Effects of rotation on convective scale and Deardorff layers

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Convective scale and subadiabatic layers in simulations of rotating compressible convection

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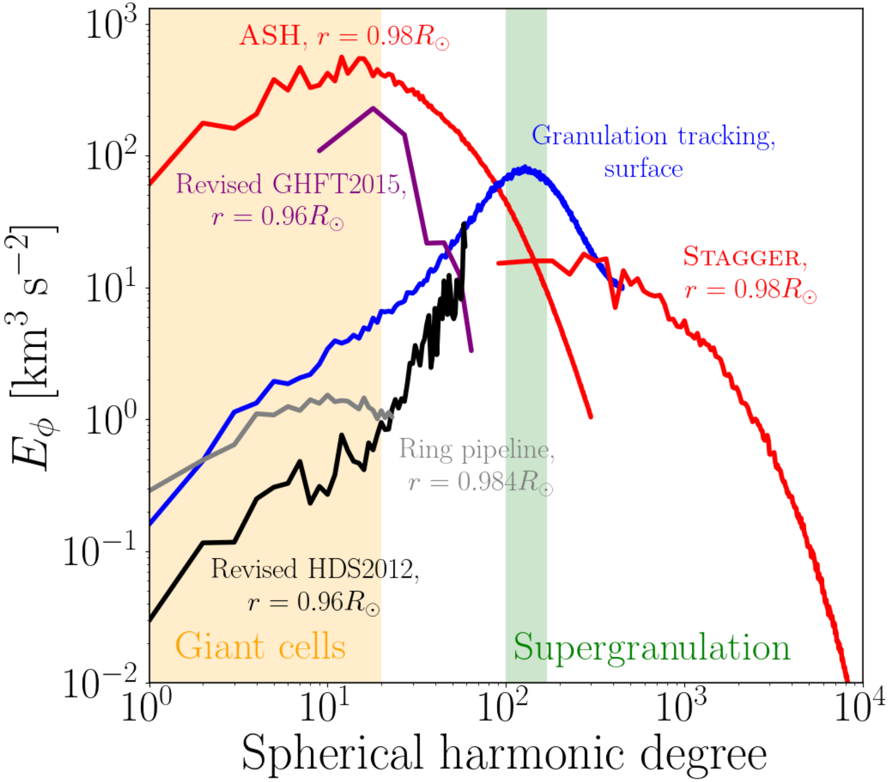
ABSTRACT

Context. Rotation is thought to influence the size of convective eddies and the efficiency of convective energy transport in the deep convection zones of stars. Rotationally constrained convection has been invoked to explain the lack of large-scale power in observations of solar flows.

Käpylä (2024), *Astron. Astrophys*, **683**, 221

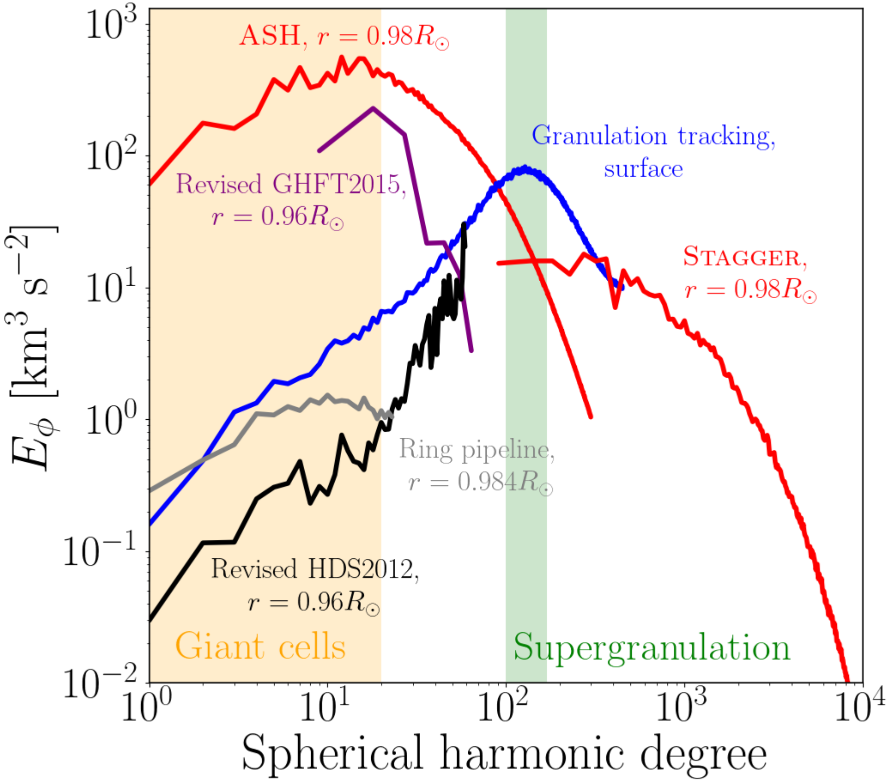
Stellar Convection: Modelling, Theory and Observations; Nordita, 29th Aug 2024

Motivation: What is the true nature of convection in the Sun?



Stellar Convection: Modelling, Theory and Observations; Nordita, 29th Aug 2024

I find your lack of supergranulation disturbing...



