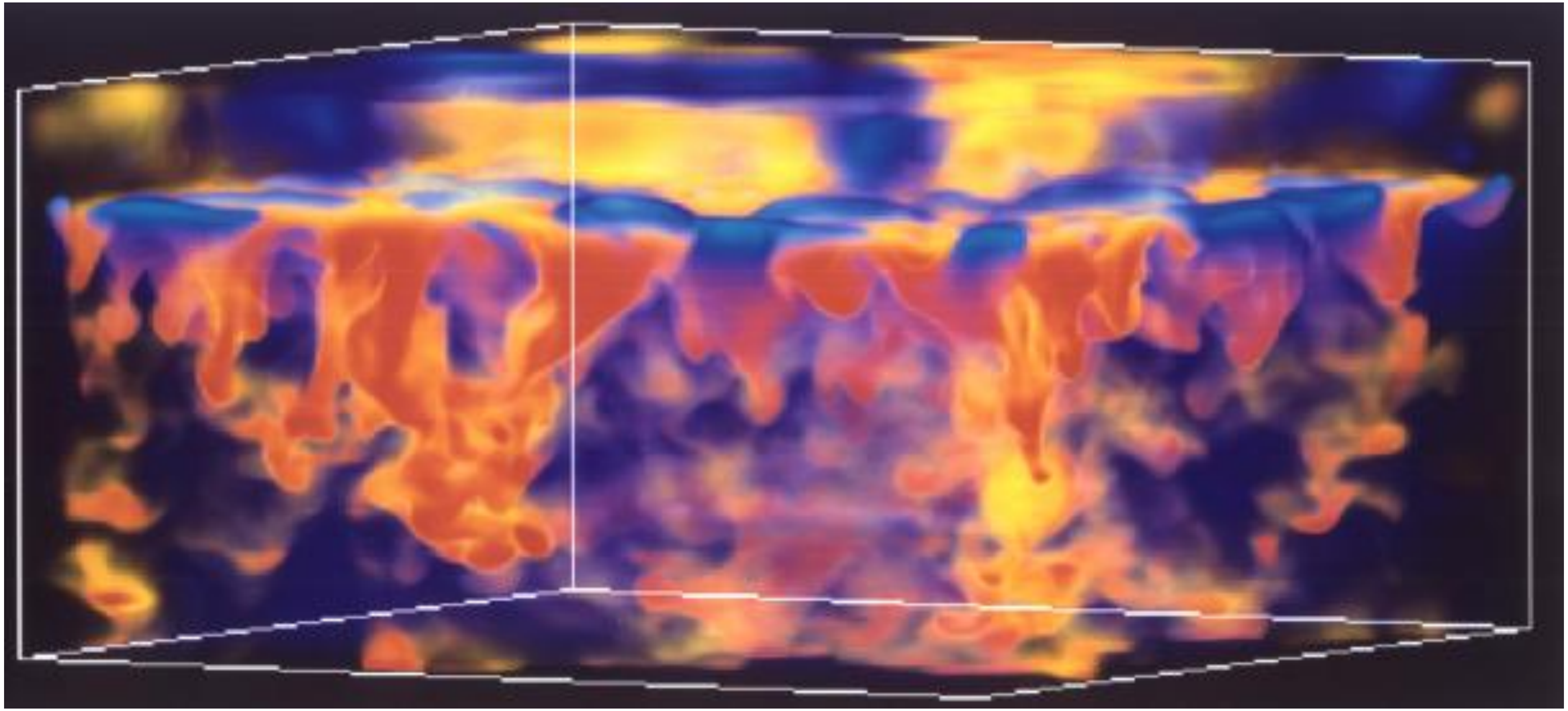
Entropy rain *- seen since Stein & Nordlund (1989)*



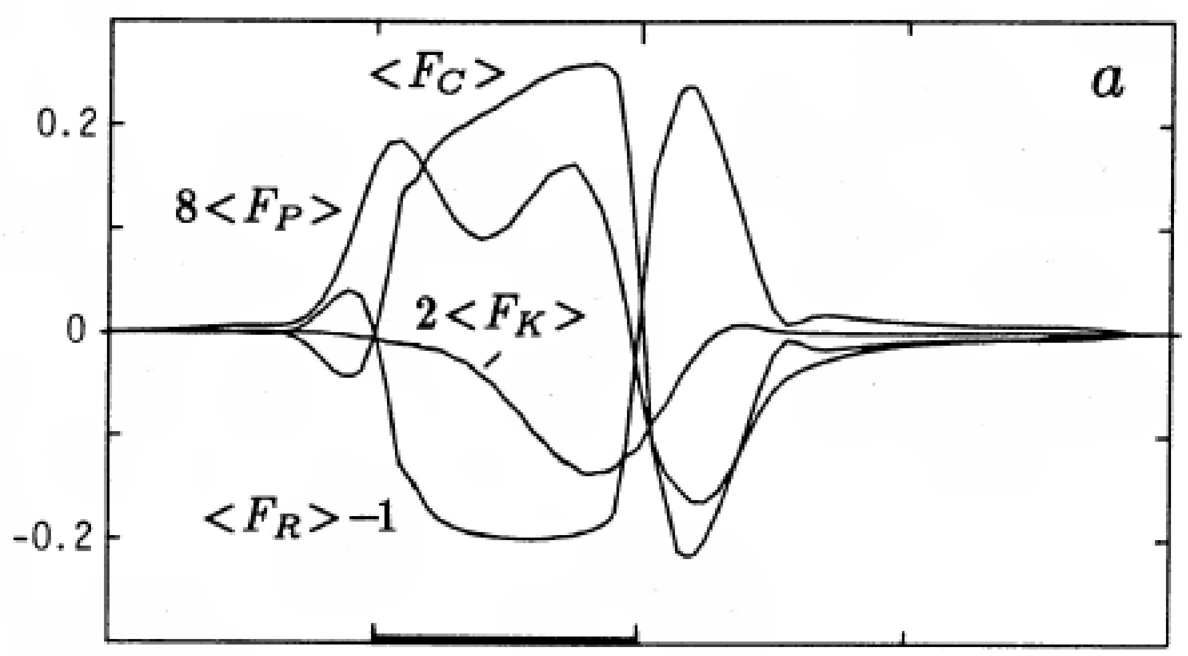
Filamentary, nonlocal

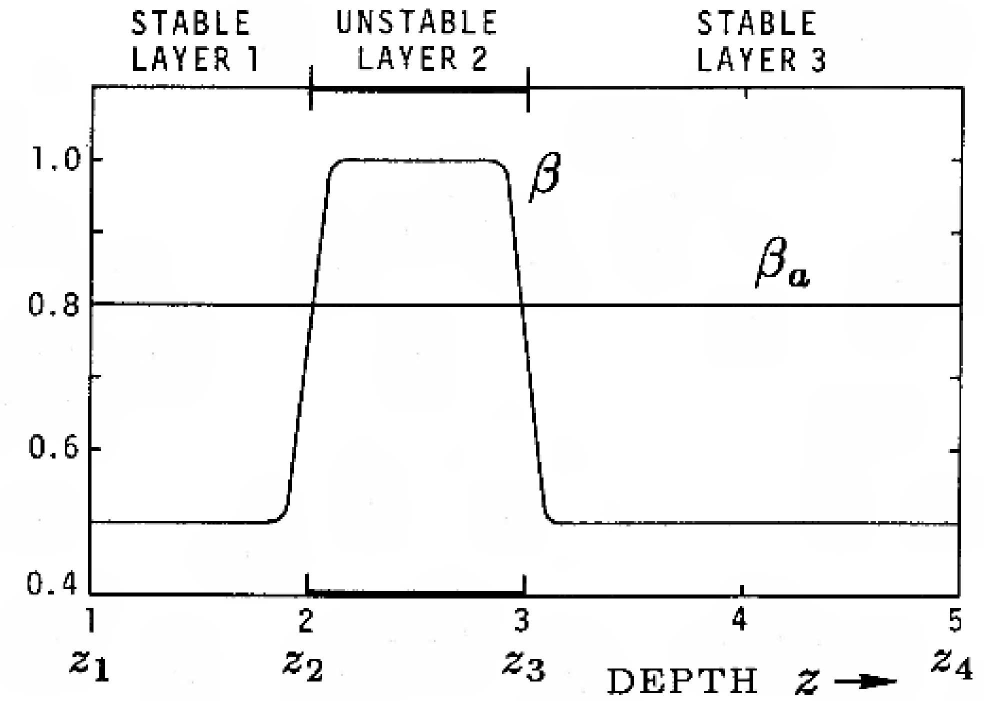
shown: entropy fluctuations pos neg

Axel Brandenburg (Nordita)

“Standard” overshooting convection

Hurlburt, Toomre, & Massaguer (1986)





→ flawed for stellar applications

Structure of my talk

- Part I: slope of opacity vs temperature matters
  - Top few Mm are Schwarzschild-unstable
  - The rest is just stirred
  - Solution to convection conundrum
- Part II: modeling this in MLT
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- Part III: Size of structures
  - Not a solution to super-small convective velocities

Brandenburg (2016, ApJ 832, 6

Near-polytropic solutions

$$\nabla \cdot \boldsymbol{F}_{\rm rad} = -\kappa\rho \oint_{4\pi} (I - S)\, {\rm d}\Omega,$$

$$\hat{\boldsymbol{n}} \cdot \nabla I = -\kappa\rho\, (I - S),$$

$$\frac{{\rm D}\ln\rho}{{\rm D}t} = -\nabla \cdot \boldsymbol{u},$$

$$\rho\frac{{\rm D}\boldsymbol{u}}{{\rm D}t} = -\nabla p + \rho\boldsymbol{g} + \nabla \cdot (2\rho\nu\mathbf{S}),$$

