Heidelberg Institute for Theoretical Studies

HITS

Convective boundary mixing: Theory and numerical simulations

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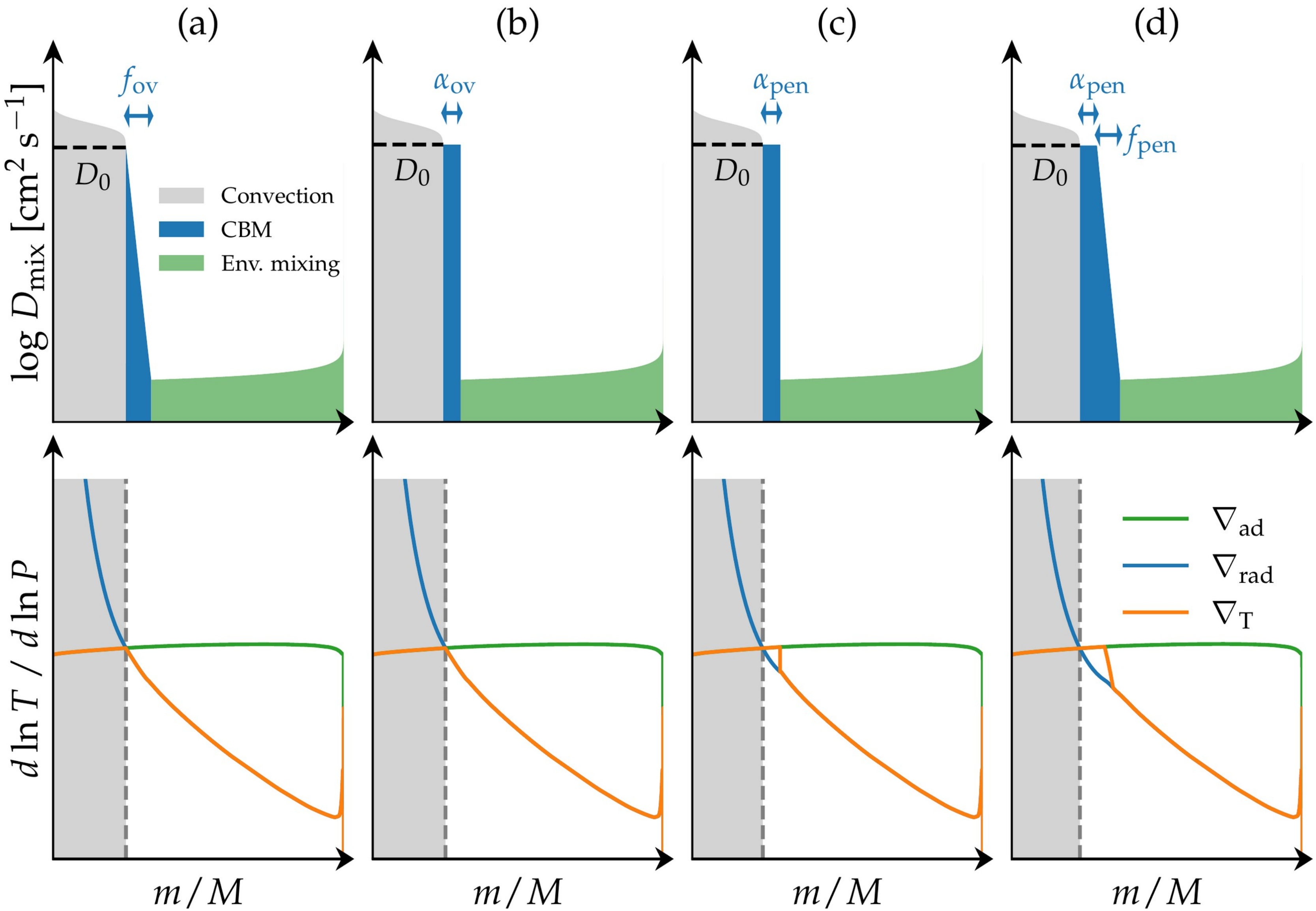
Parametrisations in 1D stellar evolution



