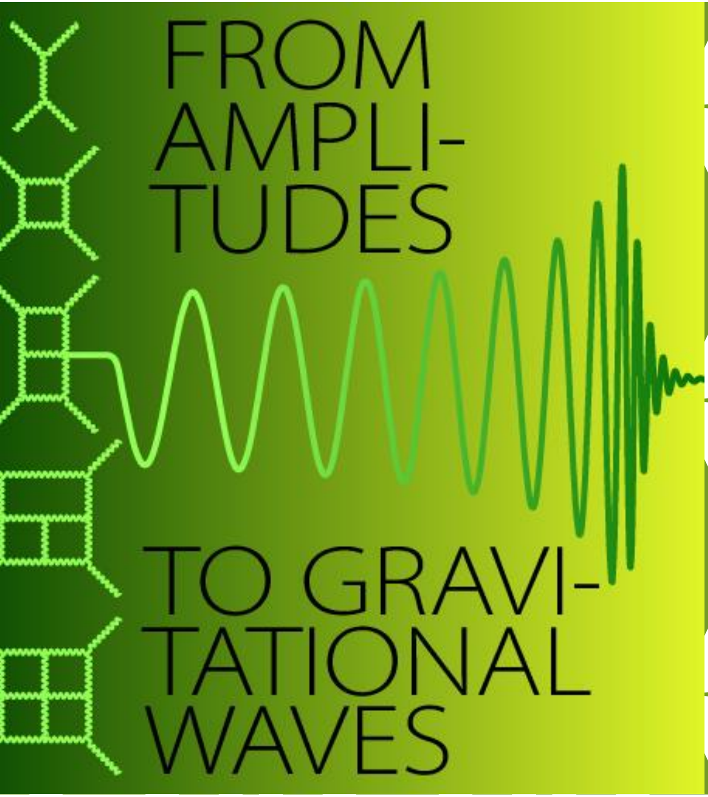
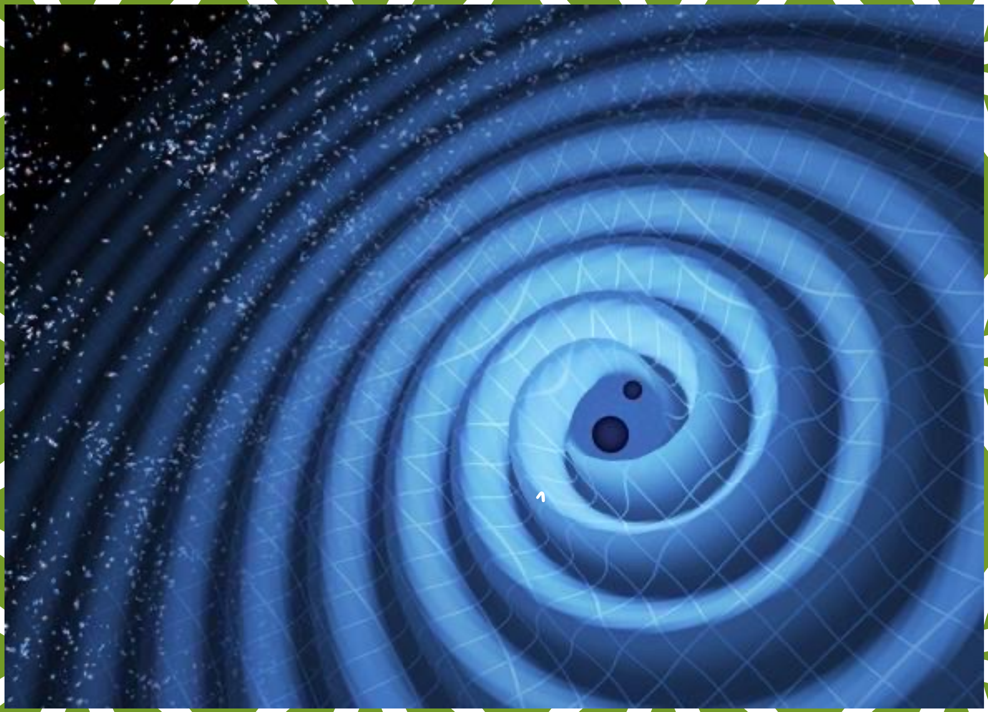
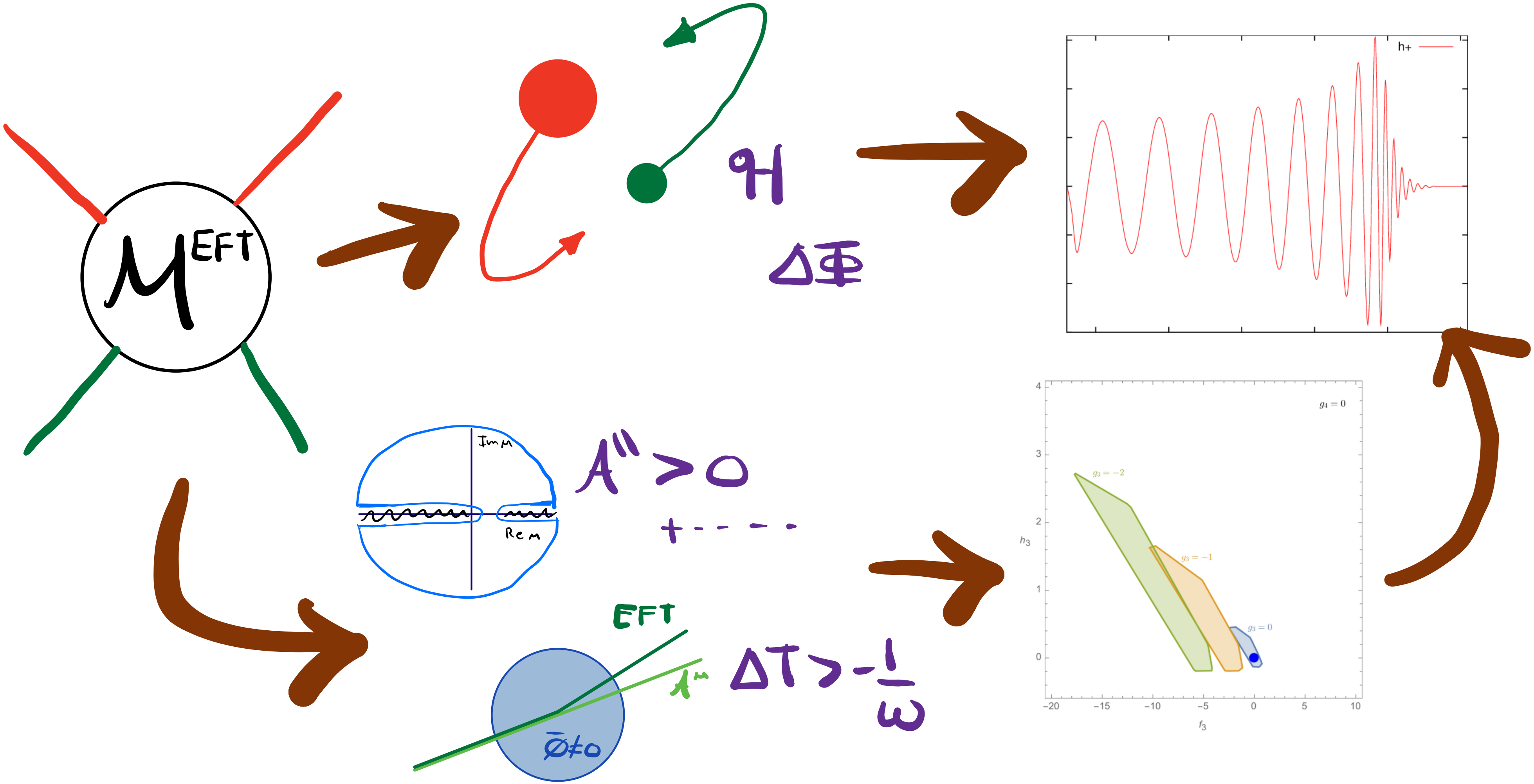




