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Hydrodynamic simulations as a test bed for turbulent convection models

Felix Ahlborn, Johann Higl
Collaborators: Achim Weiss, Friedrich Kupka

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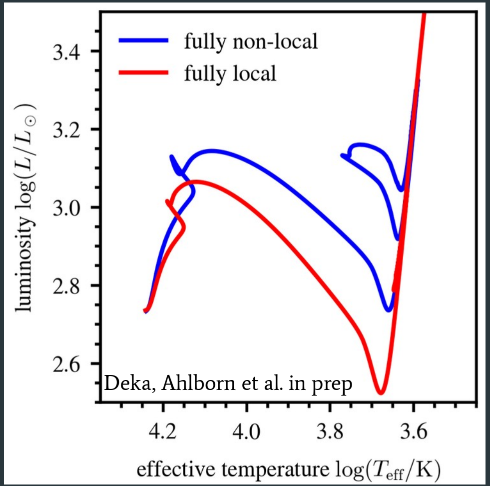
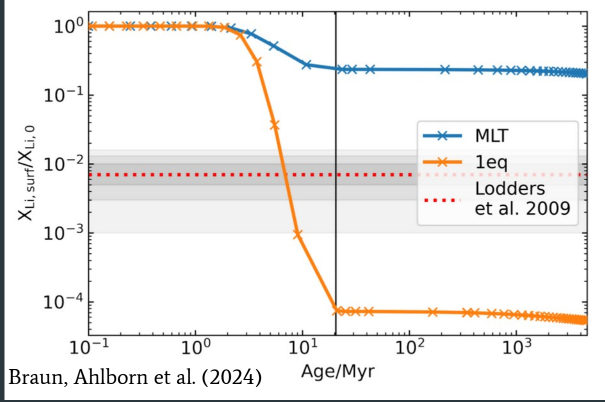
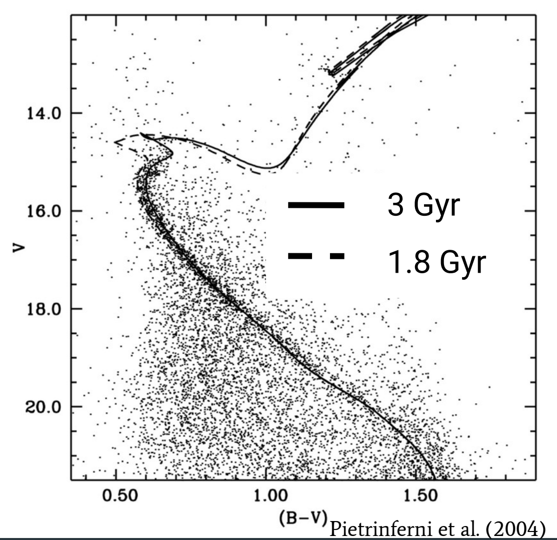


