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HIGHER SPIN COMPTON AMPLITUDES AND KERR

From Amplitudes to Gravitational Waves

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[2212.06120] & [2308.XXXX] with

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Structure

1. Introduction: 3pt amplitudes and Kerr

2. Construction of HS theories: non-chiral and chiral

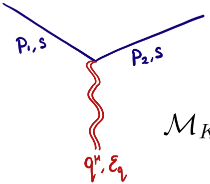
3. Compton amplitudes: $\sqrt{\text{Kerr}}$ and Kerr

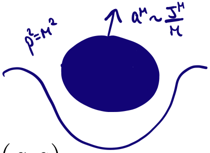
4. Classical analysis & fixing free contacts

Kerr from QFT at 3pts

3pt amp. 2 massive + 1 graviton
[Arkani-Hamed, Huang, Huang '17;
Guevara, Ochirov, Vines '18 & '19;
Chung, Huang, Kim, Lee '18]

energy-mom. tensor of Kerr BH
[Vines '18]





$$\mathcal{M}_{Kerr} = (\varepsilon\cdot p_1)^2 \left(\frac{\langle \mathbf{12}\rangle}{m}\right)^{2s} \quad \rightarrow \quad \varepsilon_{\mu\nu} T^{\mu\nu}(q) \sim (\varepsilon\cdot p_1)^2 \exp(q\cdot a)$$

make explicit using **spin variables**

$\mathcal{M}_{Kerr}$ from Higher Spin (HS) Theory [2212.06120]

HS constraints {massive gauge inv. + ...} **uniquely** predict 3pt Kerr & $\sqrt{\text{Kerr}}$ amps

Kerr from QFT at 3pts: Spin Variables

Express $\mathcal{M}_{Kerr}$ in spin variables [Maybee, O'Connell, Vines '19; Arkani-Hamed,Huang, O'Connell '19; Aoude, Haddad, Helset '20 '21]

$$\mathcal{M}_{Kerr} = (\varepsilon\cdot p_1)^2 \sum_{n=0}^{2s} \frac{1}{n!}\langle (q\cdot\hat{a})^n\rangle = (\varepsilon\cdot p_1)^2 \langle \exp(q\cdot\hat{a})\rangle,$$

truncates at $2s$, **full quantum** amp.

1. particle 2 is **Lorentz boost** of particle 1: $p_2 = \Lambda p_1 = p_1 + q$,

$$|\mathbf{2}\rangle = \frac{1}{\sqrt{1-q^2/4m^2}}\left(|\bar{\mathbf{1}}\rangle + \frac{1}{2m} q|\bar{\mathbf{1}}]\right)$$

2. re-express spinors →**expectation values of spin operator**,

$$\langle \hat{a}^\mu\rangle = \frac{1}{m^{2s+1}}\langle \bar{\mathbf{1}}|^{2s}\,\hat{S}^\mu\,|\mathbf{1}\rangle^{2s}, \qquad |\mathbf{1}\rangle = |1^a\rangle\, z_a,\ |\bar{\mathbf{1}}\rangle = |1^a\rangle\, \bar{z}_a$$

Need infinite spin

1. for **full tower** of multipoles — [Guevara, Ochirov, Vines '18 & '19; Arkani-Hamed,Huang, O'Connell '19]
2. to get correct **classical** spin multipoles — [LC, Pichini '22]

3pt amp. any theory

$$\mathcal{M} = (\varepsilon\cdot p_1)^2 \sum_{n=0}^{2s} c_n(s)\langle (q\cdot\hat{a})^n\rangle,$$

Leading Regge Superstring [LC, Pichini '22]

$c_0^{(s)} = c_1^{(s)} = 1,\ c_2^{(s)} = \frac{4s^2-7s+4}{(2s)(2s-1)}\dots$

Classical matching only for

$$(\varepsilon\cdot p_1)^2 \sum_{n=0}^{\infty} c_n^{(\infty)}\langle (q\cdot\hat{a})^n\rangle \sim \varepsilon_{\mu\nu} T^{\mu\nu}_{\text{cl. string}}$$

where classical coefficients are $c_n^{(\infty)} := \lim_{s\to\infty} c_n^{(s)}$

Kerr

spin multipole coeffs.

