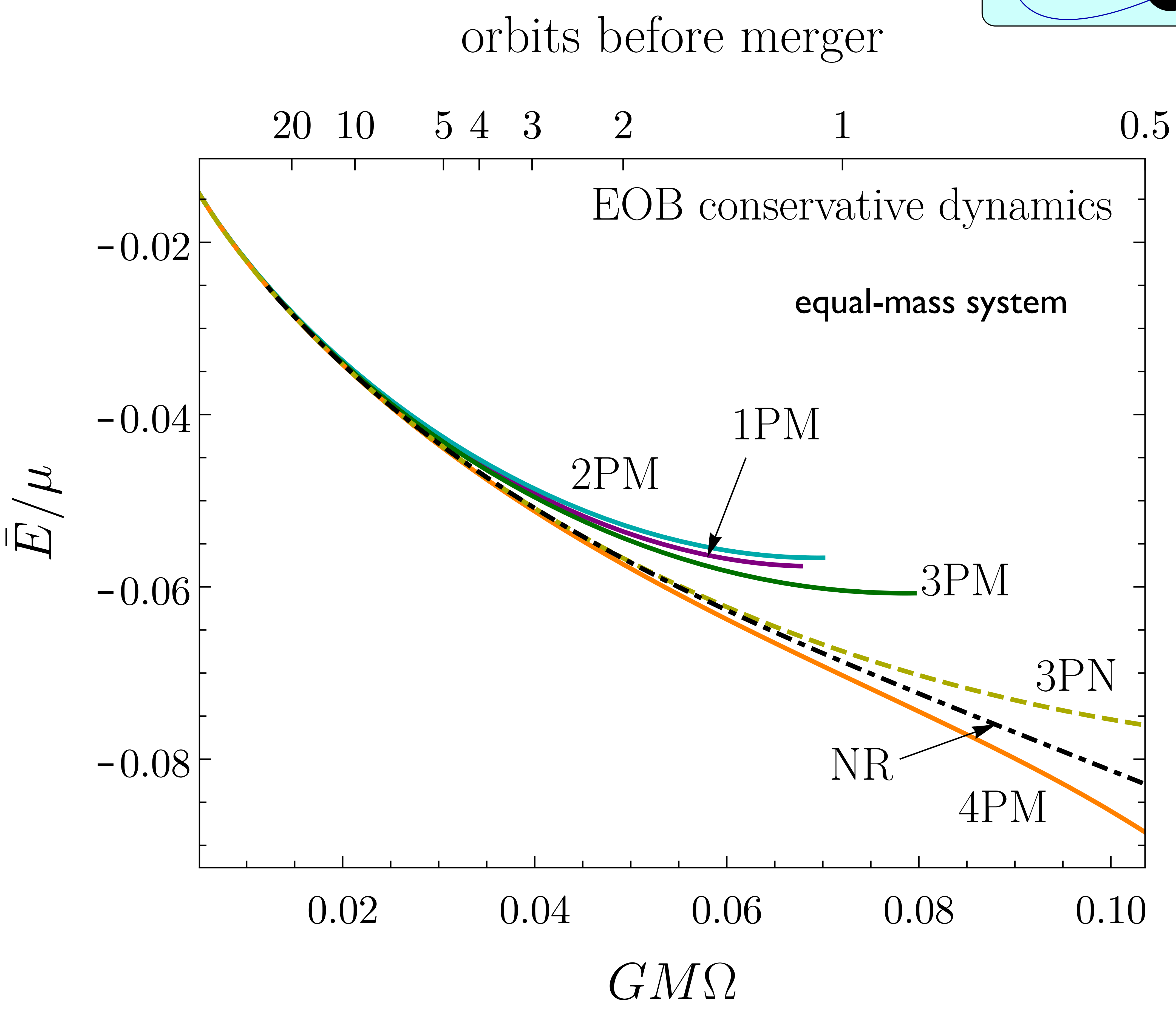
Toward Improving Waveform Accuracy: PM/EOB nonspinning

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$



nonspinning case

$$H_{\rm eff} = \sqrt{\underbrace{\left(1 - 2u + a_{\rm 2PM}\, u^2 + a_{\rm 3PM}\, u^3 + a_{\rm 4PM}\, u^4\right)}_{A_{\rm PM}} \left[\mu^2 + p_r^2\, B_{np}(r) + \frac{L^2}{r^2}\right]}$$



(Khalil, AB, Steinhoff & Vines 22)

- The coefficients $a_{\rm nPM}$ are obtained matching the scattering angles in EOB and PM.

(Antonelli, AB, Steinhoff, van de Meent & Vines 19; Khalil, AB, Steinhoff & Vines 22)

- **3PN** is slightly **better for circular orbits,** but **4PM** is **better for scattering angle** (next page!).

Toward Improving Scattering Accuracy: PM/EOB nonspinning

$$H_{\rm real}^{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu}-1\right)}$$

nonspinning case

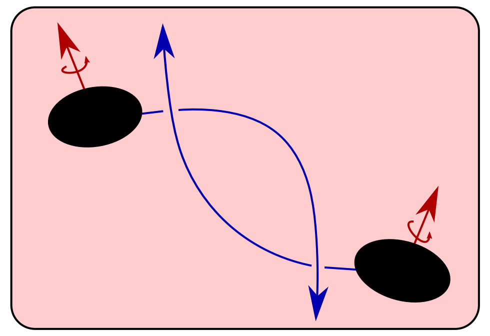
$$H_{\rm eff} = \sqrt{\underbrace{\left(1-2u+a_{\rm 2PM}\,u^2+a_{\rm 3PM}\,u^3+a_{\rm 4PM}\,u^4\right)}_{A_{\rm PM}}\left[\mu^2+p_r^2\,B_{np}(r)+\frac{L^2}{r^2}\right]}$$

