One-loop scattering waveforms from a Heavy-mass Effective Field Theory

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with

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Menu

- Classical limit for observables in General Relativity
- Heavy-mass Effective Field Theory (HEFT) for classical General Relativity (Brandhuber, Chen, GT, Wen)
  - Diagrammatic extraction of the classical limit
- One-loop gravitational bremsstrahlung (Brandhuber, Brown, Chen, De Angelis, Gowdy, GT)
  - Unbound binaries/eccentric orbits
  - KMOC & HEFT
  - Newman-Penrose scalar Ψ_4
  - Scattering waveforms in the frequency/time domain

- Based (mostly) on 2303.06111
  with Andi, Graham, Gang, Stefano & Joshua

- Related works:
  - Herdershee, Roiban & Teng 2303.06112
  - Elkidir, O'Connell, Sergola & Vazquez-Holm 2303.06211
  - Georgoudis, Heissenberg & Vazquez-Holm 2303.07006

Extracting the classical limit

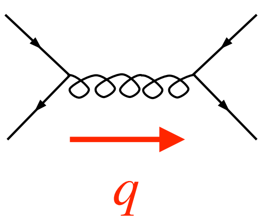
- A key question is how to extract efficiently the classical contribution from a loop computation
    - loop diagrams contain classical contributions
    - $\hbar$ expansion is not the same as the loop expansion (Holstein & Donoghue)
- What does "efficient" mean?
    - Expand as little as possible and as soon as possible…
    - …in particular before performing loop integrations
    - Ideally separate out different orders in $\hbar$ at the diagrammatic level

- Classical physics from large-charge limit
  - Large masses and large angular momenta (in $\hbar$ units) limit
  - Equivalent to scaling the momenta of external / internal massless particles and momentum transfers by $\hbar$ (Kosower, Maybee, O'Connell)

$$(k, \ell, q) \longrightarrow \hbar\,(k, \ell, q)$$

  - In addition also scale $\kappa \longrightarrow \kappa/\sqrt{\hbar}$

- Example: two-to-two scattering of heavy scalars



  - Amplitude (discarding local terms) $A_4 \sim \kappa^2/q^2 \longrightarrow \hbar^{-3}\kappa^2/q^2$
  - Newton's potential: $V \sim \frac{1}{E_1 E_2} \int d^3q\ A_4 \longrightarrow \mathcal{O}(\hbar^{3-3}) = \mathcal{O}(\hbar^0)$

Classical limit & the HEFT

- Small $\hbar$-expansion equivalent to large-mass expansion
  - Recall that the Klein-Gordon equation is $\left[\Box + \left(\frac{mc}{\hbar}\right)^2\right]\phi = 0$
- Heavy-mass Effective Field Theory (Brandhuber, Chen, GT, Wen)
  - Black holes exchange momenta that are much smaller than their masses
  - similar to Heavy-Quark Effective Theory (Georgi)
  - Earlier work by Aoude, Damgaard, Haddad, Helset
- How can this become (more) powerful?
  - Use the amplitude arsenal for the calculation of HEFT amplitudes!
  - Double copy, recursion relations, unitarity, plus diagrammatic separation of different orders in the mass expansion…

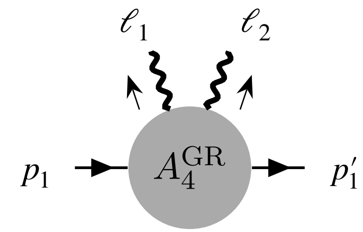
Our strategy

- Generate gravitational HEFT amplitudes efficiently
  - BCJ numerators of HEFT amplitudes in YM with two scalars + gluons from a kinematic quasi-shuffle Hopf algebra (Brandhuber, Chen, GT, Johansson, Wen)
  - # terms in a BCJ numerator is Fubini$_{n-1}$ for n gluons


  - Gravitational HEFT amplitudes via the double copy
- D-dim'l HEFT recursions with 4 scalars + gravitons (Brandhuber, Brown, Chen, De Angelis, Gowdy, GT)
  - Shifts chosen to leave linearised massive propagators unmodified, inputs: D-dim HEFT amplitudes with 2 scalars & factorisation on massless propagators
- $\hbar$ counting in HEFT manifest at the diagrammatic level
  - Pick only classically-relevant terms in the HEFT expansion of amplitudes, hyper-classical & quantum contributions dropped from the get go
- Unitarity to compute non-analytic terms in amplitudes

The idea of the HEFT

- Hard and soft particles



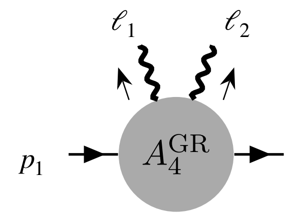
$q = \ell_1 + \ell_2$

  - Incoming heavy particle: $p^\mu = m v^\mu$
  - After the interaction with a graviton: $p_1' = p_1 + q$
  - In QCD, the momentum transfer q is of order $\Lambda_{\mathrm{QCD}} \ll m$
  - For classical gravitational physics: $\ell_{1,2} = \hbar \hat{k}_{1,2}$, with $\hat{k}_{1,2}$ fixed as $\hbar \to 0$
  - $p_1' = p_1 + \ell_1 + \ell_2 = mv + \hbar(\hat{k}_1 + \hat{k}_2)$