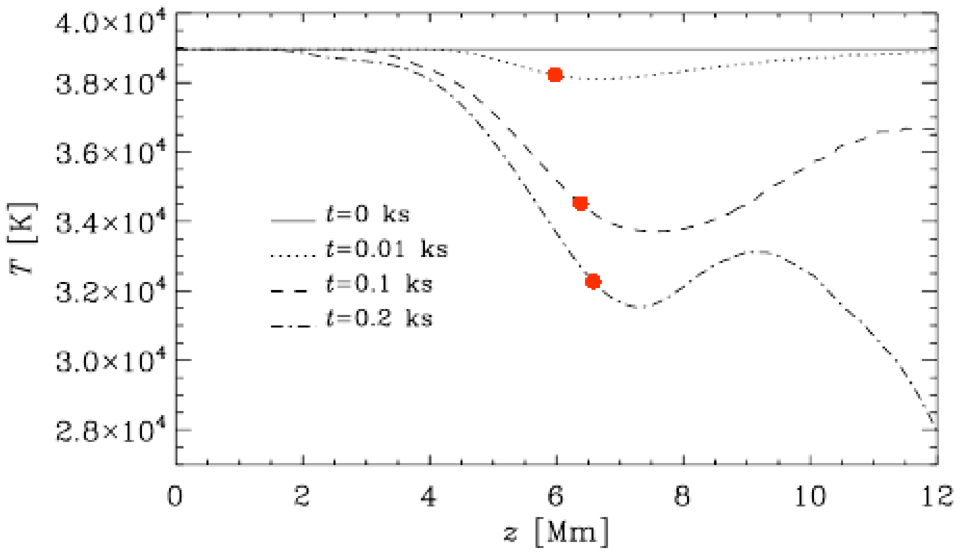
$$\rho T\frac{{\rm D}s}{{\rm D}t} = -\nabla \cdot \boldsymbol{F}_{\rm rad} + 2\rho\nu\mathbf{S}^2,$$

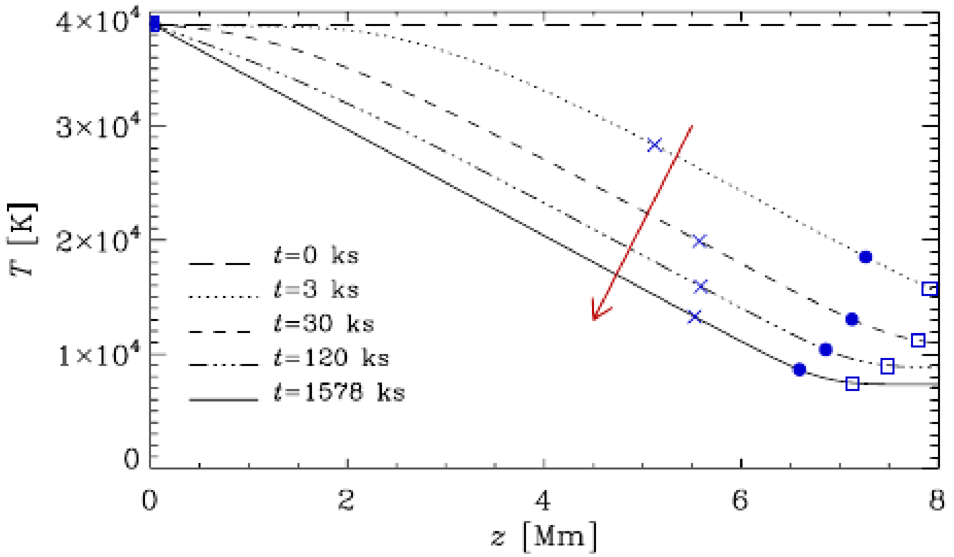
$$\kappa = \kappa_0 \rho^a T^b$$

Kramers-type opacity

$$\text{cooling rate} = \lambda = \chi k^2 \quad \Rightarrow \quad \lambda = \frac{\chi k^2}{1 + \ell^2 k^2}$$

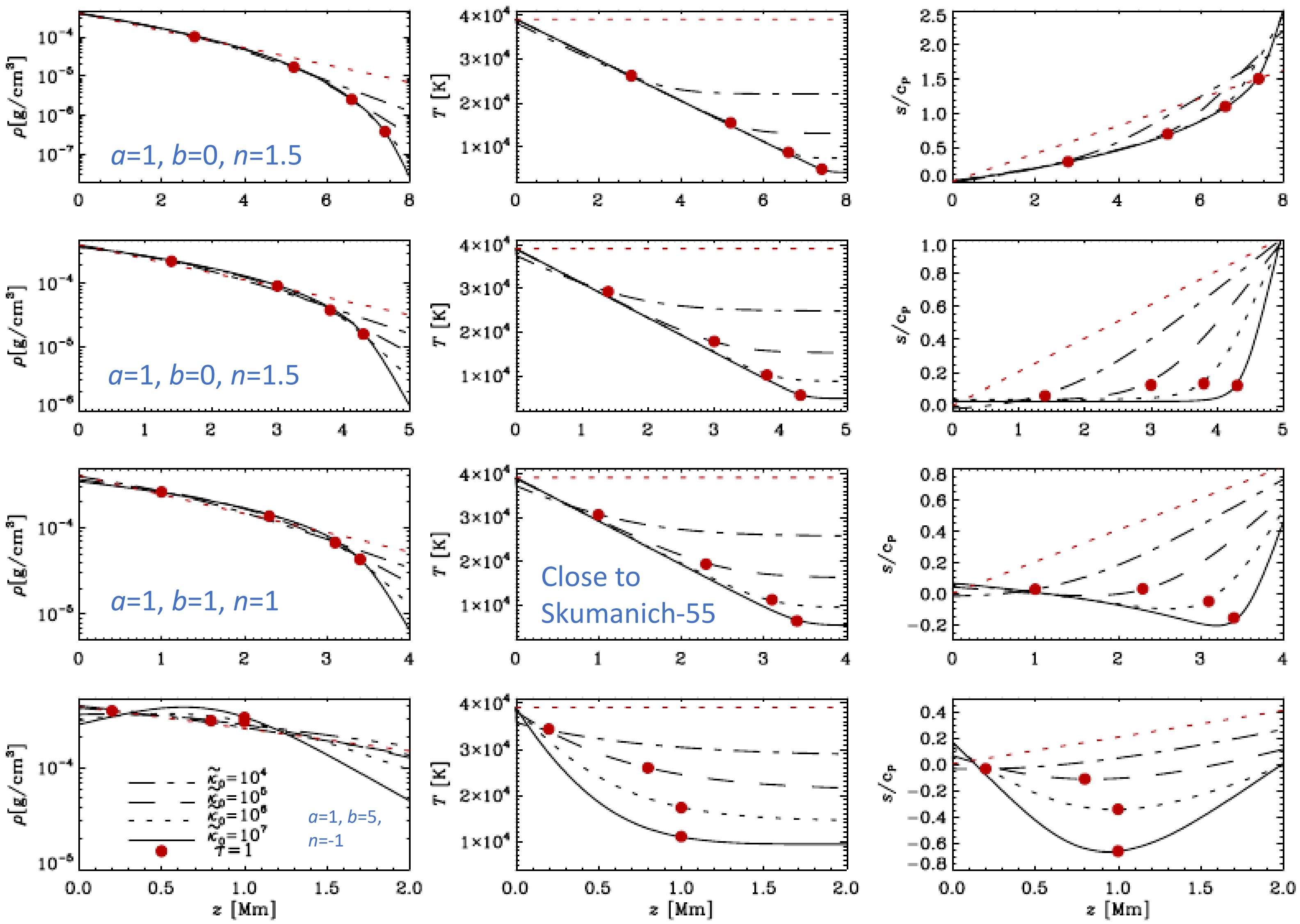
- Polytrope possible
  - dT/dz=const below photosphere
  - T = const above photosphere
- Polytropic index?
  - More complicated opacities?





Barekat & Brandenburg (2014, A&A 571, A68)

Polytropes when *n* > -1



Need:

$$F_{\mathrm{rad}} = -K\frac{dT}{dz} = \mathrm{const}$$

For example:

$$\frac{dT}{dz} = \mathrm{const}$$

and $$K = \frac{16\sigma T^3}{3\kappa\rho} = \mathrm{const}$$

Kramers type power law

$$\kappa = \kappa_0 \rho^a T^b$$

Polytropic index *n*

$$\rho = T^{\frac{3-b}{1+a}} = T^n$$

Analytic solution

Radiative flux:

$$\mathbf{F}_{\rm rad} = -K\nabla T \quad \text{with} \quad K = \frac{16\sigma_{SB}T^3}{3\kappa\rho}$$

Kramers' opacity: $\kappa = \kappa_0 (\rho/\rho_0)^a (T/T_0)^b$

Nonconvecting solution ($F_{\rm rad}$=const)

$$(T/T_0)^{4+a-b} = (n+1)\nabla_{\rm rad}^{(0)} (P/P_0)^{1+a} + (T_{\rm top}/T_0)^{4+a-b}$$