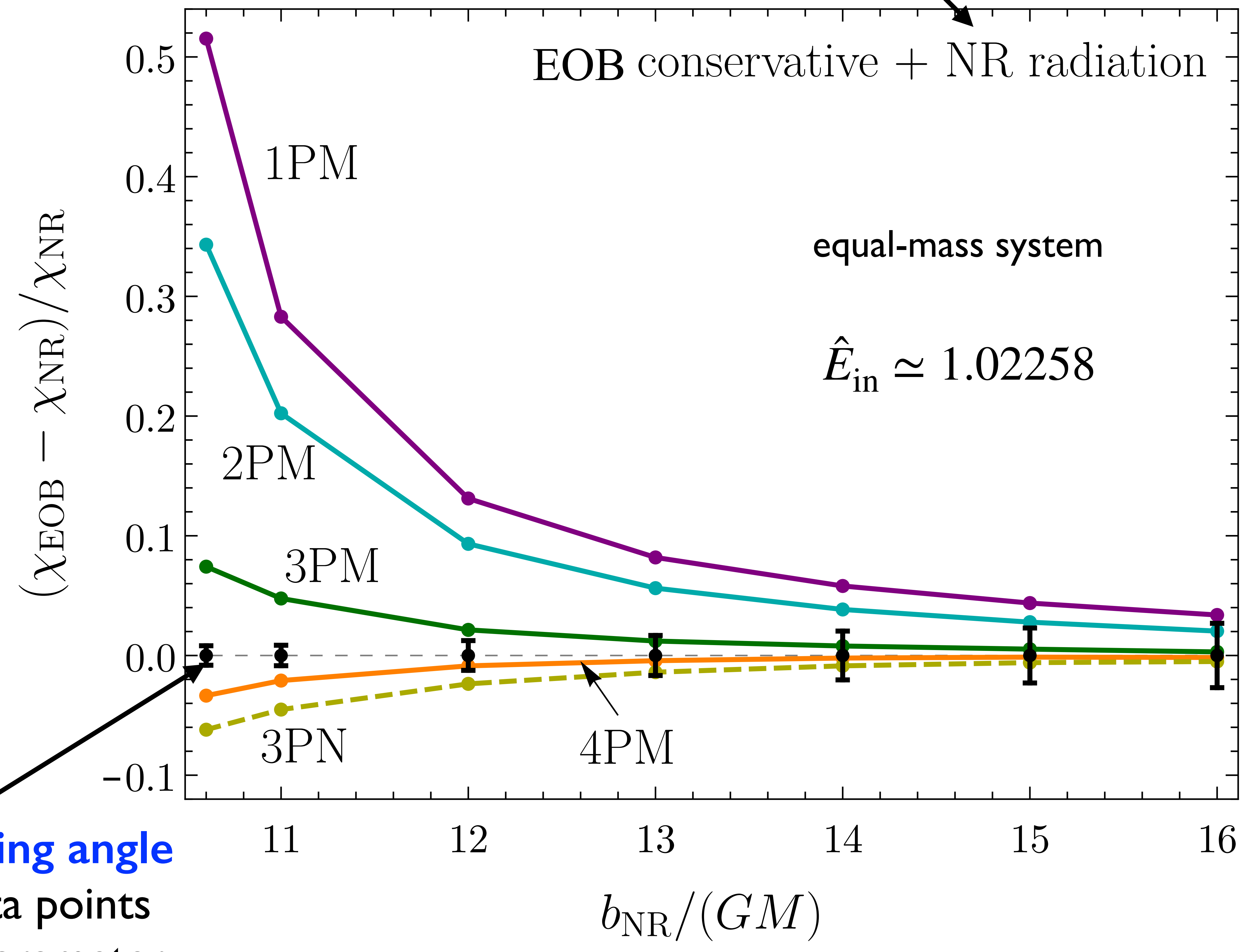
- The coefficients $a_{\rm nPM}$ are obtained matching the scattering angles in EOB and PM.

(Antonelli, AB, Steinhoff, van de Meent & Vines 19; Khalil, AB, Steinhoff & Vines 22)

It **does not include the even RR** contribution, only the odd RR part..

(Bini & Damour 12)



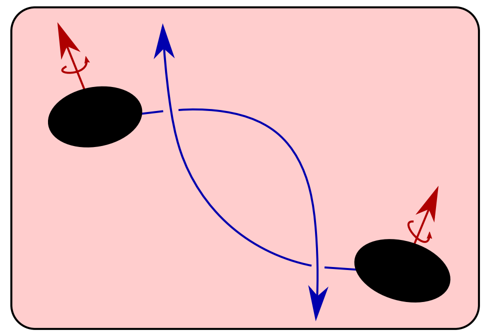


NR data of scattering angle except for three data points at smallest impact parameter.

(Damour, Guarcilena, Hinder, Hopper, Nagar & Rezzolla 14)

