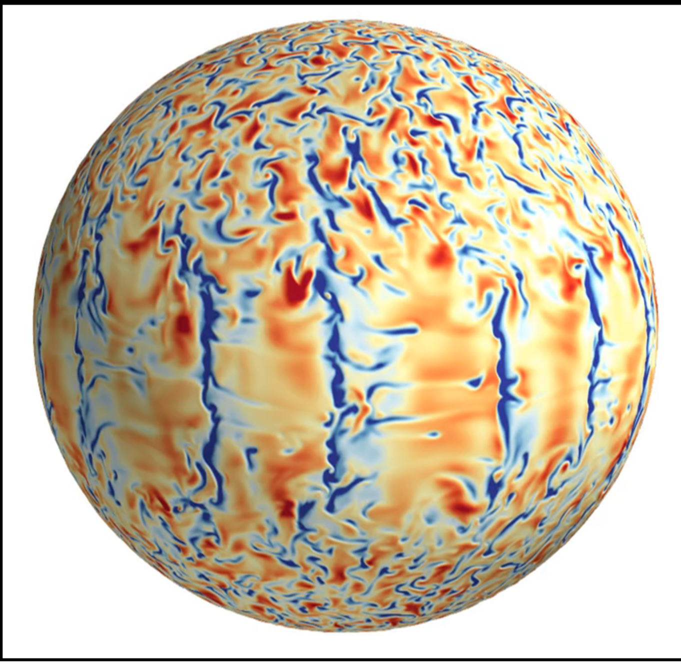
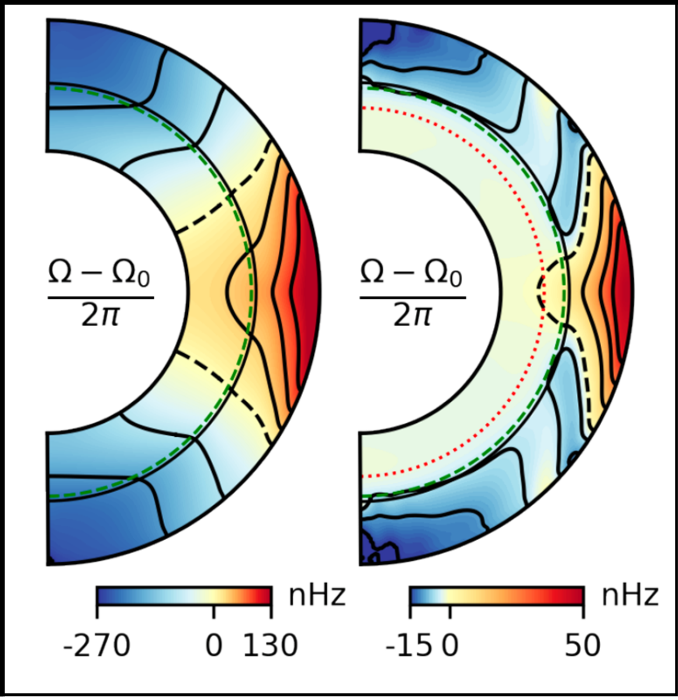
The Implications of the Sun's Differential Rotation on the Transport of Heat and Angular Momentum

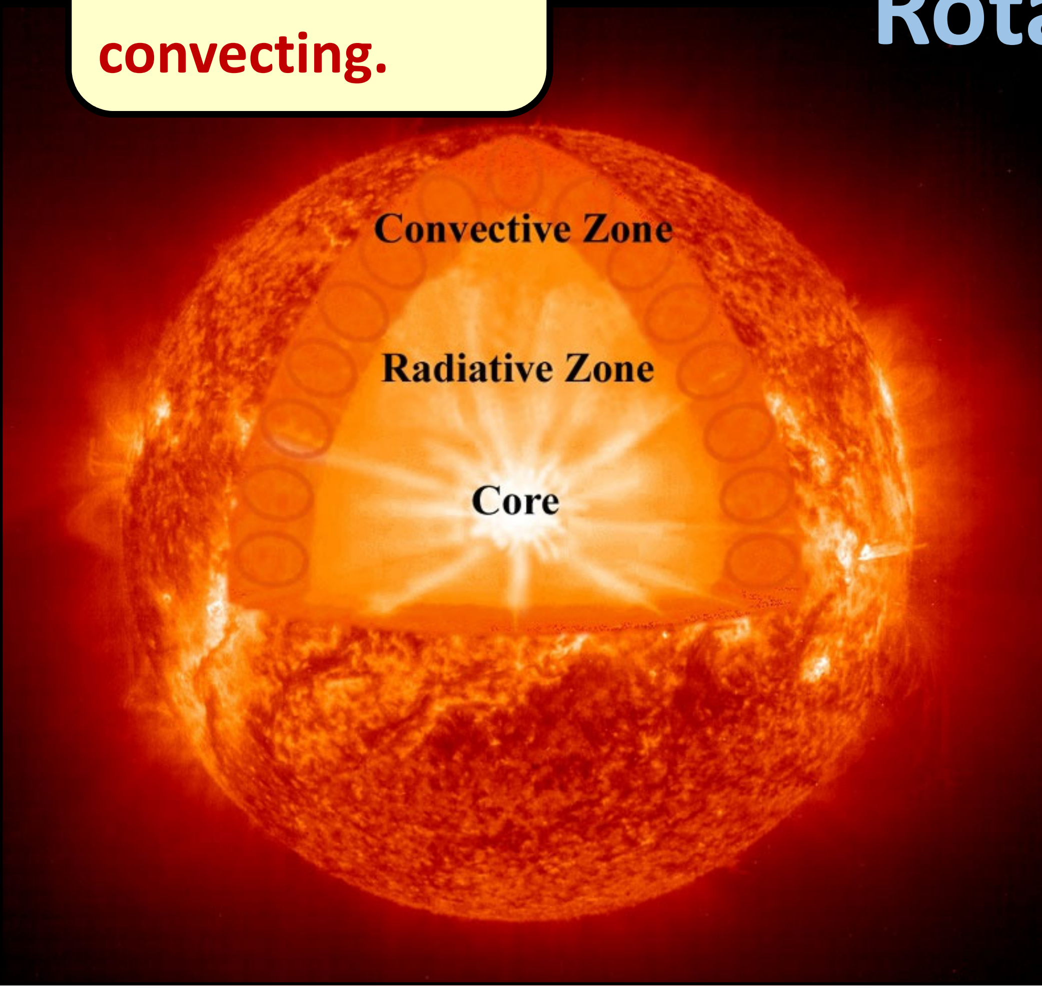


Bradley W. Hindman
University of Colorado Boulder

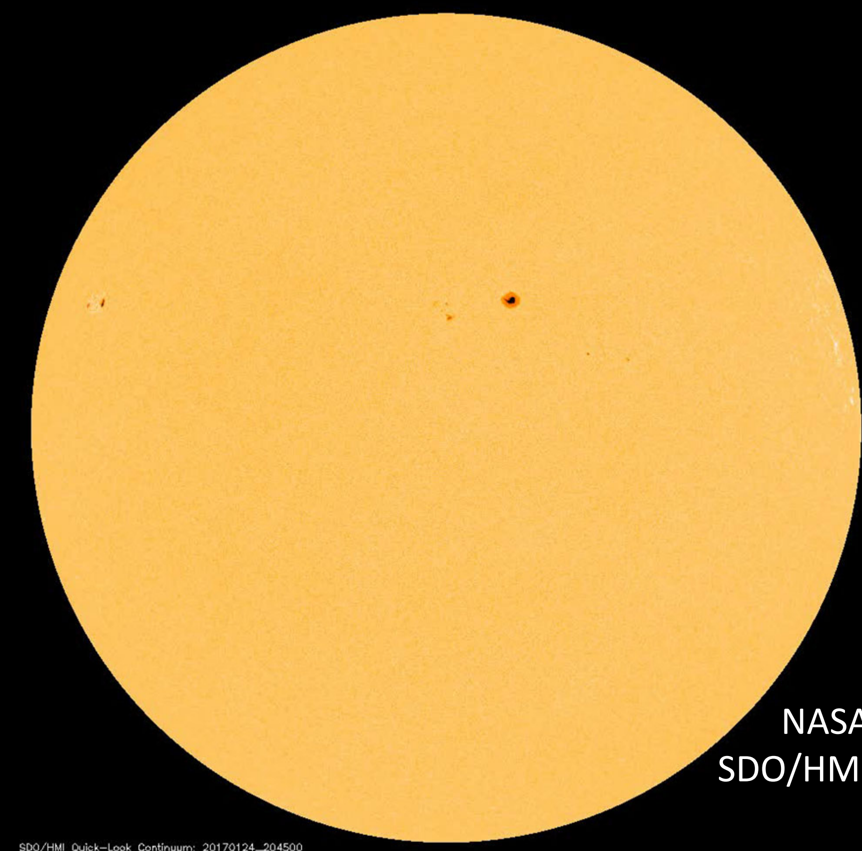


Convection and Rotation

The outer third of the Sun is convecting.



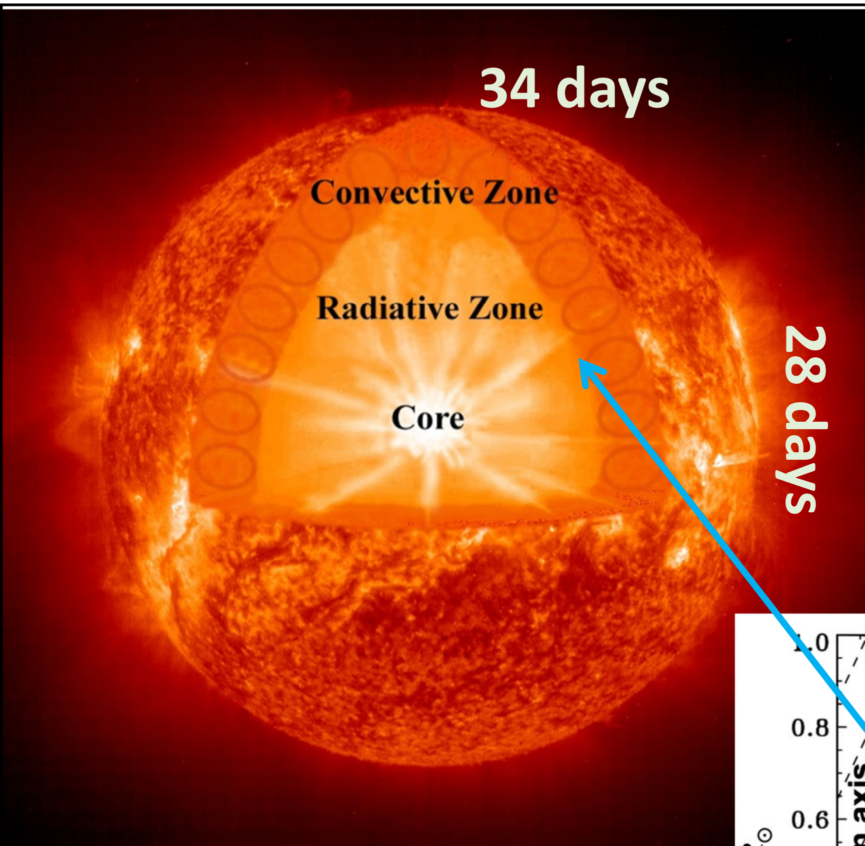
The Sun rotates with a period of roughly a month.