$c_n^{(s)} = \frac{1}{n!}$

spin universality $c_n^{(\infty)} = c_n^{(s)}$

Higher Spin Constraints: Spin 1 example

start from spontaneously broken gauge theory

$$\mathcal{L}_{SSB} = -\frac{1}{4}F^2 - 2|D_\mu W_\nu|^2 + |mW - D\phi|^2 - ieF_{\mu\nu}W^\mu \bar{W}^\nu + \dots$$

Current constraint

$\implies$ softer high energy behaviour at tree level

$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} V(W\bar{W}A)\Big|_{(2,3)} = \mathcal{O}(m)$$

where $V(W\bar{W}A)$ = offshell 3pt vertex

Massive gauge symmetry

$\mathcal{L}_{SSB}$ invariant under massive gauge symmetry

$$\delta W_\mu = \partial_\mu \xi + \dots, \qquad \delta\phi = m\xi + \dots$$

Ward identities:

$$mV(\phi\bar{W}A) - ip_1 \cdot \frac{\partial}{\partial \epsilon_1} V(W\bar{W}A)\Big|_{(2,3)} = 0$$

Bottom up construction of HS theories

Free theory for massive spin$-s$ [Zinoviev '01]

- d.o.f massless spin$-s$ $\rightarrow \begin{cases} s+1 \text{ fields } \Phi^k \\ \delta\Phi^k = \partial^{(1}\xi^{k-1)} + m\xi^k \dots \end{cases}$ $\Phi^k := \Phi^{\mu_1\cdots\mu_k}$ $\xi^k := \xi^{\mu_1\cdots\mu_k}$
- free Lag. $\mathcal{L}_0 = -\sum_{k=0}^{s}\frac{(-1)^k}{2}\left[\Phi^k(\Box + m^2)\Phi^k - \frac{k(k-1)}{4}\tilde{\Phi}^k(\Box+m^2)\tilde{\Phi}^k\right] + \mathcal{L}_{\text{off-diag.}}$

Constraining interactions $V_{\Phi^k\bar{\Phi}^s A^h}$ [2212:06120] $h = \begin{cases} 1, \text{ gauge th.} \\ 2, \text{ gravity} \end{cases}$

(MC) minimal coupling extension of $\mathcal{L}_0$

(PC) power-counting bound $V_{\Phi^k\bar{\Phi}^s A^h} \sim \partial^{k+s-2h}(F_{\mu\nu})^h$

(WI) massive Ward identities $m V_{\Phi^k\bar{\Phi}^s A^h} - p\cdot\frac{\partial}{\partial\epsilon} V_{\Phi^{k+1}\bar{\Phi}^s A^h} + \left(\frac{\partial}{\partial\epsilon}\right)^2 V_{\Phi^{k+2}\bar{\Phi}^s A^h}\Big|_{(2,3)} = 0$

(CC) current constraint $p\cdot\frac{\partial}{\partial\epsilon} V_{\Phi^s\bar{\Phi}^s A^h}\Big|_{(2,3)} = \mathcal{O}(m)$

(ND) Near-diagonal interactions: no interaction if $|s_1 - s_2| > 1$

Results from HS constraints

3pt amps. from Higher Spin (HS) Theory [2212.06120]

constraints {**(MC)**,**(PC)**,**(WI)**,**(CC)**} **uniquely** predict $\mathcal{M}_{Kerr}$
[...+ **(ND)**] **uniquely** predict $\sqrt{\text{Kerr}}$ (tested $s \leq 6$)

Beyond 3pt: Compton amp. [2212.06120, 2308.XXXX]

diag. propagator: $\Delta(\epsilon,\bar{\epsilon}) = \sum_{s=0}^{\infty}(\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar{\epsilon})^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4}\epsilon^2\bar{\epsilon}^2}{1 + \epsilon\cdot\bar{\epsilon} + \frac{1}{4}\epsilon^2\bar{\epsilon}^2}$

- gauge th. spin-2 : fixing 3pt data + 4pt **(WI)**
  $\implies$ **3 free contact terms** in $A_{\sqrt{\text{Kerr}}}(\Phi_1^2 \Phi_2^2 A_3^- A_4^+)$
- studied $s = 3$ gauge th. & gravity

- not yet unique – missing 4pt **(CC)** or other constraint?
- computationally intense for $s \geq 3$

Chiral Formalism for Higher Spins

[Ochirov, Skvortsov '22]

Trade SO$(1,3)$ tensors $\Phi_{\mu_1 \dots \mu_s} \longrightarrow$ SL$(2,\mathbb{C})$ chiral symmetric tensors $\Phi_{\alpha_1 \dots \alpha_{2s}}$

