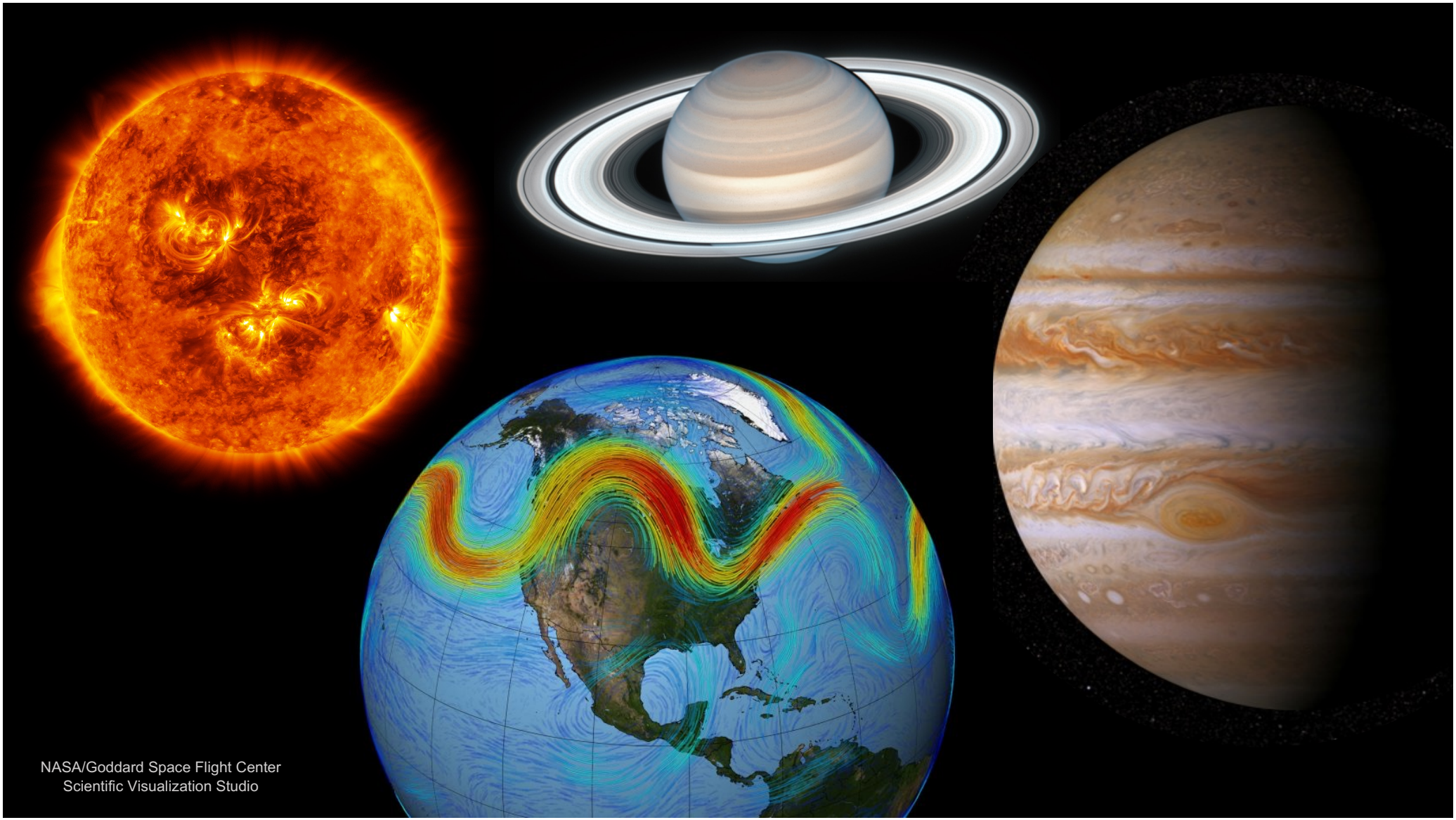
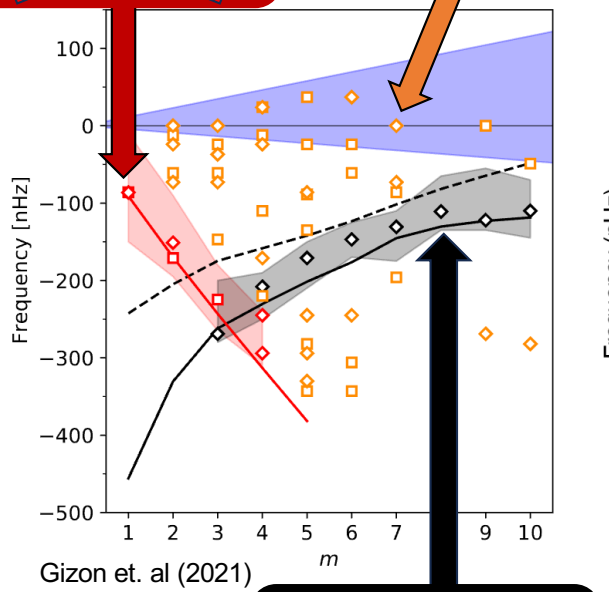
The Inertial Wave Menagerie

Catherine Blume, *University of Colorado-Boulder*
Brad Hindman, *University of Colorado-Boulder*
Loren Matilsky, *UC-Santa Cruz*
Rekha Jain, *University of Sheffield*

Stellar Convection: Modeling, Theory, and Observation
August 27, 2024


NASA/Goddard Space Flight Center
Scientific Visualization Studio

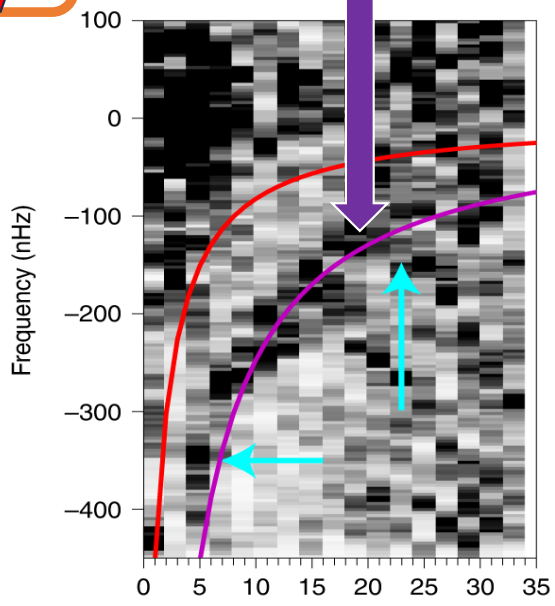
Observed



Gizon et. al (2021)

Equatorial Rossby waves

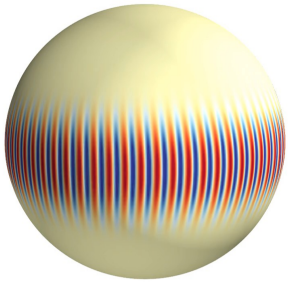
HFR modes



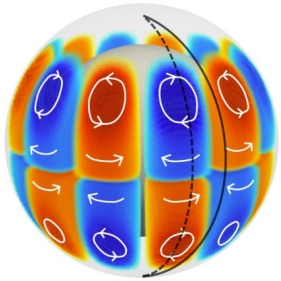
Hanson et. al (2022)

Modeled

Thermal Rossby waves

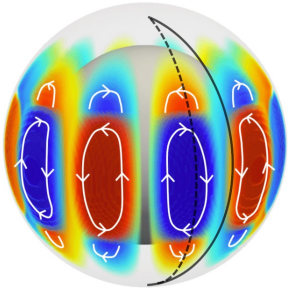


Hindman et. al (2020a)



Bekki et. al (2022a)

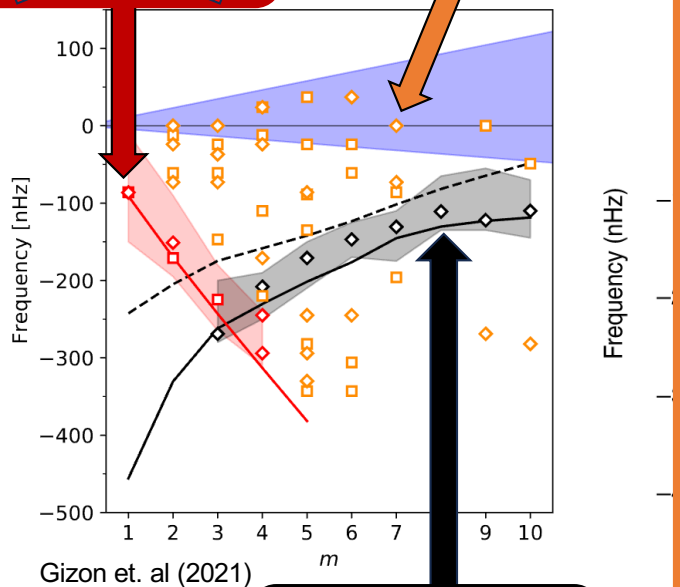
Equatorial mixed mode



Bekki et. al (2022a)

Observed

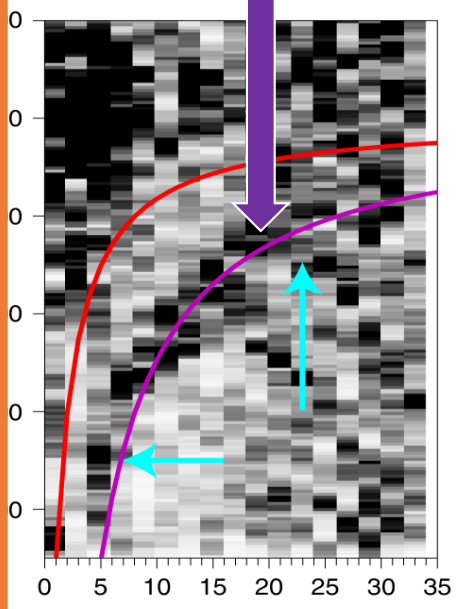
Modeled



Gizon et. al (2021)

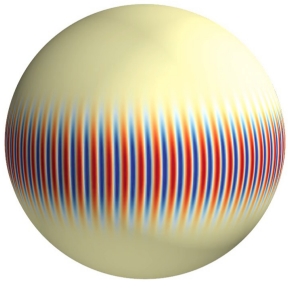
Equatorial Rossby waves

HFR modes

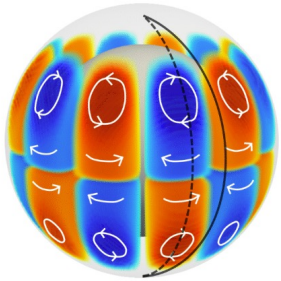


Hanson et. al (2022)

Thermal Rossby waves

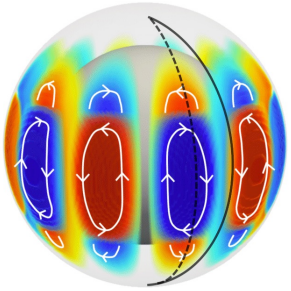


Hindman et. al (2020a)



Bekki et. al (2022a)

Equatorial mixed mode



Bekki et. al (2022a)