luminosities



chemical elements



ages

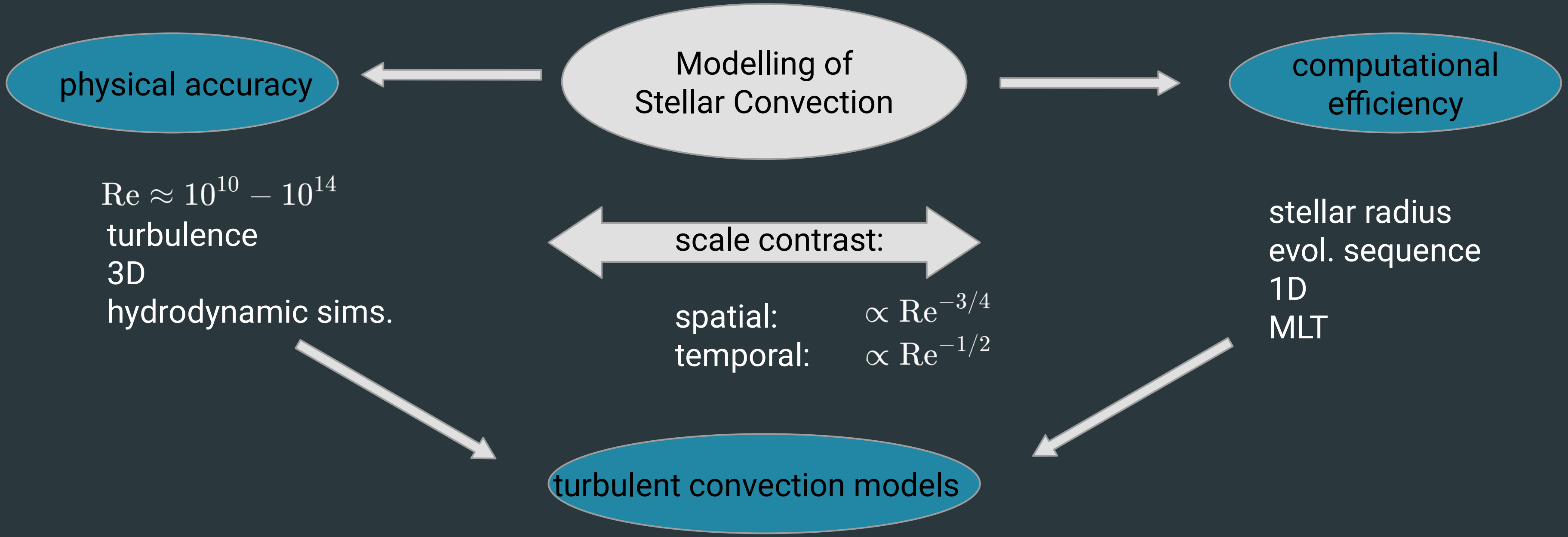
Stellar convection

exoplanet systems

distances

galactic evolution

etc.



aim: *comparison of 1D turbulent convection model to 3D hydrodynamic simulations*

Turbulent convection models I

- three equations for three convective variables (Kuhfuß 1986,1987, Flaskamp 2003)

$$\omega = \tfrac{1}{2}\overline{u'^2},\ \Pi = \overline{u's'},\ \Phi = \tfrac{1}{2}\overline{s'^2}$$

$$\mathrm{d}_t\omega = \frac{\nabla_{\mathrm{ad}}T}{H_P}\Pi \quad - \quad \varepsilon \quad - \quad \frac{1}{\rho}\mathrm{div}(-D_\omega\nabla\omega)$$

buoyant driving — dissipation — non-local flux

T = temperature
Π = convective flux
H_P = pressure scale height
ρ = density

- dissipation of kinetic energy
  - Kolmogorov cascade

$$\varepsilon = C_D\frac{\omega^{3/2}}{\Lambda}$$

in overshooting zone: $\Lambda \propto \frac{1}{\text{buoyancy freq.}} \propto \frac{1}{\nabla - \nabla_{\mathrm{ad}}}$

Kupka, Ahlborn and Weiss (2022), Ahlborn et al. (2022).

Turbulent convection models II

- different flavours
- 1-equation model: mixing length approximation for convective flux

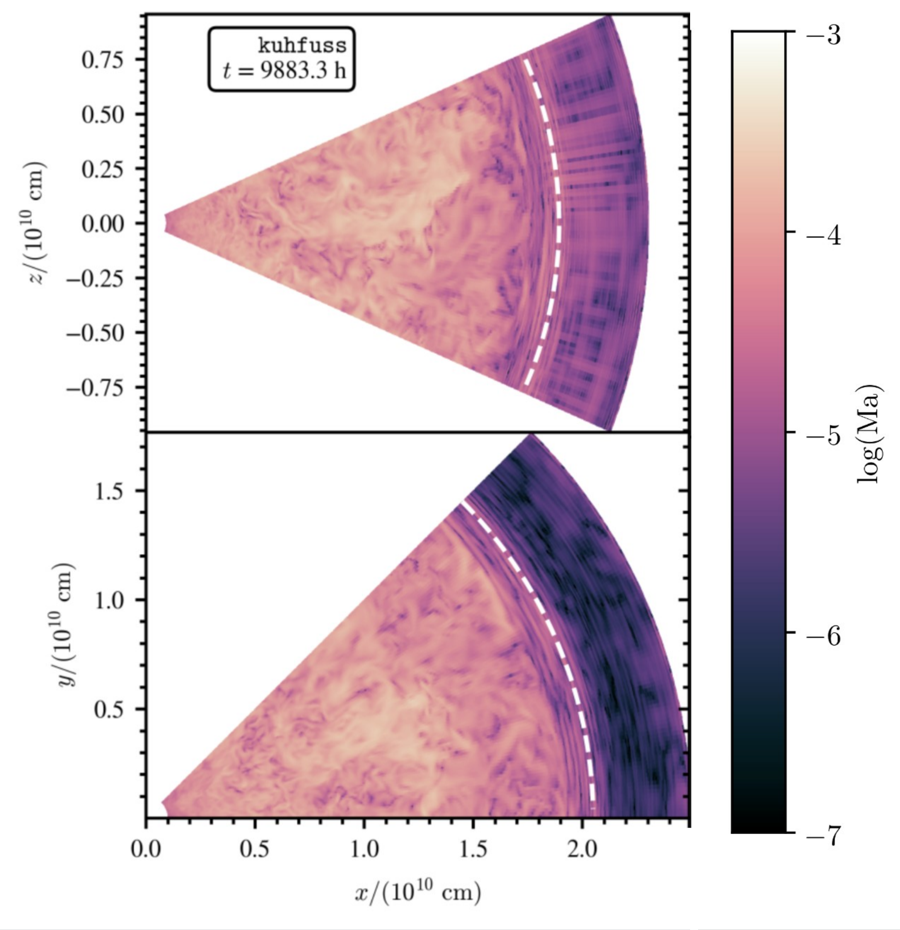
$$\Pi = \alpha_s \Lambda \sqrt{\omega} \frac{c_p}{H_p} (\nabla - \nabla_{\mathrm{ad}}) \propto -\Lambda \sqrt{\omega} \frac{\partial s}{\partial r}$$

