Inversion for Inferring Solar Meridional Circulation: The Case with Constraints on Angular Momentum Transport Inside the Sun

Stellar Convection: Modelling, Theory, and Observations (27 August, 2024)

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(yesterday)

To understand the solar large-scale flows is important!

- Large-scale flows play important roles in the solar dynamo

$$\mathscr{L} = (r\sin\theta)v_{\rm rot}$$

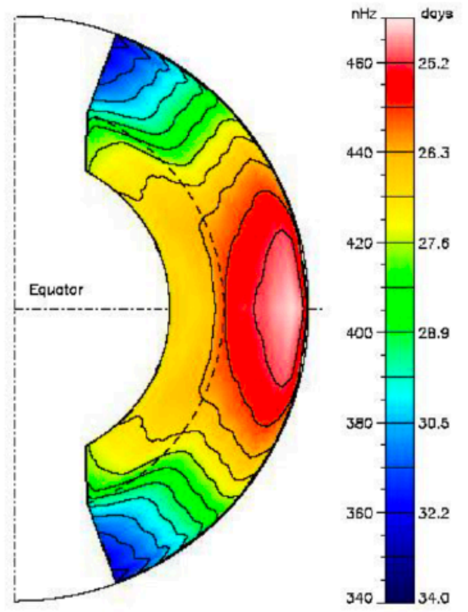
  - $\langle \rho \boldsymbol{v}_{\rm MC} \rangle \cdot \nabla \mathscr{L} = \mathscr{F}$
    ($\mathscr{L}$: specific AM; $\mathscr{F}$: torque by Reynolds stress, Maxwell stress, etc.)

    * gyroscopic pumping (e.g. Miesch & Toomre 2009)

  - A relationship among rotation ($\mathscr{L}$), meridional circulation ($\boldsymbol{v}_{\rm MC}$), and turbulence ($\mathscr{F}$)

- We know $\mathscr{L}$ very well thanks to helioseismology (e.g. Thompson+1996)
  → once we know $\boldsymbol{v}_{\rm MC}$, we can extract information on $\mathscr{F}$

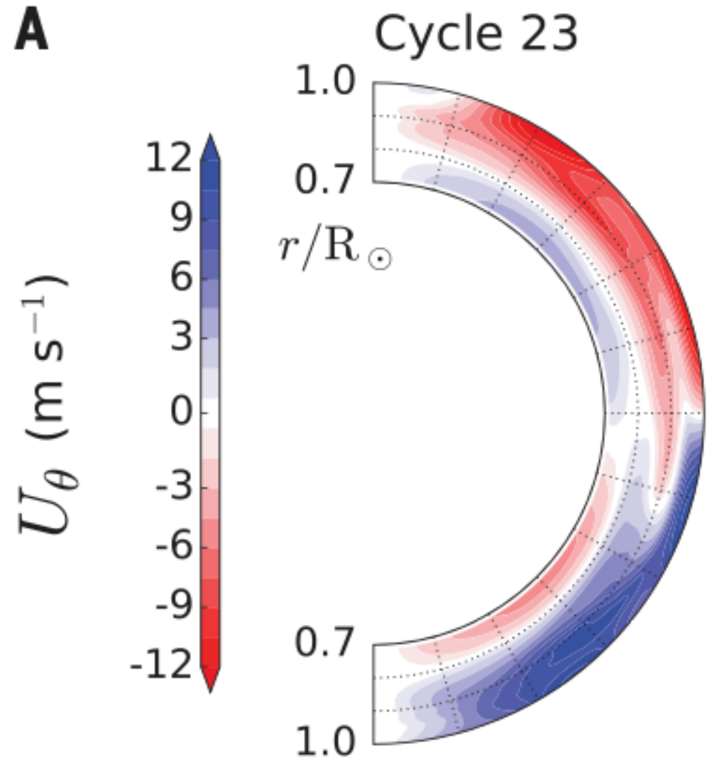
  * Observational studies of MC are important!



(from Takashi's lecture note)

Observational studies of internal MC profile

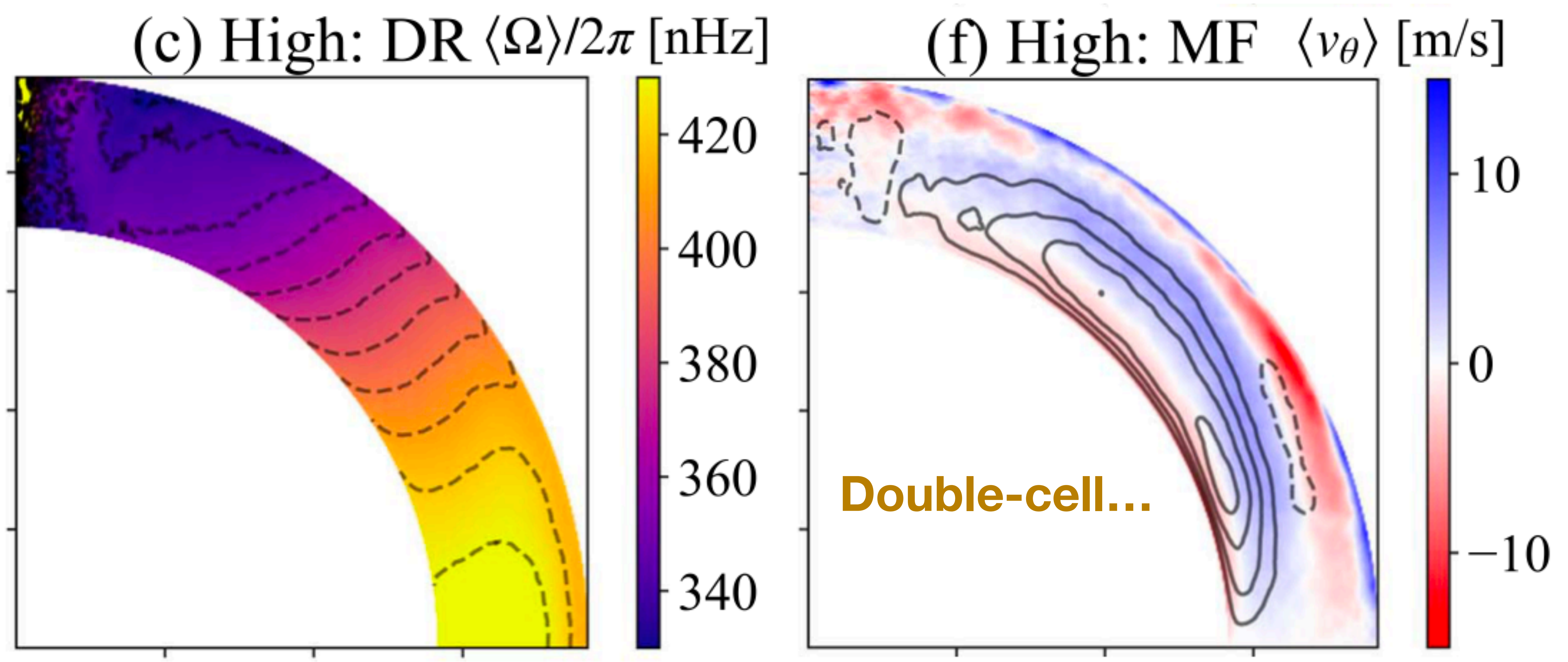
- One of the most promising ways of inferring internal MC profile is helioseismology
  - Global helioseimsology:
    eigenfunction perturbation analysis (e.g., Schad+2012, 2013,)
  - Local helioseismology:
    time-distance helioseismology (e.g., Giles 2000, Zhao+2013, Rajaguru and Basu 2015, Chen+2017, Mandal+2018, Gizon+2020, Herczeg and Jackiewicz 2023)
- But NOT CONCLUSIVE yet
  - In recent studies, a single-cell MC profile is obtained as a solution (Gizon+2020, Herczeg and Jackiewicz 2023)



(Gizon+2020)

From a theoretical perspective

- 3-d numerical simulation of the solar convection zone (e.g. Miesch 2005)
- “It is difficult for us to reproduce the solar equator-fast rotation while maintaining a single-cell MC profile” (according to Hideyuki)



Rotation profile **MC** **(Hotta and Kusano 2021; HK21)**

* In HK21, MC transports the angular momentum (AM) toward the equator

Then, what happens if we assume equatorward AMT by MC?