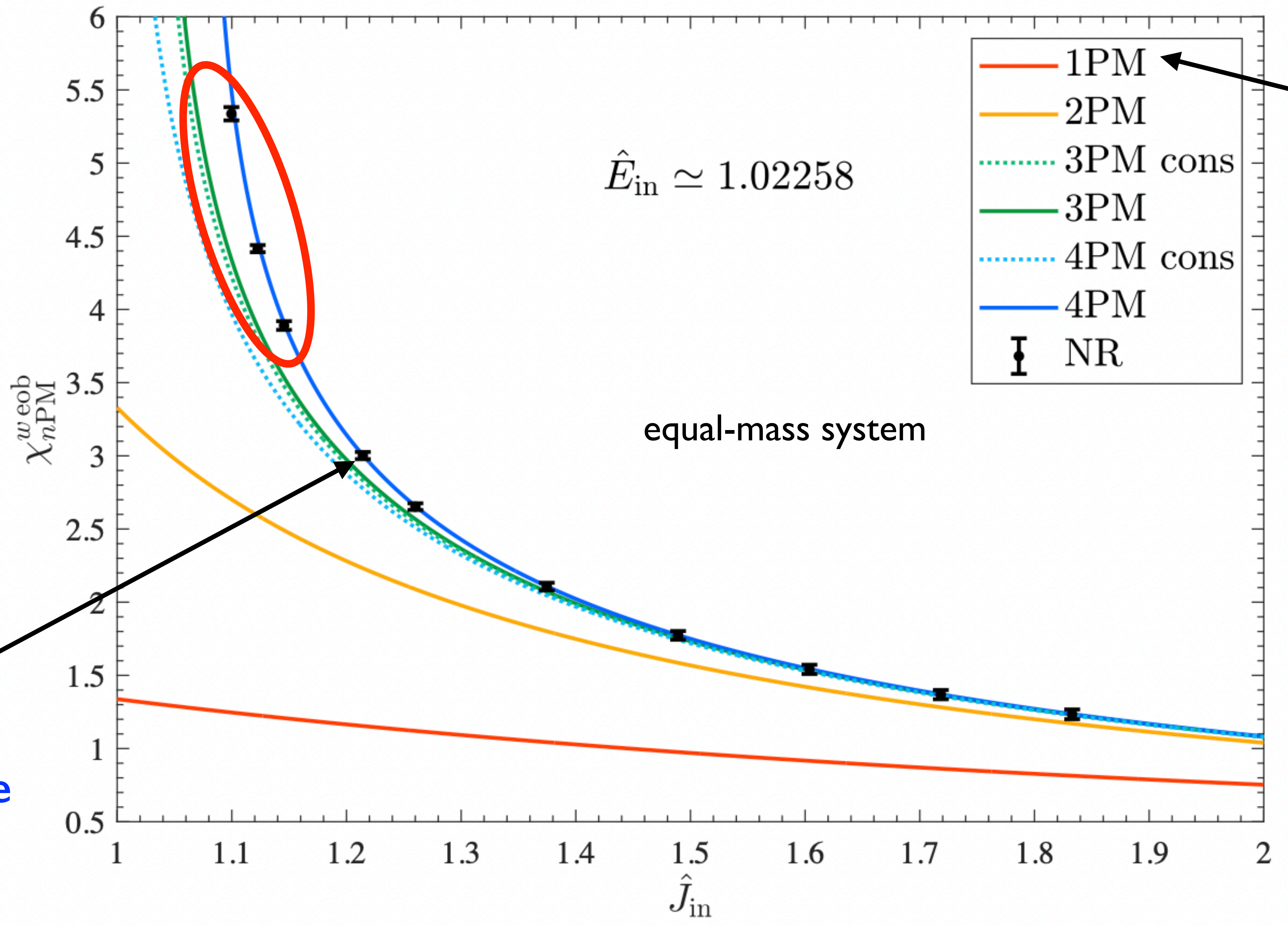
(Khalil, AB, Steinhoff & Vines 22)

Toward Improving Scattering Accuracy: PM/EOB nonspinning



(Damour & Rettegno 22)

- Including **even and odd RR effects** in scattering angle.



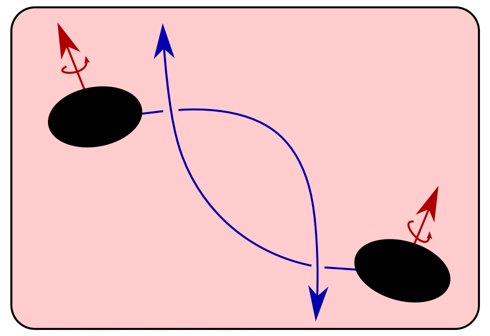
$w^{\rm EOB}$ is similar to the **resummation à la Firsov.**

(Kälin & Porto 20; Dlapa, Kälin, Liu & Porto 23)

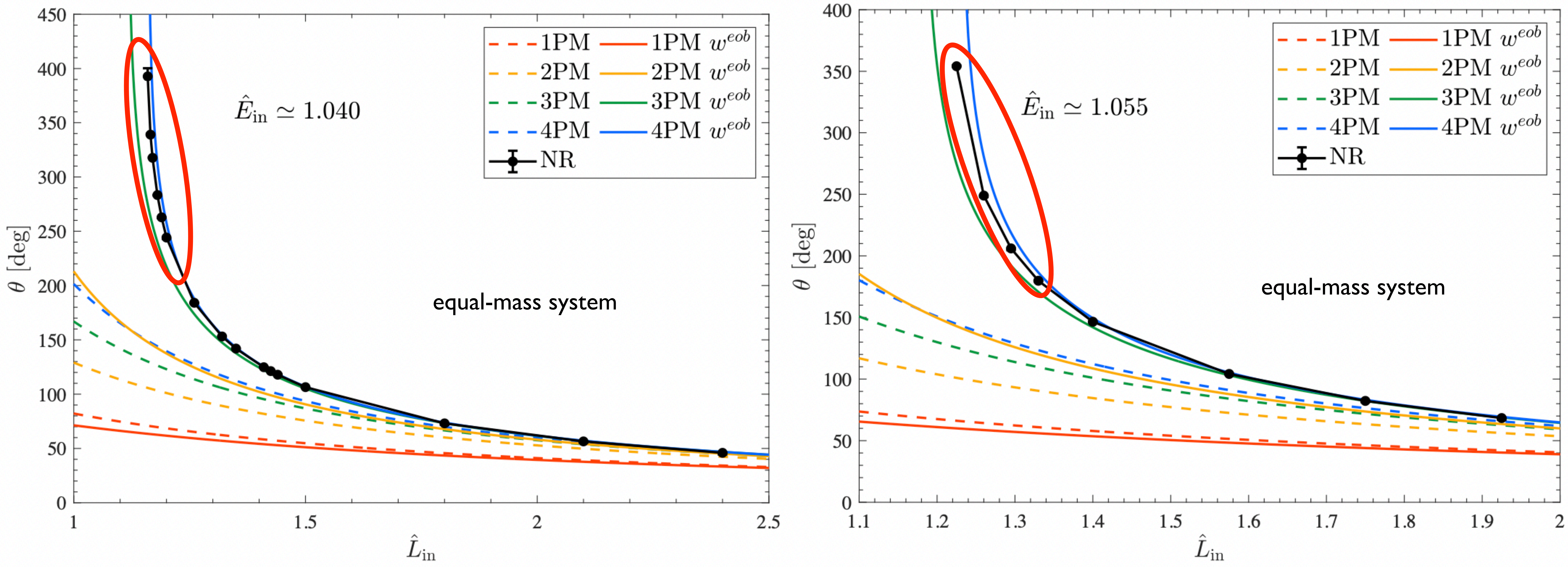
NR data of scattering angle

(Damour, Guarcilena, Hinder, Hopper, Nagar & Rezzolla 14)

Toward Improving Scattering Accuracy: PM/EOB nonspinning (contd.)