TESTING EFTS: AMPLITUDES, GRAVITATIONAL WAVES, AND CAUSALITY

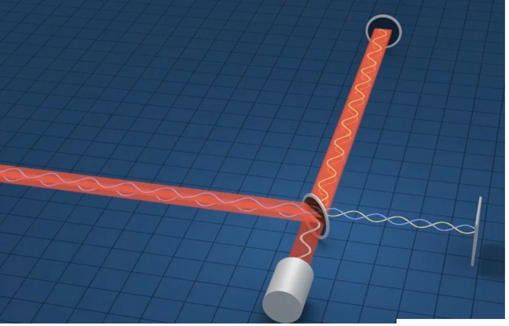
Mariana Carrillo González

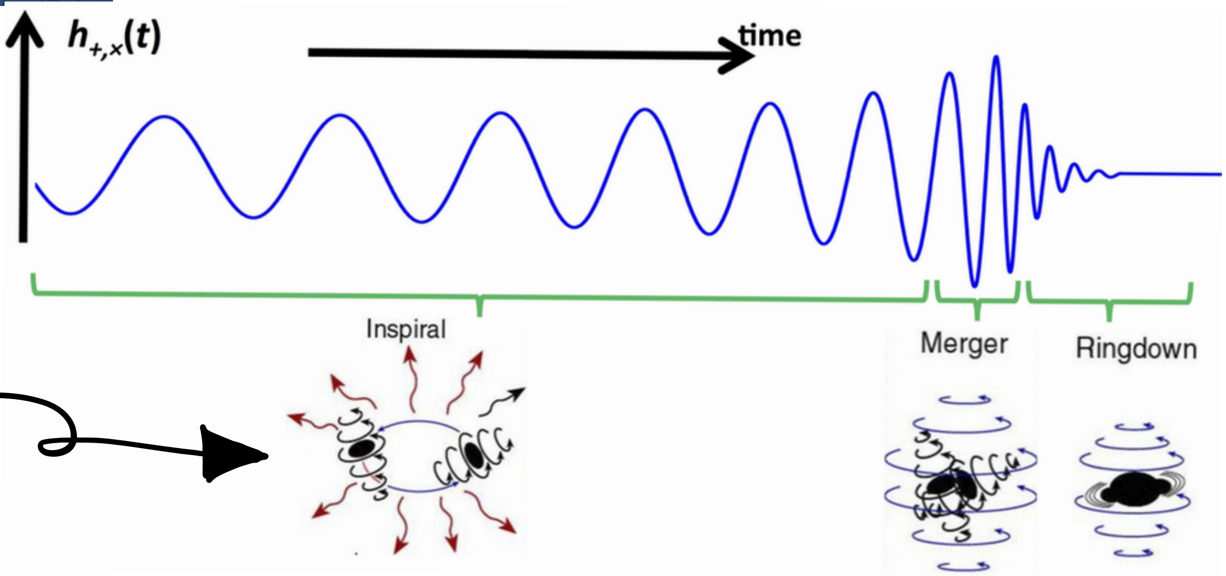
Imperial College London



M^EFT
ΔΦ
h+
Im μ
Re μ
A'' > 0
+ . . . .
EFT
A''
ΔT > -1/ω
Φ̄≠0
g4 = 0
g3 = −2
g3 = −1
g3 = 0
h3
f3

MOTIVATION





Perturbative gravity

Can test GR extensions

PN

$$\frac{Gm}{r} \sim v^2 \ll 1$$

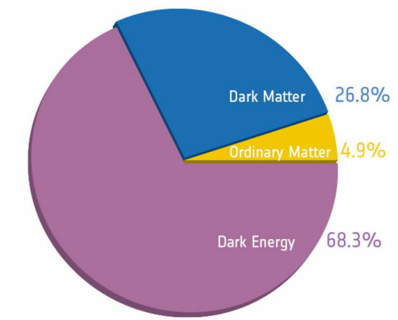
EFT methods

PM

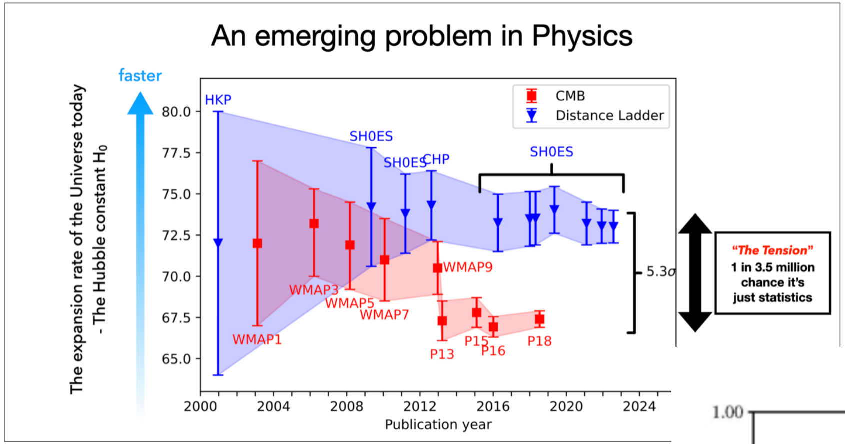
$$\frac{Gm}{r} \ll v^2 \sim 1$$ + perturbative QFT regime

WHY GO BEYOND GR?

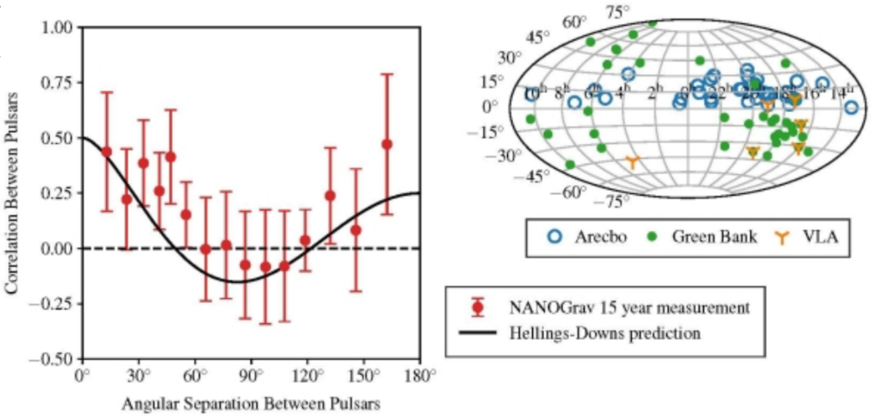
Missing explanation for dark matter and dark energy



Cosmological tensions



Observed stochastic gravitational wave background of unknown origin



Beyond GR

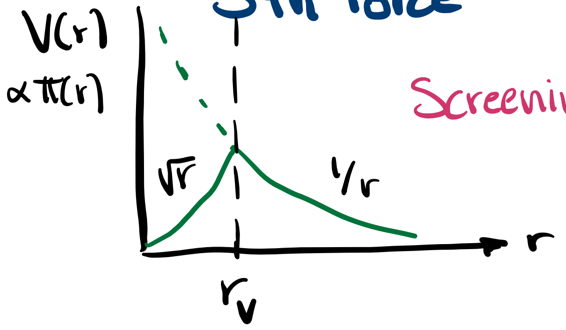
$h: \pm 2 \quad \pm 1 \quad 0 \star$

Generically arise in theories with:

- higher dim.
- higher spin states
- massive gravitons

Strong constraints from Solar System tests

5th Force



Screening

or

Weak coupling

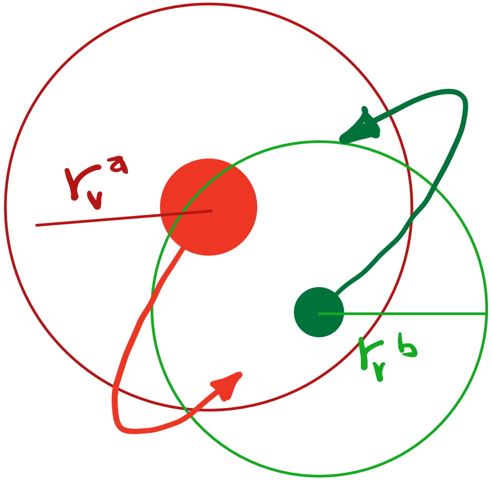
eg. Cubic Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{c}{\Lambda^3}\Box\pi(\partial\pi)^2$$

shift symmetry / soft limit

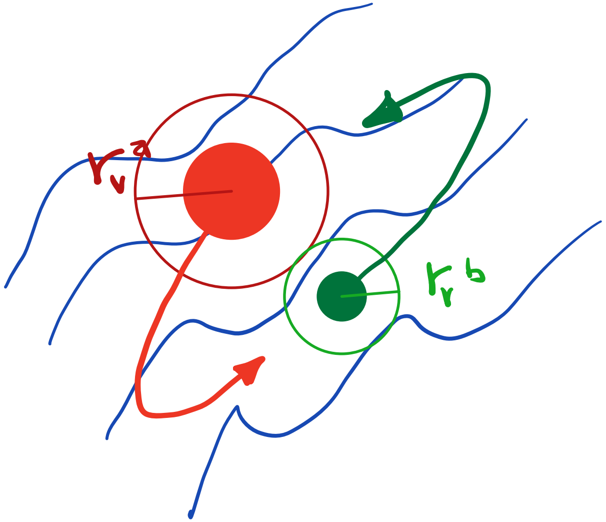
$$\delta\pi = c + b_\mu x^\mu \Rightarrow A \sim p^2$$

Expectation in vacuum

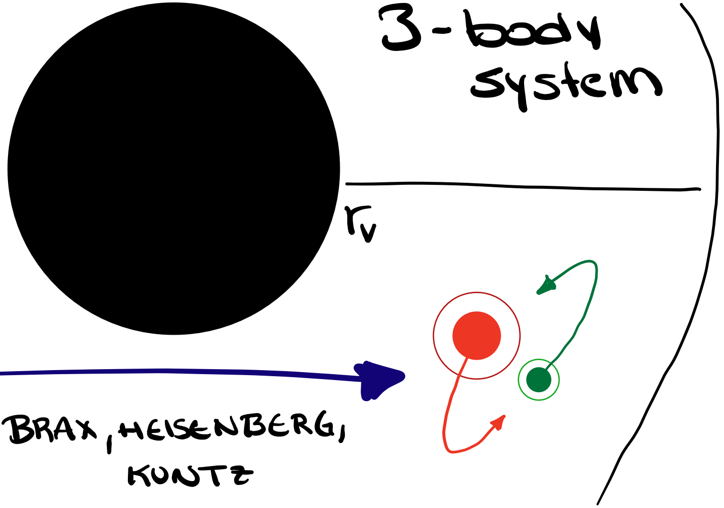


Redressed $r_v(g)$ in a background:

$\pi = \pi_0 + \delta\pi$



3-body system



Perturbative regime

BRAX, HEISENBERG, KUNTZ

Classical GR from Amplitudes

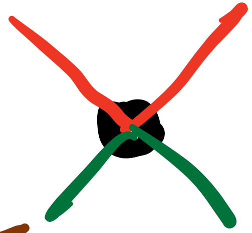
See Monday talks

Post-Minkowskian expansion

1

classical loops

2



Hamiltonian

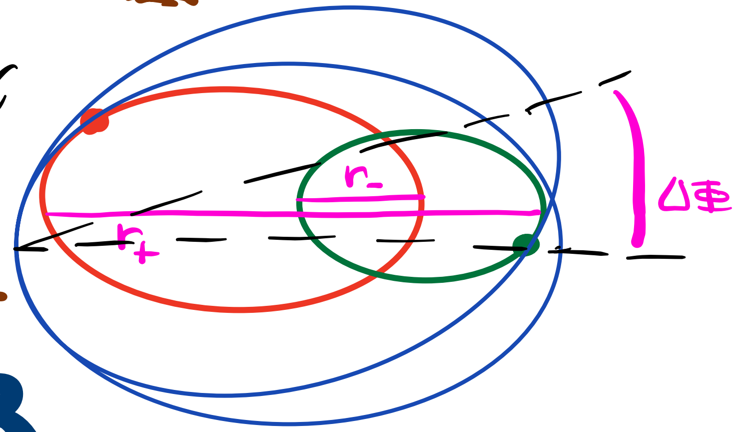
matching EFT's

Other related approaches:

- Eikonal
- KMOC
- Worldline

Δx

B2B



3

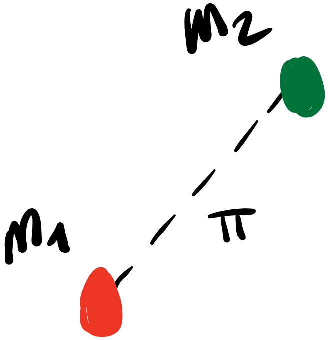
BINARY SYSTEMS BEYOND GR

2107.11384:

MCG, de Rham, Tolley

Amplitude methods beyond minimal couplings and GR

Test simple scenario: Cubic Galileon



Decoupling Limit $M_{Pl} \to \infty$, Λ fixed

→ Conformally coupled matter

Spinless compact objects

Identify Classical Contributions

Classical non-linearities

$$\epsilon^{NL} = \frac{h}{M_{pl}} \propto r_s\, q\, \hbar^0 \ll 1$$

Quantum corrections

$$\epsilon^{Q} = \frac{\partial^2}{M_{pl}^2} \propto q^2 G \hbar \ll \epsilon^{NL}$$

$$\epsilon^{Qm} = \frac{\partial}{m} \propto \frac{q}{m} \hbar \ll \epsilon^{NL}$$

Post-Minkowskian expansion

Classical limit

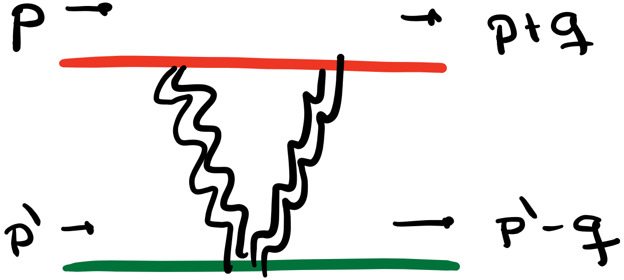
$$\epsilon^{NL} = \frac{\partial\partial\pi}{\Lambda^3} = r_v\, q\, \hbar^0 \ll 1$$

$$r_v = \left(\frac{gm}{M_{pl}}\right)^{1/3} \frac{c}{\Lambda}$$

$$\epsilon^{Q} = \frac{\partial^2}{\Lambda^2} = \frac{q^2}{\Lambda^2} \hbar^{1/3} \ll \epsilon^{NL}$$