- $|\Phi\rangle := \Phi_{\alpha_1 \dots \alpha_{2s}} \rightarrow$ **correct dof's** for a massive spin-s field: $2s+1$
- simple free Lagrangian:

$$\mathcal{L}_0^{(s)} = \langle \partial_\mu \Phi | \partial^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle \,,$$

- chiral formulation maps spinor-helicity formulae to Lagrangians
  $\implies$ external wavefunctions $|\Phi\rangle \rightarrow \frac{1}{m^s} |\mathbf{1}\rangle^{\odot 2s}$
- $\mathcal{L}_0$ not parity invariant $\rightarrow$ needs to be enforced

Chiral Lagragians

$\sqrt{\text{Kerr}}$ Lagrangian [2308.XXXX]

$$\mathcal{L}_{\sqrt{\text{Kerr}}} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle - \underbrace{g \sum_{k=0}^{2s-1} \frac{1}{m^{2k}} \langle \Phi | \left\{ \left(|\overleftarrow{D}|\overrightarrow{D}| \right)^{\odot k} \odot |F^-| \right\} |\Phi\rangle}_{\text{non-min. term fixed by parity}} + \mathcal{L}_4$$

resummed non-minimal term:

$$\frac{1}{m^{4s-2}} \langle \Phi | \left\{ \frac{m^{4s} - |\overleftarrow{D}|\overrightarrow{D}|^{\odot 2s}}{m^2 - |\overleftarrow{D}|\overrightarrow{D}|} \odot |F^-| \right\} |\Phi\rangle$$

$\mathcal{L}_4 \to$ general 4pt operators in indep. helicity sectors:

- $\mathcal{L}_4^{++} = 0$
- $\mathcal{L}_4^{-+} \propto |F^-| \odot |\overleftarrow{D}|F^+|\overrightarrow{D}|$
- $\mathcal{L}_4^{--} \propto |F^-| \odot |F^-|$

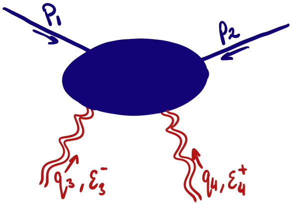
General spin $\sqrt{\text{Kerr}}$ color-ordered amplitudes

$\sqrt{\text{Kerr}}$ abelian Compton Amplitudes [2308.XXXX]

$$A^{\text{abel.}}(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^2(U+V)^{2s}}{m^{4s}t_{13}t_{14}} - \frac{\langle\mathbf{1}3\rangle\langle 3|1|4][\mathbf{2}4]}{m^{4s}t_{13}}P_2^{(2s)} + \frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s}}P_2^{(2s-1)}$$
$$- \frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s-2}}\langle\mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-1)} + C^{(s)},$$

- matches $s \leq 3$ results from massive gauge inv.
- simple polynomials $P_n^{(k)}$ encode general spin
- contact terms shown generated by $\mathcal{L}_3$ & consistent with non abel.
- $\mathcal{L}_4 \to C^{(s)}$ taken to be $C^{(s)}(P_n^{(k)})$

$$U = \tfrac{1}{2}\left(\langle\mathbf{1}|4|\mathbf{2}] - \langle\mathbf{2}|4|\mathbf{1}]\right) - m[\mathbf{12}]$$
$$V = \tfrac{1}{2}\left(\langle\mathbf{1}|4|\mathbf{2}] + \langle\mathbf{2}|4|\mathbf{1}]\right), \quad t_{ij} = 2p_i\cdot p_j$$



Kerr Variables and Polynomials

- 4 local helicity-indep. spin-$1/2$ variables

$$\varsigma_1 = U+V, \quad \varsigma_2 = U-V, \quad \varsigma_3 = -m\langle\mathbf{12}\rangle, \quad \varsigma_4 = -m[\mathbf{12}],$$

- **polynomials $P_n^{(k)}$ of degree $k-n+1$**

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1-\varsigma_2)(\varsigma_1-\varsigma_3)\ldots(\varsigma_1-\varsigma_n)} + \mathrm{perm}(\varsigma_1,\varsigma_2,\ldots\varsigma_n)\,.$$

-$n=2$ example:

$$P_2^{(2s)} = \frac{\varsigma_1^{2s}}{\varsigma_1-\varsigma_2} + \frac{\varsigma_2^{2s}}{\varsigma_2-\varsigma_1} = \sum_{i=0}^{2s-1}\varsigma_1^i\varsigma_2^{2s-1-i}$$