Anders & Pedersen (2023)

Ballistic overshoot

- Early estimates: Roxburgh (1965), Saslaw & Schwarzschild (1965)

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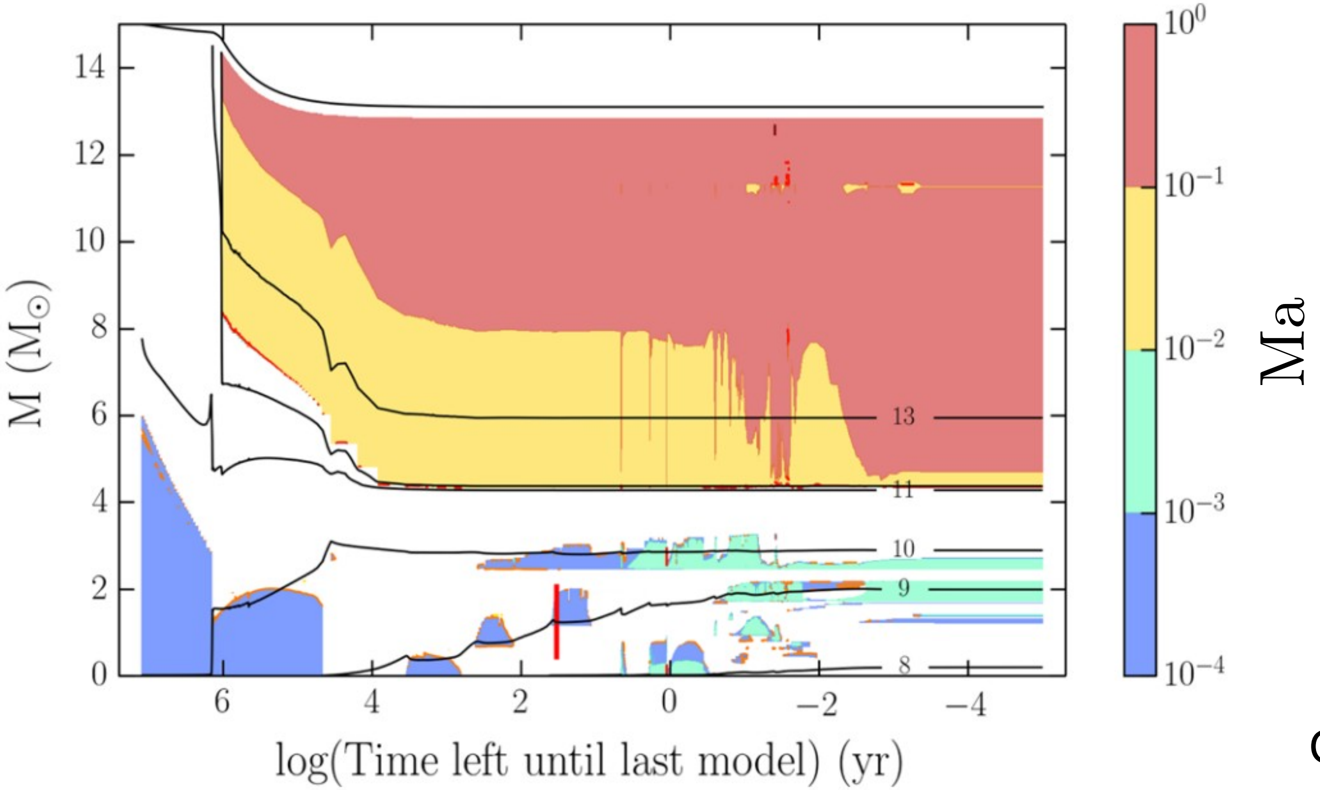
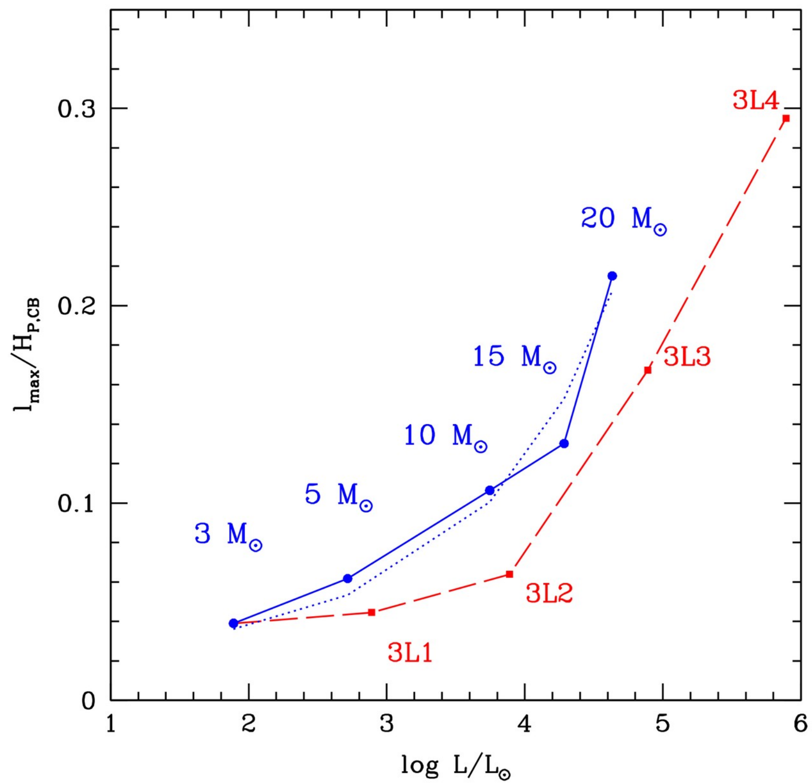


