Measurement induced dynamics of a qubit — emergent resetting dynamics

Abhishek Dhar
International centre for theoretical sciences (ICTS-TIFR), Bangalore

Varun Dubey (ICTS-TIFR),

Raphael Chetrite (Université Côte d'Azur, Nice)

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Outline of talk

- MOTIVATION: Consider the dynamics of a quantum system whose unitary dynamics is interrupted by a sequence of projective measurements. Outcome of measurements is probabilistic, hence we obtain a stochastic dynamics — Quantum Trajectories.
- We discuss a simple example: Two qubits, System (S) and Detector (D); evolve unitarily for time τ; make measurement on D-qubit and record the outcome; Study the effective stochastic dynamics of the S-qubit wavefunction and the record statistics.
- The wavefunction dynamics of the S-qubit is given by a drift-jump process for a single angle variable — can also be thought of as a resetting process!
- Exact results for steady state, time evolution, spectrum of Fokker-Planck operator, and the counting statistics.

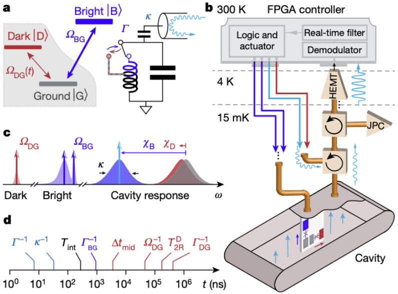
Quantum Trajectories

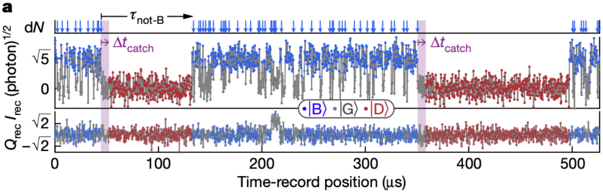
To catch and reverse a quantum jump mid-flight

Z. K. Minev[1,5]*, S. O. Mundhada[1], S. Shankar[1], P. Reinhold[1], R. Gutiérrez-Jáuregui[2], R. J. Schoelkopf[1], M. Mirrahimi[3,4], H. J. Carmichael[2] & M. H. Devoret[1]*

In quantum physics, measurements can fundamentally yield discrete and random results. Emblematic of this feature is Bohr's 1913 proposal of quantum jumps between two discrete energy levels of an atom[1]. Experimentally, quantum jumps were first observed in an atomic ion driven by a weak deterministic force while under strong continuous energy measurement[2–4]. The times at which the discontinuous jump transitions occur are reputed to be fundamentally unpredictable. Despite the non-deterministic character of quantum physics, is it possible to know if a quantum jump is about to occur? Here we answer this question affirmatively: we experimentally demonstrate that the jump from the ground state to an excited state of a superconducting artificial three-level atom can be tracked as it follows a predictable 'flight', by monitoring the population of an auxiliary energy level coupled to the ground state. The experimental results demonstrate that the evolution of each completed jump is continuous, coherent and deterministic. We exploit these features, using real-time monitoring and feedback, to catch and reverse quantum jumps mid-flight—thus deterministically preventing their completion. Our findings, which agree with theoretical predictions essentially without adjustable parameters, support the modern quantum trajectory theory[5–9] and should provide new ground for the exploration of real-time intervention techniques in the control of quantum systems, such as the early detection of error syndromes in quantum error correction.

First, we developed a superconducting artificial atom with the necessary V-shaped level structure (see Fig. 1a and Methods). It consists, besides the ground level |G⟩, of one protected, dark level |D⟩—engineered to couple only minimally to any dissipative environment or any measurement apparatus—and one ancilla level |B⟩, whose occupation is monitored at rate Γ. Quantum jumps between |G⟩ and |D⟩ are induced by a weak Rabi drive Ω_{DG}—although this drive can eventually be turned off during the jump, as explained later. Because a direct measurement of the dark level is not feasible nor desired, the jumps are monitored using the Dehmelt shelving scheme[2]. Thus, the occupation of |G⟩ is linked to that of |B⟩ by the strong Rabi drive Ω_{BG} ($\Omega_{DG} \ll \Omega_{BG} \ll \Gamma$). In the atomic physics shelving scheme[2–4], an excitation to |B⟩ is recorded by detecting the emitted photons from |B⟩ with a photodetector. From the detection events—referred to in the following as 'clicks'—one infers the occupation of |G⟩. On the other hand, from the prolonged absence of clicks (to be defined precisely below; see also Supplementary Information section II), one infers that a quantum jump from |G⟩ to |D⟩ has occurred. Owing to the poor collection efficiency and the dead time of photon counters in atomic physics[29], it is exceedingly difficult to detect every individual click required to faithfully register the origin in time of the advance warning signal. However, superconducting systems present the advantage of high collection efficiencies[30–32], as their microwave photons are emitted into one-dimensional waveguides and are detected with the same detection efficiencies as optical photons. Furthermore, rather than monitoring the direct