- To see how HEFT amplitudes arise, perform a heavy-mass expansion on the amplitudes
  - For explanation only, in practice we derive them using recursive techniques and the double copy without reference to exact amplitudes

Example: Compton amplitude

- Start from exact amplitude



$$= i\,4\kappa^2 \frac{(p_1\cdot F_1\cdot F_2\cdot p_1)^2}{D_{12}D_{21}D}$$

  - $D_{12} = -2(p_1\cdot\ell_1) + i\varepsilon,\quad D_{21} = -2(p_1\cdot\ell_2) + i\varepsilon,\quad D = q^2 + i\varepsilon,\ F$ = field strength

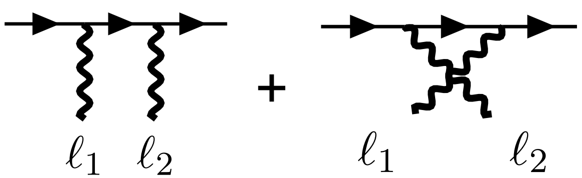
- Expand massive propagators

  - Introduce barred variables: $p_1 = \bar{p} + \frac{q}{2}\,,\quad p_1' = \bar{p} - \frac{q}{2}$ with $\bar{p}\cdot q = 0$

Expand for small q:

$$\frac{1}{D_{12}} = \frac{1}{-2(\bar{p}\cdot\ell_1) + i\varepsilon} + \frac{q^2}{2(-2\bar{p}\cdot\ell_1 + i\varepsilon)^2} + \cdots,$$

$$\frac{1}{D_{21}} = \frac{1}{2(\bar{p}\cdot\ell_1) + i\varepsilon} + \frac{q^2}{2(2\bar{p}\cdot\ell_1 + i\varepsilon)^2} + \cdots$$



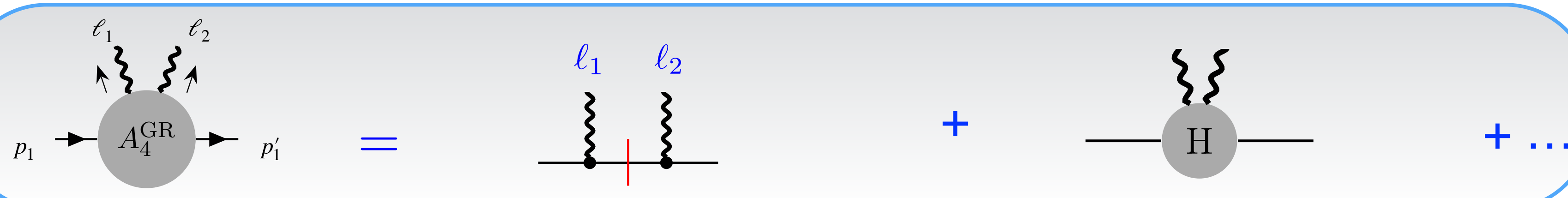
- Result:

$$\frac{1}{D_{12}D_{21}D} = -\frac{1}{(q^2)^2}\left(\frac{1}{D_{12}} + \frac{1}{D_{21}}\right) = \frac{1}{(q^2)^2}\left[i\pi\,\delta(\bar{p}\cdot\ell_1) - \frac{q^2}{(2\bar{p}\cdot\ell_1)^2}\right] + \cdots$$

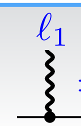
  - Unbarred variables spoil this by adding δ' terms from $p\cdot q = q^2/2 = \mathcal{O}(\hbar^2) \neq 0$
  - Set $\bar{p} = \bar{m}v,\quad \bar{m} = \left(m^2 - \frac{q^2}{4}\right)^{\frac{1}{2}}$ and count mass powers (m and $\bar{m}$ coincide at large m)

HEFT amplitude

$$A_4^{\mathrm{GR}} \to -i\kappa^2\Big[-(i\pi)\,\bar{m}^3\delta(v\cdot\ell_1)\,(v\cdot\varepsilon_1)^2(v\cdot\varepsilon_2)^2 + \frac{\bar{m}^2}{\ell_{12}^2}\Big(\frac{v\cdot F_1\cdot F_2\cdot v}{v\cdot\ell_2}\Big)^2 + \cdots\Big]$$



- three-point:

$A_3^{\mathrm{GR}} =$  $= -i\kappa\,\bar{m}^2(\varepsilon_1\cdot v)^2$

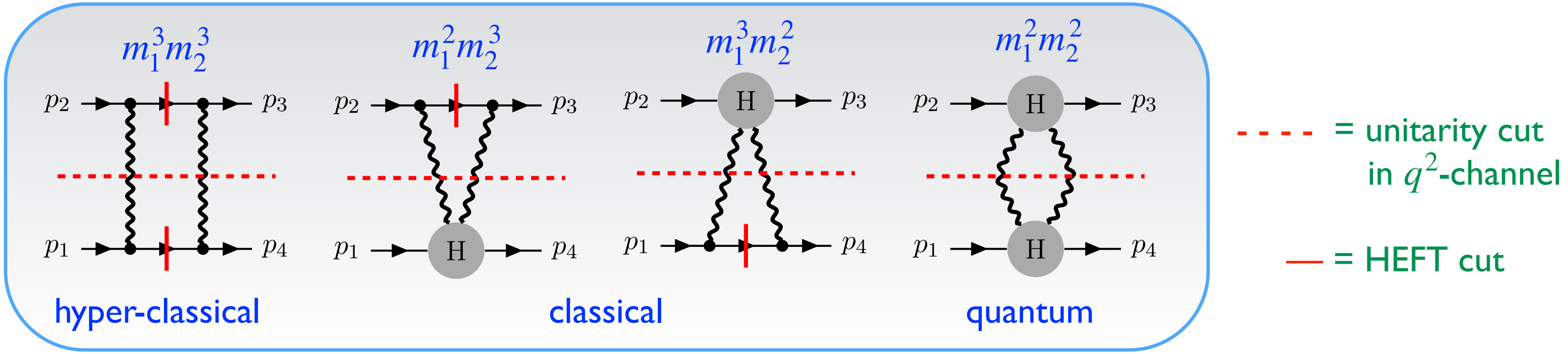
- — = cut massive propagator from $i\varepsilon$, important also at tree level !
  dots = higher-order terms in the masses

