Dynamo Confinement of the Solar Tachocline

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29 August 2024, Nordita

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Consequences Of Fields and Flows in the Interior and Exterior of the Sun

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Astronomy & Astrophysics
Postdoctoral Fellow

Paper 1 (shortish letter):
Matilsky et al. 2022, ApJL, 940, L50

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Confinement of the Solar Tachocline by Dynamo Action in the Radiative Interior

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Paper 2 (longer, in-depth paper):
Matilsky et al. 2024, ApJ, 962, 189

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Solar Tachocline Confinement by the Nonaxisymmetric Modes of a Dynamo Magnetic Field

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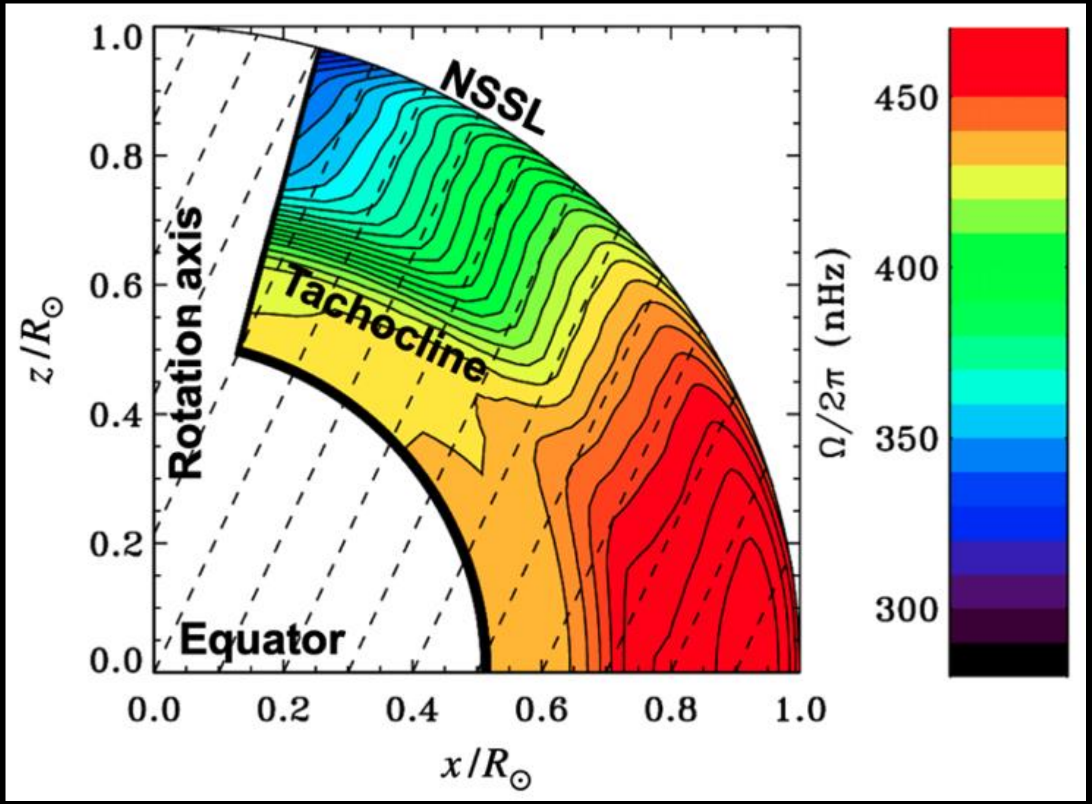
The solar rotation rate and tachocline

- Strong (20-30%) surface differential rotation
- ...increases with depth in outer 5% (NSSL)
- ...imprints along contour lines tilted $\sim 25^{\circ}$ from z-axis
- ...until solid-body rotation in radiative interior
- Transition location ($r_t \sim r_{bcz} \sim 0.7R_{\odot}$)
- Transition width $\Delta \sim 0.05R_{\odot}$
- Tachocline could be "seat" of the dynamo

Helioseismic rotation rate:
Adapted from Howe et al. 2009, LRSP, 6, 1



Time scales in the radiative zone (RZ)

- Across upper ~2 scale heights of RZ ($H = 0.2R_\odot$; recall $r_t \sim r_{bcz} \sim 0.7R_\odot$):
  - (Radiative) thermal diffusion time $H^2/\kappa_{\rm rad} \sim 7$ Myr $\kappa_{\rm rad} \sim 10^7\,{\rm cm^2 s^{-1}}$
  - Magnetic diffusion time $H^2/\eta \sim 20$ Gyr $\eta \sim 400\,{\rm cm^2 s^{-1}}$
  - Viscous diffusion time $H^2/\nu \sim 2$ Tyr $\nu \sim 4\,{\rm cm^2 s^{-1}}$
  - Eddington-Sweet time ${\rm Bu}\, H^2/\kappa_{\rm rad} \sim 60$ Gyr ${\rm Bu} = (N/2\Omega)^2 \sim 7 \times 10^4$

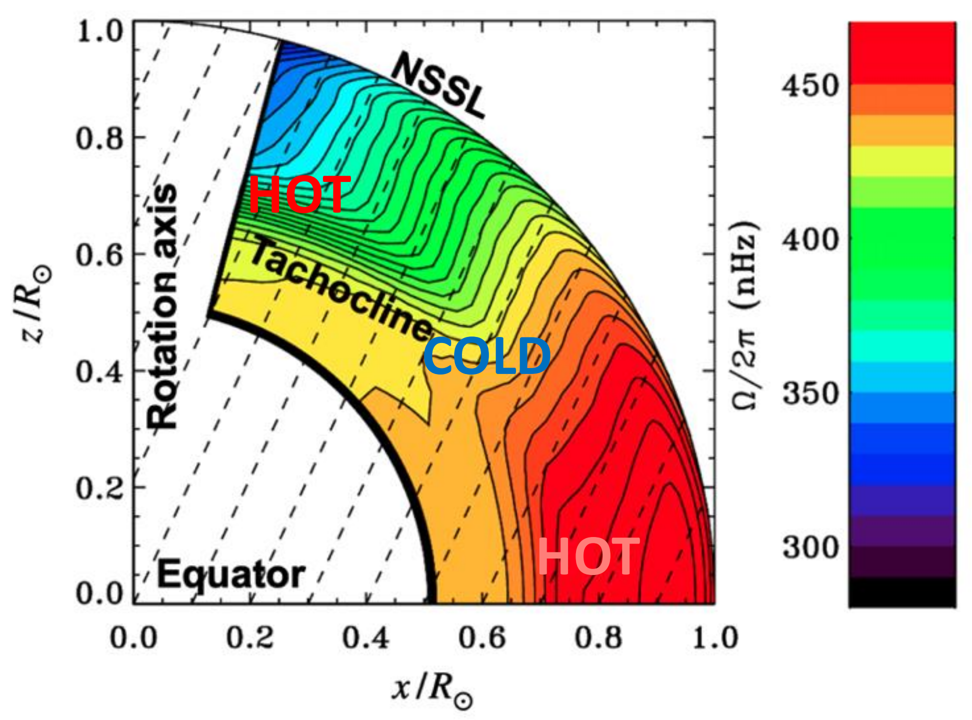
$$\sigma = \left(\frac{\text{Eddington−Sweet time}}{\text{Viscous diffusion time}}\right)^{1/2} = 0.2 \ll 1$$

Viscous versus "radiative" spread

- Tachocline can spread viscously (lots of $\nabla\Omega$)
- Also a "radiative" spreading process:
  - Thermal wind balance in tachocline
    - E.g., $\partial\Omega^2/\partial z < 0$ near poles → hot pole
  - Tachocline has baroclinic latitudinal temperature gradients
  - These diffuse inward via κ_{rad}
  - Diffuses meridional circulation inward
  - …and with it, differential rotation

