Nordita Program 2024 on Stellar Convection

# Vortices in turbulent rotating convection: A new challenge in stellar convection theories

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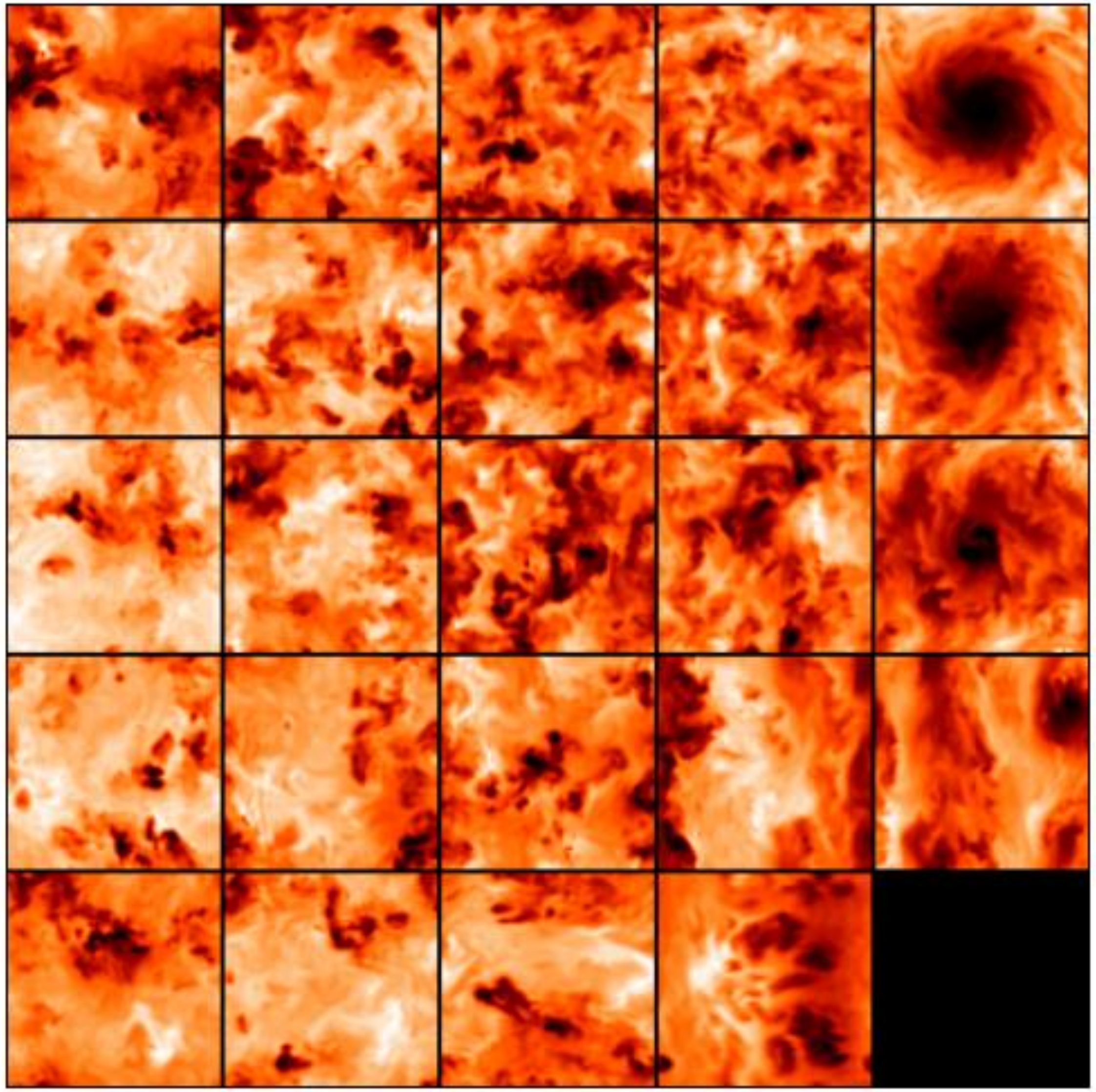
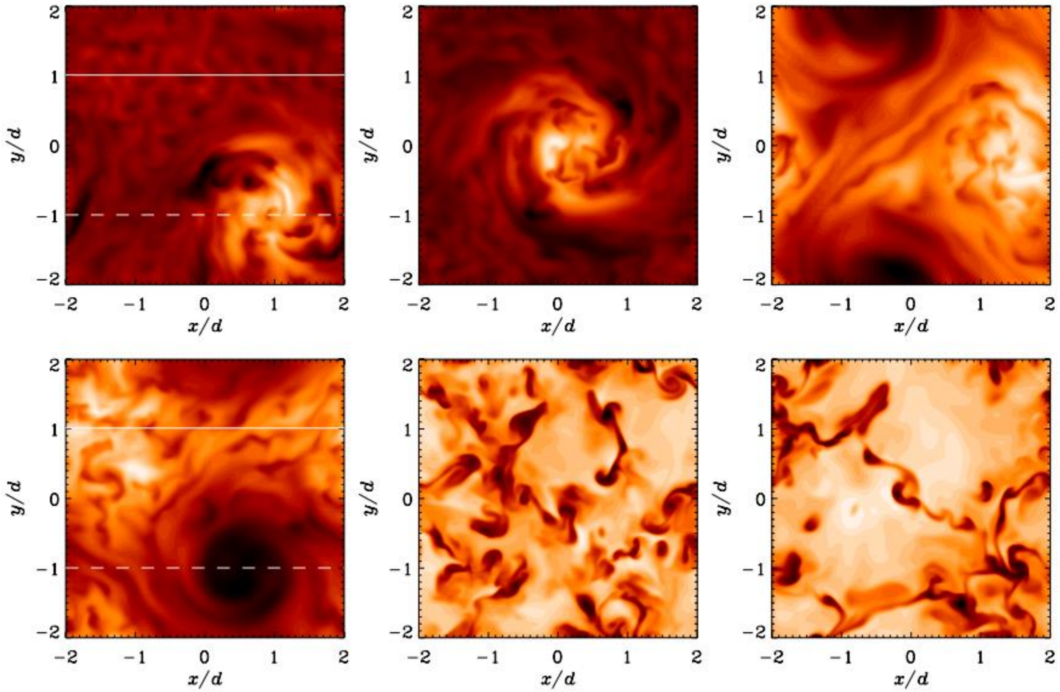
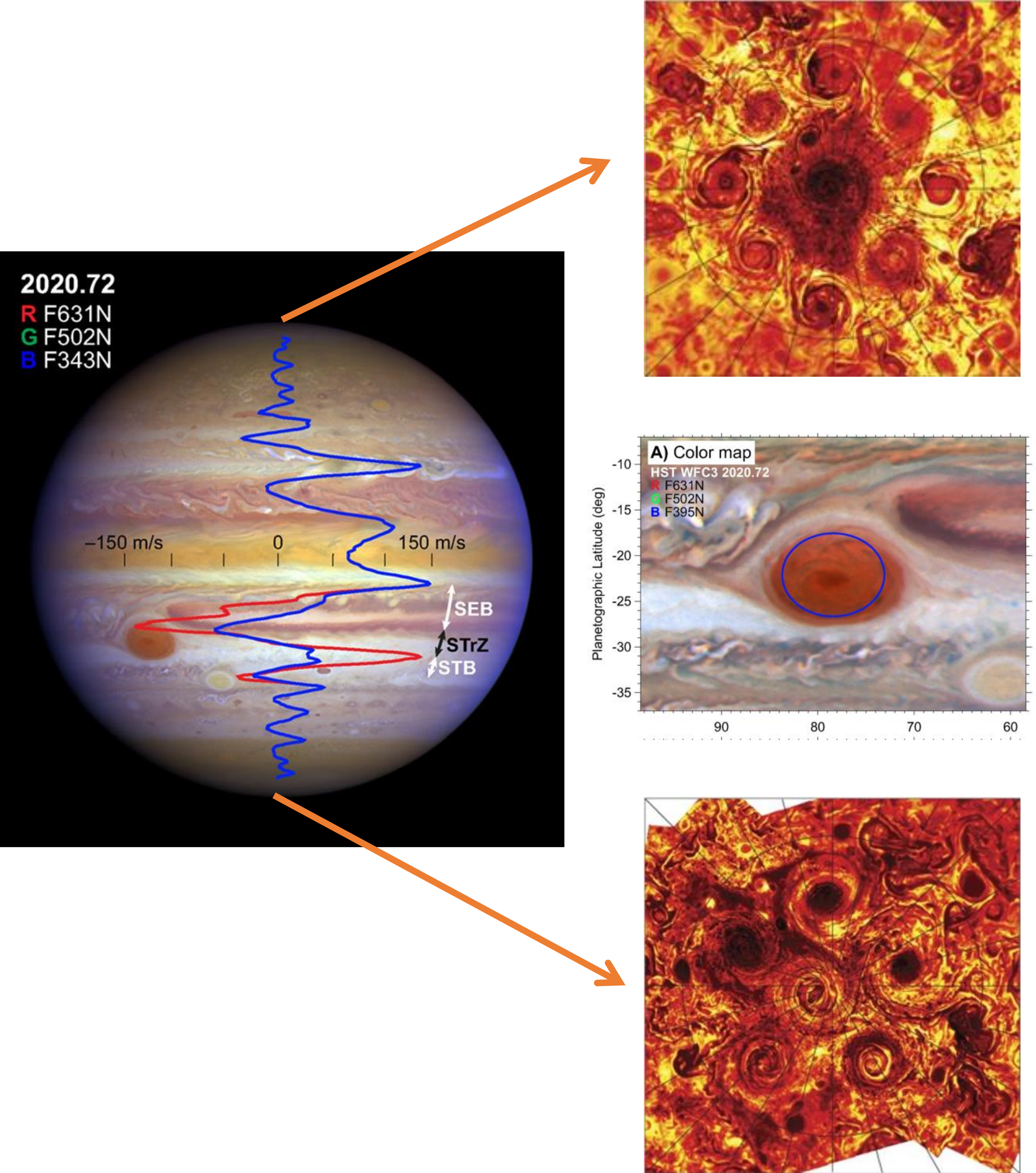


