Gauge choices, kinematic algebras, and (A)dS

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based on:

- 2306.08558 with Roberto Bonezzi and Felipe Diaz-Jaramillo
- 2304.07141 with Arthur Lipstein

Structure

Self-dual sector:

- invitation to homotopy algebras via kinematic algebras
- a toy model for curved space

Kinematic Algebras in YM

- Color-kinematic duality observed in scattering amplitudes [Bern, Carrasco,Johansson]
- Kinematic factors n_i (momenta, polarisation vectors) obey the same relations as color factors c_i (structure constants)

$$c_i + c_j + c_k = 0 \tag{1}$$
$$n_i + n_j + n_k = 0 \tag{2}$$

- (1) follows from Jacobi identity of gauge Lie algebra of YM
- Another algebra giving (2)? - "kinematic algebra"

The self-dual sector is a great toy model

- Subsector of a theory where only one of the helicities survives (e.g. YM $+1$, gravity $+2$).
- Integrable: infinite tower of charges/symmetries.
- Relation to $w_{1+\infty}$ (talk [Raclariu]) and sub^n-leading soft theorems.
- Very simple expressions for (a subset) of scattering amplitudes.
- Alternative perturbation scheme.

Self-dual YM in light-cone gauge[Monteiro,O'Connell]

- Self-duality relation

$$F_{\mu\nu} = \frac{\sqrt{g}}{2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda},$$

- Work in light-cone coordinates: $u = it + z$, $v = it - z$, $w = x + iy$, $\bar{w} = x - iy$,
- Then, choosing light-cone gauge , we are left with

$$A_w = 0, \quad A_{\bar{w}} = \partial_u\Phi, \quad A_v = \partial_w\Phi,$$

where

$$\Box_{\mathbb{R}^4}\Phi + i\,[\partial_u\Phi, \partial_w\Phi] = 0$$

- Introduce Poisson bracket

$$\{f, g\} := \partial_w f \partial_u g - \partial_u f \partial_w g$$

and notice that it appears naturally in the scalar equation of motion:

$$\Box_{\mathbb{R}^4}\Phi - \frac{i}{2}[\{\Phi, \Phi\}] = 0,$$

- Poisson bracket automatically satisfies Jacobi,

$$\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0.$$

kinematic algebra (area preserving diffeomorphisms).

Kinematic algebras

- Chern Simons [Ben-Shahar,Johansson] - volume preserving diffeo, Hopf algebras in YM from Heavy Mass EFT [Brandhuber,Browna,Chen,Gowdy,Johansson,Lin,Travaglini,Wen], via twistor theory and/or pure spinors [Borsten,Jurco,Kim,Macrelli,Saemann,Wolf], beyond MHV [Chen,Johansson,Teng,Wang],[Lee,Mafra,O. Schlotterer], [Ben-Shahar,Guillen], self-dual extensions/deformations [Chacon, Garcıa-Compean, Luna, Monteiro,White,Armostrong-Williams, Wikeley]...

- **Role of the gauge choice** in constructing a (proper) kinematic algebra ?

Homotopy algebras

- Color-kinematics, double copy[Reiterer],[Zwiebach],[Lada,Stasheff],[Hohm,Zwiebach], [Borsten,Jurco,Kim,Macrelli,Saemann,Wolf], [Bonezzi,Chiaffrino,Diaz-Jaramillo,Hohm], [Szabo,Trojani].
- Connections to holography [Chiaffrino,Ersoy,Hohm], higher spins [Sharapov,Skvortsov], etc...
- Can think of them as generalisations of BV (BRST) formalism, but can be simpler in some ways.
- Give a general way to construct kinematic algebras.

Self-dual general

- General SD relation

$$F_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}\,F^{\rho\sigma}\ ,$$

  fully gauge covariant.
- Rewrite as:

$$(1-\star)F \equiv 2\,P_{-}F = 0\ ,$$

- Separating the orders

$$2\,P_{-}dA + P_{-}[A,A] = 0\ .$$

L_∞ algebras and homotopy

- An L_∞ algebra is a graded vector space $\mathcal{X} = \bigoplus_i X_i$ equipped with a (possibly infinite) set of graded symmetric multilinear maps $B_n : \mathcal{X}^{\otimes n} \to \mathcal{X}$.
- The maps obey a (possibly infinite) set of quadratic relations.
- Nilpotency:

$$B_1\Big(B_1(\psi)\Big) \equiv \mathbb{d}\Big(\mathbb{d}(\psi)\Big) = 0$$

- Leibniz rule

$$\mathbb{d}B_2(\psi_1, \psi_2) + B_2\Big(\mathbb{d}(\psi_1), \psi_2\Big) + (-1)^{\psi_1} B_2\Big(\psi_1, \mathbb{d}(\psi_2)\Big) = 0$$