Matilsky 2023, MNRASL, 526, L100

$$\frac{\partial \Omega_*^2}{\partial z} = \frac{\overline{g}}{r^2 \sin\theta}\overline{\beta_T}\left(\frac{\partial T'}{\partial\theta}\right)_P$$



Spiegel & Zahn (1992)

- Viscous spread time: 2 Tyr
- Radiative spread time: $t_{ES} \sim 60$ Gyr
- BUT radiative spread is very fast initially
  - $\Delta(t) \sim (t/t_{ES})^{1/4}$
  - $\Delta = 0.05\mathrm{R}_{\odot}$ at $t = 0$
  - $\Delta = 0.40R_{\odot}$ at $t = 4.6$ Gyr
- Must be some other active torque in RZ

The solar tachocline

E. A. Spiegel [1] and J.-P. Zahn [1,2]

[1] Astronomy Department, Columbia University, New York, NY 10027, USA
[2] Observatoire Midi-Pyrénées, 14 avenue E. Belin, F-31400 Toulouse, France

Received June 5, accepted July 20, 1992

Spiegel & Zahn 1992, A&A, 265, 106

$$r^2\rho u = \frac{\partial\Psi}{\partial x}, \qquad r\rho\sin\theta\, v = \frac{\partial\Psi}{\partial r} \tag{2.7}$$

with $x = \cos\theta$.

Equations (2.1)–(2.3) are then distilled down to

$$-\frac{1}{\rho}\frac{\partial\widehat{P}}{\partial r} + g\frac{\widehat{T}}{T} = 0 \tag{2.8}$$

$$-2\Omega r x\widehat{\Omega} = \frac{1}{\rho r}\frac{\partial\widehat{P}}{\partial x}, \tag{2.9}$$

$$\rho r^2(1-x^2)\frac{\partial\widehat{\Omega}}{\partial t} + 2\Omega x\frac{\partial\Psi}{\partial r} = \frac{(1-x^2)}{r^2}\frac{\partial}{\partial r}\left[\rho\nu_V r^4\frac{\partial\widehat{\Omega}}{\partial r}\right] + \rho\frac{\partial}{\partial x}\left[\nu_H(1-x^2)^2\frac{\partial\widehat{\Omega}}{\partial x}\right], \tag{2.10}$$

$$\frac{\partial\widehat{T}}{\partial t} + \frac{N^2}{g}\frac{T}{\rho r^2}\frac{\partial\Psi}{\partial x} = \frac{1}{\rho C_P r^2}\frac{\partial}{\partial r}\left(\chi r^2\frac{\partial\widehat{T}}{\partial r}\right). \tag{2.11}$$

$$\frac{\partial\widetilde{\Omega}}{\partial t} + \frac{4\Omega^2}{\lambda^2}\frac{1}{\rho r^2}\frac{\partial}{\partial r}\left\{\frac{g}{N^2 C_P T}\frac{\partial}{\partial r}\left[\chi\frac{\partial}{\partial r}\left(\frac{r^2 P T}{\rho g}\frac{\partial}{\partial r}\frac{\rho r^2\widetilde{\Omega}}{P}\right)\right]\right\} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho\nu_V r^4\frac{\partial\widetilde{\Omega}}{\partial r}\right) = 0. \tag{4.9}$$

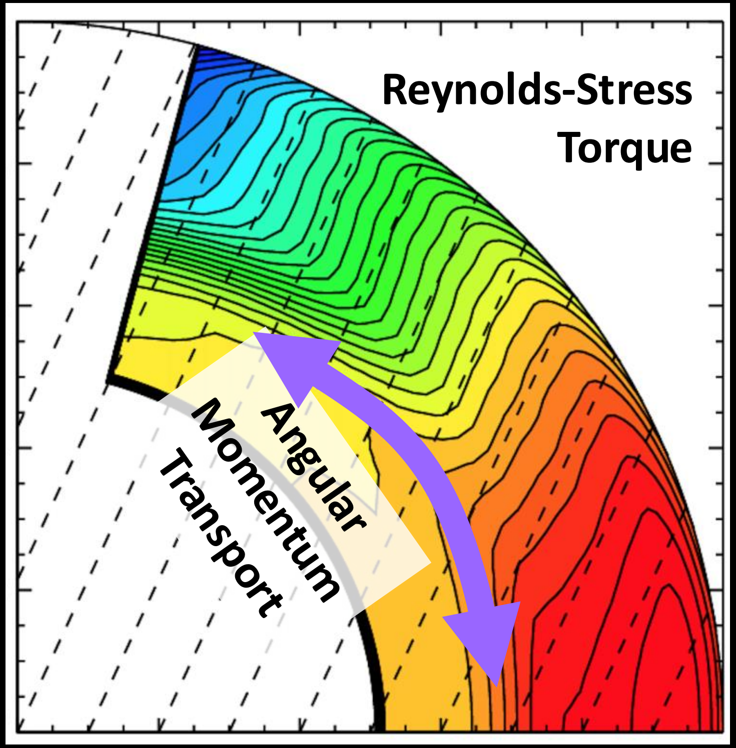
Confinement Scenarios

Two primary origin stories for torque:

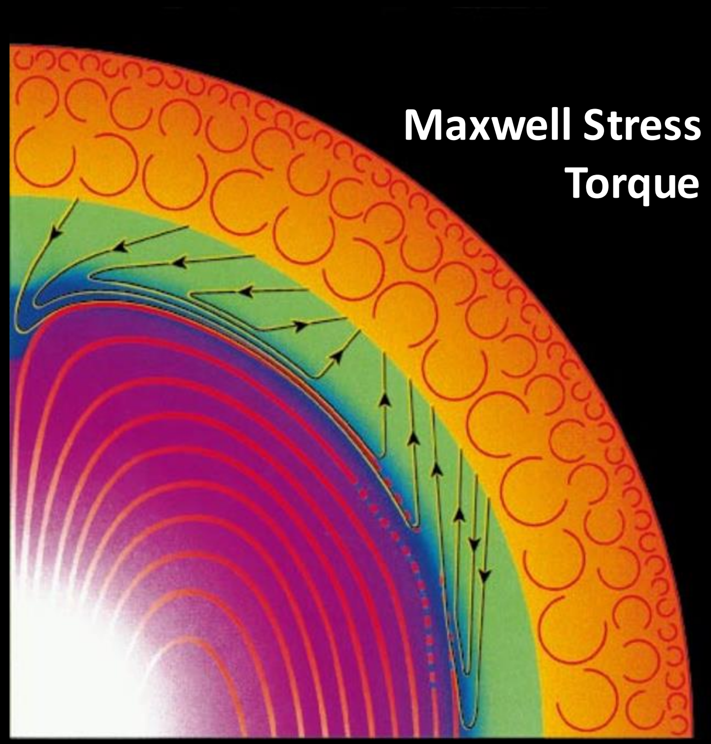
Fast HD ➢ Shear instabilities → horizontal turbulent mixing (Spiegel & Zahn 1992)

Slow MHD ➢ Magnetic field → "Stiff" poloidal field lines (Gough & McIntyre 1998)

Shear Instabilities



Magnetic Field



Flavors of Magnetic Field

- **Slow: Primordial Magnetic Field** in the radiative interior
  Gough & McIntyre 1998, Nat, 394, 755
- **Fast: Cyclic Magnetic Field** that diffuses downward into the radiative interior
  Forgács-Dajka & Petrovay 2001, SoPh, 203, 195