(Rettegno, Pratten, Thomas, Schmidt & Damour 23)



- Agreement of $w^{\rm EOB}$ **with NR data becomes worse** for larger energies.

Toward Improving Waveform Accuracy: PM/EOB spinning

$$H^{\rm EOB}_{\rm real} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu}-1\right)} \qquad H^{\rm eff} = H^{\rm eff}_{\rm odd} + H^{\rm eff}_{\rm even}$$

$$H^{\rm eff}_{\rm even} = \sqrt{A_{\rm PM}\left[\mu^2 + p_r^2\,(1+B_{np}) + \frac{L^2}{r^2}(1+a_+^2\,B_{npa})\right]}$$

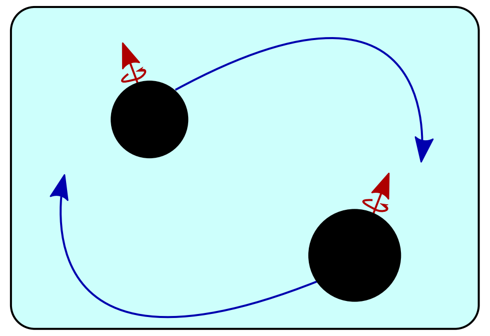
$$H^{\rm eff}_{\rm odd} = \frac{M\,L\left[g^{\rm PM}_{a_+}\,a_+ + g^{\rm PM}_{a_-}\,\delta\,a_- - a_+^2/(4r^2)\,\left(a_+ - a_-\delta\right)\right]}{a_+^2\,(r+2M)+r^3}$$

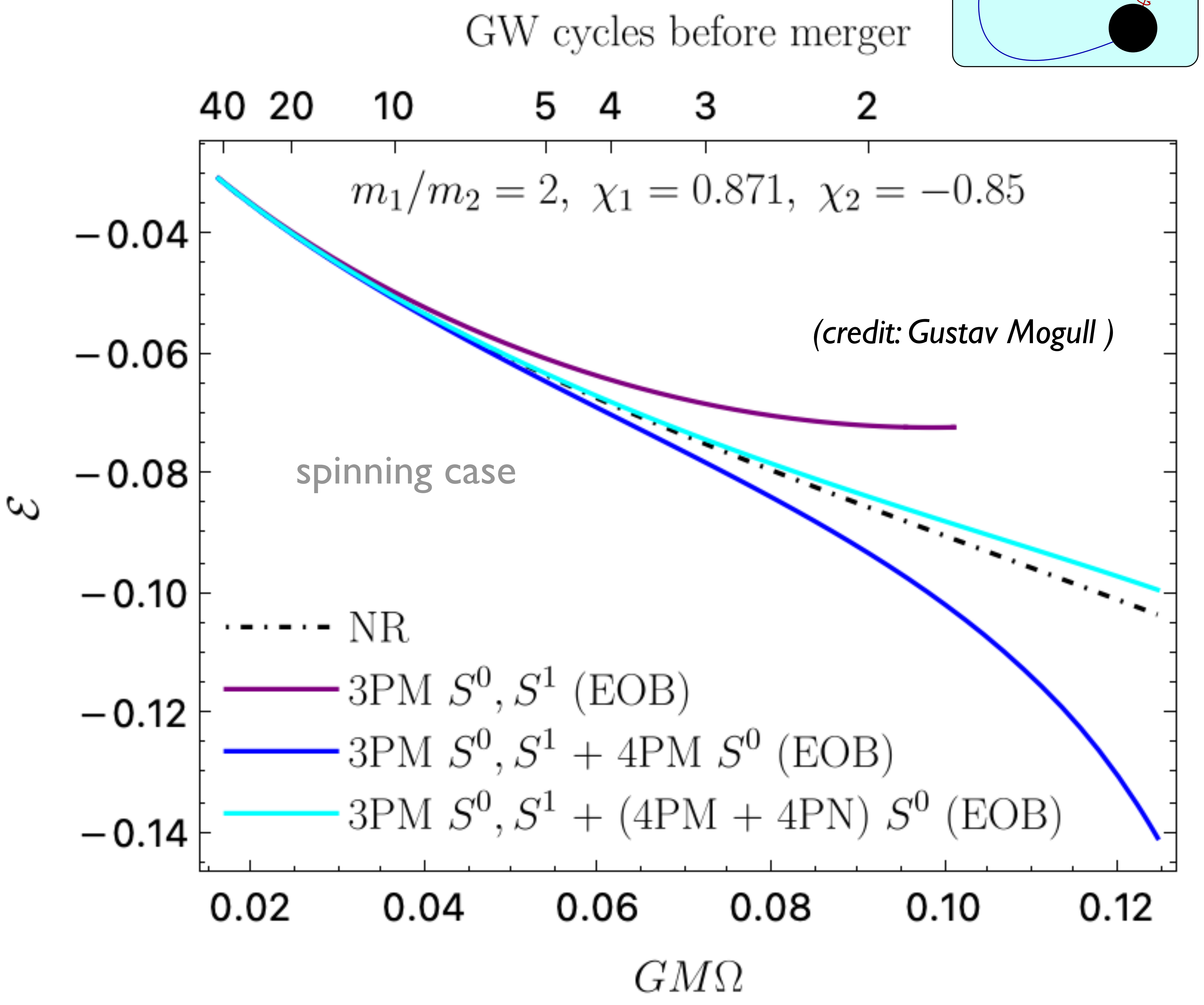
gyro-gravitomagnetic functions

- Implement PM corrections in **pySEOBNR code** publicly available, **build PM/EOB waveform** model and **compare/calibrate to NR.**

https://git.ligo.org/waveforms/software/pyseobnr

(Mihaylov, Ossokine, AB, Estelles, Pompili, Purrer & Ramos-Buades 23)





- **Linear-in-spin couplings** at 3PM order.