- 3-equation model: two additional equations

$$\mathrm{d}_t \Pi = \frac{2 \nabla_{\mathrm{ad}} T}{H_P} \Phi + \frac{2}{3} \frac{c_p}{H_p} (\nabla - \nabla_{\mathrm{ad}}) \omega - \frac{1}{\rho} \mathrm{div}(-D_\Pi \nabla \Pi) - \frac{\Pi}{\tau_{\mathrm{rad}}}$$

$$\mathrm{d}_t \Phi = \frac{c_p}{H_p} (\nabla - \nabla_{\mathrm{ad}}) \Pi - \frac{1}{\rho} \mathrm{div}(-D_\Phi \nabla \Phi) - \frac{2\Phi}{\tau_{\mathrm{rad}}}$$

- implemented into the Garching stellar evolution code (Weiss and Schlattl 2008)



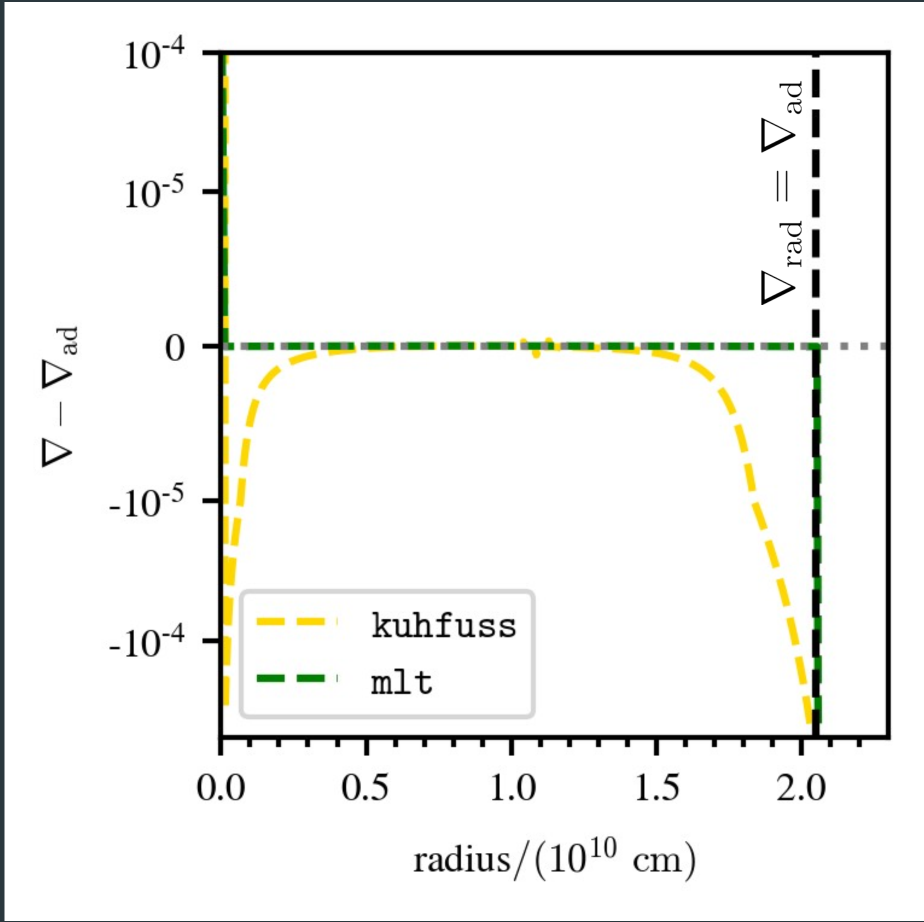
Hydrodynamic simulations

- Seven Leagues Hydro (SLH) (e.g. Miczek 2015, Edelmann et al. 2021)
- three-dimensional wedge geometry
  - 384 x 96 x 96
- nominal luminosity
- ~10000 h simulation time
- Reynolds Averaged Navier Stokes (RANS) analysis

Ahlborn and Higl in prep.