Outline

Part I **Simulation background**

Part II **Equatorial Rossby waves**

Part III **Other inertial waves in the convection zone**

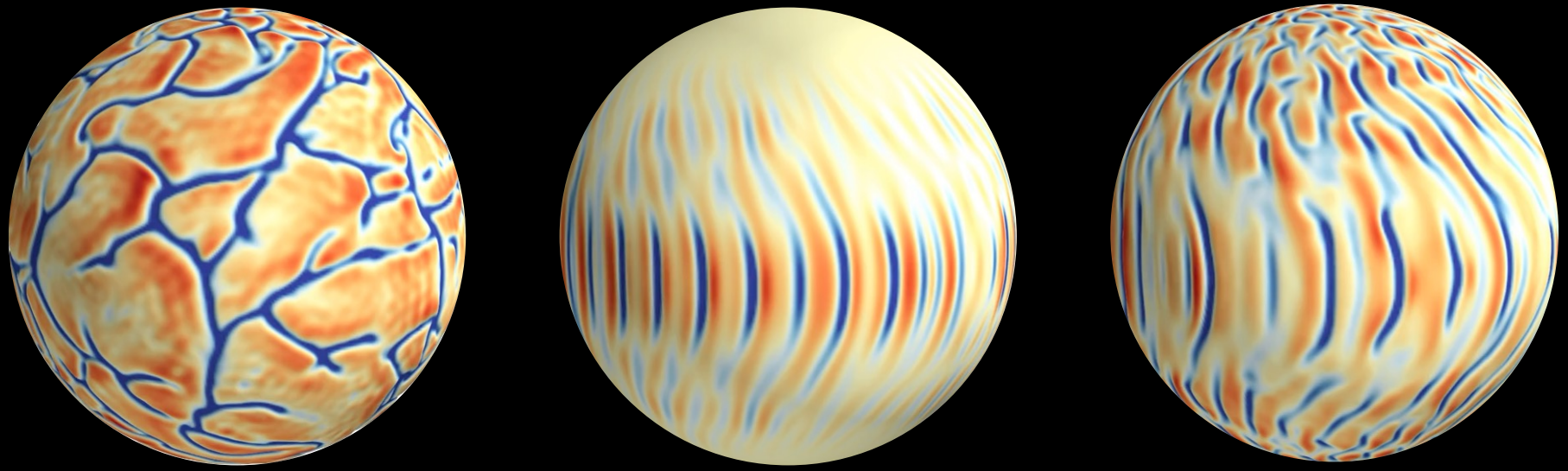
Part IV **A unifying model**

Rayleigh

3D convection code in rotating spherical shell geometry

COMPUTATIONAL INFRASTRUCTURE for GEODYNAMICS

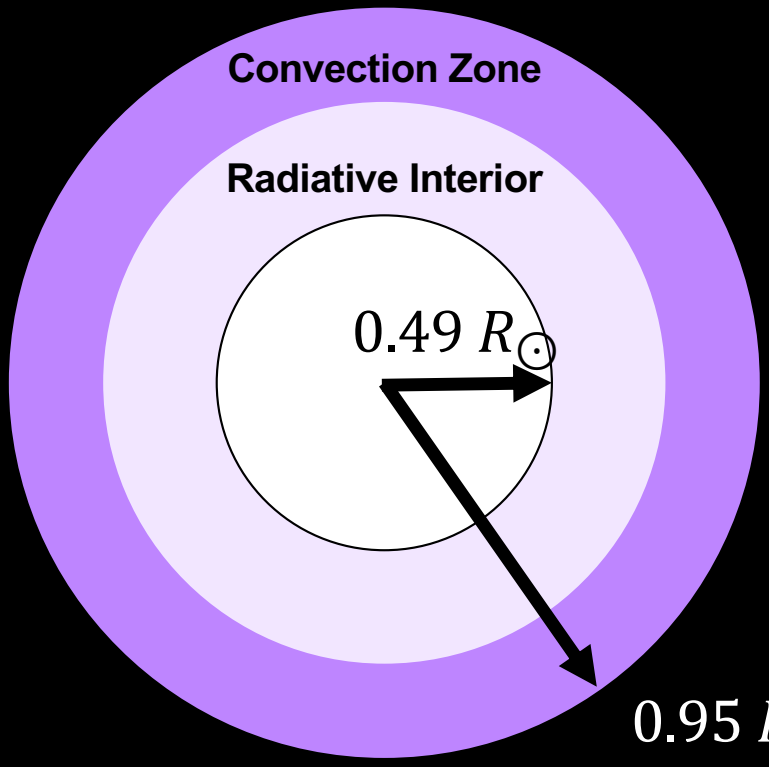
www.github.com/geodynamics/Rayleigh



Hindman and Featherstone (2020)

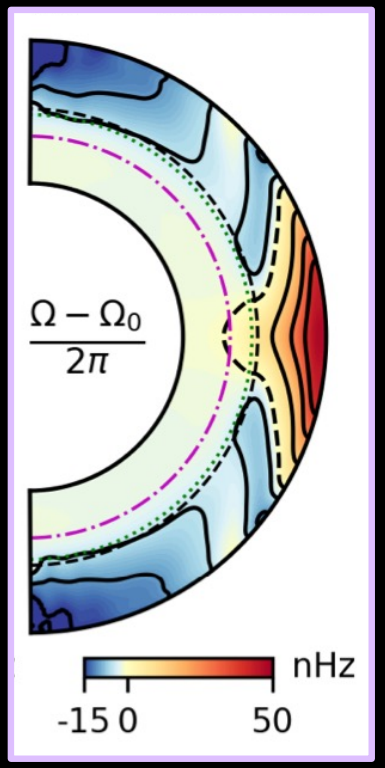
Simulation Characteristics

Matilsky et. al (2022)
Blume et. al (2024)



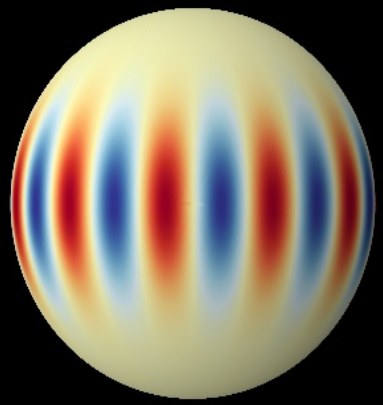
Anelastic MHD

A magnetic field generates a torque that forces the radiative interior to rigidly rotate



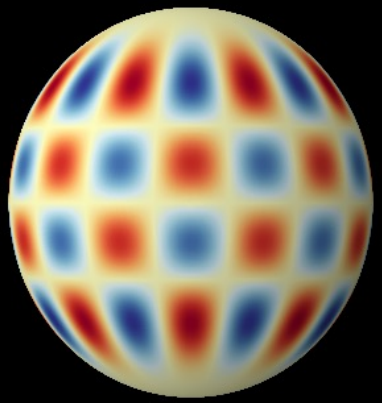
Spherical Harmonics $Y_l^m(\theta, \phi)$