- Jacobi up to homotopy :

$$B_2\Big(B_2(\psi_1, \psi_2), \psi_3\Big) + (-1)^{\psi_1(\psi_2+\psi_3)} B_2\Big(B_2(\psi_2, \psi_3), \psi_1\Big) + (-1)^{\psi_3(\psi_1+\psi_2)} B_2\Big(B_2(\psi_3, \psi_1), \psi_2\Big)$$

$$= -\mathbb{d}B_3(\psi_1, \psi_2, \psi_3) - B_3(\mathbb{d}(\psi_1), \psi_2, \psi_3) - (-1)^{\psi_1} B_3(\psi_1, \mathbb{d}(\psi_2), \psi_3) - (-1)^{\psi_1+\psi_2} B_3(\psi_1, \psi_2, \mathbb{d}(\psi_3))$$

- higher order relations...

Explicit realisation for self-dual YM

- We have 3 vector spaces: gauge parameters, gauge fields, equations:

$$X_{-1} \xrightarrow{\mathbb{d}} X_0 \xrightarrow{\mathbb{d}} X_1$$

$$\Lambda \qquad\qquad A \qquad\qquad E$$

- We extract the explicit expressions for $B_1 \equiv \mathbb{d}$, $B_2 \ldots$ from the e.o.m., symmetry transformations, gauge algebra:

$$\mathbb{d}(A) = 2\,P_{-} dA \in X_1\,, \qquad \mathbb{d}(\Lambda) = d\Lambda \in X_0$$
$$B_2(A_1, A_2) = 2\,P_{-}[A_1, A_2] \in X_1\,, \quad B_2(A, \Lambda) = [A, \Lambda] \in X_0\,.$$

- B_3 and above vanish in this case
- Consistency relations ensure gauge covariance of e.o.m., closure of gauge algebra.

Color-stripping

- The L_∞ algebra $\mathcal{X}^{\mathrm{SDYM}}$ takes the form of a tensor product:

$$\mathcal{X}^{\mathrm{SDYM}} = \mathcal{K} \otimes \mathfrak{g}\ ,$$

- Expand an arbitrary element $\psi(x)$ of $\mathcal{X}^{\mathrm{SDYM}}$ in a basis $\{T_a\}$ of $\mathfrak{g}$, and write it as

$$\psi(x) = u^a(x) \otimes T_a\ ,\ \ u^a(x) \in \mathcal{K}\ ,\ \ T_a \in \mathfrak{g}\ .$$

- Go from B_n to m_n maps via

$$\mathbb{d}\Big(\psi(x)\Big) = \mathbb{d}\Big(u^a(x)\Big) \otimes T_a\ , \qquad\qquad B_1 = \mathbb{d} = m_1$$

$$B_2(\psi_1, \psi_2) = (-1)^{\psi_1} m_2(u_1^a, u_2^b) \otimes f_{ab}{}^c\, T_c$$

Color-stripping

- Now we have

$$\begin{array}{ccccc} K_0 & \xrightarrow{\mathbb{d}} & K_1 & \xrightarrow{\mathbb{d}} & K_2 \\ \lambda & & \mathcal{A} & & \mathcal{E} \end{array} ,$$

with explicit maps

$$\mathbb{d}\mathcal{A} = 2\,P_- d\mathcal{A} \in K_2\;, \qquad \mathbb{d}\lambda = d\lambda \in K_1$$

$$m_2(\mathcal{A}_1, \mathcal{A}_2) = 2\,P_-\left(\mathcal{A}_1 \wedge \mathcal{A}_2\right) \in K_2\;, \qquad m_2(\lambda, \mathcal{A}) = \lambda \wedge \mathcal{A} \in K_1$$