Classical physics from loops

2PM



$$A \sim \frac{G m_1^2 m_2^2}{q^2} (G m_1 q)$$

Generic EFT
classical expansion

$$A \sim \frac{G m_1^2 m_2^2}{q^2} (r_s q)^n (r_v q)^{3m}$$

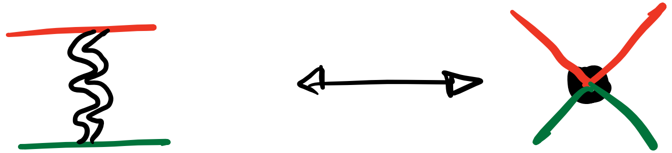
GR

Galileon

Matching EFTs

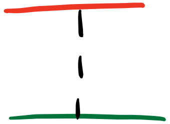
$$\frac{\mathcal{M}^{clas}}{4E_1E_2} = \mathcal{M}^{EFT} \longrightarrow V\,\Phi_{\bar p}\,\Phi_{\bar p'}\,\Phi_{-\bar p}\,\Phi_{-\bar p'}$$

$\mathcal{O}(G)$

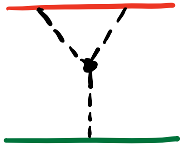


V

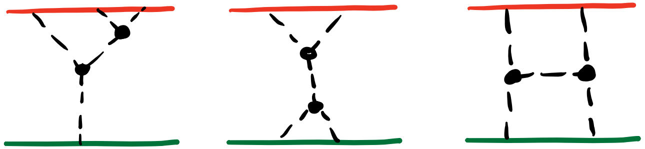
$\mathcal{O}(r_v^0)$



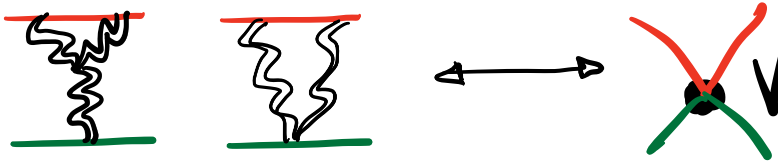
$\mathcal{O}(r_v^3)$



$\mathcal{O}(r_v^6)$



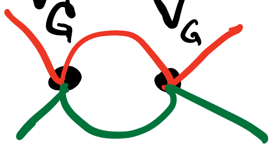
$\mathcal{O}(G^2)$



V_{G^2}

G

V_G V_G



$\to \mathcal{H}$

. . . . .

★ Consider canonically normalized states for matching

New contribution to $\Delta\Phi$

$$\Delta\Phi = C_{Gr_v^3}\left(\frac{gGM^2}{J}\right)\left(\frac{r_v^3 |P_\infty|^2 M}{J^3}\right) \quad \underline{\underline{1PM}}$$

$$+ C_{G^2}\left(\frac{gGM^2}{J}\right)^2 \; 2PM \; + \cdots$$

$+$ gravitons contribution $2PM$

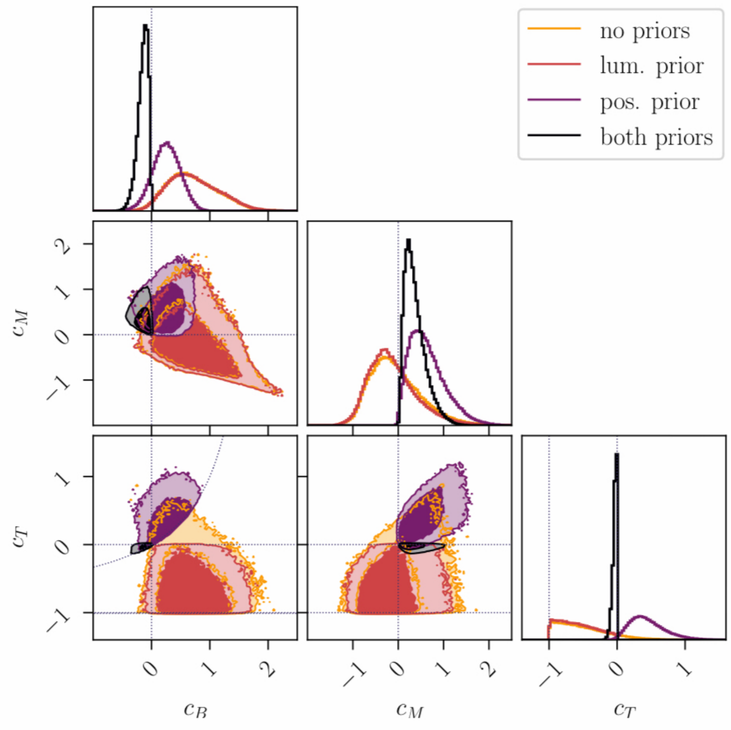
Resummation ?

Leading contribution depends on the size of $g\,C_{Gr_v^3}\left(\frac{r_v^3 |P_\infty|^2 M}{J^3}\right)$

Bounds on $C_{Gr_v^3}$?

Importance of bounds on Wilson coefs.

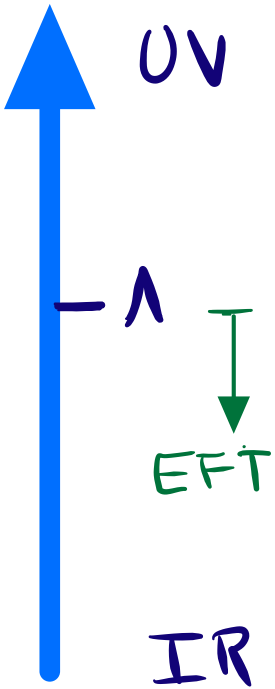
Theoretical priors can drastically change estimations of parameters



Melville, Noller

CAUSALITY BOUNDS ON EFTS

CAUSAL EFFECTIVE FIELD THEORIES



$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

1) UV = string theory $\to$ Swampland Conjectures

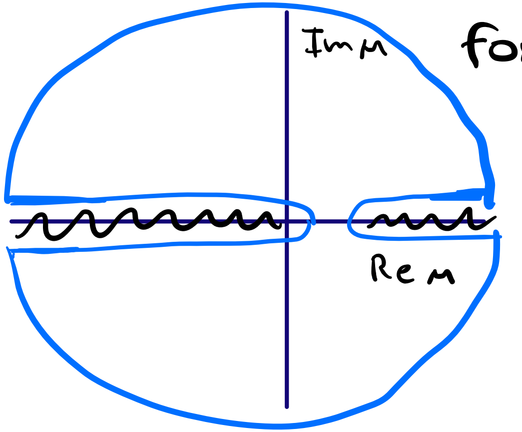
CAUSAL EFFECTIVE FIELD THEORIES

$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

2) UV = local, unitary, causal, Lorentz invariant → Positivity bounds

$\qquad\quad$ 1 $\qquad$ 2 $\qquad$ 3 $\qquad$ 4



forward lim. $(t \to 0)$

$$A''(s) = \oint \frac{d\mu}{2\pi i} \frac{A(\mu)}{(\mu - s)^3} \overset{1,3}{=} \Bigg(\underbrace{\int_{-\infty}^{0} + \int_{4m^2}^{\infty}}_{\text{related by 4}} \Bigg) \frac{\text{Im}\, A}{(\mu - s)^3} \overset{2}{>} 0$$