$\lambda = l - m$ is the number of nodes in latitude



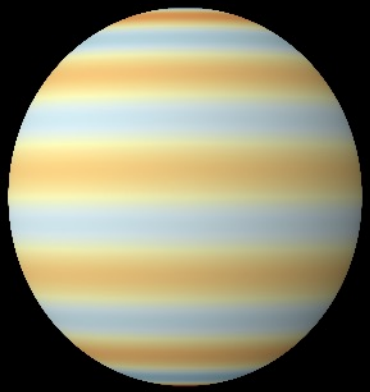
$l = 11$
$m = 11$

sectoral modes
$l = m$



$l = 11$
$m = 8$

tesseral modes
$l \neq m \neq 0$

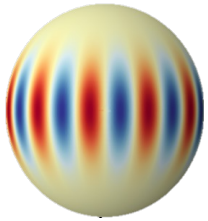


$l = 11$
$m = 0$

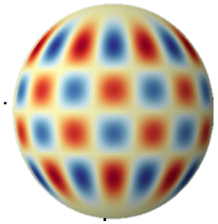
zonal modes
$m = 0$

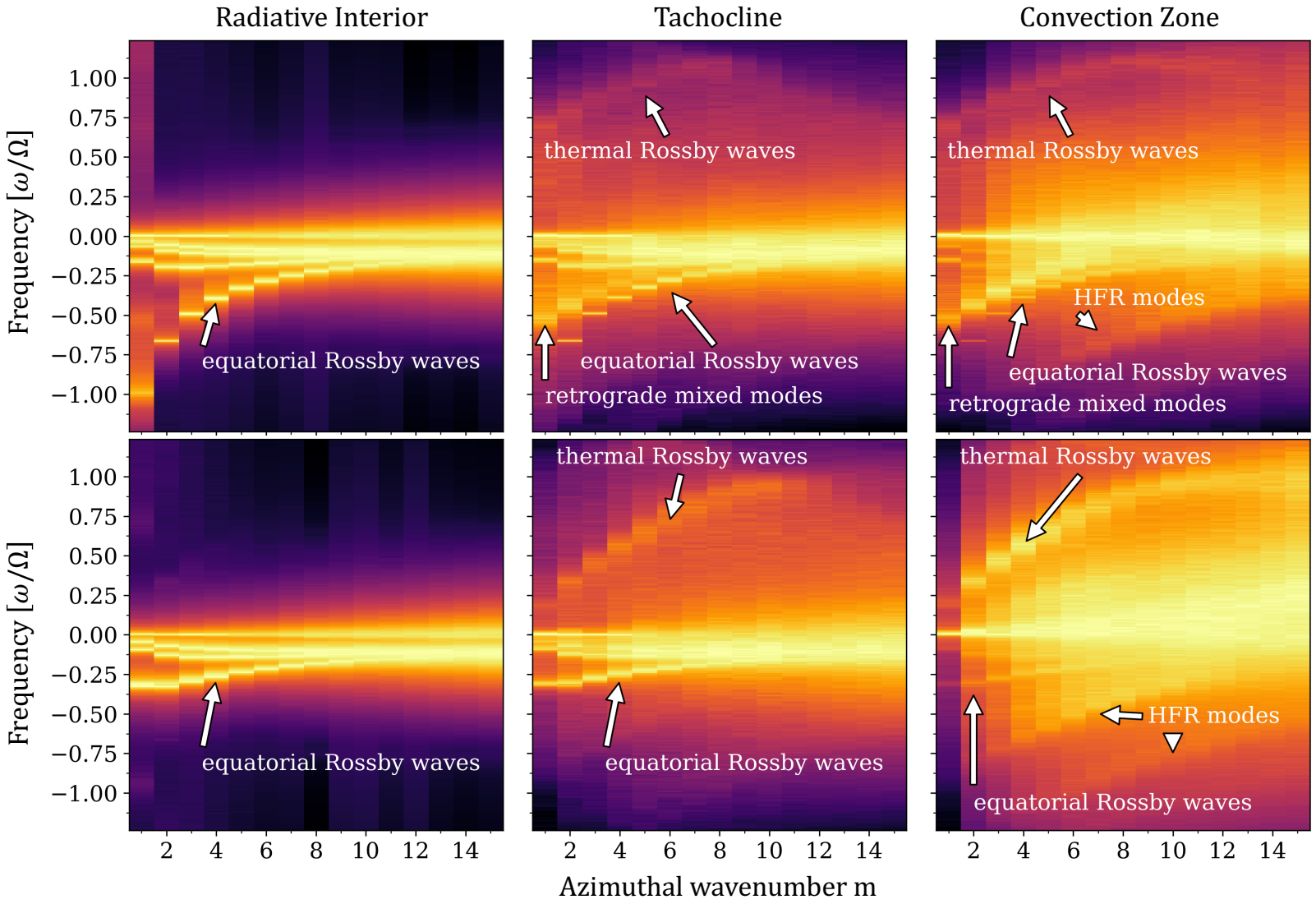
ζ_r Spectra

Symmetric modes
$\lambda = 0, 2, 4, 6$



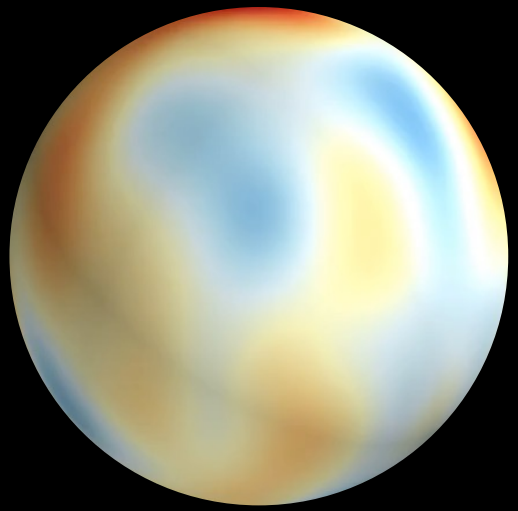
Antisymmetric modes
$\lambda = 1, 3, 5$





Part II

Classical Rossby waves



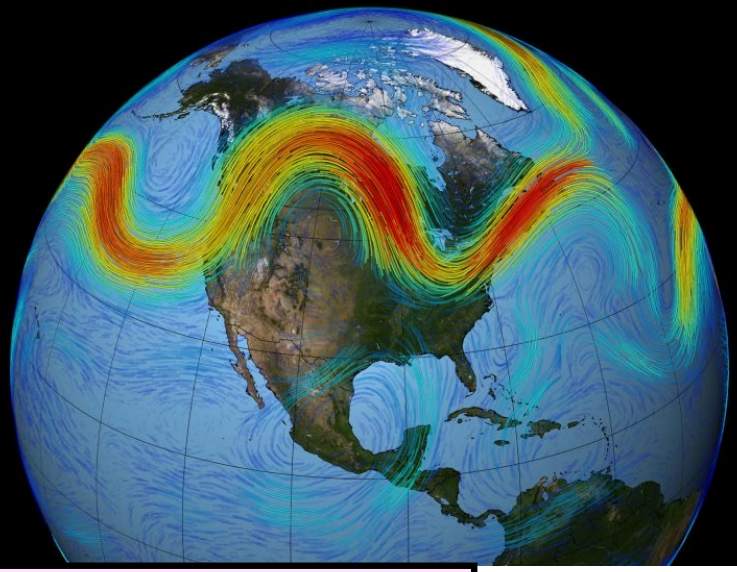
Radiative interior

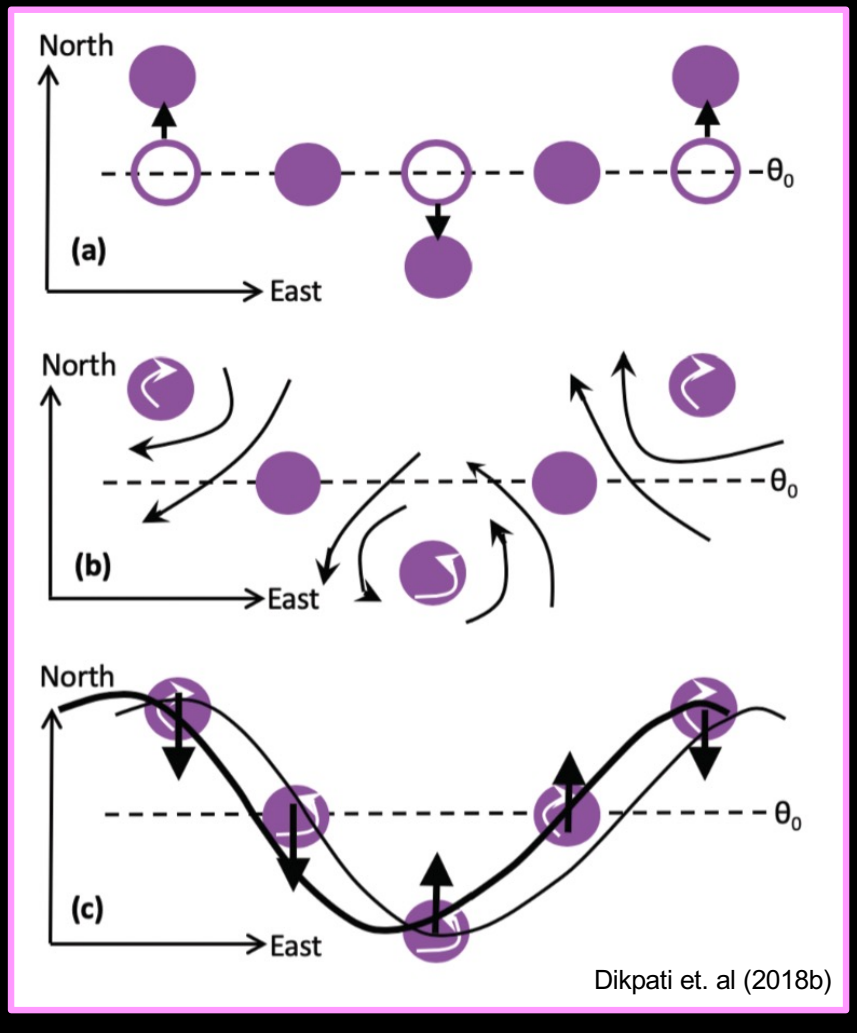
Convection zone

Two wave cavities

Rossby waves arise from the conservation of potential vorticity $\zeta_{a,r}$

$$\zeta_{a,r} = 2\Omega\cos\theta + (\nabla \times \vec{v})_r$$





Dikpati et. al (2018b)

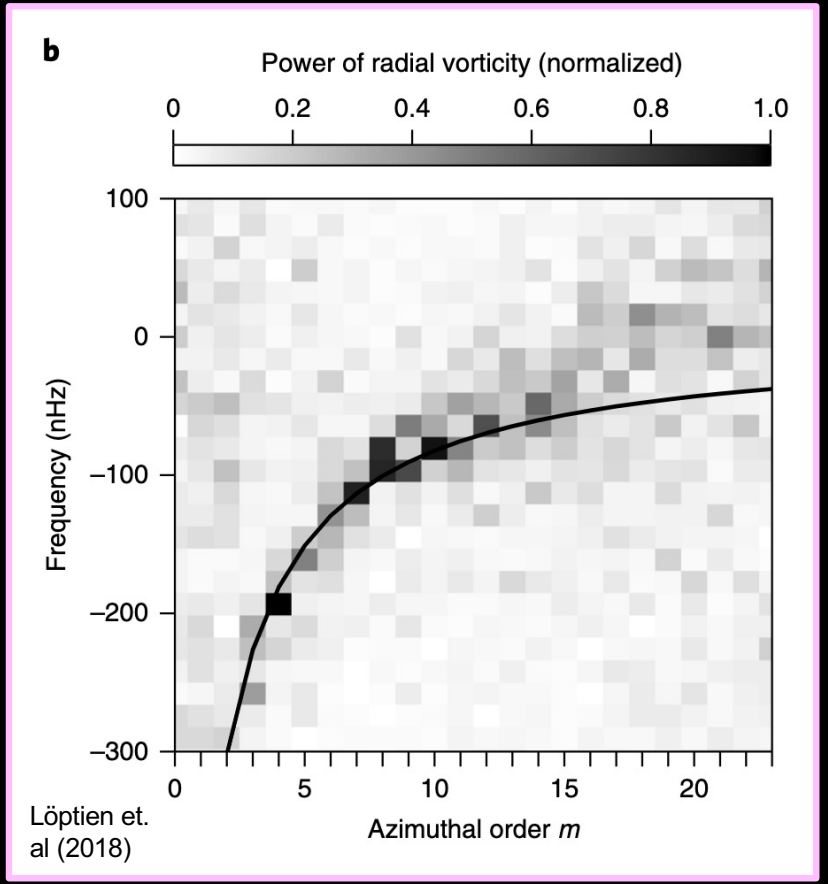
Classical Rossby Waves

Dispersion relation for 2D, hydrodynamic, solid-body case

$$\omega = -\frac{2\Omega m}{l(l+1)}$$

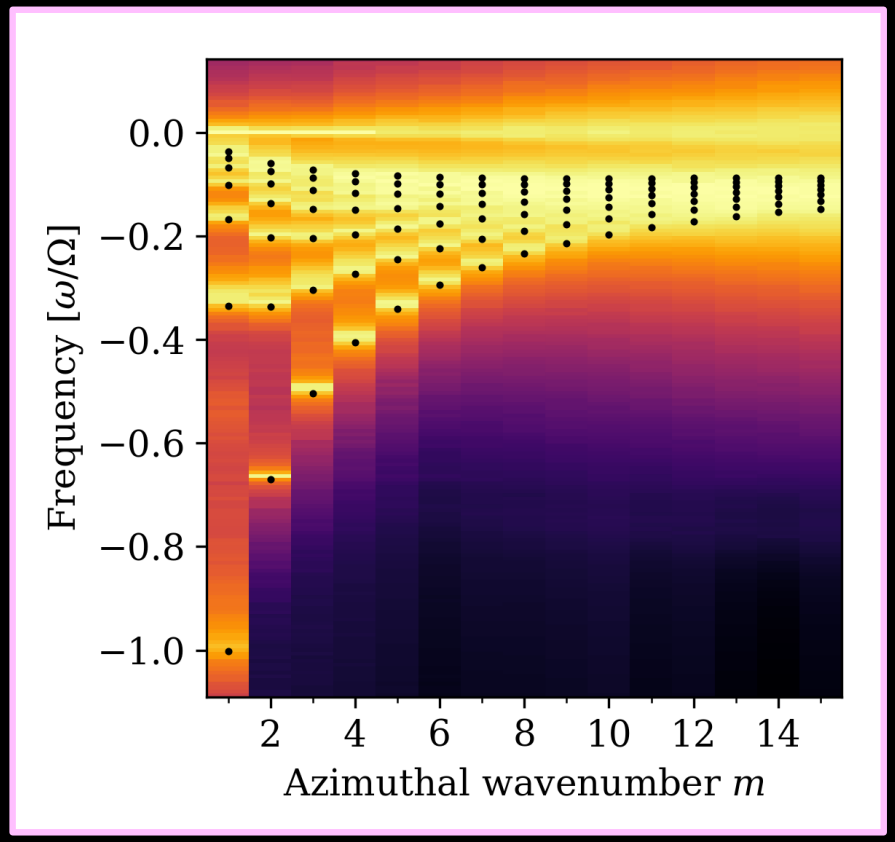
Eigenfunctions

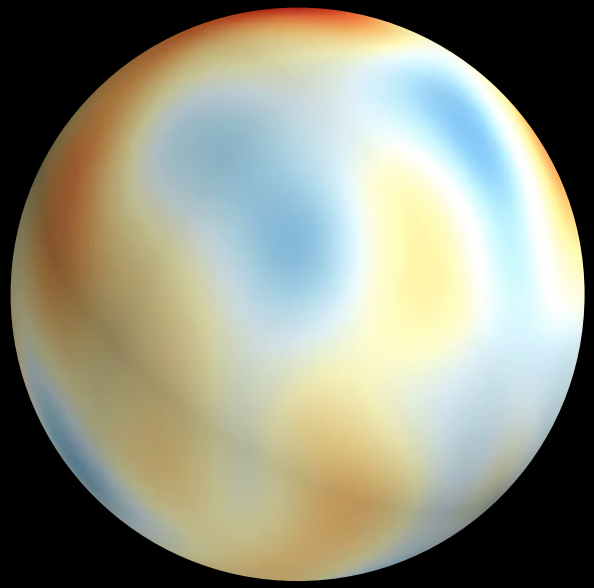
$$P_l^m(\cos\theta)e^{i(m\phi-\omega t)}$$



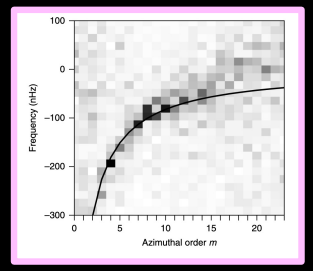
Löptien et. al (2018)

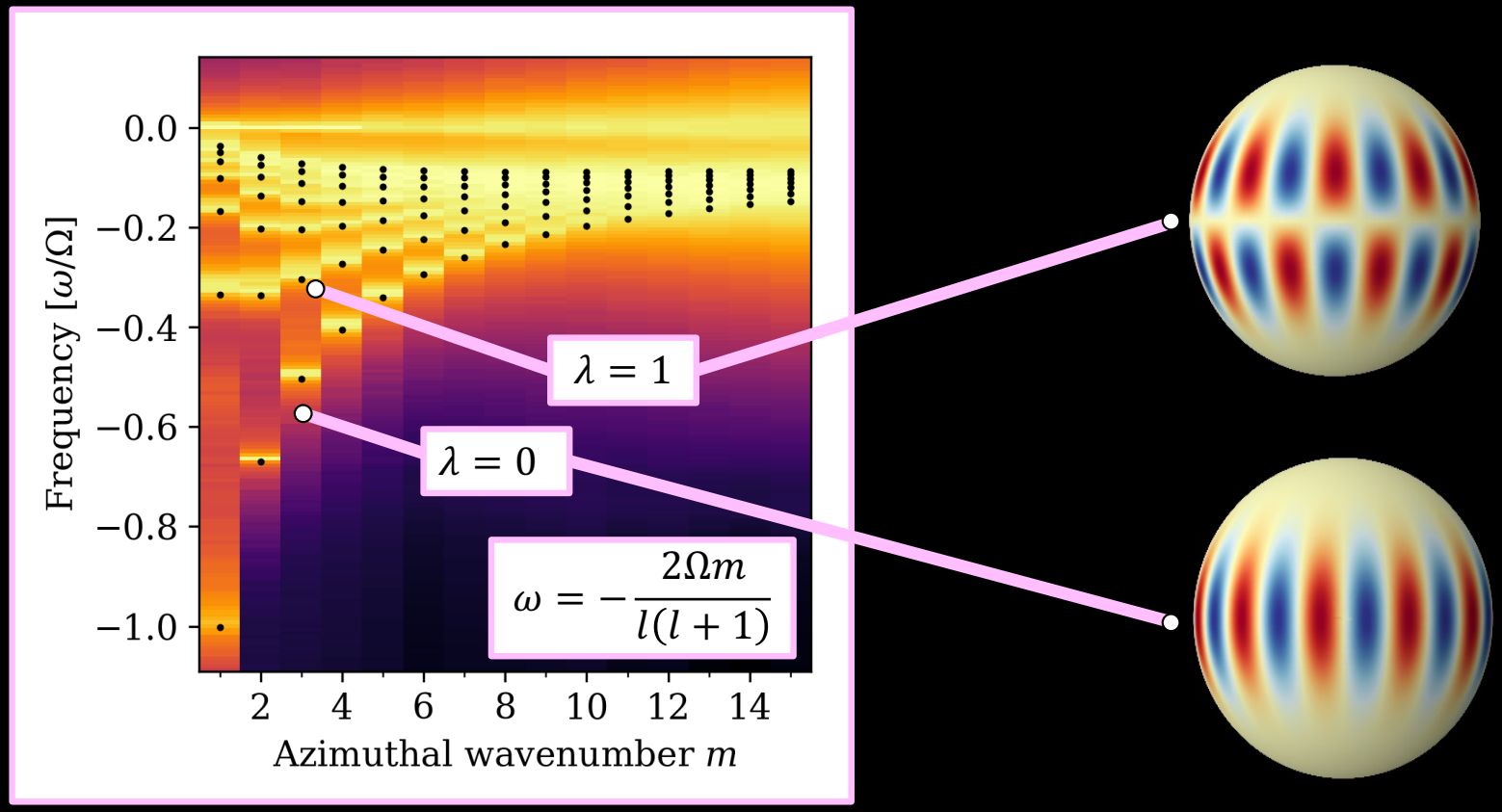
Classical Rossby waves in the radiative interior





