Numerical studies of inertial modes in the Sun using Dedalus

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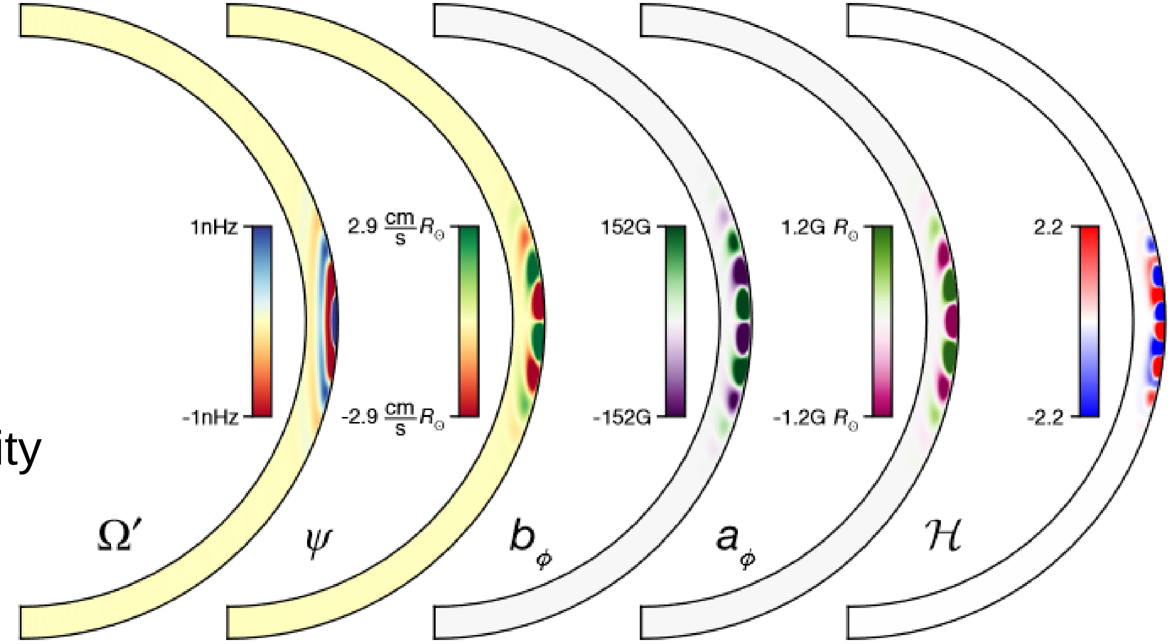
Eigenvalue problem using the Dedalus code

- **Open source**, **Python**, MPI-parallelized code (*Burns+ Phy. Rev. Res, 2020*).
- **Spectral** code: uses various basis like Fourier, Chebyshev, etc.
- Can **flexibly** solve for **various geometries**: shell, sphere, ball, annuli, and cartesian.
- Many examples: dedalus-project.org

Vasil+ Nature, 2024

Alternative solar dynamo model using near-surface instability



Previous theoretical studies of solar inertial modes

Linear studies:

Literature	Model	Stratification	Rotation	Buoyancy effects	Latitudinal Entropy gradient	Deviation from adiabaticity
Bekki+ A&A, 2022	Compressible	Solar-like	Solar differential/ uniform rotation	Included	From Thermal wind balance	Different profiles tested
Bhattacharya+ ApJ, 2024	Anelastic $\nabla \cdot (\rho_0 \mathbf{u}) = 0$	Solar-like	Solar differential/ uniform rotation	Included	Absent	Analytical model (fixed)
Jain+ ApJ, 2024	Anelastic (cylindrical)	Stratified	Uniform rotation only	Included	Absent	Discussed (not used)
Triana+ ApJ, 2022	Incompressible $\nabla \cdot \mathbf{u} = 0$	Uniform (unstratified)	Uniform rotation only	Not Boussinesq	Absent	None
Fournier+ A&A, 2022	Incompressible (surface)	None	Solar differential/ uniform rotation	Not Boussinesq	Absent	None

Non-linear simulations: *Bekki+ A&A, 2022, Bekki+ Sci. Adv. 2024, Blume+ ApJ, 2024*