Constructing the kinematic algebra

- Need to introduce another operator

$$\begin{array}{ccccc} K_0 & \underset{b}{\overset{\mathbb{d}}{\rightleftarrows}} & K_1 & \underset{b}{\overset{\mathbb{d}}{\rightleftarrows}} & K_2 \\ \lambda & & \mathcal{A} & & \mathcal{E} \end{array} \quad .$$

We require b to be nilpotent: $b^2 = 0$, and to obey the defining relation

$$\mathbb{d} b + b\, \mathbb{d} = \Box \, ,$$

- In our case

$$b = d^\dagger$$

- If b does not obey the Leibniz rule with respect to m_2, its failure to do so can be used to define a graded symmetric bracket b_2 as

$$b_2(u_1, u_2) := b\, m_2(u_1, u_2) - m_2(bu_1, u_2) - (-1)^{u_1} m_2(u_1, bu_2) \, ,$$

- In an amplitudes context, the bracket $b_2(\mathcal{A}_1, \mathcal{A}_2)$ between color-stripped fields gives the contribution to the kinematic numerator arising from a cubic vertex joining the external particles 1 and 2.

Constructing the kinematic algebra

- b_2 is our candidate for the generalisation of the Poisson bracket.
- Check Jacobi

$$b_2(b_2(u_1,u_2),u_3)+(-1)^{u_1(u_2+u_3)}b_2(b_2(u_2,u_3),u_1)+(-1)^{u_3(u_1+u_2)}b_2(b_2(u_3,u_1),u_2)$$
$$=[\mathbb{d},[b,\theta_3]](u_1,u_2,u_3)-[\Box,\theta_3](u_1,u_2,u_3)\ ,$$

- We can directly compute

$$\theta_3(\mathcal{A}_1,\mathcal{A}_2,\mathcal{A}_3)=-\star\left(\mathcal{A}_1\wedge\mathcal{A}_2\wedge\mathcal{A}_3\right)\ ,$$
$$\theta_3(\mathcal{E},\mathcal{A}_1,\mathcal{A}_2)=2\,P_-\left\{\star\left(\mathcal{E}\wedge\mathcal{A}_{[1}\right)\wedge\mathcal{A}_{2]}\right\}\ ,$$

- Jacobi follows from Leibniz rule.

Making the right gauge choices

- Obstruction to proper ("strict") algebra

$$\theta_3(\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3) = -\star\left(\mathcal{A}_1 \wedge \mathcal{A}_2 \wedge \mathcal{A}_3\right) ,$$
$$\theta_3(\mathcal{E}, \mathcal{A}_1, \mathcal{A}_2) = 2\,P_-\left\{\star\left(\mathcal{E} \wedge \mathcal{A}_{[1}\right) \wedge \mathcal{A}_{2]}\right\} ,$$

- In LC gauge, the self-duality constraint $A_u = 0$ additionally implies $A_w = 0$, i.e. two of the components vanish, so

$$\theta_3 = 0$$

e.g. $\star\left(\mathcal{A}_1 \wedge \mathcal{A}_2 \wedge \mathcal{A}_3\right) \propto \varepsilon^{\mu\nu\rho\sigma} A_{1\nu} A_{2\rho} A_{3\sigma}$

- One can extract the Poisson bracket of area-preserving diffeos [Monteiro,O'Connell]

$$b_2^{\alpha}(\mathcal{A}_1, \mathcal{A}_2) = -2\,\epsilon^{\alpha\beta}\partial_\beta\left\{\Phi_1, \Phi_2\right\} .$$

- Apply to other theories to find the best gauge choice, or extract kinematic algebra without gauge fixing.

Curved space

- Add a non-zero cosmological constant Λ
- cosmology (dS), holography (AdS)
- Everything is more difficult!
- Amplitudes $\rightarrow$ correlators (see talks Wed-Fri)
- Self-dual toy model: integrability, simple kinematic algebras, $w_{1+\infty}$, closed-form for amplitudes/correlators ... ?
- See related works [Przanowski,Krasnov,Skvortsov,Neiman,Tran,Shaw,Herfray,] and relation to twistors [Adamo,Mason,Sharma]

Self-dual YM

- Consider four dimensional Euclidean AdS_4 with unit radius in the Poincaré patch:

$$ds^2_{\mathrm{AdS}} = \frac{dt^2 + dx^2 + dy^2 + dz^2}{z^2},$$

- SD condition

$$F_{\mu\nu} = \frac{\sqrt{g}}{2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda},$$