Cristini+ (2017)

Mixing in extreme events?

- Extreme-value statistics (Pratt+ 2017, Baraffe+ 2023).
- Based on the radial fluxes f_k and $f_{\delta T}$.
- Do these extreme events represent mixing or waves?
- Thermal stratification evolves: initial transient?

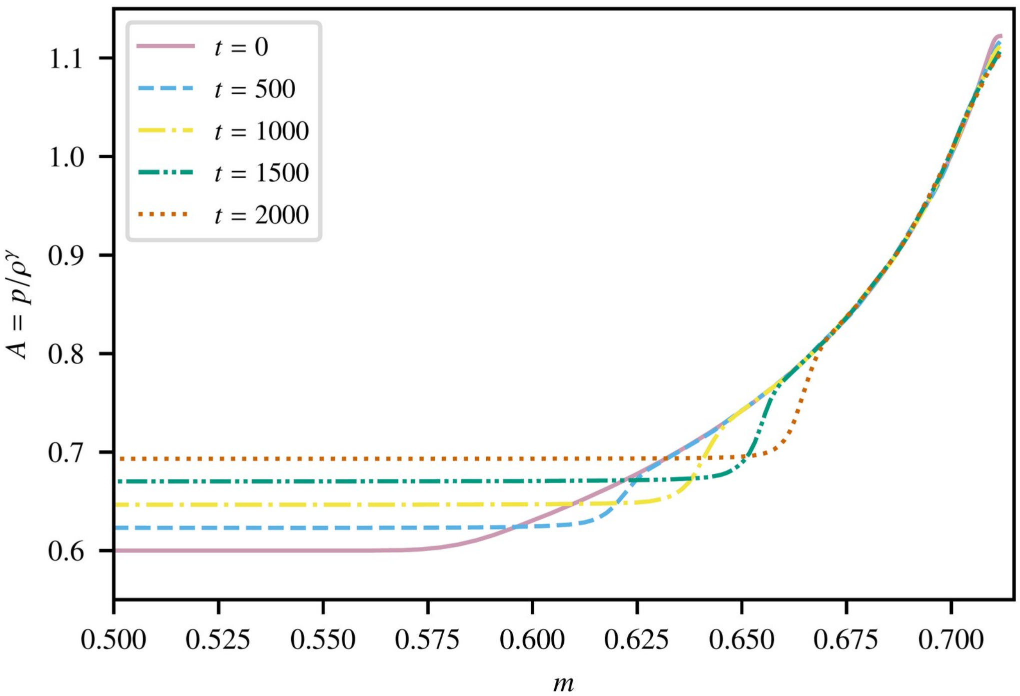


$$d_{\rm ov} \propto L^{1/3} \propto {\rm Ma}$$

Baraffe+ (2023)

Entropy increase in a convective layer

- Heating + adiabatic flows.
- Implications:
  - Mass entrainment at the top.
  - Increase in stability at the bottom.



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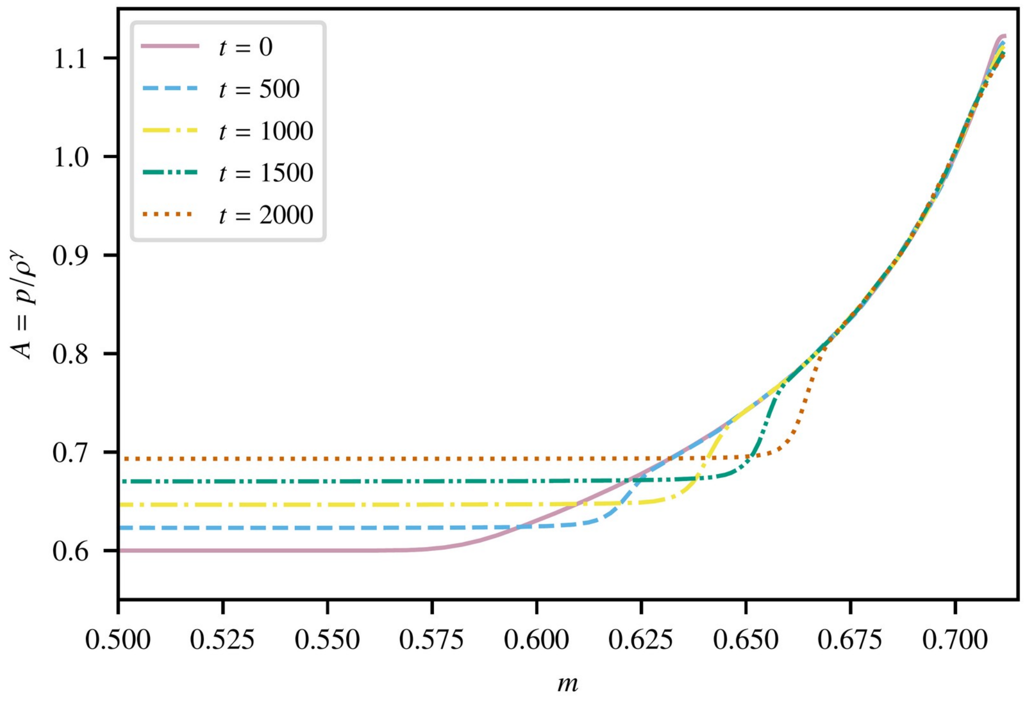