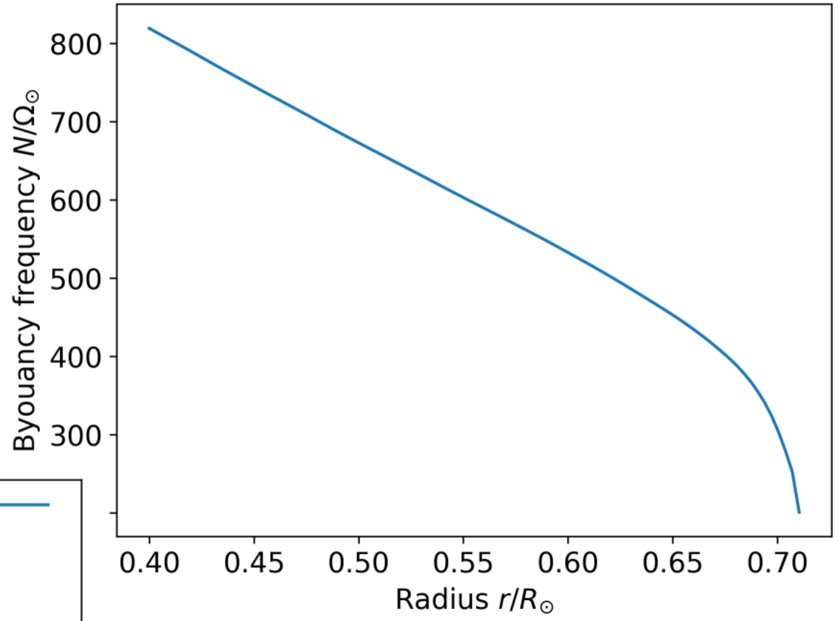
Issues with modeling solar inertial modes

- Effects of extending the **domain** have not been tested. How are they affected by radiative interior or near-surface shear layers? → **Inclusion of Radiative Interior**
- **Different** model setups have been used, but the effects of their simplifying **assumptions** have not been tested. → **Comparison of the 3 Models**

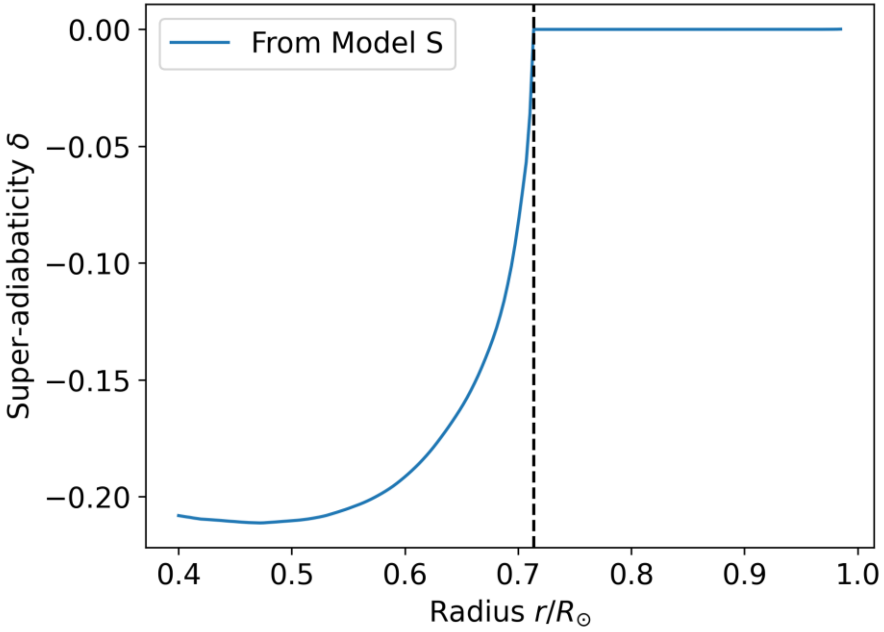
Coupling of the inertial modes with the radiative interior

Note: We ignore differential rotation for simplicity

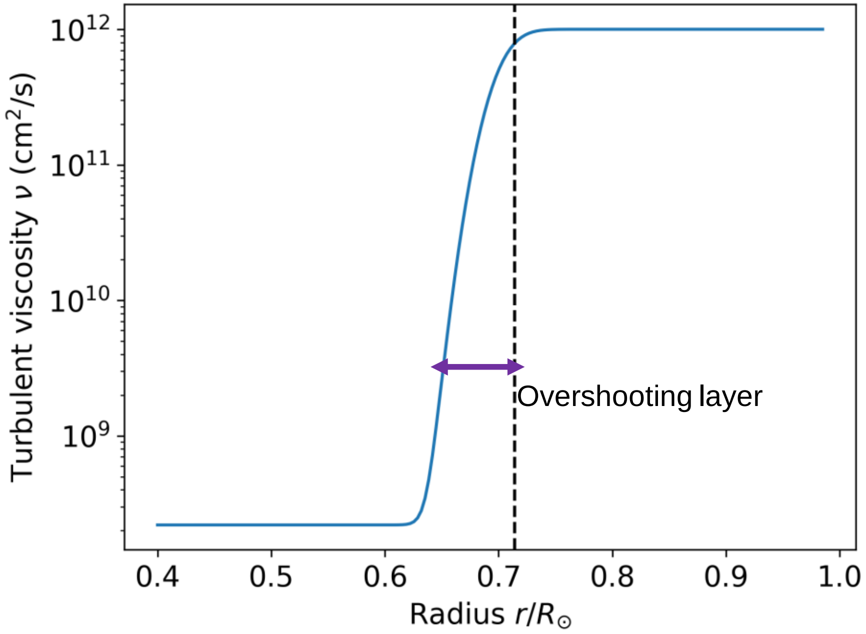
What are the differences in the radiative interior?