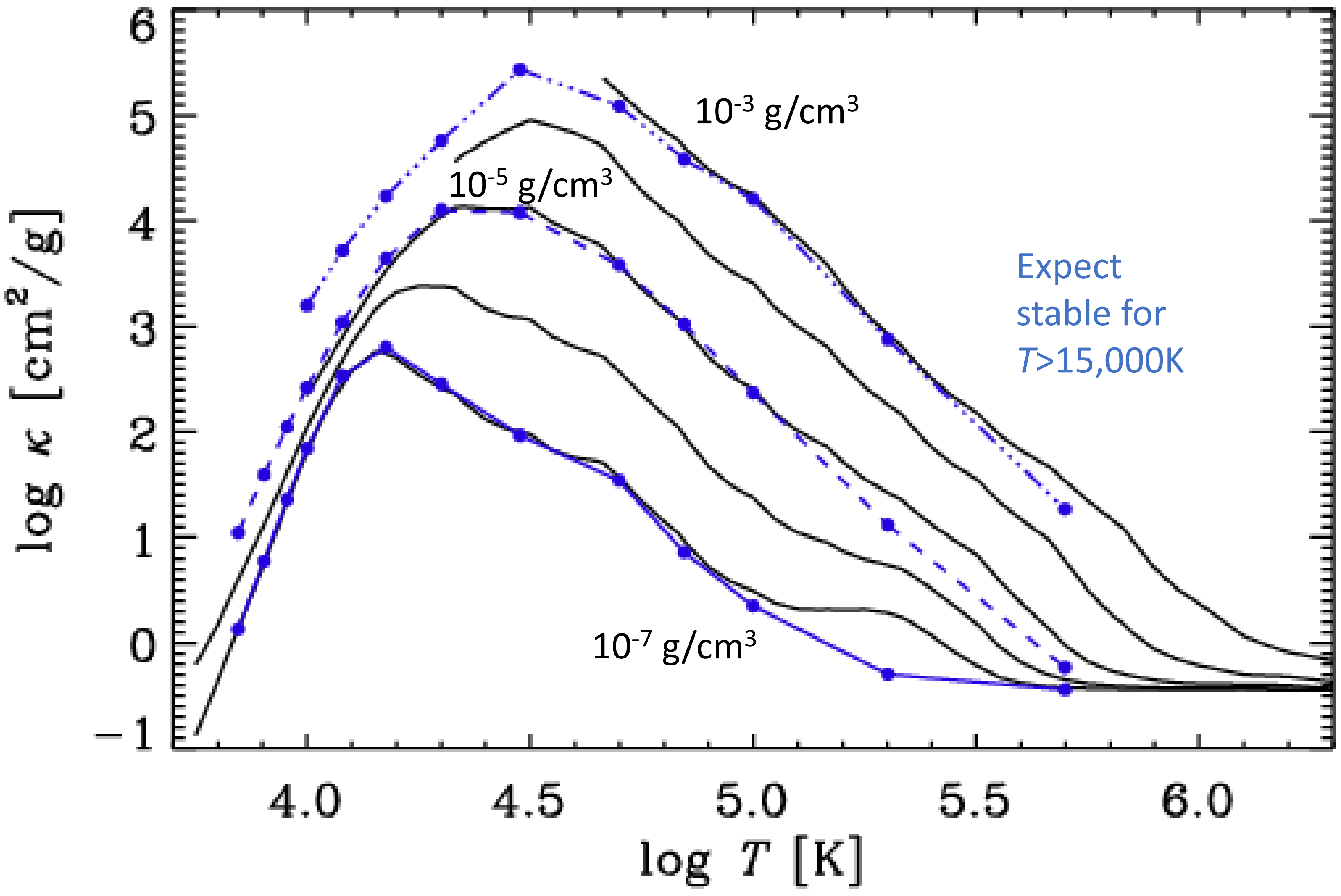
Brandenburg (2016)

Polytropic index for Kramers opacity:

$$n = \frac{3-b}{1+a} = \frac{3+3.5}{1+1} = 3.25$$

>1.5 (→ stable)

OPAL vs. old Cox & Stewart opacities

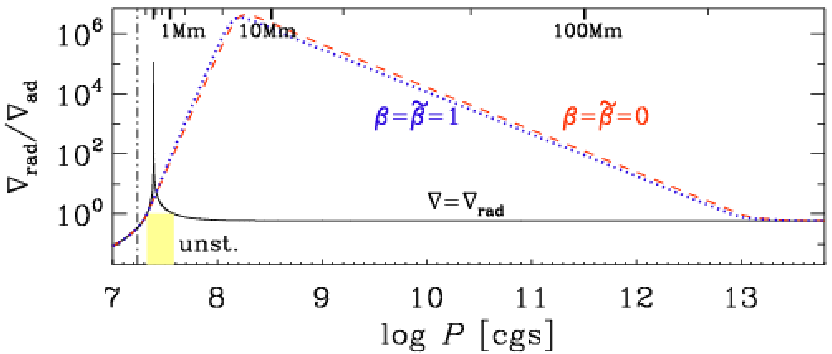


- 2 branches
- Rising branch from H^- opacity at low T
- Decreasing branch from bound-free & free-free opacity
- Kramers type opacity