NASA
SDO/HMI

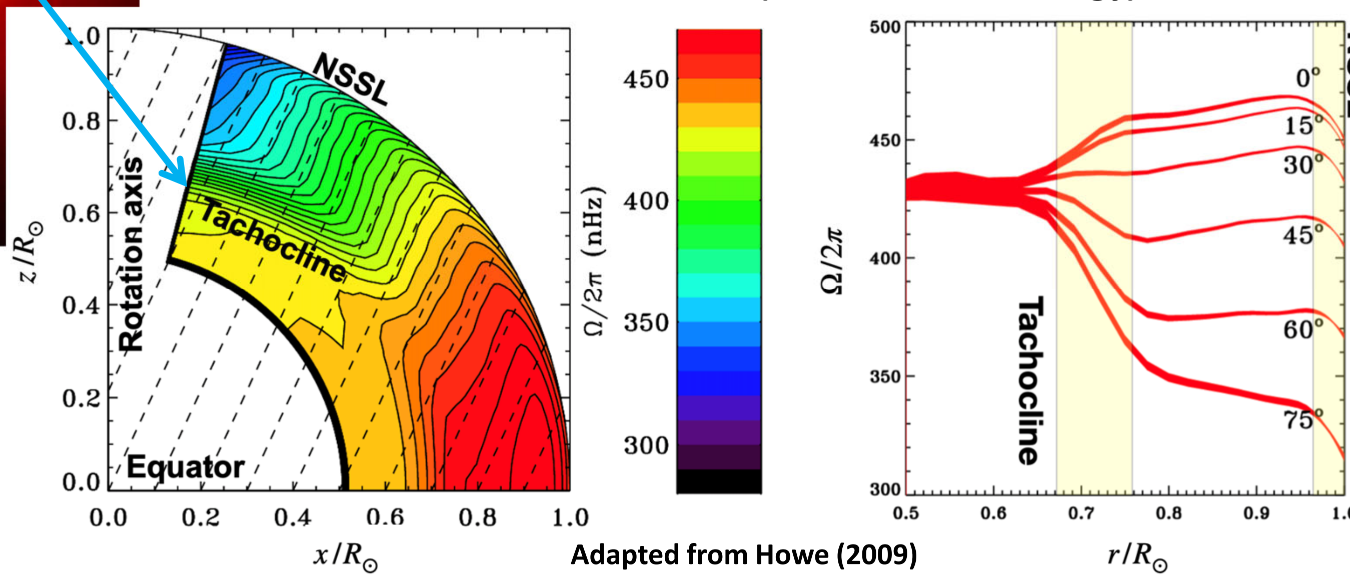
Differential Rotation

- **Differential rotation** in the convection zone
- **Solid-body rotation** in the radiative interior
- **Tachocline** width $\Delta \lesssim 0.05\ R_{\odot}$



Solar-like Differential Rotation
(within the convection zone)
- equator rotates fast
- poles rotate slow

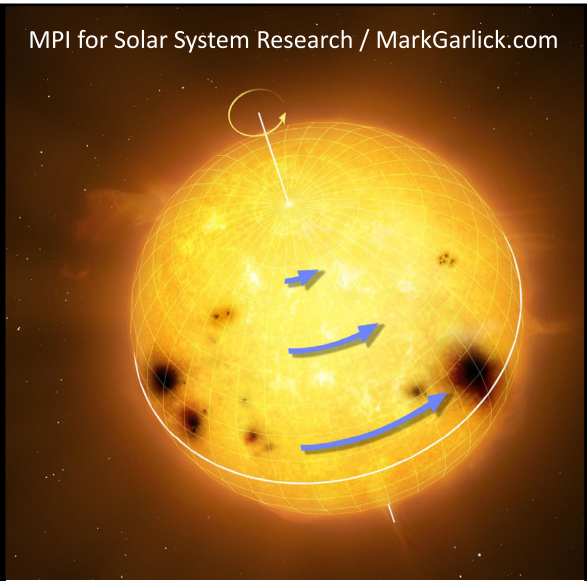
Rotation Rate of the Sun (from Helioseismology)



Adapted from Howe (2009)

Differential Rotation in Other Stars

Differential rotation has been detected in other stars with a variety of techniques

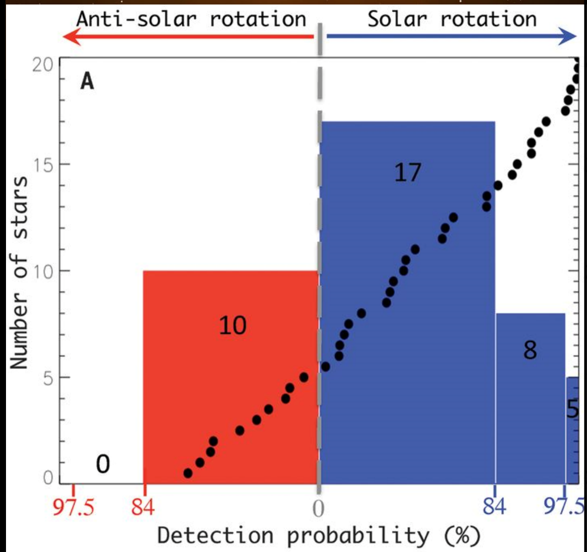


Solar-Like Differential Rotation

- Many main-sequence stars appear to have solar-like differential rotation (e.g., Marsden et al. 2006, 2011; Reiners 2006; Benomar et al. 2018, Bazot et al. 2019)
- The observations are difficult so . . . arguments ensue

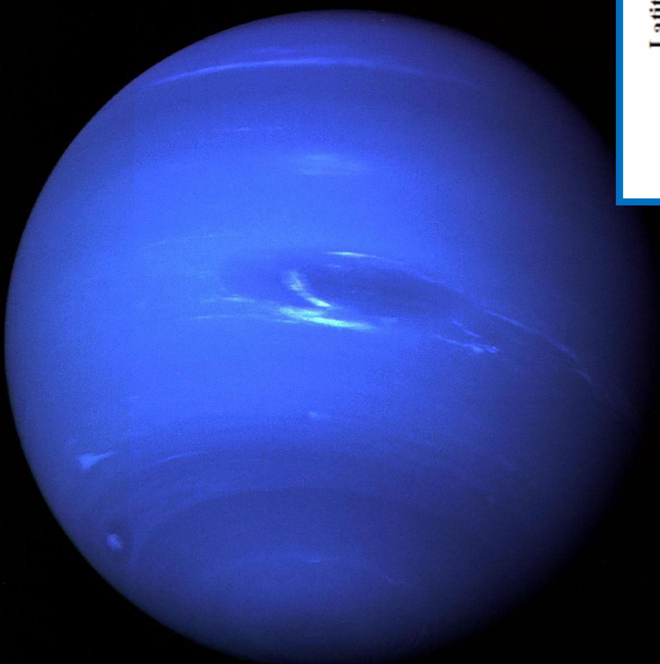
Antisolar Differential Rotation

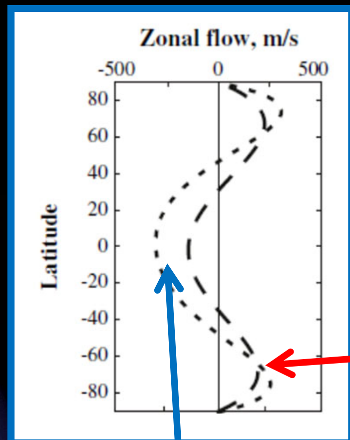
- A few evolved stars (giants and subgiants) appear to have antisolar differential rotation (Strassmeier et al. 2003; Kővári et al. 2015, 2017; Harutyunyan et al. 2016; Benomar et al. 2018)

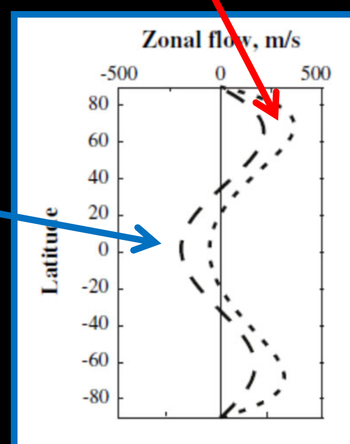


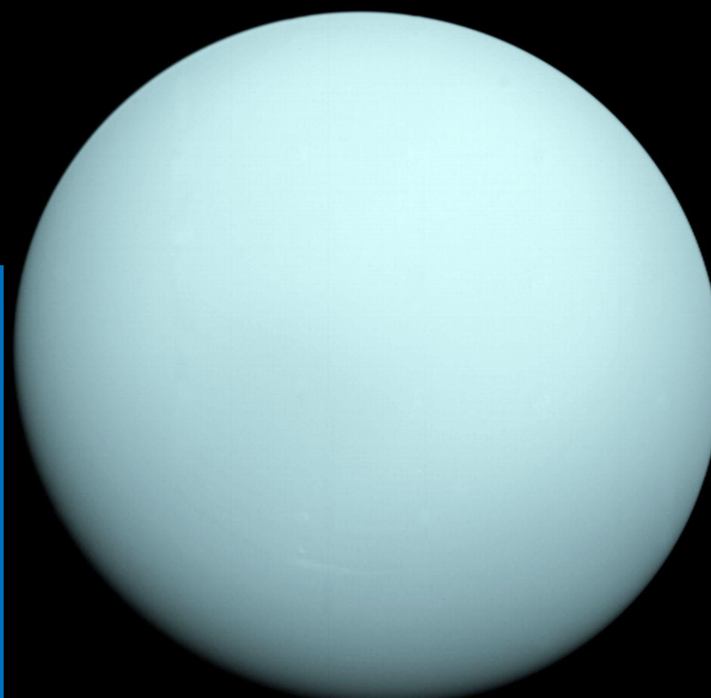
Benomar et al. 2018

The Ice Giants Have "Antisolar" Differential Rotation







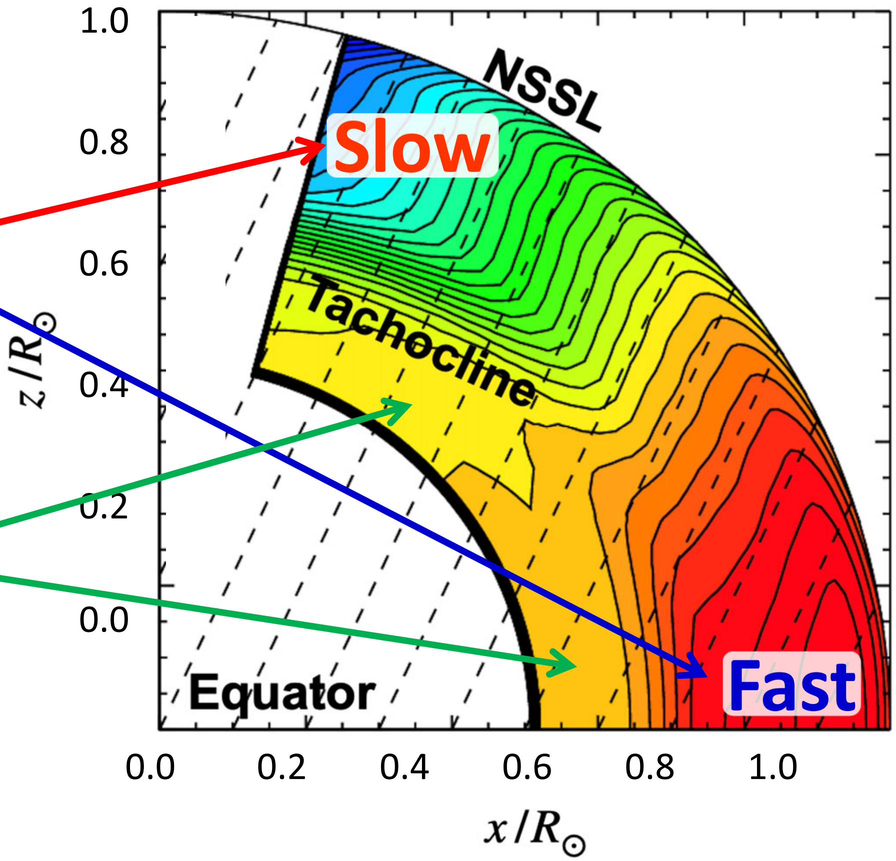


Soderlund et al. 2013, Icarus, 224, 97

Current Puzzles Concerning the Sun's Differential Rotation

1) **Why does the Sun's convection zone have *solar-like differential rotation*?**
   (i.e., why does the equator rotate fast?)

