Stochastic Resetting

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Plan

- Stochastic Resetting $\Longrightarrow$ a brief introduction

- Diffusion with Resetting: A simple model

  $\Rightarrow$ new Nonequilibrium Steady State

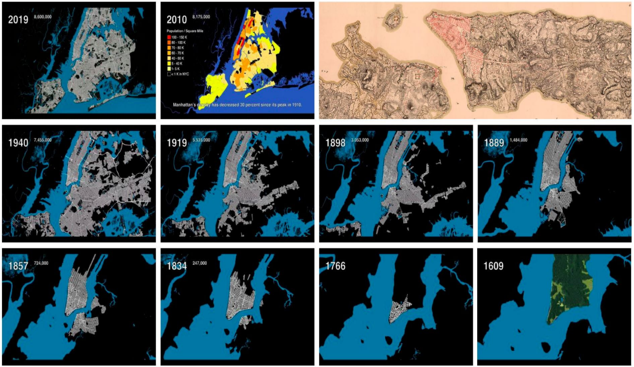
  $\Rightarrow$ unusual temporal relaxation

  $\Rightarrow$ optimal search time to find a target

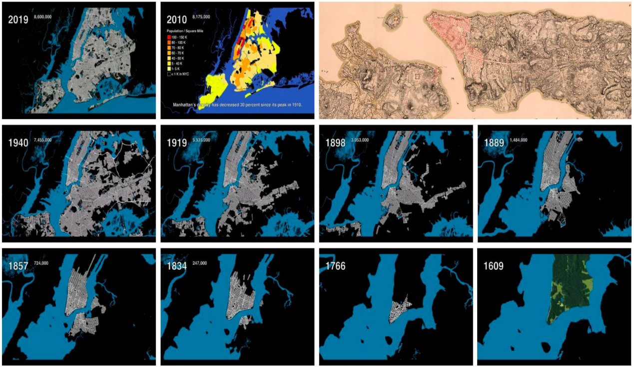
- Recent experiments using optical tweezers

- Generalisation to many-body systems

- Summary and Conclusions



Growth of NYC since 1609



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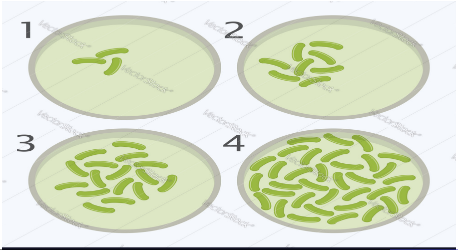
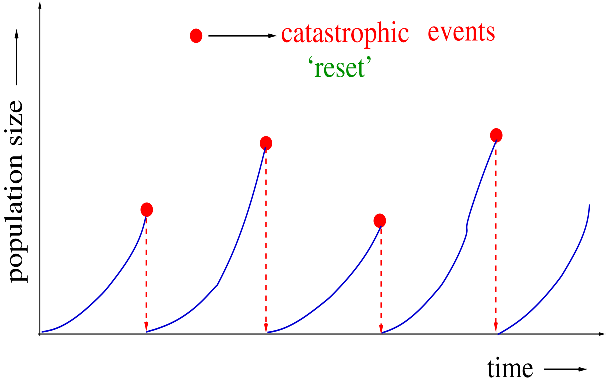


Growth of bacteria on a petri dish

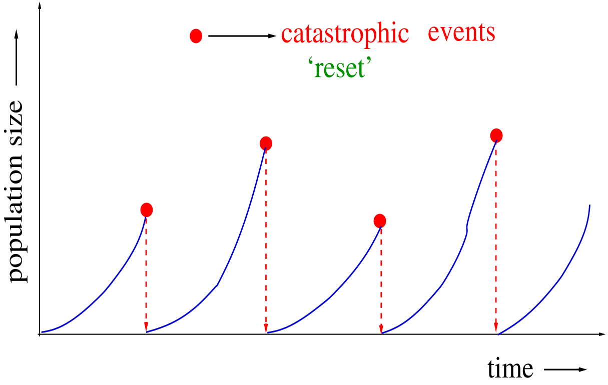
Extreme Events: rare but devastating

Population size gets reset by random catastrophes



population growth & reset $\Longrightarrow$ competing effects

Population size gets reset by random catastrophes



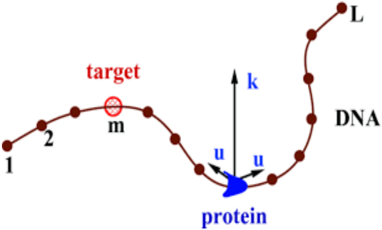
population growth & reset $\Longrightarrow$ competing effects

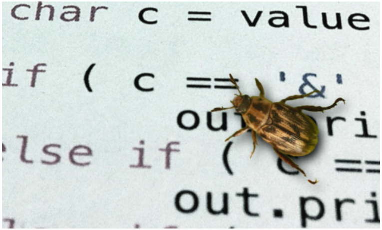
Q: Will the population size stabilize at long times **?**

Is there a **stationary state** at long time **?**

[Manrubia & Zanette, 1999]

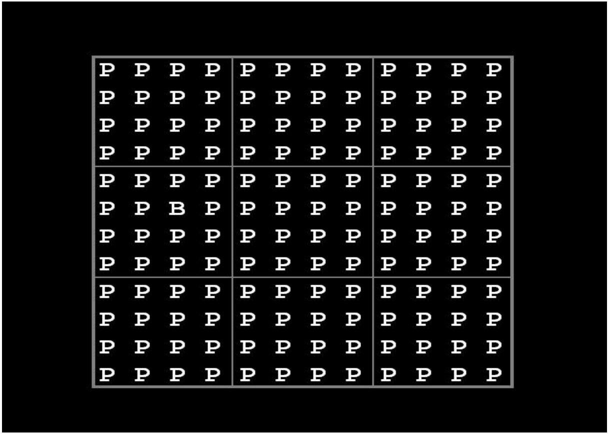
Search problems are ubiquitous





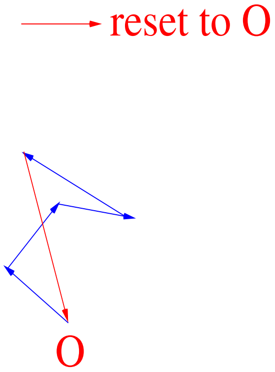
Visual search: a face in a crowd

Visual search in psychology



Search via diffusion and resetting

Schematic search trajectory



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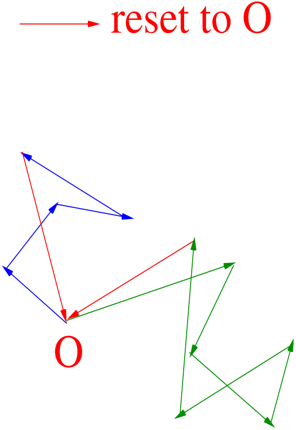
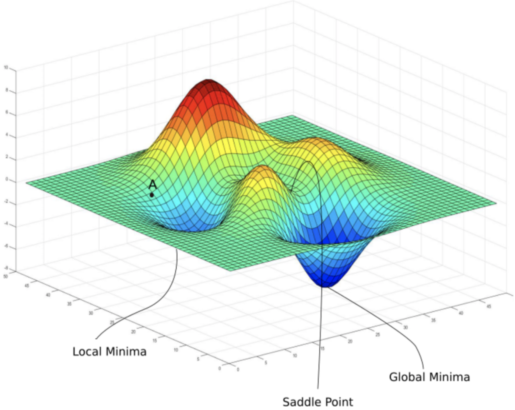