(Jakobsen & Mogull 22)

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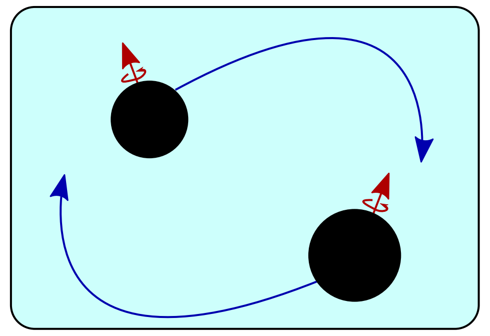
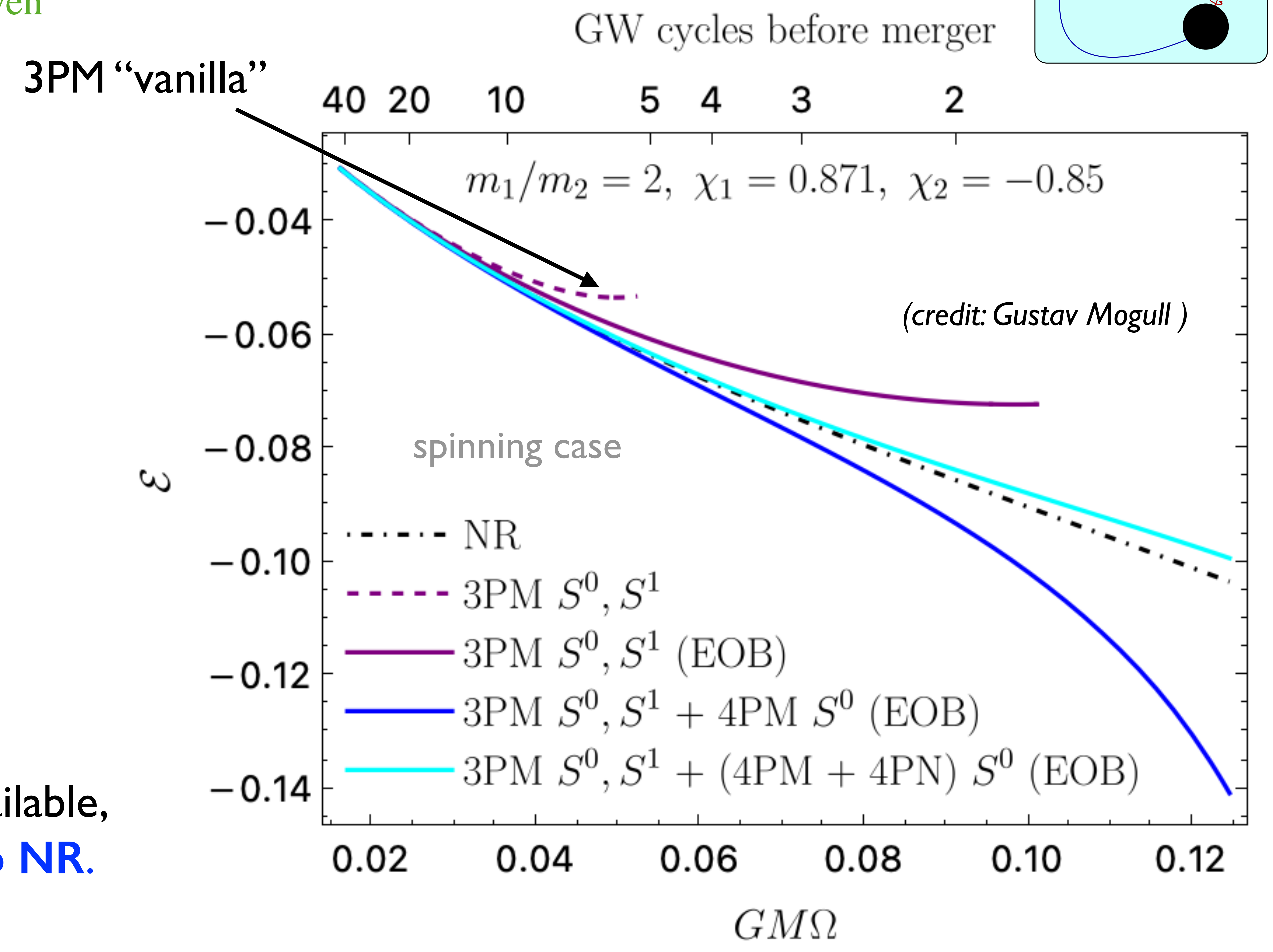
gyro-gravitomagnetic functions

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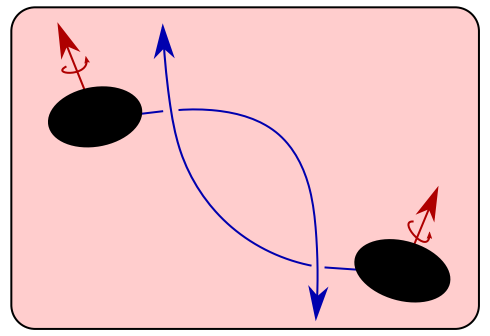
https://git.ligo.org/waveforms/software/pyseobnr

(Mihaylov, Ossokine, AB, Estelles, Pompili, Purrer & Ramos-Buades 23)