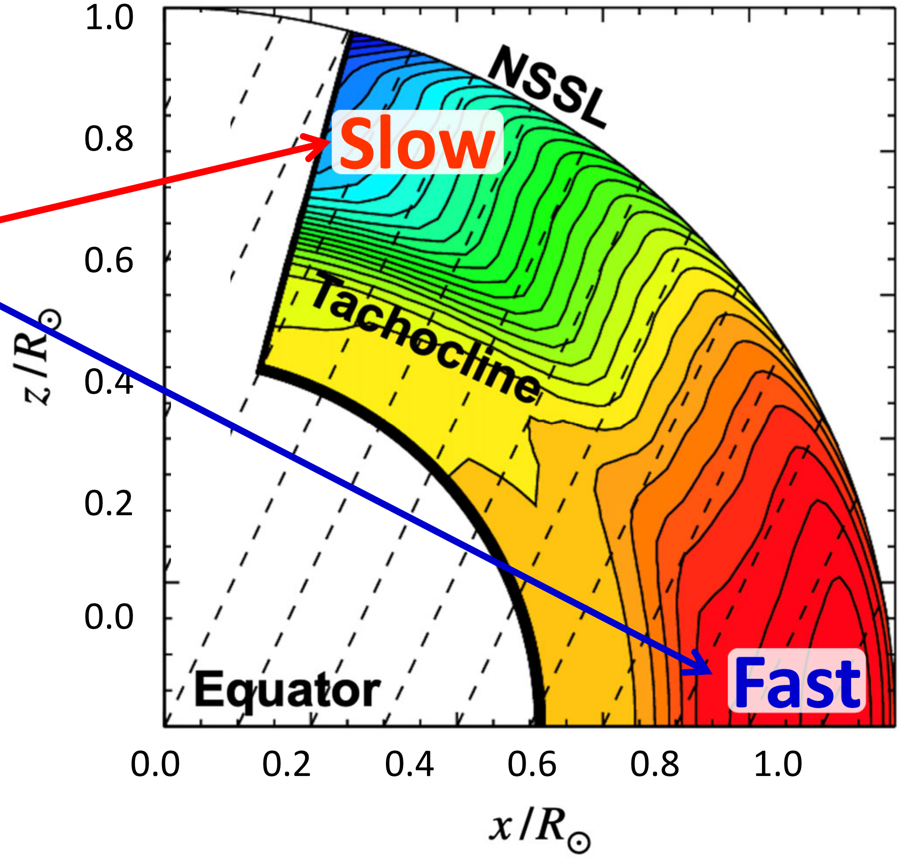
2) **Why does the Sun's radiative interior rotate like a *solid body*?**
   (i.e., no differential rotation)

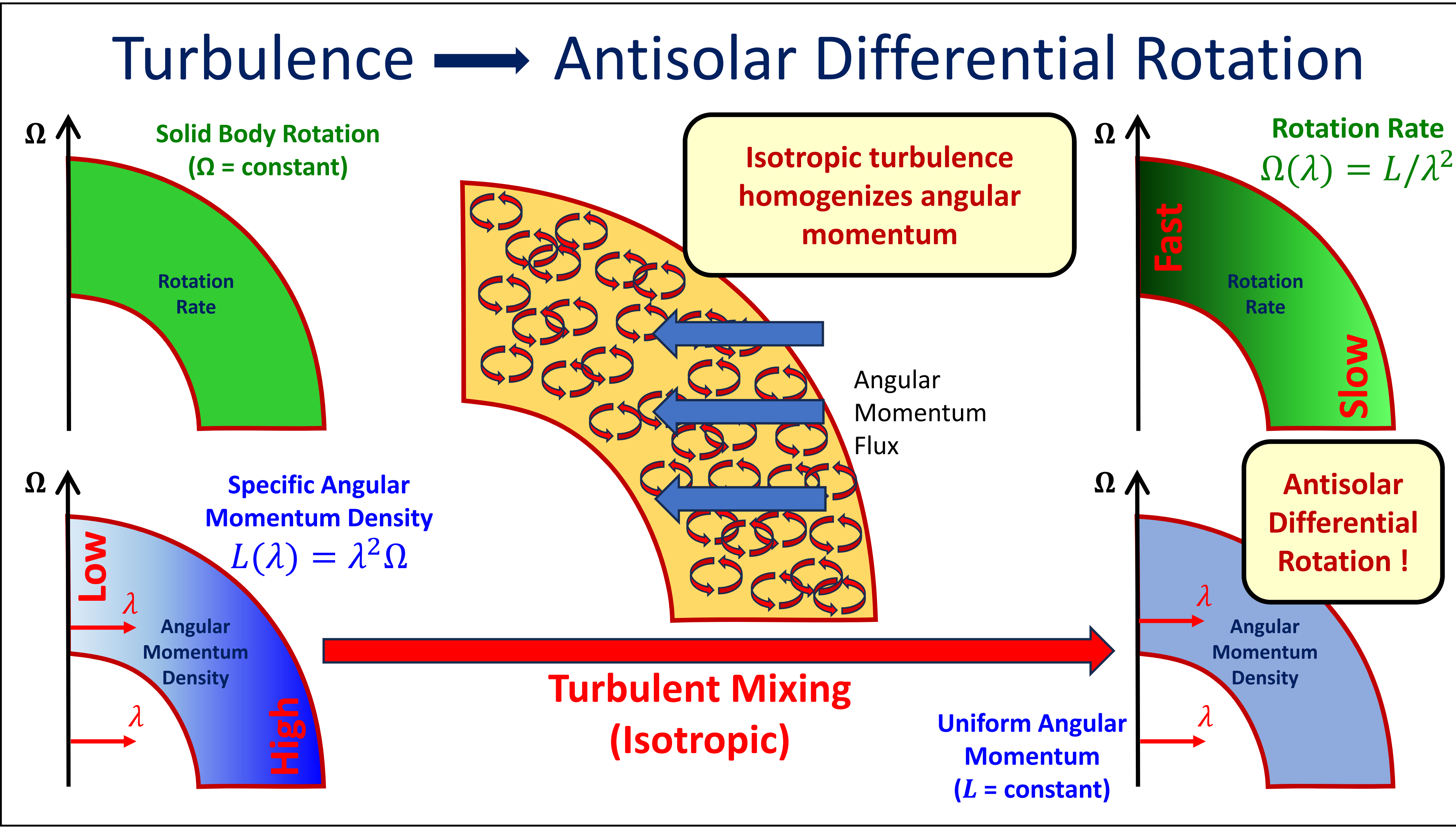


Current Puzzles Concerning the Sun's Differential Rotation

1) **Why does the Sun's convection zone have *solar-like differential rotation*?**
   (i.e., why does the equator rotate fast?)

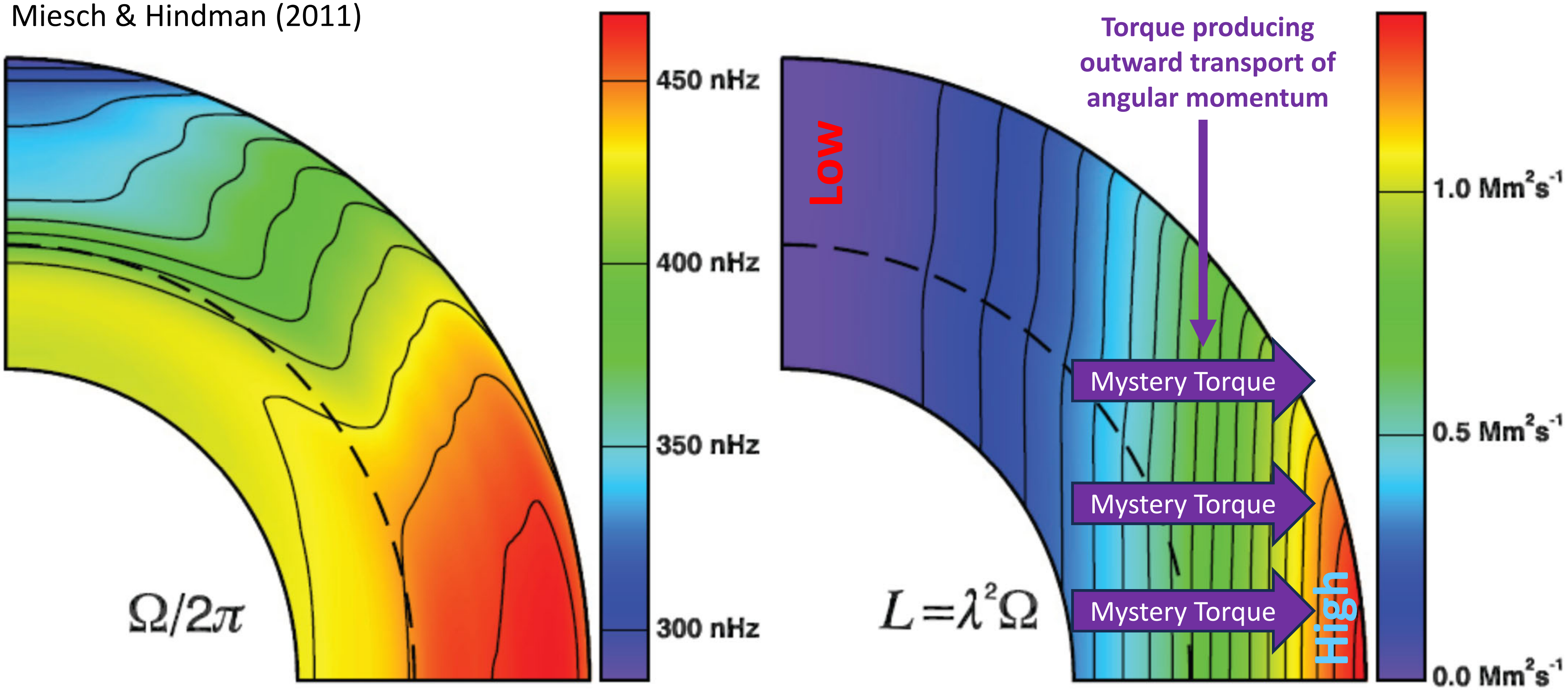
Why is this a puzzle?