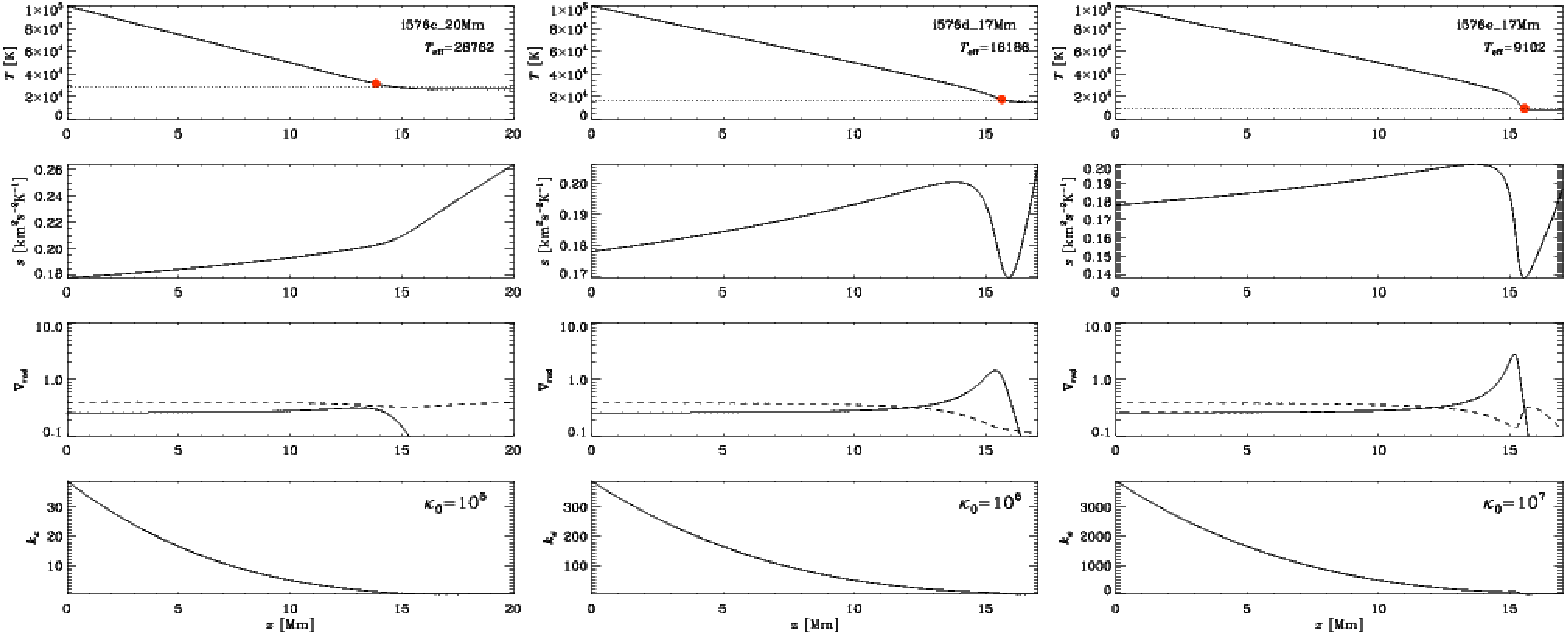
$$\kappa = \kappa_0 \rho^a T^b$$

- *a*=1, *b*=-3.5

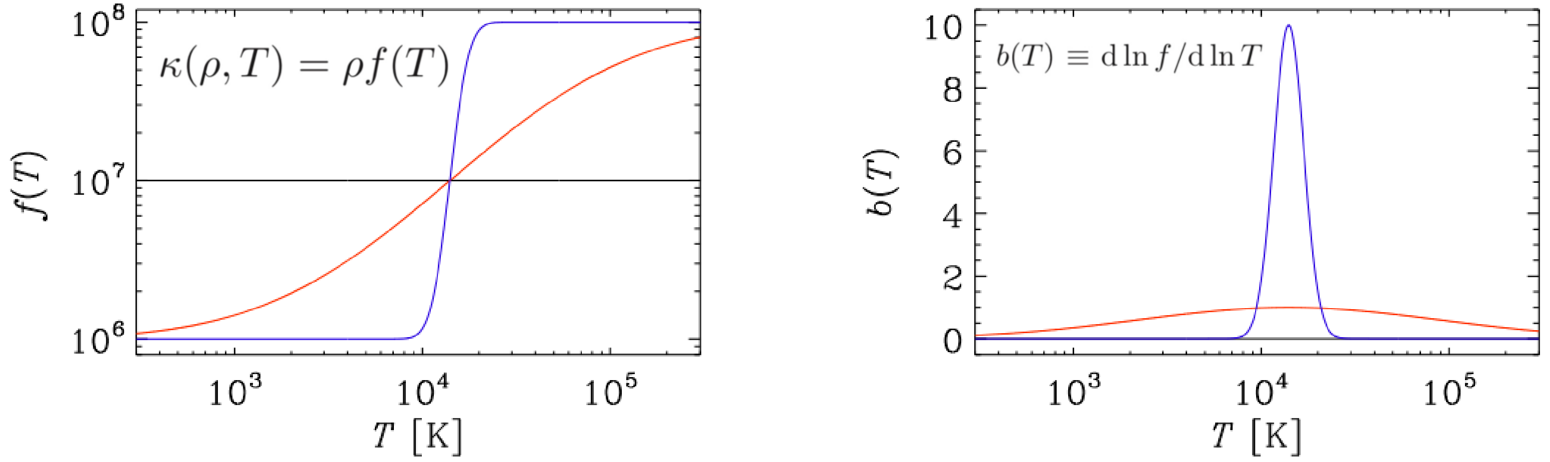
Hydrostatic reference solutions



Bimodal opacity profile: increase opacity prefactor

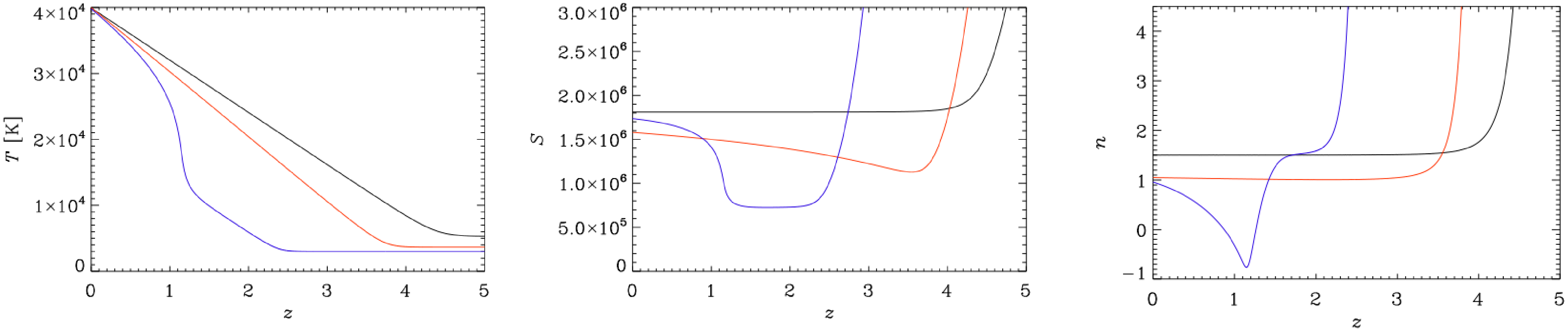


What matters? Actual opacity or its derivative?

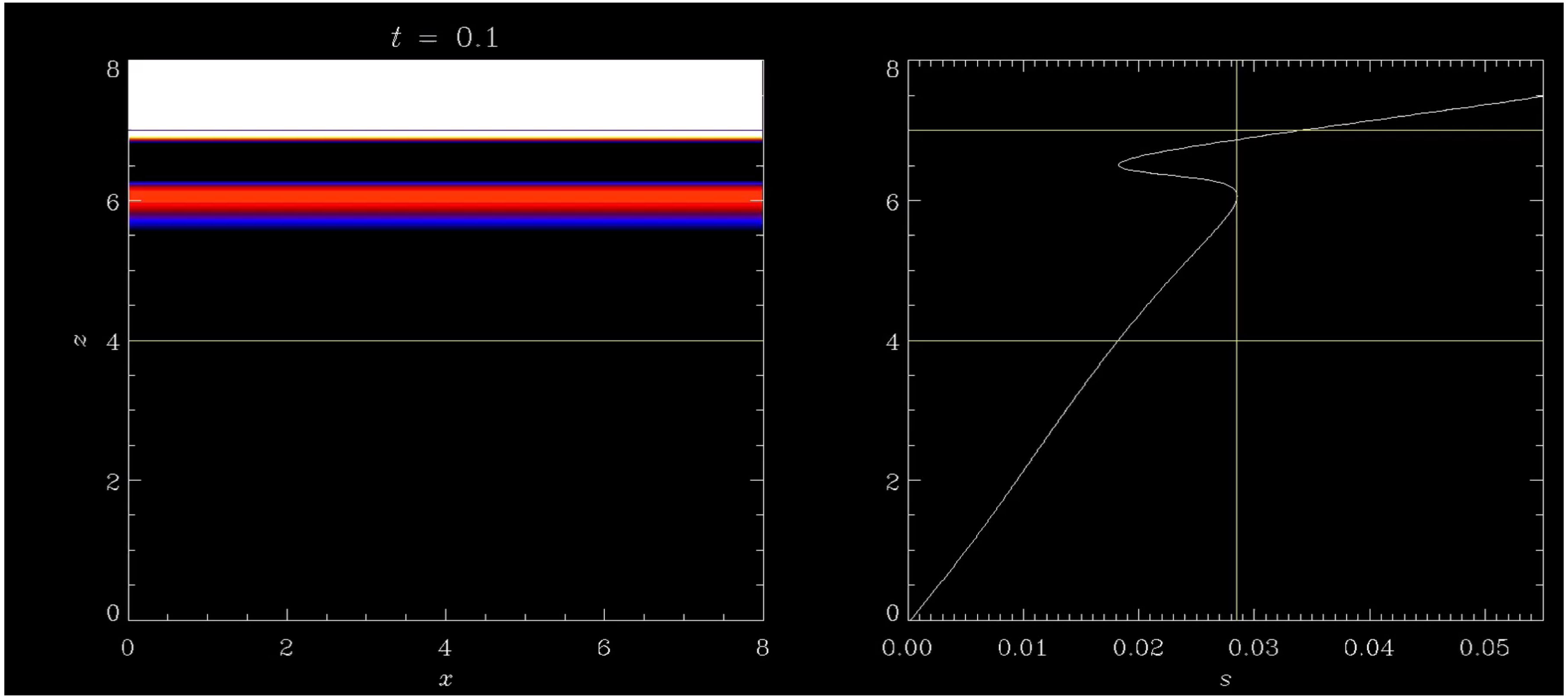


- b_{max} = 0, 1, 10
- b = 0 means n=1.5

Set	a	b	n	Schwarzschild
A	1	−3.5	3.25	stable
B	1	0	1.5	marginally stable
C	1	1	1	unstable
D	1	5	−1	ultra unstable
E	−1	3	0/0	undefined



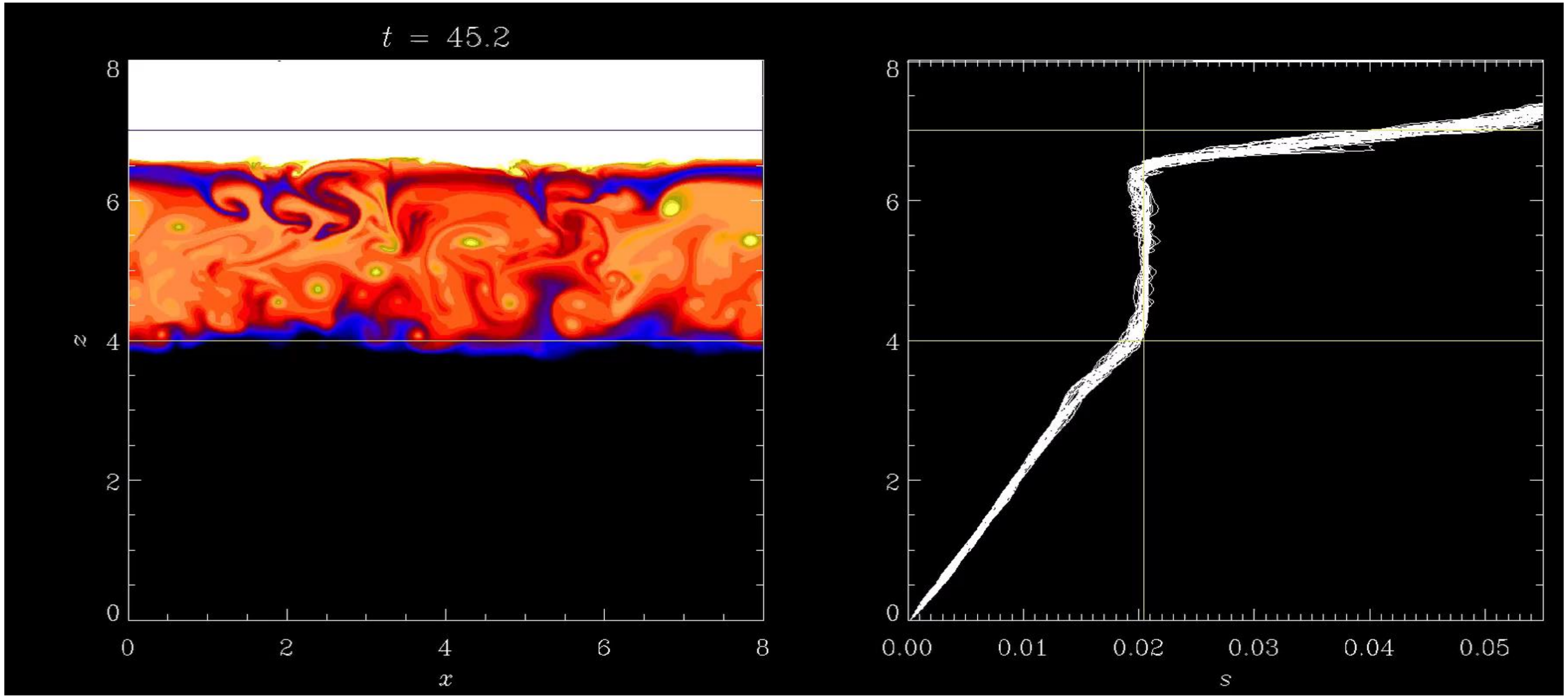
Illustrative simulations



Brandenburg (2016)