Rossby waves in the radiative interior

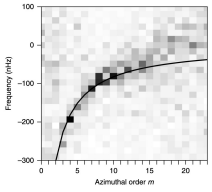


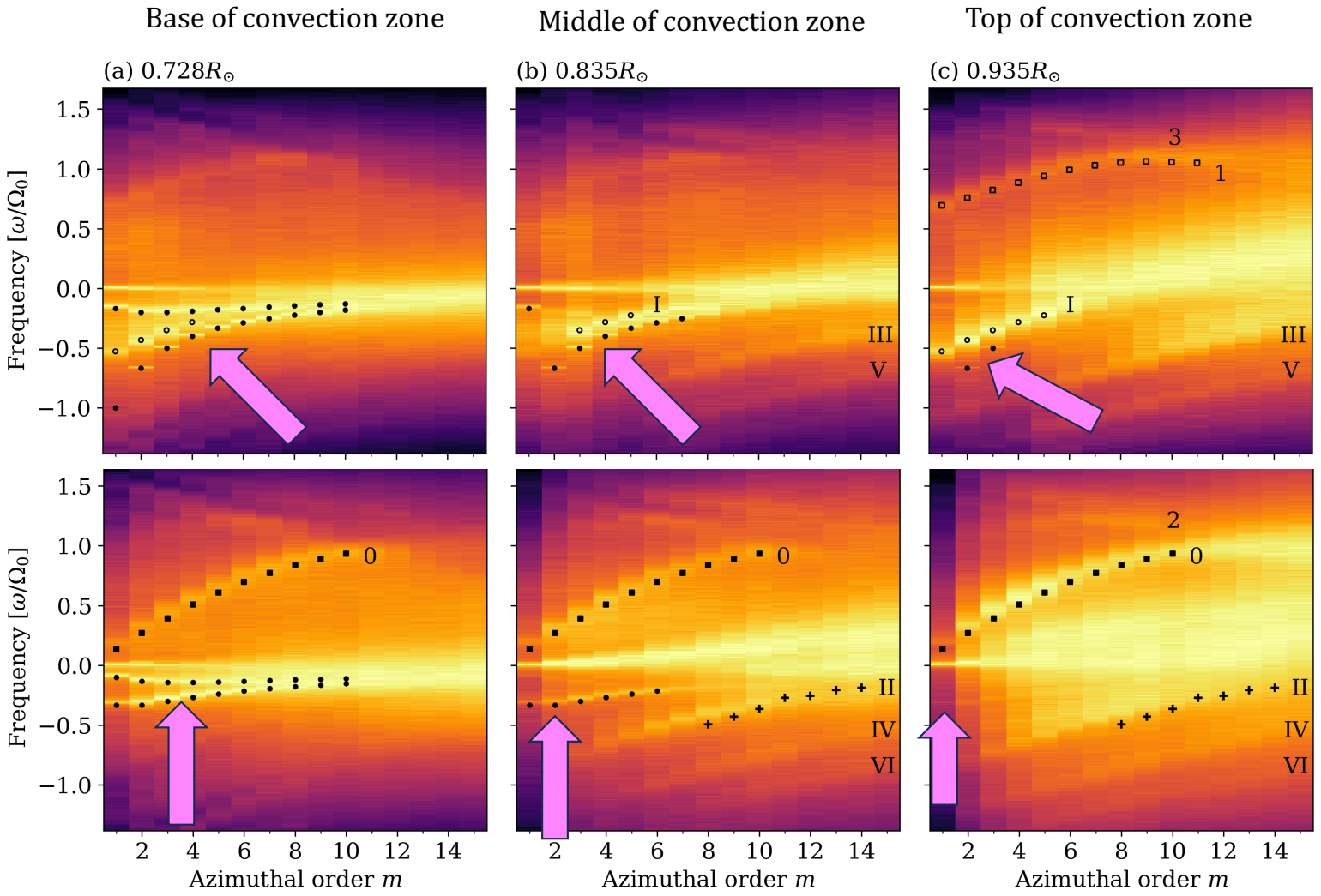


ζ_r Spectra

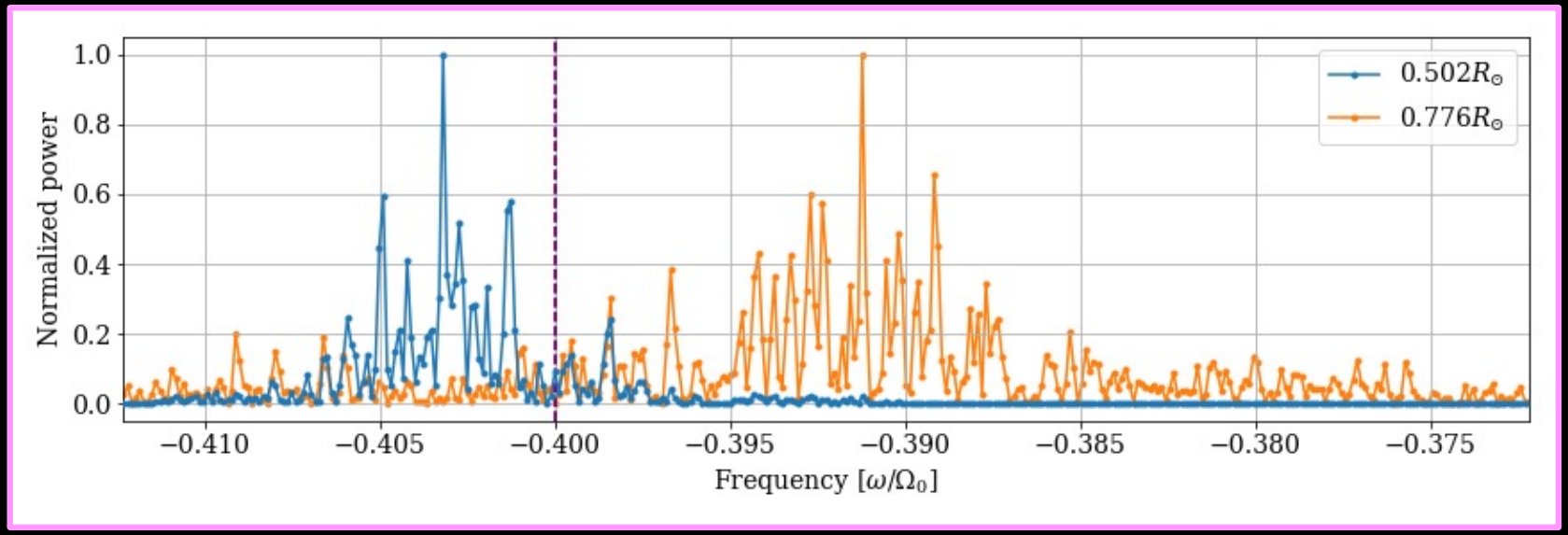
Symmetric modes
$\lambda = 0, 2$

Antisymmetric modes
$\lambda = 1, 3$





RI vs. CZ line profiles



Radiative interior and convection zone line profiles occur at different frequencies

Equation set and approximations

Mom. $\hat{x}$

$$\frac{\partial}{\partial t}(\rho_0 v_x) - f(\rho_0 v_y) = -\frac{\partial P_1}{\partial x}$$

Mom. $\hat{y}$

$$\frac{\partial}{\partial t}(\rho_0 v_y) + f(\rho_0 v_x) = -\frac{\partial P_1}{\partial y}$$

Mom. $\hat{z}$

$$g\rho_1 = -\frac{\partial P_1}{\partial z}$$

Cont.

$$\frac{\partial \rho_1}{\partial t} - \frac{N^2}{g}(\rho_0 v_z) - \frac{1}{gH_*}\frac{\partial P_1}{\partial t} = 0$$

Energy

$$\frac{\partial P_1}{\partial t} + H_* N^2 (\rho_0 v_z) + gH_* \nabla \cdot (\rho_0 \vec{v}) = 0$$

Approximations

- Beta plane
- Hydrostatic balance
- v_z is small

A separable equation

Horizontal equation

$$\left\{\frac{1}{\omega^2 - f^2}\left(\frac{\partial}{\partial y} + \frac{fk}{\omega}\right)\left(\frac{\partial}{\partial y} - \frac{fk}{\omega}\right) - \frac{k^2}{\omega^2}\right\} V_y$$

$$= \left\{\frac{1}{N^2}\frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{1}{H_* N^2}\right) + \left[\frac{1}{HN^2} + \frac{\partial}{\partial z}\left(\frac{1}{N^2}\right)\right]\frac{\partial}{\partial z}\right\} V_y$$

$$= -\Lambda$$

Vertical equation

Separation constant

Much algebra later,

$$\frac{\partial^2 \widehat{V_y}}{\partial z^2} + N^2 \left\{ \Lambda - \left[\frac{1}{4H^2N^2} - \frac{1}{2}\frac{\partial}{\partial z}\left(\frac{1}{HN^2}\right) - \frac{N^2}{4}\left[\frac{\partial}{\partial z}\left(\frac{1}{N^2}\right)\right]^2 + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left(\frac{1}{N^2}\right)\right]\right\} \widehat{V_y} = 0$$

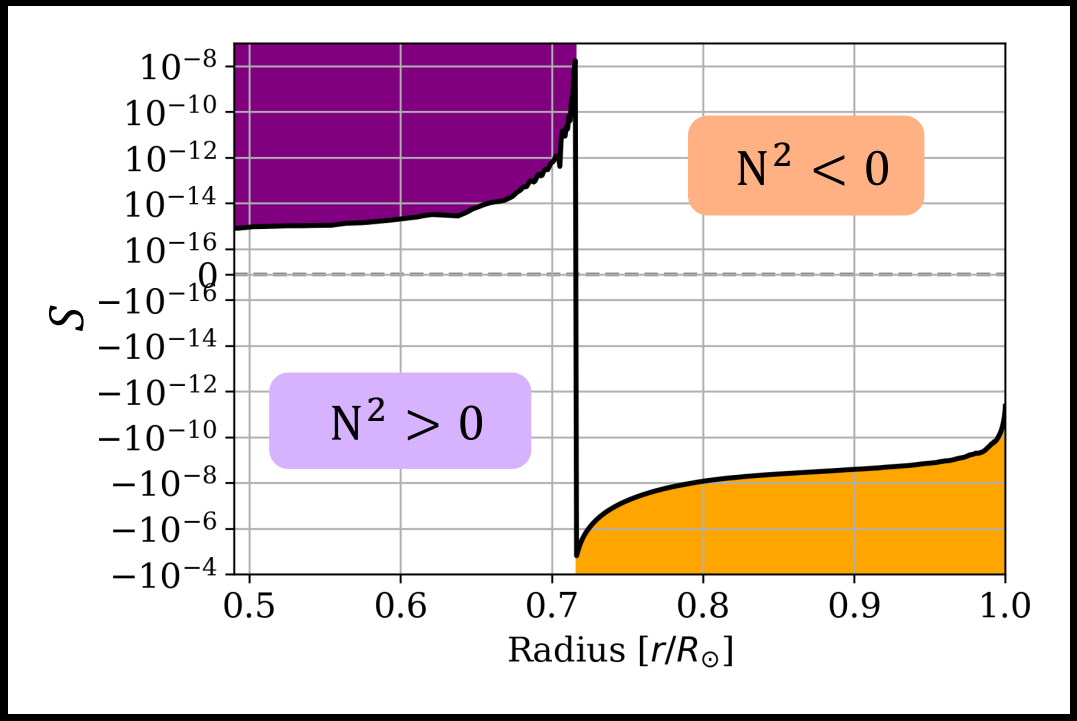
$$\frac{\partial^2 \widehat{V_y}}{\partial z^2} + N^2(\Lambda - S(z))\widehat{V_y} = 0$$

Propagation when $\mathrm{N}^2(\Lambda - \mathrm{S}) > 0$

Model S propagation diagram

$$\frac{\partial^2 \widehat{V}_y}{\partial z^2} + N^2[\Lambda - S(z)]\widehat{V}_y = 0$$

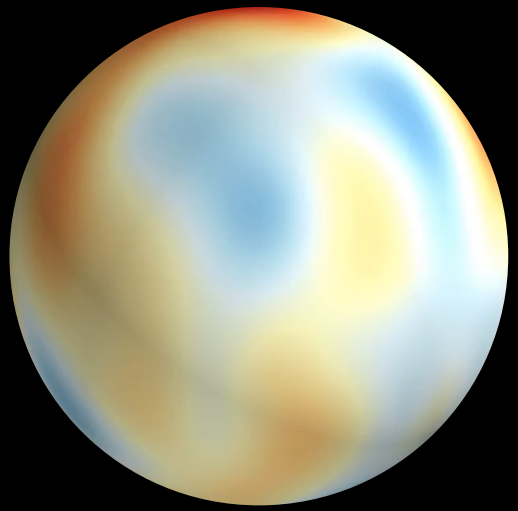
RI cavity requires $\Lambda > 0$



CZ cavity requires $\Lambda < 0$

Part II

Equatorial Rossby waves



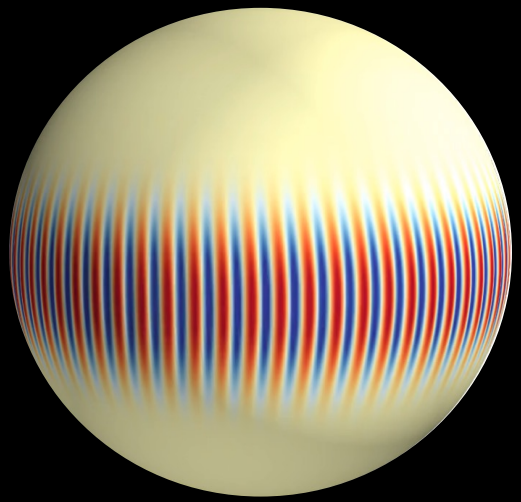
Lots of equatorial Rossby waves throughout the domain