- Reduces to flat equation in AdS_4 (due to conformal flatness).
- In light cone coordinates $u = it + z,\ v = it - z,\ w = x + iy,\ \bar{w} = x - iy,$ the metric is

$$ds^2_{\mathrm{AdS}} = \frac{4\,(dw\,d\bar{w} - du\,dv)}{(u-v)^2},$$

Self-dual YM in light-cone gauge

- In LC gauge, the self-duality constraint is solved by

$$A_u = 0, \quad A_w = 0, \quad A_{\bar{w}} = \partial_u \Phi, \quad A_v = \partial_w \Phi$$

with

$$\Box_{\mathbb{R}^4} \Phi + i \left[\partial_u \Phi, \partial_w \Phi \right] = 0$$

- Split spacetime as

$$x^i = (u, w), \qquad y^\alpha = (v, \bar{w})$$

- Introduce the operators

$$\Pi_\alpha = (\Pi_v, \Pi_{\bar{w}}) = (\partial_w, \partial_u)$$

then

$$A_i = 0, \quad A_\alpha = \Pi_\alpha \Phi$$

- Poisson bracket

$$\{f, g\} := \partial_w f \partial_u g - \partial_u f \partial_w g = \varepsilon^{\alpha\beta} \Pi_\alpha f \Pi_\beta g,$$

so finally

$$\Box_{\mathbb{R}^4} \Phi - \frac{i}{2} [\{\Phi, \Phi\}] = 0,$$

where we introduced the notation

$$[\{f, g\}] = \varepsilon^{\alpha\beta} \left[\Pi_\alpha f, \Pi_\beta g \right].$$

Self-dual gravity in curved backgrounds

- In flat background

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu}{}^{\eta\lambda}R_{\eta\lambda\rho\sigma}.$$

Upon contracting two of the idices

$$R_{\mu\rho} = \frac{1}{2}\epsilon_{\mu}{}^{\sigma\eta\lambda}R_{\eta\lambda\rho\sigma} = 0$$

i.e. e.o.m=Bianchi !

Self-dual gravity in curved backgrounds

- In flat background

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu}{}^{\eta\lambda}R_{\eta\lambda\rho\sigma}.$$

Upon contracting two of the idices, get e.o.m=Bianchi

$$R_{\mu\rho} = \frac{1}{2}\epsilon_{\mu}{}^{\sigma\eta\lambda}R_{\eta\lambda\rho\sigma} = 0$$

- In curved background, introduce the tensor

$$T_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{3}\Lambda(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma}),$$

- Self-duality relation

$$T_{\mu\nu\rho\sigma} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu}{}^{\eta\lambda}T_{\eta\lambda\rho\sigma}.$$

Upon contracting two indices:

$$R_{\mu\rho} - \Lambda g_{\mu\rho} = \frac{1}{2}\sqrt{g}\epsilon_{\mu}{}^{\sigma\eta\lambda}R_{\eta\lambda\rho\sigma} = 0,$$

- LHS reproduces e.o.m. $R_{\mu\nu} = \Lambda g_{\mu\nu}, R = 4\Lambda$, and RHS is again Bianchi.

Relation to Weyl tensor

- Recall

$$C_{\mu\nu}{}^{\rho\sigma} = R_{\mu\nu}{}^{\rho\sigma} - 2R_{[\mu}{}^{[\rho}g_{\nu]}{}^{\sigma]} + \frac{1}{3}Rg_{[\mu}{}^{[\rho}g_{\nu]}{}^{\sigma]}$$

- In flat space, on the support of the e.o.m. $R_{\mu\nu} = R = 0$

$$C_{\mu\nu\rho\sigma} \to R_{\mu\nu\rho\sigma}$$

- In curved space, on the support of the e.o.m. $R_{\mu\nu} = \Lambda g_{\mu\nu}, R = 4\Lambda$

$$C_{\mu\nu\rho\sigma} \to T_{\mu\nu\rho\sigma}$$

Scalar description flat

- Notation (recall $x^i = (u, w),\ y^\alpha = (v, \bar{w})$)

$$\Pi_\alpha = (\Pi_v, \Pi_{\bar{w}}) = (\partial_w, \partial_u)$$

- In flat space, write the metric as (non-perturbative)