- HEFT neatly separates different orders in masses and $\hbar$!
  - First term: hyper-classical, scales as $\hbar^{-1-1} = \hbar^{-2}$ and $\mathcal{O}(\bar{m}^3)$
  - Second term: HEFT amplitude, classical, scales as $\hbar^{-1-2+2} = \hbar^{-1}$ and $\mathcal{O}(\bar{m}^2)$
  - Generalises to higher points, HEFT amplitude is the term without delta functions

Example: one-loop scattering angle

(Brandhuber, Chen, GT, Wen)

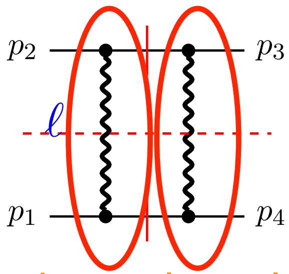
- Two-to-two scattering of spinless heavy objects:



  - Four diagrams, different dependence on the masses and hence $\hbar$
  - What happens to the hyper-classical term?

- Go to impact parameter space:
  - relevant to compute the scattering angle

- In impact parameter space, convolution $\Longrightarrow$ product:

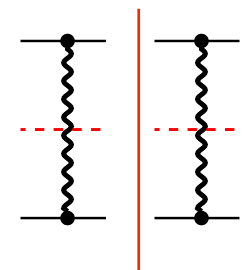


b=impact parameter

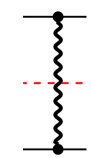
$$\sim \int \frac{d^D\ell}{(2\pi)^{D-2}}\, \delta(2\bar{p}_1\cdot\ell)\delta(2\bar{p}_2\cdot\ell)\; e^{iq\cdot b}\, \mathcal{M}_L(\ell)\mathcal{M}_R(q-\ell) \xrightarrow{\text{IPS}} \widetilde{\mathcal{M}}_L(b)\widetilde{\mathcal{M}}_R(b)$$

hyper-classical

  - Hyper-classical diagrams are two-massive-particle reducible



- Reconstruct exponentiation of tree-level phase $\delta^{(0)}_{\text{HEFT}}\cdots$



  - $$e^{i(\delta^{(0)}_{\text{HEFT}}+\delta^{(1)}_{\text{HEFT}}+\cdots)} = 1 + i\,\delta^{(0)}_{\text{HEFT}} + \left(\delta^{(1)}_{\text{HEFT}} - \frac{1}{2}\left(\delta^{(0)}_{\text{HEFT}}\right)^2\right) + \cdots$$
  - manifest in HEFT, no need to check it. Simply drop hyper-classical diagrams!

- HEFT allows to compute the exponent diagrammatically
  - Compute only two-massive-particle irreducible diagrams
  - One- and two-loop scattering angle checked (Brandhuber, Chen, GT, Wen '21)

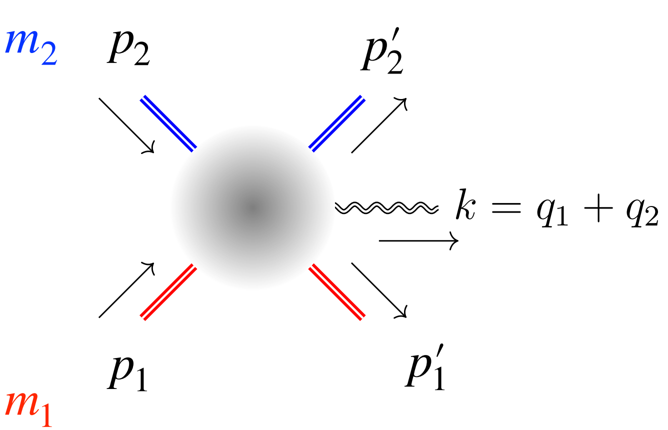
Gravitational Bremsstrahlung

(Brandhuber, Brown, Chen, De Angelis, Gowdy, GT)

- Scattering of two spinless black holes
  - Newman-Penrose scalar ψ_4
  - Spectral and far-field time-domain gravitational waveforms
  - Main computation is the five-point HEFT amplitude at one loop

- Combine the HEFT expansion with the KMOC approach

- Kinematics:
  - $y := v_1 \cdot v_2$ relative velocity
  - $q_{1,2}^2 \leq 0$ momentum transfers
  - $w_{1,2} := v_{1,2} \cdot k \geq 0$