Turbulence → Antisolar Differential Rotation
Ω
Solid Body Rotation
(Ω = constant)
Rotation Rate
Isotropic turbulence homogenizes angular momentum
Angular Momentum Flux
Ω
Rotation Rate
$\Omega(\lambda) = L/\lambda^2$
Fast
Rotation Rate
Slow
Ω
Specific Angular Momentum Density
$L(\lambda) = \lambda^2\Omega$
Low
λ
Angular Momentum Density
λ
High
Turbulent Mixing
(Isotropic)
Ω
Antisolar Differential Rotation !
λ
Angular Momentum Density
λ
Uniform Angular Momentum
(L = constant)

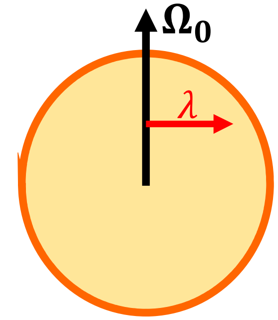
There Must be an Additional Torque !

Miesch & Hindman (2011)



The Sun's Angular momentum density is NOT uniform

What are the Potential Torques?



Torque Balance

azimuthal mean of the zonal momentum eqn

$$\langle\rho\rangle \frac{\partial}{\partial t}(\lambda^2\Omega) = -\langle\rho\boldsymbol{u}_m\rangle\cdot\nabla(\lambda^2\Omega) + \nabla\cdot\left[\langle\rho\lambda\boldsymbol{u}_m' u_\varphi'\rangle - \mu\lambda^2\nabla\Omega - \frac{1}{4\pi}\langle\lambda\boldsymbol{B}B_\varphi\rangle\right]$$

Advection by the *Meridional Flow*

Reynolds Stresses (*Turbulent Transport*)

Viscous Stress

Magnetic Stresses

Azimuthal Mean of the Mass Density

Specific Angular Momentum Density $L = \lambda^2\Omega$

Meridional Velocity Component

Fluctuating Meridional Velocity Component $\boldsymbol{u}_m = \langle\boldsymbol{u}_m\rangle + \boldsymbol{u}_m'$

Fluctuating Zonal Velocity Component $u_\varphi = \lambda(\Omega - \Omega_0) + u_\varphi'$

Rotation Rate

$$\Omega(r,\theta) \equiv \Omega_0 + \frac{\langle u_\varphi\rangle}{\lambda}$$

Frame rate

What are the Potential Torques?

Torque Balance

Steady State

Advection by the *Meridional Flow*

Reynolds Stresses (*Turbulent Transport*)

Viscous Stress

Magnetic Stresses ?

$$\langle\rho\rangle \frac{\partial}{\partial t}(\lambda^2\Omega) = -\langle\rho \boldsymbol{u}_m\rangle \cdot \boldsymbol{\nabla}(\lambda^2\Omega) + \boldsymbol{\nabla}\cdot\left[\langle\rho\lambda\boldsymbol{u}'_m u'_\varphi\rangle - \mu\lambda^2\boldsymbol{\nabla}\Omega - \frac{1}{4\pi}\langle\lambda\boldsymbol{B}B_\varphi\rangle\right]$$

Azimuthal Mean of the Mass Density

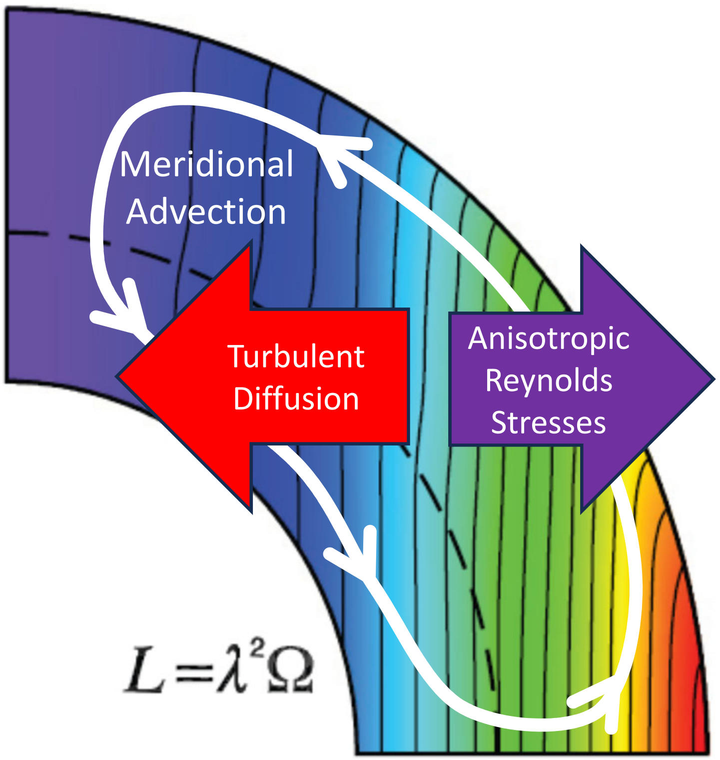
Specific Angular Momentum Density $L = \lambda^2\Omega$

A canonically held view is thus:

1) **Turbulent transport is balanced by meridional advection**
2) **The turbulence has a significant anisotropic component**

I'll come back to magnetism !

Current Paradigm



Three-way balance between

isotropic Reynolds stresses
anisotropic Reynolds stress
meridional advection

Reynolds stresses come from two types of flows:

Isotropic turbulence:

- small spatial scales
- rotationally unconstrained $\left(\mathrm{Ro} = \frac{U}{2\Omega L} > 1\right)$
- mixes angular momentum
- ***inward*** transport of angular momentum

Anisotropic convective columns

- larger spatial scales
- Rotationally constrained $(\mathrm{Ro} < 1)$
- ***outward*** transport of angular momentum

Convective Columns in Simulations

Thermal Rossby Waves

Low-Frequency Prograde Waves

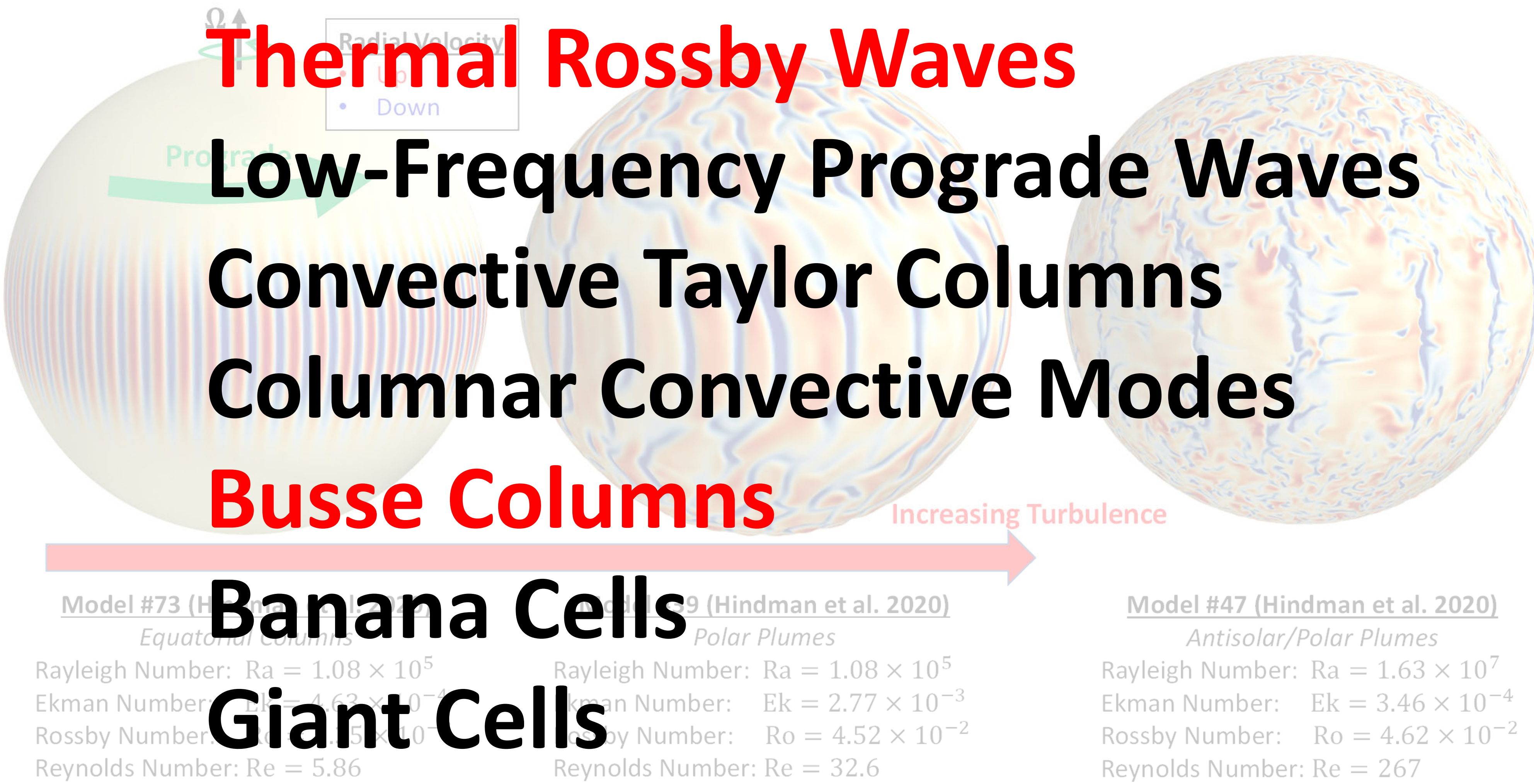
Convective Taylor Columns

Columnar Convective Modes

Busse Columns

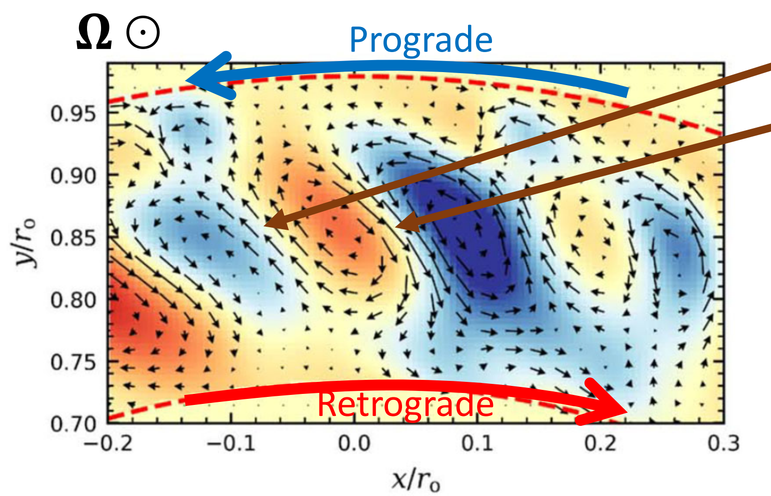
Banana Cells

Giant Cells



Angular Momentum Transport

Pressure and Mass Flux

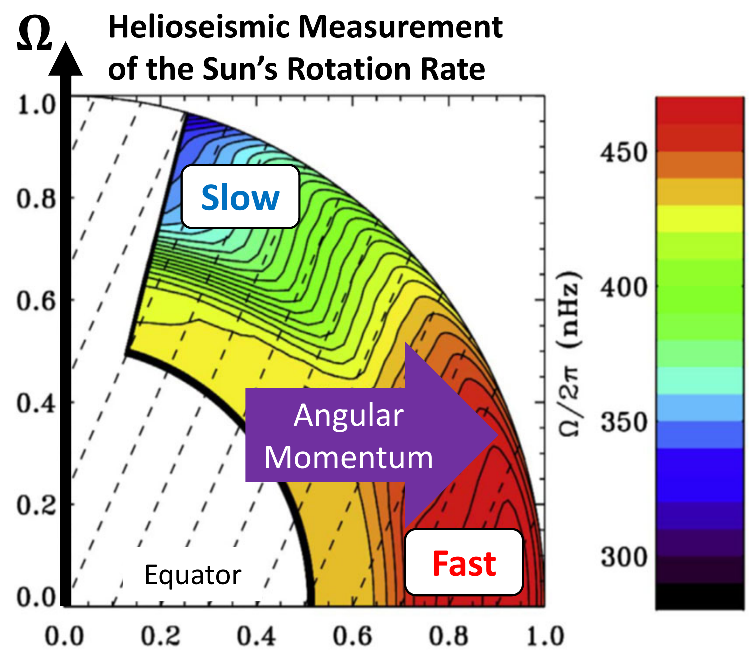


Adapted from Matilsky, Hindman, & Toomre (2020)