Progressive field-state collapse and quantum non-demolition photon counting

Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Clément Sayrin[1], Sébastien Gleyzes[1], Stefan Kuhr[1]†, Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

The irreversible evolution of a microscopic system under measurement is a central feature of quantum theory. From an initial state generally exhibiting quantum uncertainty in the measured observable, the system is projected into a state in which this observable becomes precisely known. Its value is random, with a probability determined by the initial system's state. The evolution induced by measurement (known as 'state collapse') can be progressive, accumulating the effects of elementary state changes. Here we report the observation of such a step-by-step collapse by non-destructively measuring the photon number of a field stored in a cavity. Atoms behaving as microscopic clocks cross the cavity successively. By measuring the light-induced alterations of the clock rate, information is progressively extracted, until the initially uncertain photon number converges to an integer. The suppression of the photon number spread is demonstrated by correlations between repeated measurements. The procedure illustrates all the postulates of quantum measurement (state collapse, statistical results and repeatability) and should facilitate studies of non-classical fields trapped in cavities.

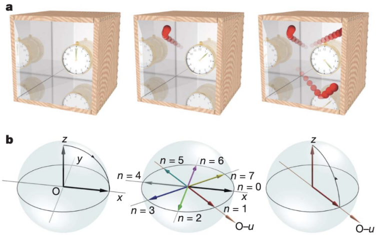


Figure 1 | Principle of QND photon counting. a, Thought experiment with a clock in a box containing n photons. The hand of the clock undergoes a $\pi/4$ phase-advance per photon ($n = 0, 1, 3$ represented). **b,** Evolution of the atomic spin on the Bloch sphere in a real experiment: an initial pulse R_1 rotates the spin from O–z to O–x (left). Light shift produces a $\pi/4$ phase shift per photon of the spin's precession in the equatorial plane. Directions associated with $n = 0$ to 7 end up regularly distributed over 360° (centre). Pulse R_2 maps the direction O–u onto O–z, before the atomic state is read out (right).

Quantum Trajectories — measurement induced phase transitions

- Measurement induced phase transitions: many body quantum systems show a transition from volume-law entanglement entropy to area-law entropy with increase in the rate of measurements.
- Entanglement has to be measured for pure states on individual trajectories . Very difficult from the experimental point of view.
- What do measurement records tell us about such transitions?
- Ask this question for few-body systems.

General setup of two qubits

1. Two qubits, System (S) and Detector (D).

$$\text{initial state} \quad |\Psi(0)\rangle = |\psi(0)\rangle^{(S)} \otimes |0\rangle^{(D)} = \left(a|0\rangle^{(S)} + b|1\rangle^{(S)}\right) \otimes |0\rangle^{(D)}$$

2. Evolve unitarily for time τ; S and D get entangled.

$$\begin{aligned}|\Psi(\tau)\rangle &= U_\tau|\Psi(0)\rangle = a_{00}|00\rangle + a_{10}|10\rangle + a_{01}|01\rangle + a_{11}|11\rangle \\ &= (a_{00}|0\rangle + a_{10}|1\rangle) \otimes |0\rangle^{(D)} + (a_{01}|0\rangle + a_{11}|1\rangle) \otimes |1\rangle^{(D)}.\end{aligned}$$

3. Make measurement on D (e.g. $\sigma_z^{(\mathrm{D})}$). Outcomes:

$$\begin{aligned}|0\rangle^D \quad &\text{implies } |\psi\rangle = a_{00}|0\rangle + a_{10}|1\rangle \quad \text{with prob. } \langle\psi|\psi\rangle = |a_{00}|^2 + |a_{10}|^2 \\ |1\rangle^D \quad &\text{implies } \quad\;\; = a_{01}|0\rangle + a_{11}|1\rangle \quad \text{with prob. } \langle\psi|\psi\rangle = |a_{00}|^2 + |a_{10}|^2.\end{aligned}$$

4. Thus we have a stochastic evolution to a new state

$$\begin{bmatrix} a \\ b \end{bmatrix} \to \begin{bmatrix} a' \\ b' \end{bmatrix}$$

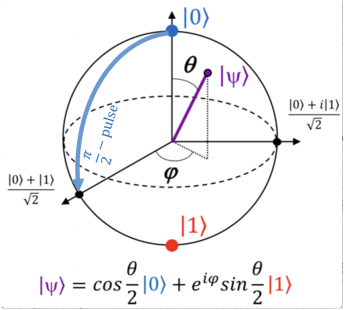
5. Reset D-qubit to state $|0\rangle^D$ —- REPEAT the above steps.