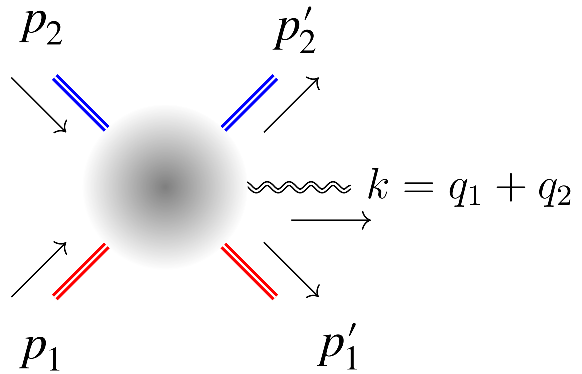
From amplitudes to classical observables

- KMOC approach allows to relate scattering data to observables in classical gravity (Kosower, Maybee, O'Connell)
  - For some operator O, compute $\langle O^{\text{out}} \rangle_\psi := {}_{\text{out}}\langle \psi | O | \psi \rangle_{\text{out}}$
  - We will be interested in $R_{\mu\nu\rho\sigma}(x)$ or $h_{\mu\nu}(x)$
  - $|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}}$ where $S = \mathbb{I} + i\,T$
  - $\langle O^{\text{out}} \rangle_\psi := {}_{\text{in}}\langle \psi | (\mathbb{I} - i\,T^\dagger) O (\mathbb{I} + i\,T) | \psi \rangle_{\text{in}}$
  - BH Initial state: $$|\psi\rangle_{\text{in}} := \int d\Phi(p_1) d\Phi(p_2) e^{i(p_1\cdot b_1 + p_2 \cdot b_2)} \phi(p_1)\phi(p_2) |p_1 p_2\rangle_{\text{in}}$$ $$d\Phi(p_i) := \frac{d^D p_i}{(2\pi)^{D-1}} \delta^{(+)}(p_i^2 - m_i^2)\,, \qquad |p\rangle := a^\dagger(\vec{p})|0\rangle$$
  - $b_1, b_2 =$ impact parameters
  - Wavepackets sharply localised: $\lambda_{1,2}^{\text{Compton}} \ll \ell_{\phi_{1,2}} \ll \sqrt{-b^2}$

- For the linearised Riemann curvature

$$\langle R^{\rm out}_{\mu\nu\rho\lambda}(x)\rangle_\psi = \kappa\,{\rm Re}\left\{\int\prod_{j=1}^{2} d\Phi(p_j)\,|\phi(p_1)|^2|\phi(p_2)|^2\sum_h\int d\Phi(k)e^{-ik\cdot x}\,k_{[\mu}\varepsilon^{(h)*}_{\nu]}(\vec{k})k_{[\rho}\varepsilon^{(h)*}_{\lambda]}(\vec{k})\right.$$
$$\left.\int\prod_{j=1}^{2}\frac{d^Dq_j}{(2\pi)^{D-1}}\delta(2\bar{p}_1\cdot q_1)\delta(2\bar{p}_2\cdot q_2)e^{i(q_1\cdot b_1+q_2\cdot b_2)}\Big[\langle p_1'p_2'k^h|iT|p_1p_2\rangle+\langle p_1'p_2'|T^\dagger a_h(\vec{k})T|p_1p_2\rangle\Big]\right\}$$

- From now on omit the spectator red part
- A certain subtracted five-point amplitude has appeared…
- …transformed to impact parameter space $q_{1,2} =$ momentum transfers, $q_1+q_2=k$



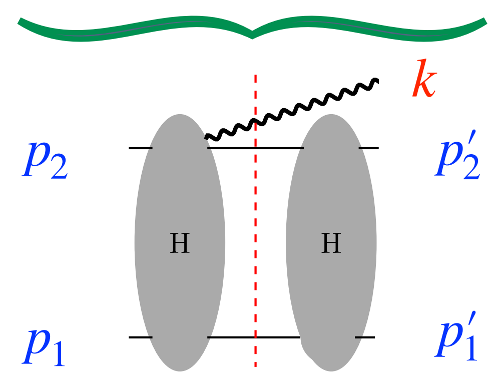
From KMOC to HEFT

- Subtracted amplitude:

$$\langle p_1' p_2' k^h | iT | p_1 p_2 \rangle + \langle p_1' p_2' | T^\dagger a_h(\vec{k}) T | p_1 p_2 \rangle = \langle p_1' p_2' k^h | iT | p_1 p_2 \rangle + \sum_n \langle p_1' p_2' | T^\dagger | n \rangle \langle n, k^h | T | p_1 p_2 \rangle$$

- 1st term is the complete (one-loop) amplitude
  - contains everything from hyper-classical to quantum

- 2nd term on RHS at one loop: $\sum_{r_1, r_2} \langle p_1' p_2' | T^\dagger | r_1, r_2 \rangle^{(0)} \langle r_1, r_2, k^h | T | p_1 p_2 \rangle^{(0)}$
  - r_1, r_2 scalars
  - 2-massive particle reducible
  - $\mathcal{O}(\bar{m}_1^3 \bar{m}_2^3)$ and thus hyper-classical
  - Factorises in IPS



- Subtracted amplitude is the one-loop HEFT amplitude!

$$\langle p_1' p_2' k^h | i\,T | p_1 p_2 \rangle^{(1)} + \sum_{r_1,r_2} \langle p_1' p_2' | T^\dagger | r_1, r_2 \rangle^{(0)} \langle r_1, r_2, k^h | T | p_1 p_2 \rangle^{(0)} = (2\pi)^D \delta^{(D)}(q_1 + q_2 - k)\, \mathcal{M}^{(1)}_{5,\mathrm{HEFT}}$$