Prograde flows move up
Retrograde flows move down

Outward angular momentum transport

Helioseismic Measurement of the Sun's Rotation Rate



Adapted from Howe et al. (2005)

Thermal Rossby waves cause "solar-like" differential rotation (fast equator & slow poles)

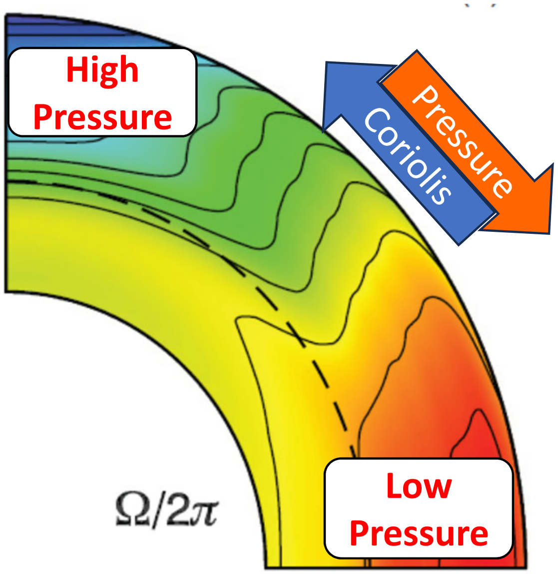
__Model FF4-3__

Rayleigh Number: $\mathrm{Ra} = 1.07 \times 10^5$
Ekman Number: $\mathrm{Ek} = 9.23 \times 10^{-4}$
Rossby Number: $\mathrm{Ro} = 2.46 \times 10^{-2}$
Reynolds Number: $\mathrm{Re} = 22.4$

Differential Rotation ⟷ Thermal Gradients

For low Rossby number flows, like the Sun's differential rotation, we expect geostrophic balance in the latitudinal direction.

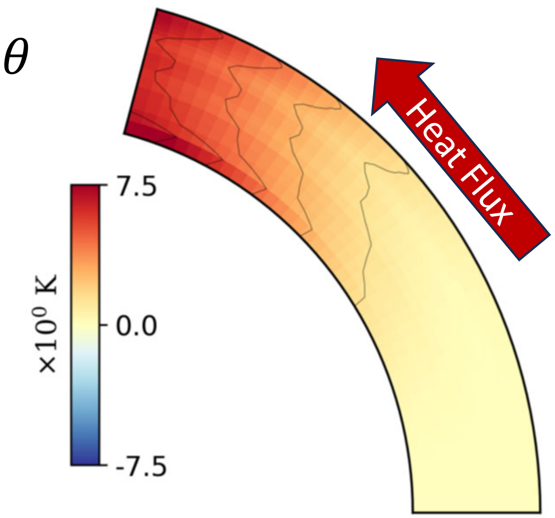
Solar-like differential rotation requires both *angular momentum and heat transport*



$$\frac{1}{r}\frac{\partial\langle P\rangle}{\partial\theta} = \langle\rho\rangle\Omega_0(\Omega-\Omega_0)r^2\sin 2\theta$$