Other examples of stochastic resetting

- Searching for the global minimum in a complex energy landscape via simulated annealing

  empirical observation: Resetting to the initial configuration from time to time (and starting afresh) helps finding new pathways out of a metastable configuration



Random Search Problems

In the context of random search problems, a natural question thus emerges:

Q: Does stochastic resetting help in searching a target **?**

Does it really reduce the mean search time of a target **?**

Two principal issues concerning stochastic resetting

To summarize:

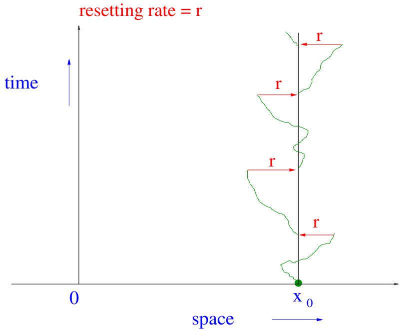
Two principal issues when stochastic resetting is switched on in a system evolving under its own natural dynamics:

- Does the system reach a **stationary state ?**
- Does stochastic resetting make a random search process **efficient ?**

Diffusion with stochastic resetting

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Diffusion with stochastic resetting: The model



Poissonian resetting

Time intervals between successive resettings distributed as:

$$p(\tau) = r\, e^{-r\tau}$$

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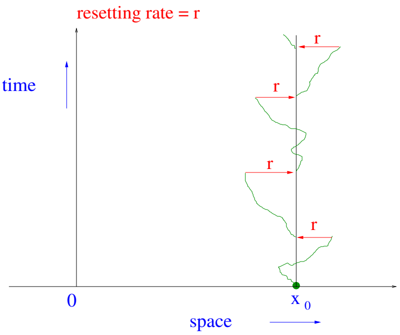

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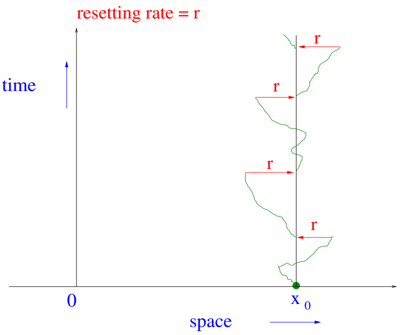
Dynamics: In a small time interval Δt

$$x(t+\Delta t) = x_0 \qquad \text{with prob. } r\Delta t \qquad \text{(resetting)}$$

$$= x(t) + \eta(t)\,\Delta t \qquad \text{with prob. } 1 - r\Delta t \qquad \text{(diffusion)}$$

Diffusion with stochastic resetting: The model



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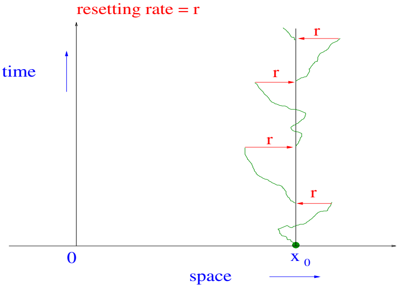
$\eta(t) \to$ Gaussian white noise: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = 2\,D\,\delta(t-t')$

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Prob. density $p_r(x,t)$ with resetting rate $r > 0$



$p_r(x,t) \rightarrow$ prob. density at time t,

given $p_r(x,0) = \delta(x - x_0)$

- In the absence of resetting ($r = 0$):

$$p_0(x,t) = \frac{1}{\sqrt{4\pi\, D\, t}}\, \exp[-(x - x_0)^2/4Dt]$$

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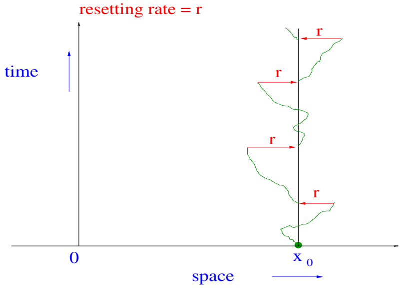

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- In the presence of resetting ($r > 0$):

$$p_r(x,t) = ?$$

Fokker-Planck (Master) Equation

Fokker-Planck Equation:

$$\boxed{\partial_t p_r(x,t) = D\,\partial_x^2 p_r(x,t) - r\,p_r(x,t) + r\,\delta(x-x_0)}$$

Initial Cond.: $p_r(x,0) = \delta(x-x_0)$

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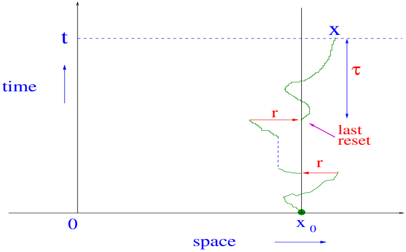
This linear equation can be solved at all t exactly by Fourier transform

Exact solution valid at all times t



- Exact solution at all times t:

$$p_r(x,t) = e^{-r\,t}\, p_0(x,t) + \int_0^t d\tau\, (r\, e^{-r\,\tau})\, p_0(x,\tau)$$

where $p_0(x,\tau) = \text{diffusion propagator} = \frac{1}{\sqrt{4\pi\, D\, \tau}}\, \exp[-(x-x_0)^2/4D\tau]$

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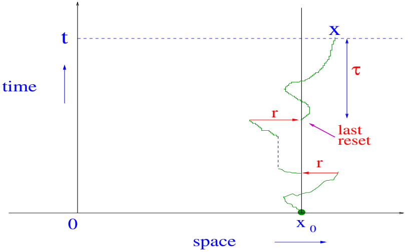

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Renewal interpretation: $\tau \to$ time since the last resetting during which

$\Longrightarrow$ free diffusion

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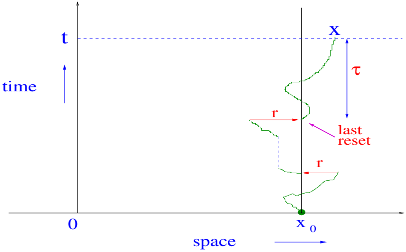

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Renewal interpretation: $\tau \to$ time since the last resetting during which $\Longrightarrow$ free diffusion

- As $t \to \infty$, $p_r^{\rm st}(x) = r\, \int_0^\infty p_0(x,\tau)\, e^{-r\,\tau}\, d\tau = \frac{\alpha_0}{2}\, \exp[-\alpha_0\, |x-x_0|]$ where $\alpha_0 = \sqrt{r/D}$