- What we would like to do is:
  - based on an assumption that the HK21 regime is correct, we carry out inversion (of travel times) to infer the solar MC profile with an additional constraint that AMT by MC is equatorward
- Which type of MC profile would we obtain, single-, double-, or multiple-cell structure?

What we invert is

- Travel time is related to meridional flow field (see Gizon+2017):

$$\tau_i = \int_0^{\pi} \int_{r_b}^{R_\odot} (K_i^r U_r + K_i^\theta U_\theta) \mathrm{d}r \mathrm{d}\theta + e_i$$

(K_i^r and K_i^θ are sensitivity kernels)

($i = 1,\ldots,9120$ in the case of the G20 data)

- Discretizing the integral equation above leads to:

$$\boldsymbol{\tau} = \boldsymbol{K}\boldsymbol{u} + \boldsymbol{e}$$

- We have used data of Gizon+2020 (G20) such as travel times, sensitivity kernels, and covariance matrix

Inversion method used in G20

$$\boldsymbol{\tau} = K\boldsymbol{u} + \boldsymbol{e}$$

- What they minimize is:

mass conservation in the CZ

$$A' = |\boldsymbol{\tau} - K\boldsymbol{u}|^2 + \alpha |D\boldsymbol{u}|^2 + \boldsymbol{\kappa} \cdot (C\boldsymbol{u}) + \boldsymbol{\mu} \cdot (S\boldsymbol{u})$$

Assuming that the vorticity is small
α: trade-off parameter

- The solution $\hat{\boldsymbol{u}}$ is obtained by inverting the matrix equation as below:

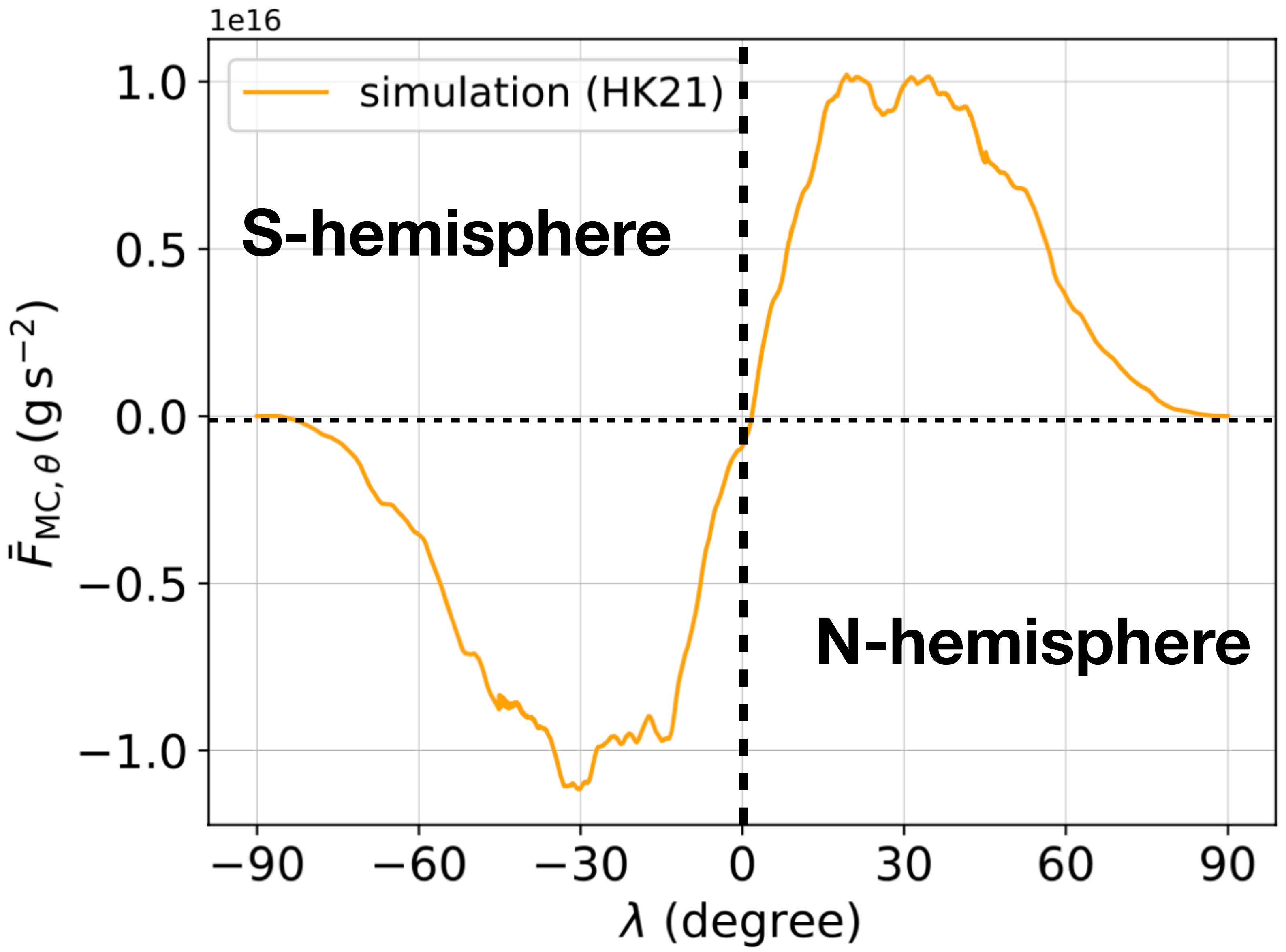
$$\begin{pmatrix} K^TK + \alpha D^TD & C^T & S^T \\ C & O & O \\ S & O & O \end{pmatrix} \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{\kappa} \\ \boldsymbol{\mu} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{pmatrix}$$

* G20 provide the matrices D, C and S

Additional constraint in terms of AMT by MC: HK21-type constraint

- AMT by MC is equator-ward (HK21)
- AM flux by MC is given as: $F_{\mathrm{MC},\theta} = \rho_0 u_\theta \mathscr{L} \quad (\mathscr{L} = (r\sin\theta)v_\phi)$

Radially averaged AMF $\bar{F}_{\mathrm{MC},\theta}$ (HK21)