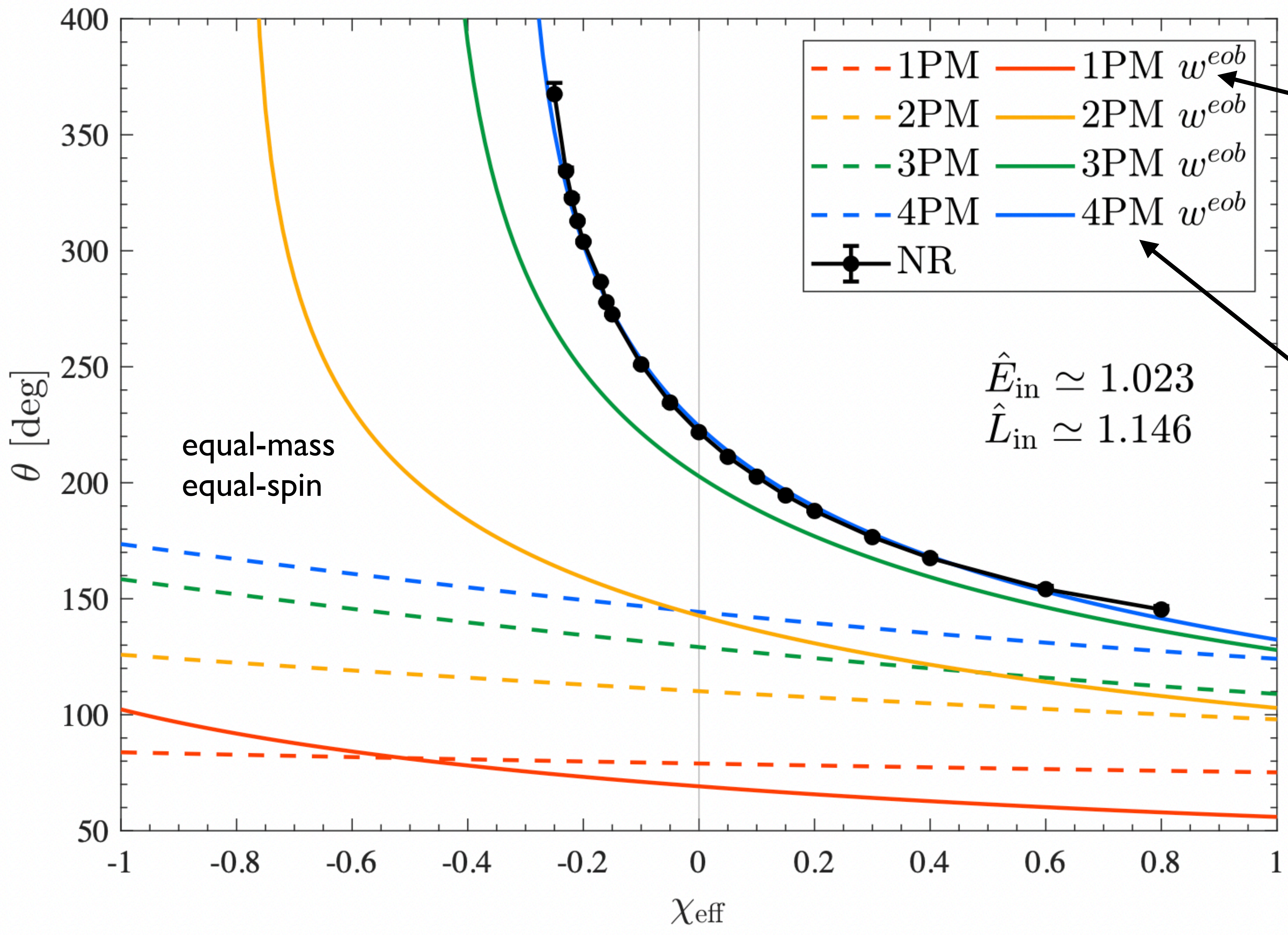
- **Linear-in-spin couplings** at 3PM order.

(Jakobsen & Mogull 22)

Toward Improving Scattering Accuracy: PM/EOB spinning



(Rettegno, Pratten, Thomas, Schmidt & Damour 23)



w^{EOB} is similar to the **resummation à la Firsov**.

(Kälin & Porto 20; Dlapa, Kälin, Liu & Porto 23)

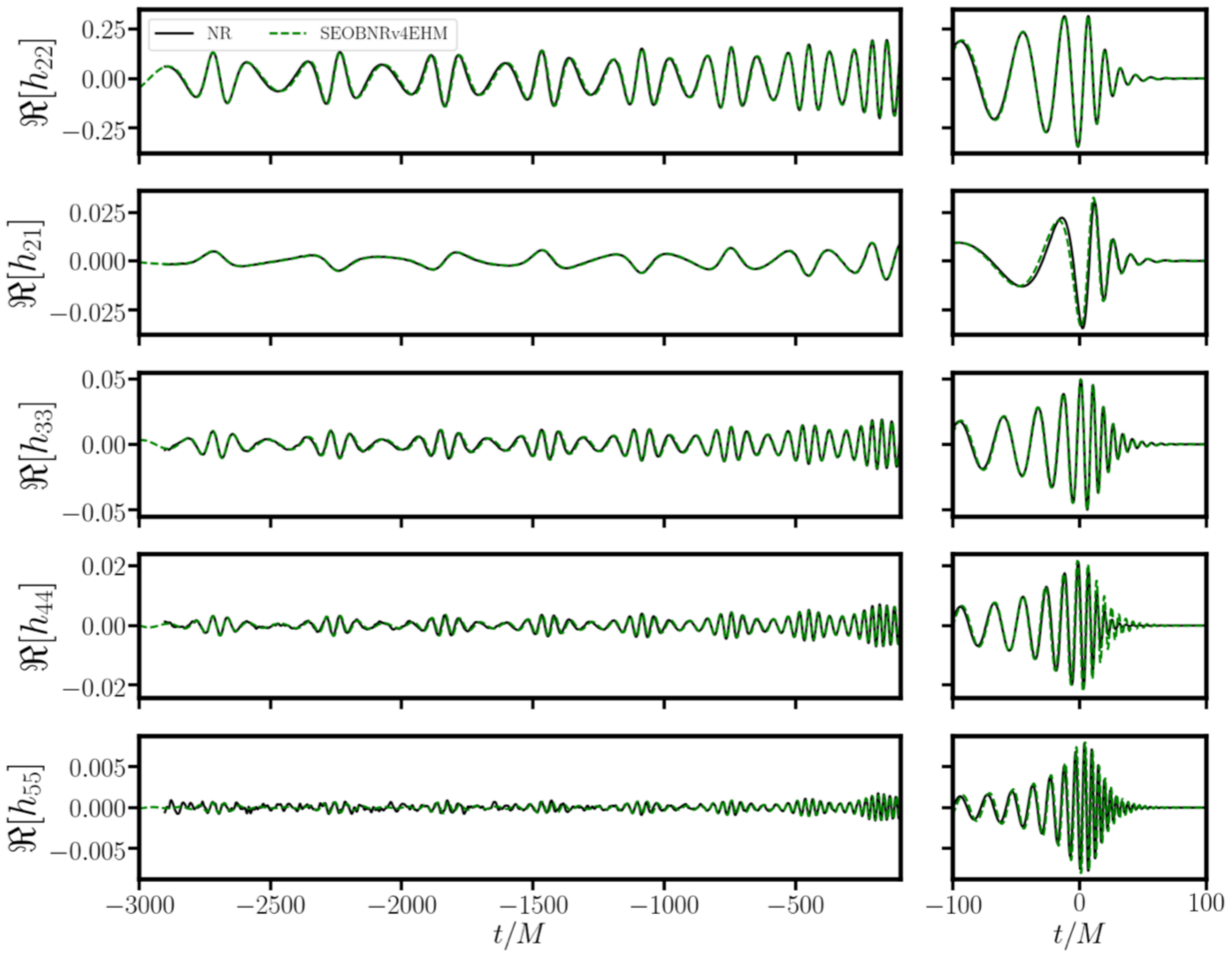
It includes nonspinning up to 4PM,
linear-in-spin up to 3PM,
quadratic in-spin up to 3PM,
cubic- and quartic-in-spin up to 2PM.