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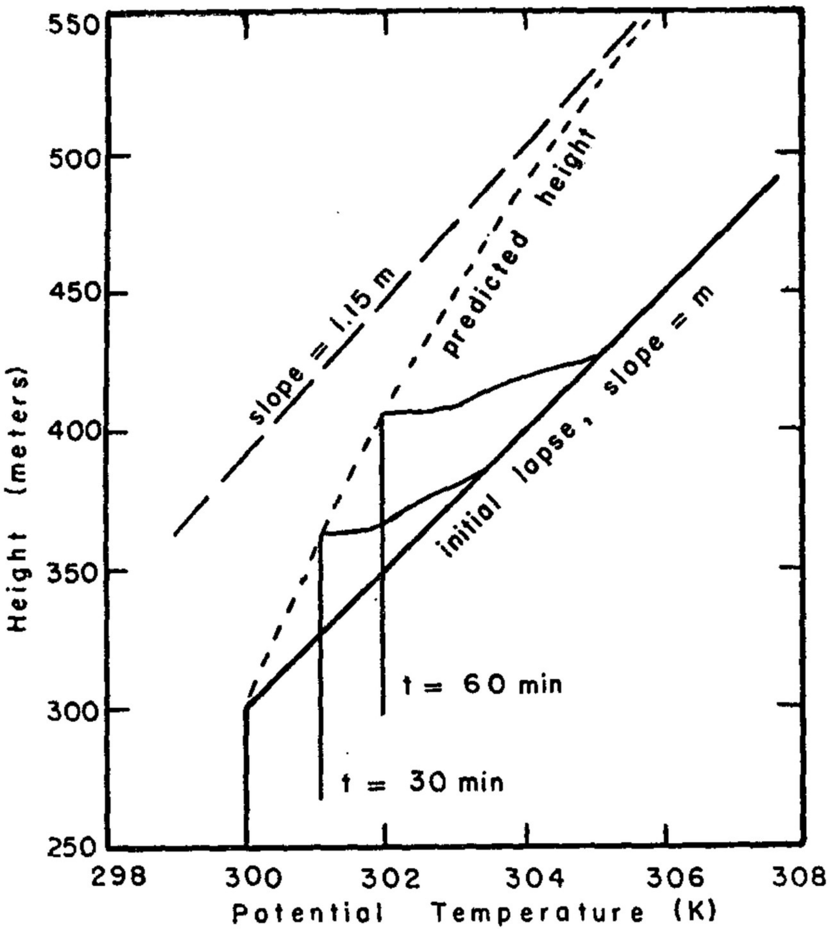

Stull (1973)