We have assumed:

that the latitudinal average of $\bar{F}_{\mathrm{MC},\theta}$ is positive (negative) for the northern (southern) hemisphere

and that the latitudinal derivative of $\bar{F}_{\mathrm{MC},\theta}$ is small

The assumptions can be expressed as:

$$D_{\mathrm{HK1}}\boldsymbol{u} \sim \boldsymbol{b} \text{ and } D_{\mathrm{HK2}}\boldsymbol{u} \sim \boldsymbol{0}$$

because AMF by MC is linear in terms of MC velocity u_θ

(the vector $\boldsymbol{b}$ is determined based on results of HK21)

What to solve when we add D_{MC}

- What we minimize is:

$$A' = |\boldsymbol{\tau} - K\boldsymbol{u}|^2 + \alpha|D\boldsymbol{u}|^2 + \beta|D_{\mathrm{HK1}}\boldsymbol{u} - \boldsymbol{b}|^2 + \gamma|D_{\mathrm{HK2}}\boldsymbol{u}|^2 + \boldsymbol{\kappa}\cdot(C\boldsymbol{u}) + \boldsymbol{\mu}\cdot(S\boldsymbol{u})$$

Assuming that the vorticity is small ($\alpha|D\boldsymbol{u}|^2$)

Assuming the equatorward AMT by MC ($\beta|D_{\mathrm{HK1}}\boldsymbol{u} - \boldsymbol{b}|^2$)

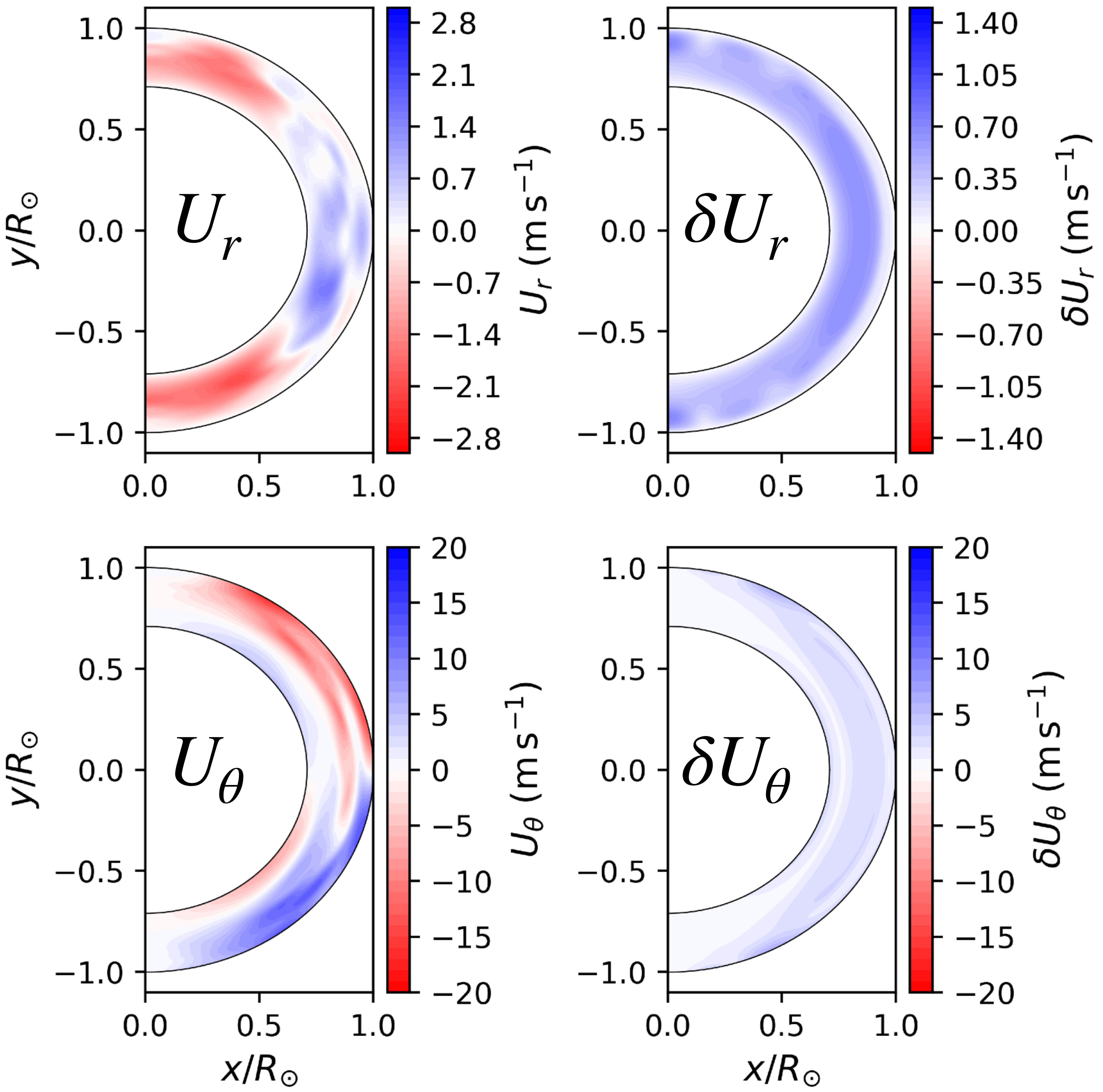
Assuming that $\partial_\theta \bar{F}_{\mathrm{MC},\theta}$ is small ($\gamma|D_{\mathrm{HK2}}\boldsymbol{u}|^2$)

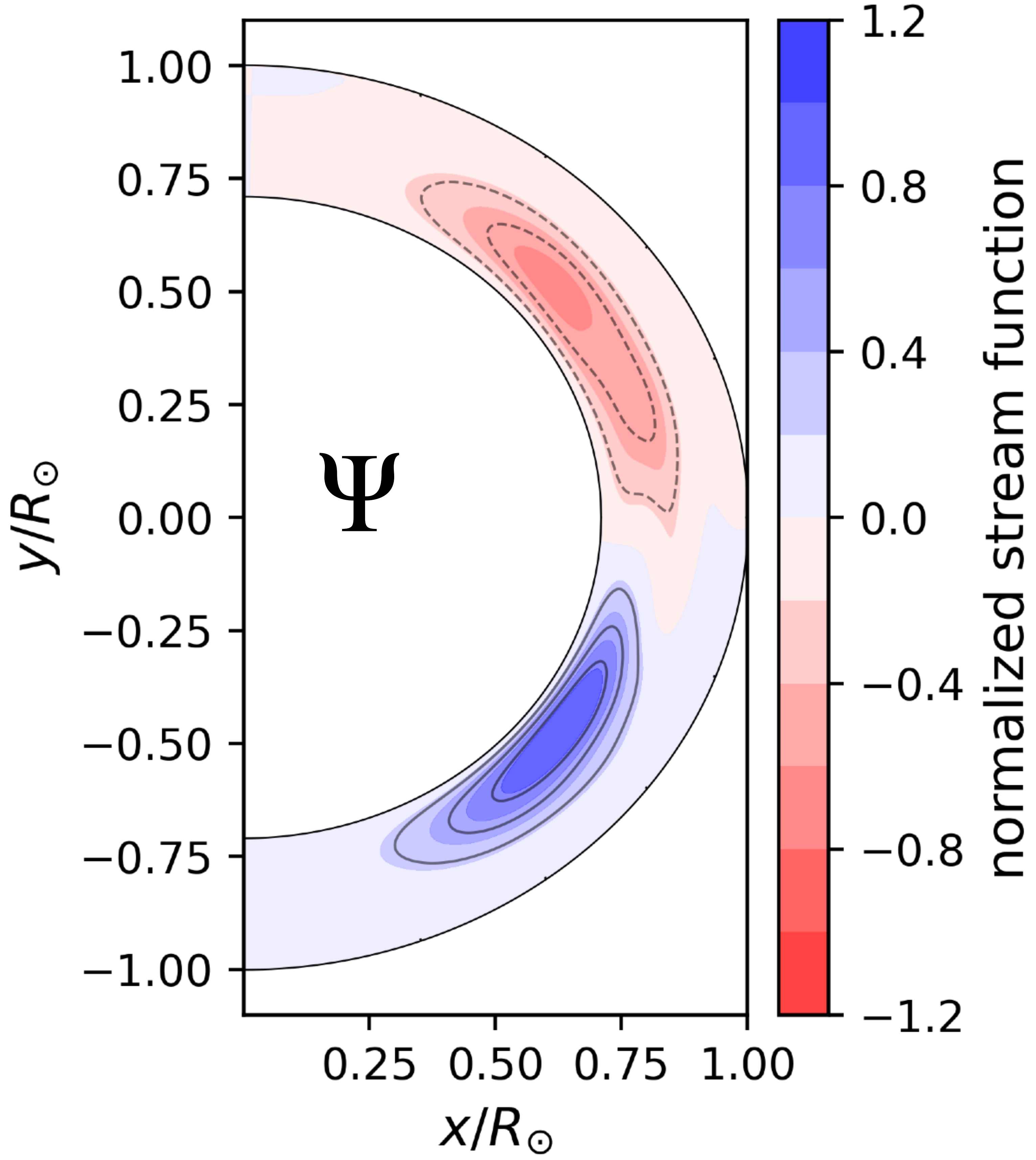
mass conservation in the CZ ($\boldsymbol{\kappa}\cdot(C\boldsymbol{u}) + \boldsymbol{\mu}\cdot(S\boldsymbol{u})$)