Stationary State

Exact solution $\rightarrow$ $\boxed{p_r^{\rm st}(x) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x-x_0|]}$ with $\alpha_0 = \sqrt{r/D}$

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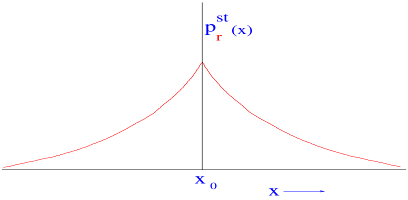

$\rightarrow$ nonequilibrium stationary state (NESS)

$\Rightarrow$ current carrying with detailed balance $\rightarrow$ violated

$p_r^{\mathrm{st}}(x) = \alpha_0\, \exp[-V_{\mathrm{eff}}(x)]$
effective potential: $\alpha_0 |x - x_0|$

Stationary State

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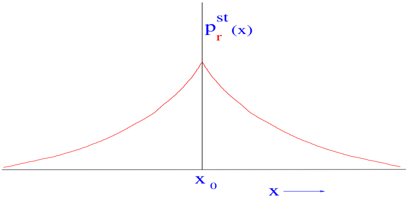

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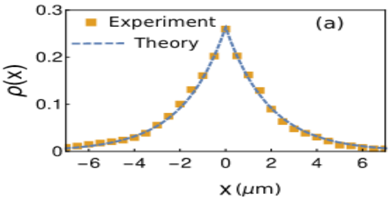
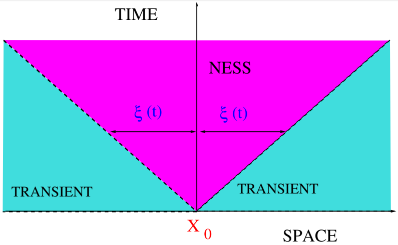


Experimental verificaion using holographic optical tweezers

Tal-Friedman, Pal, Sekhon, Reuveni, & Roichman
J. Phys. Chem. Lett. 11, 7350 (2020)

Unusual temporal relaxation

Dynamical phase transition

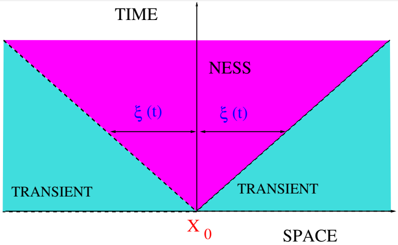


$$p_r(x,t) \sim \exp[-\alpha_0\,|x-x_0|] \quad \text{for } |x-x_0| \le \xi(t) \quad \text{(NESS)}$$

$$\sim \exp[-r\,t - |x-x_0|^2/4Dt] \quad \text{for } |x-x_0| \ge \xi(t) \quad \text{(TRANSIENT)}$$

where $\alpha_0 = \sqrt{r/D}$ and $\xi(\mathbf{t}) = \sqrt{\mathbf{4\,D\,r}}\ \mathbf{t} \Rightarrow$ growing length scale

Dynamical phase transition



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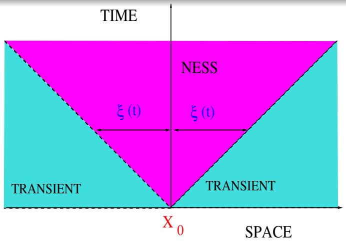
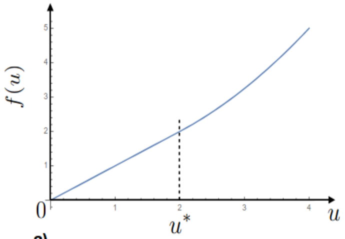
$\Longrightarrow$ NESS gets established on larger and larger length scales

Dynamical phase transition

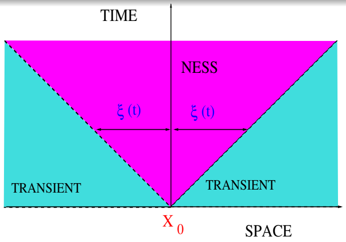


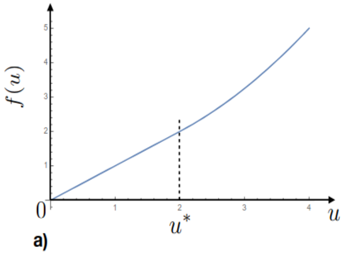


a)

Large deviation form: $$p_r(x,t) \sim \exp\left[-t\, f\left(\frac{|x-x_0|}{t}\right)\right]$$

Dynamical phase transition





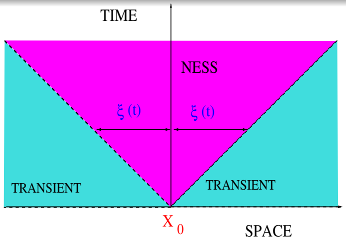
Large deviation form: $$p_r(x,t) \sim \exp\left[-t\, f\left(\frac{|x-x_0|}{t}\right)\right]$$

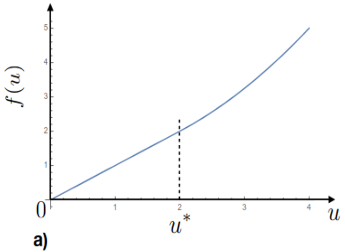
where the rate function

$$f(u) = \alpha_0\, |u| \qquad \text{for } |u| \le u^* = \sqrt{4Dr}$$

$$= r + u^2/4D \qquad \text{for } |u| \ge u^* = \sqrt{4Dr}$$

Dynamical phase transition





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second derivative $f''(u)$ is discontinuous at $u = u^*$

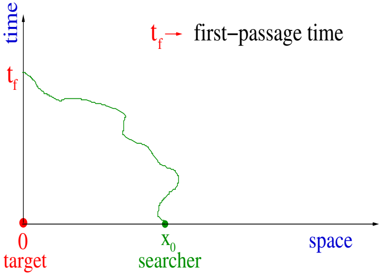
$\Longrightarrow$ 2-nd order dynamical phase transition

[S.M., S. Sabhapandit, G. Schehr, PRE, 91, 052131 (2015)]

Target Search: First-passage properties

Search of a fixed target via pure diffusion ($r = 0$)

Consider a 1-d Brownian walker, starting at x_0, searching for a fixed target at 0



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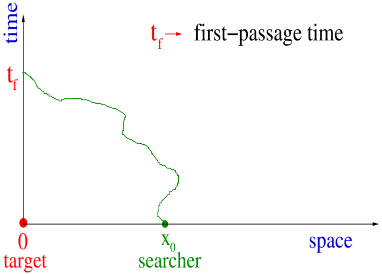

first-passage time t_f

$\Longrightarrow$ random variable

$F_0(t_f|x_0) \rightarrow$ Prob. distribution of t_f given x_0

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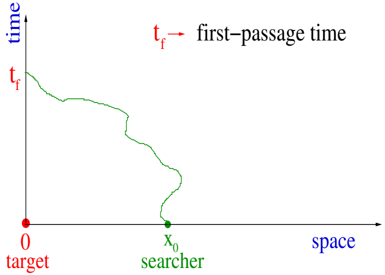