- Result: $$\langle R^{\mathrm{out}}_{\mu\nu\rho\lambda}(x) \rangle_\psi = \kappa\, \mathrm{Re} \left[i \sum_h \int d\Phi(k) e^{-ik\cdot x}\, k_{[\mu} \varepsilon^{(h)*}_{\nu]}(\vec{k}) k_{[\rho} \varepsilon^{(h)*}_{\lambda]}(\vec{k})\, \widetilde{W} \right]$$

- $\widetilde{W} =$ spectral waveform

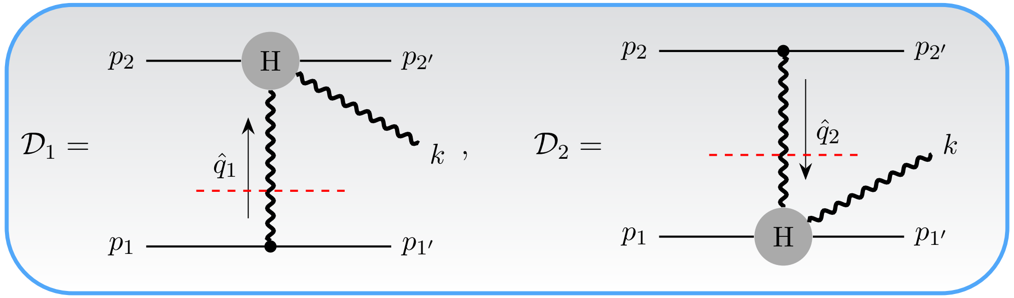
$$\widetilde{W}(\vec{b}, k^h) := -i \int d\mu^{(D)}\; e^{i(q_1 \cdot b_1 + q_2 \cdot b_2)}\; \mathcal{M}_{5,\mathrm{HEFT}}(q_1, q_2; h)$$

$q_1 + q_2 = k$

  - Waveform from five-point HEFT amplitude in Impact Parameter Space (IPS)
  - IPS measure $$d\mu^{(D)} := \frac{d^D q_1}{(2\pi)^{D-1}} \frac{d^D q_2}{(2\pi)^{D-1}} (2\pi)^D \delta^{(D)}(q_1 + q_2 - k) \delta(2p_1 \cdot q_1) \delta(2p_2 \cdot q_2)$$
  - Under a shift $b_{1,2} \to b_{1,2} + a$, we have $\widetilde{W} \to e^{ik\cdot a}\widetilde{W}$ hence we can set $b_2 \to 0$

Five-point HEFT amplitude

- **Tree-level from recursion:** (Brandhuber, Brown, Chen, De Angelis, Gowdy, GT)



$$\hat{q}_1 = q_1 + zr$$
$$\hat{q}_2 = q_2 - zr$$

  - Conditions on shifts: 1. $r^2 = 0$
    2. $r\cdot k = r\cdot\varepsilon_k = r\cdot\bar{v}_{1,2} = 0$
  - 1. for linear poles in z, 2. for shifts to appear only in polarisation vectors
  - Need HEFT amplitudes with 2 scalars and many gravitons in arbitrary dimension (known from double copy and kinematic algebra construction)
  - Result:

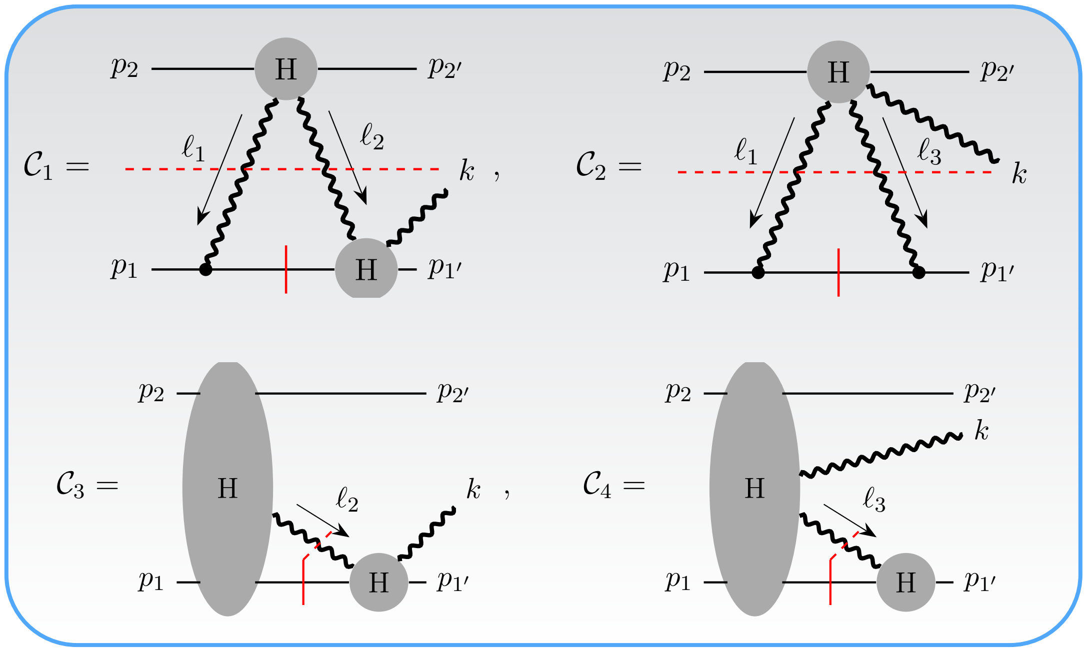
$$\mathcal{M}(k,\bar{p}_1,\bar{p}_2) = i\frac{N_1}{q_1^2} + i\frac{N_2}{q_2^2}$$