Most dynamically important horizontal phenomena in the radiative interior

Most likely two families of Rossby waves present

Part III

Other waves in the convection zone



Thermal Rossby waves

Thermal Rossby waves/mixed mode

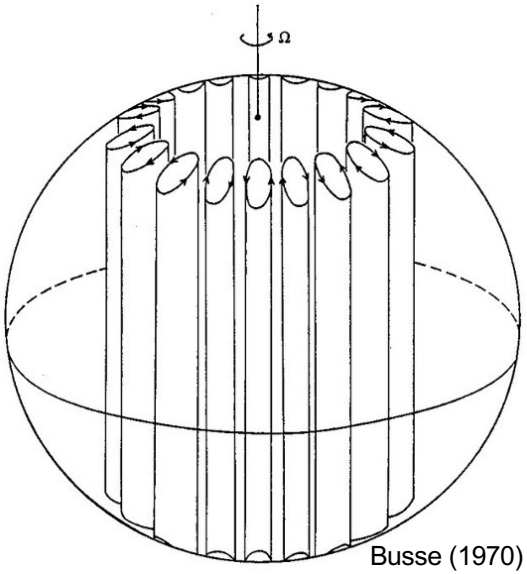
HFR modes

Thermal Rossby Waves (prograde)

Hindman and Jain (2022)

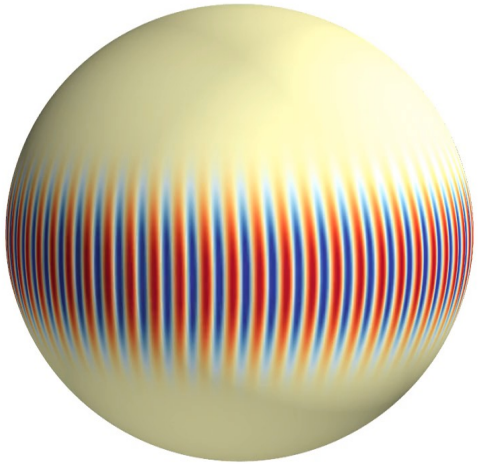
A different conservation of potential vorticity

$$\frac{D}{Dt}\left(\frac{\vec{\zeta}_a \cdot \vec{\Omega}}{\rho L}\right) = \frac{D}{Dt}\left(\frac{\zeta_y + 2\Omega}{\rho L}\right) = 0$$

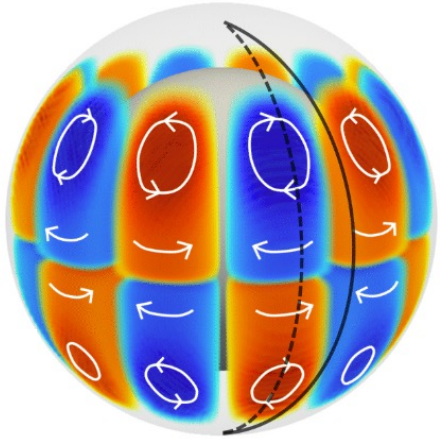


Busse (1970)

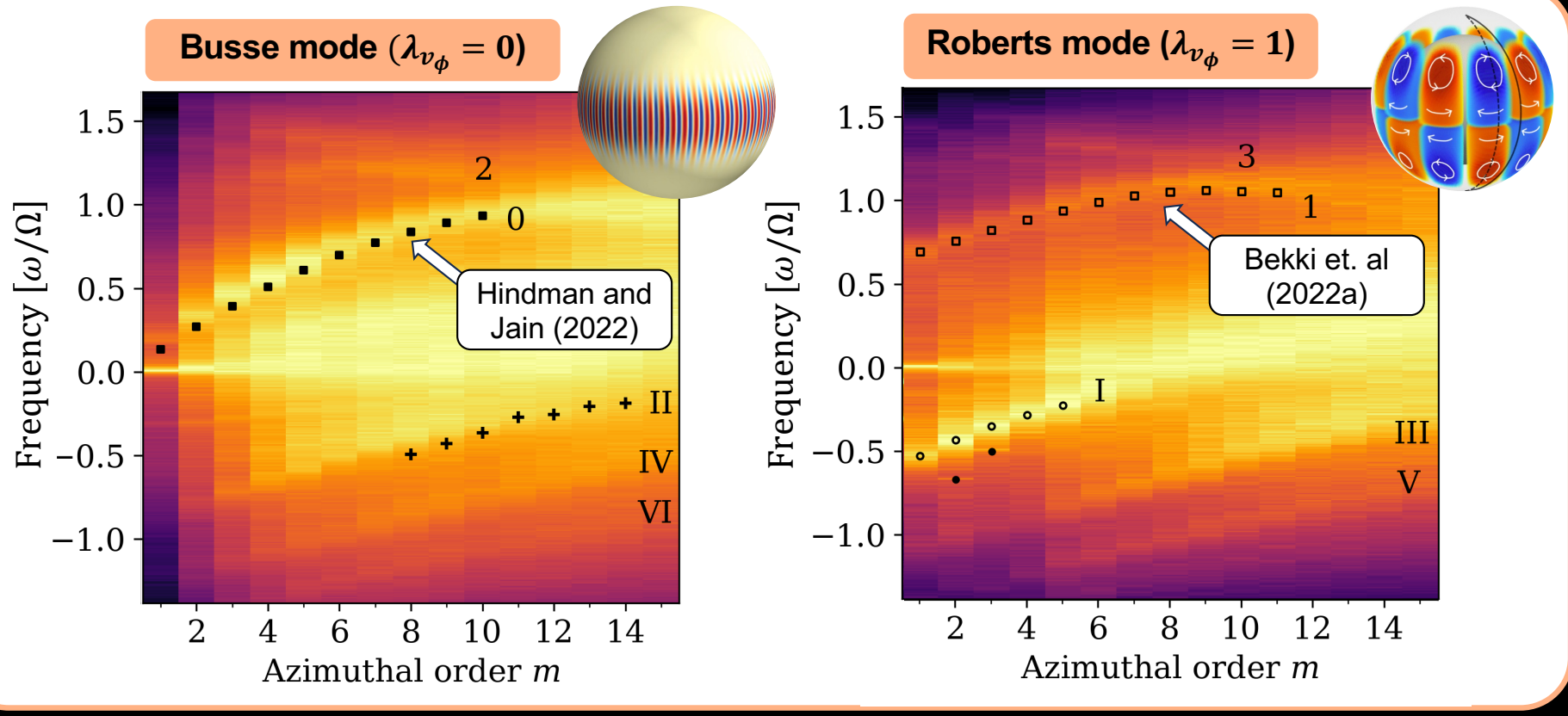
Busse mode ($\lambda_{v_\phi} = 0$)


Hindman et. al (2020a)

Roberts mode ($\lambda_{v_\phi} = 1$)

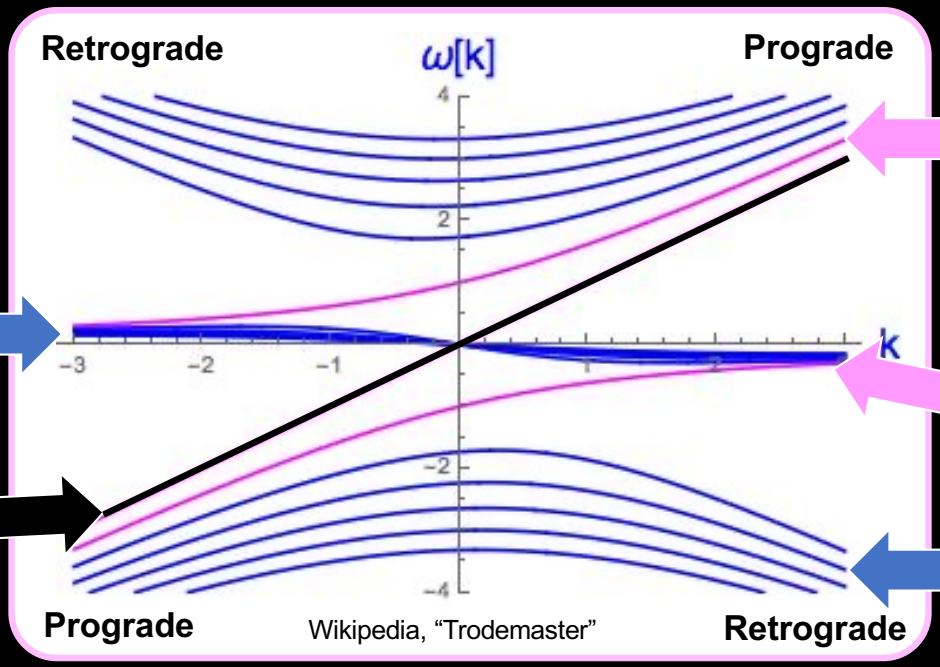

Bekki et. al (2022a)

Thermal Rossby wave spectra



Mixed modes

Modes which feature different retrograde and prograde behavior



Wikipedia, "Trodemaster"

Classical Rossby Waves

Kelvin Wave

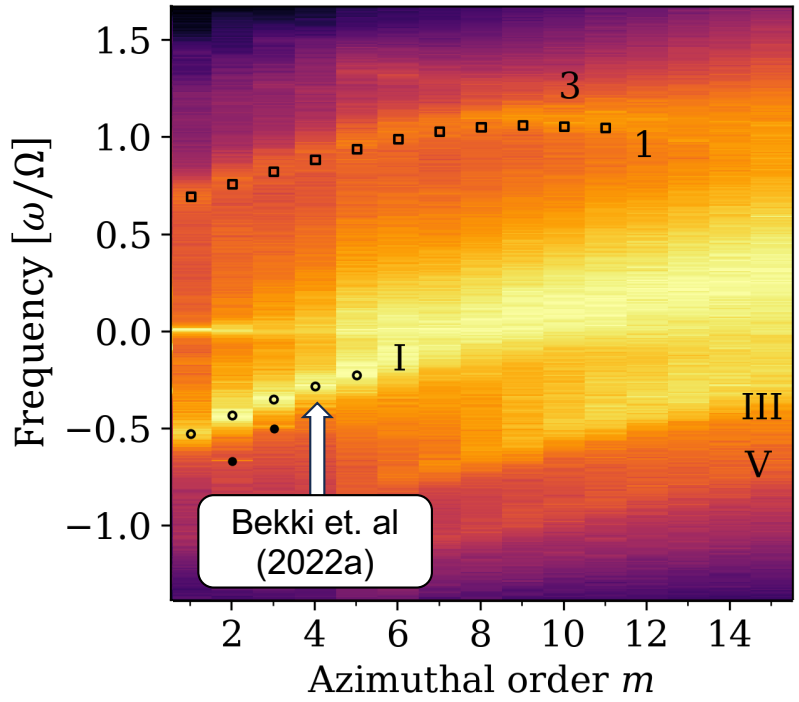
Yanai Wave

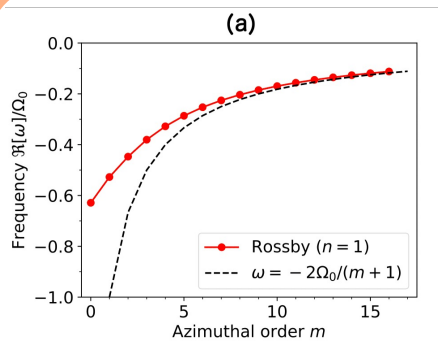
Yanai Wave

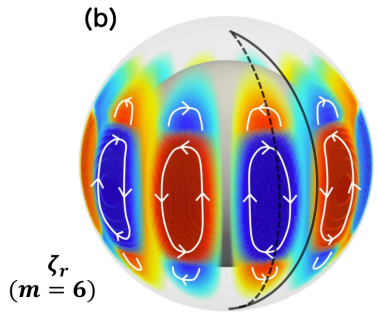
Poincare Waves

Equatorial mixed modes

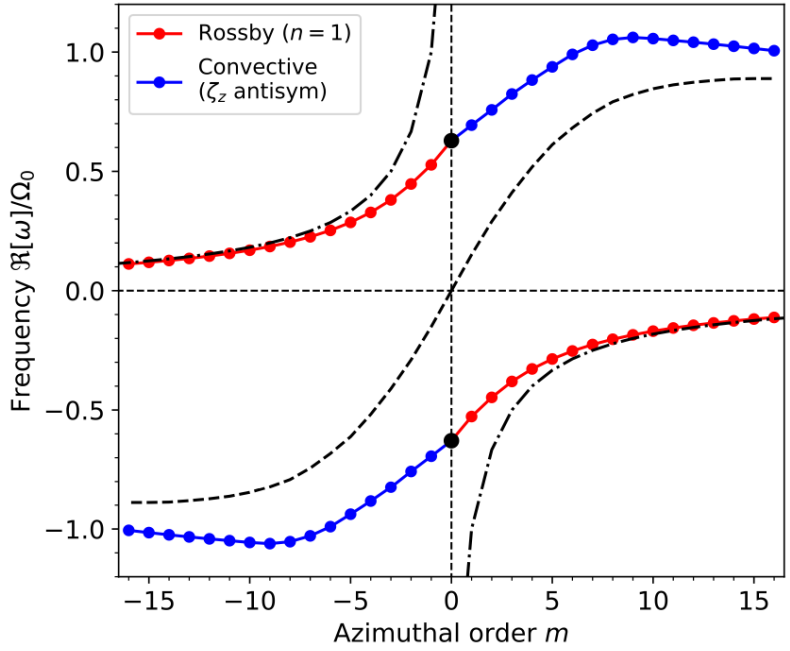
3D motions
$\lambda = 0$







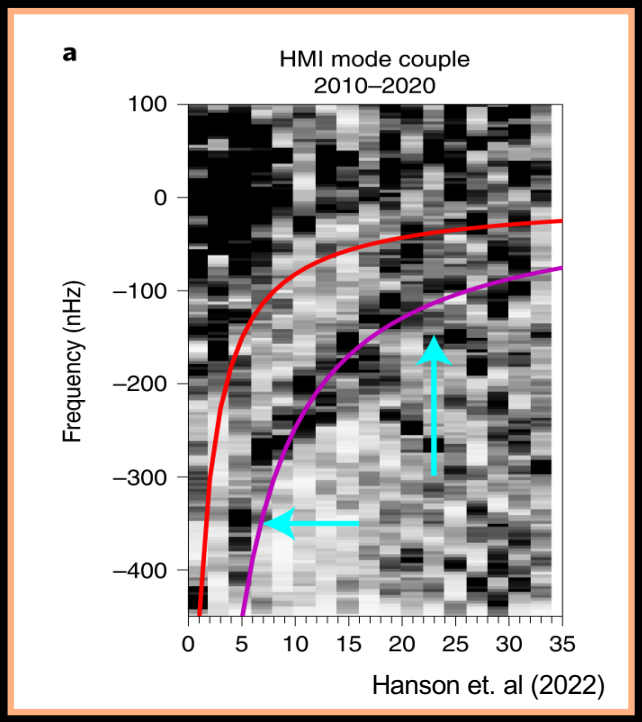
Bekki et. al (2022a)