- The solution $\hat{\boldsymbol{u}}$ is obtained by inverting the matrix equation as below:

$$\begin{pmatrix} K^TK + \alpha D^TD + \beta D_{\mathrm{HK1}}^T D_{\mathrm{HK1}} + \gamma D_{\mathrm{HK2}}^T D_{\mathrm{HK2}} & C^T & S^T \\ C & O & O \\ S & O & O \end{pmatrix} \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{\kappa} \\ \boldsymbol{\mu} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\tau} + \beta D_{\mathrm{HK1}}^T \boldsymbol{b} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

Result: in the case **without** the HK21-type constraint

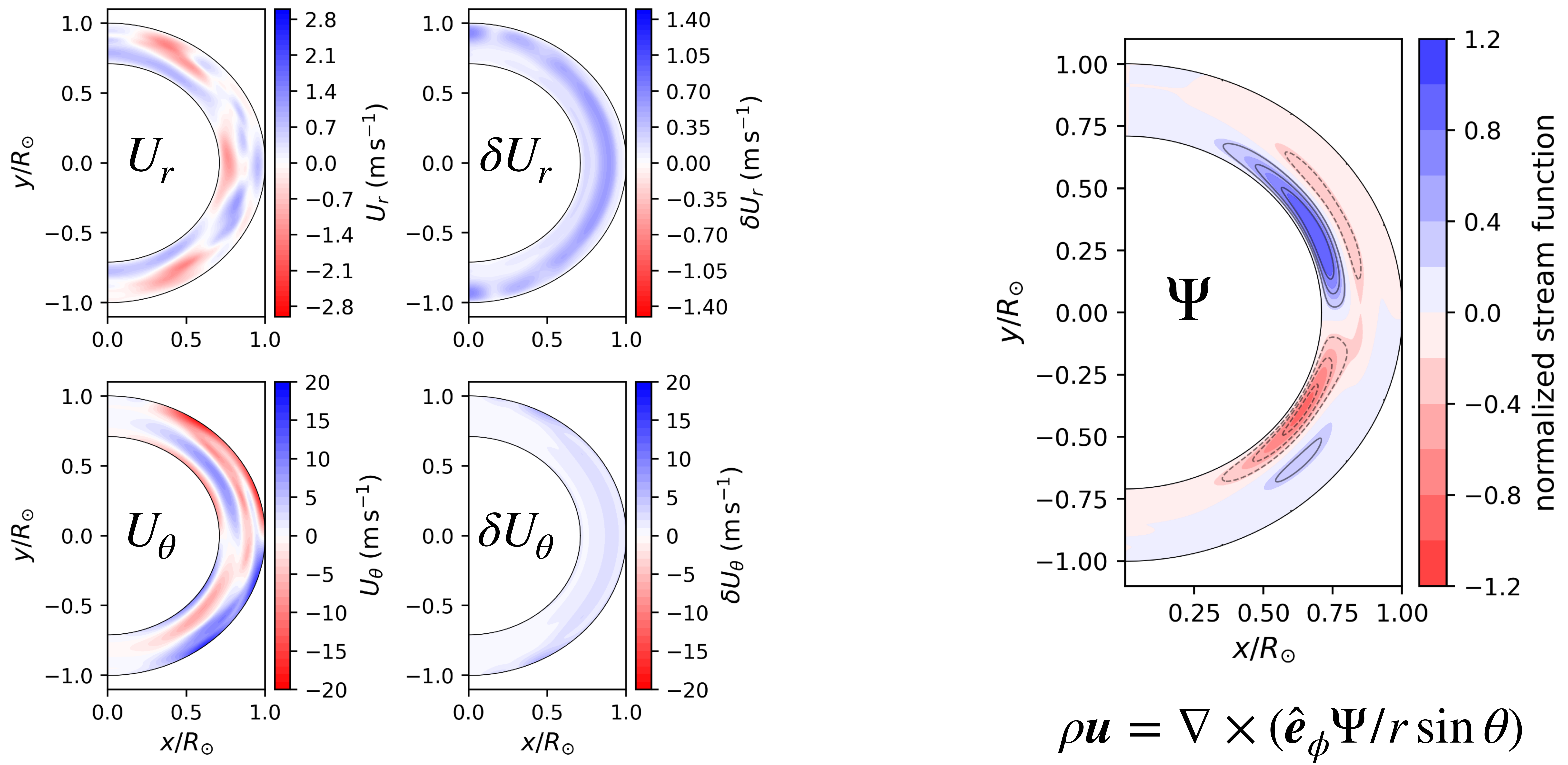




$$\rho \boldsymbol{u} = \nabla \times (\hat{\boldsymbol{e}}_\phi \Psi / r \sin\theta)$$

Single-cell (G20's result has been confirmed)