Ansatz I: Subadiabatic deep convection

The Schwarzschild criterion:

$$\Delta\nabla = \nabla - \nabla_{\rm ad} = -\frac{H_{\rm P}}{c_{\rm P}}\frac{{\rm d}s}{{\rm d}r} > 0,$$

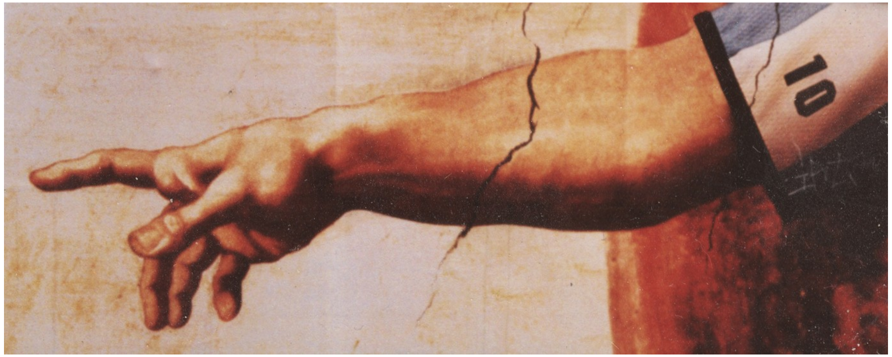
is encoded in mixing-length theory (MLT; cf. Vitense 1953) of convection: $F_{\rm conv} \propto (\Delta\nabla)^{3/2}$.

Therefore MLT implies local driving of convection up to a scale of $200\,{\rm Mm}$ (giant cells).

This has been internalized by many simulation setups such that the depth of the CZ is predetermined and fixed, and in mean-field theory by assuming $\overline{\boldsymbol{F}}_{\rm conv} = -\chi_{\rm t}\rho T\boldsymbol{\nabla}\overline{s}$.

The classics state that a convection zone (CZ) is always superadiabatic. So are my simulations. Ergo, so must the solar CZ!

Beware of the Maradona effect!



Ansatz I: Subadiabatic deep convection

CONVECTION IN STELLAR ENVELOPES: A CHANGING PARADIGM

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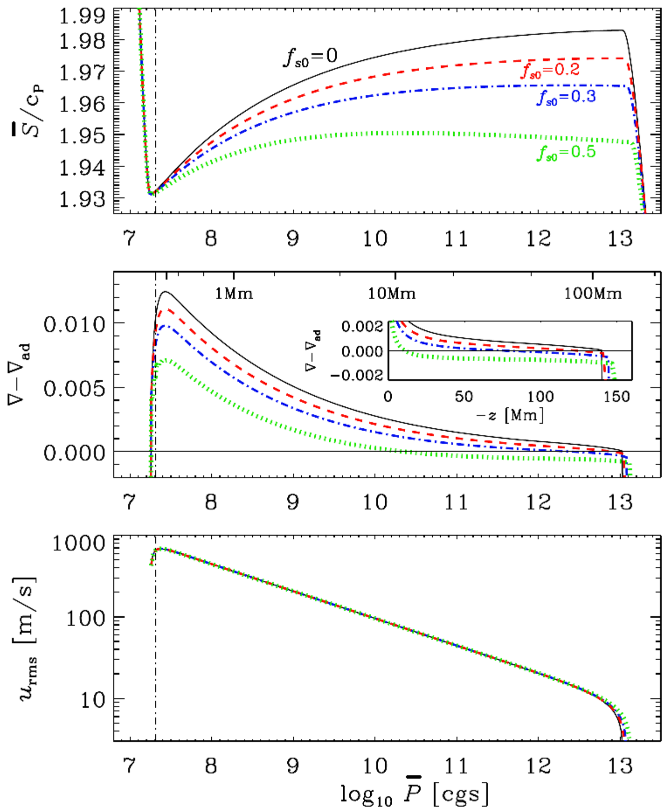
ABSTRACT. Progress in the theory of stellar convection over the past decade is reviewed. The similarities and differences between convection in stellar envelopes and laboratory convection at high Rayleigh numbers are discussed. Direct numerical simulation of the solar surface layers, with no other input than atomic physics, the equations of hydrodynamics and radiative transfer is now capable of reproducing the observed heat flux, convection velocities, granulation patterns and line profiles with remarkably accuracy. These results show that convection in stellar envelopes is an essentially non-local process, being driven by cooling at the surface. This differs distinctly from the traditional view of stellar convection in terms of local concepts such as cascades of eddies in a mean superadiabatic gradient. The consequences this has for our physical picture of processes in the convective envelope are illustrated with the problems of sunspot heat flux blocking, the eruption of magnetic flux from the base of the convection zone, and the Lithium depletion problem.

Spruit (1997), *Mem. d. Soc. Astron. Italiana*, **68**, 397



Meanwhile in atmospheric physics:

$$\overline{\boldsymbol{F}}_{\rm enth} = -\chi_{\rm t}\rho T \boldsymbol{\nabla}\overline{s} + \tau\overline{\rho}\overline{T} g \overline{s'^2}/c_{\rm P}.$$

Deardorff (1961/66), *J. Atmosph. Phys.*, **18**, 540 / **23**, 503.