Toward Addressing the Eccentric Problem

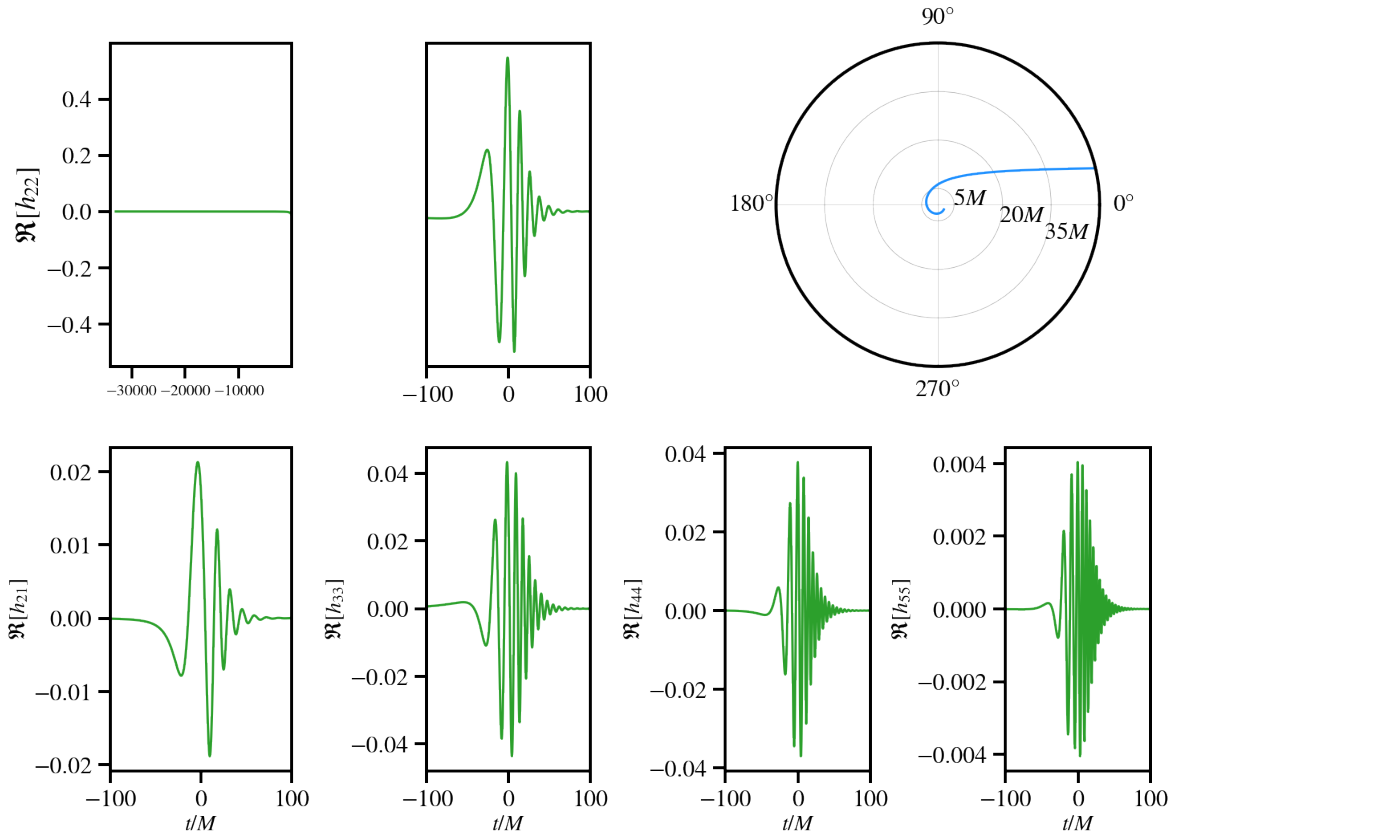
- **Measuring eccentricity** can unveil **origin of compact-binary** observed by LIGO-Virgo, and **reduce systematics.**
- **Eccentric, spinning non-precessing** SEOBNR waveforms. *(Khalil, AB, Steinhoff & Vines 21, Ramos-Buades, AB et al. 21)*