$$ds^2 = dw\, d\bar{w} - du\, dv + h_{\mu\nu}\, dx^\mu dx^\nu ,$$

- In LC gauge ($h_{u\mu} = 0$), the self-duality constraint is then solved by

$$h_{i\mu} = 0, \quad h_{\alpha\beta} = \Pi_\alpha \Pi_\beta \phi,$$

with ϕ satisfying

$$\Box_{\mathbb{R}^4} \phi - \{\{\phi, \phi\}\} = 0,$$

where we introduced the notation

$$\{\{f, g\}\} = \frac{1}{2} \varepsilon^{\alpha\beta} \{\Pi_\alpha f, \Pi_\beta g\},$$

Scalar description (flat with YM)

- Notation (recall $x^i = (u, w)$, $y^\alpha = (v, \bar{w})$)

$$\Pi_\alpha = (\Pi_v, \Pi_{\bar{w}}) = (\partial_w, \partial_u)$$

- In flat space, write the metric as (non-perturbative)

$$ds^2 = dw\, d\bar{w} - du\, dv + h_{\mu\nu}\, dx^\mu dx^\nu ,$$

- In LC gauge ($h_{u\mu} = 0$), the self-duality constraint is then solved by

$$h_{i\mu} = 0,\ h_{\alpha\beta} = \Pi_\alpha \Pi_\beta \phi, \qquad A_i = 0,\ A_\alpha = \Pi_\alpha \Phi$$

with ϕ satisfying

$$\Box_{\mathbb{R}^4} \phi - \{\{\phi, \phi\}\} = 0, \qquad \Box_{\mathbb{R}^4} \Phi - \frac{i}{2} [\{\Phi, \Phi\}] = 0,$$

where we introduced the notation

$$\{\{f, g\}\} = \frac{1}{2} \varepsilon^{\alpha\beta} \{\Pi_\alpha f, \Pi_\beta g\}, \qquad [\{f, g\}] = \varepsilon^{\alpha\beta} \left[\Pi_\alpha f, \Pi_\beta g\right] .$$

Scalar description AdS$_4$

- Notation (recall $x^i = (u, w)$, $y^\alpha = (v, \bar{w})$)

$$\tilde{\Pi} = (\tilde{\Pi}_v, \tilde{\Pi}_{\bar{w}}) = \left(\partial_w, \partial_u - \tfrac{4}{u-v}\right),$$

- Write the metric as (non-perturbative)

$$ds^2 = \tfrac{4}{(u-v)^2}\left(dw\, d\bar{w} - du\, dv + h_{\mu\nu}\, dx^\mu dx^\nu\right)$$

- In LC gauge ($h_{u\mu} = 0$), the self-duality constraint is then solved by

$$h_{i\mu} = 0, \quad h_{\alpha\beta} = \Pi_{(\alpha}\tilde{\Pi}_{\beta)}\phi$$

with ϕ satisfying

$$\sqrt{g}\left(-\Box_{\mathrm{AdS}} + m^2\right)\phi + 4\left\{\left\{\frac{\phi}{u-v}, \frac{\phi}{u-v}\right\}\right\}_* = 0$$

where $m^2 = -2$ corresponding to a conformally coupled scalar in AdS$_4$ and we introduced the notation

$$\{\{f, g\}\}_* = \frac{1}{2}\varepsilon^{\alpha\beta}\{\Pi_\alpha f, \Pi_\beta g\}_*$$

Modified Poisson bracket

- Notation (recall $x^i = (u, w)$, $y^\alpha = (v, \bar{w})$)

$$\Pi_\alpha = (\Pi_v, \Pi_{\bar{w}}) = (\partial_w, \partial_u), \quad \tilde{\Pi} = (\tilde{\Pi}_v, \tilde{\Pi}_{\bar{w}}) = \left(\partial_w, \partial_u - \tfrac{4}{u-v}\right)$$

- The modified Poisson bracket is given by

$$\{f, g\}_* = \frac{1}{2}\varepsilon^{\alpha\beta}(\Pi_\alpha f \tilde{\Pi}_\beta g - \Pi_\alpha g \tilde{\Pi}_\beta f).$$