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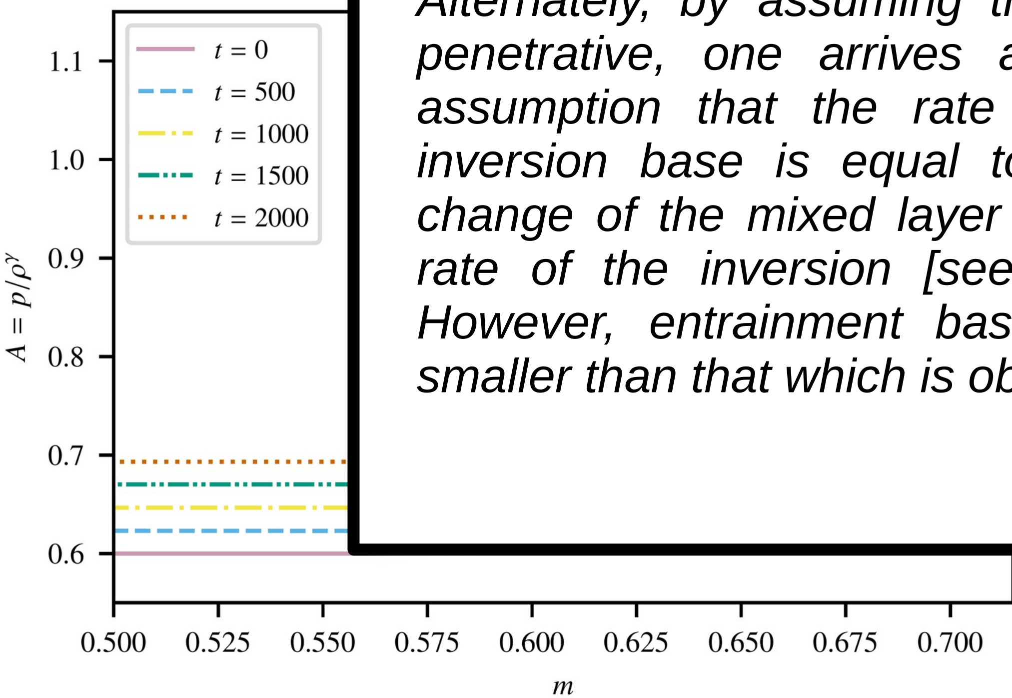
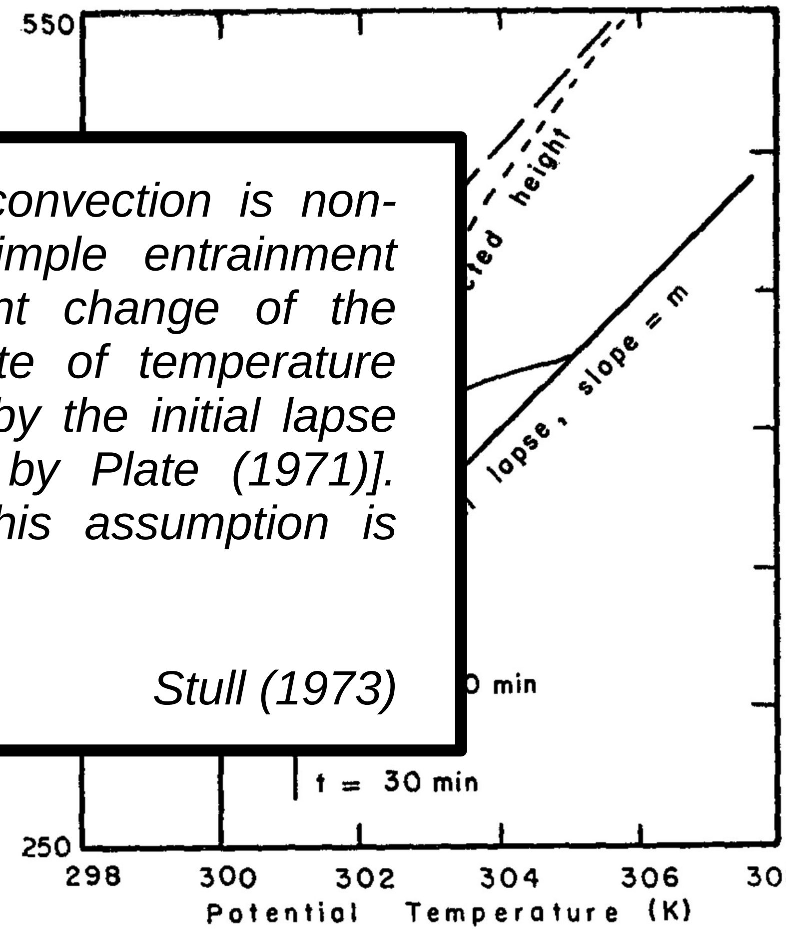


Andrassy+ (2022)

> *Alternately, by assuming that the convection is non-penetrative, one arrives at the simple entrainment assumption that the rate of height change of the inversion base is equal to the rate of temperature change of the mixed layer divided by the initial lapse rate of the inversion [see review by Plate (1971)]. However, entrainment based on this assumption is smaller than that which is observed.*
>
> *Stull (1973)*



Stull (1973)

Hydrodynamic entrainment

- Many experimental, atmospheric, and oceanic studies (e.g. Linden 1975, Deardorff 1980, Boers & Eloranta 1986, Strang & Fernando 2001, Jonker & Jiménez 2014)
- In the limit of a stiff interface:

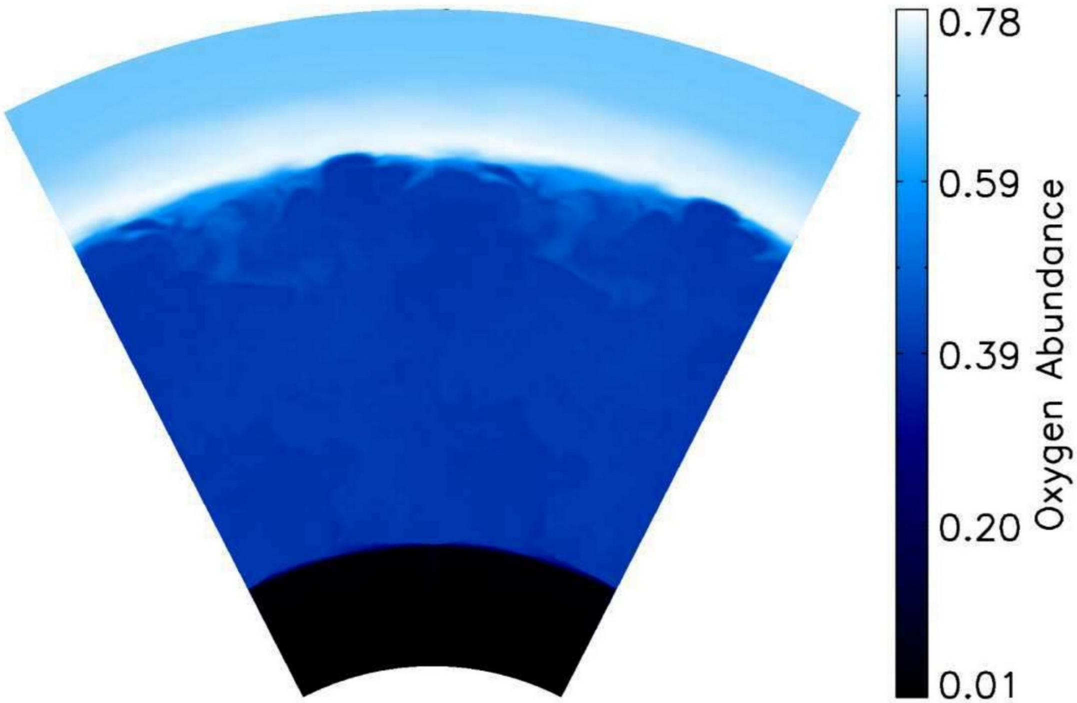
> *Since the eddies were too feeble to scour dense fluid from across the interface into the upper layer, the entrainment occurred by local mixing induced by interfacial instabilities that caused the development of an intermediate layer of partially mixed fluid which, in turn, could be engulfed by turbulent eddies.*
>
> Strang & Fernando (2001)