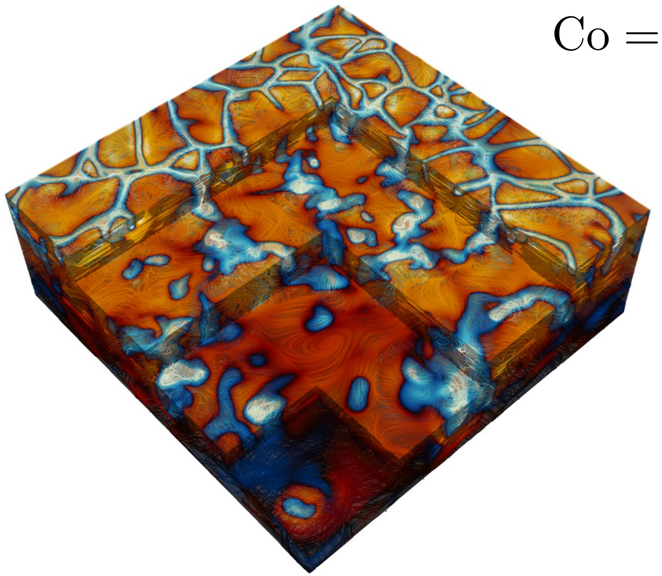
Addition of such non-locality *can* lead to a very thin superadiabatic layer + deep Deardorff zone.

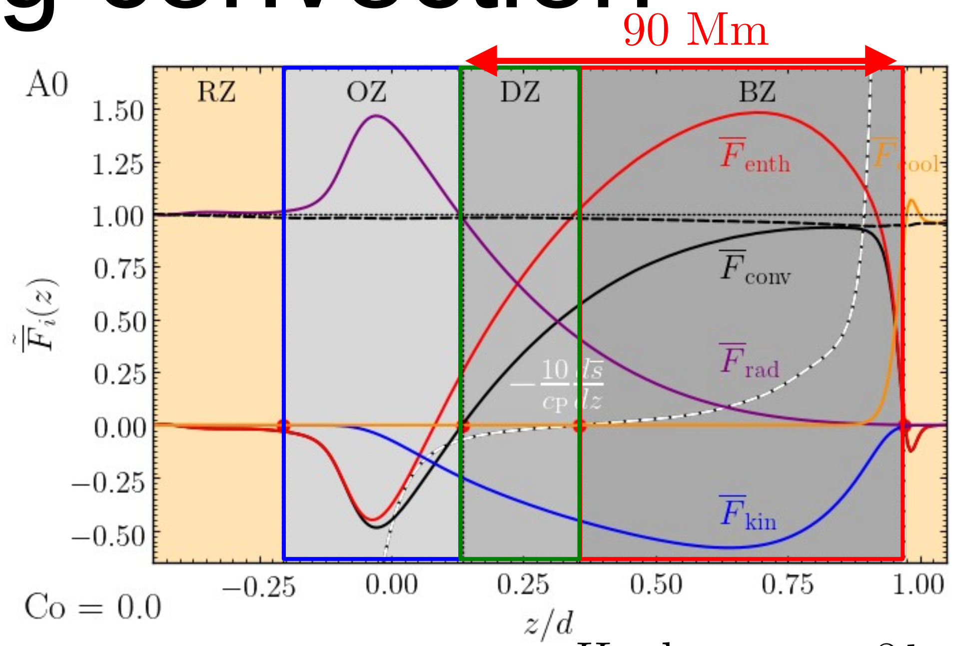
Brandenburg (2016), *Astrophys. J.*, **832**, 6

Chan & Gigas (1992), *Astrophys. J.*, **389**, 87

Roxburgh & Simmons (1993), *Astron. Astrophys.*, **277**, 93

Non-rotating convection





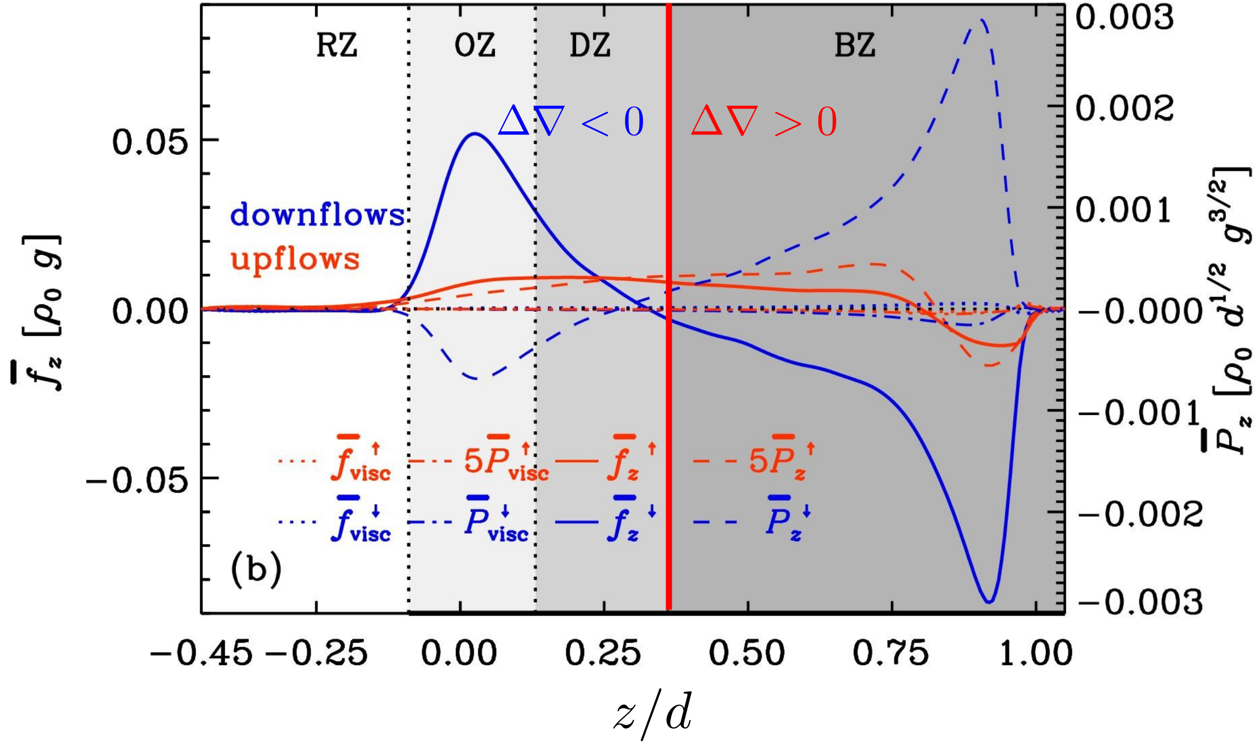
$$\overline{F}_{\rm conv} = \overline{F}_{\rm kin} + \overline{F}_{\rm enth} = \overline{\tfrac{1}{2}\rho\boldsymbol{u}^2 u_z} + c_{\rm P}\overline{(\rho u_z)'T'}, \quad \Delta\nabla = \nabla - \nabla_{\rm ad} = -\frac{H_{\rm P}}{c_{\rm P}}\frac{{\rm d}s}{{\rm d}z}, \quad \nabla = \frac{\partial \ln T}{\partial \ln p}.$$