Result: in the case with the HK21-type constraint

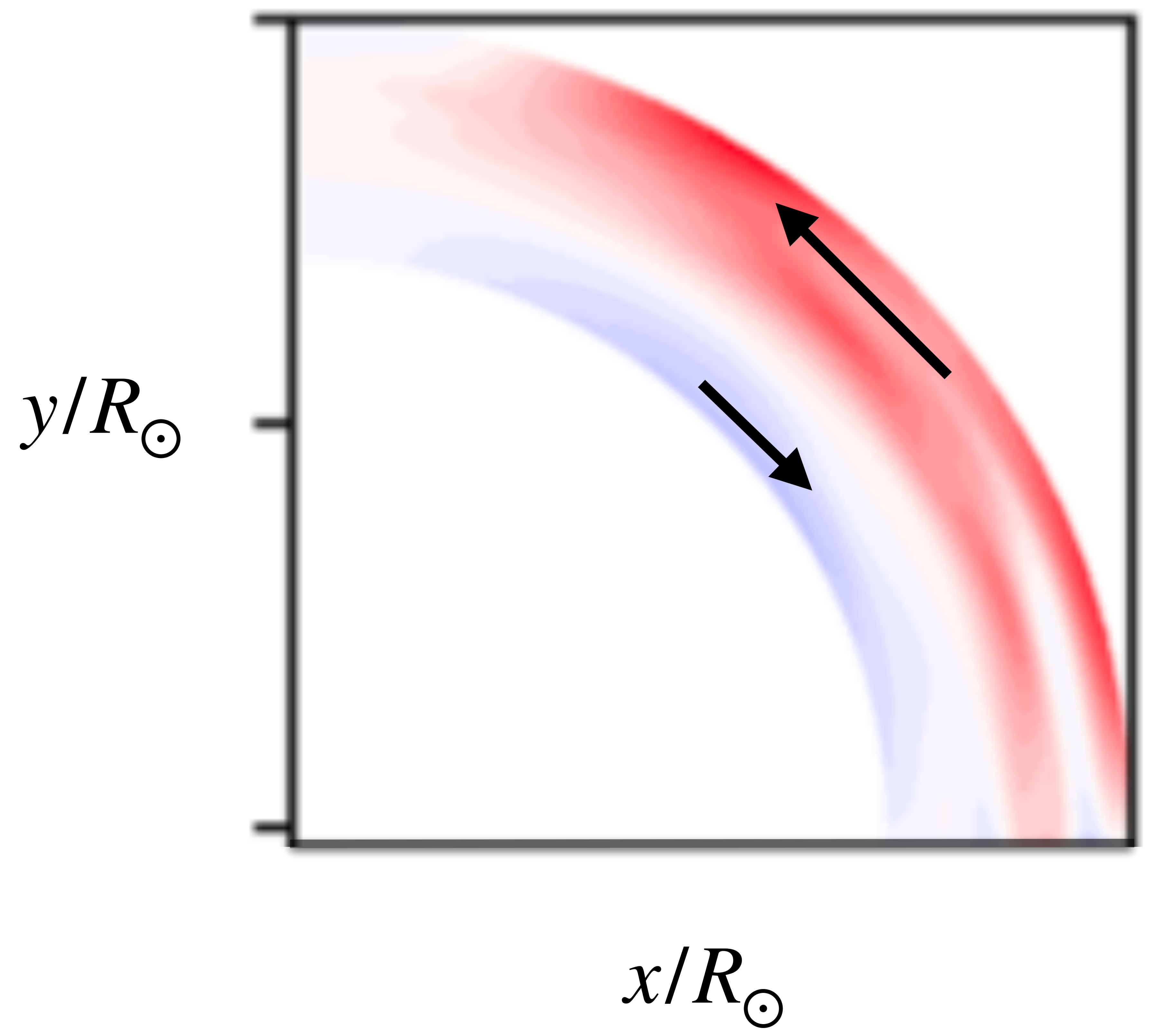


$$\rho \boldsymbol{u} = \nabla \times (\hat{\boldsymbol{e}}_\phi \Psi / r \sin\theta)$$

Double-cell (different from G20's result)