Chan 2007 AN




Kapyla 2011 ApJ




- Vortex crystals in the south and north poles
- Great Red Spot at 22 degrees south

# Questions

- What is the driving mechanism of the LSVs?
- How deep are they?
- How are these vortex crystals formed?

# Rotating Rayleigh-Bénard Convection （RRBC）

incompressible flow

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \frac{\nabla p}{\rho} + T\hat{z} + \upsilon \nabla^2 \boldsymbol{u} - 2\boldsymbol{\Omega} \times \boldsymbol{u}$$

Advection term

Coriolis term

$$\frac{\partial T}{\partial t} = -(u \cdot \nabla)T + \kappa \nabla^2 T$$

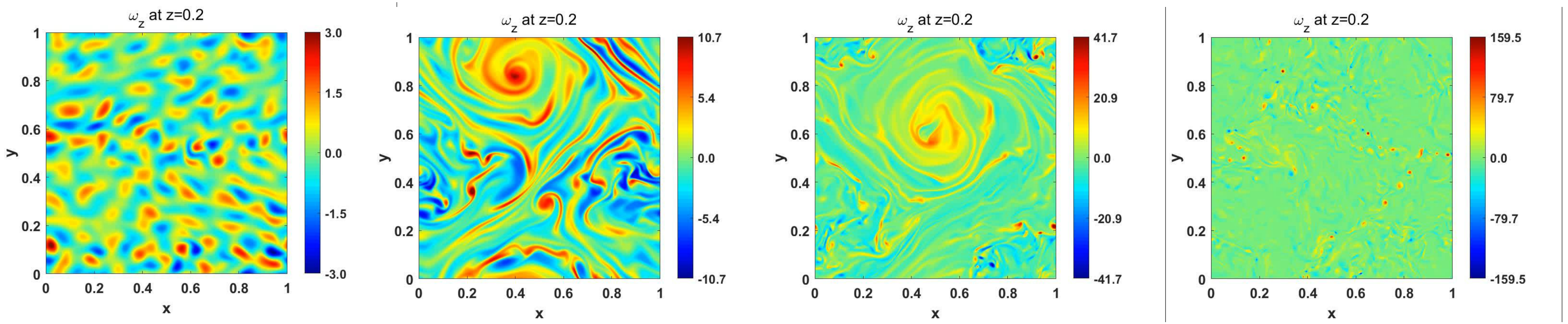
$$\frac{UU}{H} \sim 2\Omega U$$

$$Ro = \frac{U}{2\Omega\, H} = \frac{\tau_\Omega}{\tau_c}$$

Small Rossby number, strong rotational effect

Cai 2021 ApJ

Rotation Rate




Regime I:
Multiple small-scale vortices

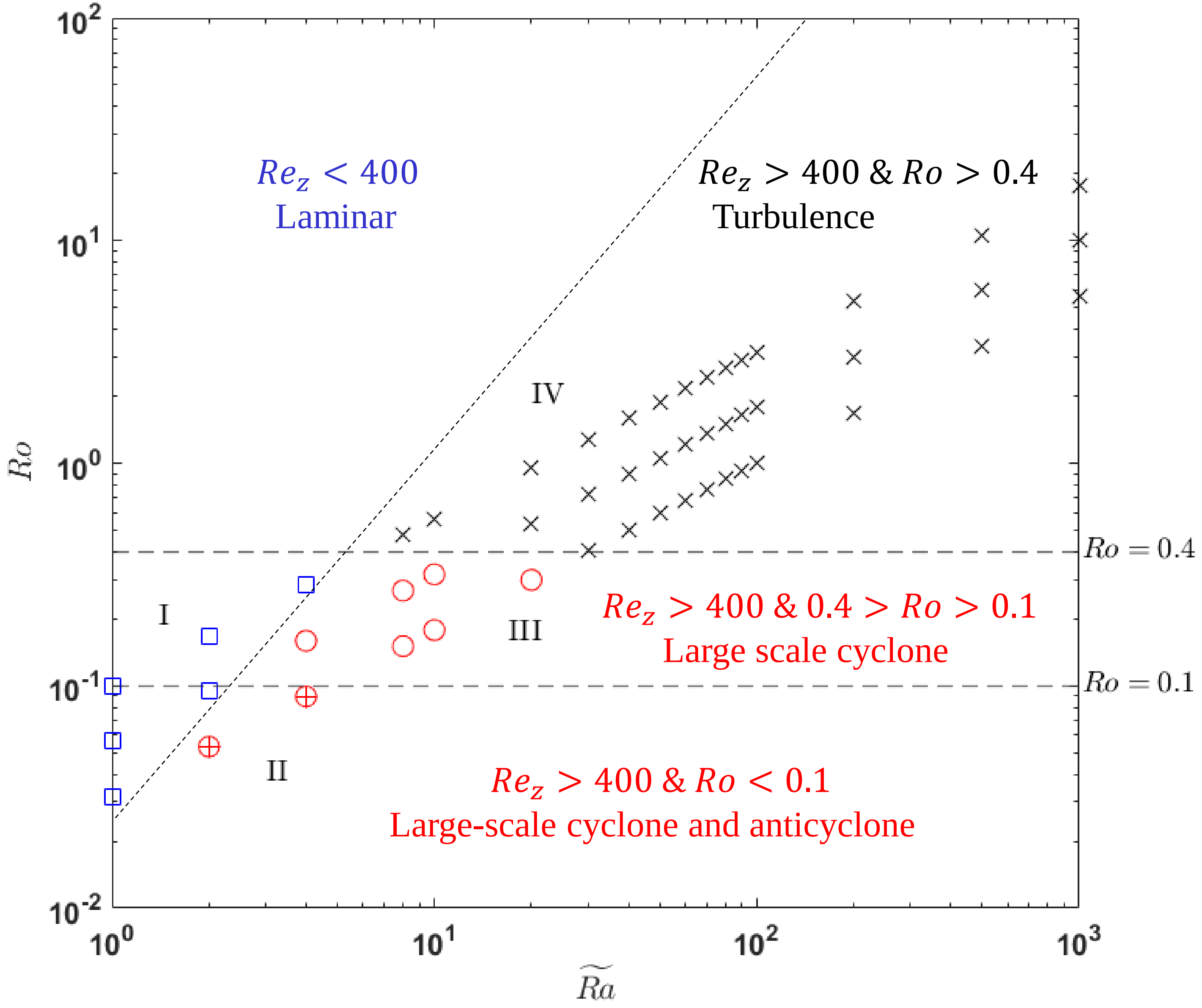
Regime II:
Coexisting large-scale cyclone and anticyclone

Regime III:
Large-scale cyclone

Regime IV:
Geostrophic turbulence

Fast rotation

Turbulent flow

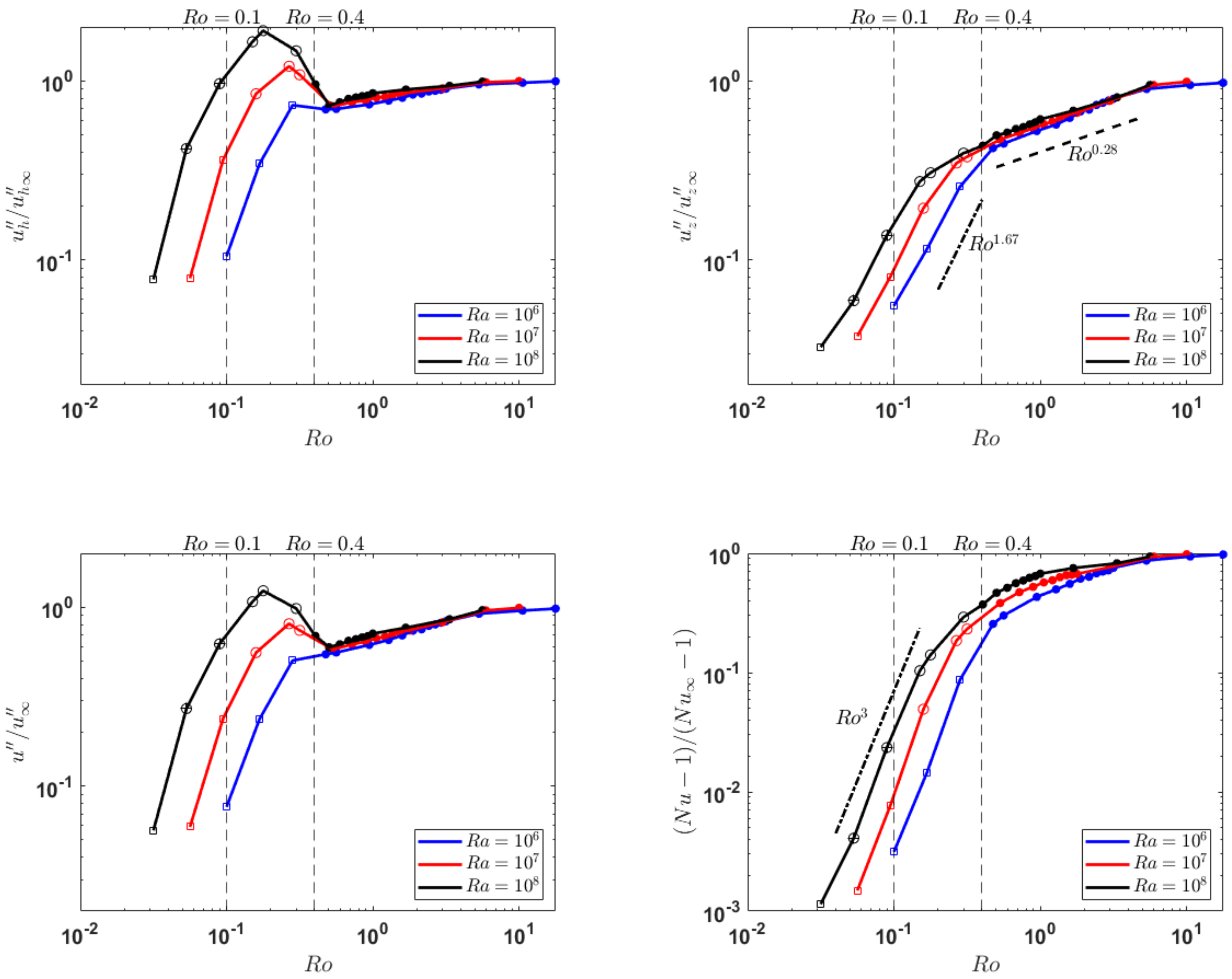



$Ra = 10^{6-8}$ & $Pr = 0.1$ & $\Gamma = 1$

Criterion on generating LSVs:

1. $Ro$<0.4 (fast rotating)
2. $Re_z$>400 (strong turbulence)

Cai 2021 ApJ




Scalings are modified by LSVs

Cai 2021ApJ

# Compressible flow

$$\partial_t \rho = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{u})$$

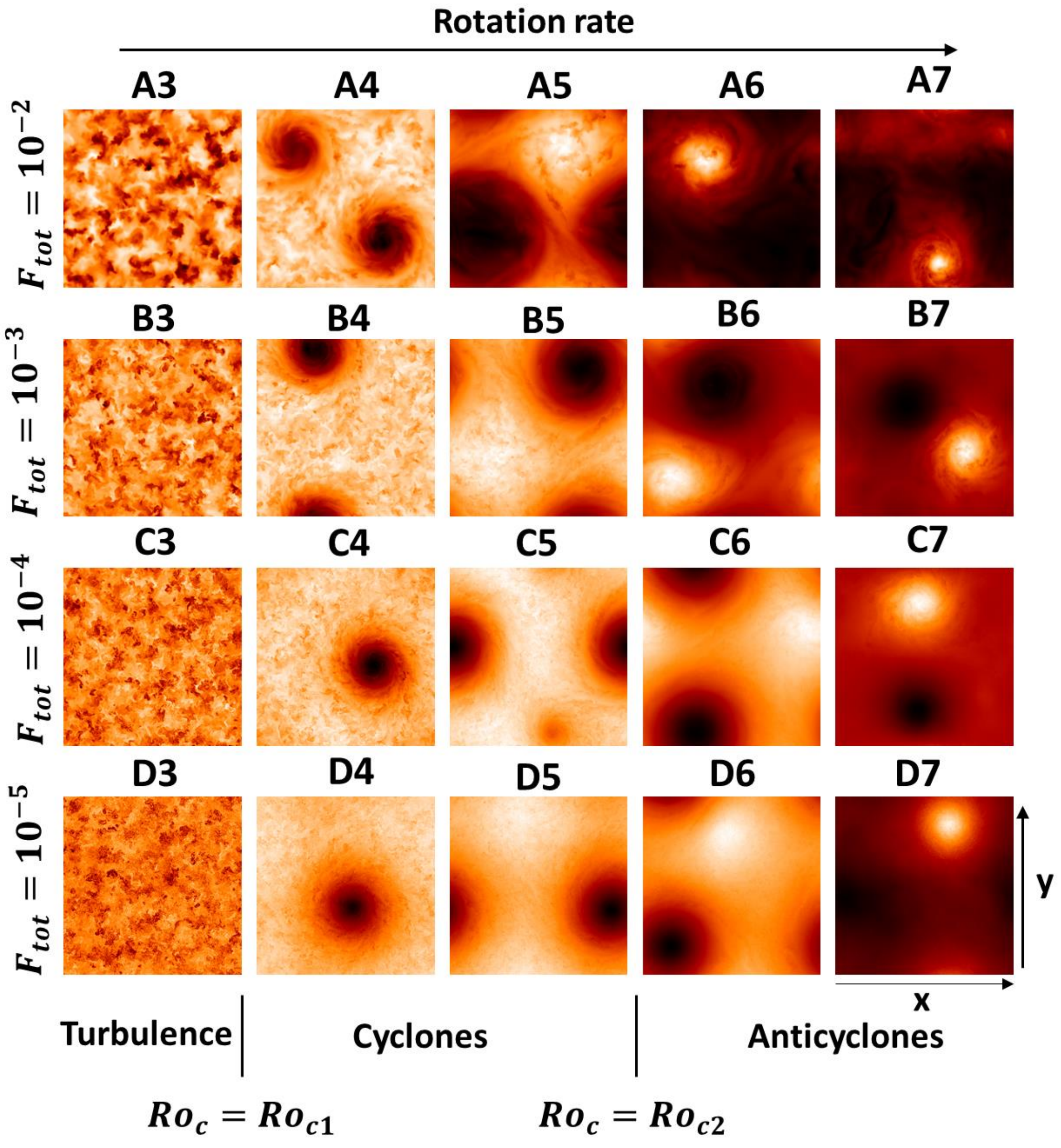
$$\partial_t (\rho \boldsymbol{u}) = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \boldsymbol{\nabla} \cdot \boldsymbol{\Sigma} - \boldsymbol{\nabla} p + \rho \boldsymbol{g} - 2\rho \boldsymbol{\Omega} \times \boldsymbol{u}$$

$$\partial_t E = -\boldsymbol{\nabla} \cdot [(E + p)\boldsymbol{u} - \boldsymbol{u} \cdot \boldsymbol{\Sigma} + \boldsymbol{F}] + \rho \boldsymbol{u} \cdot \boldsymbol{g}$$

$$Ro \sim \frac{\rho U U / H}{\rho \Omega U} = \frac{U}{2\Omega H}$$

$$F \sim p U \sim \rho U^3 \Rightarrow U \sim (\frac{F}{\rho})^{1/3}$$

Cai 2016 JCP




$Ro < Ro_{c1} = 0.11$ cyclone

$Ro < Ro_{c2} = 0.023$ anticyclone

Cai, Chan & Chow 2022 ApJ

# Inference of the depth from the simulations

Critical Rossby numbers:

$Ro < Ro_{c1} = 0.11$ Cyclone

$Ro < Ro_{c2} = 0.023$ Anticyclone

$$U \sim 1.11 \left(\frac{F}{\rho}\right)^{1/3} = 1.11 \times \left(\frac{7.48\mathrm{W}m^{-2}}{0.167kgm^{-3}}\right)^{1/3} \approx 4ms^{-1}$$