Initial stellar models

- $3M_\odot$ stellar model as initial model
  - beginning of main-sequence
- two 1D stellar models for comparison
  - MLT
  - Kuhfuss 3-equation
- different final states
  - thermal timescale too long
- which one is correct?
  - probably none



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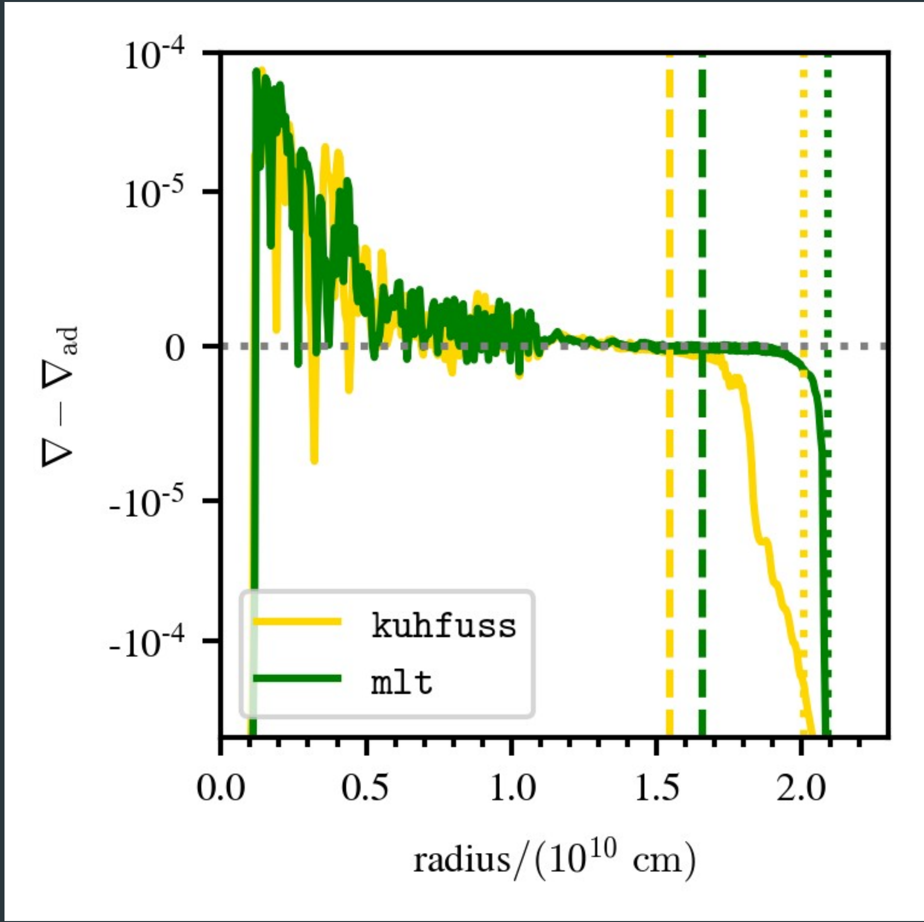
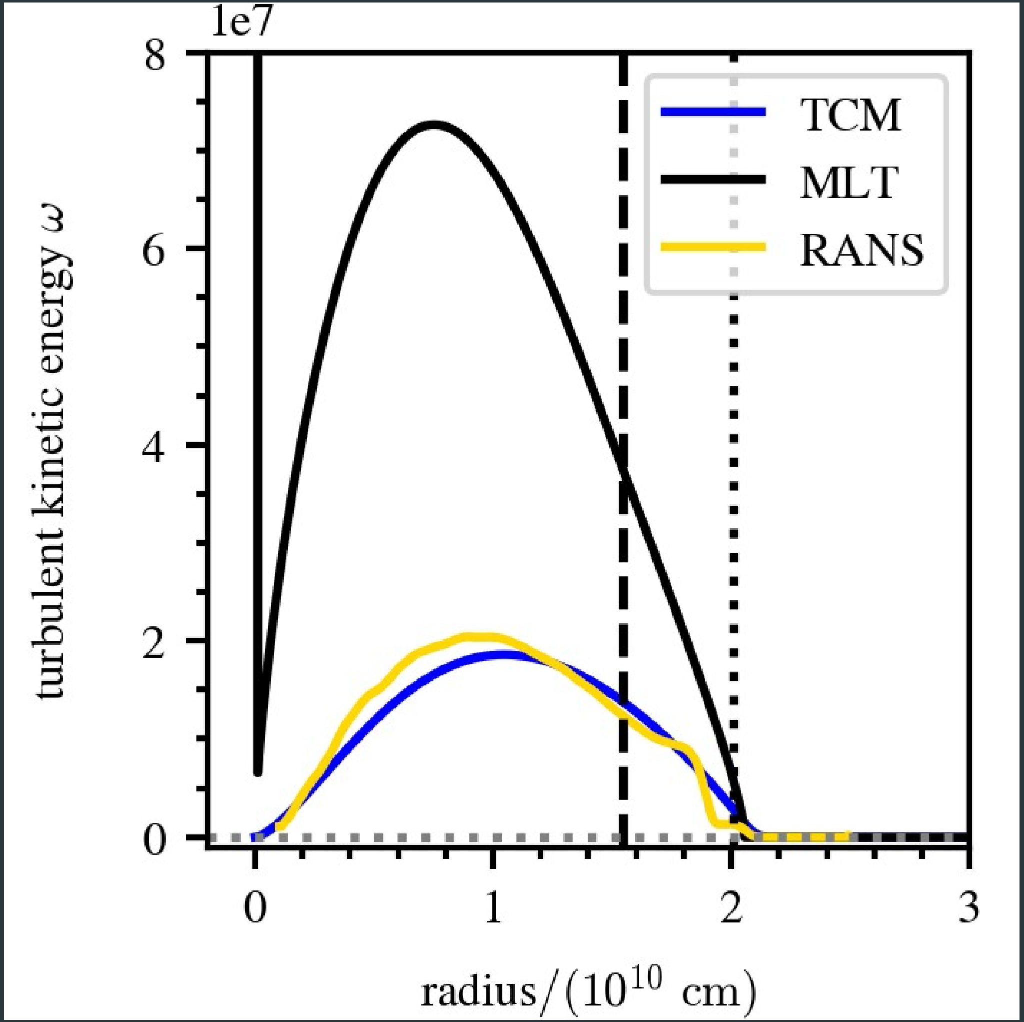
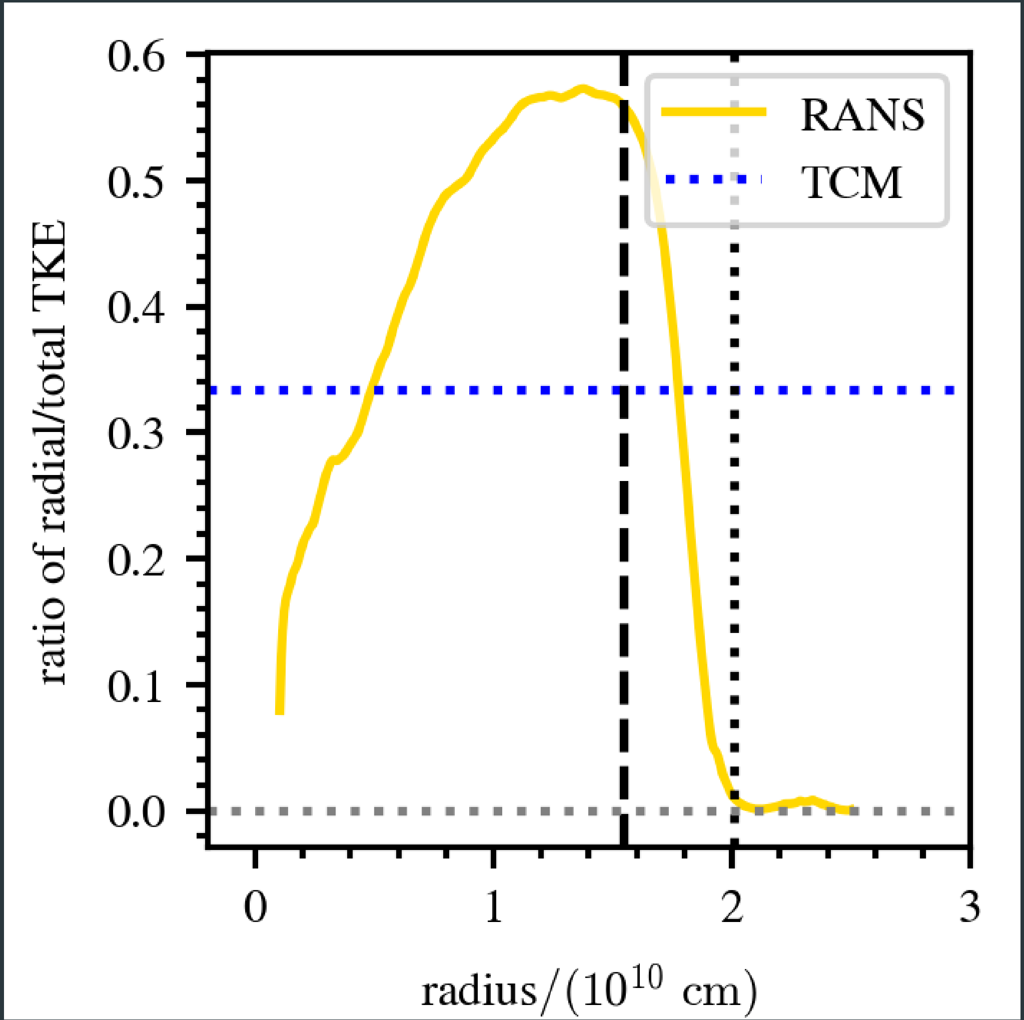