- complete homogeneous symmetric polynomials
- naturally generated by $\mathcal{L}_{\sqrt{\text{Kerr}}}$
- comes from factorisation properties for $\sqrt{\text{Kerr}}$
- generalises to Kerr

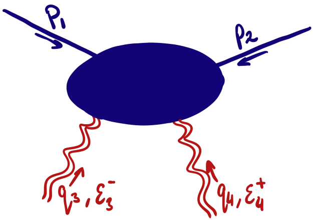
we will construct general spin contact terms out of $P_n^{(k)}$

General spin Kerr amplitudes

Kerr Compton [2308.XXXX]

$$M(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+)=\frac{\langle 3|1|4]^4(U+V)^{2s}}{m^{4s}s_{12}t_{13}t_{14}}+\frac{\langle 3|1|4]^3\langle \mathbf{13}\rangle[\mathbf{42}]}{m^{4s}s_{12}t_{13}}P_2^{(2s)}+\frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s}s_{12}}\langle 3|1|4]^2P_2^{(2s-1)}$$
$$+\frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]\langle 3|1|4]\langle 3|\rho|4]}{m^{4s-2}s_{12}}\left(P_2^{(2s-2)}-m^2\langle \mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-2)}\right)$$
$$+\frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]\langle 3|\rho|4]^2}{m^{4s-4}s_{12}}P_4^{(2s-1)}+C^{(s)}$$

- matches known $s\leq 2$ Compton amplitudes
  [Arkani-Hamed, Huang, Huang '17]
- matches previous $s=5/2$ result
  [Chiodaroli, Johansson, Pichini '21]
- obtained from 3pt vertices
- $C^{(s)}=C^{(s)}(P_n^{(k)})$ for $s\geq 3$

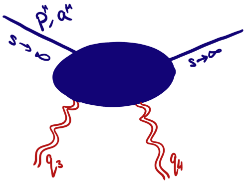


Classical Limits at 4pt

classical scaling: small deflection $\implies$ $q, q_\perp \sim \mathcal{O}(\hbar)$

Large spin limit

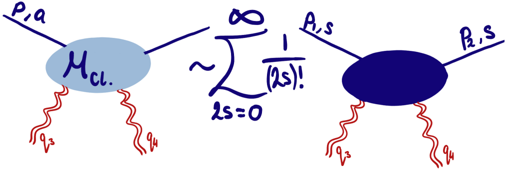
[Guevara, Ochirov, Vines '18 '19; Arkani-Hamed,O'Connell, Huang '19]



- $s \to \infty$ s.t. $\hbar s \sim \mathcal{O}(1)$
- note $m|a| = m\sqrt{-\langle \hat{a} \rangle^2} = \hbar s$
- cl. ring radius: $a^\mu \dots a^\nu = \langle \hat{a}^\mu \dots \hat{a}^\nu \rangle$

Coherent spin states

[Aoude, Ochirov '22]



- classical amp.
  $$\sum_{2s=0}^{\infty} e^{-|z|^2} \frac{1}{(2s)!} \mathcal{M}(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+)$$
- scaling: $z^a, \bar{z}^a \sim \frac{1}{\sqrt{\hbar}}$ and $|z|^2 = \frac{2m^2|a|}{\hbar}$

the two approaches generate the same classical amplitude!

Fixing $\sqrt{\text{Kerr}}$ Contact terms

Fixing $C^{(s)}$ by

[2308.XXXX]

1. ansatz using $P_n^{(k)}$
2. require $A^{\text{abel.}}$ is finite in classical limit (for $C^{(s)} = 0$ it diverges)
3. not modifying $s = 3/2$ & compatible with HS symm. at $s \leq 3$

$\sqrt{\text{Kerr}}$ abelian contact term

$$C^{(s)} = \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{2m^{4s-3}}(\langle \mathbf{12}\rangle + [\mathbf{12}])\left(P_4^{(2s)} - P_2^{(2s-2)}\right)$$

Not unique for quantum contacts but **unique in classical limit** given our ansatz space