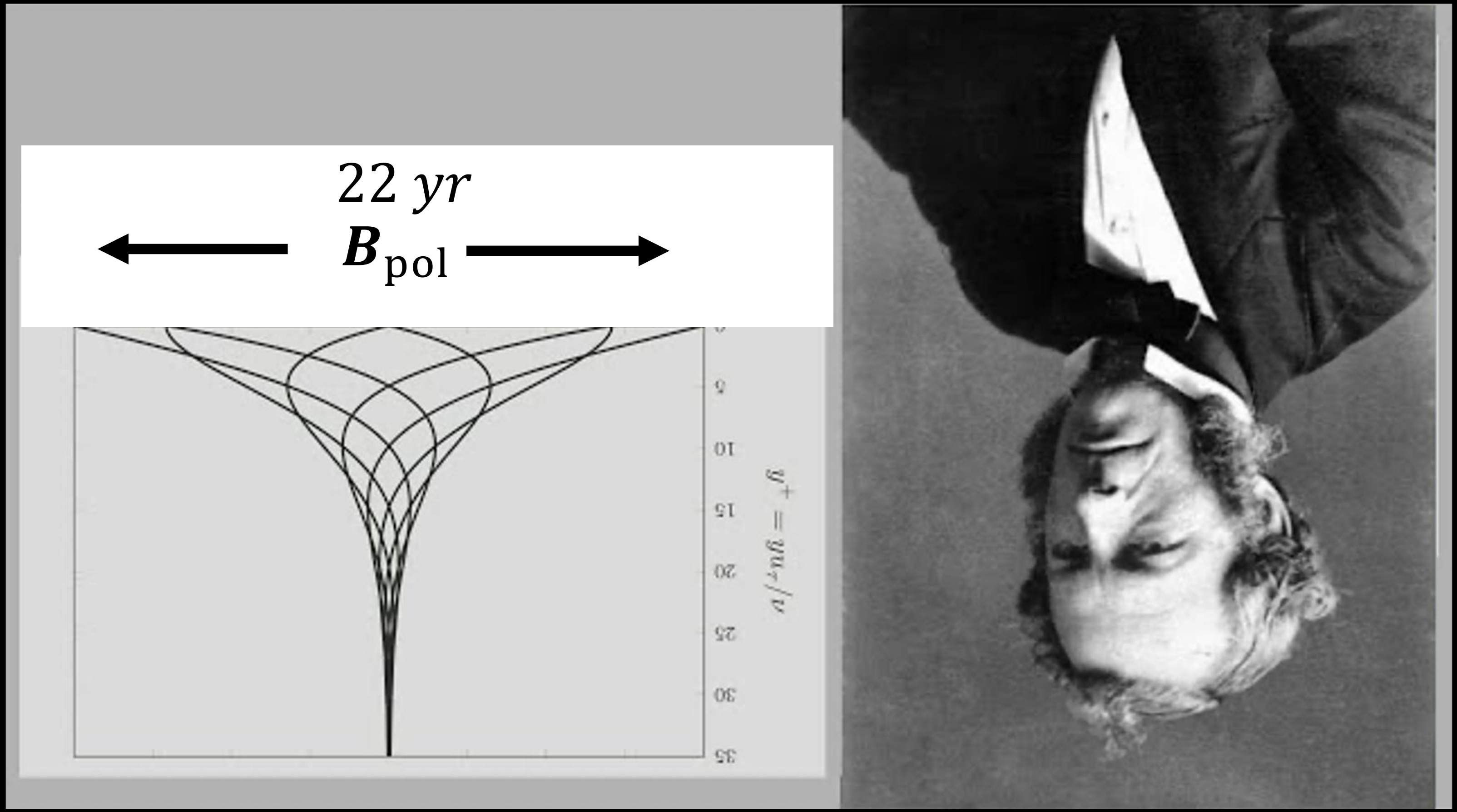
Fast magnetic confinement scenario

$$\frac{\partial \boldsymbol{B}_{\mathrm{pol}}}{\partial t} \approx \bar{\eta}\frac{\partial^2 \boldsymbol{B}_{\mathrm{pol}}}{\partial r^2}$$

Convection zone

$r = r_{bcz}$

Radiative zone



Simulated dynamos

Matilsky & Toomre (2020), ApJ, 892, 106

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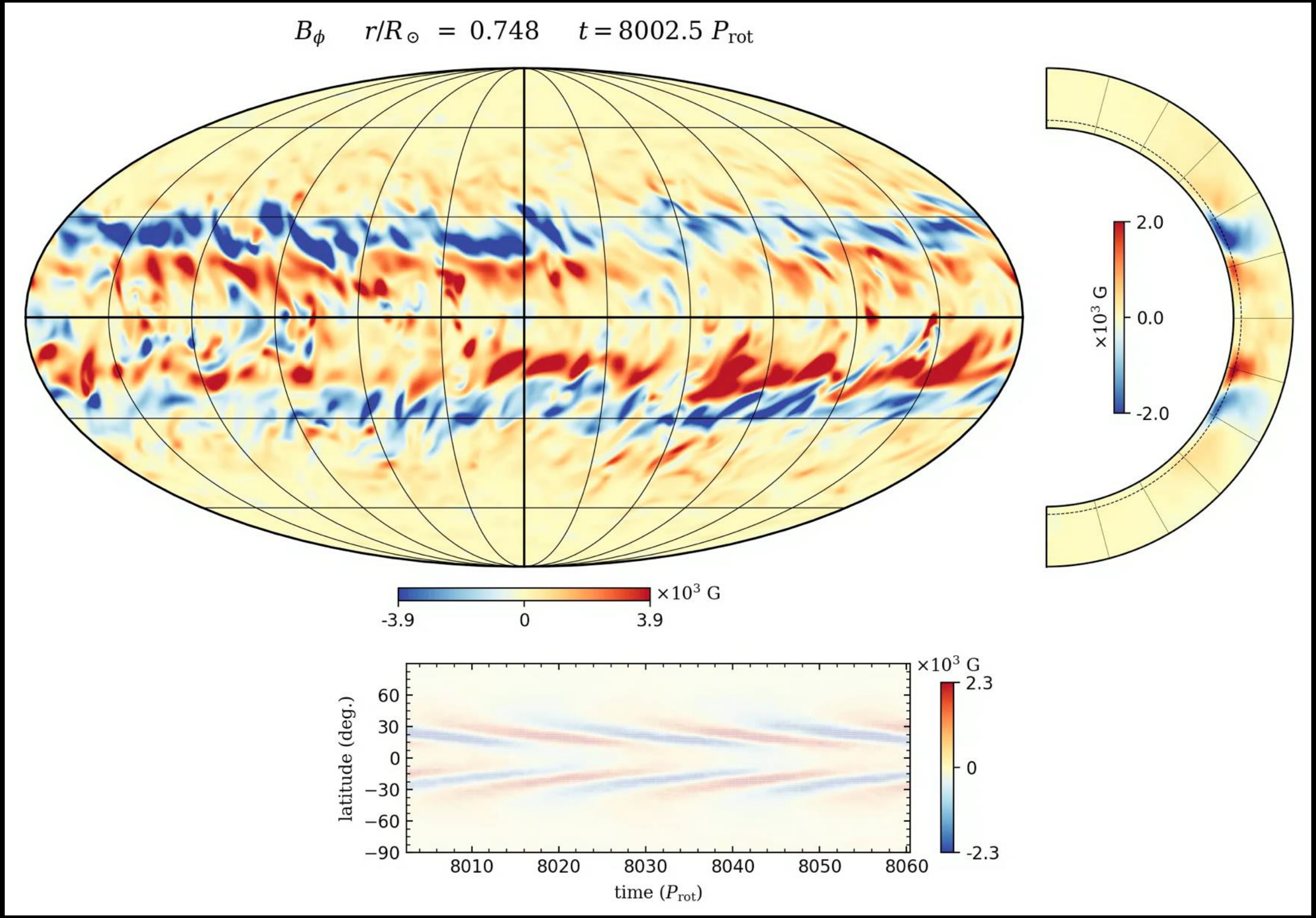