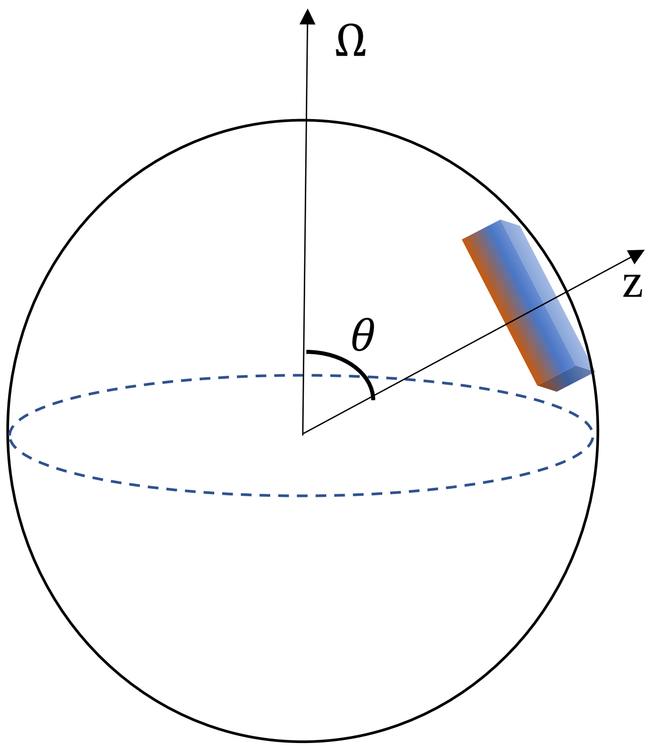
for non-rotating Jupiter

$$\Omega \sim \left(\frac{2\pi}{36000s}\right) = 1.75 \times 10^{-4}\ s^{-1}$$

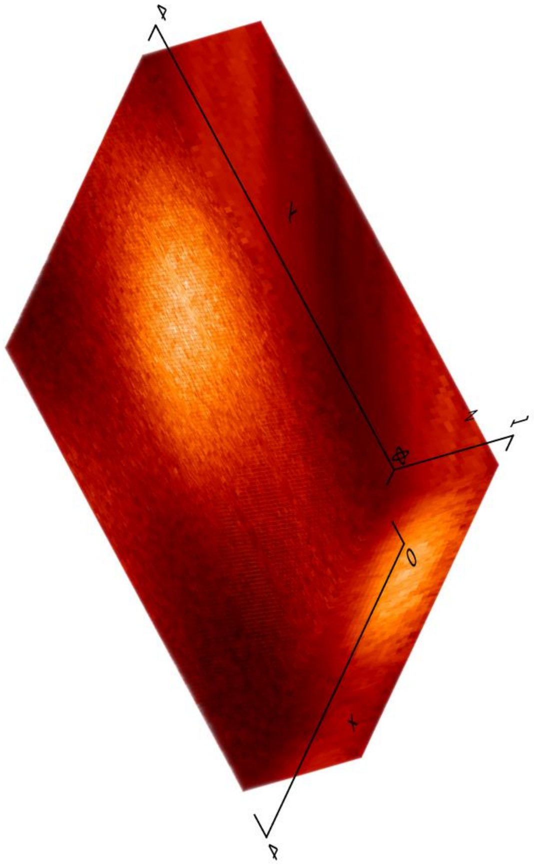
$$Ro = \frac{U}{2\Omega H} < Ro_{c2} \Longrightarrow\ H > \frac{U}{2\Omega Ro_{c2}} \approx 500km$$

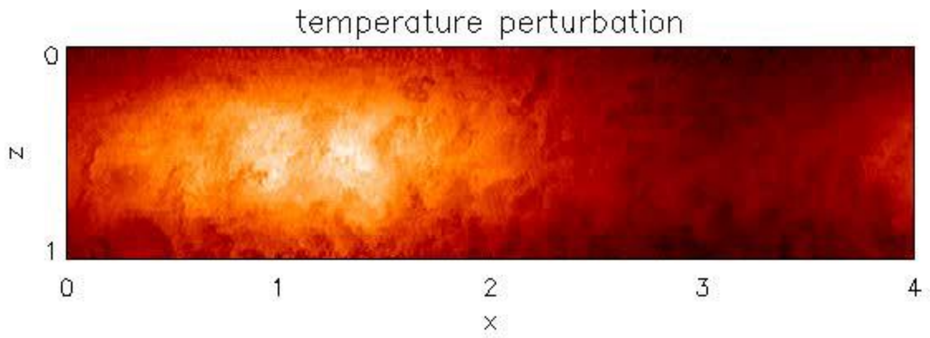
Cai, Chan & Chow 2022 ApJ

# GRS-like large-scale anticyclone at low latitude




Simulation at $\theta = 67.5^{\circ}$



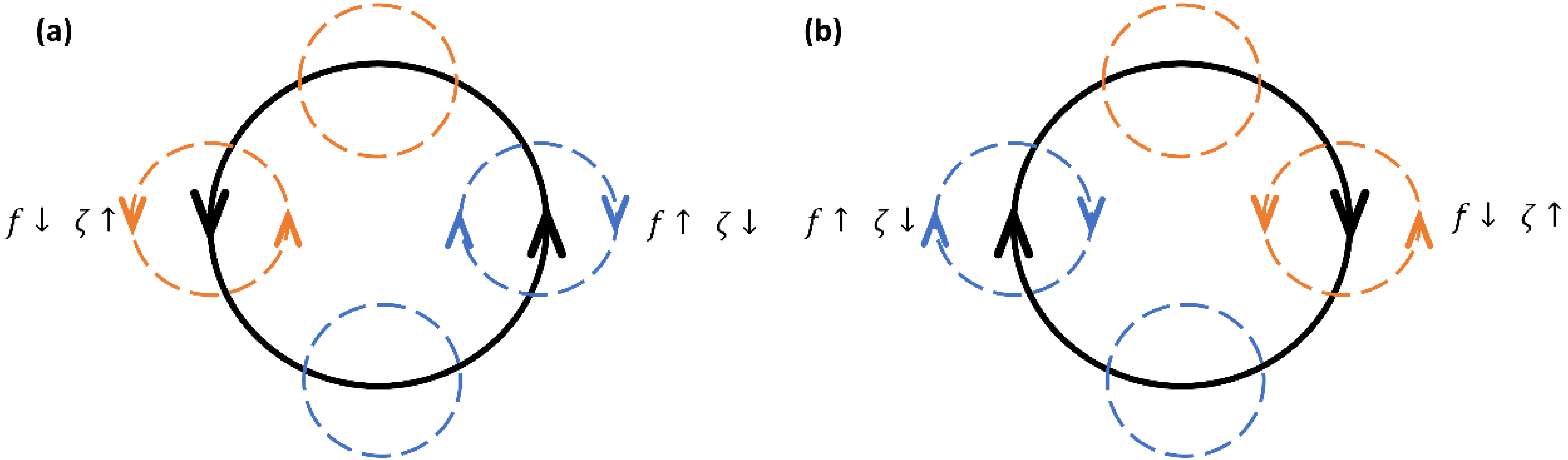



Taylor-Proudman column (LSV is parallel to the rotational axis)