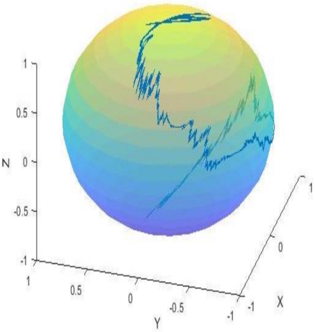
General setup

1. Bloch-Sphere representation:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \end{bmatrix}$$

2. A stochastic trajectory on the Bloch sphere





Specific model: Snizhko, Kumar, Romito (2020) Roy, Chalker, Gornyi, Gefen (2020)

- Unitary evolution of 2-qubit system is given by Hamiltonian:

$$H = \gamma_0 \sigma_x \otimes I + \sqrt{\frac{\gamma}{\tau}} \pi_1 \otimes \sigma_y, \qquad \pi_1 = (1 - \sigma_z)/2$$
$$U_\tau = e^{-iH\tau}.$$

- Consider continuous time limit $\tau \to 0$.

- Dynamics is confined to $y - z$ plane of Bloch-sphere
  $\implies$ single parameter, θ, characterizes the quantum state.

- Stochastic dynamics is given by the following drift-jump equation

$$d\theta_t = \Omega(\theta_t)\, dt + \left(\pi - \theta_{t\_}\right) dN_t, \quad \mathbb{E}[dN_t] = \alpha(\theta_t)\, dt = \gamma \sin^2 \frac{\theta_t}{2}\, dt.$$
$$\Omega(\theta) = -2\gamma_0 [1 + \lambda \sin\theta]. \quad \lambda = \frac{\gamma}{4\gamma_0}.$$

N_t is a counting process with θ-dependent rate function α_t.

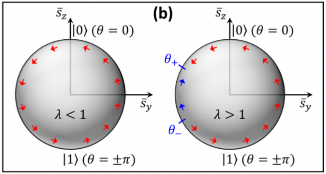
Quantum Trajectories

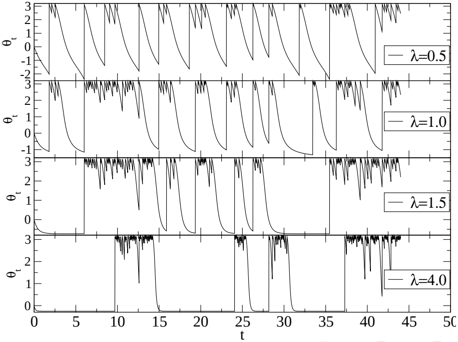
Equivalently, the dynamics is given by:

$$\theta_{t+dt} = \begin{cases} \theta_t + \Omega(\theta_t)\, dt & \text{with prob. } 1-\alpha(\theta_t)dt \\ \pi & \text{with prob. } \alpha(\theta_t)dt \end{cases} .$$

$$\Omega(\theta) = -2\gamma_0\,[1+\lambda \sin\theta]$$

- Single parameter, $\lambda = \gamma/(4\gamma_0)$, controls the measurement strength.
- Drift + position-dependent RESET.



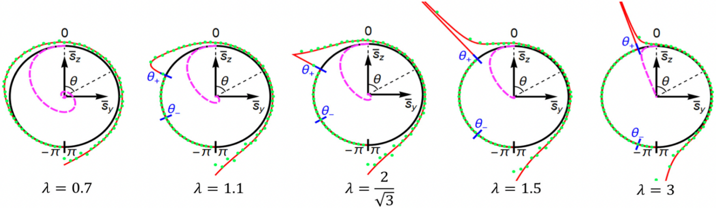


- Master Equation:

$$\frac{\partial P(\theta,t)}{\partial t} = -\frac{\partial}{\partial \theta}[\Omega(\theta)P(\theta,t)] - \gamma \sin^2(\theta/2)P(\theta,t) + \gamma\delta(\theta-\pi)\int_0^{2\pi} \sin^2(\theta'/2)P(\theta',t)\,d\theta'.$$

$$\Omega(\theta) = -2\gamma_0\left[1+\lambda\sin\theta\right]$$

- Exact steady state distribution: interesting transitions



- Exact expression for survival probability [Prob. of no resets]: $S(t) = e^{-\int_0^t dt' \alpha(\theta(t'))}$, starting from $|0\rangle$. Transition at $\lambda = 1$, from oscillatory to pure exponential decay.
- Few eigenfunctions of the Fokker-Planck operator.