Convectively stable
(*Christensen-Dalsgaard+ Sci, 1996*)



Less turbulent motions



$$N = \sqrt{-\frac{g\,\delta}{H_p}}$$

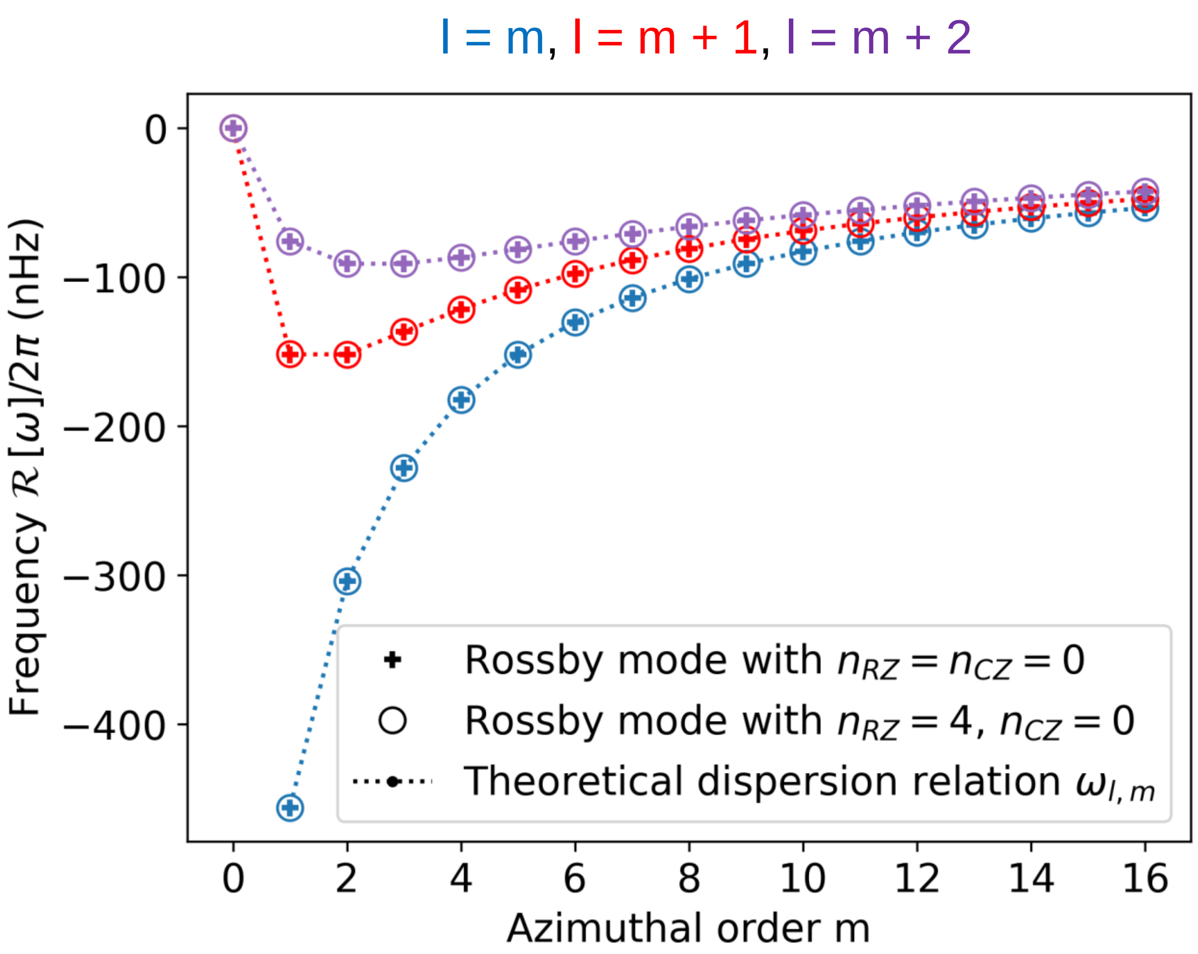
$$\delta \equiv \frac{H_p}{T}\left[\left(\frac{dT}{dr}\right)_{\rm ad} - \frac{dT}{dr}\right]$$

Rossby modes in setup including radiative zone

- Both sectoral (l = m) and **non-sectoral** (l ≠ m) Rossby modes are present on including RZ, unlike in CZ (*Blume+ ApJ,2024*).
- They follow analytical dispersion relation (e.g. *Zaqarashvili+ Space Sci. Rev, 2021*):