$$N_{1,2} = \pm\frac{\kappa^3\bar{m}_1^2\bar{m}_2^2}{(2k\cdot q)(\bar{v}_{2,1}\cdot k)^2}\left\{\left[\bar{y}(\bar{v}_{2,1}\cdot F_k\cdot q) + (\bar{v}_{2,1}\cdot k)(\bar{v}_1\cdot F_k\cdot \bar{v}_2)\right]^2 - \frac{1}{D-2}(\bar{v}_{2,1}\cdot F_k\cdot q)^2\right\}$$

  - agrees with Luna, Nicholson, O'Connell & White

One-loop five-point HEFT amplitude

- One-loop cut diagrams, to be merged:



- - - - = unitarity cuts

— = HEFT cut

the snail diagrams

- Plus diagrams where graviton is emitted from an incoming leg, plus 1-2 flip
- Diagrams shown are of $\mathcal{O}(m_1^3 m_2^2)$ (cfr. hyper-classical, $\mathcal{O}(m_1^3 m_2^3)$)
- Linearised massive propagators with PV prescription (from HEFT expansion)
- Infrared-divergent results, compare then with Weinberg's 1965 formula

- **Weinberg's formula (schematically):**

$$S_{\beta\alpha} \to (\mathrm{Exp}) \cdot S^{(0)}_{\beta\alpha}$$

  - Exponent contains a real and imaginary part
  - Phase usually discarded for cross sections, but relevant for us!

- **HEFT-expand Weinberg's formula:**

$$M_5^{(1)}\Big|_{\mathrm{IR}} \sim e^{-i\frac{G}{\epsilon}\left(\bar{m}_1\bar{m}_2\frac{2\bar{y}^2-1}{\sqrt{\bar{y}^2-1}}+\bar{m}_1\bar{w}_1+\bar{m}_2\bar{w}_2\right)} M_5^{(0)}$$

Hyperclassical   Classical

  - Real part of exponent vanishes at this order
  - Our result perfectly agrees with HEFT-expanded Weinberg's formula

- Infrared divergences are a useful sanity check…
  …but what do we do with them?
  - KMOC contains amplitudes, not $|A|^2$
- IR divergences removed by re-defining observer's time:
  - $t \to t - \frac{G(p_1 + p_2)\cdot n}{\epsilon}$ with $k^\mu = \omega\, n^\mu$
  - Recall $\langle R^{\rm out}_{\mu\nu\rho\lambda}(x)\rangle_\psi = \kappa\,\mathrm{Re}\left[i\sum_h \int d\Phi(k) e^{-ik\cdot x}\, k_{[\mu}\varepsilon^{(h)*}_{\nu]}(\vec{k}) k_{[\rho}\varepsilon^{(h)*}_{\lambda]}(\vec{k})\,\widetilde{W}\right]$
  - Shift is irrelevant since we measure time intervals (Porto, Ross & Rothstein 2012)
- Henceforth use $W^{(1)}(b, k^h) := -i\int d\mu^{(4)}\ e^{iq_1\cdot b}\ \mathcal{M}^{(1)}_{5,\mathrm{HEFT,fin}}(q_1, q_2; h)$
  - Set D=4, and take finite part of the amplitude

Far-field limit of waveforms

- Asymptotic waveforms at large observer distance

  - At large distances $r = |\vec{x}| \to \infty$ and $|x_0| \to \infty$, with fixed retarded time $u := t - r$, integration over angular directions of emitted graviton localises

$$\langle R^{\text{out}}_{\mu\nu\rho\lambda}(x)\rangle_\psi \overset{r\to\infty}{=} \frac{\kappa}{8\pi r}\sum_h \int_0^{+\infty}\frac{d\omega}{2\pi}\left[k_{[\mu}\varepsilon^{(h)*}_{\nu]}(\vec{k})k_{[\rho}\varepsilon^{(h)*}_{\lambda]}(\vec{k})\,W(b,k^h)e^{-i\omega u} + k_{[\mu}\varepsilon^{(h)}_{\nu]}(\vec{k})k_{[\rho}\varepsilon^{(h)}_{\lambda]}(\vec{k})\;W^*(b,k^h)e^{i\omega u}\right]_{k=\omega(1,\hat{\mathbf{x}})}$$

  - angular direction of the graviton localises along observer's direction $\hat{\mathbf{x}}$

- Newman-Penrose null tetrad formalism (Newman & Penrose 1969)

  - project on a basis of null complex vectors $L_\mu = n_\mu,\ N_\mu = \zeta_\mu,\ M_\mu = \varepsilon^{(+)}_\mu,\ M^*_\mu = \varepsilon^{(-)}_\mu$
  - $k = \omega n,\ \zeta =$ reference vector, with $\zeta\cdot\varepsilon^{(\pm)} = 0,\ \zeta\cdot n = 1$
  - $(L_\mu, N_\mu, M_\mu, M^*_\mu)$ are frequency-independent