Buoyancy zone (BZ): convective energy flux positive, superadiabatic temperature gradient.

Deardorff zone (DZ): convective energy flux positive, subadiabatic temperature gradient.

Overshoot zone (OZ): convective energy flux negative, subadiabatic temperature gradient.

Surface cooling drives convection

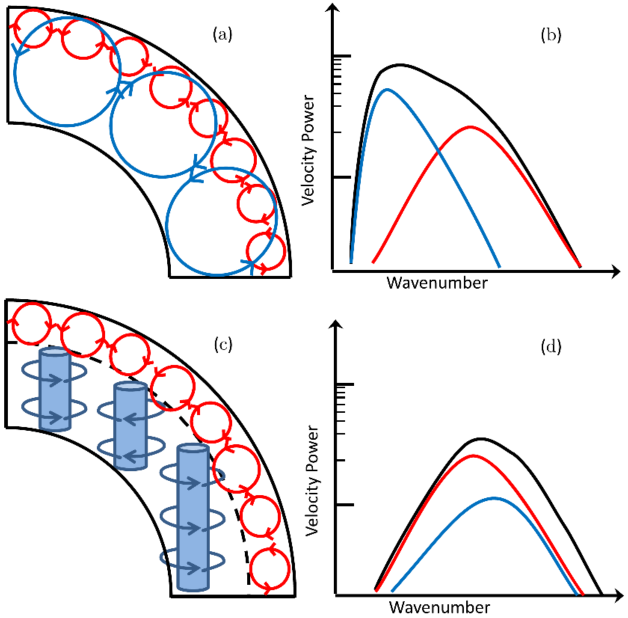


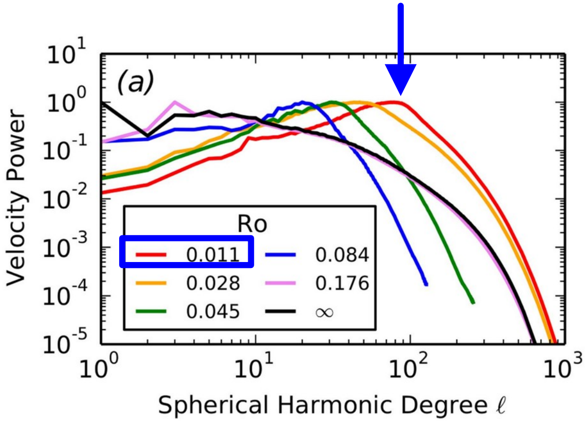
Käpylä et al. (2017), *Astrophys. J. Lett.*, **845**, L23

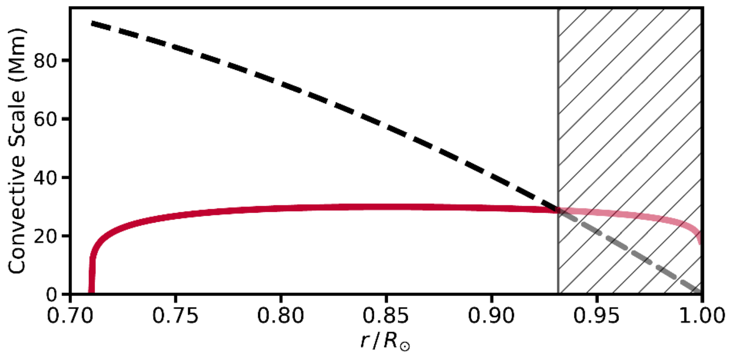
Ansatz II: Rotationally constrained convection

Conjecture: Convection in deep parts of solar CZ is strongly affected by rotation such that large-scales are suppressed. The maximum scale of convection coincides with that of supergranulation with $\ell_{\rm sph} \approx 100$.