Cai, Chan & Chow 2022 ApJ

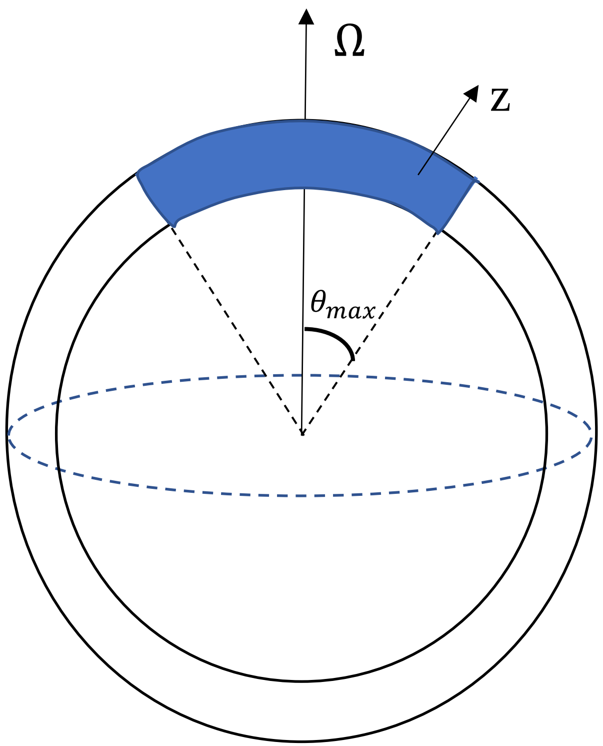
# Beta effect

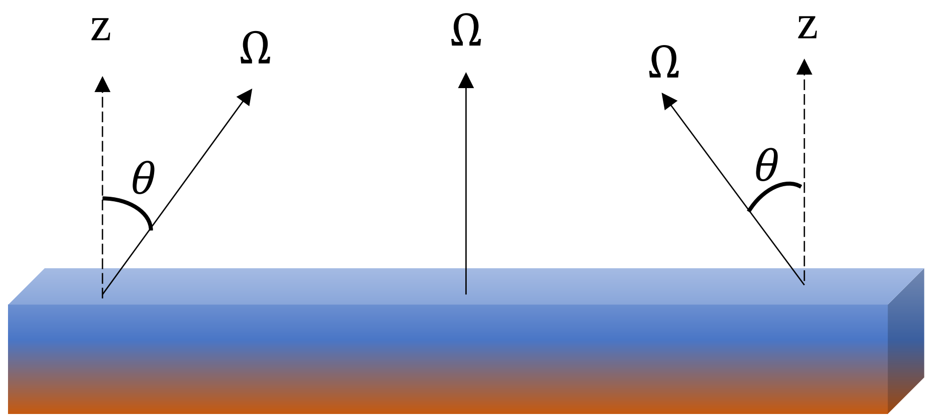
$$\frac{D}{Dt}\left(\frac{f+\xi}{h}\right)=0$$