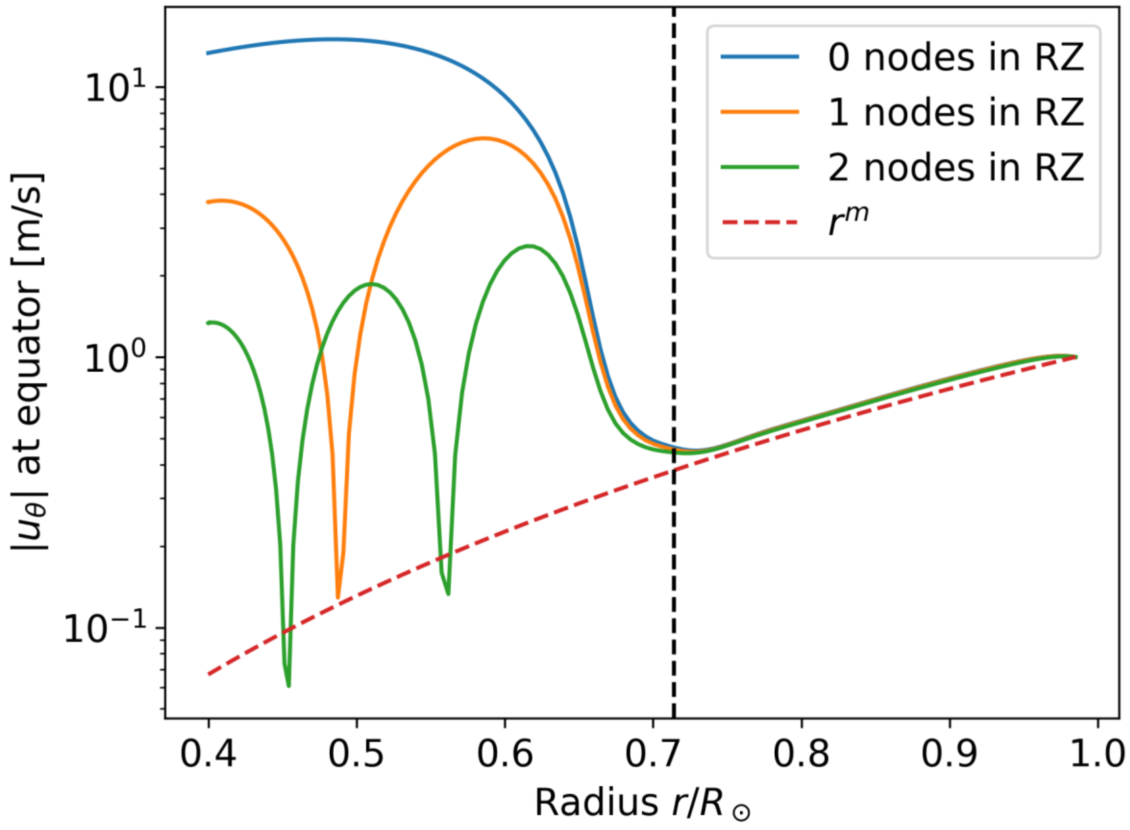
$$\omega_{l,m} = -\frac{2m\Omega_0}{l(l+1)}.$$

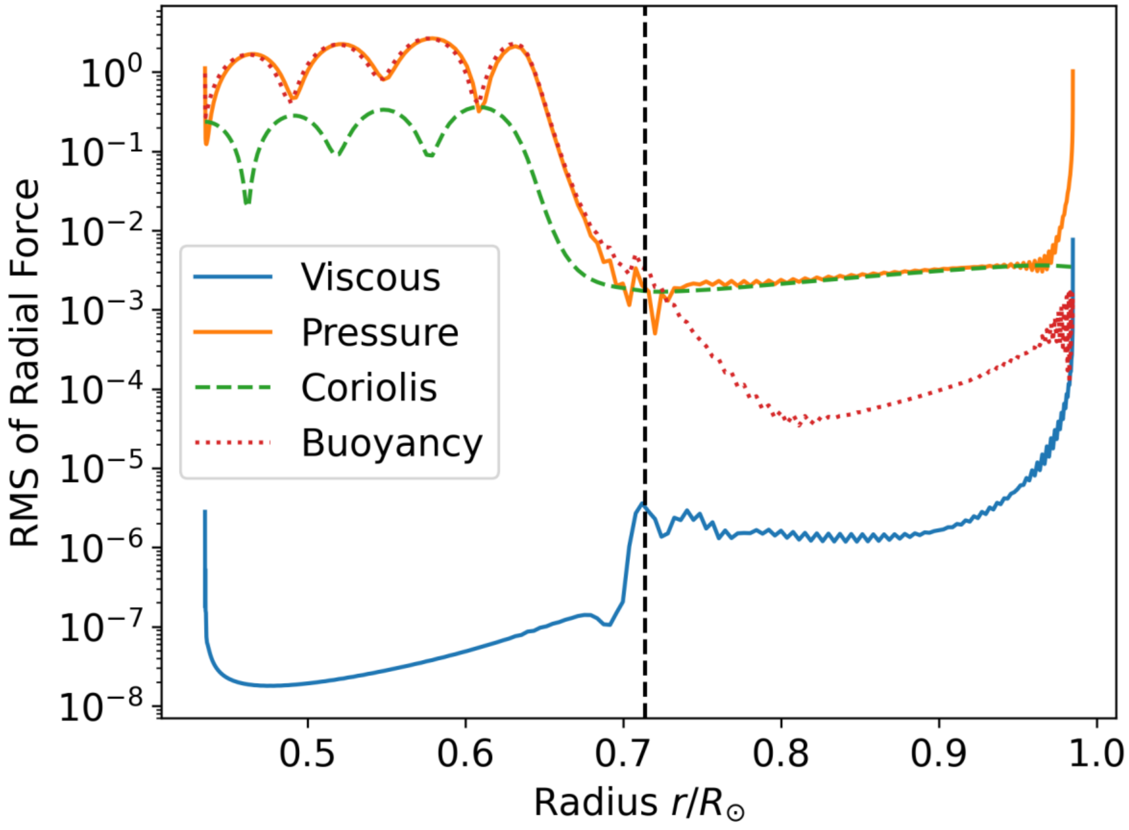
- There are **radial overtones** of the modes, with no radial nodes in the CZ (n_{CZ}) and **any** number of radial nodes in the **RZ** (n_{RZ}).
- The number of **radial nodes** in **RZ** (n_{RZ}) produces a **negligible perturbation** in the **frequency**.