Vasil et al. (2021), *Proc. Nat. Acad. Sci.*, **118**, 31







These studies assume *a priori* that rotational influence on convection is strong. But can we estimate this independently?

Featherstone & Hindman (2016), *Astrophys. J. Lett.*, **830**, 15

Simulations and relation to reality

Dimensionless numbers in simulations:

$$\mathrm{Ra} = \frac{gd^4}{\nu\chi}\left(-\frac{1}{c_\mathrm{P}}\frac{\mathrm{d}s}{\mathrm{d}z}\right), \quad \mathrm{Pr} = \frac{\nu}{\chi}, \quad \mathrm{Pr_M} = \frac{\eta}{\nu},$$

$$\mathrm{Re} = \frac{u_\mathrm{rms}\ell}{\nu}, \quad \mathrm{Re_M} = \mathrm{PmRe}, \quad \mathrm{Pe} = \mathrm{PrRe},$$

$$\Delta\rho = \frac{\rho_\mathrm{bot}}{\rho_\mathrm{top}}, \quad \mathrm{Ro} = \frac{u_\mathrm{rms}}{2\Omega\ell} = \mathrm{Co}^{-1}.$$

Simulations can only reproduce Co (Ro).

Parameter	Sun ($M_\odot$)	O9 ($20M_\odot$)	Simulations
Ra	10^{20}	10^{24}	10^{9}
Pr	10^{-6}	10^{-5}	$10^{-1}\ldots10$
$\mathrm{Pr_M}$	10^{-3}	10^{3}	$10^{-1}\ldots10$
Re	10^{13}	10^{11}	10^{4}
Pe	10^{7}	10^{6}	10^{4}
$\mathrm{Re_M}$	10^{10}	10^{14}	10^{4}
$\Delta\rho$	10^{6}	3	10^{2}
Ro	$0.1\ldots1$	1^1	$10^{-2}\ldots10^{3}$

Käpylä et al. (2023), *Space Science Rev.*, **219**, 58

The actual velocity and length scales in the deep CZ of the Sun are unknown:

$$\mathrm{Co} = \frac{2\Omega\ell}{u_\mathrm{conv}} = \mathrm{Ro}^{-1}.$$

Stating that e.g. $\mathrm{Co} \approx 10$ in the deep CZ is woefully imprecise!

Simulations and relation to reality

Let us define a hypothetical velocity measuring of the energy flux:

$$F_{\rm tot} = \rho u_\star^3, \quad u_\star = \left(\frac{F_{\rm tot}}{\rho}\right)^{1/3}.$$

Define a *flux Coriolis number*:

$${\rm Co_F} = 2\Omega H_{\rm p}\left(\frac{\rho}{F_{\rm tot}}\right)^{1/3}. \qquad {\rm Co_F^\odot} \approx 3.1.$$

This is a *hypothetical* measure of the importance of rotation based on the available energy flux. No dependence on dynamical length or velocity scales!

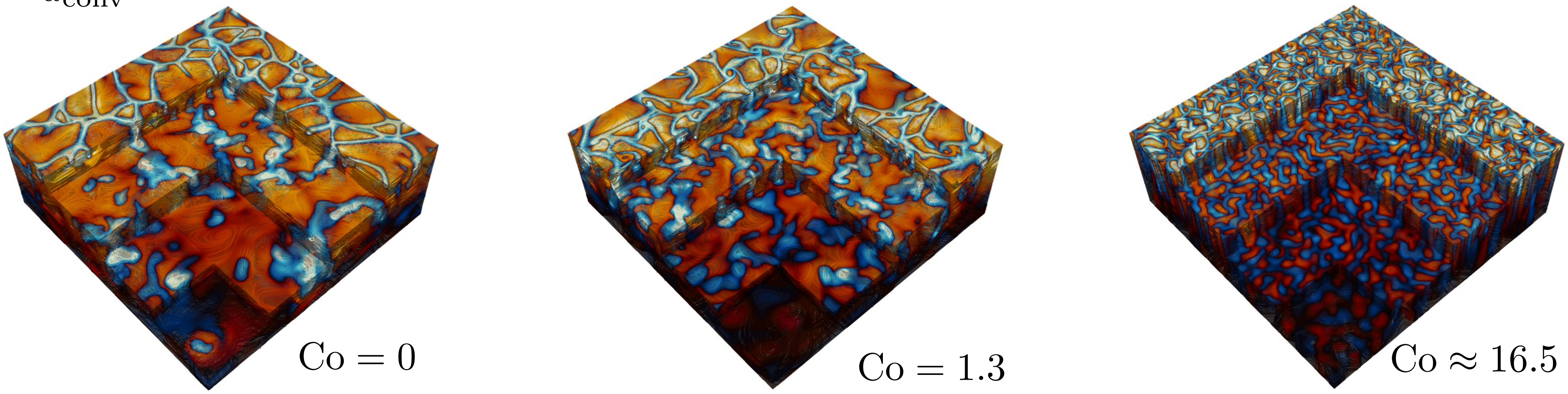
Turns out that: ${\rm Co_F} = ({\rm Ra_F^\star})^{-1/3}$, where ${\rm Ra_F^\star} = \dfrac{\rm Ra_F}{\rm Pr^2\, Ta^{3/2}} = \dfrac{F_{\rm tot}}{8\rho\Omega^3 H_{\rm p}^3}$.