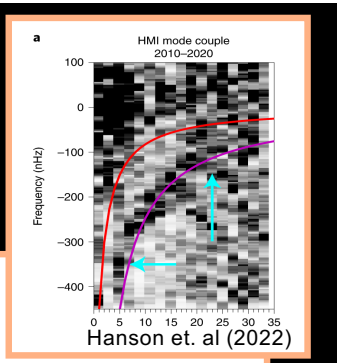
High-frequency retrograde (HFR) vorticity waves

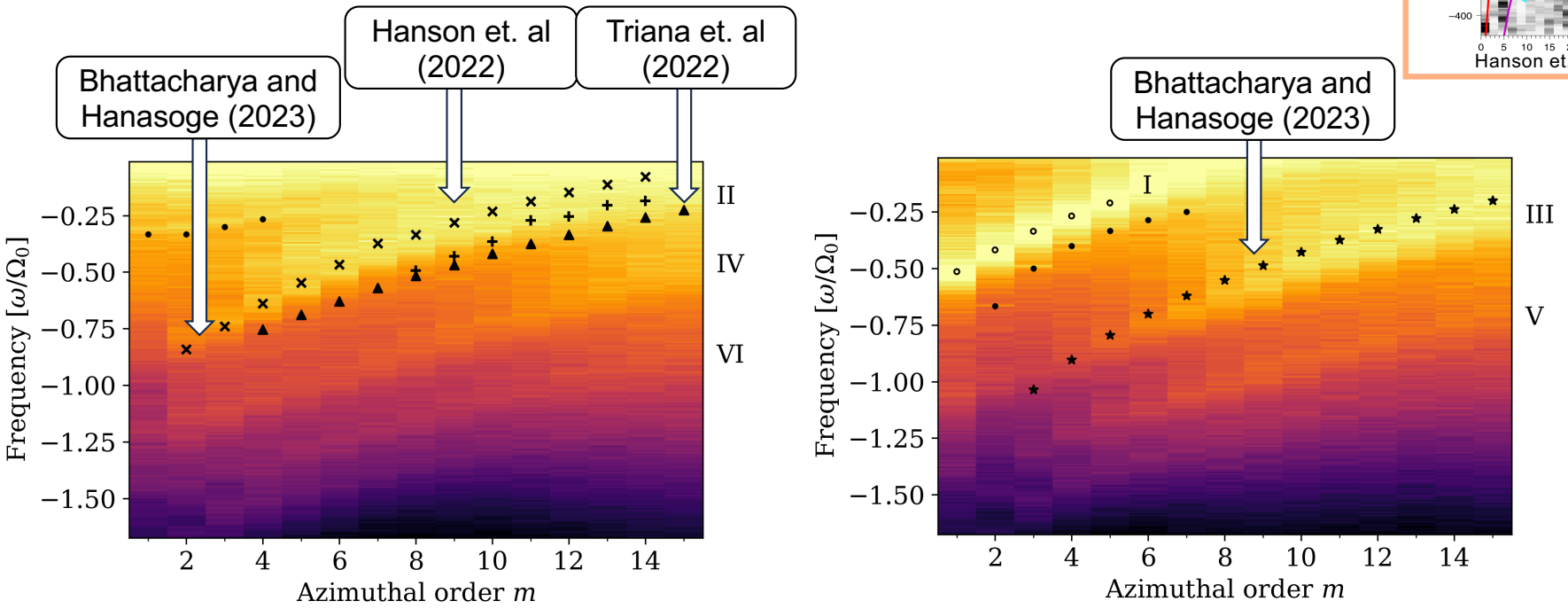
- Anti-symmetric ($\lambda = 1$)
- Seen near the surface
- 3D motions

No thorough theoretical explanation



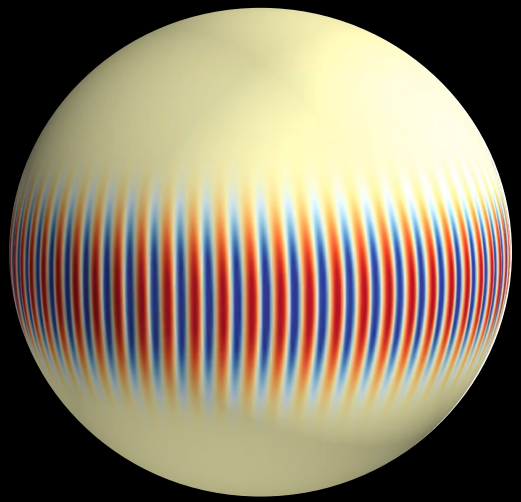
HFR spectra


a
HMI mode couple
2010–2020
Frequency (nHz)
100
0
−100
−200
−300
−400
0 5 10 15 20 25 30 35
Hanson et. al (2022)


Bhattacharya and Hanasoge (2023)
Hanson et. al (2022)
Triana et. al (2022)
Bhattacharya and Hanasoge (2023)
Frequency $[\omega/\Omega_0]$
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
2 4 6 8 10 12 14
Azimuthal order m
I
II
III
IV
V
VI

Part III

Other waves in the convection zone



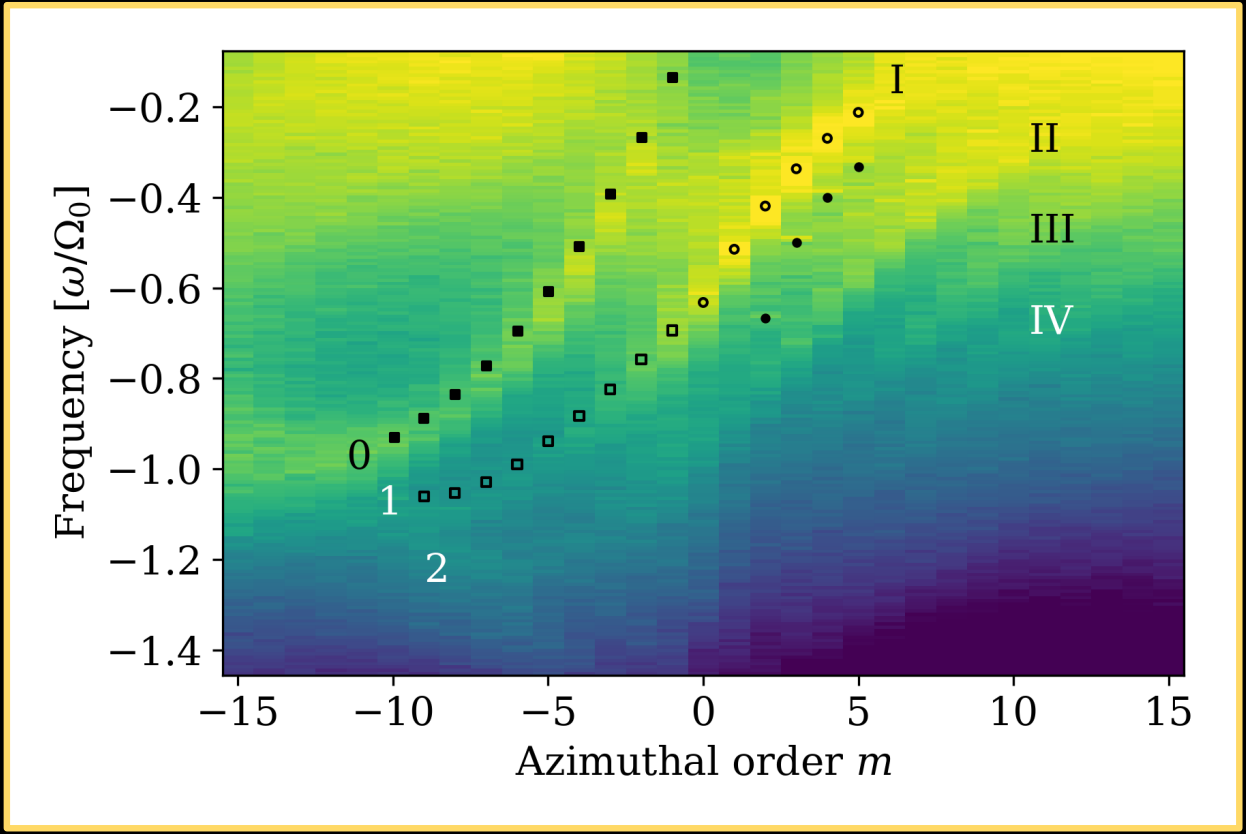
- Thermal Rossby waves
- Thermal Rossby waves/mixed mode
- HFR modes

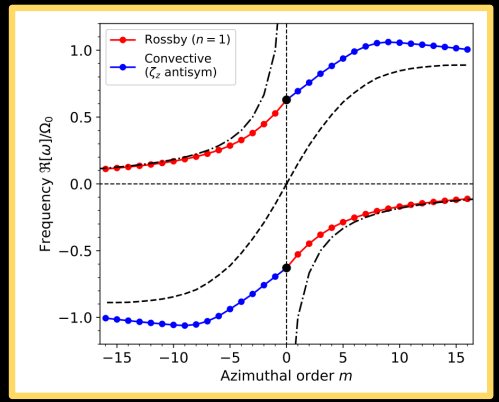
Part IV

All waves from the previous section are related to each other

- Everything is a mixed mode
- Physical intuition
- A theoretical explanation

Mixed modes?

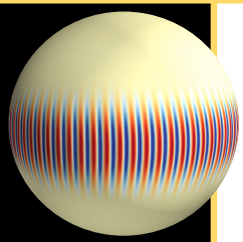




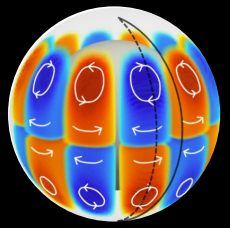
Bekki et. al (2022a)

Mixed modes?