Numerical setup for tachocline simulations

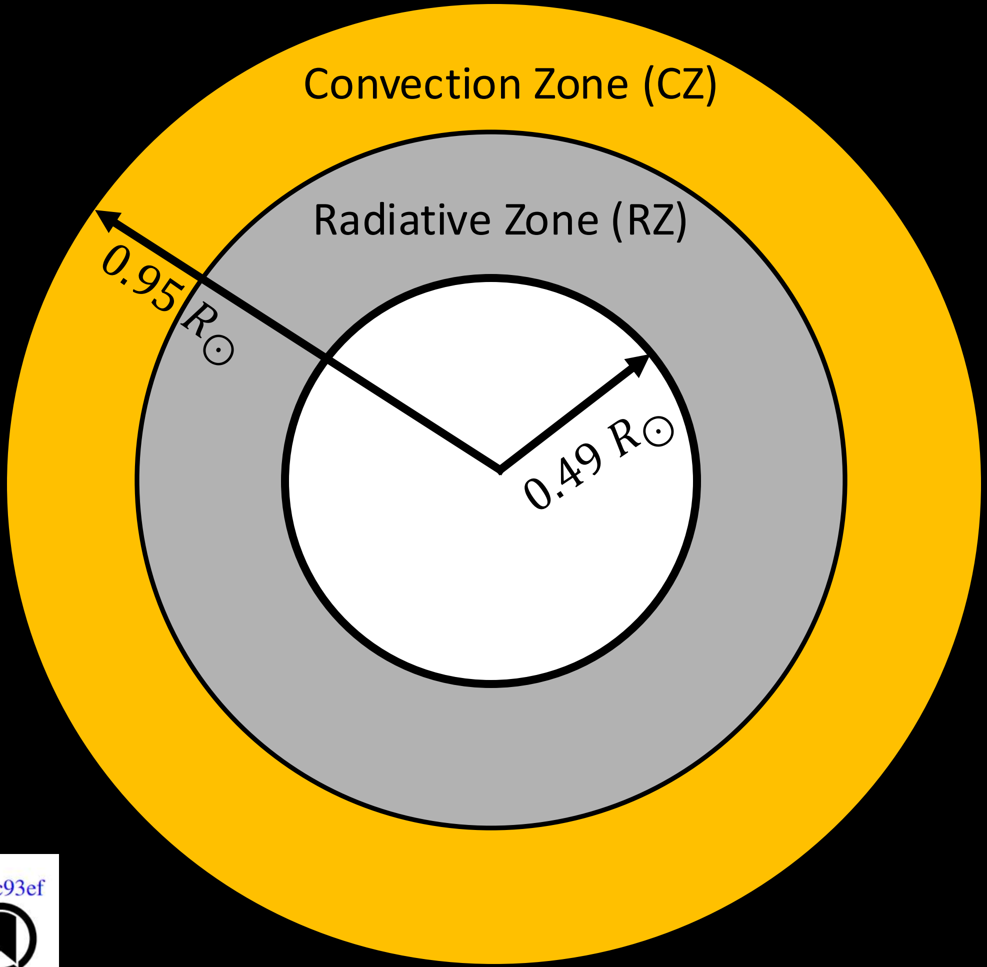
- Use Rayleigh to simulate CZ-RZ system in MHD
- Allow for tachocline spread
- Dynamo efficiency increases with $\mathrm{Pr_m}$
- Adjust field strength by varying $\mathrm{Pr_m}$
- Case name = magnetic Prandtl number (e.g., case 4.00)

Input Model Parameters

$\mathrm{Ra} = 5.7 \times 10^5$ $\quad$ $\mathrm{Pr} = 1$

$\mathrm{Ek} = 1.1 \times 10^{-3}$ $\quad$ $\mathbf{Pr_m = 1 - 8}$

$\sigma = 80 \gg 1$



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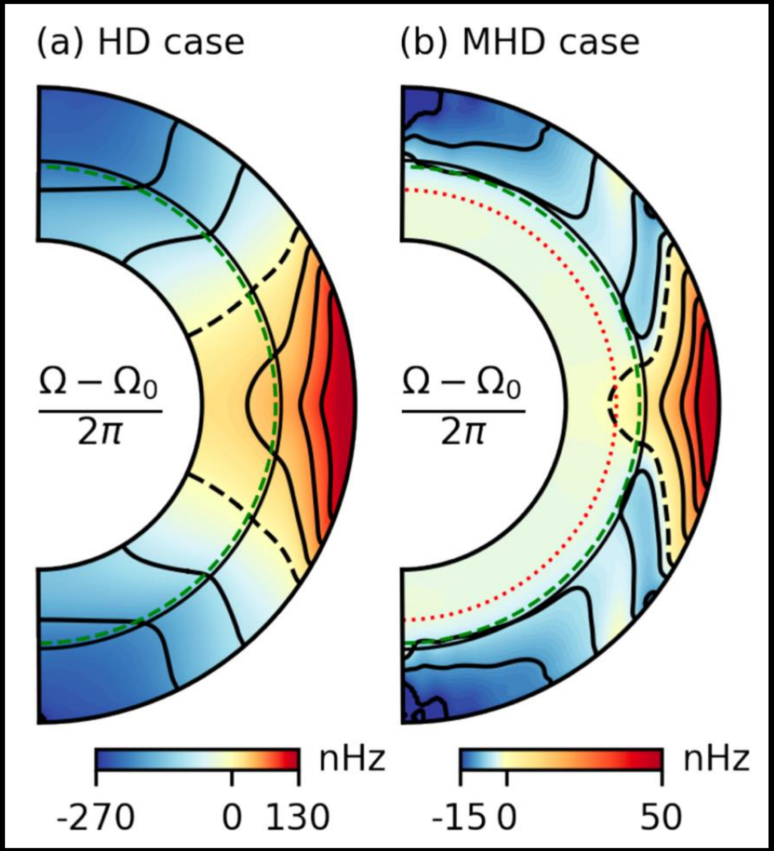
Dynamo tachocline confinement

- Two cases: hydrodynamic (HD) and magnetohydrodynamic (MHD)
- MHD case confined tachocline against viscous spread



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Natural followup

- Process most similar to fast magnetic confinement scenario:
  - ➢ How sensitive is confinement to (many) input parameters?
  - ➢ How does confinement depend on field strength?
  - ➢ How does confinement depend on cycling behavior?