IR

UV

CAUSAL EFFECTIVE FIELD THEORIES

UV

$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

Λ

1) UV = string theory $\rightarrow$ Swampland

EFT

2) UV = local, unitary, causal, Lorentz invariant

$\rightarrow$ Positivity bounds

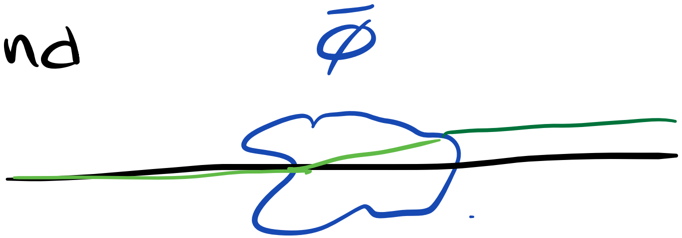
IR

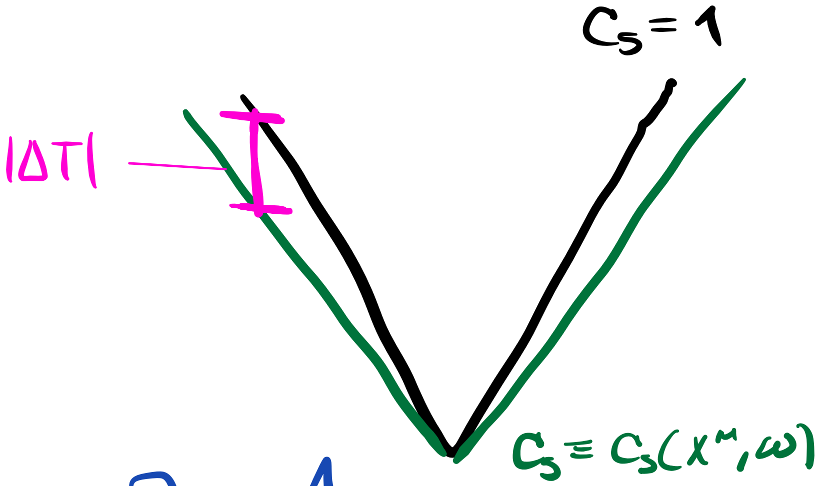
3) Causal IR propagation $\rightarrow$ CAUSALITY BOUNDS

CAUSALITY

$$\boxed{G_R(x-y) = 0 \quad \text{for} \quad (x-y)^2 > 0}$$

- Consider local propagation of information around a fixed background





Diagnose acausality by looking at time delay

$$\Delta T = -i \langle in | \hat{S}^\dagger \frac{\partial}{\partial \omega} \hat{S} | in \rangle$$

$$|\Delta T| \lesssim \lambda \sim \frac{1}{\omega}$$

Unresolvable

$$\boxed{\Delta T \gtrsim -\frac{1}{\omega}}$$

CAUSALITY + WKB + EFT

Consider $\phi = \bar{\phi} + \varphi$. Find ΔT experienced by φ.
Solve linearized φ eom using WKB approximation.

$$\boxed{\underbrace{\frac{\lambda^{\text{background}}}{\lambda^{\text{perturbation}}}}_{\gg 1 \text{ WKB}} \int_{X \subset \mathbb{R}^{1+3}} \underbrace{\left(1 - c_s(\lambda^{\text{pert.}})\right)}_{= -\epsilon \text{ EFT} \to |\epsilon| \ll 1} \gtrsim -1}$$

$$\boxed{\omega \Delta T \gtrsim -1}$$

CAUSALITY BOUNDS ON SCALAR EFTS

2207.03491:

MCG, de Rham, Pozsgay, Tolley

CAUSALITY INSIGHTS ON SCALAR EFTS

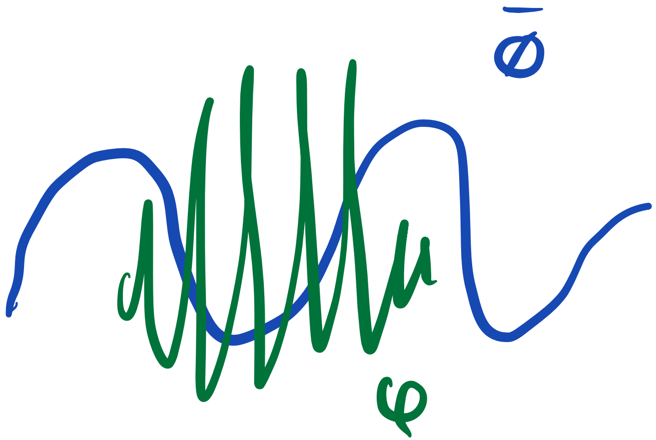
$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}\left((\partial\phi)^2\right)^2$$

$$+ \frac{g_{10}}{\Lambda^6}(\partial\phi)^2\left((\partial\partial\phi)^2 - (\Box\phi)^2\right) + \frac{g_{12}}{\Lambda^8}\left((\partial\partial\phi)^2\right)^2 - g_{\text{matter}}\,\phi J$$

quartic galileon

external source

CAUSALITY INSIGHTS ON SCALAR EFTS



$\phi = \bar{\phi} + \varphi$ $\qquad$ $\partial_\mu \varphi = i k_\mu \varphi$ plane waves

EOM + Disp. rel $\to c_s$

Adams et al

$$c_s^2 \simeq \Bigg(1 - \# \frac{g_8}{\Lambda^4} \overbrace{\frac{(k\cdot\partial\bar{\phi})^2}{|\vec{k}|^2}}^{>0} + \# \frac{g_{10}}{|k|^2\Lambda^6}\Big((k_\mu \partial^\mu \partial_\rho \phi)^2 - \Box\phi\,(k_\mu k_\nu \partial^\mu \partial^\nu \phi)\Big) - \# \frac{g_{12}}{\Lambda^8} \overbrace{\frac{(kk\cdot\partial\partial\phi)^2}{|k|^2}}^{>0}\Bigg)$$

$\to$ Work with

$$g_8 = \begin{cases} 1 \\ 0 \end{cases}$$

PROPAGATION AROUND SPHERICALLY SYMMETRIC BACKGROUNDS

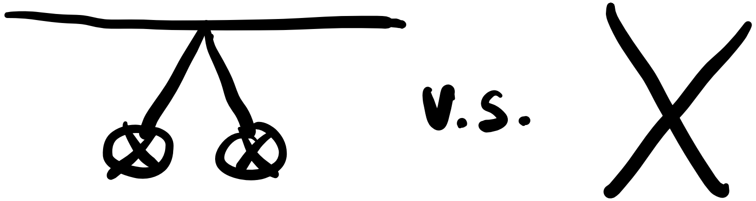
Spherically-symmetric background $\bar{\phi} \equiv \Phi_0 f(r/r_0)$