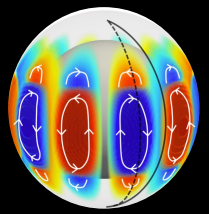
Busse mode ($\lambda = 0$)

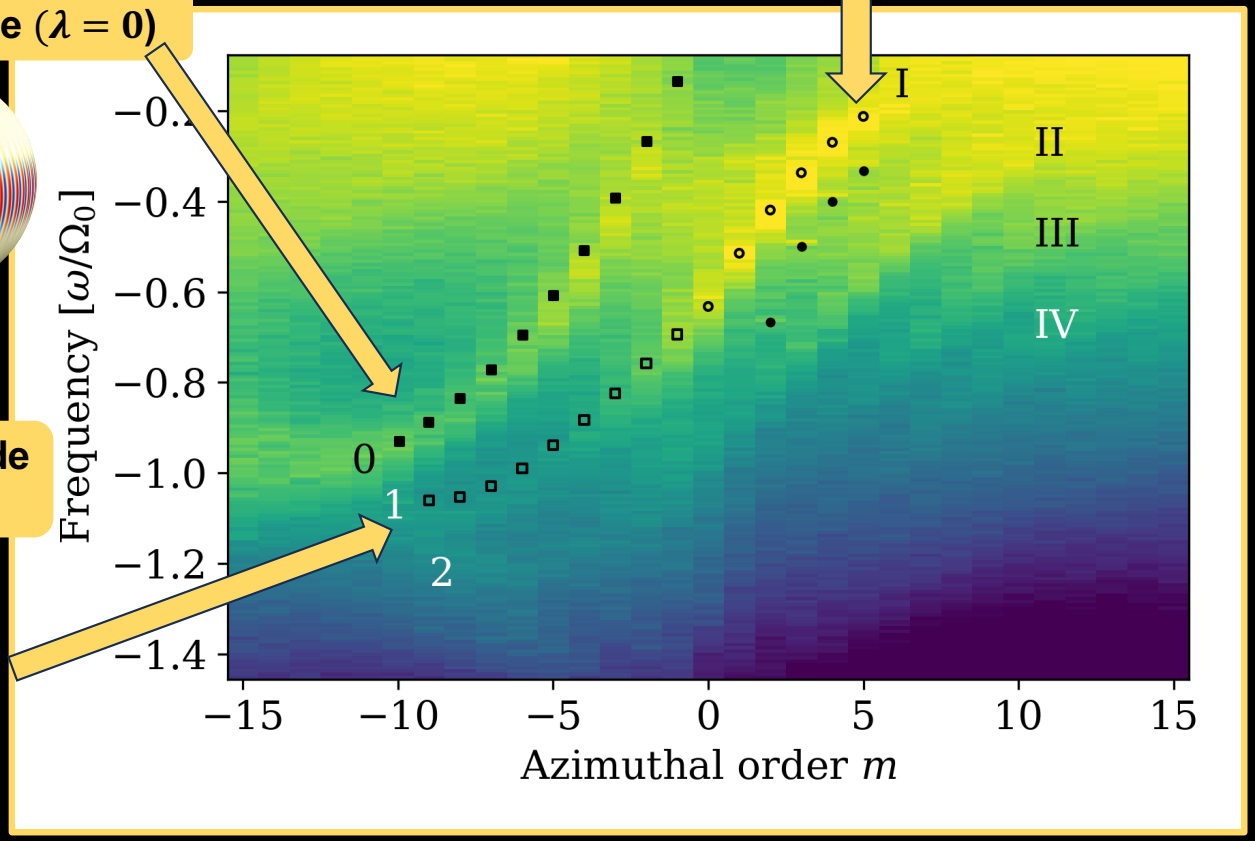


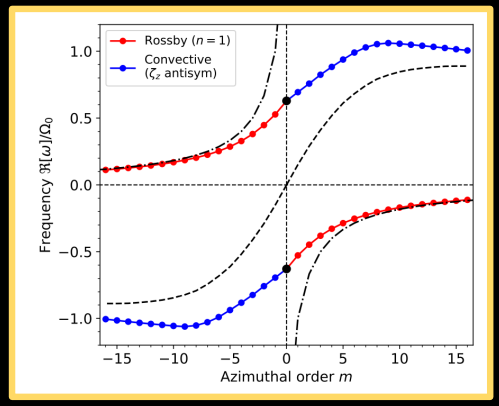
Roberts mode ($\lambda = 1$)



Equatorial mixed mode



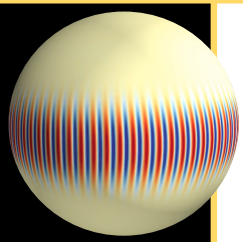




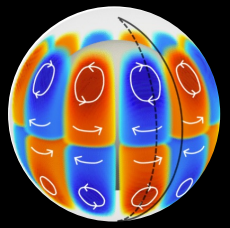
Bekki et. al (2022a)

Mixed modes?

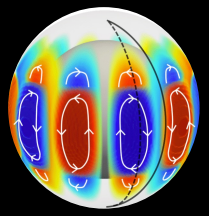
Busse mode ($\lambda = 0$)

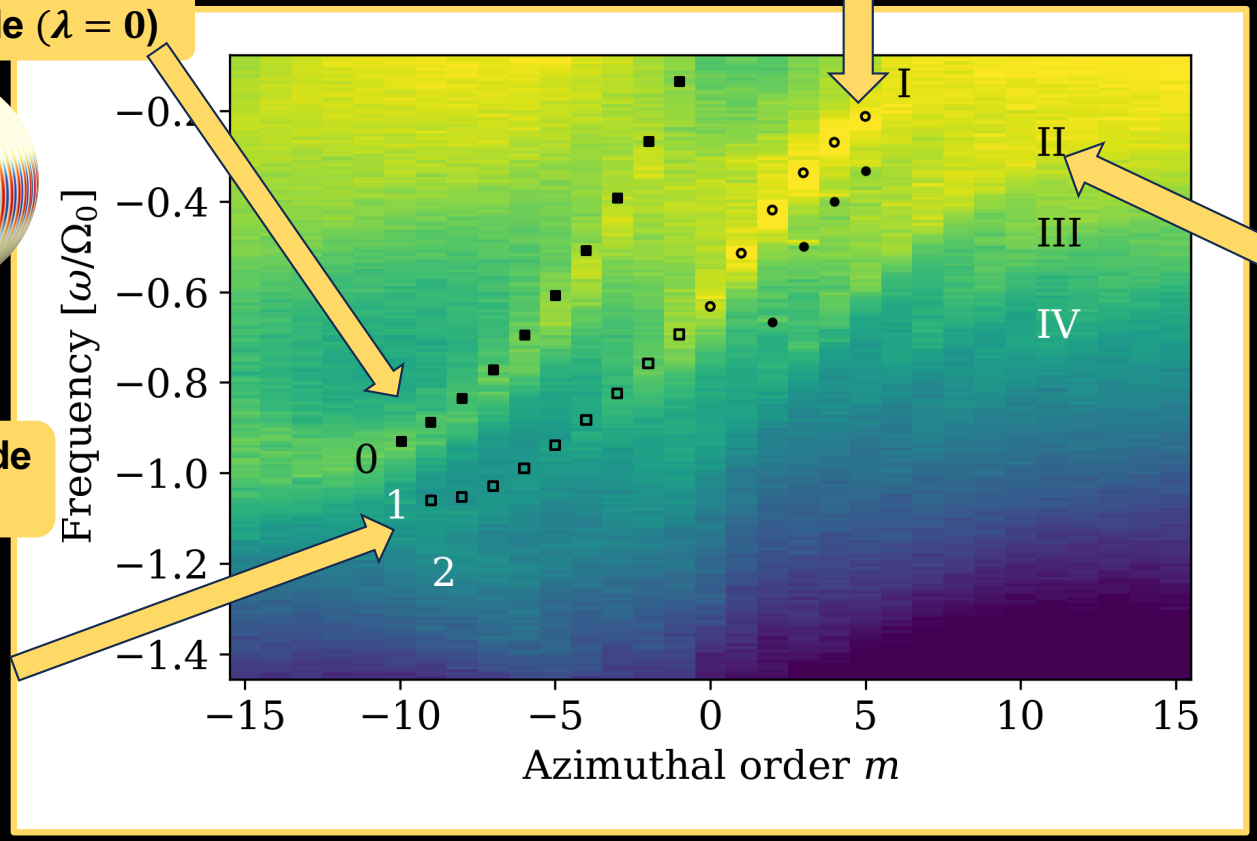


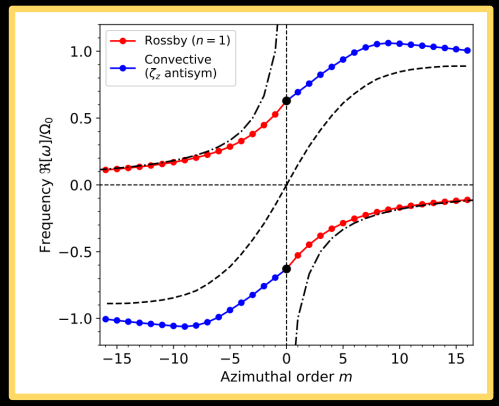
Roberts mode ($\lambda = 1$)



Equatorial mixed mode

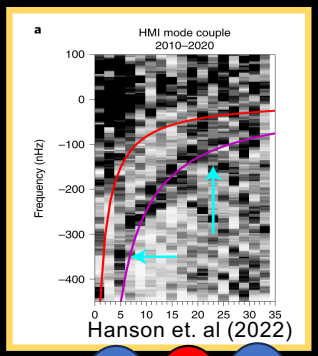




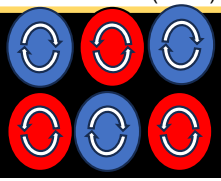


Bekki et. al (2022a)

HFR mode

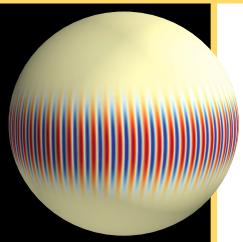


Hanson et. al (2022)

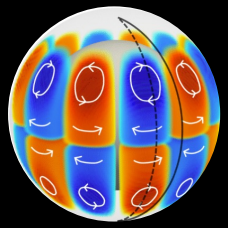


Mixed modes?

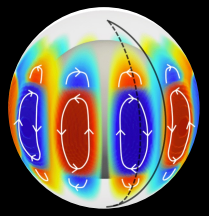
Busse mode ($\lambda = 0$)

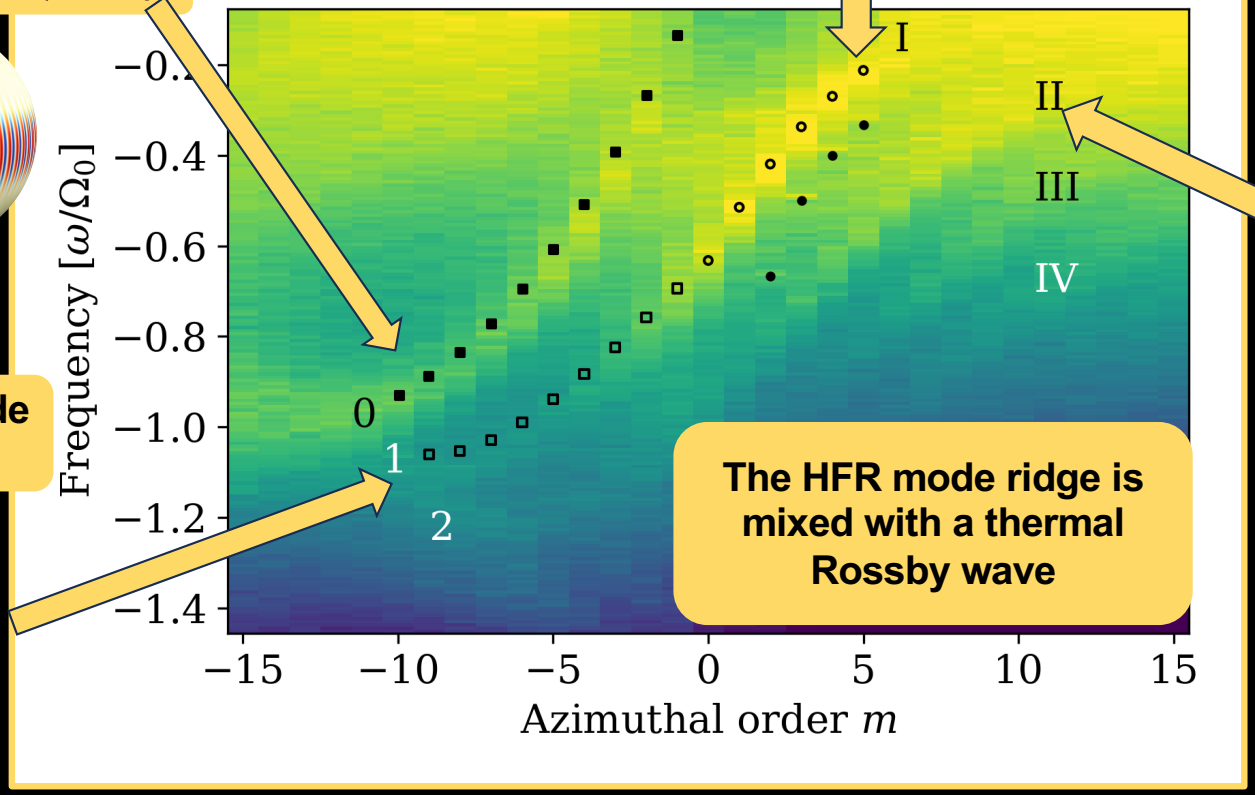


Roberts mode ($\lambda = 1$)