- Extended subadiabatic layer
- Yet upward enthalpy flux

Illustrative simulations



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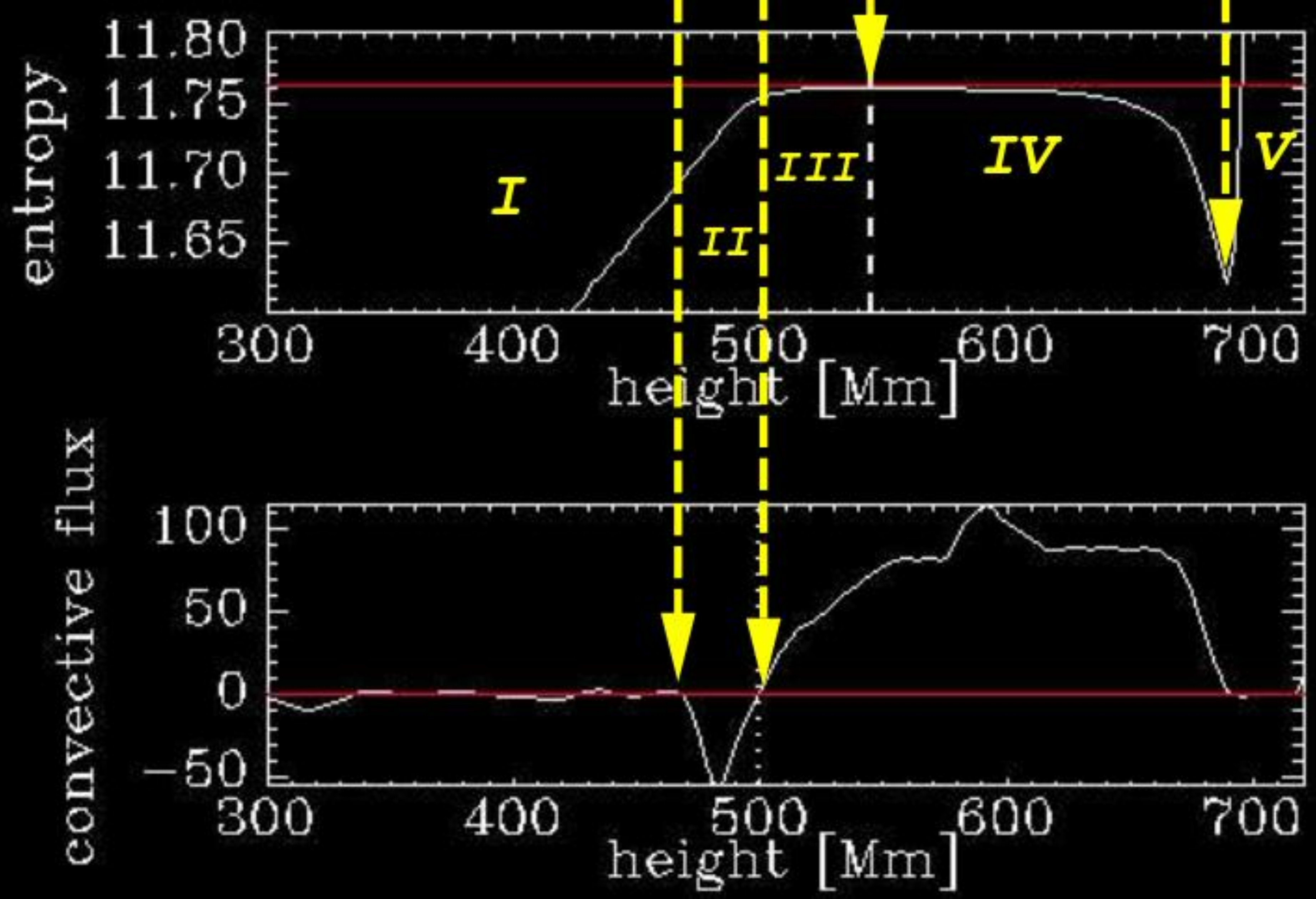
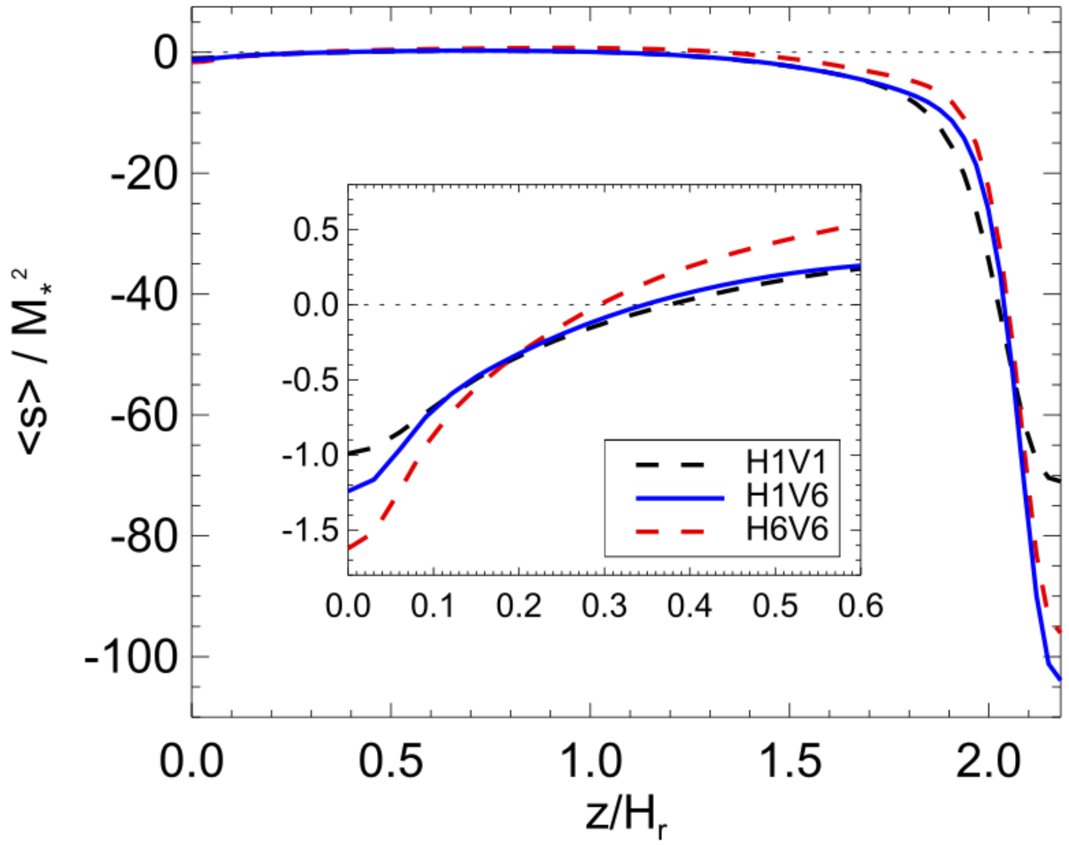
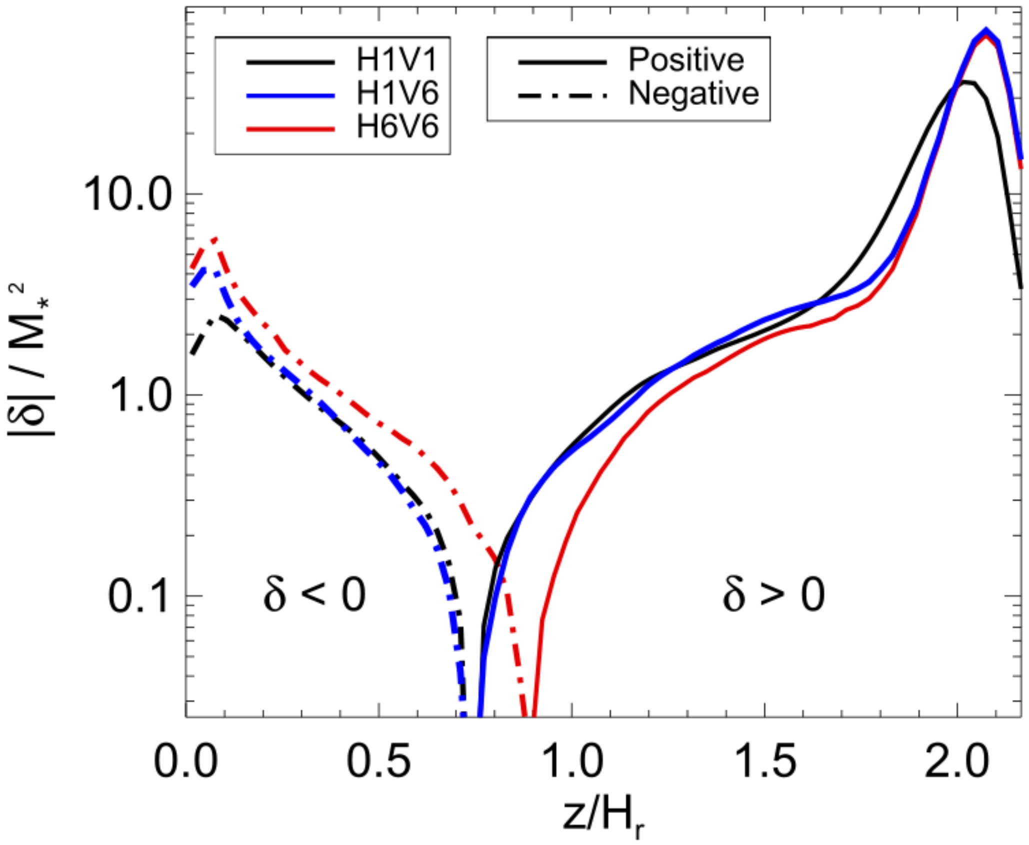