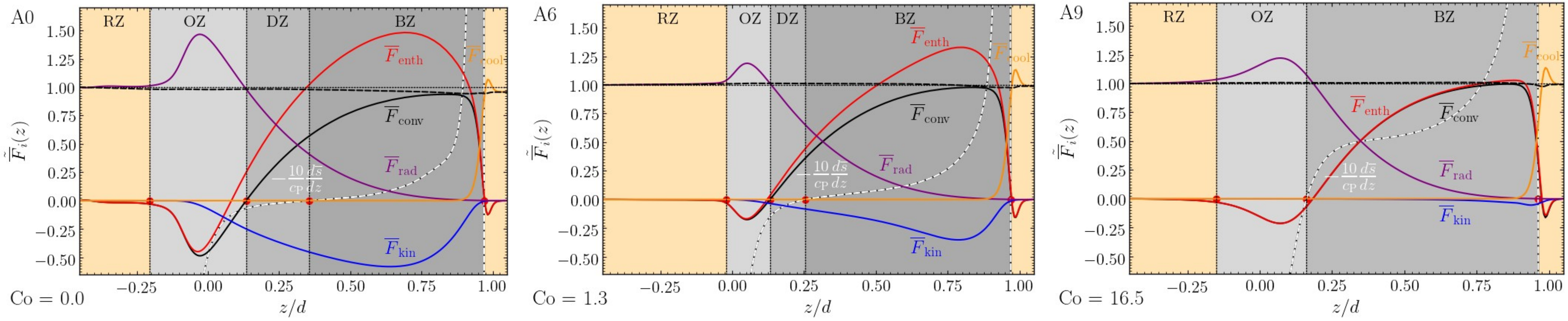
It is necessary (but not sufficient!) to reproduce ${\rm Co_F}$ in a numerical simulation claiming to target the Sun.