- Note $\{f, g\}_*\big|_{\tilde{\Pi}\to\Pi} = \{f, g\} = \varepsilon^{\alpha\beta}\Pi_\alpha f \Pi_\beta g$
- Alternative forumula

$$\{f, g\}_* = \{f, g\} + \tfrac{2}{u-v}\left(f\partial_w g - g\partial_w f\right).$$

- In the flat-space limit $z = (u - v) \to \infty$, we get $\tilde{\Pi} \to \Pi$, and $\{,\} \Rightarrow \{,\}_*$
- Crucially, it satifies jacobi

$$\Big\{f, \{g, h\}_*\Big\}_* + \Big\{g, \{h, f\}_*\Big\}_* + \Big\{h, \{f, g\}_*\Big\}_* = 0.$$

Correlators

- Scalar SD AdS equation dmits the following solutions, which are related to planewave solutions by a Weyl rescaling:

$$\phi = (u - v)e^{ik\cdot x},$$

- Extract structure "constants":

$$\left\{e^{ik_1\cdot x}, e^{ik_2\cdot x}\right\} = X\left(k_1, k_2\right) e^{i(k_1+k_2)\cdot x},$$
$$\left\{e^{ik_1\cdot x}, e^{ik_2\cdot x}\right\}_* = \tilde{X}\left(k_1, k_2\right) e^{i(k_1+k_2)\cdot x},$$

where

$$X\left(k_1, k_2\right) = k_{1u}k_{2w} - k_{1w}k_{2u},$$
$$\tilde{X}\left(k_1, k_2\right) = X\left(k_1, k_2\right) - \frac{2i}{u-v}\left(k_1 - k_2\right)_w .$$

- computing three-point boundary correlators:

$$V_{\mathrm{SDYM}} = \frac{1}{2}X\left(k_1, k_2\right) f^{a_1a_2a_3},$$
$$V_{\mathrm{SDG}} = \frac{1}{2}X\left(k_1, k_2\right)\tilde{X}\left(k_1, k_2\right),$$

Correlators

- The objects X and $\tilde{X}$ obey Jacobi identities analogous to $f^{a_1 a_2 a_3}$ and can therefore be thought of as structure constants of kinematic Lie algebras:

$$\begin{aligned} 0 &= X(k_1, k_2)\, X(k_3, k_1 + k_2) + \text{cyclic} \\ &= \tilde{X}(k_1, k_2)\, \tilde{X}(k_3, k_1 + k_2) + \text{cyclic.} \end{aligned}$$

  which follow from

$$\left\{f, \{g, h\}_*\right\}_* + \left\{g, \{h, f\}_*\right\}_* + \left\{h, \{f, g\}_*\right\}_* = 0.$$

- Currently computing higher orders (with [Chowdhury,Lipstein,Monteiro,Singh])

Deformed $w_{1+\infty}$

- One can extract the $w_{1+\infty}$ from the Poisson breacket [Monteiro]
- For an on-shell state, the momentum satisfies $k_{\bar{w}}/k_u = k_v/k_w = \rho$, where ρ is some number. It is then possible to expand an on-shell plane wave as follows:

$$e^{ik\cdot x} = \sum_{a,b=0}^{\infty} \frac{(ik_u)^a\,(ik_w)^b}{a!b!}\mathfrak{e}_{ab},$$

where $\mathfrak{e}_{ab} = (u+\rho\bar{w})^a\,(w+\rho v)^b$. This is naturally interpreted as an expansion in soft momenta. Letting $w^p_m = \frac{1}{2}\mathfrak{e}_{p-1+m,p-1-m}$ and plugging this into the Poisson bracket:

$$\left\{w^p_m, w^q_n\right\} = (n(p-1) - m(q-1))\,w^{p+q-2}_{m+n}$$

- For our modified Poisson bracket

$$\left\{w^p_m, w^q_n\right\}_* = \left\{w^p_m, w^q_n\right\} + \frac{(m+q-p-n)}{u-v}w^{p+q-3/2}_{m+n+1/2}$$

- Local deformation, falls outside the classification of global deformations [Pope,Bittleston, Heuveline, Skinner, Bu, Etingof, Kalinov, Rains]

Conclusions and future directions

- Correlators - simple formula at n points ?
- Connection to asymptotic symmetries.
- Integrability and connection to AdS/CFT.
- Full theory from expansion around self-dual sector.

Thank You !