cyclone moves poleward and anticyclones move equatorward

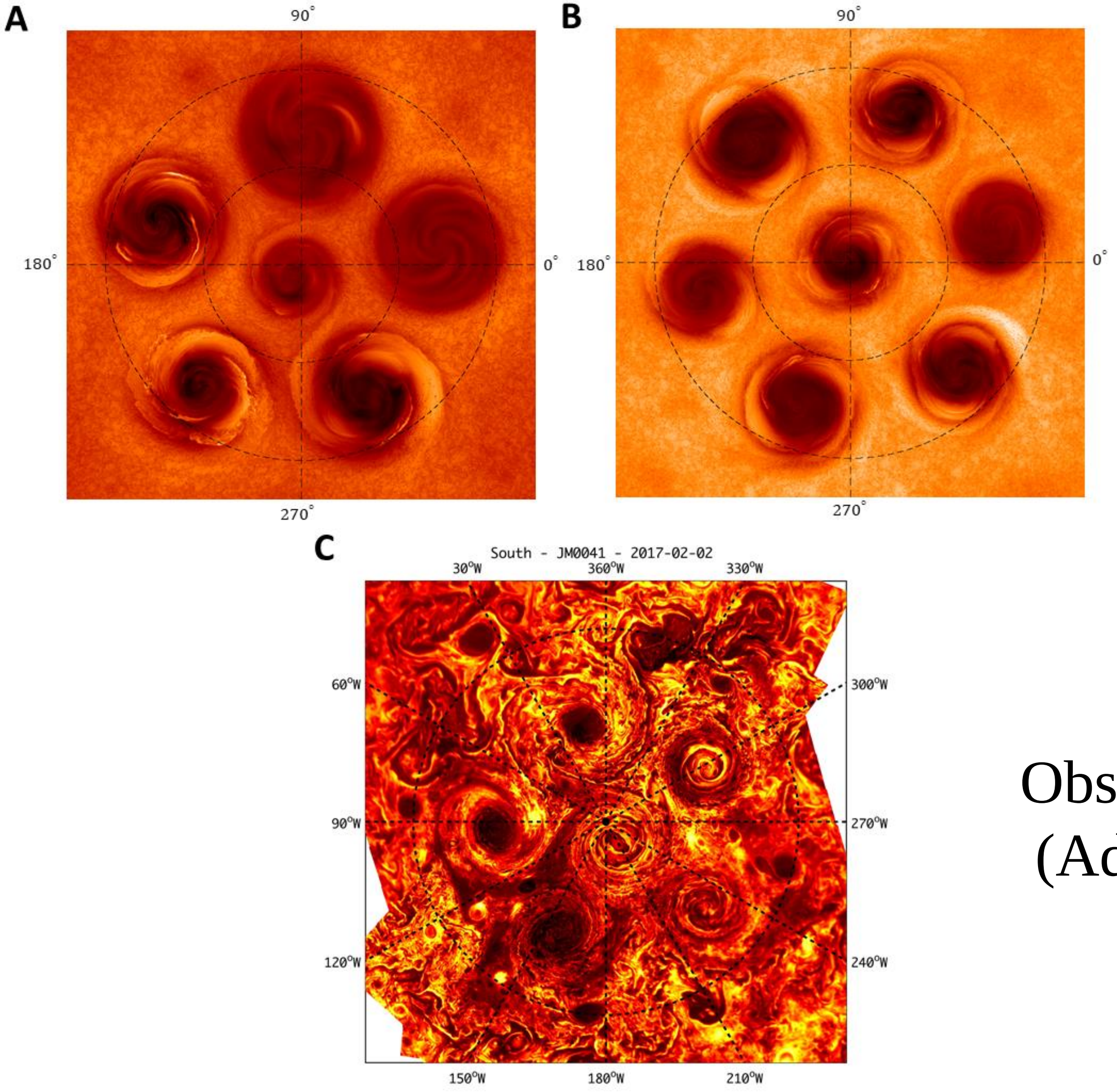
# Polar gamma box model



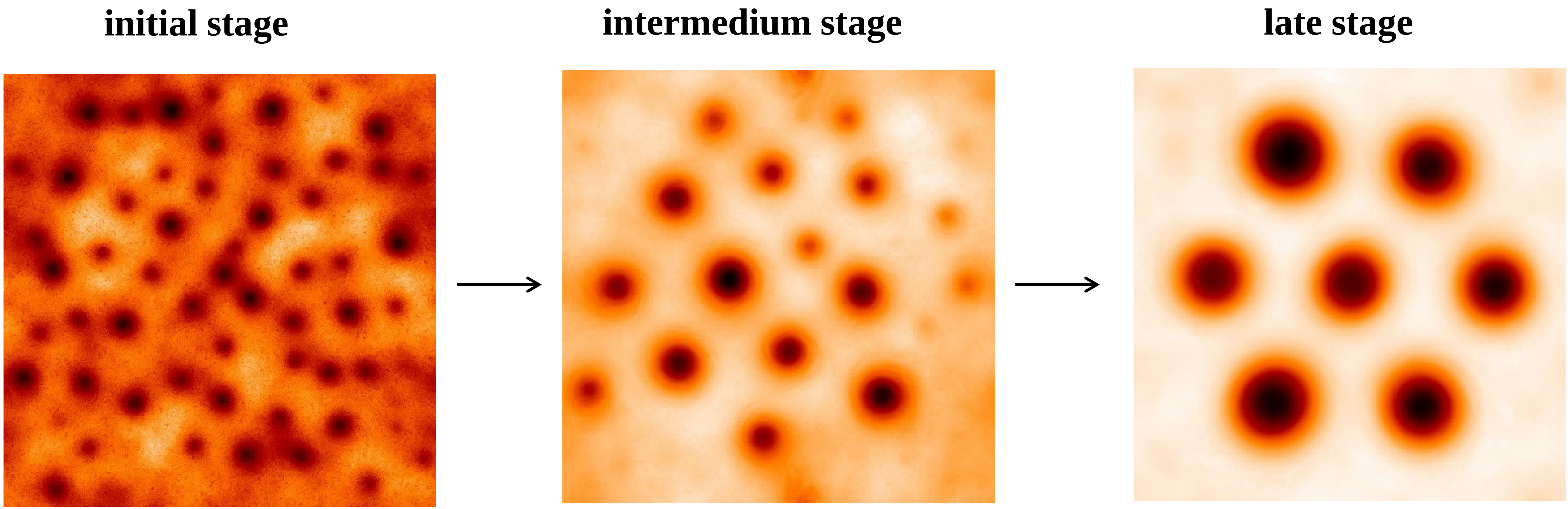

Polar gamma plane

Cai, Chan & Mayr 2021 PSJ




Simulation
(Cai, Chan & Mayr 2021 PSJ)

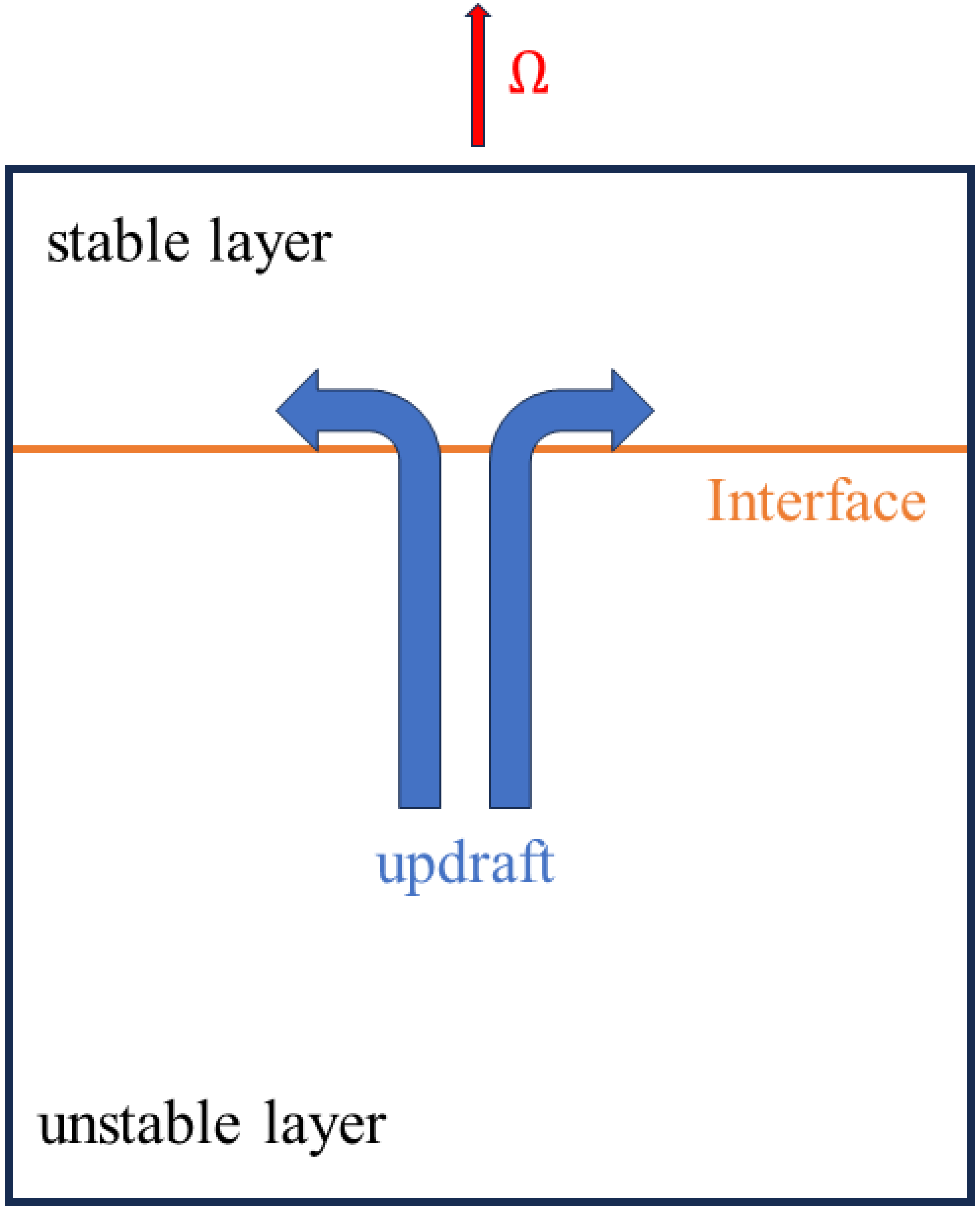
Observation
(Adriani et al. 2018 Nature)