Classical $\sqrt{\text{Kerr}}$ amplitudes

$$\mathcal{A}_{\text{cl}}=-(p\cdot\chi)^2\left(\frac{[T^{c_3},T^{c_4}]}{q^2\,p\cdot q_\perp}+\frac{\{T^{c_3},T^{c_4}\}}{2(p\cdot q_\perp)^2}\right)\left(e^x\cosh z-we^x\text{sinhc}z+\frac{w^2-z^2}{2}E(x,y,z)\right)$$
$$+(p\cdot\chi)^2\frac{[T^{c_3},T^{c_4}]}{q^2\,(p\cdot q_\perp)^2}\frac{w^2-z^2}{2}i\epsilon(q_\perp,p,q,a)\tilde{E}(x,y,z)$$

Compton kinematics:

$q=p_3+p_4,\;\; q_\perp=p_4-p_3,$

+ helicity dep. vector $\chi=\langle 3|\sigma|4]$

$\implies$ 4 spin-dep. vars.

$$x=a\cdot q_\perp\,, y=a\cdot q\,,$$
$$z=|a|\frac{p\cdot q_\perp}{m}\,, w=\frac{a\cdot\chi\, p\cdot q_\perp}{p\cdot\chi}.$$

Entire functions:

$$E(x,y,z)=\frac{e^y-e^x\cosh z+(x-y)\,e^x\,\text{sinhc}\,z}{(x-y)^2-z^2}\;+\;(y\to -y)$$

$$\tilde{E}(x,y,z)=\frac{4xe^x\cosh z-2(x^2-y^2+z^2)\,e^x\,\text{sinhc}\,z}{\big((x-y)^2-z^2\big)\big((x+y)^2-z^2\big)}$$
$$-\frac{4x\cosh y+2(x^2+y^2-z^2)\,\text{sinhc}\,y}{\big((x-y)^2-z^2\big)\big((x+y)^2-z^2\big)}.$$

Classical $\sqrt{\text{Kerr}}$ amplitude

$\mathcal{A}_{\text{cl}}$ up to $\mathcal{O}(a^4)$

$$\mathcal{A}_{\text{cl}}^{\text{abel.}} = -\frac{(p\cdot\chi)^2}{2(p\cdot q_\perp)^2}\Big(\underbrace{1+(x-w)+\frac{1}{2}(x-w)^2+\frac{1}{3!}(x-w)^3}_{\text{low order exponential terms } e^{x-w}} - \frac{1}{3!}((x-w)^2-y^2)w + \mathcal{O}(a^4)\Big)$$

Compton kinematics:

$q = p_3+p_4,\ \ q_\perp = p_4-p_3, \chi = \langle 3|\sigma|4]$

spin-dep. vars.

$$x = a\cdot q_\perp\,, y = a\cdot q\,,$$
$$z = |a|\frac{p\cdot q_\perp}{m}\,, w = \frac{a\cdot\chi\, p\cdot q_\perp}{p\cdot\chi}\,.$$

- matches exponential terms up to $\mathcal{O}(a^2)$
- $\mathcal{O}(a^3)$ break of spin shift symmetry
  $a \to a+q \implies x\to x, y\to y+q^2, w\to w,$
  $z^2 \to z^2 + \frac{(p\cdot q_\perp)^2}{m^2}(2y+q^2)$
- even in $z \implies$ no $|a|$ terms

Classical Kerr amplitudes

Classical Kerr amplitude

$$\mathcal{M}_{\text{Kerr}}(\mathbf{1},\mathbf{2},3^-,4^+) = \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\left(e^x\cosh z - w\,e^x\text{sinhc}\,z + \frac{w^2-z^2}{2}E(x,y,z)\right) + \frac{(p\cdot\chi)^3}{q^2\,(p\cdot q_\perp)^2}\frac{w^2-z^2}{2}i\epsilon(\chi,p,q,a)\tilde{E}(x,y,z) + C^{(s)}$$

Compton kinematics:

$q = p_3 + p_4,\;\; q_\perp = p_4 - p_3,$

+ helicity dep. vector $\chi = \langle 3|\sigma|4]$

$\implies$ 4 spin-dep. vars.

$$x = a\cdot q_\perp\,, y = a\cdot q\,,$$
$$z = |a|\frac{p\cdot q_\perp}{m}\,, w = \frac{a\cdot\chi\, p\cdot q_\perp}{p\cdot\chi}.$$

- same E, $\tilde{E}$ as $\sqrt{\text{Kerr}}$
- 1st line: classical double copy ✓
  2nd line: broken by pole term
- even in z
- free contact terms $C^{(s)}$