Pressure = Coriolis

Geostrophic Balance &
Hydrostatic Balance

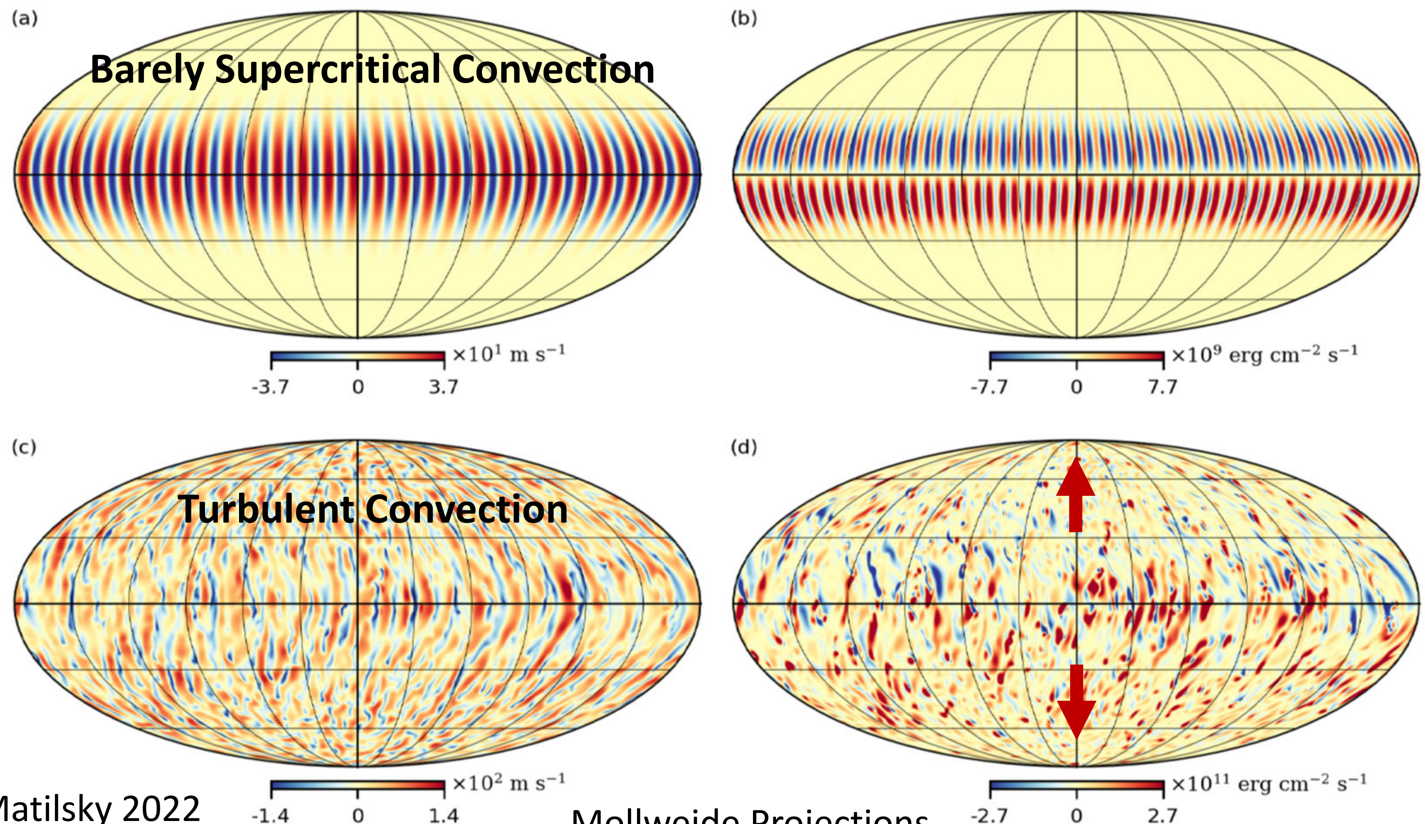


Matilsky 2022

Poleward Enthalpy Flux by Convective Columns

Radial Velocity

Enthalpy Flux

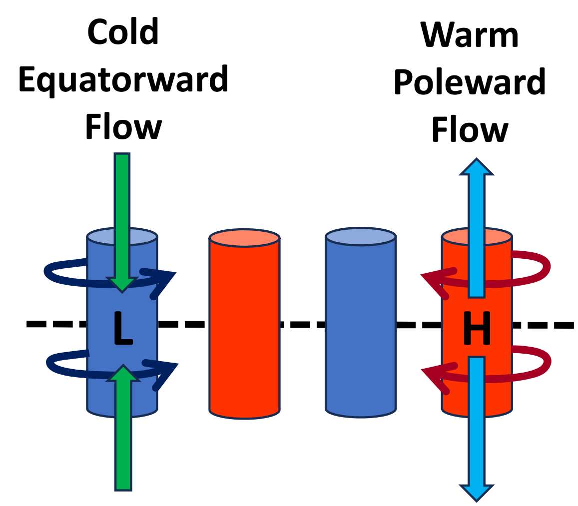


Matilsky 2022

Mollweide Projections

Convective columns have a heat flux towards the poles

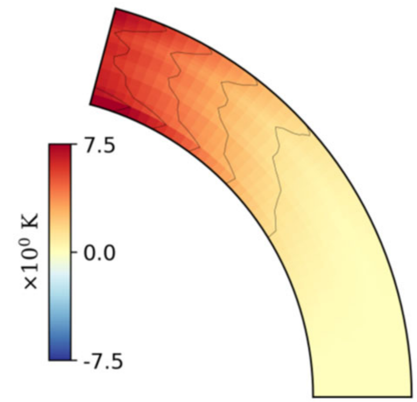
Poleward Heat Flux



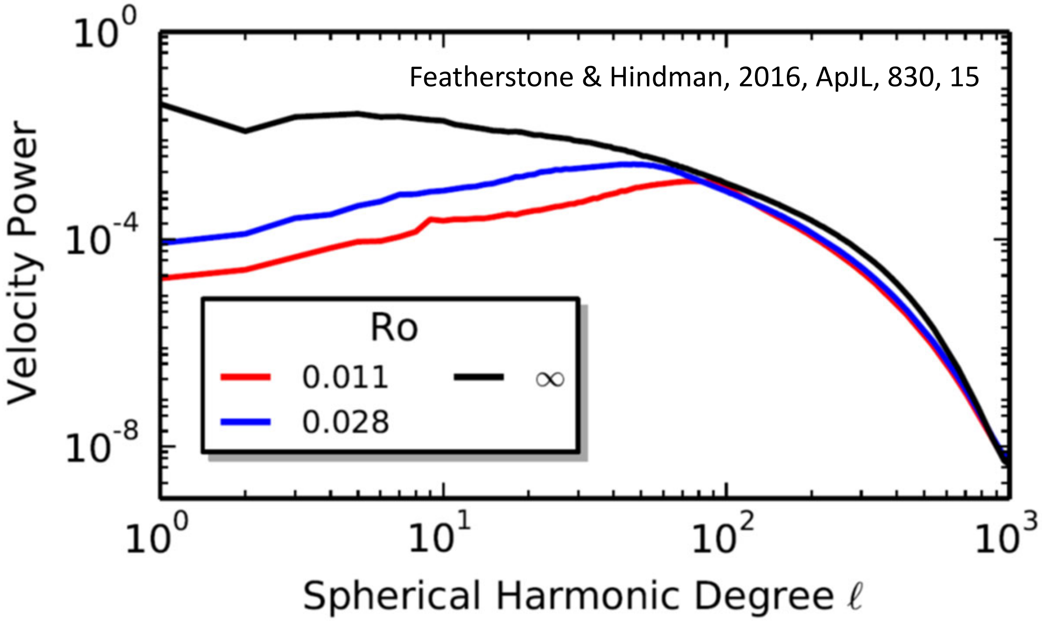
Outstanding Problems

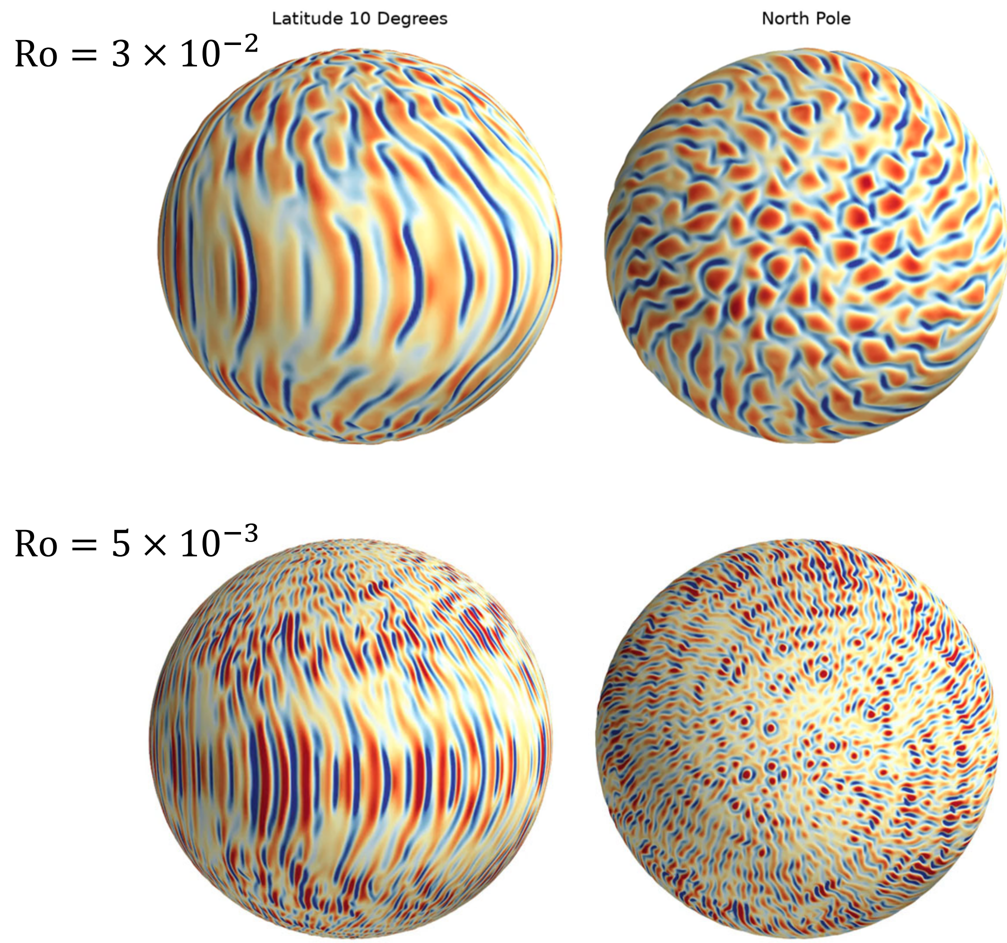
- **Thermal Rossby Waves (or Busse Columns): Where are they?**
  - Could they be hiding? (perhaps they don't extend to the surface)
  - Could they have a smaller spatial scale than expected? (next slide)
  - Could they be malarkey (or more likely, they are low amplitude)?

- **Geostrophic Balance: Where's the temperature gradient?**
  - Perhaps there's an equator-to-pole contrast at the photosphere of 1.5 K (Kuhn et al. 1997)
  - Could the contrast be 10K below the photosphere?



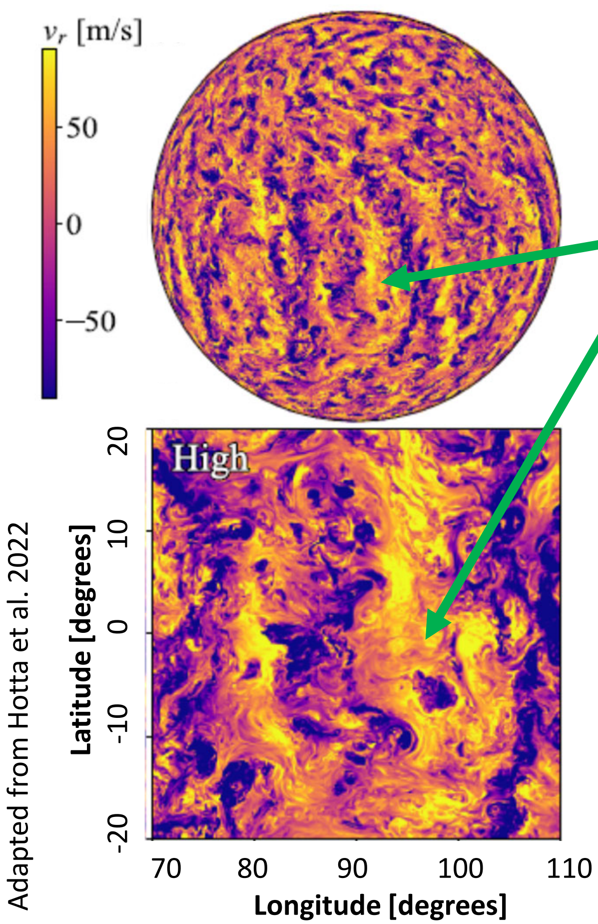
Size of the Convective Structures



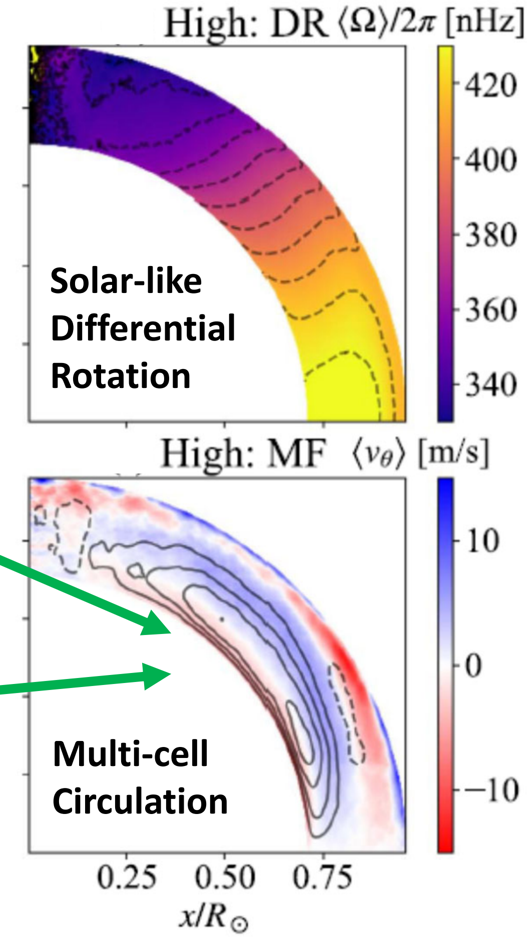


- Spectral peak moves with Rossby number
- Convective columns shrink as the Rossby number decreases

Recent Suggestion: Magnetism



- Hotta et al. 2021, 2022 suggest that magnetism is crucial
- The Busse columns still exist, but they have been circularized and their Reynolds stress diminished
- Maxwell stresses modify the meridional circulation
- The meridional circulations then transport angular momentum equatorward and heat poleward



Current Puzzles Concerning the Sun's Differential Rotation

1) Why does the Sun's convection zone have *solar-like differential rotation*?
(i.e., why does the equator rotate fast?)

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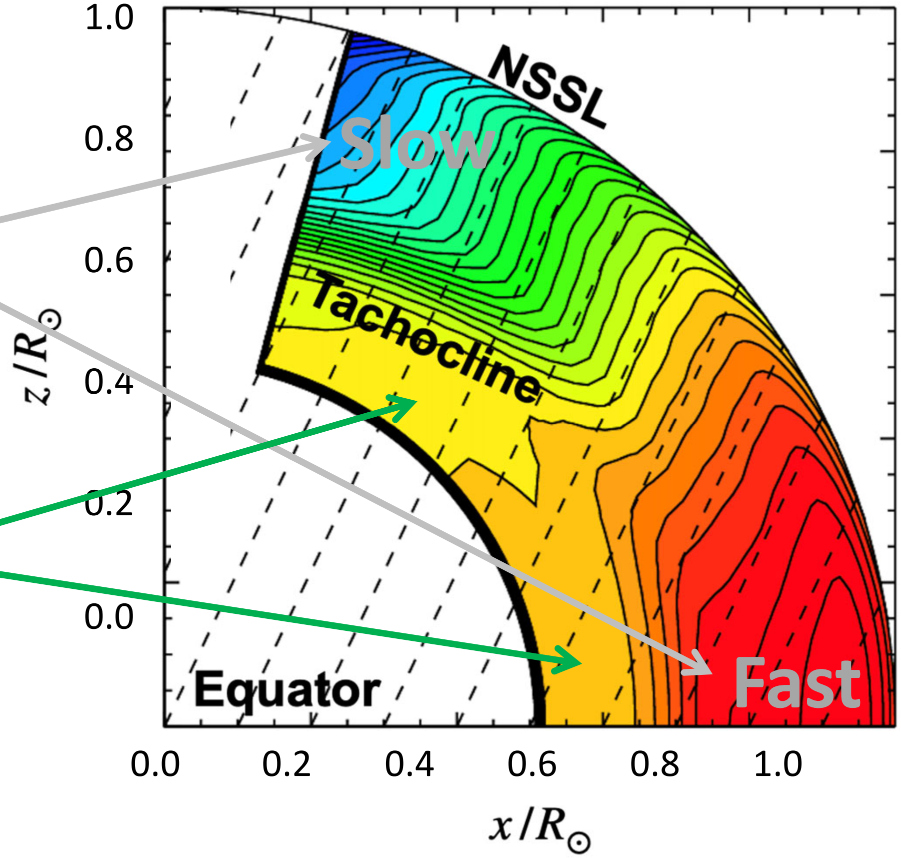
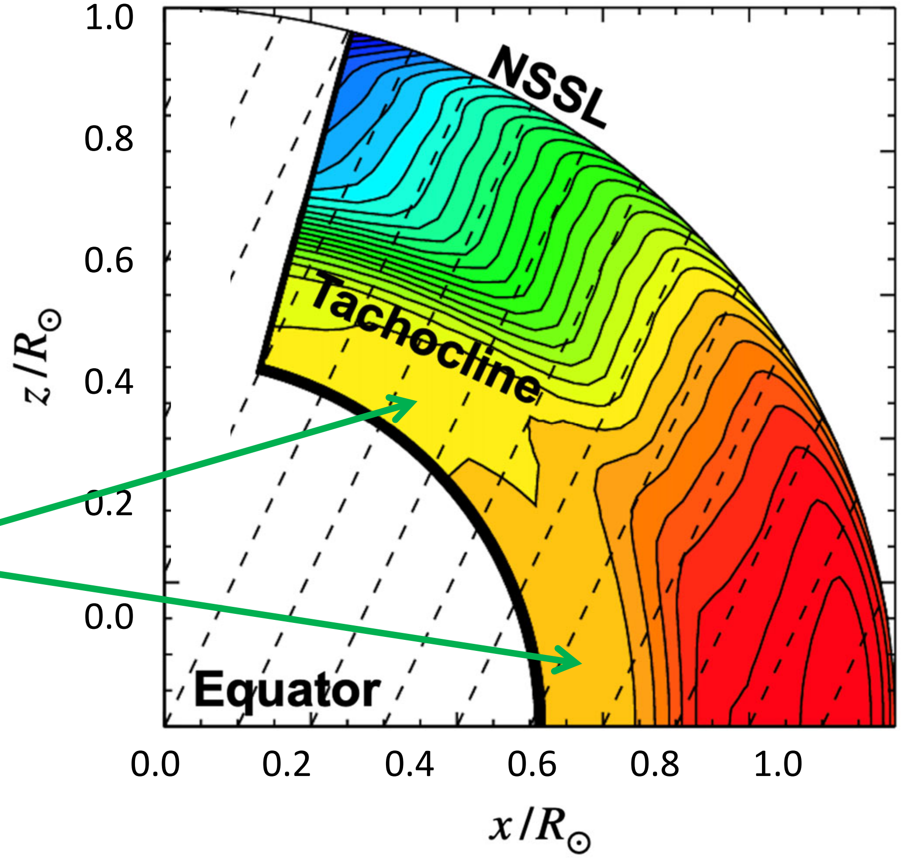