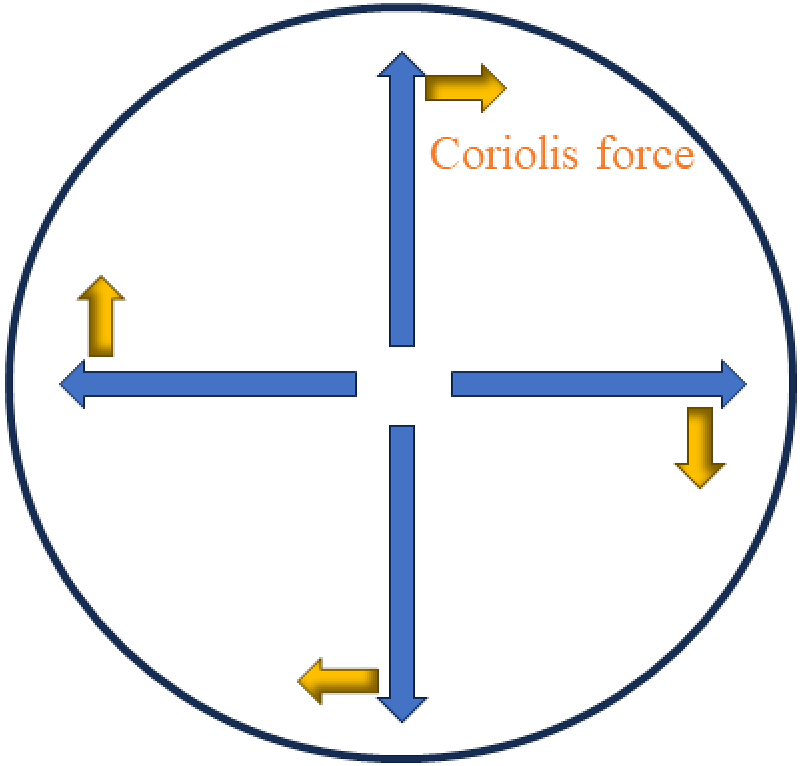

initial stage
intermedium stage
late stage

divergence and vorticity correlation

Northern hemisphere $\delta\xi < 0$

Southern hemisphere $\delta\xi > 0$

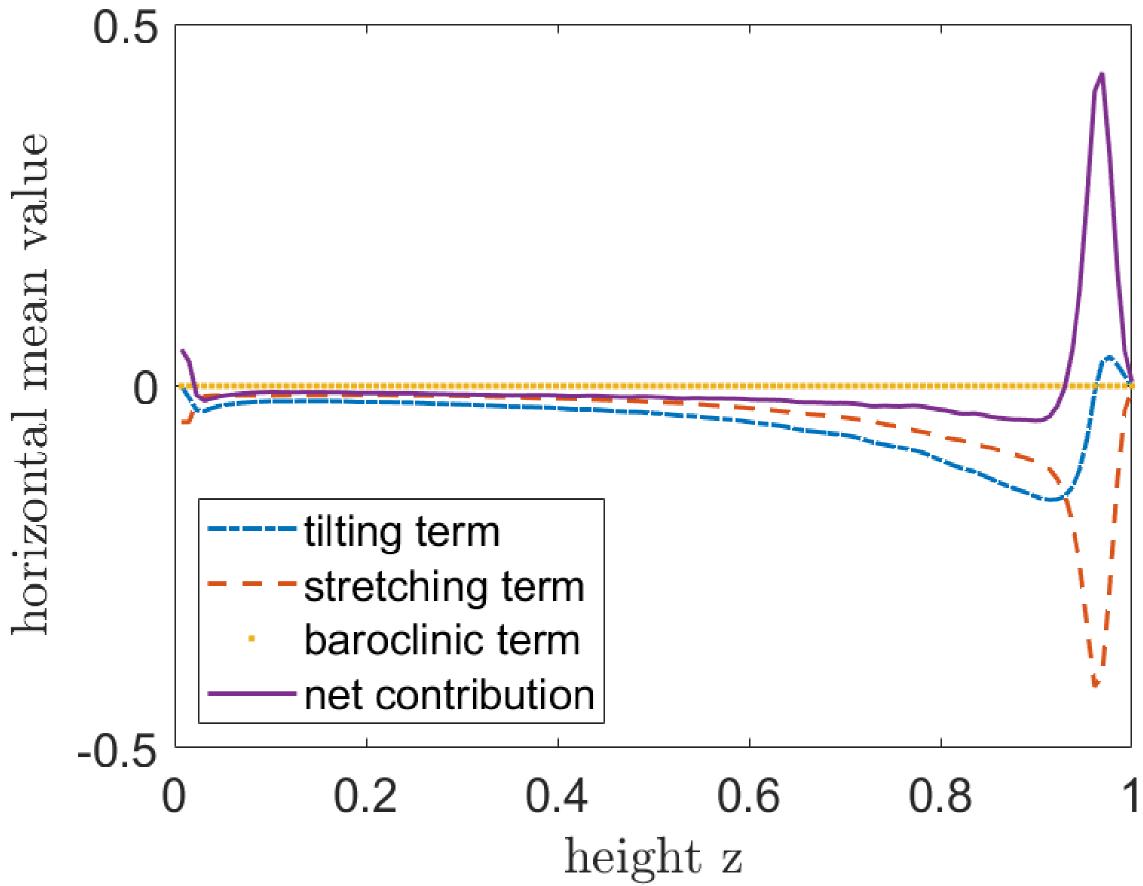



Valid in Sun (Gizon & Duvall 2003)

# How to maintain the LSVs in the stable layer?

$$\overline{D_t\xi} = \underbrace{\overline{(\boldsymbol{\xi}_h \cdot \boldsymbol{\nabla}_h)w}}_{\text{tilting term}} - \underbrace{\overline{\xi\delta}}_{\text{stretching term}} + \underbrace{\overline{\rho^{-2}\boldsymbol{\nabla}_h\rho \times \boldsymbol{\nabla}_h p}}_{\text{baroclinic term}} .$$




- Tilting effect dominates in the convective layer
- Stretching effect dominates in the radiative layer
- Vorticity is transferred from convective layer to radiative layer

Cai 2024 JGR Planets

# Challenges in rotating convection theories

- What determines the size of the LSV? How do the LSVs affect mixing, momentum and energy transportations?

- Shall the theoretical models be different for rapidly and slowly rotating convection?

- Is it possible to develop a generalized rotating Reynolds stress model?

Thank you for attention!

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