Difference of sectoral Rossby modes in RZ and CZ

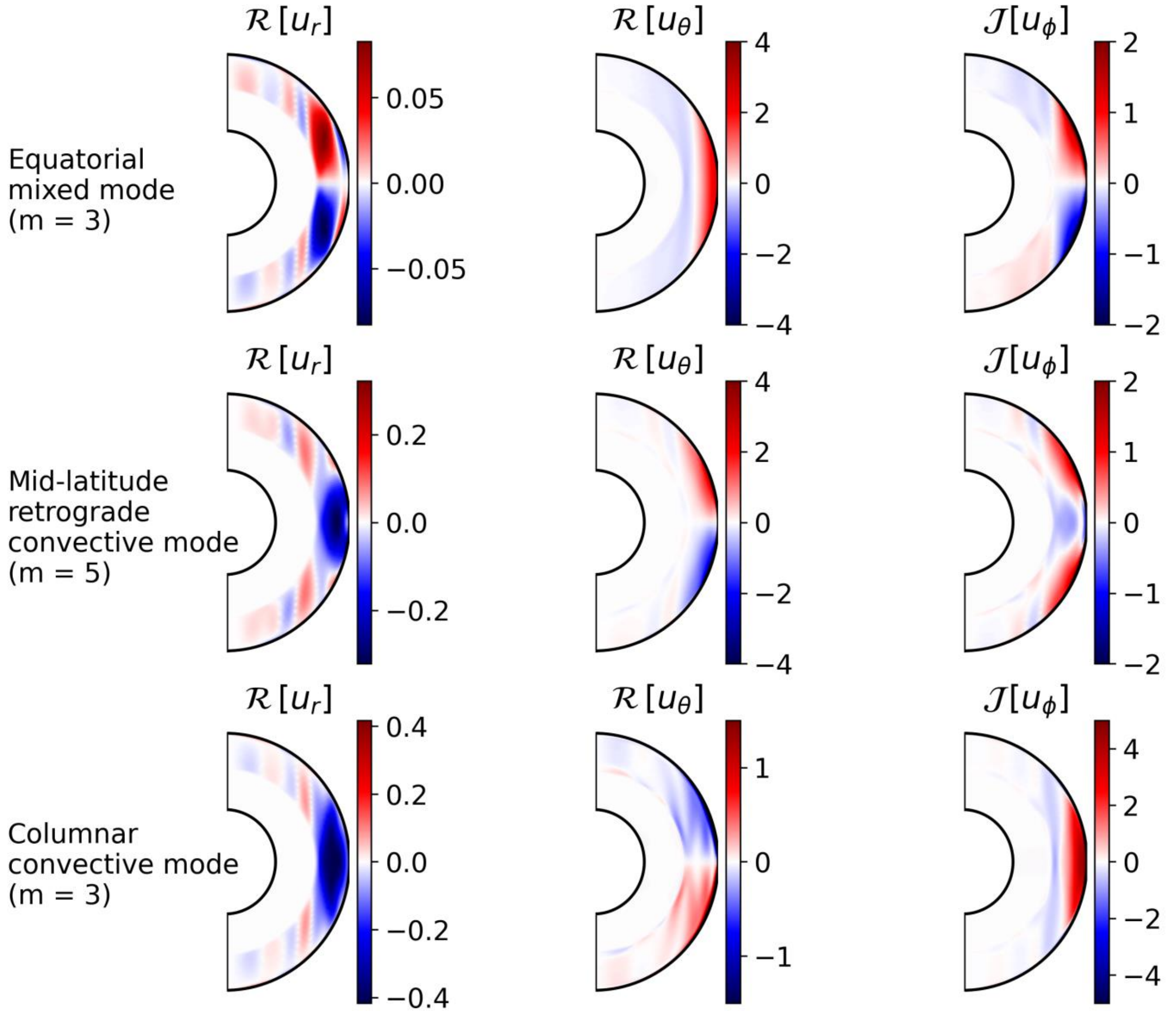
- They have high velocities in RZ, **deviating** from the r^m behavior of u_θ in **CZ**(*Provost+ A&A, 1981*).
- They behave very differently in RZ and CZ due to change in the **Force balance**.
- The behaviour of the mode in CZ is unaffected by RZ, i.e. **very weakly coupled**.