Aurnou et al. (2020), *Phys. Rev. Research*, **2**, 043115

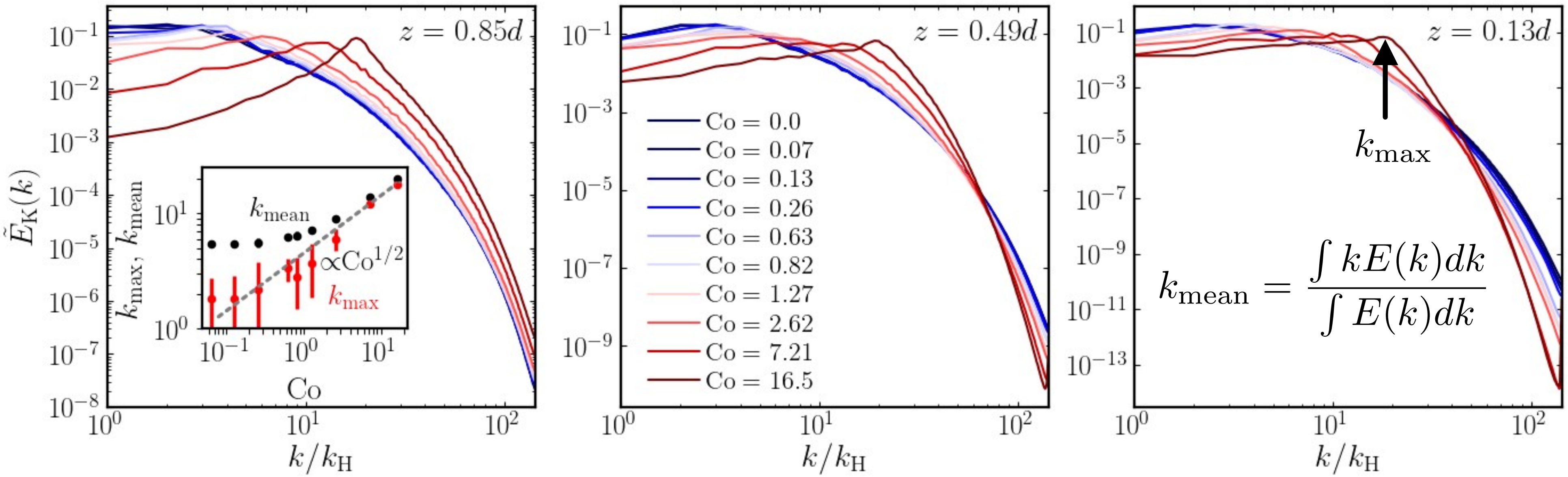
Rotating convection

$$\mathrm{Co} = \frac{2\Omega\ell}{u_{\rm conv}}.$$





Convective scale as a function of Ω



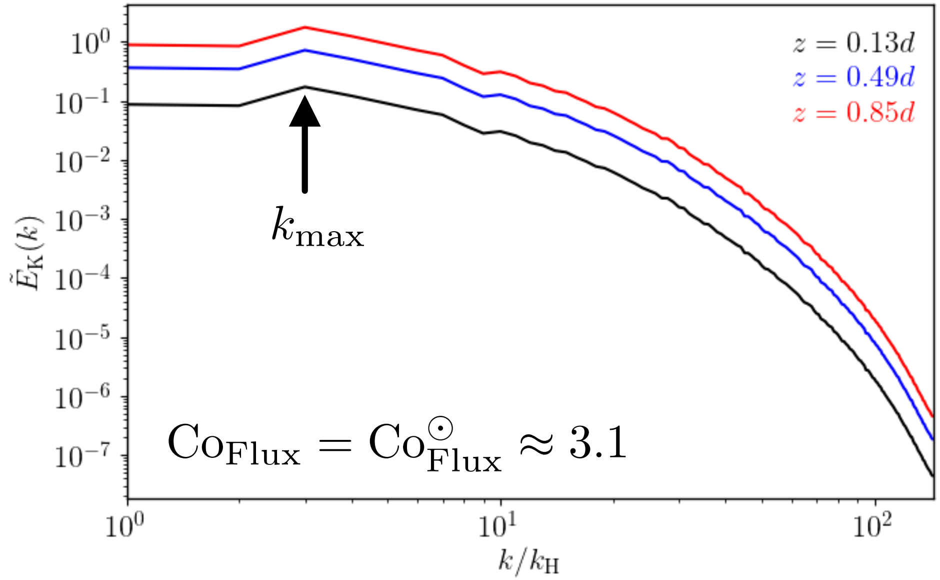
$$\ell_{\rm max} = \frac{2\pi}{k_{\rm max}}, \quad \ell_{\rm mean} = \frac{2\pi}{k_{\rm mean}}.$$

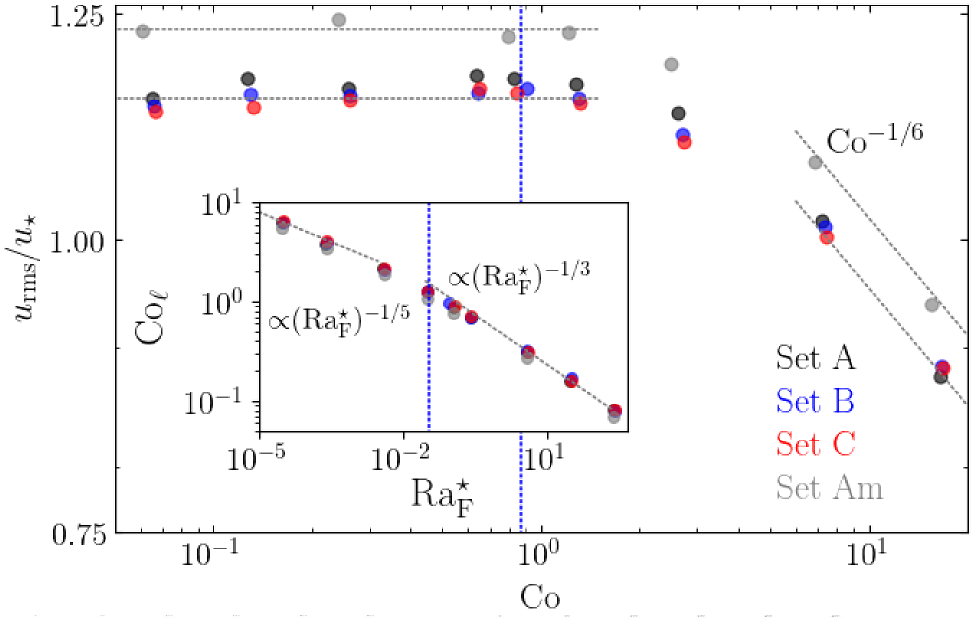
$$k_{\rm H} = 2\pi/L_{\rm H}, \quad H_{\rm p} \approx 0.49d \; (= 50 \text{ Mm}).$$

Featherstone & Hindman (2016) ${\rm Ro}_{\rm FH16} = 0.011$ run: ${\rm Co} = (2\pi {\rm Ro}_{\rm FH16})^{-1} \approx 14.5$. Current run with ${\rm Co} = 17$: $k_{\rm max} = 17k_{\rm H}$, $\ell_{\rm max} \approx 0.235d \approx 0.48H_{\rm p}$, corresponding to 24 Mm.

But this corresponds to $\Omega \approx 15\Omega_\odot$!

So what about the Sun?





Convective scale in the Sun is affected by rotation but only mildly even in the deep convection zone!

Rotationally constrained convection ruled out as a solution to the convective conundrum?

Conclusions

Non-rotating and moderately rotating convection zones have stably stratified, yet convective layers (Deardorff zones). But these do not solve the convective conundrum!

For rapid enough rotation the Deardorff zone vanishes in simulations. Is this still the case in real stars where convection and surface forcing is more vigorous?

Current results suggest that the solar convection zone is not strongly rotationally constrained anywhere. This rules out another way to solve the convective conundrum?

What is it then? Surface effects, magnetic fields, Prandtl number, resolution,
or unknown unknowns?

Overshooting depth vs. rotation

$d_{\rm OZ}^{\rm c}$: depth of OZ based on $F_{\rm conv}$.

$d_{\rm OZ}^{\rm k}$: depth of OZ based on $F_{\rm kin}$.

$d_{\rm DZ}$: depth of DZ.