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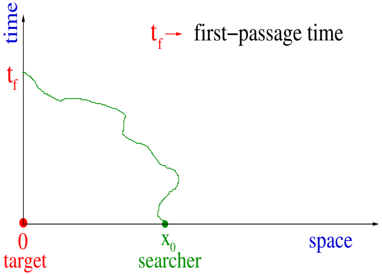

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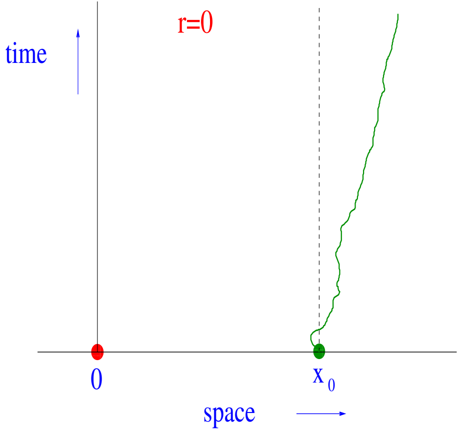
- Mean capture time $\rightarrow$ $\bar{T} = \int_0^\infty t_f \, F_0(t_f|x_0) \, dt_f = \infty$

Lévy '40, Chandrasekhar '43, Feller's book, Redner's book, ...

Diverging mean capture time for pure diffusion

The diverging mean capture time $\bar{T} \to \infty$ can traced back to trajectories that typically wander away in the direction opposite to that of the target



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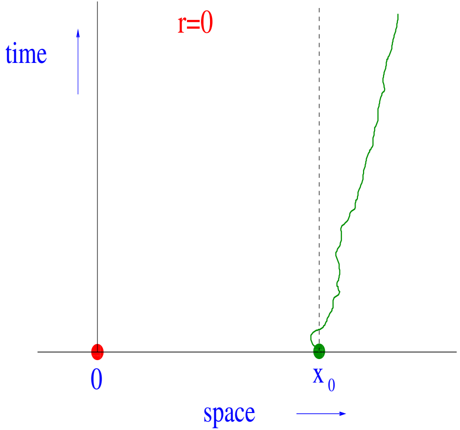
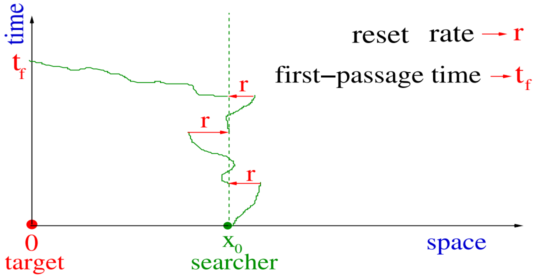


Such wandering trajectories get cut-off by resetting ($r > 0$)

Target search via diffusion with resetting ($r > 0$)



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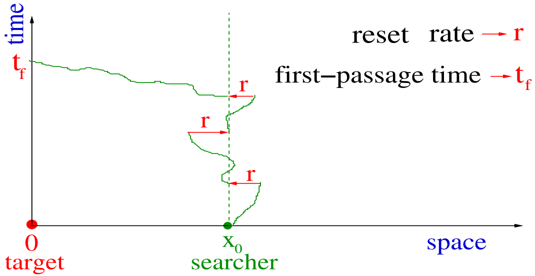
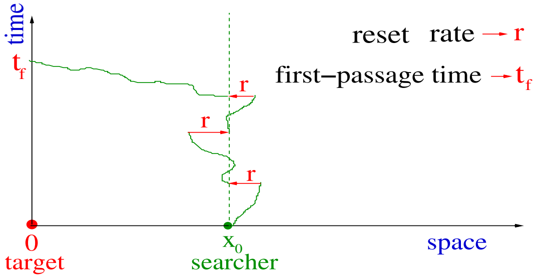


Exact result for the Mean capture time:

$$\bar{T} = \frac{1}{r}\left[\exp\left(\sqrt{r/D}\,x_0\right) - 1\right]$$

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Target search via diffusion with resetting ($r > 0$)



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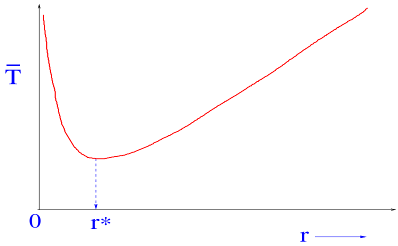
[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

$\Longrightarrow$ Mean capture time is ∞ for $r = 0$, but finite when $r > 0$

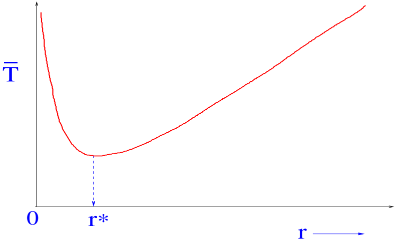
Optimal resetting rate



$$\bar{T}(r,x_0)=\frac{1}{r}\left[\exp\left(\sqrt{r/D}\,x_0\right)-1\right]$$

diverges as $r\to 0$ and $r\to\infty$

Optimal resetting rate



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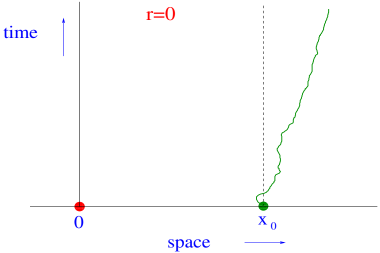
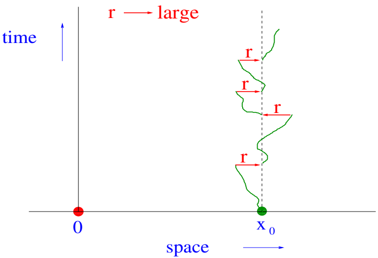


$\bar{T}(r \to 0) \to \infty$



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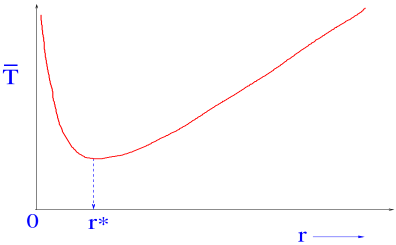

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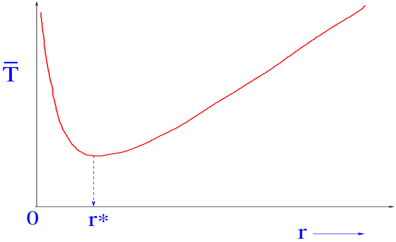

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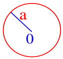
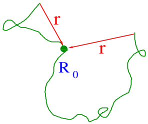
optimal resetting rate r^* is given by:

$$\boxed{r^* = \gamma^2 \frac{D}{x_0^2}} \text{ where } \boxed{\gamma - 2\left(1 - e^{-\gamma}\right) = 0} \Rightarrow \gamma = 1.59362\ldots$$