Discussion 1:
single-cell MC always transports AM toward the poles

- Let us consider a single-cell MC in the norther hemisphere…

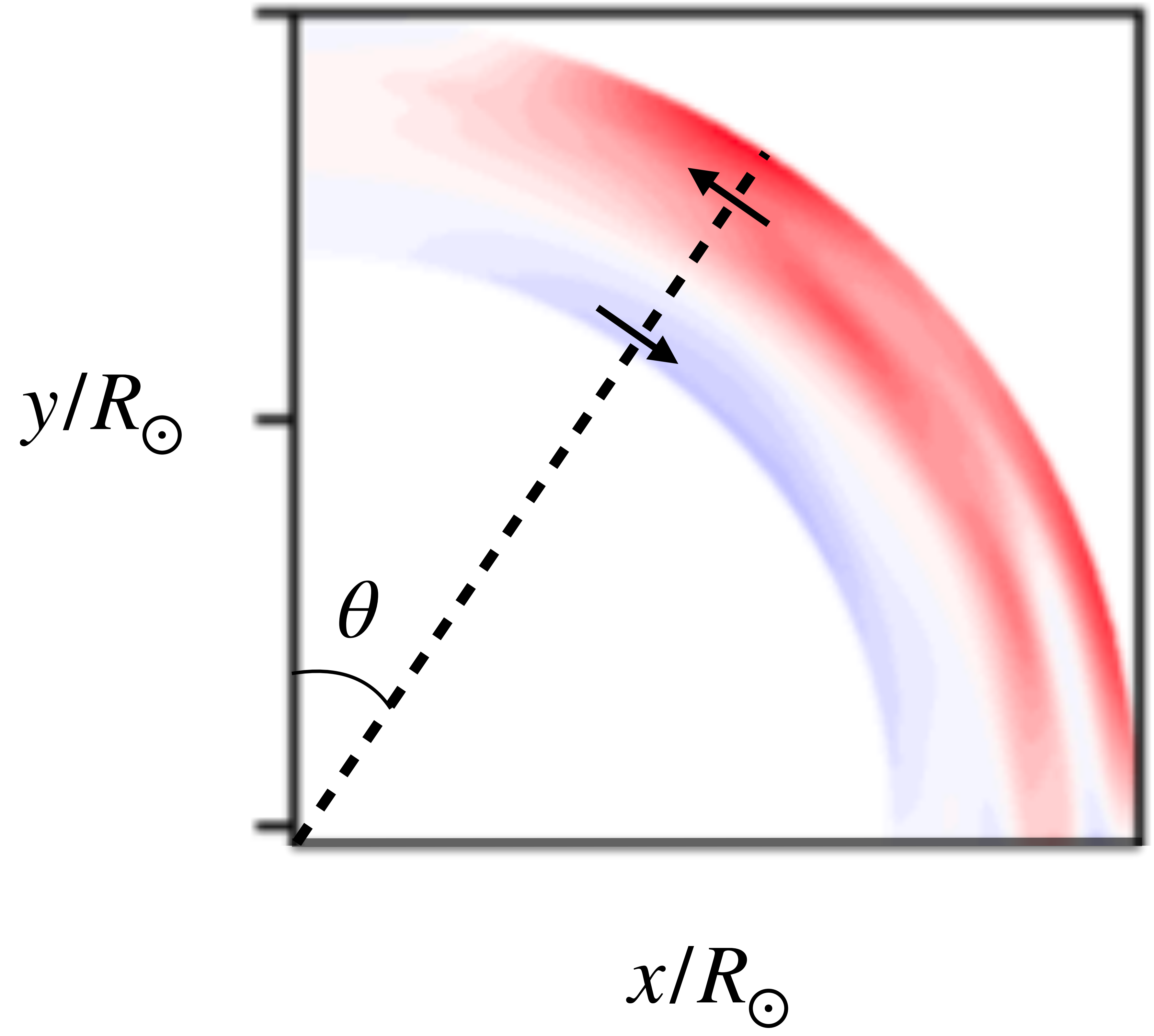


Inner convective zone → equatorward

Outer convective zone → poleward

Single-cell MC always transports AM toward the poles

- For an arbitrary colatitude θ, the mass flux M_{net} is given as below

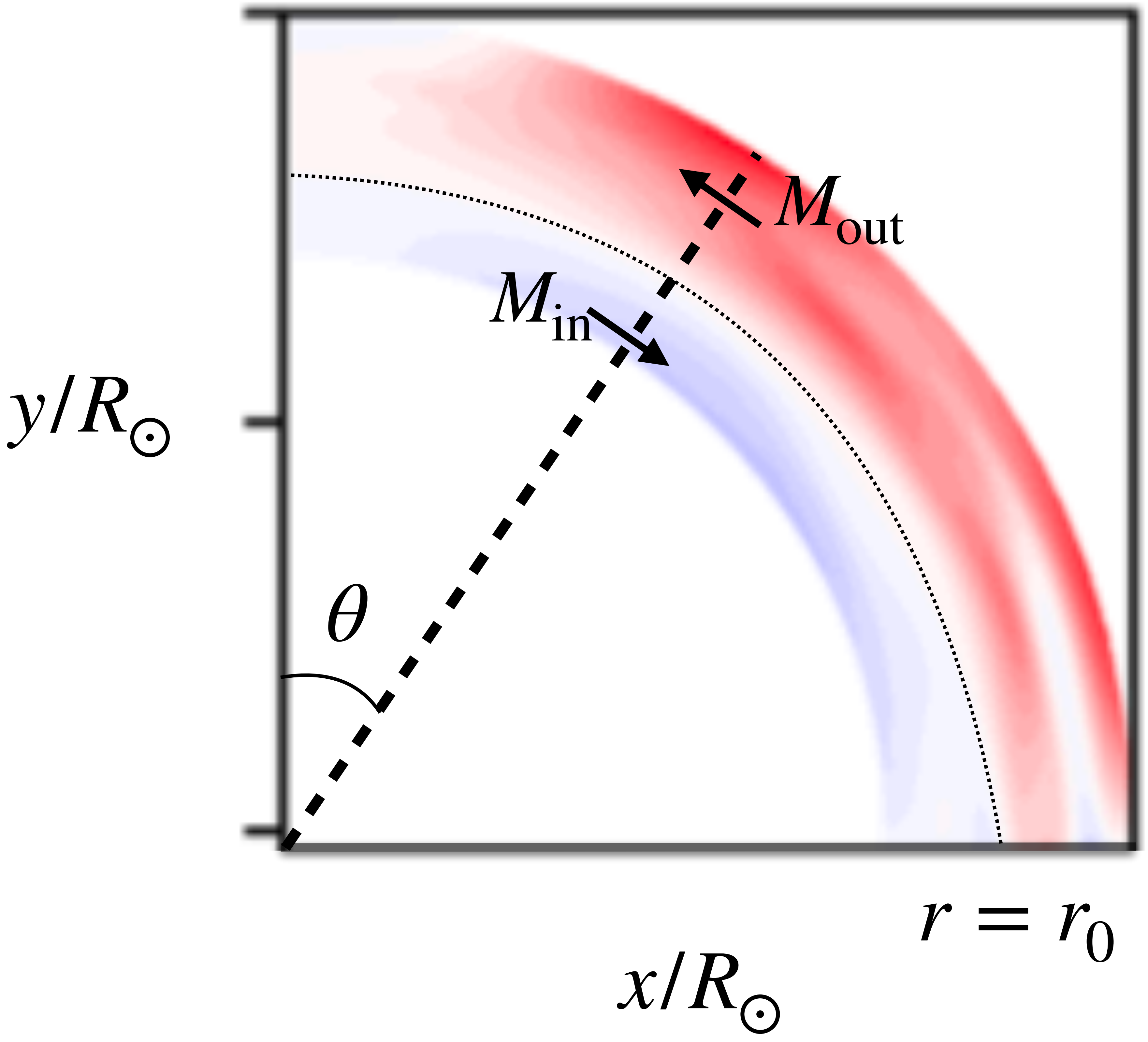


$$M_{\mathrm{net}} = \int_{r_{\mathrm{czb}}}^{R_\odot} \int_0^{2\pi} \rho U_\theta r \sin\theta \mathrm{d}r \mathrm{d}\phi$$