Brandenburg, Nordlund, & Stein (2000) using Kramers opacity

Subadiabatic layers now seen routinely

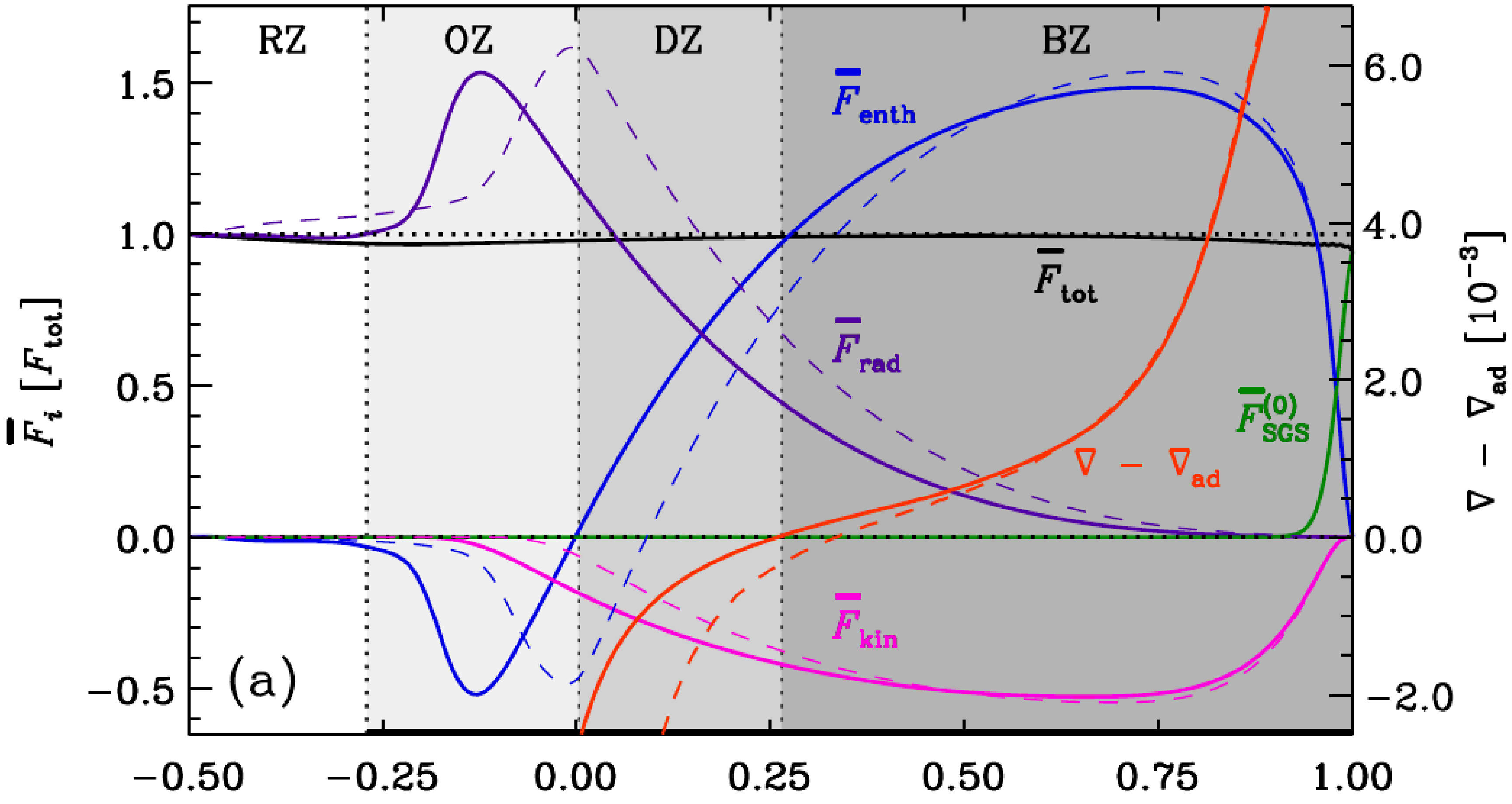




Bekki, Hotta, & Yokoyama (2017)

- Lower 1/3 subadiabatic
- But overshoot layer not included

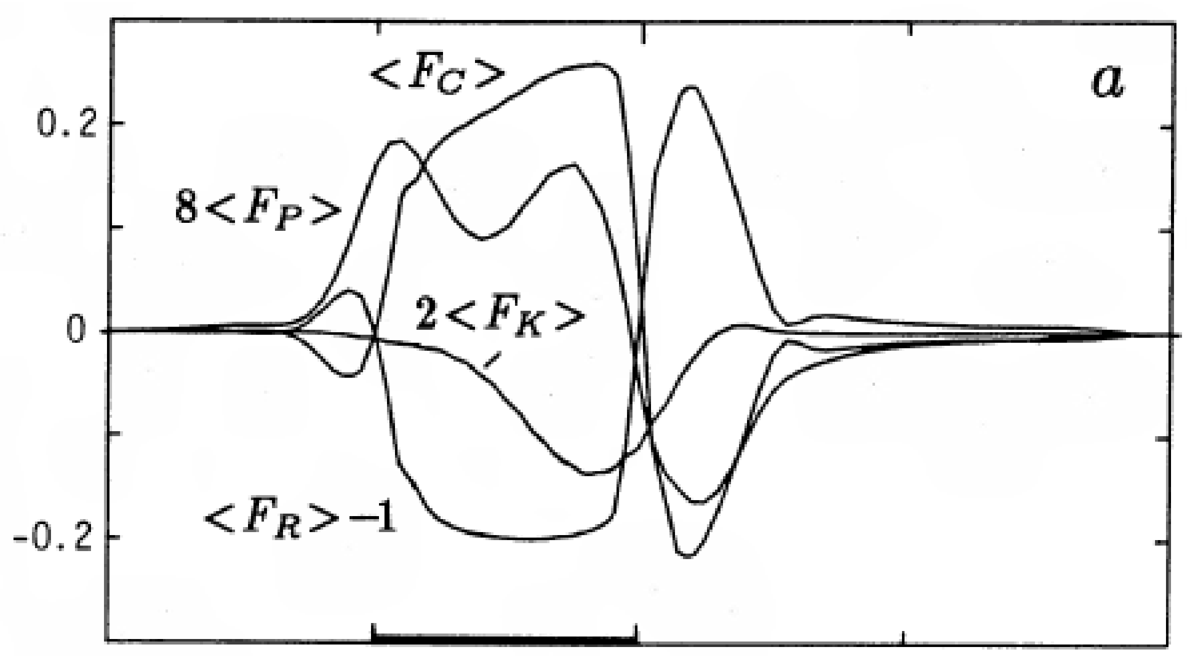
Confirmed by simulations (Käpylä+17)

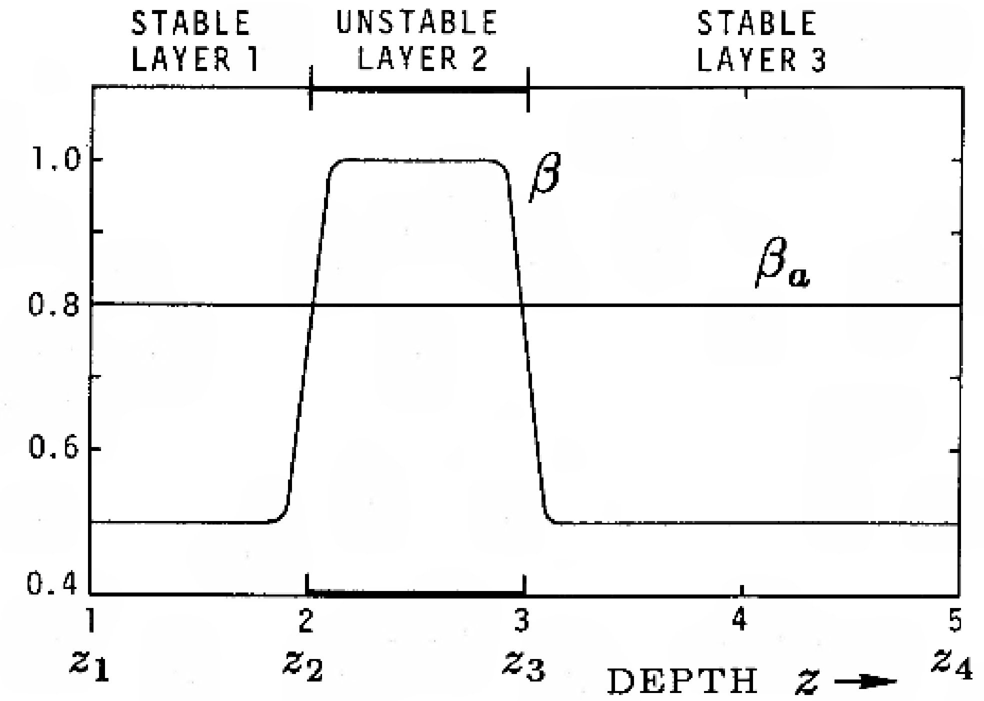


- Extended subadiabatic layer
- Yet upward enthalpy flux
- Distinct from usual overshoot layer (where enthalpy flux is downward!)

“Standard” overshooting convection

Hurlburt, Toomre, & Massaguer (1986)





Explained by Deardorff term

tau approximation

$$\partial F_{\rm enth}/\partial t = \overline{\rho}\,\overline{T}\,(\overline{u_z \dot{s}} + \overline{\dot{u}_z s})$$

energy & momentum eqn

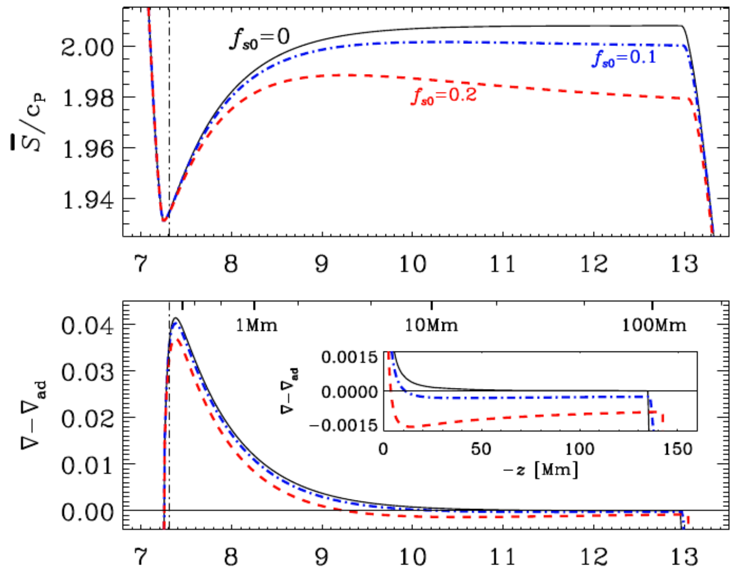
$$\dot{s} = -u_j \nabla_j \overline{S} - s/\tau_{\rm cool}\ \ldots,$$

$$\dot{u}_i = -g_i s/c_P + \ldots,$$

gradient & Deardorff terms

$$\boldsymbol{F}_{\rm G} = -\tfrac{1}{3}\tau_{\rm red} u_{\rm rms}^2 \overline{\rho}\ \overline{T}\ \boldsymbol{\nabla}\overline{S},$$

$$\boldsymbol{F}_{\rm D} = -\tau_{\rm red}\overline{s^2}\ \boldsymbol{g}\ \overline{\rho}\ \overline{T}/c_P$$



extra nabla term
in standard MLT

$$F_{\rm enth} = \frac{1}{3}\overline{\rho}c_P\overline{T}\ (\tau_{\rm red}u_{\rm rms}^2/H_P)(\nabla - \nabla_{\rm ad} + \nabla_{\rm D})$$