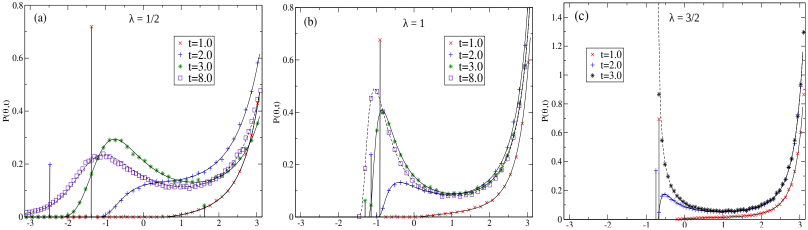
Space-dependent resetting — Use approaches from:
Majumdar, Evans (PRL, 2011); Majumdar, Evans (J. Phys. A , 2011)
Pal, Kundu, Evans (J. Phys. A, 2016)
Roldan, Gupta (PRE, 2017).

- Exact solution for time evolution of $P(\theta, t)$ by solution of the renewal equation:

$$P(\theta,t) = P_0^t[0]\delta(\theta - \theta_t(0,0)) + \int_0^t \overline{\alpha}_{t-\tau} P_0^\tau[0||\pi]\,\delta(\theta - \theta_\tau(0,\pi))\,d\tau,$$

where $\overline{\alpha}_{t-\tau}$ is the mean transition rate:

$$\overline{\alpha}_t = \int_0^{2\pi} \gamma \sin^2\left(\frac{\theta}{2}\right)\, P(\theta,t)\, d\theta.$$



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Record statistics

- Full spectrum obtained for $\lambda \leq 1$. Time evolution in terms of spectral expansion.
- Counting statistics: The probability of registering exactly n counts in the interval $(0, t]$ is given by

$$P_0^t[n] = \int_0^t dt_n \int_0^{t_n} dt_{n-1} \cdots \int_0^{t_2} dt_1 \, p_0^t[t_1, \ldots, t_n].$$

- Can obtain exact expression for Laplace-Generating function

$$Z(s, \sigma) = \int_0^\infty dt e^{-\sigma t} \sum_{n=0}^{\infty} e^{-ns} P_0^t[n]$$

.

- All cumulants of N scale linearly with time t (at large times).

Click statistics

- Explicit formula for mean number of clicks:

$$\langle N_t \rangle = \begin{cases} 2\lambda\gamma_0 t + \lambda^2 \left[-1 + e^{-\lambda\gamma_0 t} \frac{\sin(\omega t+\varphi)}{\sin\varphi} \right] & 0 \le \lambda < 2, \\ 2\lambda\gamma_0 t + \lambda^2 \left[-1 + e^{-\lambda\gamma_0 t} \frac{\sinh(\omega' t+\varphi')}{\sinh\varphi'} \right] & \lambda > 2, \\ 4\left(-1 + \gamma_0 t + e^{-2\gamma_0 t}(1+\gamma_0 t)\right) & \lambda = 2, \end{cases}$$

- Probability of no clicks — survival probability

$$S(t,\lambda) = \frac{e^{-\gamma t}/2}{\beta^2} \left(\sin^2(\beta\gamma_0 t) + \sin^2(\beta\gamma_0 t + \phi) \right) \quad \text{for } 0 \le \lambda < 1$$

$$= \frac{e^{-\gamma 2t}}{\beta'^2} \left(\sinh^2(\beta'\gamma_0 t) + \sinh^2(\beta'\gamma_0 t + \phi') \right) \quad \text{for } \lambda > 1$$

$$= e^{-\gamma 2t} \left((\gamma_0 t)^2 + (1+\gamma_0 t)^2 \right) \text{ for } \lambda = 1$$

- The clicks are the only experimental observables. Their statistics allow us to see various transitions of the system dynamics.

Conclusions

- We studied the dynamics of a qubit that is continuously monitored via measurements on a detector qubit with which it interacts strongly so as to avoid the zeno limit.
- For the case considered here, the qubit state remains confined at all times on the *yz* plane of the Bloch sphere so that it can represented by a single angle variable. The state $|\psi(t)\rangle$ follows a stochastic dynamics with drift and jump terms.
- The stochastic wavefunction dynamics can be naturally interpreted as a resetting process, with a resetting rate that depends on the instantaneous state. The strength of the resetting rate λ quantifies the strength of measurements.
  [Quantum resetting: B Mukherjee, K Sengupta, SN Majumdar (PRB, 2018)]
  Measurement dynamics leads to many possibilities: Drift+Jump+Diffusion
- Exact results on the number of resetting events, N_t, in a specified time t. The no-click probabilty shows a transition at $\lambda = 1$. [this is the variable accessible to experiments].
- Solving the renewal equation, we obtain the exact form of the probability distribution $P(\theta, t)$ for the system to be in the quantum state, $|\theta\rangle = \begin{bmatrix} \cos(\theta/2) \\ i\sin(\theta/2) \end{bmatrix}$, at time t.
- Average density matrix of the qubit, $\hat{\rho}(t) = \int_0^{2\pi} d\theta P(\theta, t)|\theta\rangle\langle\theta|$, satisfies a Lindblad equation and has much less information about the dynamics. The stochastic dynamics is a “unraveling” of the Lindblad.