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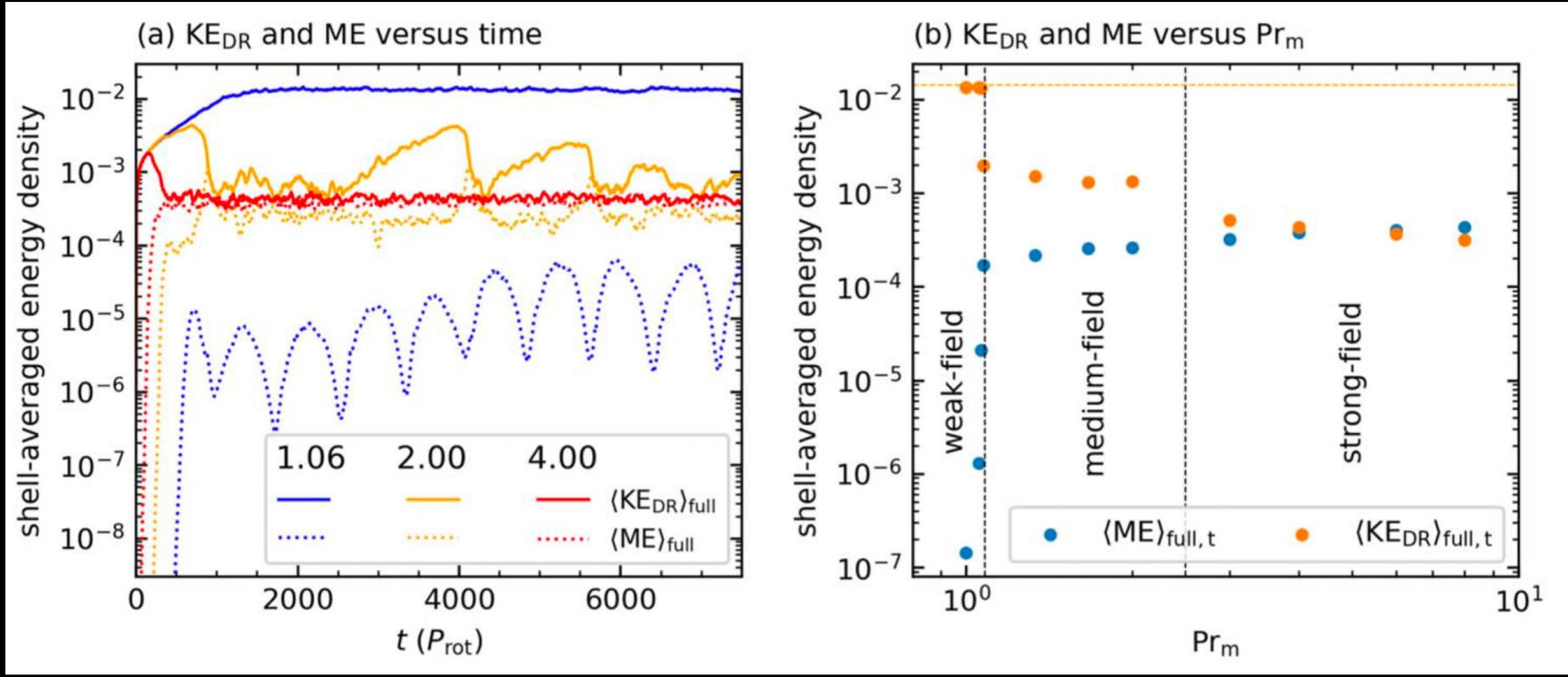
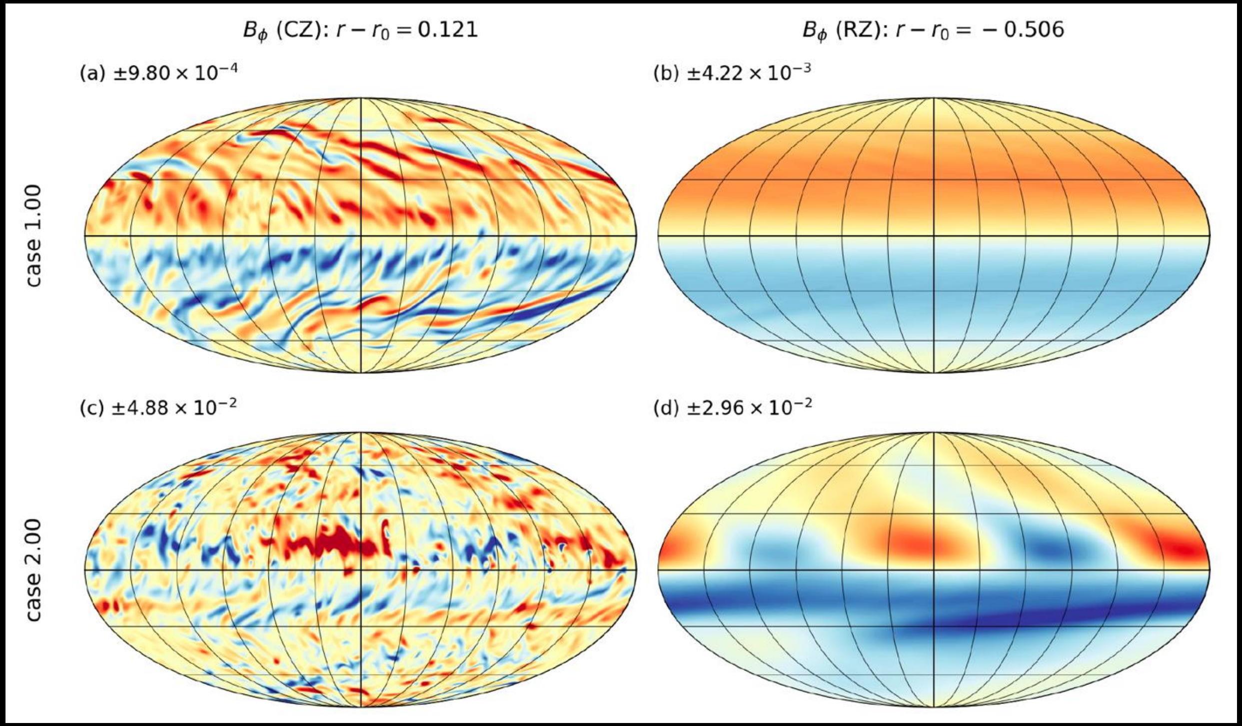
Followup paper: crank up $\mathrm{Pr_m}$

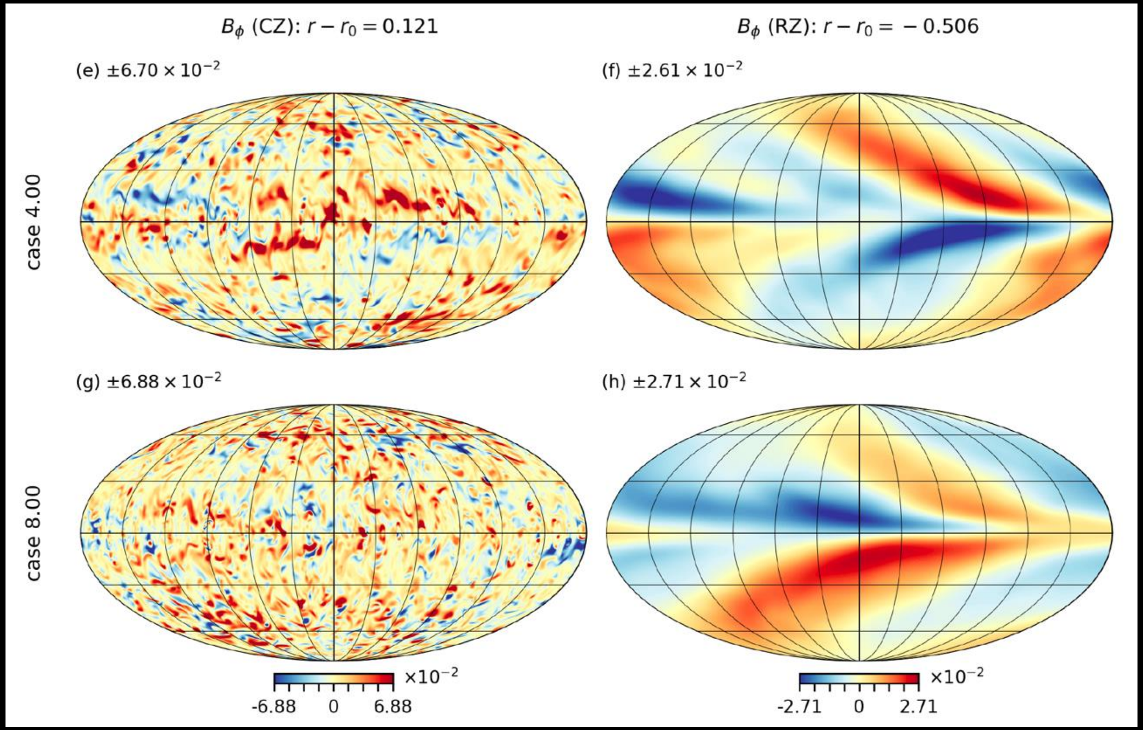
- At low $\mathrm{Pr_m}$, weak-field dynamo: $\mathrm{ME} \ll \mathrm{KE}$
  - Highly regular periodic energy cycles
- At high $\mathrm{Pr_m}$, strong-field dynamo: $\mathrm{ME} \sim \mathrm{KE}$
  - Irregular, aperiodic cycles
- Maybe medium-field regime as well