Turbulent kinetic energy

- compare turbulent convection model and MLT with RANS data
- good agreement between TCM and RANS

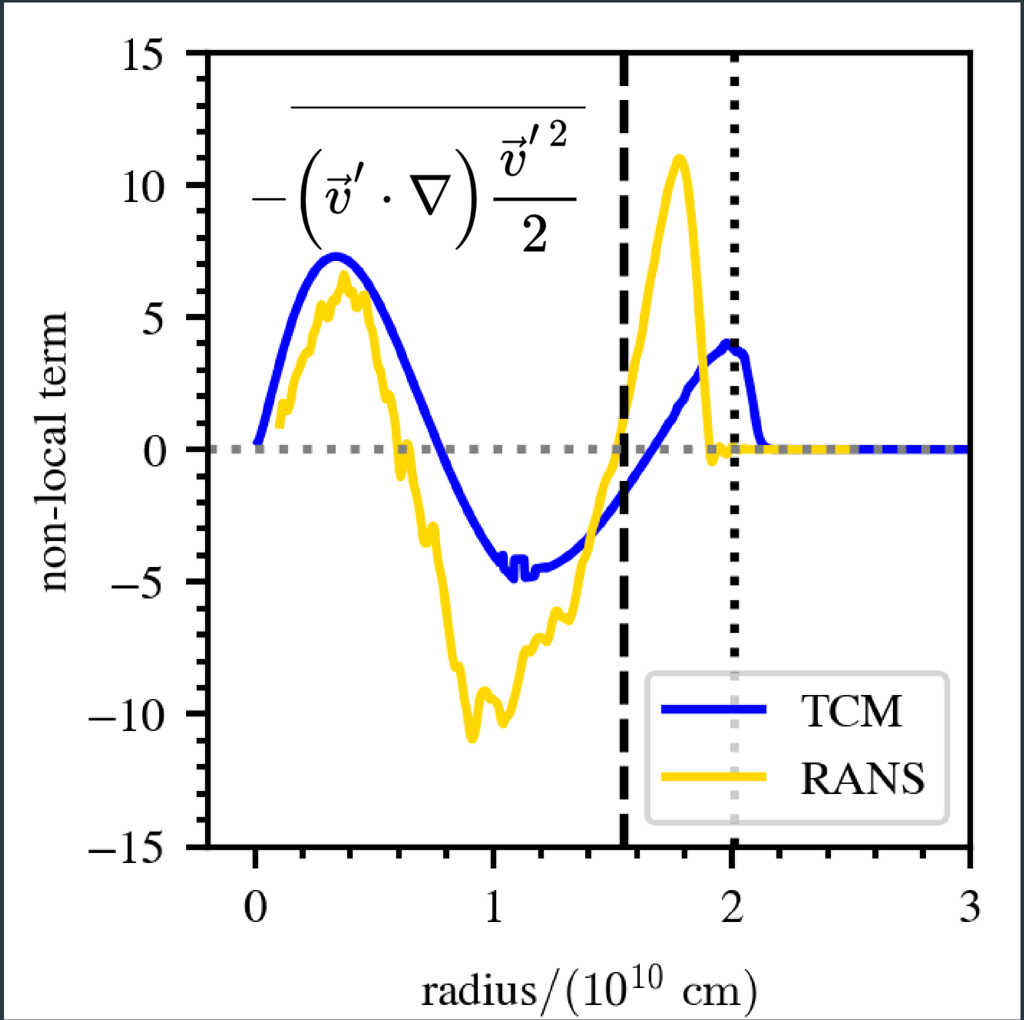
Ahlborn and Higl in prep.