[M.R. Evans and S.M., Phys. Rev. Lett. 106, 160601 (2011)]

Target search via diffusion with resetting in $d > 1$





stationary target of radius a at 0 in $d > 2$

searcher starts at $R_0 > a$, diffuses, and resets with rate r

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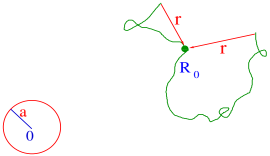

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$K_\nu(z) \longrightarrow$ modified Bessel function

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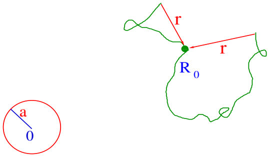

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- Once again, there is an optimal r^* that minimizes $\bar{T}(r, R_0)$ in all d

[M.R. Evans and S.M., J. Phys. A: Math. Theo. 47, 285001 (2014)]

Three important facts from the simple toy model

diffusion + **stochastic resetting**

$\Longrightarrow$

- Nonequilibrium **stationary** state (NESS)

- Unusual **temporal** relaxation

  $\Longrightarrow$ accompanied by a **dynamical phase transition**

- The existence of an **optimal** resetting rate r^*

  $\Longrightarrow$ renders a diffusive search **efficient**

Toy model $\Longrightarrow$ explosion of activities

- Enzymatic reactions in biology (Michaelis-Menten reaction)
- Diffusion in a confining potential/box
- Lévy flights, Lévy walks, fractional BM with resetting
- Space-time dependent resetting rate $r(x, t)$
- Search via nonequilibrium reset dynamics vs. equilibrium dynamics
- Resetting dynamics of extended systems
- Memory dependent reset
- Quantum dynamics with reset
- Active particles with reset
- Cost of resetting
- Optimization of random search algorithms
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... $\Longrightarrow$ a long list **!**

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Reviews: "Stochastic resetting and applications",

M.R. Evans, S.M., & G. Schehr, J. Phys. A. : Math. Theor. 53, 193001 (2020)

"The inspection paradox in stochastic resetting",

A. Pal, S. Kostinski & S. Reuveni, J. Phys. A. : Math. Theor. 55, 021001 (2022)

Experiments

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Theory of resetting $\Longrightarrow$ rapidly developing

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How about experiments?

Experiments

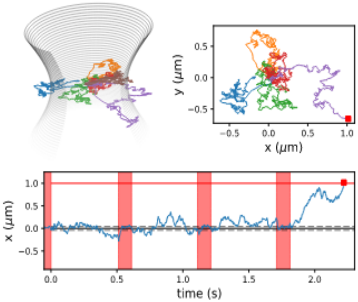
Theory of resetting $\Longrightarrow$ rapidly developing

How about experiments?

Experiments on target search via diffusion with resetting using optical traps set-up:

Tal-Friedman, Pal, Sekhon, Reuveni, Roichman, J. Phys. Chem. Lett. 11, 7350 (2020)

Besga, Bovon, Petrosyan, S.M., Ciliberto, Phys. Rev. Res. 2, 032029 (2020)

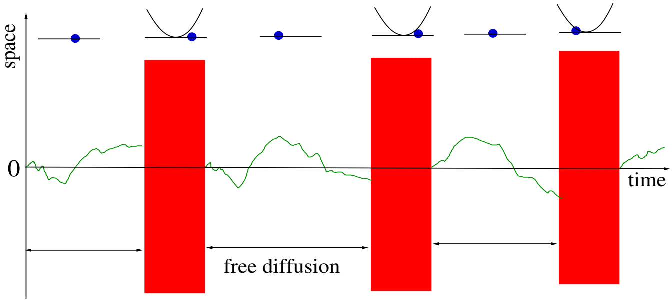


Experimental protocol for resetting

1. Free diffusion for an exponentially distributed period

2. Switch on an optical harmonic trap and the let the particle relax to its equilibrium distribution using Engineered Swift Equilibration (ESE) technique $\Longrightarrow$ mimics instantaneous resetting

Steps 1 and 2 alternate ...



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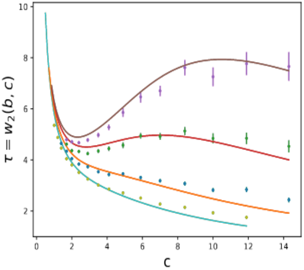
$\bar{T}(r, b = L/\sigma) \longrightarrow$ non-monotonic function of r for fixed $b = \frac{L}{\sigma} > b_c = 2.53..$

$\Longrightarrow$ spinodal transition at $b = b_c$

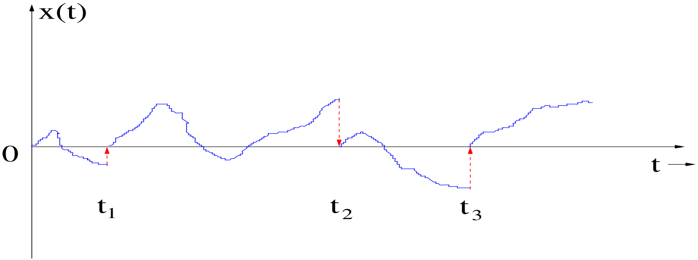


Besga, Bovon, Petrosyan, S.M. & Ciliberto
Phys. Rev. Res. 2, 032029 (2020)

Faisant, Besga, Petrosyan, Ciliberto & S.M.
J. Stat. Mech. 113203 (2021)

Generalisation to many-body systems

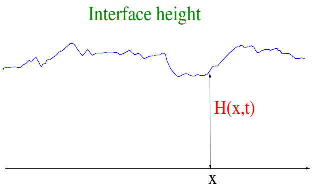
Stochastic Resetting in a nutshell

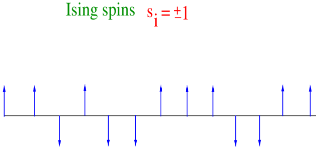


- Natural dynamics $\Longrightarrow$ deterministic/stochastic/classical/quantum
- Resetting at random times and then natural dynamics restarts afresh
- Interval between resettings $\Longrightarrow$ $p(\tau)$ independently

  $\Longrightarrow$ renewal process
- If $p(\tau) = r\, e^{-r\,\tau}$ $\Longrightarrow$ Poissonian resetting

Generalisation to many-body systems

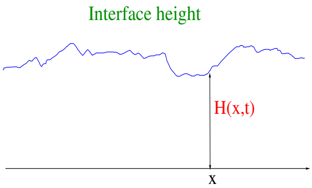


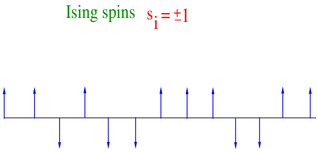


Any many-body system evolving under its own stochastic dynamics:

Ex: (i) fluctuating interface with EW/KPZ dynamics
(ii) Ising model with Glauber dynamics

Generalisation to many-body systems





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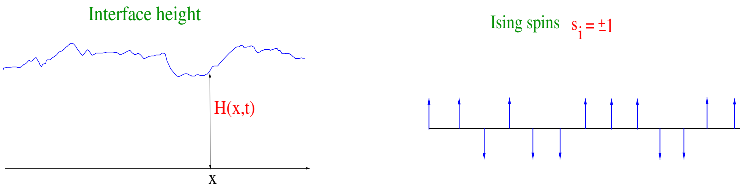

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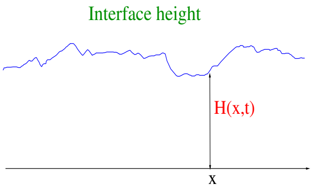

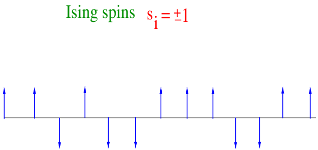

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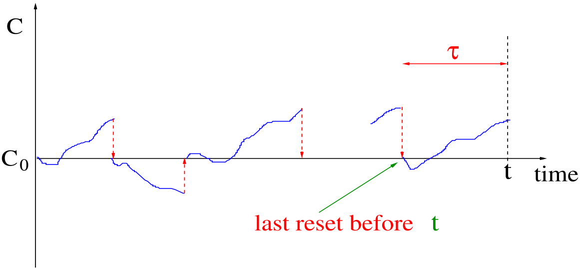
$P_r(C,t) \longrightarrow$ Prob. that the system is in config. C at time t **?**

General Renewal Equation



Renewal equation: Setting $\tau \to$ time since last resetting before t

$$P_r(C,t) = e^{-r\,t}\,P_0(C,t) + \int_0^t d\tau\,(r\,e^{-r\,\tau})\,P_0(C,\tau)$$

[S. Gupta, S.M., G. Schehr, PRL, 112, 220601 (2014)]

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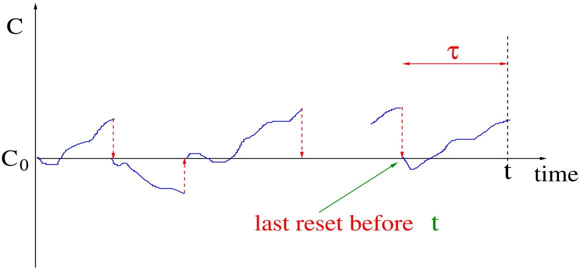

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As $t \to \infty$, the nonequilibrium stationary state:

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Nonequilibrium Stationary State

At long times, the system reaches a nonequilibrium stationary state

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$\Longrightarrow$ makes it **hard**

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Few cases where analytical progress can be made

Examples: Fluctuating interfaces, Exclusion processes, N independent Brownian motions, Ising model etc.

S. Gupta, S.M., G. Schehr, PRL, 112, 220601 (2014); U. Basu, A. Kundu, A. Pal, PRE, 100, 032136 (2019); M. Magoni, S.M., G. Schehr, PRR, 2, 033182 (2020)...

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Example: N **noninteracting** particles in a switching optical trap

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Exp. protocol for resetting using optical traps

1. Free diffusion of N noninteracting particles during an exponentially distributed period
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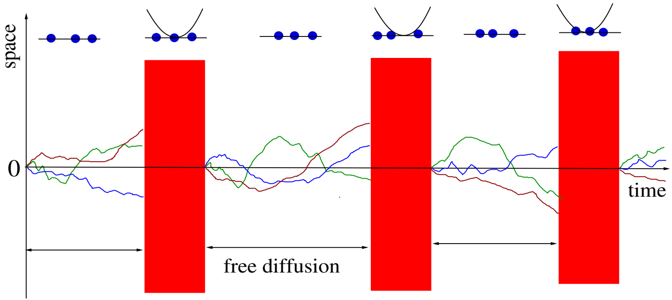
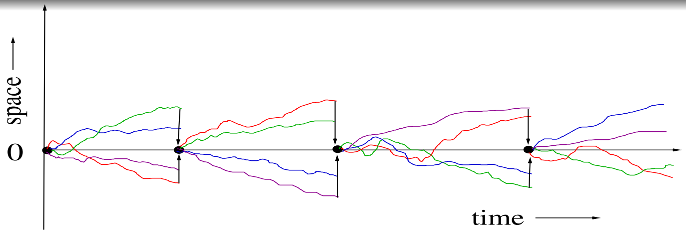
Steps 1 and 2 alternate ...



A simple model of Correlated resetting gas



Consider N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

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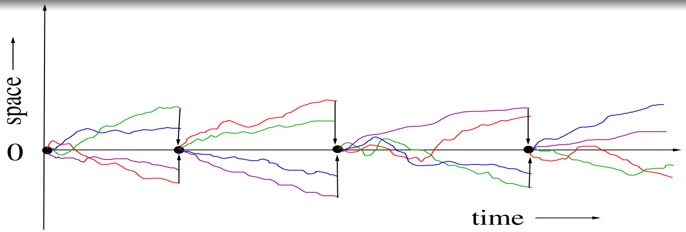

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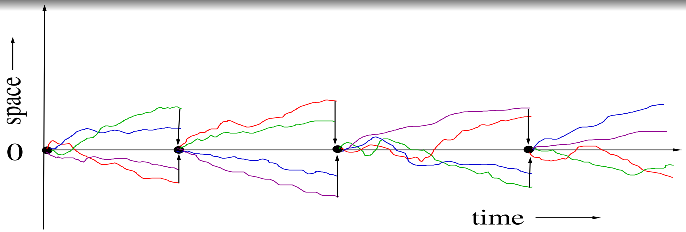

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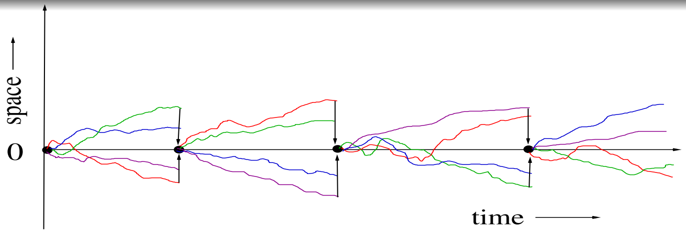

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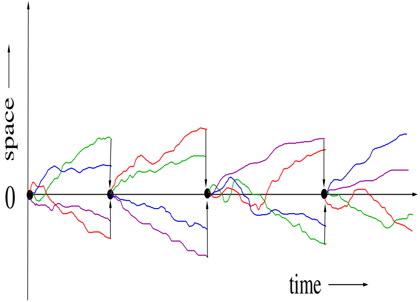
The joint distribution does not **factorize** $\Longrightarrow$ **correlated** resetting gas

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Solvable Correlated Gas



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In this model, interactions between particles are not built-in, but the correlations are generated by the dynamics (**simultaneous resetting**), that persist all the way to the stationary state