Weak- and strong-field dynamos

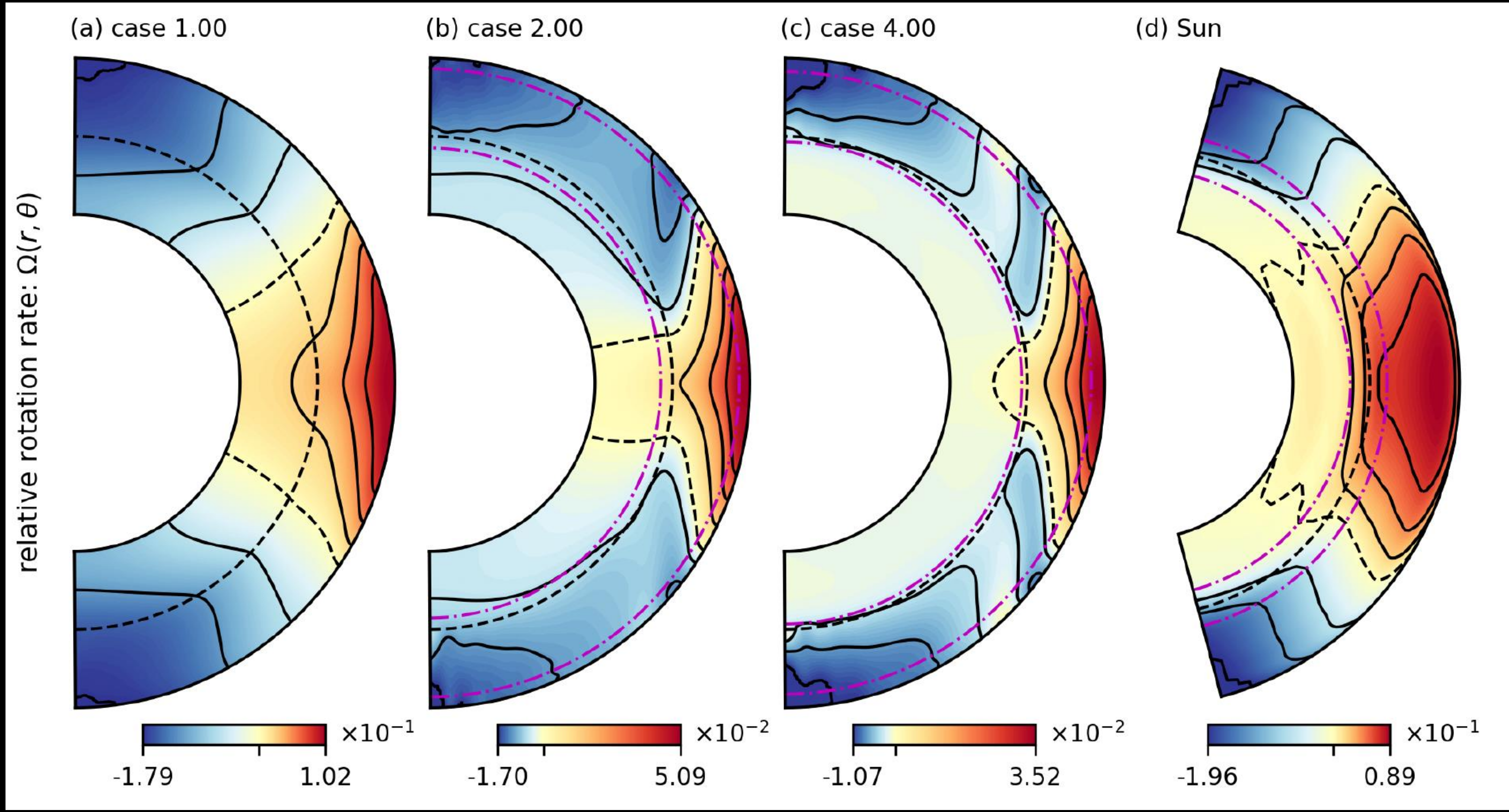
- Weak-field modes are mainly axisymmetric
- Strong-field modes are mainly nonaxisymmetric
    - ➢ Still large-scale ($m = 1,2$)
- In all cases, only largest scales (lowest m) print through to stable layer
- Suggestive of diffusive downward spread





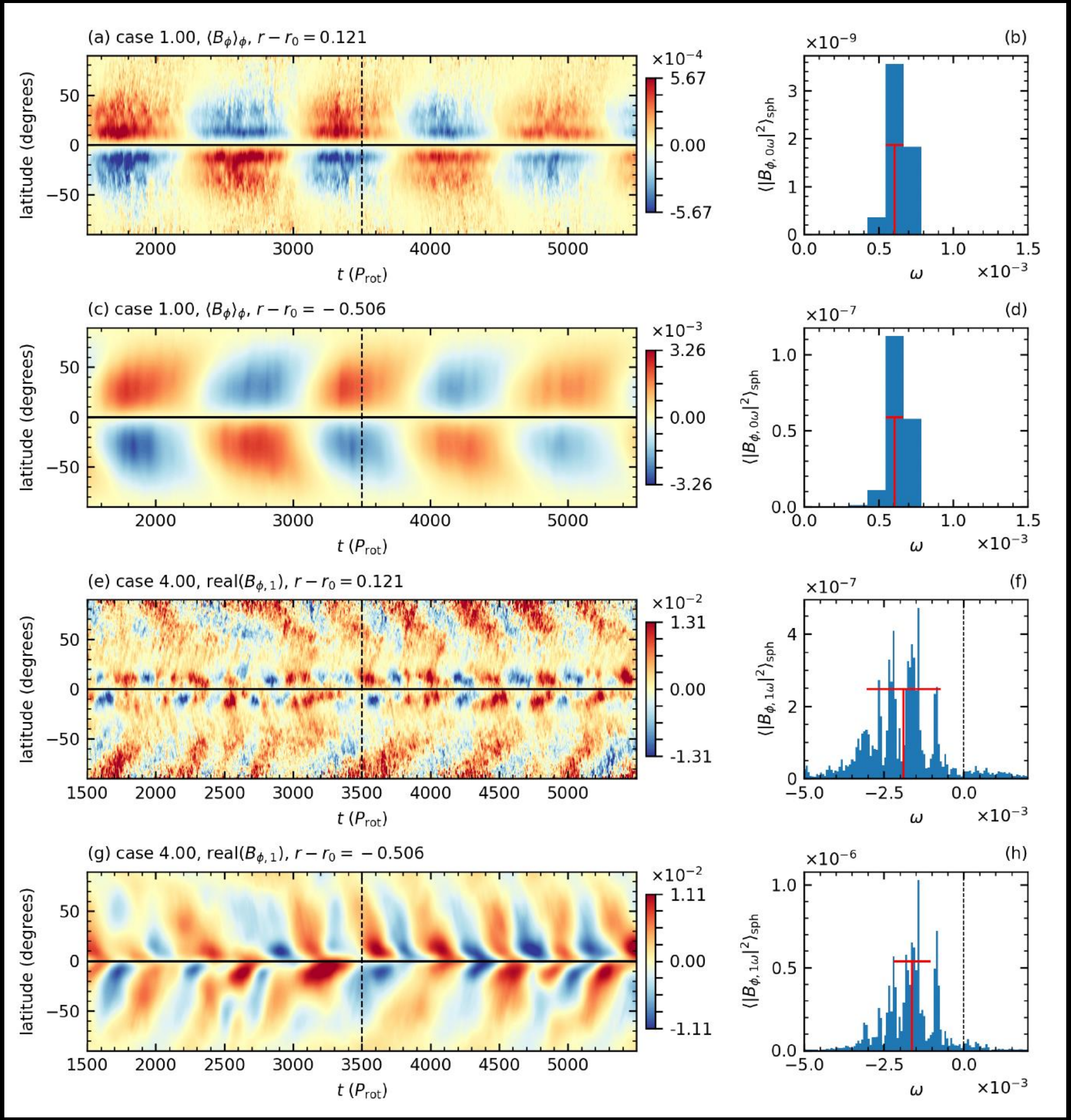
Tachoclines in strong-field cases

- Weak-field, axisymmetric fields do not confine tachocline
- Strong-field, non-axisymmetric fields do confine tachocline
  - ➢ But substantially diminish differential rotation in CZ as well
- Case 4.00 from tachocline letter ≥ *somewhat* robust!