binary black-hole coalescence

mass ratio = 2, non-spinning, e = 0.06



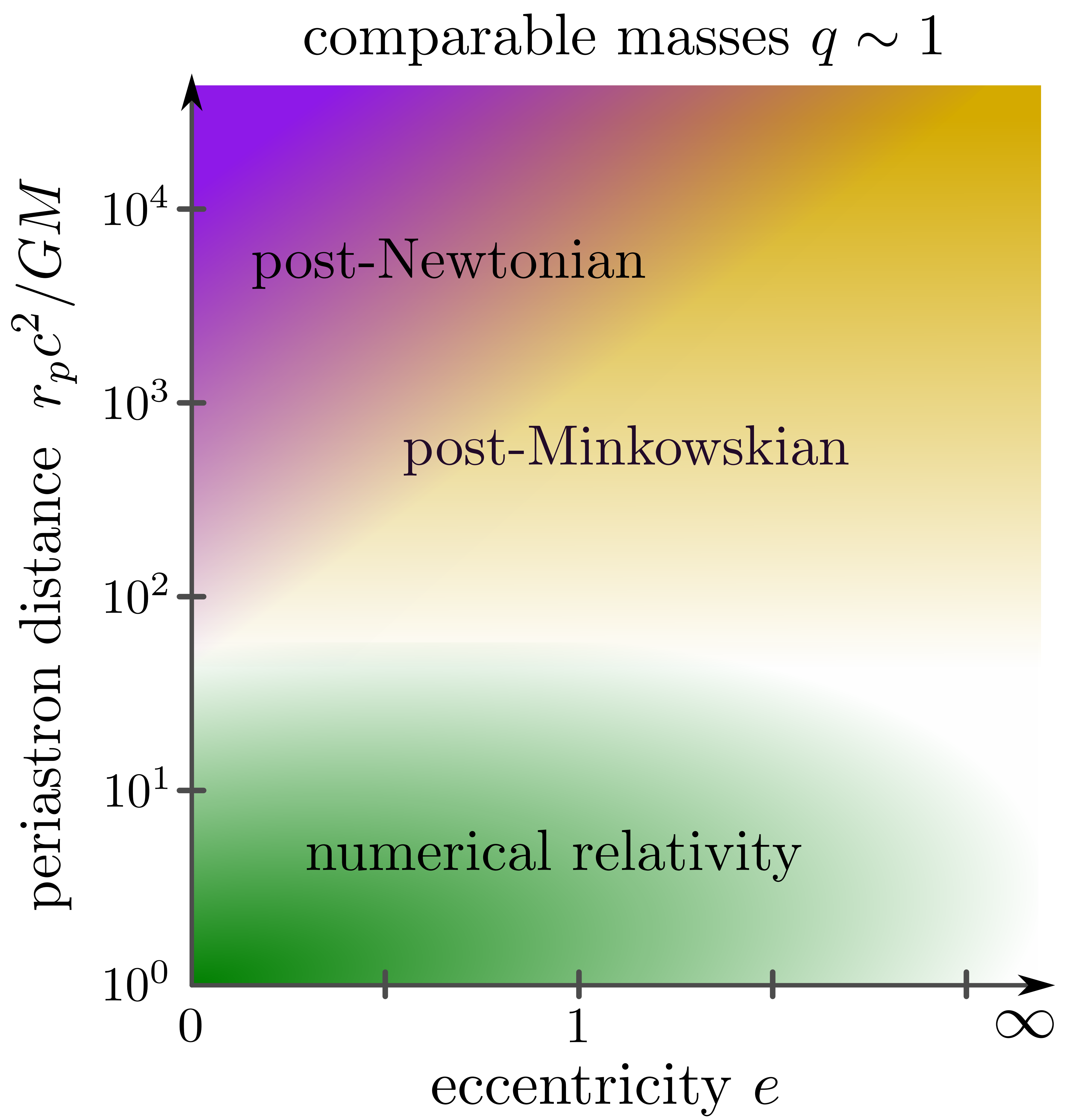
dynamical capture



(see also Huerta et al. 14-19, Hinder et al. 17, Cao & Han 17; Loutrel & Yunes 16, 17, Ireland et al. 19, Moore & Yunes 19, Tiwari et al. 19, Chiaramello & Nagar 20, Ramos-Buades et al. 20, Liu et al. 21, Nagar et al. 20, 21, Islam et al. 21, Nagar & Rettegno 21, Gamba et al. 21, Placidi et al. 21, Albanesi et al. 22)

Toward Addressing the Eccentric Problem (contd.)

(Khalil, AB, Steinhoff & Vines 22)



- The **PM approximation is more accurate than PN approximation** for scattering encounters **at large velocities,** or equivalently **large eccentricities at fixed periastron distance**.

How Scattering Amplitudes/EFT/QFT May Improve Waveform Modeling

- Upcoming **LIGO-Virgo-Kagra runs, and next decade GW detectors** have set **ever more stringent requirements** on the accuracy and precision of waveform models.

- **Amplitude/EFT/QFT** methods have **brought fresh perspectives** (and tools) to solve 2-body problem.

- Besides progress in the non-spinning case, **perturbative results in PM** have also been **extended to the spin sector** (spin-orbit and spin-spin-…) and **radiation**.

- Until recently, **EOB Hamiltonians/fluxes have been mostly based on PN** results (except for SEOBNRv5 which uses 2GSF). Given the recent important developments in PM and GSF, relevant to **explore EOB Hamiltonians/fluxes resummations based on PM, GSF and PN**.

- **Scattering amplitudes/EFT/QFT** may be **more effective in pushing perturbative calculations** (PM, PN) at higher order, and may suggest **new ways of resuming the building blocks** of 2-body dynamics/radiation.

- Until comparisons and full calibration of EOB waveforms against NR simulations is performed, it is **difficult to assess the actual gain of a new higher-order result in PN/PM/GSF**.

The "Astrophysical and Cosmological Relativity" Division

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LIGO
VIRGO
KAGRA

Thank You!