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Theoretical Expression for the Countergradient Vertical Heat Flux

J. W. Deardorff

National Center for Atmospheric Research, Boulder, Colorado 80302

A theoretical expression is derived from the heat-flux conservation equation for the counter potential-temperature gradient that can sustain an upward flux of sensible heat. This gradient is found to be $\gamma_c = (g/\theta)\ \langle\theta'^2\rangle/\langle w'^2\rangle$, where $\langle\theta'^2\rangle$ is the potential temperature variance and $\langle w'^2\rangle$ is the vertical velocity variance. The usual down-gradient eddy coefficient expression for the heat flux is obtained from the derivation only if γ_c is set to zero. Aircraft measurements of $(g/\theta)\ \langle\theta'^2\rangle/\langle w'^2\rangle$ in the middle and upper portions of convective planetary boundary layers indicate that this expression for γ_c is of the same order of magnitude (near 0.7×10^{-5} °K cm^{-1}) as the value deduced previously for γ_c from completely different considerations.

et al. [1971], and *Donaldson* [1972] that utilize equations for the second moments and closure assumptions for third moments. The equation, which makes use of the Boussinesq approximation, is

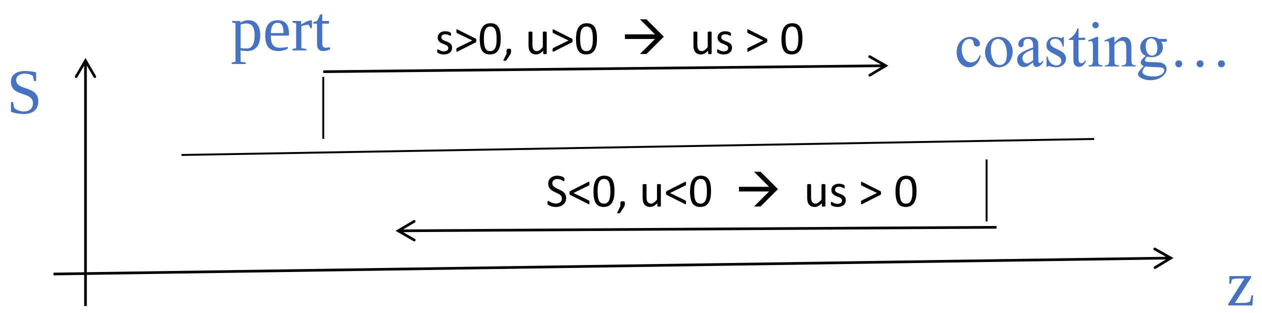
$$\frac{\partial}{\partial t}\langle w'\theta'\rangle = -\langle u_j\rangle\frac{\partial}{\partial x_j}\langle w'\theta'\rangle - \langle w'u_j'\rangle\frac{\partial\langle\theta\rangle}{\partial x_j} - \langle u_j'\theta'\rangle\frac{\partial\langle w\rangle}{\partial x_j} - \frac{\partial}{\partial x_j}\langle w'u_j'\theta'\rangle + \frac{g}{\theta_0}\langle\theta'^2\rangle - \frac{1}{\rho_0}\left\langle\theta'\frac{\partial p'}{\partial z}\right\rangle \quad (3)$$

ion.

Nearly constant entropy through mixing from the top

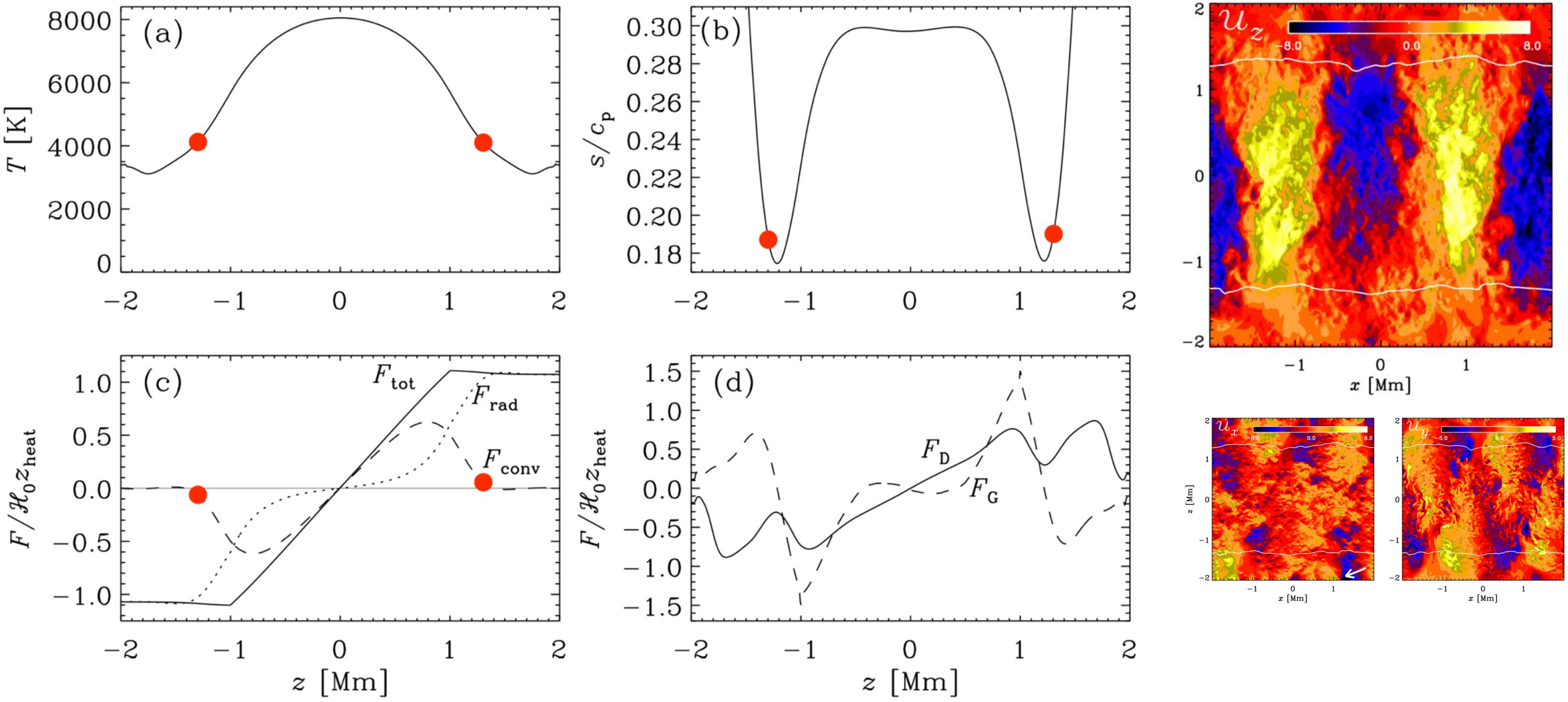
$$F_G = -\frac{1}{3}\tau_{red}\, u_{rms}^2\, \overline{\rho}\, \overline{T}\, \nabla \overline{S},$$

$$F_D = -\tau_{red}\, \overline{s^2}\, g\, \overline{\rho}\, \overline{T}/c_P$$



- Enthalpy flux without gradient term
  - *Non-local* phenomenon
- Convection instability not by local Schwarzschild criterion
  - But stirring from above → drives Deardorff
  - No giant cells expected (→ global simulations assumed MLT)
  - Stability depends on *local* opacity law

Also seen in accretion disc simulations



Brandenburg & Das (2020, GAFD 114, 162)