Cycling properties

- Weak field
  - Regular polarity reversals
  - Small spread in frequency
- Strong-field
  - Irregular polarity reversals
  - Large spread in frequency
  - Preferentially *negative* frequency
- In all cases
  - Cycle in RZ occurs *after* cycle in CZ
  - Higher frequencies less prominent in RZ
  - Again suggestive of diffusive spread



Skin effect for axisymmetric fields

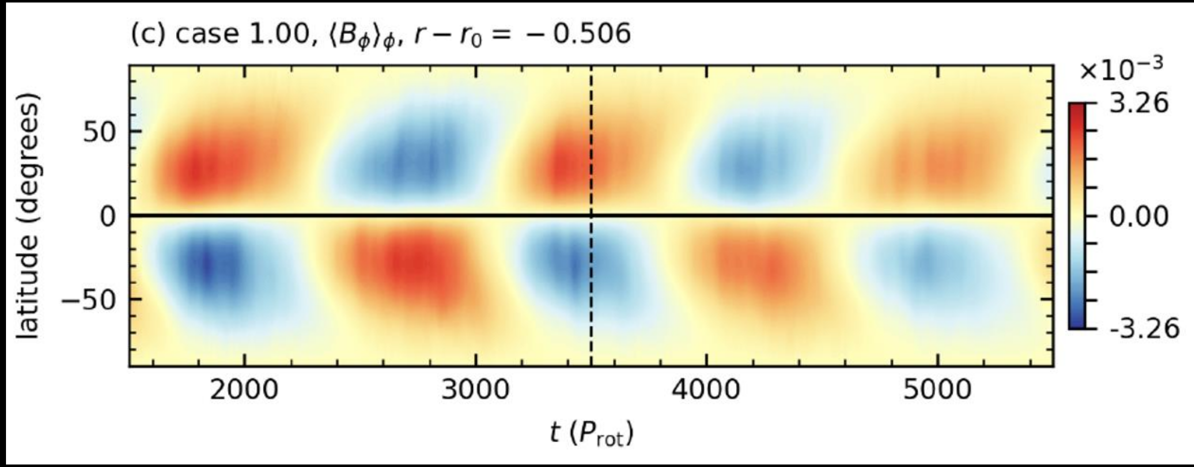
- "Traditional" skin effect:
  - Reversing axisymmetric field at top
  - Single cycle frequency ($\omega = 2\pi/22\ \mathrm{yr}$)
  - Skin depth: $\delta = \sqrt{2\bar{\eta}/\omega}$
  - Rotation rate of RZ (Ω_{RZ}) does not matter

Diffusion-only induction equation:

$$\frac{\partial \boldsymbol{B}_{\mathrm{pol}}}{\partial t} = -i\omega \boldsymbol{B}_{\mathrm{pol}} \approx \bar{\eta}\frac{\partial^2 \boldsymbol{B}_{\mathrm{pol}}}{\partial r^2}$$

Exponential damping of field strength over skin-depth

$$\left\langle |\boldsymbol{B}_{\mathrm{pol}}|^2 \right\rangle_{\mathrm{sph}} \approx \left\langle |\boldsymbol{B}_{\mathrm{pol}}|^2 \right\rangle_{\mathrm{sph}} \exp[-2(r_0 - r)/\delta]$$

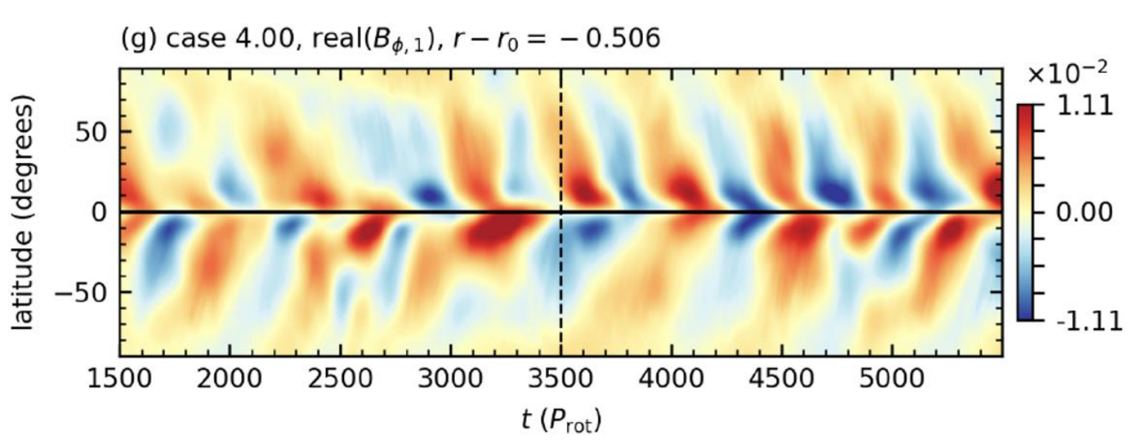


Skin effect for nonaxisymmetric fields

- Nonaxisymmetric skin effect:
  - Ω_{RZ} *does* matter
  - skin depth set by $\omega - m\Omega_{\mathrm{RZ}}$
  - Multiple frequencies (aperiodic)

Diffusion-only induction equation (*must* be in frame of RZ):

$$-i(\omega - m\Omega_{\mathrm{RZ}})\boldsymbol{B}_{\mathrm{pol},m\omega} \approx \frac{\mathrm{Ek}}{\mathrm{Pr_m}}\overline{\eta}(r)\frac{\partial^2 \boldsymbol{B}_{\mathrm{pol},m\omega}}{\partial r^2} \quad (17)$$