* $M_{\mathrm{net}} = 0$ because of the assumption of the mass conservation

Single-cell MC always transports AM toward the poles

- Take a point r_0 at which U_θ changes the sign, then

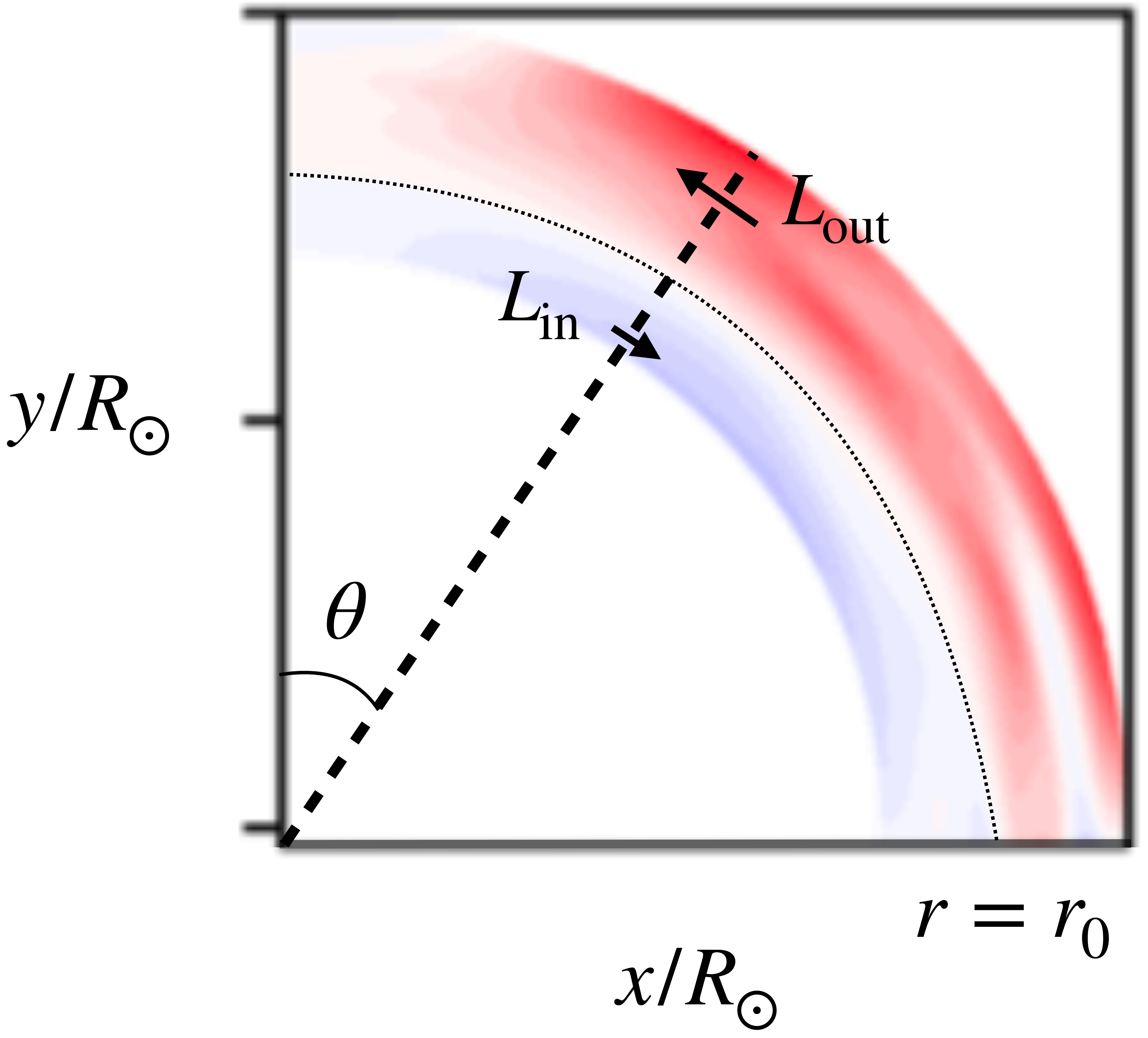


$$M_{net} = M_{in} + M_{out}$$

* $M_{net} = M_{in} + M_{out} = 0$

Single-cell MC always transports AM toward the poles

- For the same colatitude θ, the AM flux L_{net} is given as below



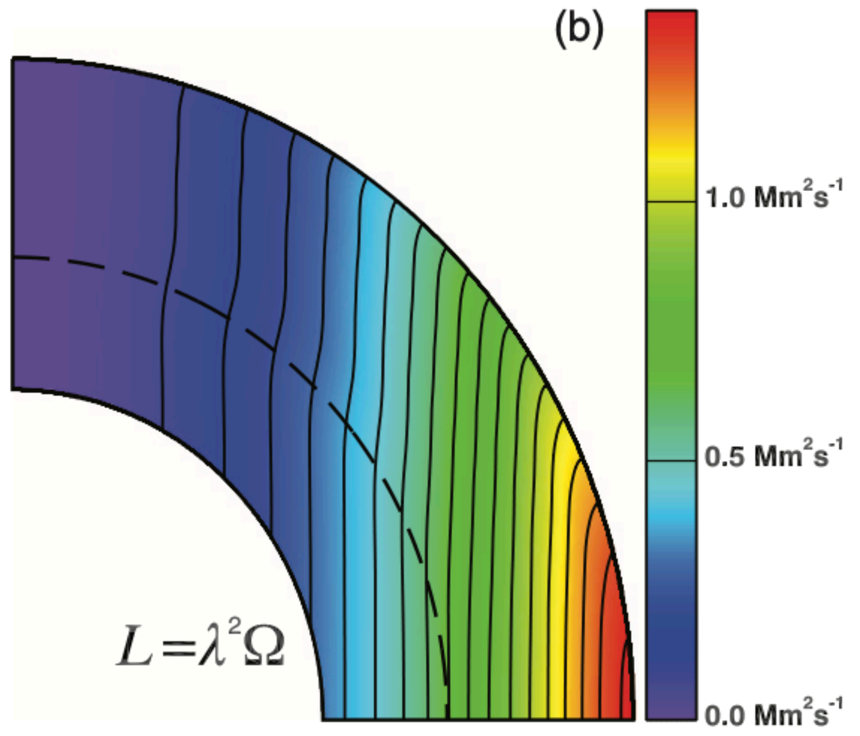
$L_{\mathrm{net}} = L_{\mathrm{in}} + L_{\mathrm{out}}$, where

$$L_{\mathrm{in}} \sim \bar{\mathscr{L}}_{\mathrm{in}} \times M_{\mathrm{in}} \text{ and } L_{\mathrm{out}} \sim \bar{\mathscr{L}}_{\mathrm{out}} \times M_{\mathrm{out}}$$

Since $\bar{\mathscr{L}}_{\mathrm{in}} < \bar{\mathscr{L}}_{\mathrm{out}}$,

$$L_{\mathrm{net}} < 0$$

AMT by single-cell MC is poleward!

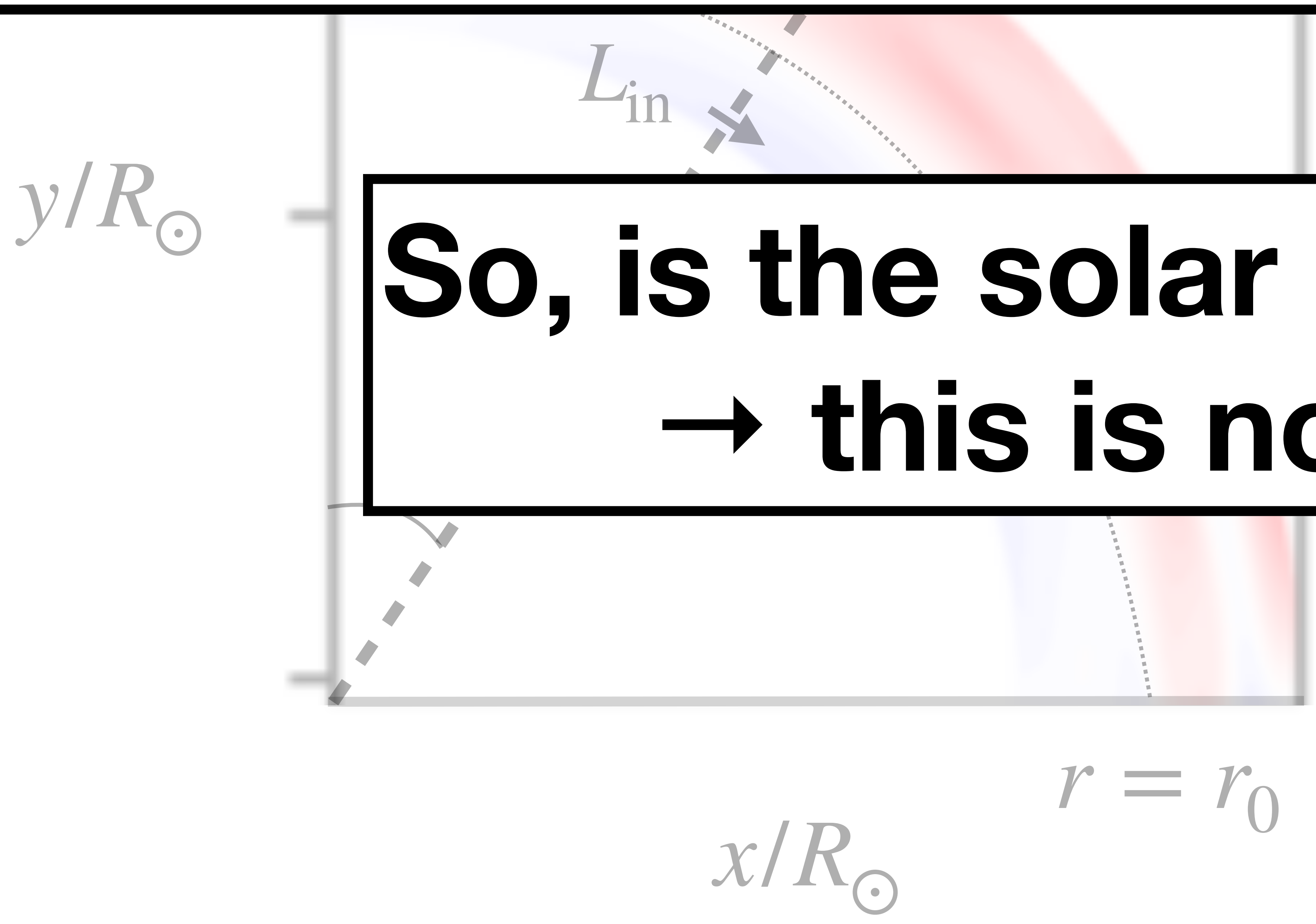


(Miesch & Hindman 2011)

Single-cell MC always transports AM toward the poles

- For the same colatitude θ, the AM flux $L_{\rm net}$ is given as below

Inferring a single-cell MC profile is not possible when we add the constraint that AMT by MC is equatorward



$$< \bar{\mathscr{L}}_{\rm out}(M_{\rm in} + M_{\rm out}) = 0$$

...gle-cell MC is ...eward!