- **Newman-Penrose scalar** $\Psi_4(x) = N^\mu M^{\nu\,*} N^\rho M^{\sigma\,*} \langle W^{\rm out}_{\mu\nu\rho\sigma}(x)\rangle$
  - Of the NP scalars, $\Psi_4(x)$ has the slowest decay at large $|\vec{x}\,|$,

$$\Psi_4(x) \stackrel{|\vec{x}\,|\to\infty}{\longrightarrow} \frac{\Psi_4^0(x)}{|\vec{x}\,|}$$

- **NP scalar in terms of waveforms in the far-field domain:**

$$\Psi_4^0(x) = \frac{\kappa}{8\pi}\int_0^{+\infty}\frac{d\omega}{2\pi}\,\omega^2\Big[W(b;k^-)e^{-i\omega u} + [W(b;k^+)]^* e^{i\omega u}\Big]_{k=\omega(1,\hat{\mathbf{x}})}$$

  - Far from the sources, it is related to the second (retarded-)time derivative of the two polarisation of the gravitational wave

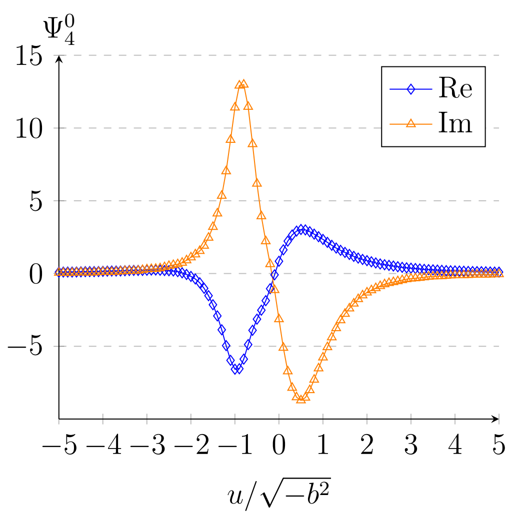
$$\Psi_4(x) = -\frac{1}{2}\varepsilon^{(--)}_{\mu\nu}\ddot{h}^{\mu\nu} = -\frac{1}{8\pi|\vec{x}\,|}(\ddot{h}^\infty_+ - i\ddot{h}^\infty_\times)$$

  - $\varepsilon_{++,--} \sim \varepsilon_+ \pm i\varepsilon_\times$ (plus and cross polarisations)

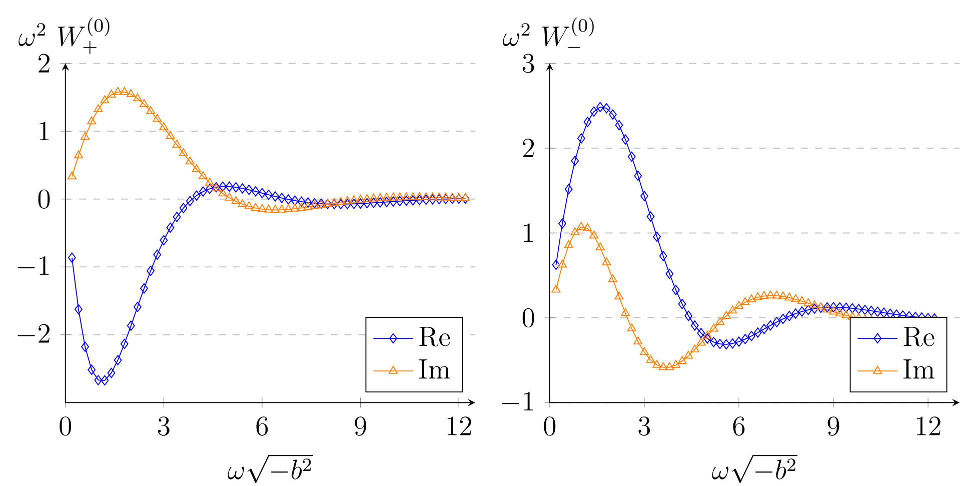
Tree-level waveforms

- Earlier computations:
  - Time domain, asymptotic metric computed by Kovacs and Thorne (1978)
  - Analytic computation with worldline formalism by Jakobsen, Mogull, Plefka and Steinhoff '21(see also Mougiakakos, Riva & Vernizzi for a semi-analytic result)

- Time-domain:



Frequency domain:



One-loop waveforms

- One-loop frequency-domain (or spectral) waveform:

$$\widehat{W}^{(1)}(b,k^h) := -i\int d\mu^{(4)}\; e^{iq_1\cdot b}\left[\mathcal{M}^{(1)}_{5,\mathrm{HEFT,fin}} - iG\left(m_1 w_1 \log\frac{w_1^2}{\mu_{\mathrm{IR}}^2} + m_2 w_2 \log\frac{w_2^2}{\mu_{\mathrm{IR}}^2}\right)\mathcal{M}^{(0)}_{\mathrm{HEFT}}\right]$$