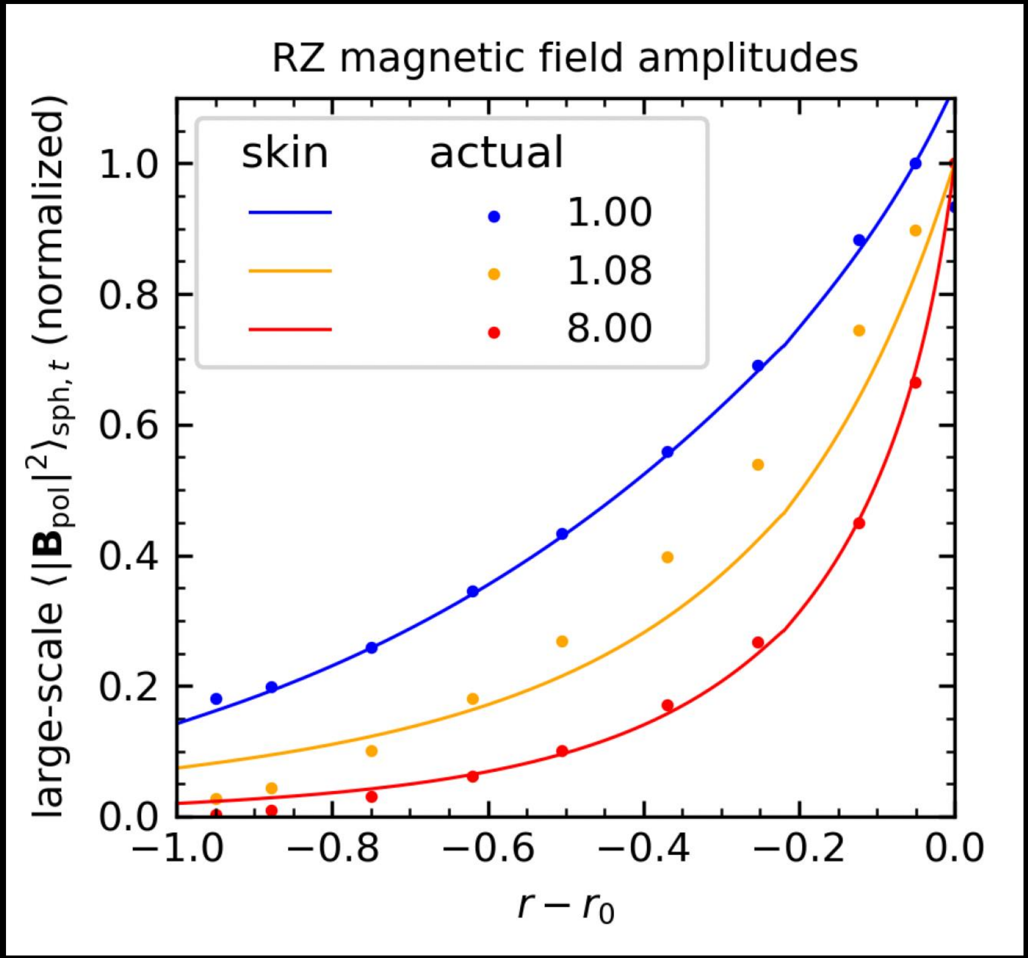


$$\left\langle|\boldsymbol{B}_{\mathrm{pol},m\omega}|^2\right\rangle_{\mathrm{sph}}(r) = \left\langle|\boldsymbol{B}_{\mathrm{pol},m\omega}|^2\right\rangle_{\mathrm{sph}}(r_0)\times \exp\left[-2\left(\frac{r_0 - r_\eta}{\delta_{m\omega}}\right)\right], \quad (20a)$$

$$\text{where} \quad \delta_{m\omega} \equiv \sqrt{\frac{2\mathrm{Ek}\overline{\eta}_{\mathrm{const}}}{\mathrm{Pr_m}|\omega - m\Omega_{\mathrm{RZ}}|}} \quad (20b)$$

Skin-depth explains field strength very well

- For all cases
  - Take frequency spectrum of $|\boldsymbol{B}_{\mathrm{pol}}|^2$ at r_0
  - Compute damping profile for each ω
  - Add weighted profiles
  - Magnetic diffusion accounts for most of strong $\boldsymbol{B}_{\mathrm{pol}}$ in deep interior



What have we learned?

- The tachocline case of Matilsky et al. (2022) is at least somewhat robust
- The fast magnetic confinement scenario works in a wider context:
  - Include large-scale **nonaxisymmetric fields**
  - Overall rotation rate of the RZ matters
  - Scenario includes aperiodic dynamos
  - Any field nearly corotating with RZ should penetrate very deeply

End of talk; extra slides follow

Open question 1: constraints on interior solar field strength

- Need $\boldsymbol{B}$ to halt radiative spread in tachocline
- …But not disturb balance in CZ

$$0 = \begin{cases} \underbrace{-\frac{4\Omega_\odot^2}{N^2} r_0^2 \overline{\rho}\, \overline{\kappa} \frac{\partial^4 \langle \mathcal{L} \rangle_t}{\partial r^4}}_{\tau_{\rm rad}\ \text{(radiative spread)}} + \tau_{\rm mag} & \text{in the RZ} \\ \tau_{\rm rs} + \tau_{\rm mc} & \text{in the CZ,} \end{cases} \quad (23)$$

- Lower bound on $\boldsymbol{B}$ using observed $\Delta \sim 0.05\ R_\odot$

$$\tau_{\rm mag} \sim \tau_{\rm rad} \sim 0.84\ \text{dyn cm}^{-2} \qquad \text{in the RZ.}$$

- Upper bound on $\boldsymbol{B}$ using observed circulation torque

$$\tau_{\rm rs} \sim \tau_{\rm mc} \sim 1.2 \times 10^6\ \text{dyn cm}^{-2} \qquad \text{in the CZ.}$$

- Could solar interior $\boldsymbol{B}$ obey (something like) following constraint?

$$4.8\ \text{G} \lesssim |B_\phi| \ll 5800\ \text{G.}$$

Open question 2: Can the dynamo field really penetrate?

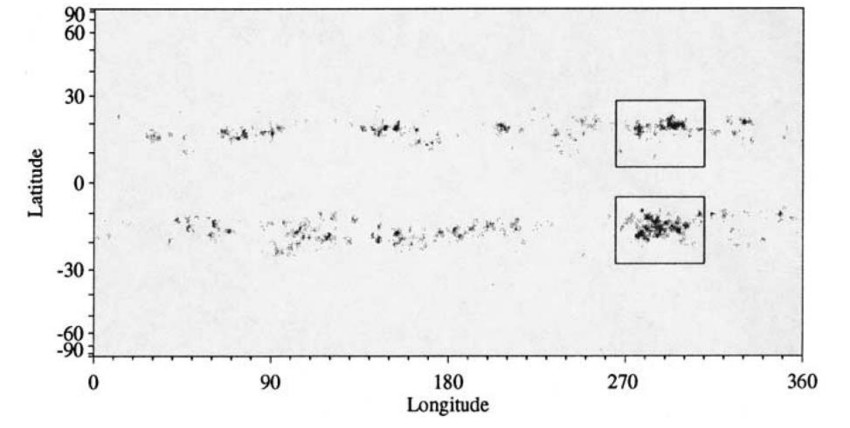
- Formally, η and δ are very small in the Sun
  - 2a: Could η be turbulently enhanced?
  - 2b: Could there be long-lived dynamo component?
    - Either ~permanent axisymmetric field
    - Or corotating nonaxisymmetric field

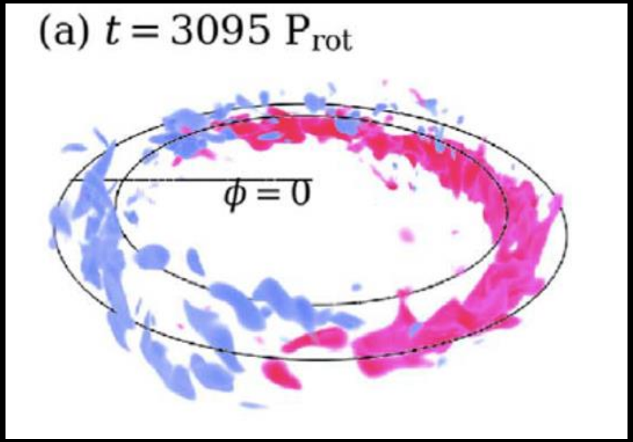
$$\delta_{m\omega} = \left(\frac{2\langle \eta \rangle_{\rm RZ}}{|\omega - m\Omega_{\rm RZ}|} \right)^{1/2} = (0.027 R_\odot) P_{\rm cyc}^{1/2}, \qquad (30)$$

where $P_{\rm cyc} \equiv 2\pi / |\omega - m\Omega_{\rm RZ}|$ and is measured in Gyr. If we require diffusive spread over (say) $\Gamma_\odot = 0.05 R_\odot$, we need $P_{\rm cyc} \sim 1.4$ Gyr. With the solar age at $t_\odot = 4.6$ Gyr, such a high

Sources of long-lived solar fields

- Where can we get ~Gyr-time-scale fields?
  - Primordial field (Gough + McIntyre 1998)
  - Diffusive field from random (Garaud 1999, A&A, MNRAS, 304, 583)
  - Corotating nonaxisymmetric field (Matilsky et al. 2024)
  - Fun to speculate about “active longitudes”
    - In our models, $\omega_{cyc} \sim \Omega_{RZ}$
    - Nonaxisymmetric advect , similar to active longitudes
    - Could rotation rate of observed active longitudes *determine* rotation rate of RZ?





Nonaxisymmetric dynamo cycles

- Primary dynamo mode is nonaxisymmetric ($m = 1, 2$)
- B_θ and B_ϕ are clearly correlated
- Resultant Maxwell stress always opposes differential rotation
  - ➢ Result of magnetic tension
- Downward spread of cycle