So, is the solar MC a double-cell structure?
→ this is not necessary the case…

** $M_{\rm net} = M_{\rm in} + M_{\rm out} = 0$

Discussion 2: averaging kernel

- An estimate (of the latitudinal component) of the meridional flow field is given as a linear combination of the data:

$$\hat{U}_\theta(r_0,\theta_0) = \sum_i c_i \tau_i = \sum_i c_i \left[\int_0^\pi \int_{r_b}^{R_\odot} (K_i^r(r,\theta) U_r(r,\theta) + K_i^\theta(r,\theta) U_\theta(r,\theta)) \mathrm{d}r\mathrm{d}\theta + e_i \right]$$

$$= \int_0^\pi \int_{r_b}^{R_\odot} \left[\left(\sum_i c_i K_i^r(r,\theta) \right) U_r(r,\theta) + \left(\sum_i c_i K_i^\theta(r,\theta) \right) U_\theta(r,\theta) \right] \mathrm{d}r\mathrm{d}\theta + \sum_i c_i e_i$$

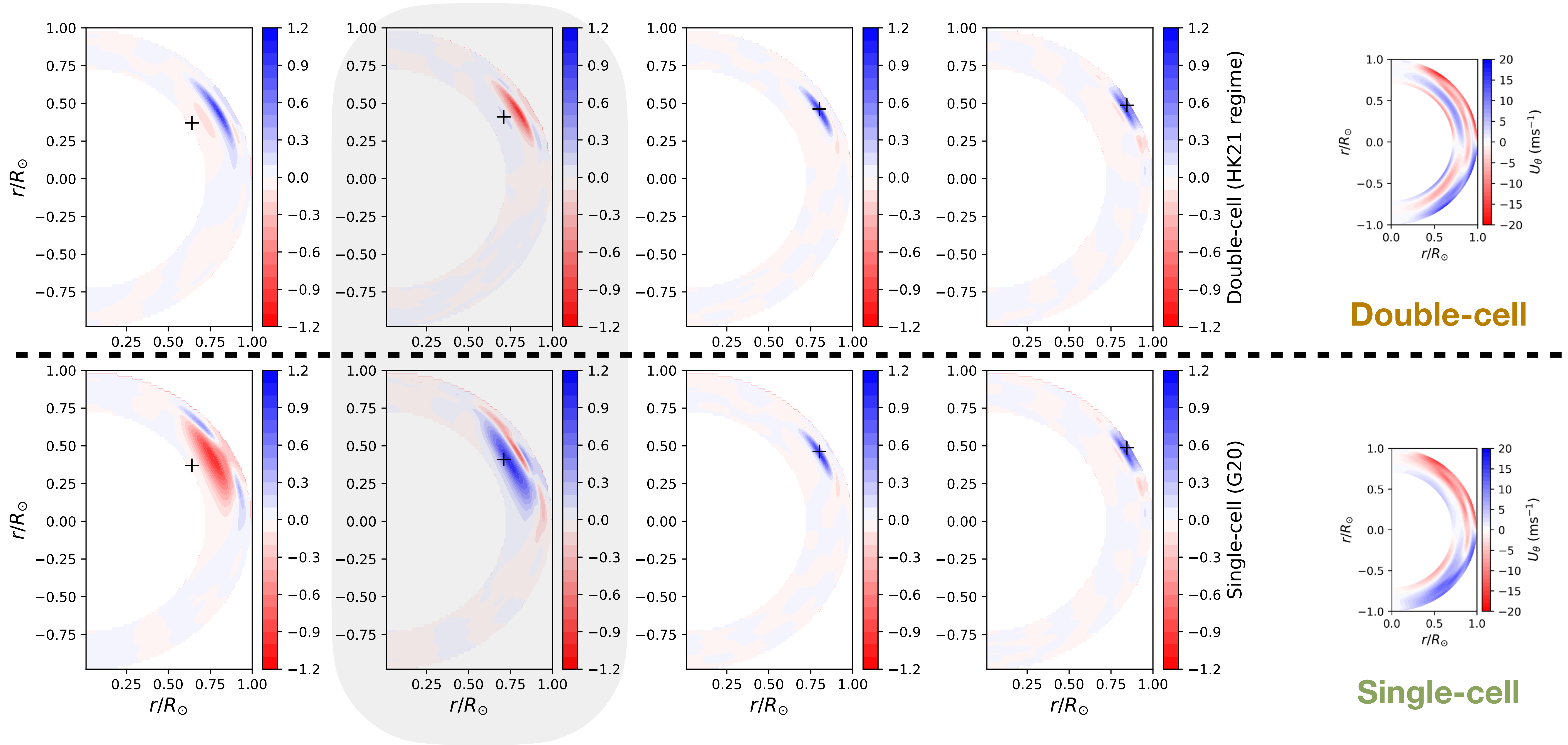
$$= \int_0^\pi \int_{r_b}^{R_\odot} \left[D_r(r,\theta) U_r(r,\theta) + D_\theta(r,\theta) U_\theta(r,\theta) \right] \mathrm{d}r\mathrm{d}\theta + \sum_i c_i e_i$$

* If $D_r \to 0,\ D_\theta \to \delta(r - r_0, \theta - \theta_0),\ \ \hat{U}_\theta(r_0,\theta_0) = U_\theta(r_0,\theta_0) + \sum_i c_i e_i$

Averaging kernels for the inversion results

deep target point ⟷ **shallow target point**

Blue: Positive
Red: Negative



Double-cell

Single-cell

* **Averaging kernels are not localized well for shallow target points**

* **Localization of the averaging kernels for the single-cell MC solution is better than that for the double-cell solution**

Summary of the results and future prospects

- If the HK21 regime is correct and the solar equator-fast rotation is sustained by AM transport by MC, the inferred MC profile is double-cell
- If we put more priority on the localization of the averaging kernels, the inferred MC profile is single-cell

→ Dilemma!

- There is another promising regime for achieving the equator-fast rotation, where the AM transport by Reynolds stress is crucial (RS regime)
- Should we test the RS regime? (what we are going to do next)

Conclusions

- Double-cell MC profile has been inferred (by inverting the travel times measured by G20 who concluded that the MC profile is single-cell) when we put the HK21-type constraint
    - We can reasonably explain the travel times based on the assumption that HK21 regime (in which the magnetic field plays an important role in AMT) is correct
    - Putting such a physics constraint on MC inversion has never been attempted
- However, localization of the averaging kernels for the single-cell MC solution is better than that for the double-cell solution
- We cannot determine the large-scale morphology of the solar MC profile at the moment
    - We will carry out MC inversion with the RS-type constraint