Flow anisotropy

- ratio of radial to total kinetic energy
- contradicts assumption of isotropic flow
- 4th dynamic equation? (e.g. Xiong or Canuto)

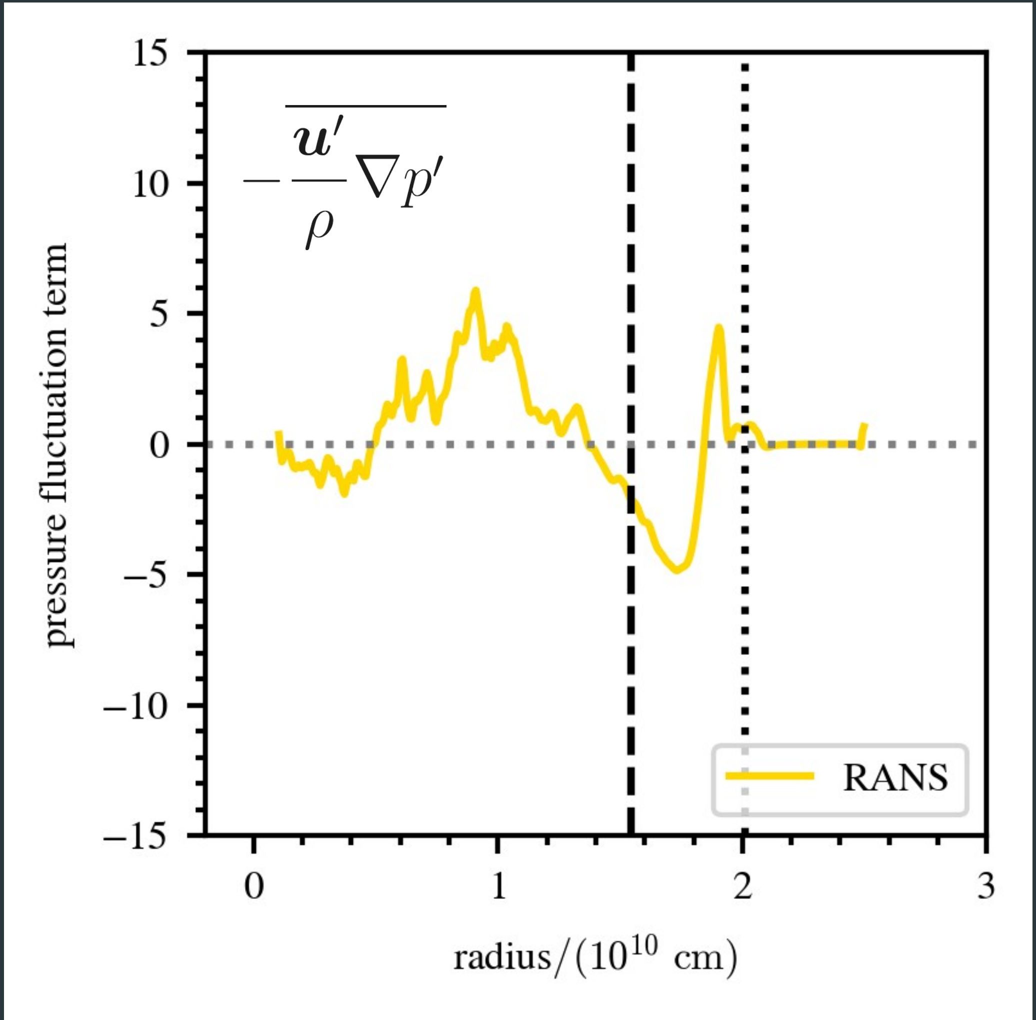
Ahlborn and Higl in prep.