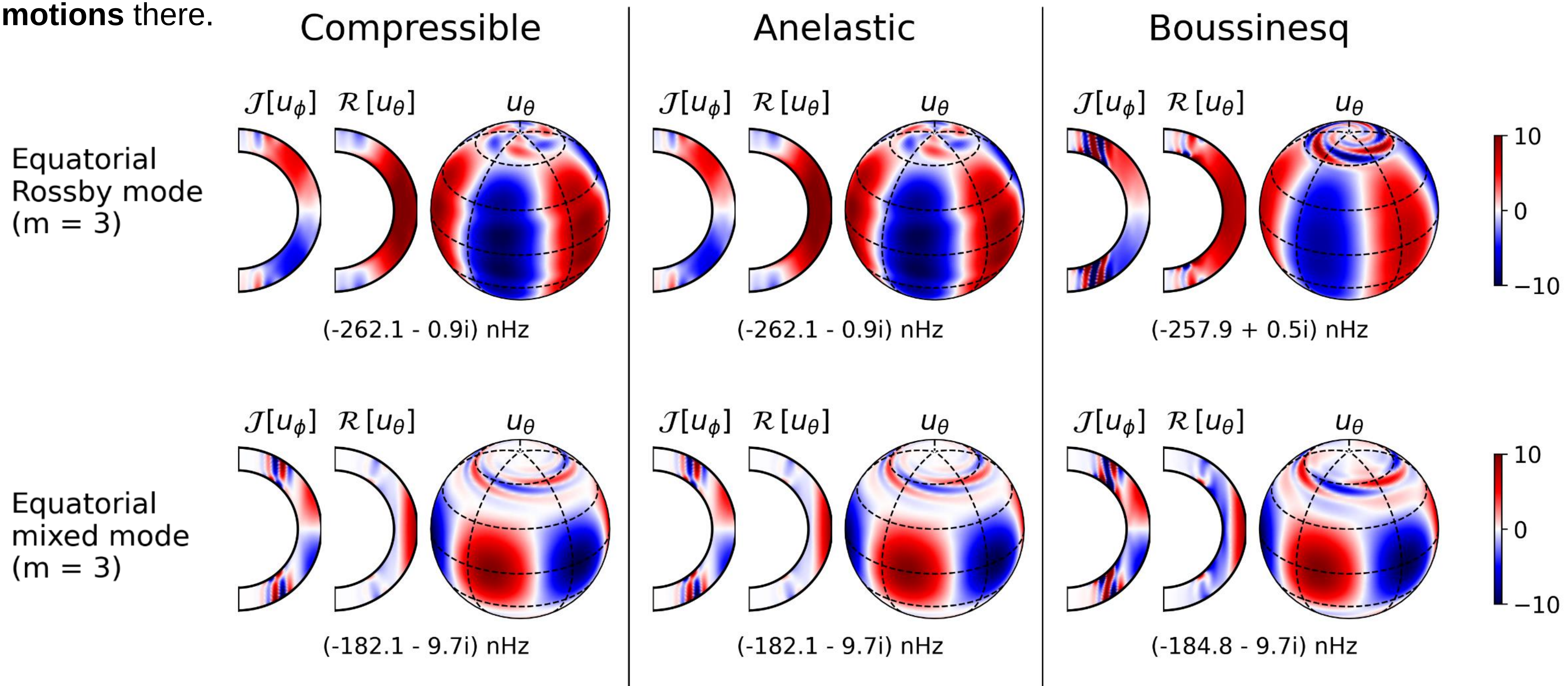
What happens to other inertial modes?

- Modes with significant **convective motions** are **evanescent** in the convectively stable RZ.
- The portion of these modes in the CZ is **unaffected** by the presence of the radiative zone
- These modes can be considered completely **decoupled** from RZ.