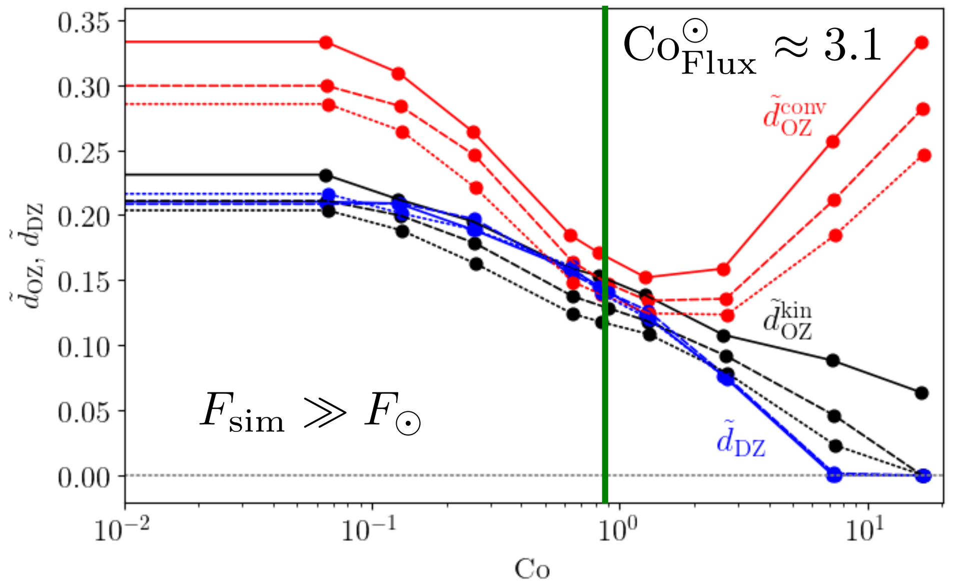
Deardorff zone vanishes for rapid enough rotation, overshooting?

OZ decreases until around $\mathrm{Co}\approx 1$ by both measures.

The solar case only mildly affected by rotation, OZ depth between a third and a half reduced.

→ $d_{\rm os}^{\odot} \approx 0.05 H_{\rm p}$.

(Käpylä 2019, *Astron. Astrophys*, **631**, 122)



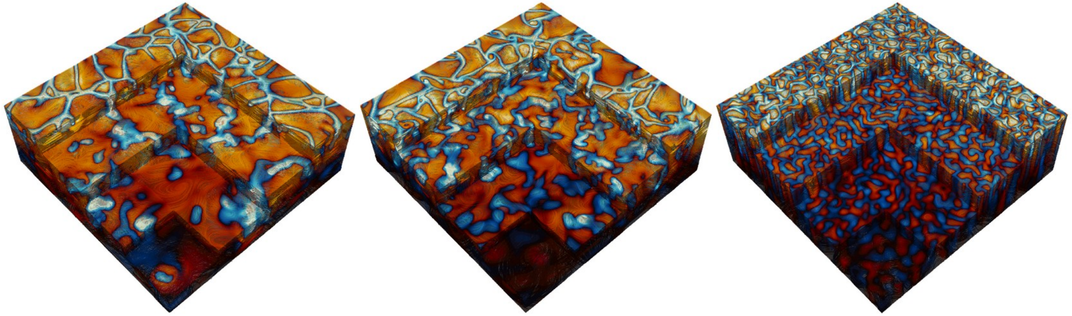
For $\mathrm{Co} > 1$ the Richardson number is:

$$\mathrm{Ri}_{\Omega} = \frac{N^2}{\Omega_0^2} \lesssim 1, \ \ \text{where}\ N^2 = \frac{g}{c_{\rm P}}\frac{\mathrm{d}s}{\mathrm{d}z}.$$

In the Sun, however: $\mathrm{Ri}_{\Omega} \approx 10^4$.

Numerical simulations

Cartesian setup with the Pencil Code; fully compressible equations in rotating frame:



https://github.com/pencil-code

The Pencil Code Collaboration (2021), *J. Open Source Softw.*, **6**, 2807

$$\frac{D\ln\rho}{Dt} = -\boldsymbol{\nabla}\cdot\boldsymbol{u},$$

$$\frac{D\boldsymbol{u}}{Dt} = \boldsymbol{g} - \frac{1}{\rho}(\boldsymbol{\nabla}p - \boldsymbol{\nabla}\cdot 2\nu\rho\mathbf{S}) - 2\boldsymbol{\Omega}\times\boldsymbol{u},$$

$$T\frac{Ds}{Dt} = -\frac{1}{\rho}\left[\boldsymbol{\nabla}\cdot(\boldsymbol{F}_{\rm rad} + \boldsymbol{F}_{\rm SGS}) - \mathcal{C}\right] + 2\nu\mathbf{S}^2,$$

$$\boldsymbol{F}^{\rm rad} = -K\boldsymbol{\nabla}T, \quad K = K_0\rho^{-2}T^{6.5}.$$

$$\boldsymbol{F}^{\rm SGS} = -\chi_{\rm SGS}\rho\boldsymbol{\nabla}s'.$$

$$\mathrm{Re} = \frac{u_{\rm rms}\ell}{\nu} \approx 40,\ \mathrm{Pe} = \mathrm{PrRe}, \quad \mathrm{Pr} = \frac{\nu}{\chi_{\rm SGS}} = 1,\ \ell = k_1^{-1}. \quad \mathrm{Co} = \frac{2\Omega\ell}{u_{\rm conv}} = 0\ldots 17.$$