Perturbations

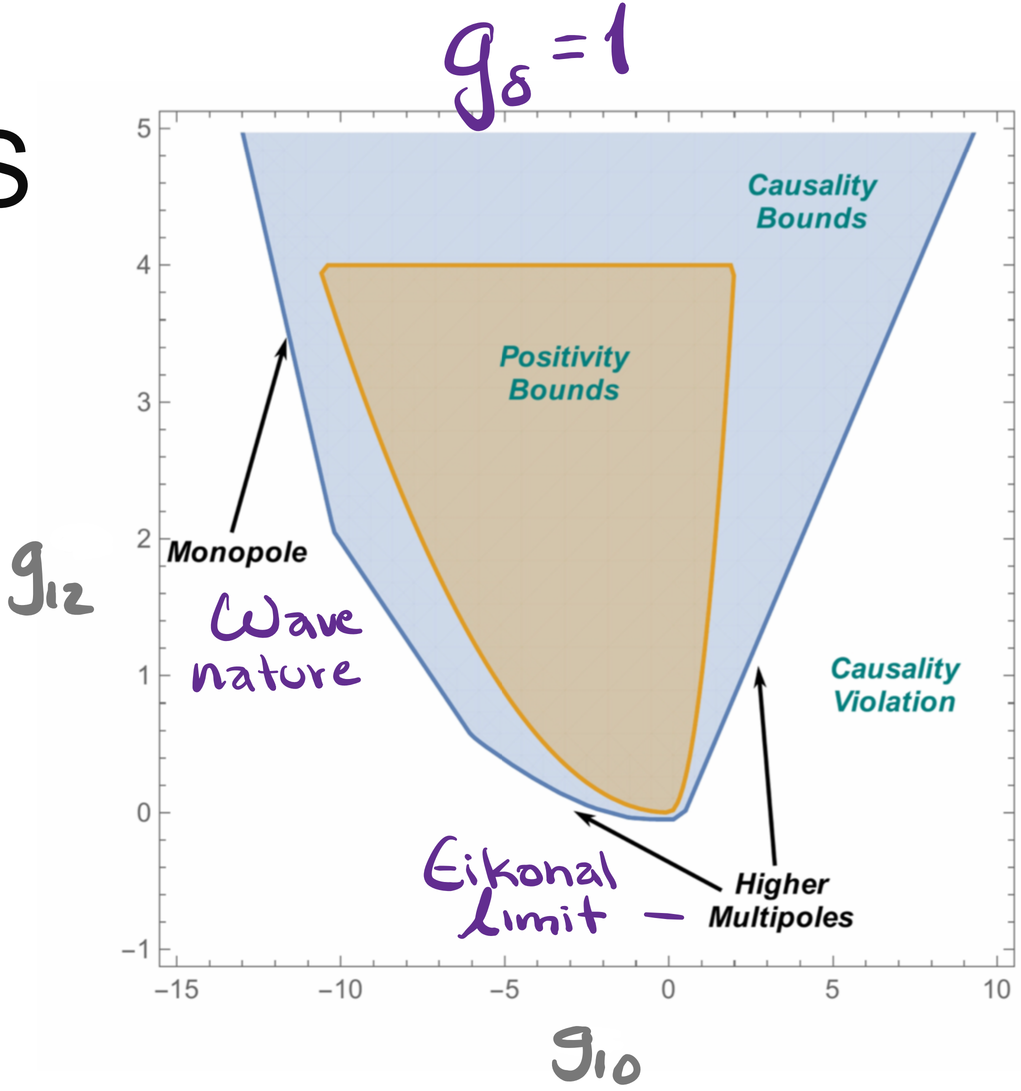
$$\rightarrow \chi_\ell''(r/r_0) + \underbrace{(\omega r_0)^2}_{\gg 1 \text{ WKB}} \underbrace{\frac{1}{c_s^2(\omega, r)}\left(1 - \frac{V_\ell^{\text{eff}}(r)}{(\omega r_0)^2}\right)}_{\equiv W_\ell(\omega, r) \;\ll 1 \text{ EFT}} \chi_\ell(r/r_0) = 0$$

$\rightarrow$ Find $\delta \rightarrow \Delta T$

CAUSALITY VS POSITIVITY



No upper bound on g_{12} from causality due to WKB technical issues.



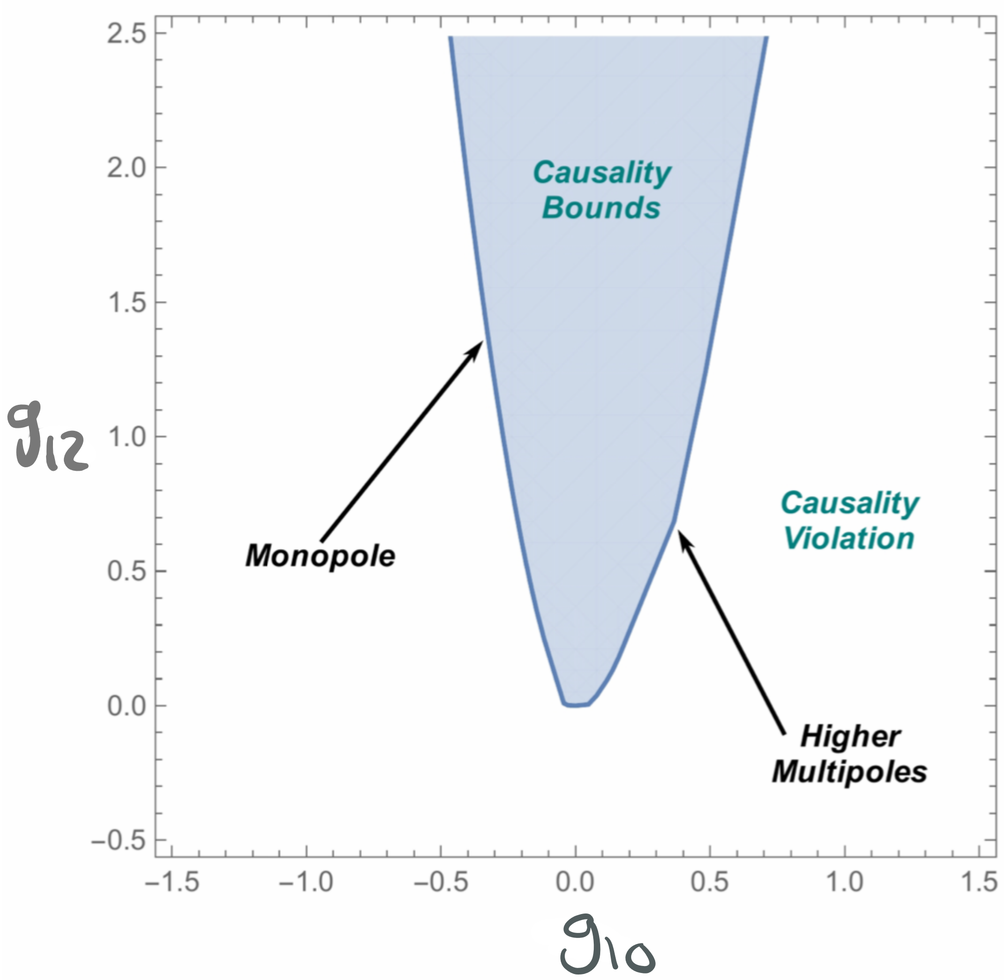
ADDITIONAL SYMMETRIES: GALILEONS

$g_8 = 0$

From positivity:

$g_8 = 0 \Rightarrow g_{10} = g_{12} = 0$

valid around $\langle \phi \rangle = 0$



CAUSALITY AND POSITIVITY BOUNDS ON PHOTON EFTS

2307.04784

MCG, de Rham, Jaitly,
Pozsgay, Tokareva

PHOTON EFT

$$\mathcal{A}^{++++} = \underline{f_2\left(s^2+t^2+u^2\right)} + \underline{f_3 stu} + \underline{f_4\left(s^2+t^2+u^2\right)^2}$$
$$\mathcal{A}^{++--} = \underline{g_2 s^2} + \underline{g_3 s^3} + \underline{g_4 s^4} + \underline{g_4' s^2 tu} \qquad + \mathcal{O}(s^5)$$
$$\mathcal{A}^{+++-} = \underline{h_3 stu}$$