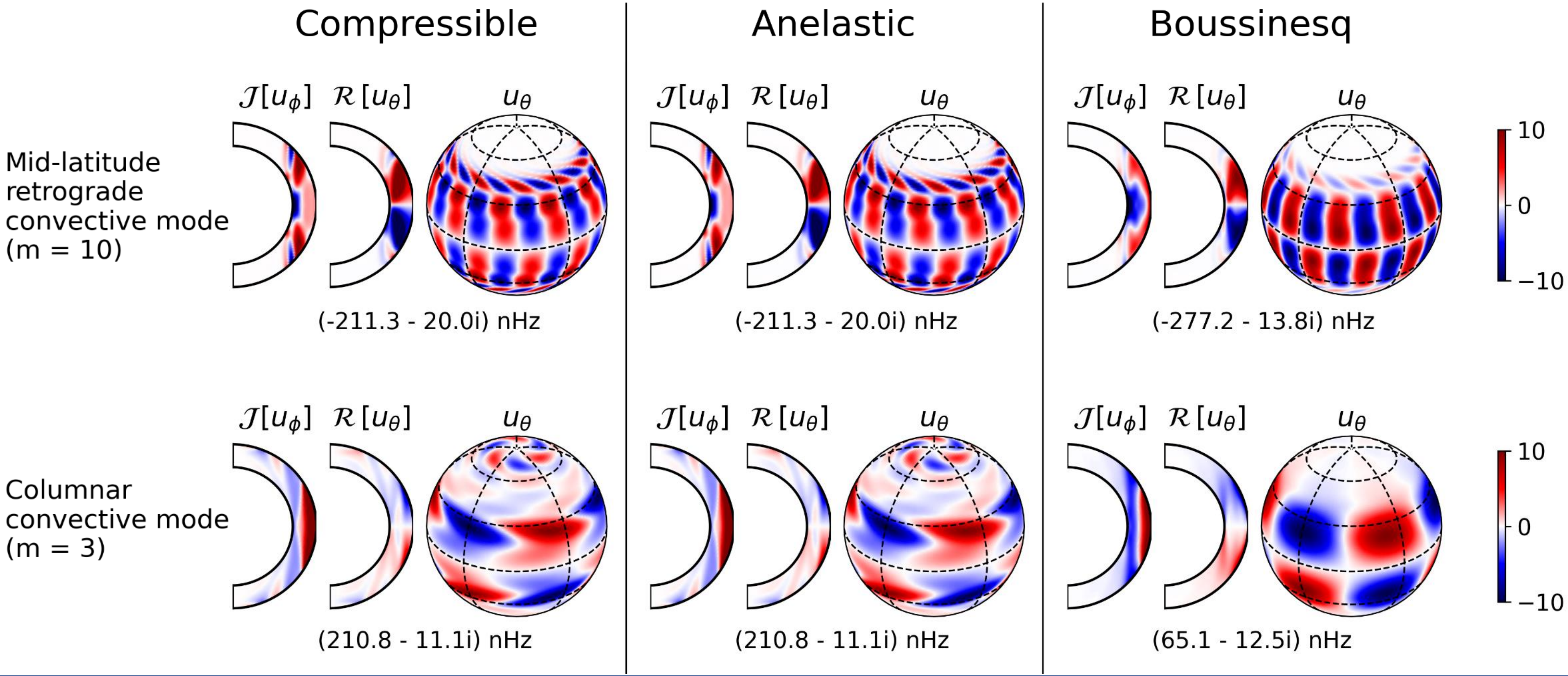
Eigenfunctions under DR in the convection zone

- Modes from the **Boussinesq model** are different near the **critical latitude** due to the presence of **radial motions** there.



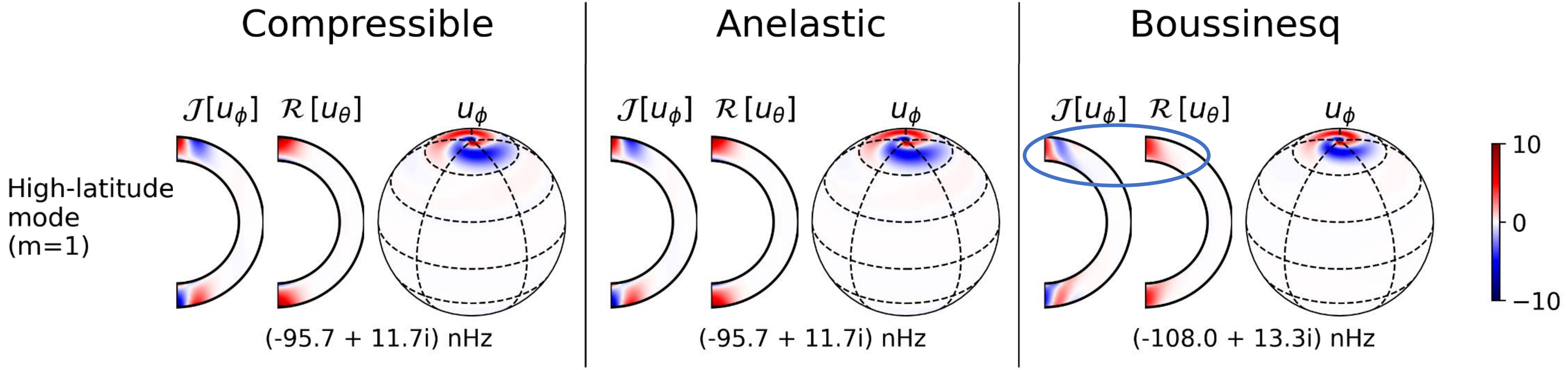
Eigenfunctions of the rotating convective modes

- The **non-toroidal** modes are significantly different for **Boussinesq** model, due to absence of **stratification**.