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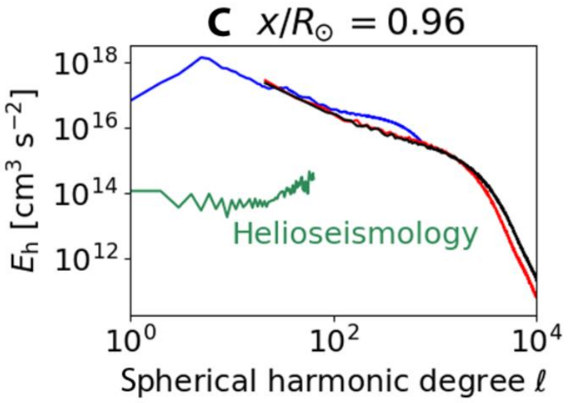
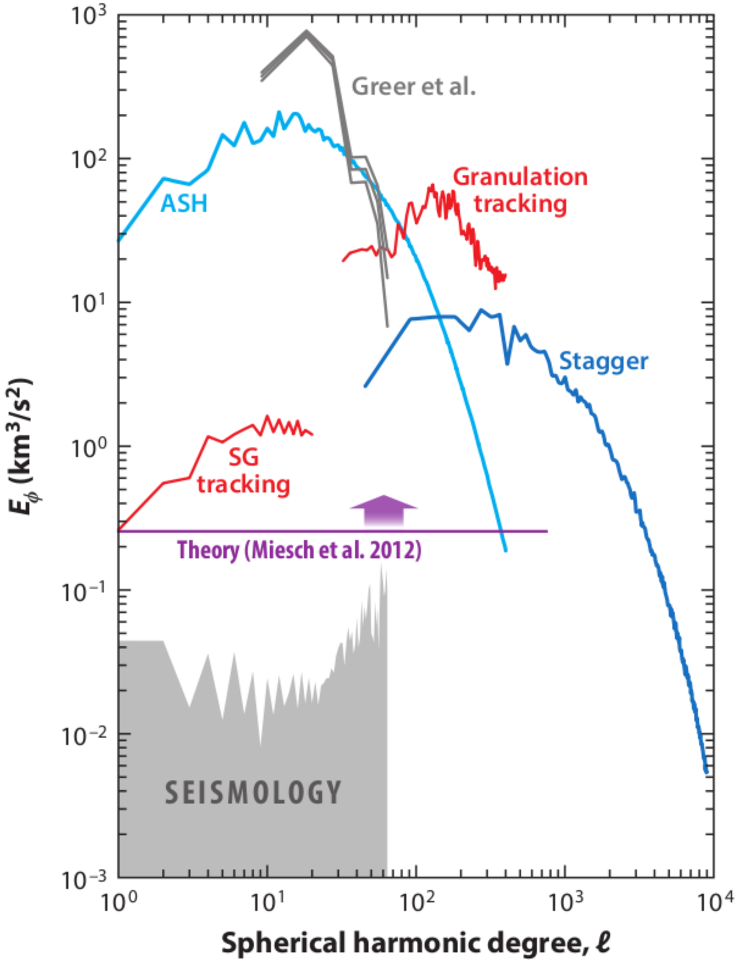
Small scales predominant?

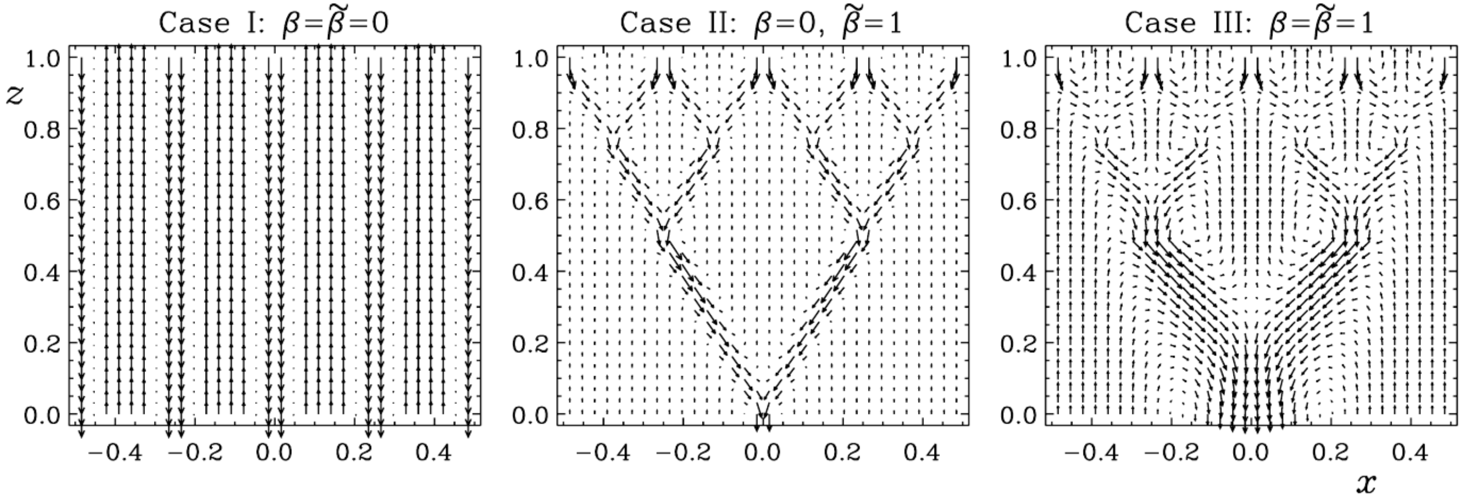
- Rapid downdrafts
  - How fast can they go?
- Are small scales unobservable?
  - Could explain helioseismic result?
  - E.g., like cases I or II?

Hotta19



Hanasoge+17





Filling factor?

f	1/2	1/3	0.14	0.015	0.0006
$\phi_{\rm kin}$	0	0.35	1	4	20
$-\overline{U}_\downarrow/u_{\rm rms}$	1	1.4	2.5	8	40
$\overline{U}_\uparrow/u_{\rm rms}$	1	0.7	0.4	0.12	0.025

$$\overline{S} = (1-f)\overline{S}_\uparrow + f\overline{S}_\downarrow = \overline{S}_\uparrow - f\,\Delta\overline{S}$$

$$\overline{s^2} = (1-f)(\overline{S}_\uparrow - \overline{S})^2 + f(\overline{S}_\downarrow - \overline{S})^2 = \hat{f}\,(\Delta\overline{S})^2$$

$$\hat{f} = (1-f)f$$

$$\overline{u_z^3} = (1-f)\overline{U}_\uparrow^3 + f\,\overline{U}_\downarrow^3 = -\hat{f}\,(1-2f)(\Delta\overline{U})^3$$

When f becomes small (<0.14), $\phi_{\rm kin}$ exceeds unity and for $f < 0.015$, $\phi_{\rm kin}$ exceeds the estimate $\phi_{\rm enth} \approx 4$ found by Brandenburg et al. (2005), so the sum of enthalpy and kinetic energy fluxes may become negative, which appears unphysical.

$$F_{\rm kin} = -\phi_{\rm kin}\,\overline{\rho}u_{\rm rms}^3$$

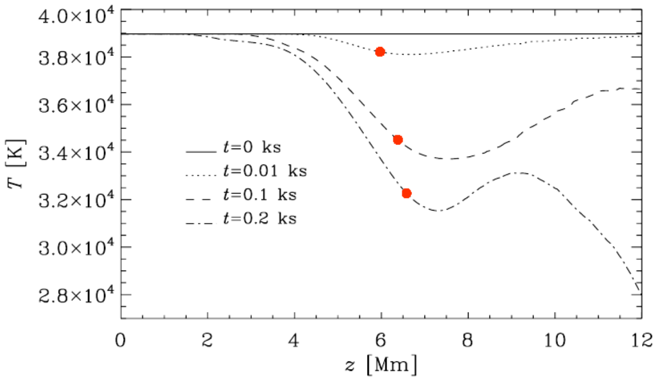
where $\phi_{\rm kin} = (1/2 - f)/\hat{f}^{1/2}$ is a positive prefactor (corresponding to downward kinetic energy flux) if $f < 1/2$. Stein et al. (2009) find $f \approx 1/3$, nearly independently of depth, which yields $\phi_{\rm kin} \approx \sqrt{2}/4 \approx 0.35$; see Table 1, where we list $\phi_{\rm kin}$ and $-\overline{U}_\downarrow/u_{\rm rms} = [(1-f)/f]^{1/2}$ for selected values of f.

$$F_{\rm enth} = \phi_{\rm enth}\,\overline{\rho}u_{\rm rms}^3$$

with $\phi_{\rm enth} = k_{\rm f} H_P/(a_{\rm MLT}\nabla_{\rm ad})$. This yields $\phi_{\rm enth} \approx 20$, which is rather large. By contrast, Brandenburg et al. (2005) determined a quantity k_u such that $\phi_{\rm enth} = k_u^{-3/2} \approx 4$.

Final remarks

- NSSL (near-surface shear layer) not (well) resolved
  - Tremendous difference in time scales: 5 min vs 12 days
  - Length scales: 300 km vs 60 Mm
- Convection instability not by *local* Schwarzschild criterion
  - But stirring from above → drives Deardorff flux
  - No giant cells expected (→ all global simulations flawed!?)
  - Stability depends on *local* opacity law



Opacity κ
Polytropic index *n*

$$\kappa = \kappa_0 \rho^a T^b \qquad n = \frac{3-b}{1+a}$$

Barekat+Brandenburg14

Gradient flux (Böhm-Vitense 1953)
Deardorff flux (Deardorff 1968)

$$\boldsymbol{F}_\mathrm{G} = -\frac{1}{3}\tau_\mathrm{red} u_\mathrm{rms}^2 \overline{\rho}\, \overline{T}\, \boldsymbol{\nabla}\overline{S},$$

$$\boldsymbol{F}_\mathrm{D} = -\tau_\mathrm{red}\, \overline{s^2}\, \boldsymbol{g}\, \overline{\rho}\, \overline{T}/c_P$$

Brandenburg (2016, ApJ 832, 6

Conclusions

- Convection dynamics not quite like mixing length theory
- Slope of entropy matters for convective stability
- Find even hot blobs in convection simulations
- Identified Deardorff term:responsible for subadiabatic conv
- Mixing length model still gives sharp bottom of CZ

Tau approximation

$$\dot{s} = -u_j \nabla_j \overline{S} + N_s$$

$$\dot{u}_i = g_i s / c_p + N_u$$

$$\frac{\partial F_i}{\partial t} \propto \overline{u_i \dot{s}} + \overline{\dot{u}_i s} = -\overline{u_i u_j} \nabla_j \overline{S} + g_i \overline{s^2} / c_p + N_{su}$$

Closure hypothesis

$$N_{su} = -\frac{F_i}{\tau}$$

Another missing piece: surface appearence

- Stratified MHD turbulence produces spots
  - Even without convection
  - Can form + disappear in days
  - Strong scale separation required
  - Best in forced turbulence
  - Unclear how important for the Sun
- Buoyant rise picture questionable
  - Expansion during ascent
  - Slender tubes not seen in simulations
  - Anticipated role of tachocline?
- Link between dynamo & butterfly
  - Must be integral part of solar dynamo
  - Surface appearance possibly shallow

Brandenburg+13