Current Puzzles Concerning the Sun's Differential Rotation

Why is this a puzzle?

2) **Why does the Sun's radiative interior rotate like a *solid body*?**
(i.e., no differential rotation)

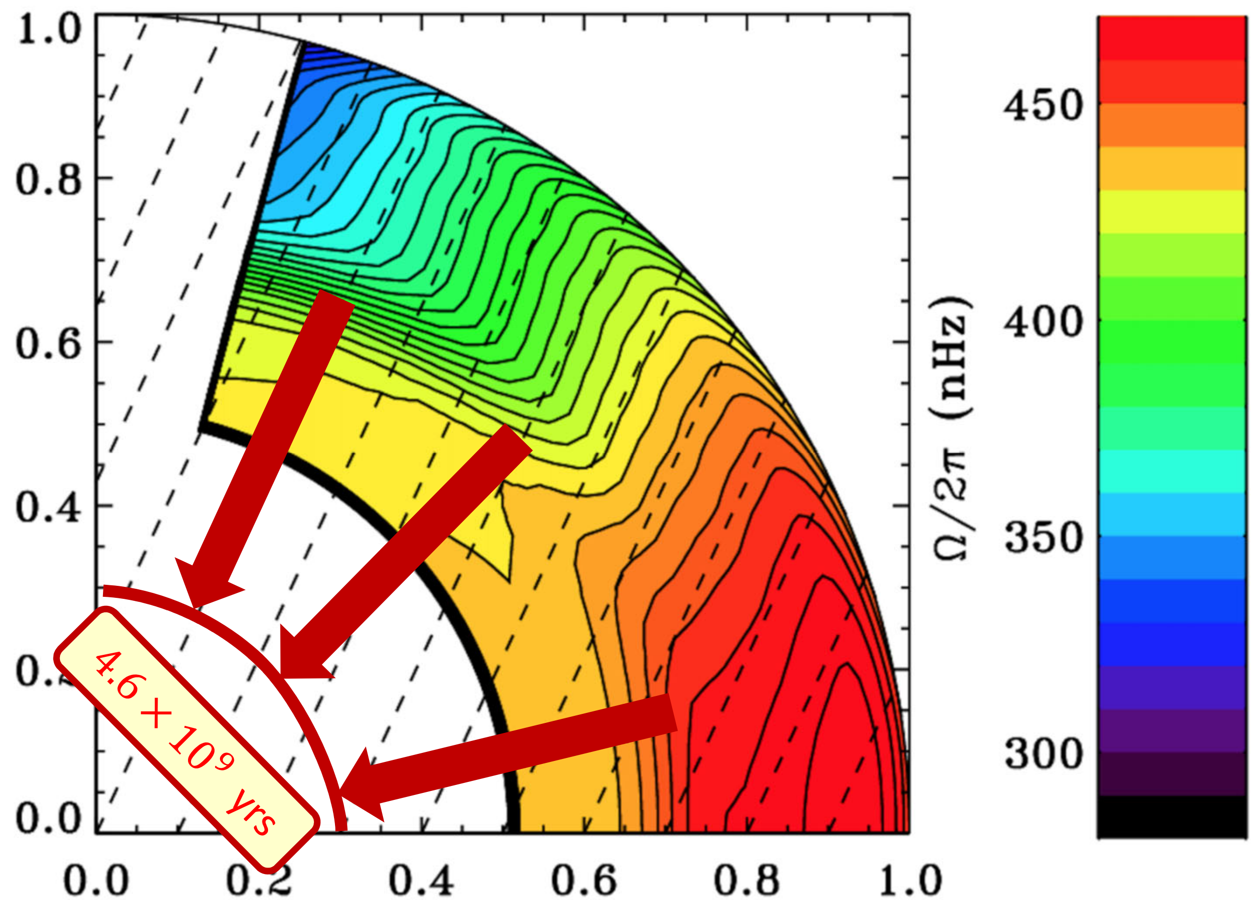


Why does the radiative interior lack shear?

Without opposition, diffusion acting over the lifetime of the Sun should have broadened the tachocline:

4.6 billion years $\rightarrow \Delta \approx 0.4\, R_{\odot}$

A torque must exist that thwarts the diffusive spread of the tachocline.



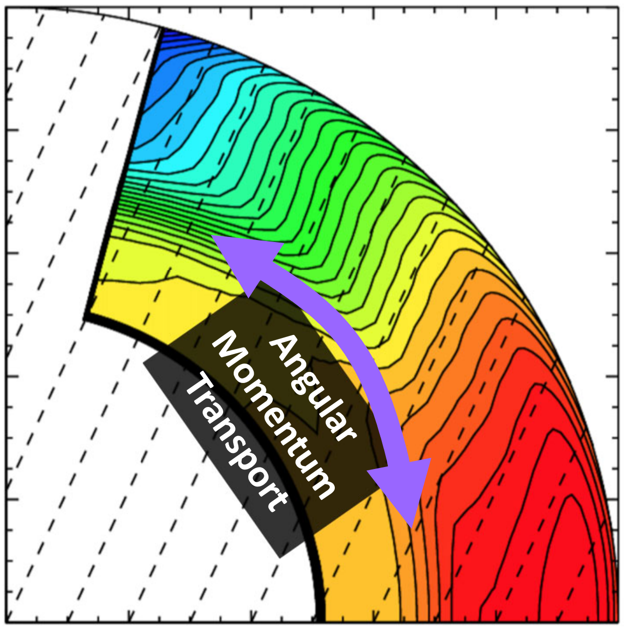
Current Paradigm

(Tachocline Confinement Scenarios)

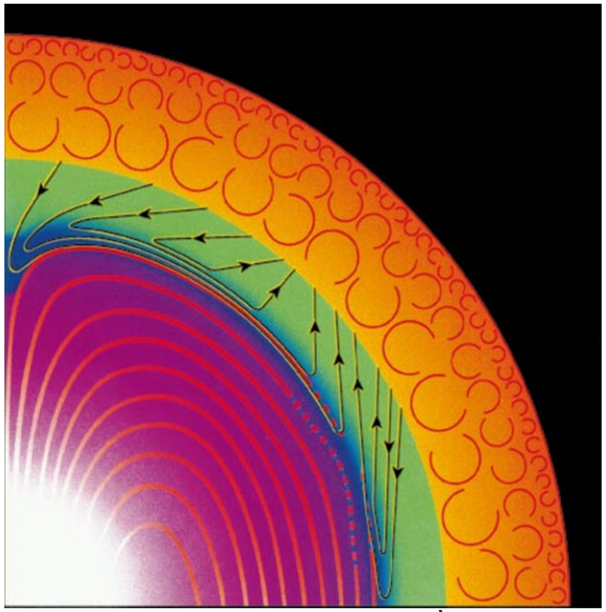
There are two primary propositions for the nature of the confining torque:

- Shear instabilities → horizontal turbulent mixing (e.g., Spiegel & Zahn 1992)
- Magnetic field → Magnetic torque (e.g., Gough & McIntyre 1998)

Shear Instabilities



Magnetic Field



)

Flavors of Magnetic Field

- **Primordial Magnetic Field** in the radiative zone
  Gough & McIntyre (1998)
- **Cyclic Magnetic Field** that diffuses downward into the radiative zone
  Forgács-Dajka & Petrovay (2001)
  Barnabé et al. (2017)

Outstanding Problems

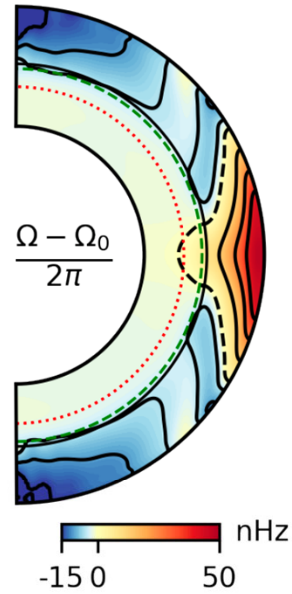
Which Mechanism?