Dim. 8

F^4 : $\underline{f_2, g_2}$ $\leftarrow = 1, 0$

Dim. 10

$\partial^2 F^4$: $\underline{f_3, g_3, h_3}$

Dim. 12

$\partial^4 F^4$: $\underline{f_4, g_4, g_4'}$

PROPAGATION AROUND SPHERICALLY SYMMETRIC BACKGROUNDS

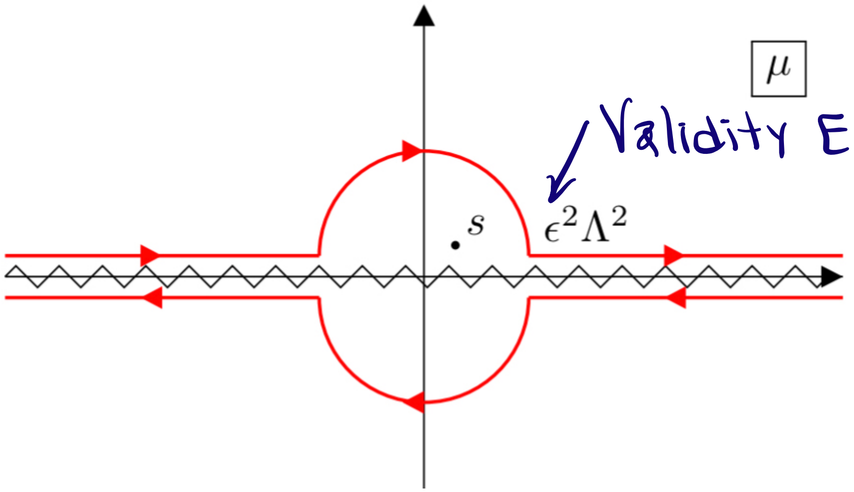
$$A = A + \delta A, \qquad \bar{A} = \Phi_0 f(r/r_0)\, dt$$

$$\chi_\ell^{even\,''} + (\omega r_0)^2 W_\ell^{even}(\omega, r)\, \chi_\ell^{even} = 0 \qquad \Delta T^{even} > -1/\omega$$

$$\Longrightarrow$$

$$\chi_\ell^{odd\,''} + (\omega r_0)^2 W_\ell^{odd}(\omega, r)\, \chi_\ell^{odd} = 0 \qquad \Delta T^{odd} > -1/\omega$$

Positivity Bounds



Validity EFT

$$\boxed{\frac{1}{2\pi \mathrm{i}}\int_\delta \frac{\mathcal{A}(\mu,t)}{(\mu-s)^3}\,\mathrm{d}\mu = \int_{\epsilon^2\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi}\,\frac{\mathrm{Disc}_s \mathcal{A}_s(\mu,t)}{(\mu-s)^3} + \int_{\epsilon^2\Lambda^2-t}^{\infty} \frac{\mathrm{d}\mu}{\pi}\,\frac{\mathrm{Disc}_s \mathcal{A}_u(\mu,t)}{(\mu-u)^3}}$$

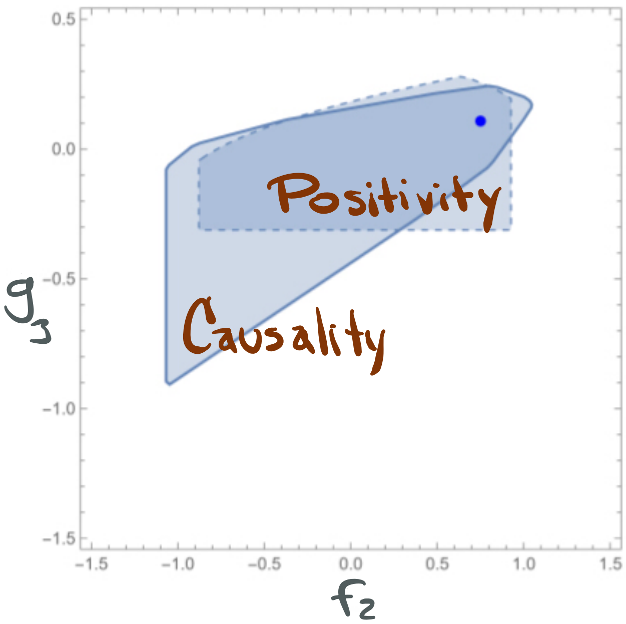
Simple example: forward limit

$$\frac{1}{2\pi i}\int_\delta \frac{A(\mu,0)}{\mu^3}\,d\mu > 0$$

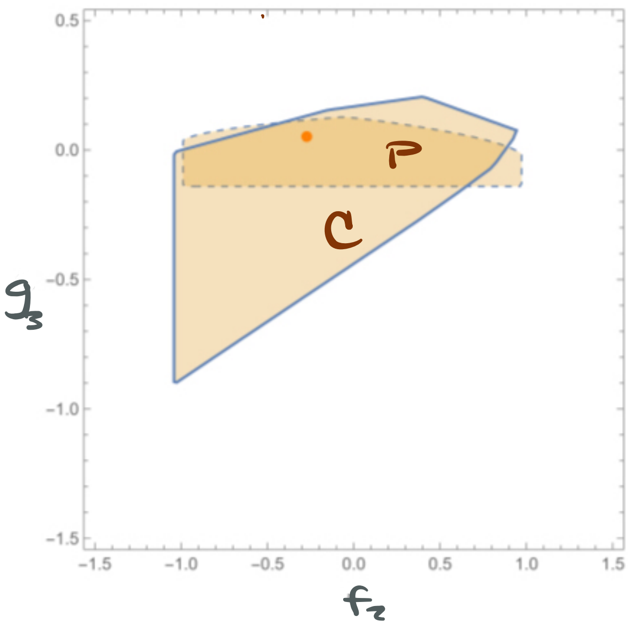
For $A(\mu) = g_2 \mu^2 + \alpha \mu^4 \log(\mu)$