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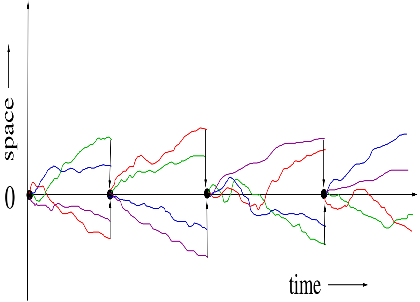

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The gas is **strongly** correlated in the NESS

$$\langle x_i^2 x_j^2\rangle - \langle x_i^2\rangle\langle x_j^2\rangle = 4\frac{D^2}{r^2} \Longrightarrow$$ **attractive all-to-all** interaction

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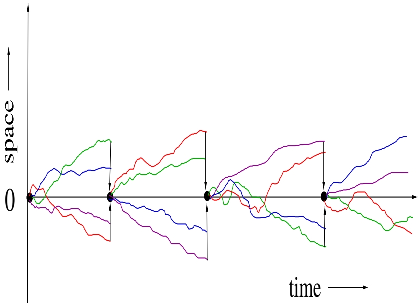

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The stationary joint distribution has a **CIID** structure $\Longrightarrow$ Solvable

$$P_r^{\rm st}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u) \prod_{i=1}^{N} p(x_i|u)$$

CIID $\Longrightarrow$ Conditionally Independent and Identically Distributed

Solvable Correlated Gas

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Despite **strong correlations**, several physical observables can be computed **exactly** in the NESS due to the **CIID** structure

- Compute any observable for the ideal gas $\Rightarrow$ I.I.D variables with distribution $p_0(x,\tau)$ parametrized by $\tau \Longrightarrow$ easy
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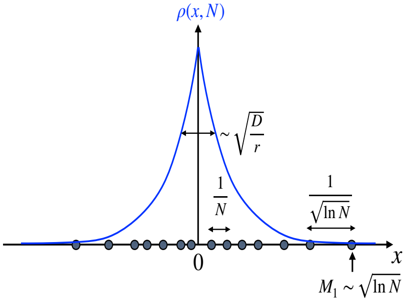
Examples:

- Average density
- Distribution of the k-th maximum: Order statistics
- Spacing distribution
- Full Counting Statistics

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Average Density



Joint distribution:

$$P_r^{\mathrm{st}}(\{x_i\}) = r\int_0^\infty d\tau\, e^{-r\tau}\prod_{i=1}^{N} p_0(x_i,\tau)$$

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Average density:

$$\rho(x,N) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x_i - x)\rangle = \int P_r^{\mathrm{st}}(x,\, x_2, \ldots, x_N)\, dx_2\, dx_3 \ldots dx_N$$

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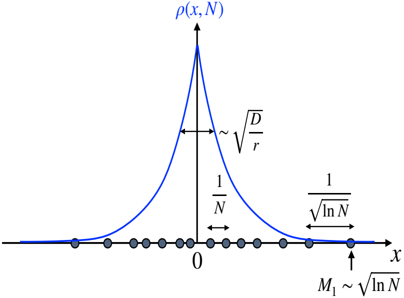

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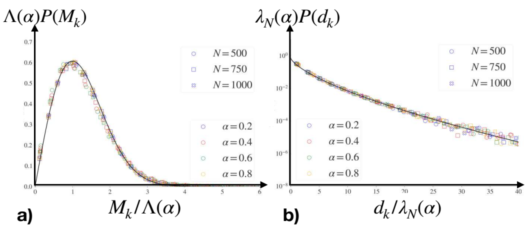
$$= r\int_0^\infty d\tau\, e^{-r\tau}\, p_0(x,\tau) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x|]$$

where $\alpha_0 = \sqrt{r/D}$

$\Longrightarrow$ same as the single particle position distribution

Extreme statistics and the gap/spacing distribution



- Global maximum (rightmost particle): $M_1 = \max\{x_1, x_2, \ldots, x_N\}$

$\mathrm{Prob.}[M_1 = w|N] \to \frac{1}{L_N} f\left(\frac{w}{L_N}\right)$ where $L_N = \sqrt{\frac{4\,D\,\ln N}{r}}$ and

$$\mathbf{f(z) = 2\,z\,e^{-z^2}} \text{ with } z \geq 0$$

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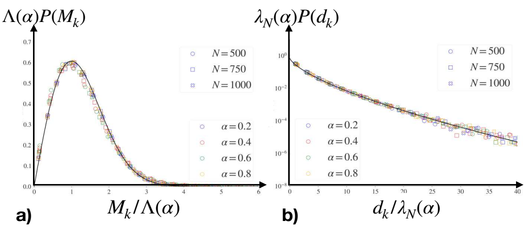

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$$\text{Prob.}[d_1 = g|N] \to \frac{1}{l_N} h\left(\frac{g}{l_N}\right) \text{ where } l_N = \sqrt{\frac{D}{r\,\ln N}} \text{ and}$$

$$\mathbf{h(z) = 2 \int_0^\infty du\, e^{-u^2 - z/u}} \text{ with } z \geq 0$$

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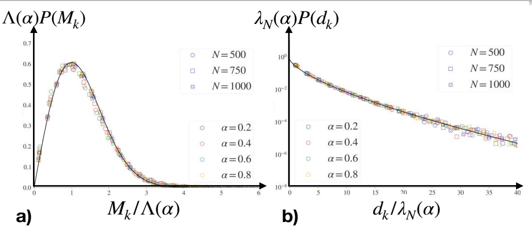

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Non-exponential tail: $\mathbf{h(z)} \sim \exp[-\mathbf{3}\,(\mathbf{z/2})^{\mathbf{2/3}}]$ as $z \to \infty$

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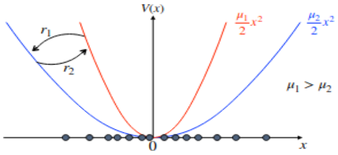
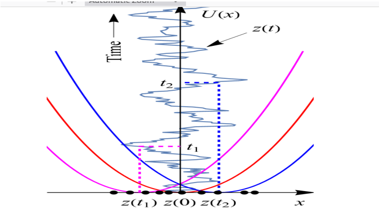
Exact stationary states for two other protocols





N noninteracting particles in a harmonic trap

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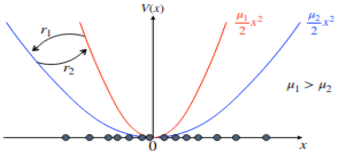

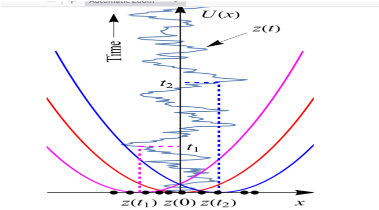

N noninteracting particles in a harmonic trap

(1) Protocol 1: Stiffness of the harmonic trap changes from $\mu_1 \to \mu_2$ with rate r_1 and $\mu_2 \to \mu_1$ with rate r_2