1. Anisotropic turbulence: What's its source?
   - Speigel & Zahn (1981) argued shear instabilities operating in a highly stratified fluid generates enhanced horizontal diffusion
   - But there is a lack of validation of this model by simulations
   - I personally have wondered about stochastically excited Rossby waves
2. Primordial magnetic field: Why hasn't the field bled into the convection zone
3. Dynamo field: The skin depth is too short to explain the tachocline

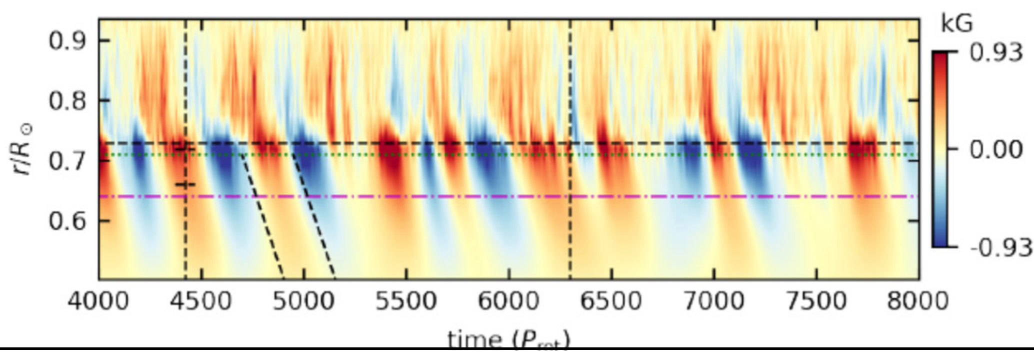
Recent Suggestion: Nonaxisymmetric, Quasi-periodic Dynamo Field

In a 3D MHD simulation, Matilsky et al. (2022) achieved:

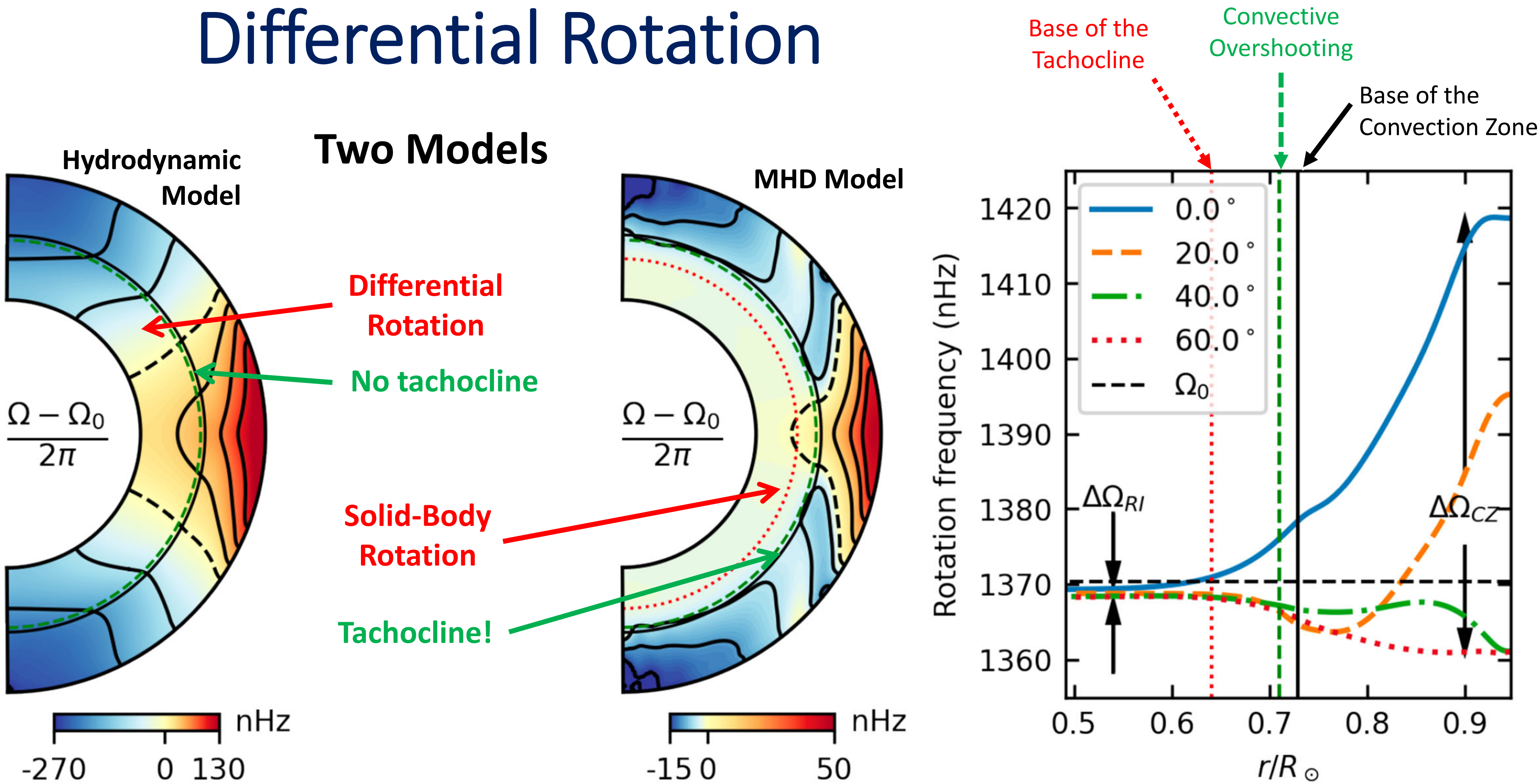
- **Tachocline formation**
  - Narrow: $\Delta \approx 0.08\, R_{\odot}$
  - Occurs spontaneously
- **Tachocline confinement by a cycling magnetic dynamo**
  - The field diffuses downward into the radiative interior.
  - Viscous spread is halted by magnetic torques.
  - The magnetic field is quasi-periodic (with many frequency components)
- **Solid-body rotation of the radiative interior**
  - The magnetic field is primarily *nonaxisymmetric*.
  - The magnetic field "stiffens" the interior and inhibits differential rotation.



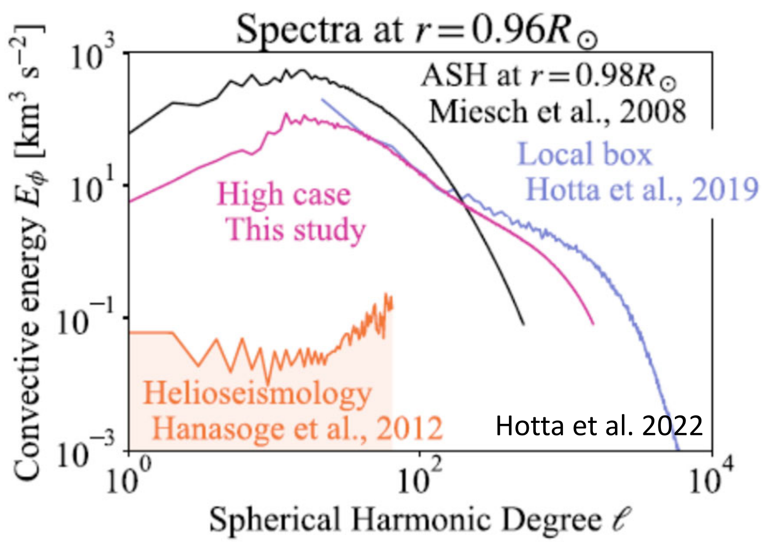
Solid-body rotation of the radiative interior of a numerical MHD simulation.



Differential Rotation



Outstanding Issues



<u>Differential Rotation in the Convection Zone</u>

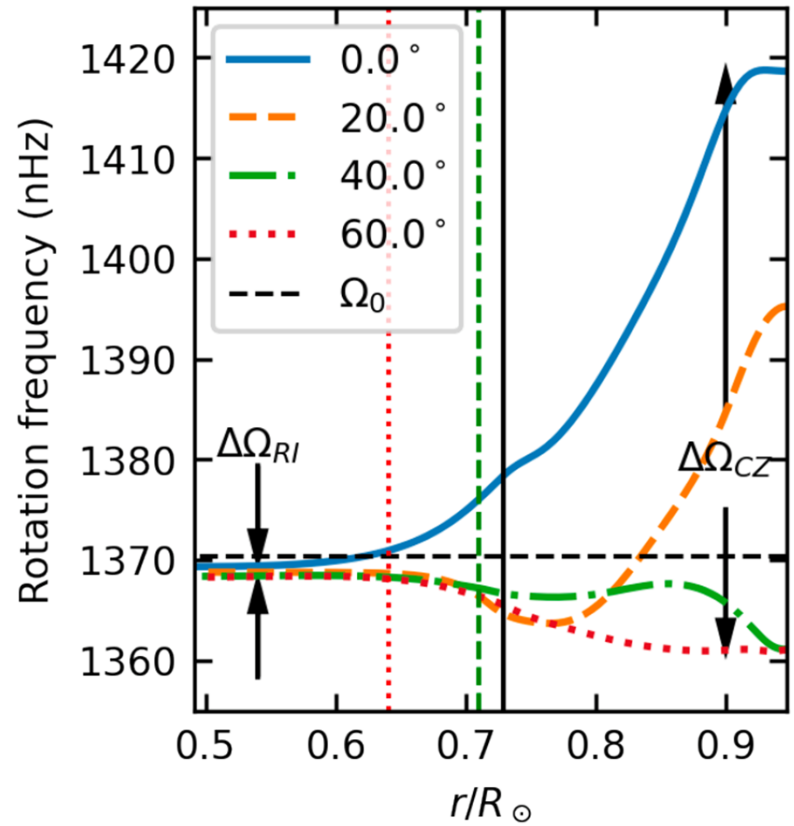
- The models of Hotta et al. 2021,2022 do a fine job of reproducing the Sun's differential rotation profile
  - Can these results be reproduced in models with a different diffusion scheme?
  - Is magnetism truly necessary to restructure the meridional circulation in a fruitful way? (Geostrophic turbulence?)
  - The convective spectrum is still wrong – the surface amplitude is still too large.

- Where is the latitudinal temperature gradient expected from thermal wind balance?
  - Perhaps the balance fails in the Near-Surface Shear Layer?
  - . . . or in the radiative boundary layer?

Outstanding Issues

Differential Rotation in the Radiative Interior

- Magnetic torques can stop the diffusive spread of the tachocline
  - Does the mechanism of Matilsky et al. 2022 (quasi-periodic, nonaxisymmetric dynamo field) work in more realistic parameter regimes (thermal instead of viscous diffusion)?
  - While the current models have a self-consistent tachocline (yeah!), they don't look particularly solar-like (boo!).
    - Too broad with latitudinal variation in the width
    - Weak differential rotation in the convection zone



Nice Review !

Dynamics of Large-Scale Solar Flows

by Hideyuki Hotta, Yuto Bekki, Laurent Gizon, Quentin Noraz, Mark Rast

- Hotta et al. 2023, Space Science Reviews, 219,77
- https://doi.org/10.1007/s11214-023-01021-6

Space Science Reviews (2023) 219:77
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Dynamics of Large-Scale Solar Flows

Hideyuki Hotta[1] · Yuto Bekki[2] · Laurent Gizon[2,3] · Quentin Noraz[4,5] · Mark Rast[6]

Received: 29 June 2023 / Accepted: 18 October 2023 / Published online: 17 November 2023

Abstract
The Sun's axisymmetric large-scale flows, differential rotation and meridional circulation, are thought to be maintained by the influence of rotation on the thermal-convective motions in the solar convection zone. These large-scale flows are crucial for maintaining the Sun's global magnetic field. Over the last several decades, our understanding of large-scale motions in the Sun has significantly improved, both through observational and theoretical efforts. Helioseismology has constrained the flow topology in the solar interior, and the growth of supercomputers has enabled simulations that can self-consistently generate large-scale flows in rotating spherical convective shells. In this article, we review our current understanding of solar convection and the large-scale flows present in the Sun, including those associated with the recently discovered inertial modes of oscillation. We discuss some issues still outstanding, and provide an outline of future efforts needed to address these.

Keywords Convection · Differential rotation · Meridional flow · Helioseismology · Numerical simulation