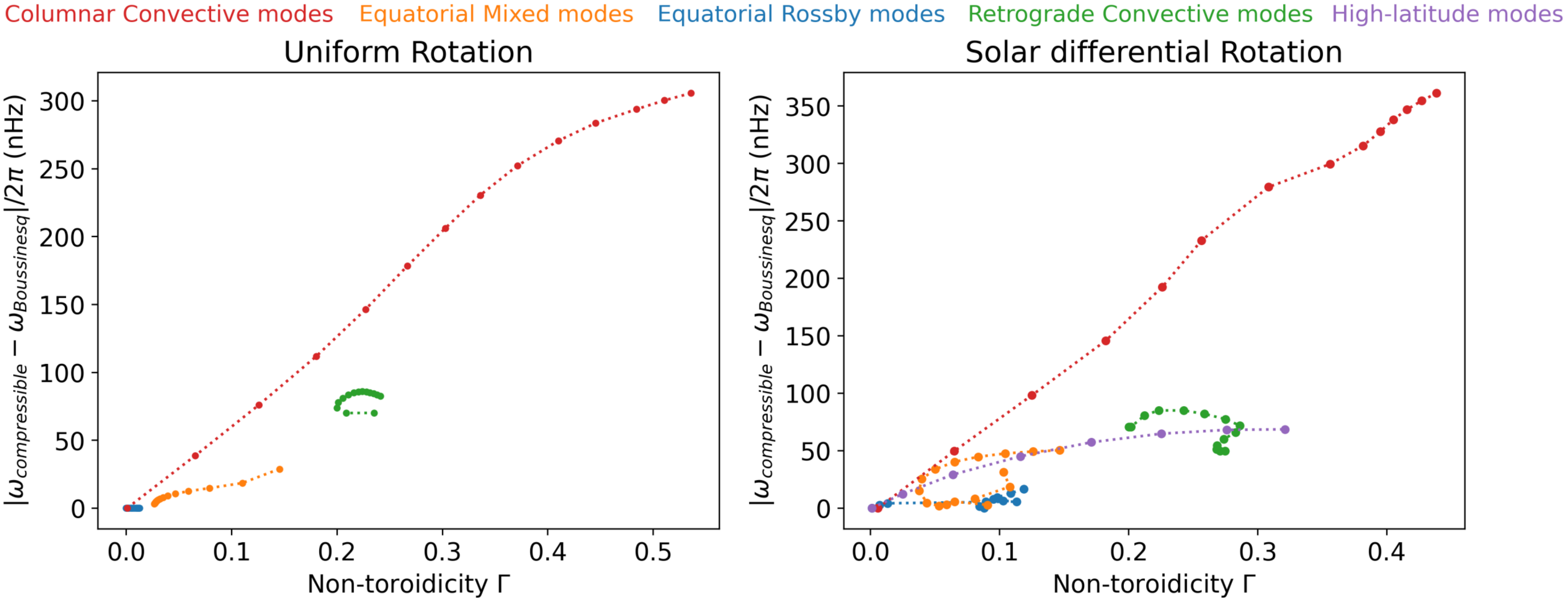


Eigenfunctions of the high-latitude modes

- Baroclinically **unstable** high-latitude modes are produced under latitudinal entropy gradient from thermal wind balance of differential rotation. (*Bekki+ A&A, 2022*)
- The high-latitude modes obtained from the **Boussinesq** model have a different **meridional structure** from the other models because baroclinical instability involves non-toroidal motions.
- It is trapped at much **higher latitudes** than the other models.
- The mode also has a more negative **frequency**.



Cause of the differences: radial motions



$$\Gamma = \sqrt{\frac{\langle \rho_0 u_r^2 \rangle}{\langle \rho_0 (u_r^2 + u_\theta^2 + u_\phi^2) \rangle}}$$

General trend: Difference in frequency between Boussinesq and compressible models increases with increase in the estimate of non-toroidicity of the mode.

Summary and Outlook

Mukhopadhyay, Bekki, Zhu, & Gizon, 2024 (Manuscript in prep.)

- Inertial modes are **very weakly coupled** or **decoupled** to the radiative interior.
- **Anelastic** model simplifies the calculations without affecting the inertial modes.
- **Boussinesq** model significantly affects the **non-toroidal** modes due to the neglect of density stratification.