Fixing Kerr contact terms

Proposal for Kerr contact terms (conservative)

$$C_1^{(s)} = \left(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\right)^2 \left(P_5^{(2s)}[\varsigma_1] + P_5^{(2s)}[\varsigma_2] - 2P_4^{(2s-1)} - (\varsigma_1 + \varsigma_2)P_4^{(2s-2)}\right)$$

Guiding constraints

$$P_5^{(2s)}[\varsigma'] := \lim_{\varsigma_5 \to \varsigma'} P_5^{(2s)}$$

1. ansatz $C^{(s)}$ as $P_n^{(k)}$
2. unmodified for $s = 5/2$
3. matching to s.f. based calc. of $\mathcal{O}(a^4)$ [Siemonsen, Vines '19] [GOV '18 '19; Chung et al. '18, Aouade et al. '20 '21]
4. agrees with Teukolsky results of $\mathcal{O}(a^6)$ for $\alpha = 0$ [Bautista, Guevara, Kavanagh, Vines '22]

Fixing $\mathcal{O}(a^4)$

self-force based calc. [Siemonsen, Vines '19] $\Longrightarrow$ $\mathcal{M}_{\text{Kerr}} = \mathcal{M}_{\text{exp}}$ for $\mathcal{O}(a^{\leq 4})$

$$\mathcal{M}_{\text{exp}} = \mathcal{M}_0 e^{x-w}$$
$$= \mathcal{M}_0 \left(1 + (x-w) + \frac{1}{2}(x-w)^2 + \frac{1}{3!}(x-w)^3 + \frac{1}{4!}(x-w)^4 + \mathcal{O}(a^5)\right)$$

Mismatch between $\mathcal{M}_{\text{Kerr}}$ and $\mathcal{M}_{\text{exp}}$

$$\mathcal{M}_{\text{Kerr}} - \mathcal{M}_{\text{exp}} = \mathcal{M}_0 \left(-\frac{1}{24}(w^2 - z^2)((x-w)^2 - y^2) + \mathcal{O}(a^5)\right)$$

$x = a \cdot q_\perp,\, y = a \cdot q,\, z = |a| \frac{p \cdot q_\perp}{m},\, w = \frac{a \cdot \chi}{p \cdot \chi} p \cdot q_\perp,\, \xi^{-1} = \frac{q^2 m^2}{(p \cdot q_\perp)^2}$

- ansatz for $C_1^{(s)}$:

  helicity term fixed $\mathcal{M}_0 (w^2 - z^2)^2 \Longrightarrow \left(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\right)^2$

  1 free parameter in spin - dep. ansatz $\rightarrow$ **fixed by $\mathcal{O}(a^4)$**

Comparing to Teukolsky Results

Teukolsky soln. given up to $\mathcal{O}(a^6)$ [Bautista, Guevara, Kavanagh, Vines '22]

$$\mathcal{M}_{\text{Teuk.}} = \mathcal{M}_0 \times \left(e^{x-w} + P_\xi(x,y,w)\right)$$

Mismatch for $C^{(s)} \neq 0$ (conservative sector $\eta = 0$)

$$\mathcal{M}_{\text{cl}} - \mathcal{M}_{\text{Teuk.}}\Big|_{\eta=0} = -\mathcal{M}_0 \frac{q^2}{(p\cdot q_\perp)^2}\Big[(w^2 - z^2)^2\left(\frac{1}{24} + \frac{x}{40} + \frac{1}{720}(6x^2 + y^2 + 2z^2)\right)$$
$$+ \frac{\alpha w z^2}{30}(3w^2 + z^2) + \frac{\alpha z^2}{180}\left(4xw(3w^2+z^2) + 3w^4 + 12w^2z^2 + z^4\right) + \mathcal{O}(a^7)\Big]$$

$$x = a\cdot q_\perp,\ y = a\cdot q,\ z = |a|\frac{p\cdot q_\perp}{m},\ w = \frac{a\cdot\chi}{p\cdot\chi}p\cdot q_\perp,\ \xi^{-1} = \frac{q^2 m^2}{(p\cdot q_\perp)^2}$$

- including $C_1^{(s)}$ **improved matching!**
- identify 2 more contact terms: $C_2^{(s)} + C_3^{(s)}$ both $\propto \alpha$