$\implies$ drives the system into a correlated NESS

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

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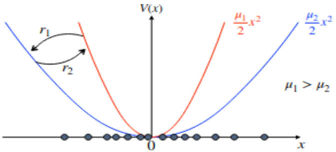

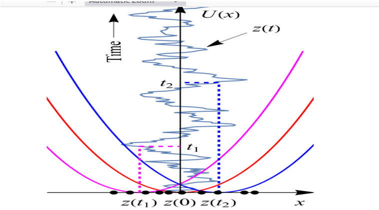

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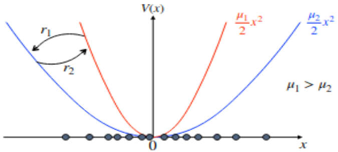
Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

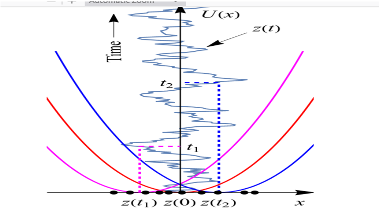
(2) Protocol 2: The center of the harmonic trap performs a stochastic motion

$\Longrightarrow$ drives the system into a correlated NESS

Sabhapandit, S.M., J. Phys. A: Math. Theor. 57, 335003 (2024)

Exact stationary states for two other protocols





In both protocols, the NESS has the CIID (conditionally independent and identically distributed) structure

$$P_{\rm st}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u) \prod_{i=1}^{N} p(x_i|u)$$

This CIID structure $\Longrightarrow$ **solvable** for various observables

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024); Sabhapandit, S.M., J. Phys. A: Math. Theor. 57, 335003 (2024); Kulkarni, S.M., Sabhapandit, arXiv: 2407.20342

Summary and Conclusions

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Recent directions

- Geometrical properties of the territory covered by a resetting Brownian motion

  Span, Convex hull, no. of distinct sites visited etc.

  S.M., F. Mori, H. Schawe, G. Schehr, PRE, 103, 022135 (2021); M. Biroli, F. Mori, S.M., J. Phys. A 55, 244001 (2022)

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- First-passage resetting

  De Bruyne, Randon-Furling, Redner (2020)

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- Resetting in classical **many-body** systems

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- Resetting in Quantum **many-body** systems

  Mukherjee, Sengupta, S.M. (2018); Rose, Touchette, Lesanovsky, Garrahan (2018); Perfetto, Carollo, Magoni & Lesanovsky (2021); Yin, Barkai (2022); Dubey, Chetrite, Dhar (2022); Turkeshi, Dalmonte, Fazio, Schiro (2022); Kulkarni, S.M. (2023), Kulkarni, S.M. & Sabhapandit (2024) . . .

  $\Longrightarrow$ **An optimally chosen resetting rate enhances quantum entanglement between qubits**

Ongoing collaboration between LPTMS, Munich, Tel-Aviv and ICTS (Bangalore) using IBM quantum computer

Stochastic Resetting $\rightarrow$ rich and interesting static/dynamic phenomena

Stochastic Resetting: Theory and Applications

In Celebration of the 10th Anniversary of 'Diffusion with Stochastic Resetting'

Guest Editors

Anupam Kundu *International Centre for Theoretical Sciences, Bengaluru, India*
Shlomi Reuveni *Tel Aviv University, Israel*

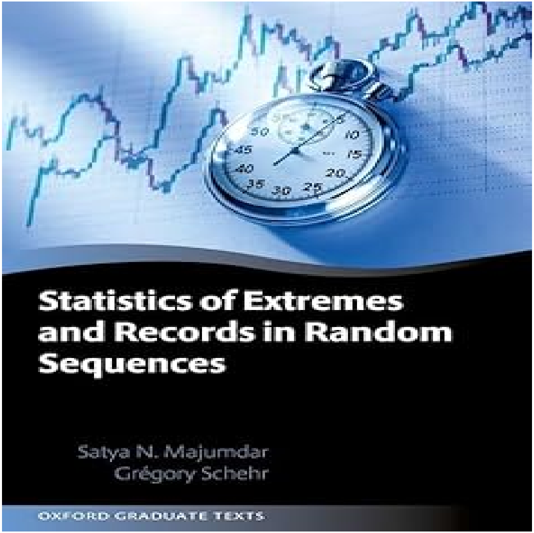
Scope

Restart is a simple and natural mechanism that has emerged as an overreaching topic in physics, chemistry, biology, ecology, engineering and economics. Since the inaugural work of Evans and Majumdar (Evans M R and Majumdar S N 2011, *Phys. Rev. Lett.* **106**, 160601) a substantial amount of research has been carried out on stochastic resetting and its applications. This work spans different contexts starting from first-passage and search theory, stochastic thermodynamics, optimization theory, and all the way to quantum mechanics. Further connections have been made to animal foraging, protein-DNA interactions, coagulation-diffusion processes, chemical reaction processes, as well as to stock-market and population dynamics which display colossal crashes, i.e., resetting events. While most studies to date have been theoretical, several experimental groups have now entered the playing field, which marks the dawn of a new era.

The goal of this issue is to report new and original advancements made on stochastic resetting and applications, to open novel research directions and to attract additional researchers to work in the exciting field of stochastic resetting.

Collaborators

- M. Biroli, I. Burenev, B. De Bruyne, C. Di Bello, M. Gueneau, M. Magoni, F. Mori (Master & Ph. D students, LPTMS & LPTHE); **F. Mori** ($\Rightarrow$ Oxford, UK)
- D. Boyer, H. Larralde, A. Falcon-Cortes, G. Marcado-Vasquez (UNAM, Mexico)
- S. Ciliberto & group (ENS-Lyon, France)
- F. den Hollander (Leiden University, The Netherlands)
- M. R. Evans, J. Whitehouse, R. A. Blythe, J. Sunil (Edinburgh University, UK)
- L. Giuggioli (Bristol, UK)
- A. K. Hartmann (Oldenburg Univ,, Germany)
- M. Kulkarni, A. Kundu (ICTS, Bangalore), S. Gupta (TIFR, Mumbai)
- L. Kusmierz (Inst. of Phys., Krakow, Poland $\rightarrow$ Riken Center, Japan)
- K. Mallick (IPHT, Saclay, France)
- J. M. Meylahn, H. Touchette (Stellenbosch University, South Africa)
- B. Mukherjee, K. Sengupta (IACS, Kolkata, India)
- G. Oshanin (Sorbonne Université, Paris, France)
- S. Sabhapandit (RRI, Bangalore, India)
- H. Schawe (Cergy-Pontoise, France)
- G. Schehr (LPTHE, Sorbonne University, France)
- G. Tucci & A. Gambassi (SISSA, Trieste, Italy)
- N. R. Smith (Ben-Gurion Univ., Israel)



S. N. Majumdar and G. Schehr (Oxford University Press, 2024)

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