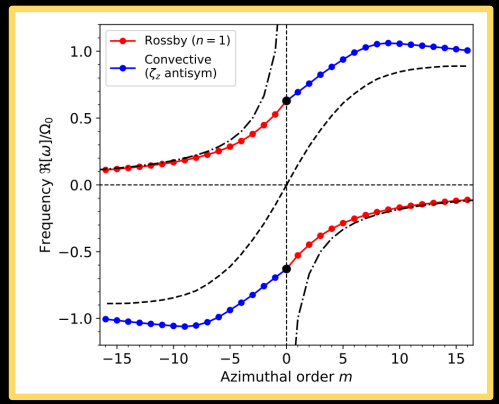
Equatorial mixed mode



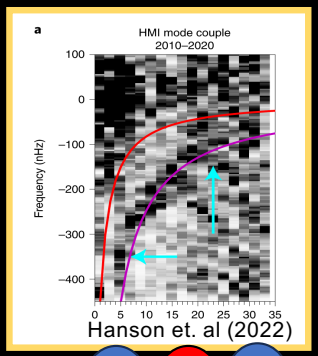


The HFR mode ridge is mixed with a thermal Rossby wave

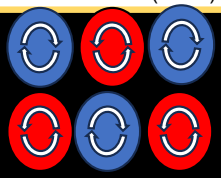
HFR mode

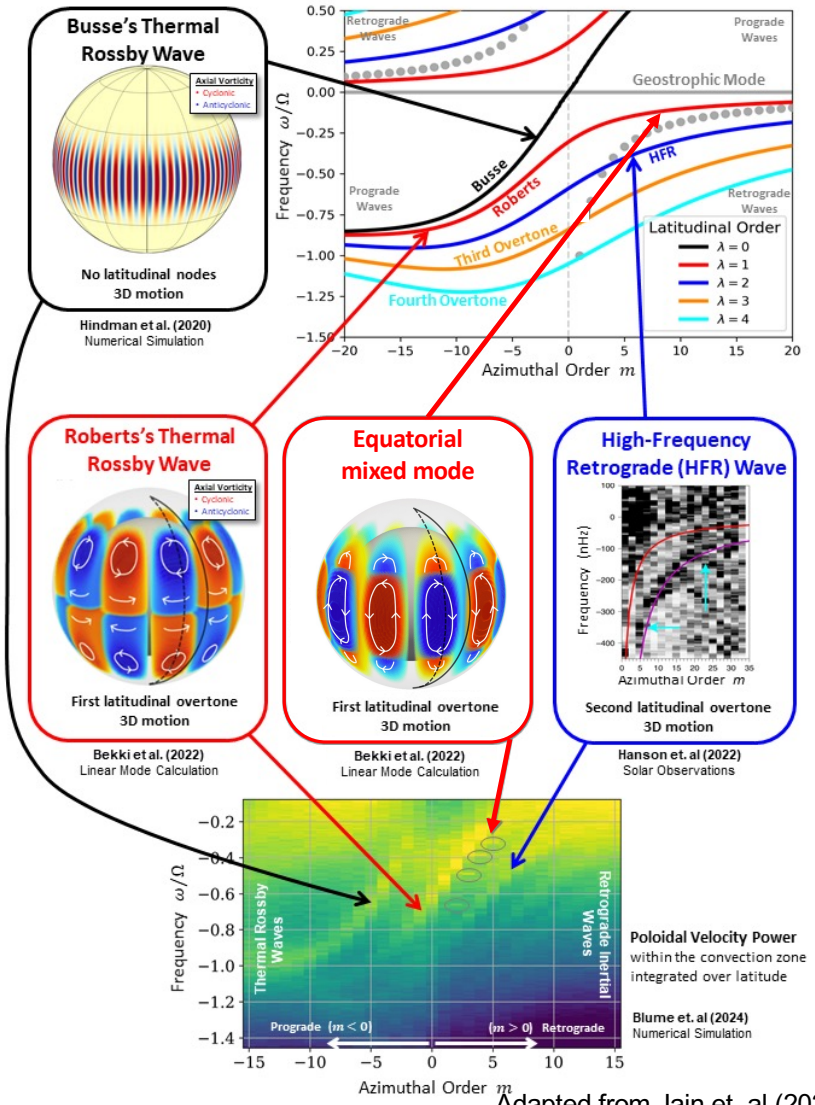


Bekki et. al (2022a)



Hanson et. al (2022)





Adapted from Jain et. al (2024)

A number of observed and modeled inertial waves can be combined into a single class of mixed modes.

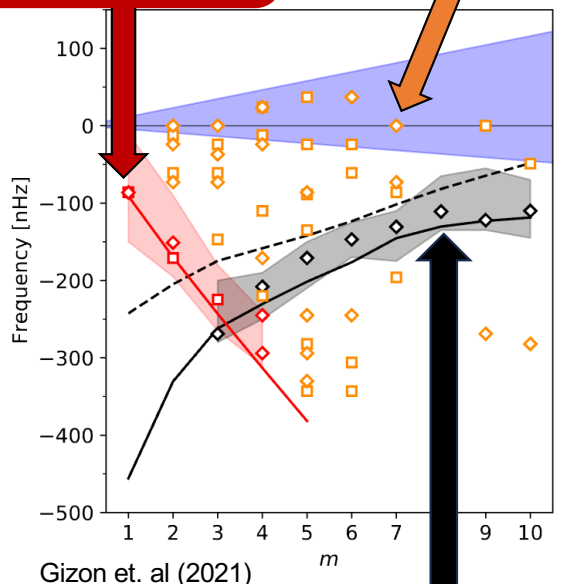
Prograde branch: thermal Rossby waves

Retrograde branch: 3D modes that include previously noted mixed mode and observed HFR modes

Observed

High-latitude modes

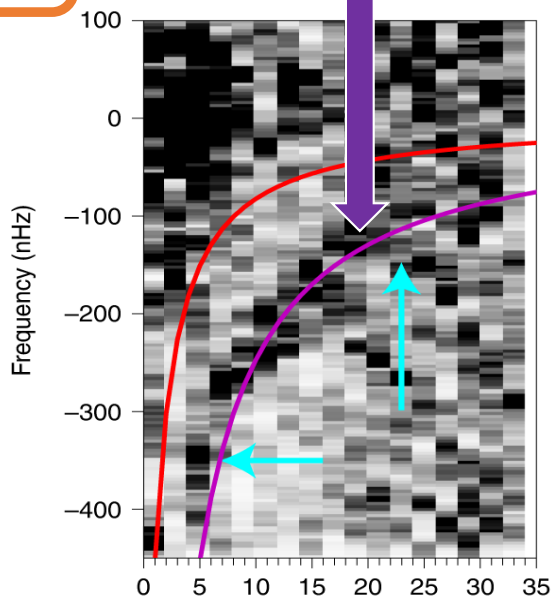
Critical-latitude modes



Gizon et. al (2021)

Classical Rossby waves

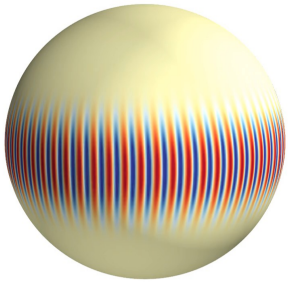
HFR modes



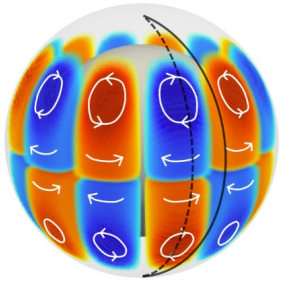
Hanson et. al (2022)

Modeled

Thermal Rossby waves

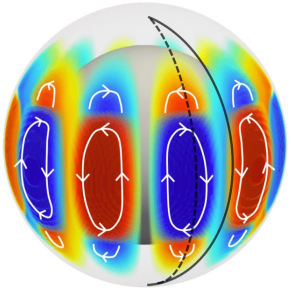


Hindman et. al (2020a)



Bekki et. al (2022a)

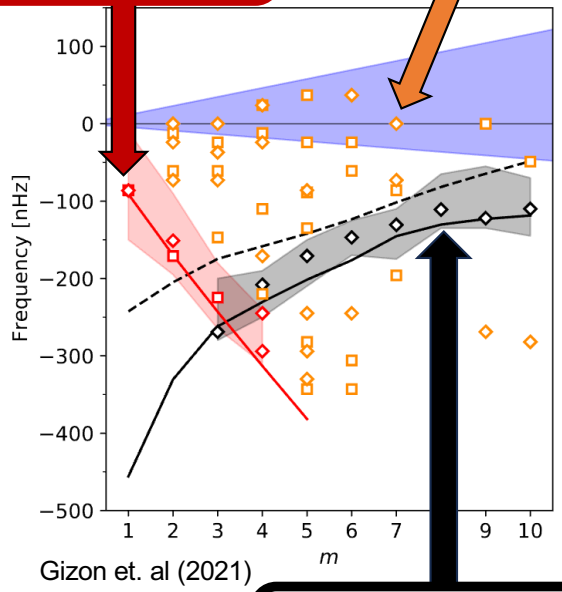
Equatorial mixed mode



Bekki et. al (2022a)

High-latitude modes

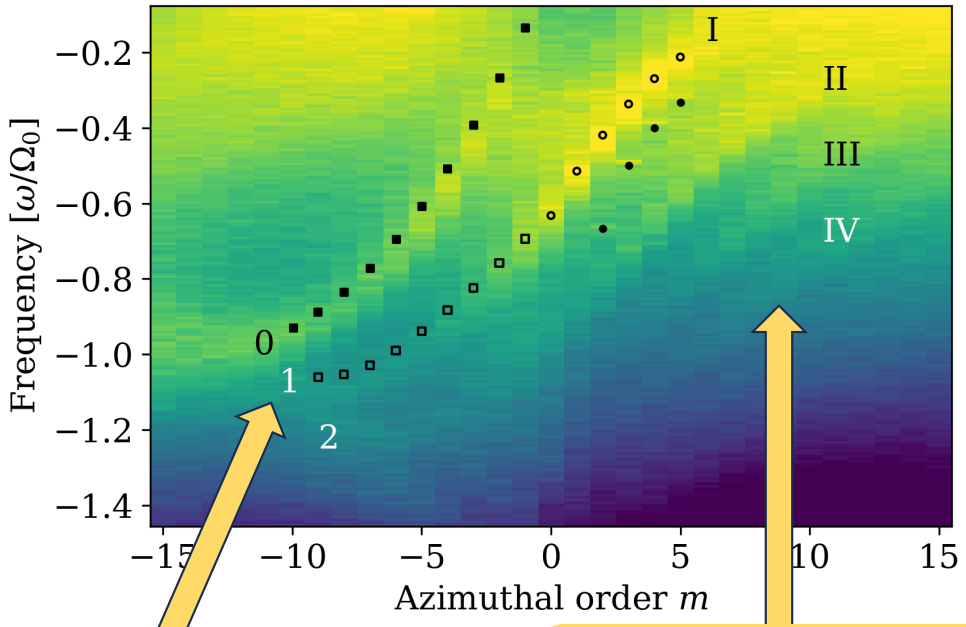
Critical-latitude modes



Gizon et. al (2021)

Classical Rossby waves

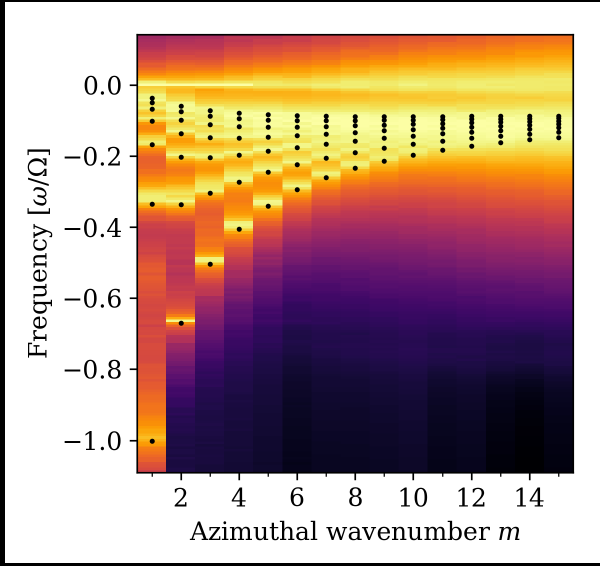
Mixed modes



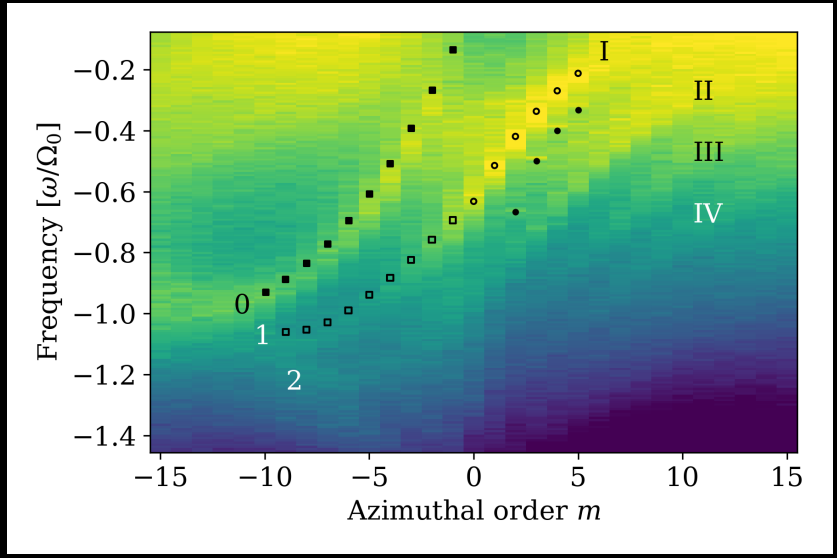
Thermal Rossby waves

Retrograde inertial waves

This simulation features equatorial Rossby waves throughout the domain, potentially living in two different wave cavities



A number of observed and modeled inertial waves can be combined into a single class of mixed modes.



Prograde branch: thermal Rossby waves

Retrograde branch: retrograde mixed mode, HFR modes

Papers!

- **Inertial Waves in a Nonlinear Simulation of the Sun's Convection Zone and Radiative Interior**
  - Blume, Hindman, and Matilsky (2024)
  - https://doi.org/10.3847/1538-4357/ad27d1.

- **A Unifying Model of Mixed Inertial Modes in the Sun**
  - Jain, Hindman, and Blume (2024)
  - https://iopscience.iop.org/article/10.3847/2041-8213/ad35c6