Conservative Contact Terms

Quantum contact terms $C^{(s)}$

$$C_1^{(s)} = \left(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\right)^2 \left(P_5^{(2s)}[\varsigma_1] + P_5^{(2s)}[\varsigma_2] - 2P_4^{(2s-1)} - (\varsigma_1+\varsigma_2)P_4^{(2s-2)}\right)$$

$$C_2^{(s)} = \alpha\langle 3|\rho|4]\langle 3|\bar{\rho}|4]\left(\langle 3|\rho|4]^2 + 3\langle 3|\bar{\rho}|4]^2\right)\left(P_5^{(2s)}[\varsigma_1] - P_5^{(2s)}[\varsigma_2] - (\varsigma_1-\varsigma_2)P_4^{(2s-2)}\right),$$

$$C_3^{(s)} = \alpha\left(\langle 3|\rho|4]^4 + 12\langle 3|\rho|4]^2\langle 3|\bar{\rho}|4]^2 + 3\langle 3|\bar{\rho}|4]^4\right)(\varsigma_1-\varsigma_2)\left(P_6^{(2s)}[\varsigma_1,\varsigma_1] - P_6^{(2s)}[\varsigma_2,\varsigma_2]\right).$$

$$P_5^{(2s)}[\varsigma] := \lim_{\varsigma_5\to\varsigma} P_5^{(2s)}, \quad P_6^{(2s)}[\varsigma,\varsigma'] := \lim_{\varsigma_5\to\varsigma,\varsigma'\to\varsigma_j} P_6^{(2s)}$$

- $C_1^{(\infty)}$ **matches up to $\mathcal{O}(a^{\leq 6})$** & **predicts $\mathcal{O}(a^{>6})$**
- $C_2^{(\infty)}$ fix with $\mathcal{O}(a^5)$ & matches $\mathcal{O}(a^6)$
- $C_3^{(\infty)}$ fixed with $\mathcal{O}(a^{\leq 6})$ **need results $\mathcal{O}(a^{>6})$ to confirm**
- does $C_4^{(\infty)}$ exist? $\to$ **need Teukolsky results $\mathcal{O}(a^7)$ + higher**

Conservative Contact Terms

Classical limit of $C^{(s)}$

$$C_1^{(\infty)} = (w^2 - z^2)^2 \frac{q^2}{2(p\cdot q_\perp)^2}\Big(\frac{e^{-y}(x+y)}{y((x+y)^2 - z^2)^2} + \frac{e^{x+z}}{z}\Big(\frac{1}{2((x+z)^2 - y^2)} - \frac{x+z}{((x+z)^2 - y^2)^2}\Big)\Big) + (y \to -y)$$

similarly for $C_2^{(\infty)}$ & $C_3^{(\infty)}$

- $C_1^{(\infty)}$ **matches up to $\mathcal{O}(a^{\leq 6})$** & **predicts $\mathcal{O}(a^{>6})$**
- $C_2^{(\infty)}$ fix with $\mathcal{O}(a^5)$ & matches $\mathcal{O}(a^6)$
- $C_3^{(\infty)}$ fixed with $\mathcal{O}(a^{\leq 6})$ **need results $\mathcal{O}(a^{>6})$ to check**
- does $C_4^{(\infty)}$ exist? $\to$ **need Teukolsky results $\mathcal{O}(a^7)$ + higher**

Short comment on dissipative terms

Mismatch for $C^{(s)} \neq 0$ (dissipative sector $\eta \neq 0$)

	$\tilde{C}_1^{(\infty)}$	$\tilde{C}_2^{(\infty)}$	$\tilde{C}_3^{(\infty)}$
a^4	0	0	0
a^5	$\eta \frac{z}{120}(w^2 - z^2)^2$	$\eta\alpha \frac{zw^2}{30}(w^2 + 3z^2)$	0
a^6	$\eta \frac{xz}{180}(w^2 - z^2)^2$	$\eta\alpha \frac{xzw^2}{45}(w^2 + 3z^2)$	$\eta\alpha \frac{z^3 w}{90}(5w^2 + 3z^2)$

- $\tilde{C}^{(s)}$ are same functions of $P_n^{(k)}$
- **dissipative terms correspond to non-crossing symmetric quantum amps**.