$$\Rightarrow \quad g_2 + \frac{\alpha}{2}\epsilon^4 > 0$$

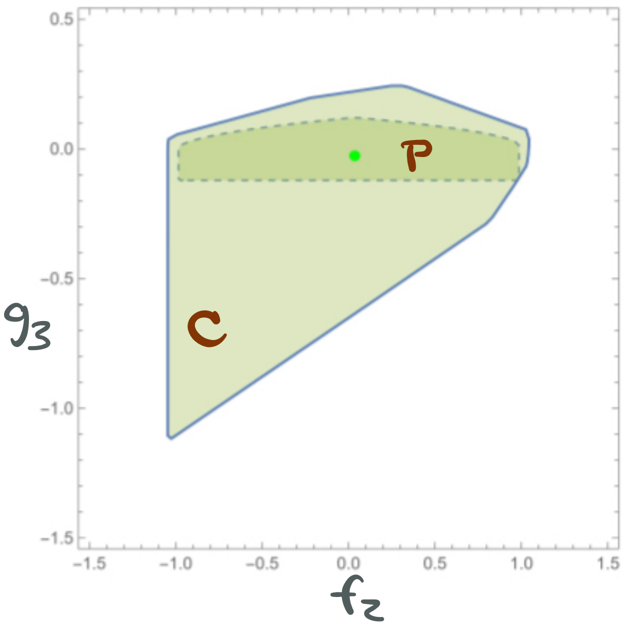
QED-like partial UV completions



(a) Scalar



(b) Spinor



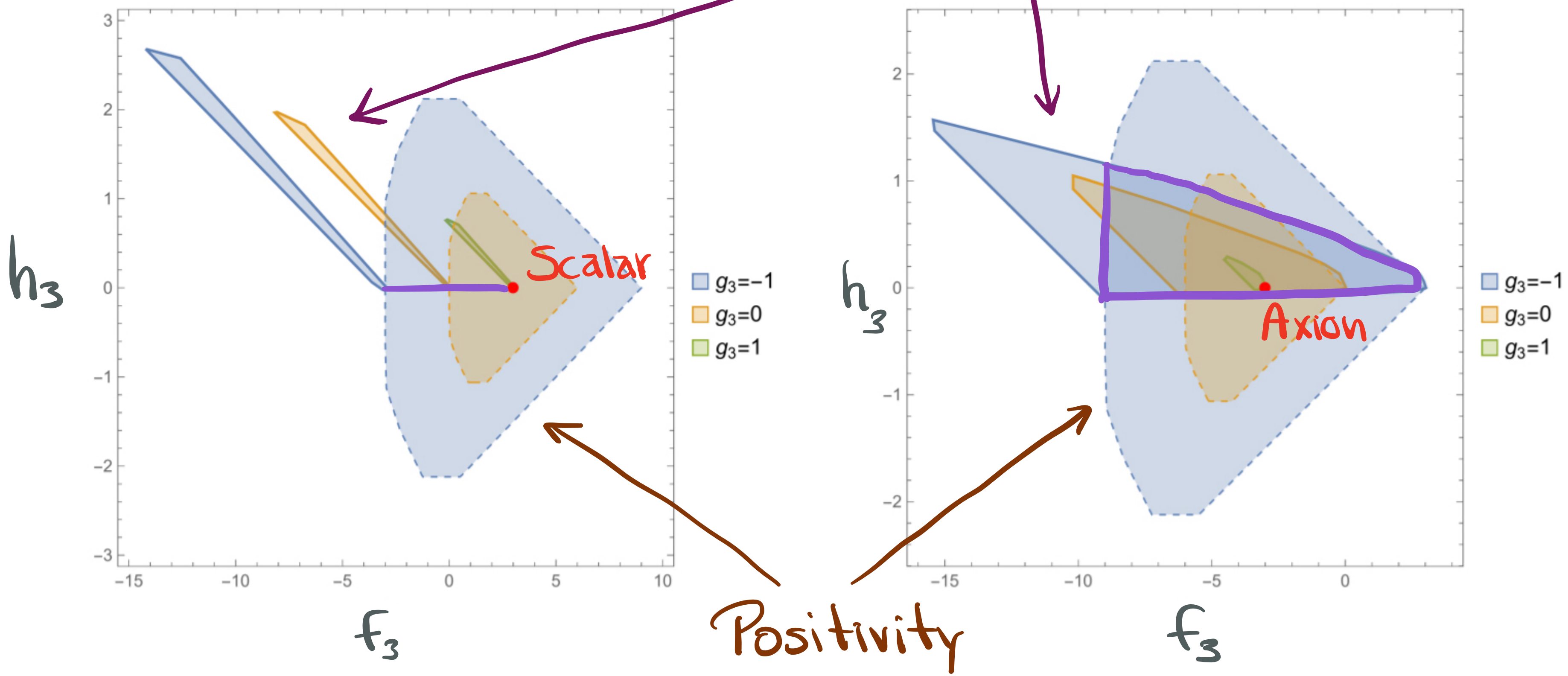
(c) Vector

DIMENSION 10

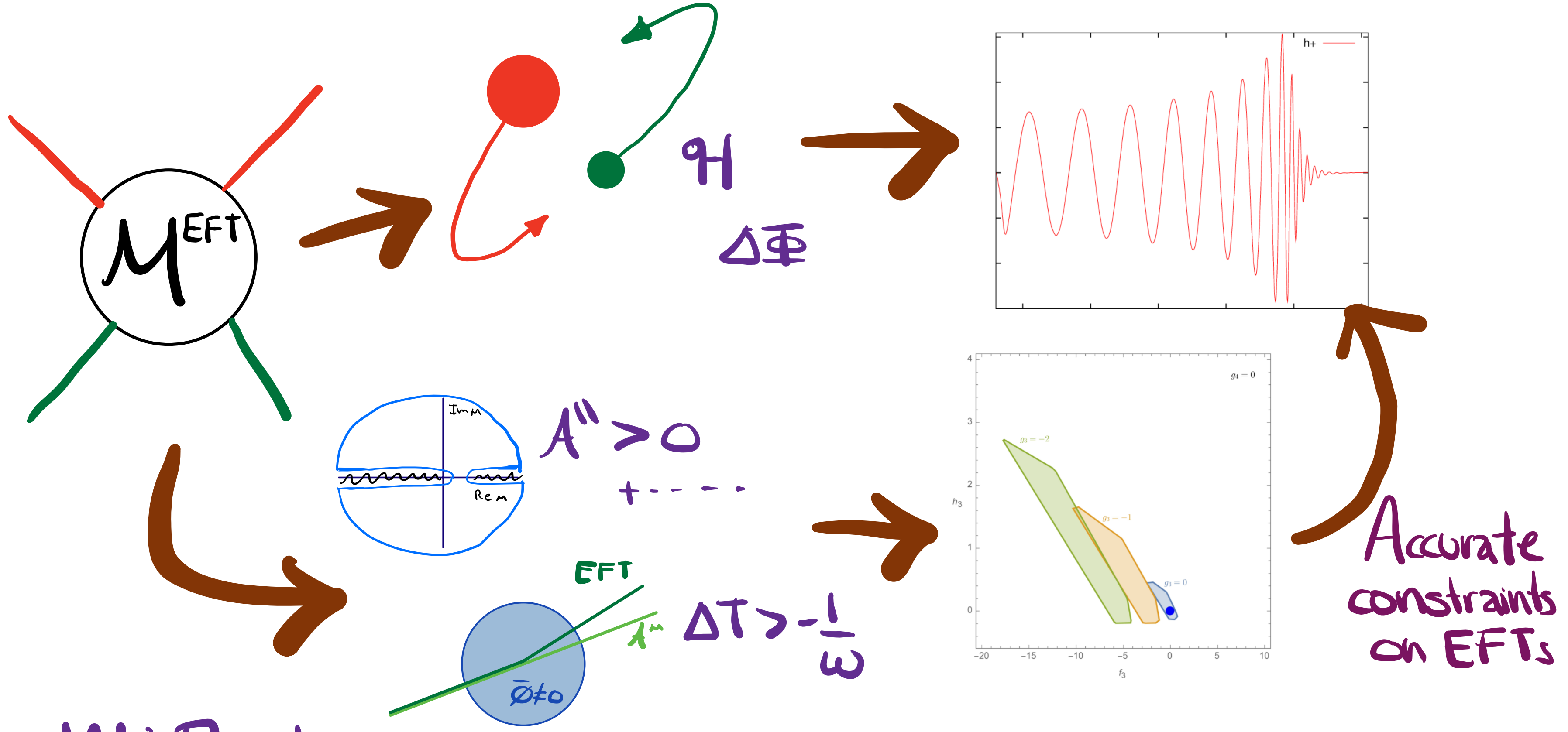
$$\boxed{(f_3 + 3g_3) < X^{even} + \xi h_3}$$

$$\boxed{-X^{odd} - \epsilon h_3 < f_3 - 3g_3 + 4h_3 < X^{odd} + \epsilon h_3}$$

Causality



Positivity



W.i.P. +

Future directions

- Test higher dimensional gravitational operators
- Cosmological backgrounds ✓ + EFT of inflation