Non-local term

- 1D non-local term:
  $$-\frac{1}{\rho}\mathrm{div}(-D_\omega \nabla \omega)$$
- agreement in terms of shape

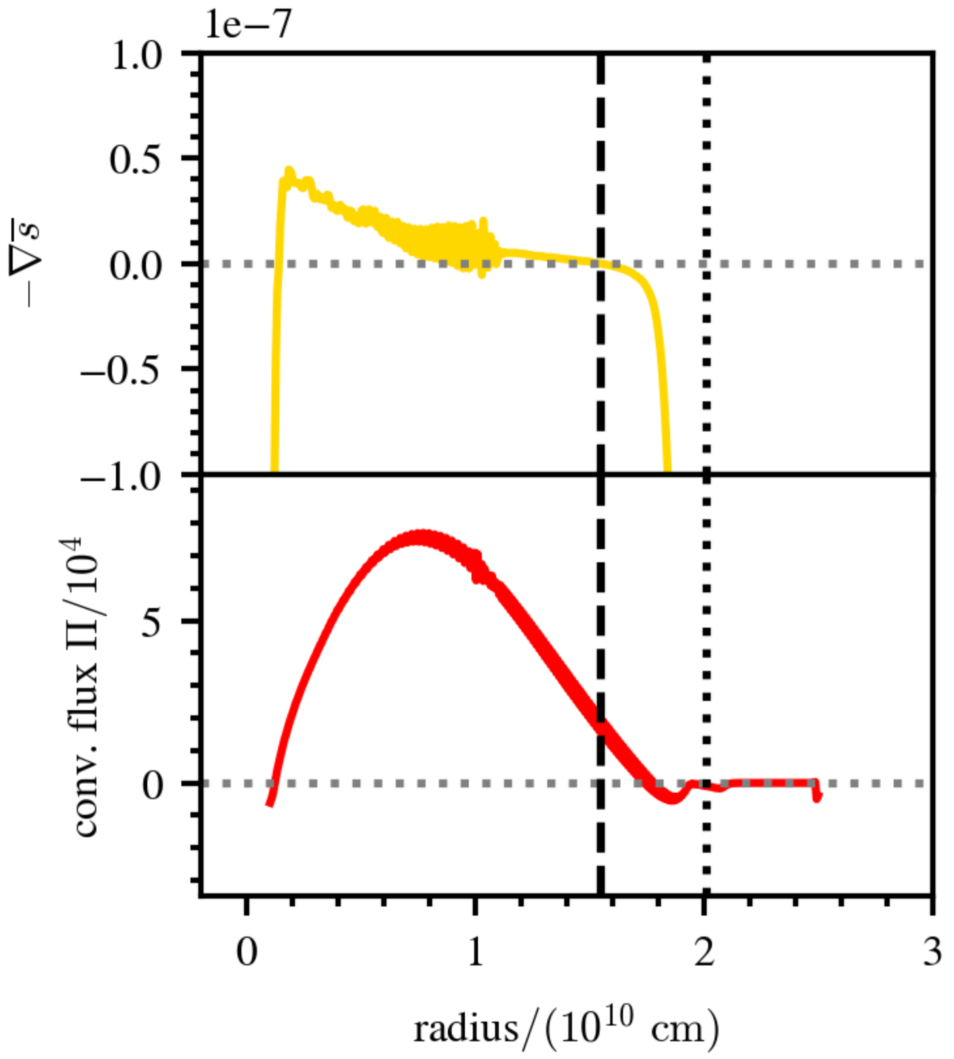
Ahlborn and Higl in prep.



Pressure fluctuation term

- non-zero
- neglected in the Kuhfuss model
- reminiscent of the non-local term
- solutions from literature (e.g. Rotta 1951, Canuto 1992, Canuto 1993, Sander 1998 and references therein)

Ahlborn and Higl in prep.



Deardorff layer

- infer thermal structure from entropy profile
  $$-\frac{\partial \overline{s}}{\partial r} = \frac{c_p}{H_p}(\nabla - \nabla_{\rm ad})$$
- subadiabatic region with positive convective flux
- convection driven by non-local effects

Ahlborn and Higl in prep.

Applications

- convective core on the main sequence
  - Ahlborn, Kupka, Weiss and Flaskamp (2022)
  - Kupka, Ahlborn and Weiss (2022)
- standard solar model
  - Braun, Ahlborn and Weiss (2024) (T. Braun will be here in the 3rd and 4th week)
- Cepheid mass discrepancy problem
  - Deka, Ahlborn, Braun and Weiss in prep.

Conclusions

- self-consistent convective boundary layers using a turbulent convection model
  - Kuhfuss model (Kuhfuss 1986, 1987)
  - implemented in the Garching Stellar evolution code (GARSTEC)
- hydrodynamic simulations of a $3M_{\odot}$ star
  - nominal luminosity
  - compute 1D averages

confirming

- turbulent kinetic energy equation
- Deardorff layer

disagreeing

- final stratification
- pressure fluctuation terms
- isotropy