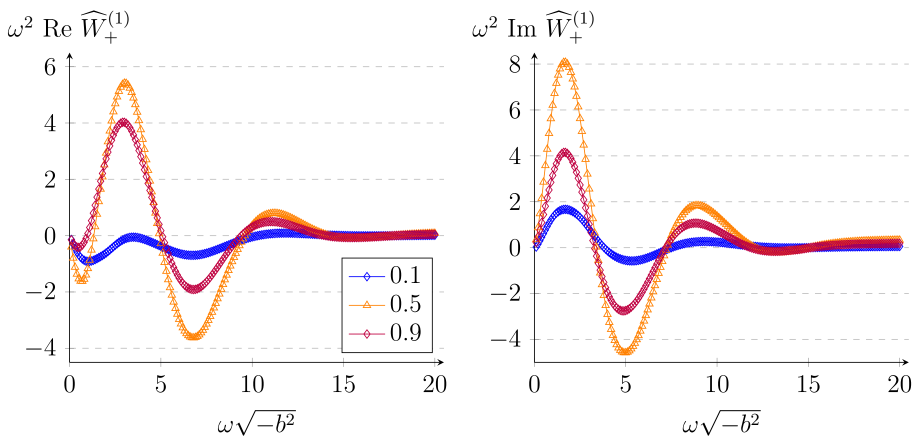
  - $w_{1,2} = k\cdot v_{1,2}$ proportional to ω
  - Exponentiation of the "tails" (Blanchet & Schaefer '93; Porto, Ross, Rothstein 2012)

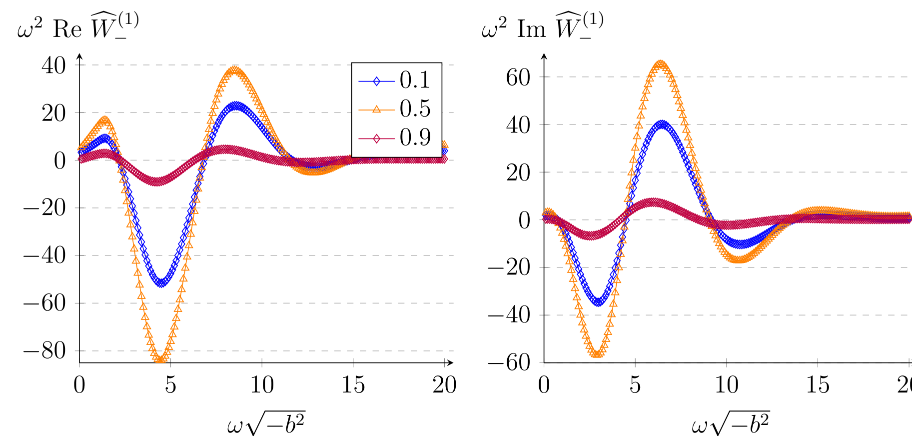
$$e^{i\theta_{\mathrm{tail}}(\mu,\omega)}\mathcal{M}^{(0)}_{\mathrm{HEFT}}$$

with $$\theta_{\mathrm{tail}}(\mu_{\mathrm{IR}},\omega) = G\left(m_1 w_1 \log\frac{w_1^2}{\mu_{\mathrm{IR}}^2} + m_2 w_2 \log\frac{w_2^2}{\mu_{\mathrm{IR}}^2}\right)$$

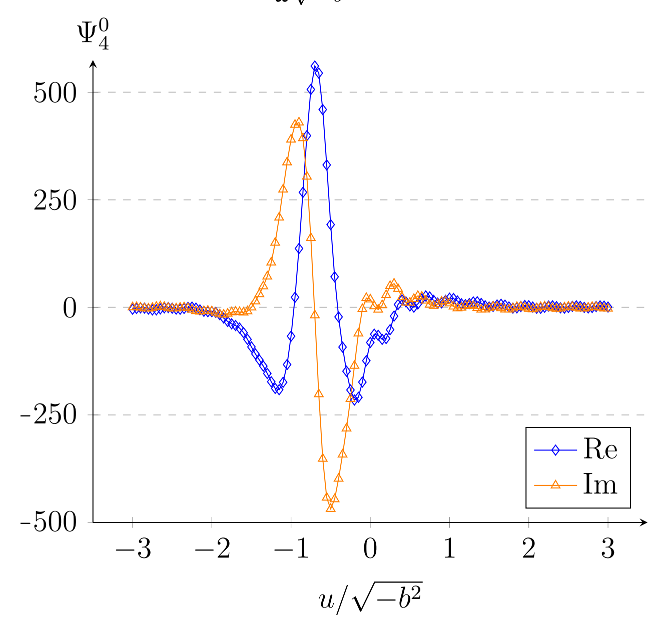
  - Natural from the QFT viewpoint
  - IR pole always accompanied by a log $\quad \dfrac{1}{\epsilon} - \log\left(-\dfrac{\omega^2}{\mu_{\mathrm{IR}}^2}\right)$

- **Frequency domain:**
  - Introduce mass ratio $\chi := \dfrac{m_2}{m_1 + m_2}$, and plot for various values of χ





- **Time domain (for $m_1 = m_2$):**



Conclusions & open problems

- Heavy-mass Effective Field Theory allows for an efficient extraction of the classical limit
    - No "artistic" element in combining diagrams, no cancellations to be found
    - Hyper-classical / quantum terms identified and dropped before integrations
    - HEFT amplitude even have their own double copy and can be computed by recursions or the algebra
    - Kinematic algebra of the HEFT and Yang-Mills theory
- One-loop scattering waveforms
    - KMOC approach leads naturally to the HEFT

- (Some) questions
  - Analytic waveforms (important to clarify shape of time-domain waveform)
  - Analytic continuation for bound orbits?
  - Apply HEFT to higher PM orders calculations
  - Spin, tidal effects
  - Similarities with the worldline formalism(s)
  - ……

Thank you!