Proposal for $\mathcal{O}(a^7)$ Teukolsky

$$\mathcal{M}_{\text{Teuk.}} = \mathcal{M}_0 \times \left(e^{x-w} + P_\xi(x, y, w) \right)$$

$\mathcal{M}_{\text{Kerr}}$ at $\mathcal{O}(a^7)$

$$\begin{aligned} P_\xi(x, y, w)|_{\mathcal{O}(a^7)} \propto -\frac{1}{5040} \Big[&(w-z)^2(w+z)^2(w(w^2-5wx+10x^2+y^2)+(w+5x)z^2) \\ &+\alpha z(w^2-z^2)^2(10x^2+y^2+2z^2) \\ &+\frac{\eta}{4} z^2(3w^3(5x(w+2x)+y^2)+w(6w^2+60wx+10x^2+y^2)z^2+(2w+5x)z^4) \\ &+\frac{\eta\alpha}{4} wz(w^3(10x^2+y^2)+w(2w^2+50wx+30x^2+3y^2)z^2+6(w+5x)z^4) \Big] \end{aligned}$$

$x = a \cdot q_\perp$, $y = a \cdot q$, $z = |a| \frac{p \cdot q_\perp}{m}$, $w = \frac{a \cdot \chi}{p \cdot \chi} p \cdot q_\perp$, $\xi^{-1} = \frac{q^2 m^2}{(p \cdot q_\perp)^2}$

Review & Outlook

In this talk

- HS theory + ... $\Longrightarrow$ **uniqueness** of 3pt, not yet 4pt
- identify **general spin structures of quantum amps.** for Kerr that is compatible with HS symm.
- proposal all-spin $\sqrt{\text{Kerr}}$ & Kerr Compton amps: **Quantum** + **Classical**
- Matching results by [Siemonson Vines '19] & [Bautista, Guevara, Kavanagh, Vines '22]

Outlook

- 4pt non-uniqueness from HS symm - need new constraints
- compare to higher orders in Teukolsky calc. $\mathcal{O}(a^{\geq 7})$
- compare to 2PM results

General spin $\sqrt{\text{Kerr}}$ amplitudes

$\sqrt{\text{Kerr}}$ Compton

$$A(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+)=\frac{\langle 3|\mathbf{1}|4]^2(U+V)^{2s}}{m^{4s}s_{12}t_{14}}-\frac{\langle\mathbf{1}3\rangle\langle 3|\mathbf{1}|4][4\mathbf{2}]}{m^{4s}s_{12}}P_2^{(2s)}+\frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s}s_{12}}t_{13}P_2^{(2s-1)}$$
$$-\frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s-2}s_{12}}\left(t_{13}\langle\mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-1)}+2\,VP_4^{(2s)}\right)+C^{(s)}$$

Proposed Contact terms

Proposal for Kerr contact terms (conservative) [2308.XXXX]

$$C_1^{(s)} = \left(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\right)^2 \left(P_5^{(2s)}[\varsigma_1] + P_5^{(2s)}[\varsigma_2] - 2P_4^{(2s-1)} - (\varsigma_1 + \varsigma_2)P_4^{(2s-2)}\right)$$

$$C_2^{(s)} = \alpha\langle 3|\rho|4]\langle 3|\bar{\rho}|4]\left(\langle 3|\rho|4]^2 + 3\langle 3|\bar{\rho}|4]^2\right)\left(P_5^{(2s)}[\varsigma_1] - P_5^{(2s)}[\varsigma_2] - (\varsigma_1 - \varsigma_2)P_4^{(2s-2)}\right),$$

$$C_3^{(s)} = \alpha\left(\langle 3|\rho|4]^4 + 12\langle 3|\rho|4]^2\langle 3|\bar{\rho}|4]^2 + 3\langle 3|\bar{\rho}|4]^4\right)(\varsigma_1 - \varsigma_2)\left(P_6^{(2s)}[\varsigma_1,\varsigma_1] - P_6^{(2s)}[\varsigma_2,\varsigma_2]\right).$$

Guiding constraints

$$P_5^{(2s)}[\varsigma_i] := \lim_{\varsigma_5\to\varsigma_i} P_5^{(2s)}, \quad P_6^{(2s)}[\varsigma_i,\varsigma_j] := \lim_{\varsigma_5\to\varsigma_i,\varsigma_6\to\varsigma_j} P_6^{(2s)}$$

1. ansatz $C^{(s)}$ as $P_n^{(k)}$
2. unmodified for $s = 5/2$
3. matching to self-force calc. of $\mathcal{O}(a^4)$ [Siemonsen, Vines '19]
4. matching to Teukolsky results [Bautista, Guevara, Kavanagh, Vines '22]