Hydrodynamic entrainment

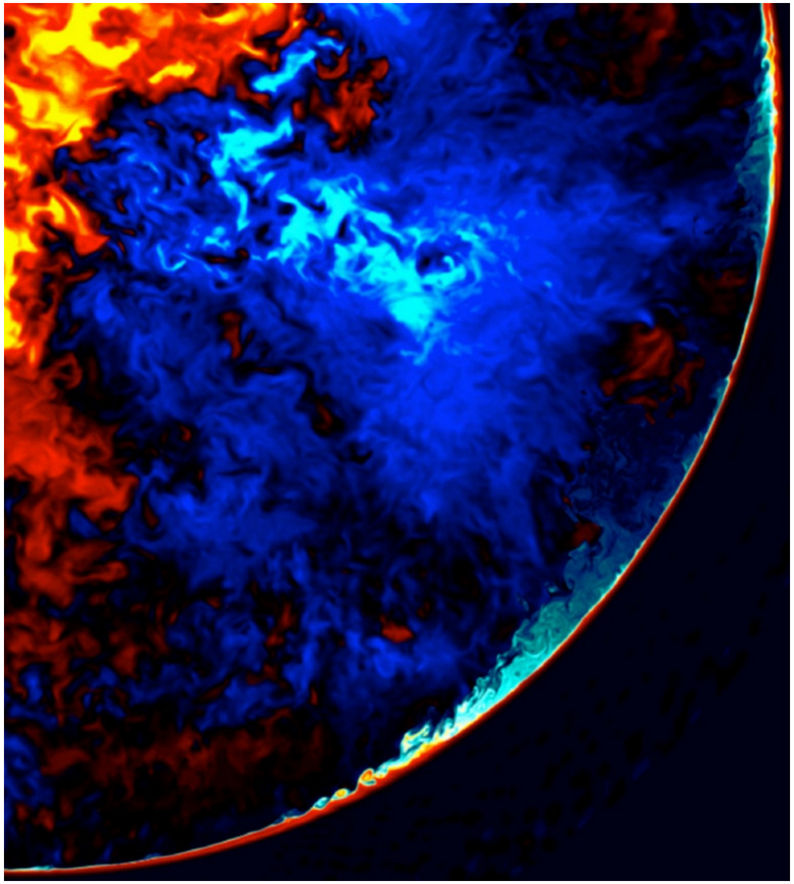
(1) Shear instabilities and breaking waves mix the boundary layer.

(2) Buoyancy of the partially mixed material reduced.

(3) Downflows pull the mixed material into the convective layer.



Meakin & Arnett (2007)



Woodward+ (2015)

Hydrodynamic entrainment

- Relevant for evolved stars, short evolutionary stages.
- Bulk Richardson number: a measure of boundary stiffness.

$$\mathrm{Ri_B} = \frac{L \Delta b}{v_{\mathrm{rms}}^2} \qquad \Delta b = \int_{r_1}^{r_2} N^2 \mathrm{d}r$$

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- Entrainment law (Meakin & Arnett 2007, inspired by lab experiments):

$$v_\mathrm{e} = A \mathrm{Ri_B}^{-n} v_\mathrm{rms}$$

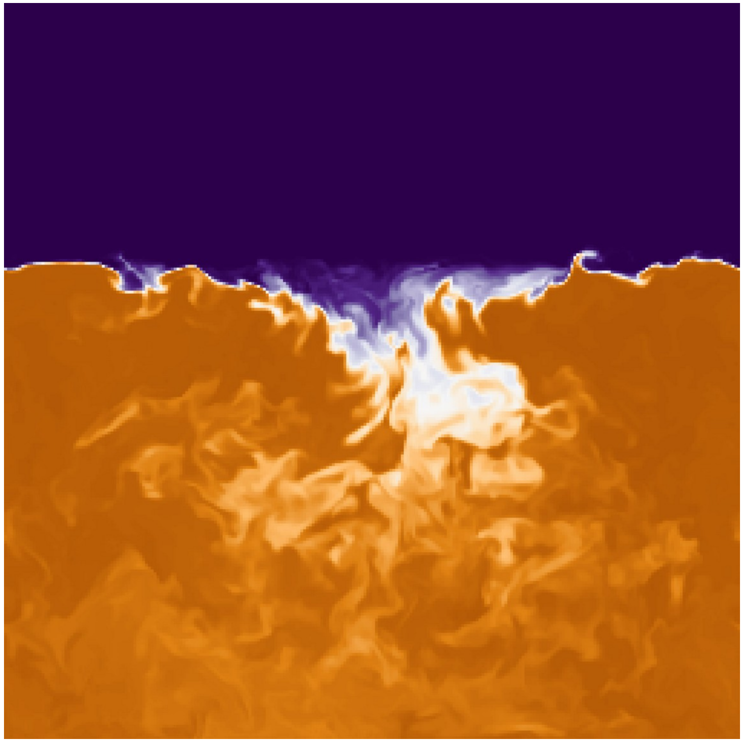
$$\dot{M}_\mathrm{e} = 4\pi r^2 \rho \, v_\mathrm{e}$$

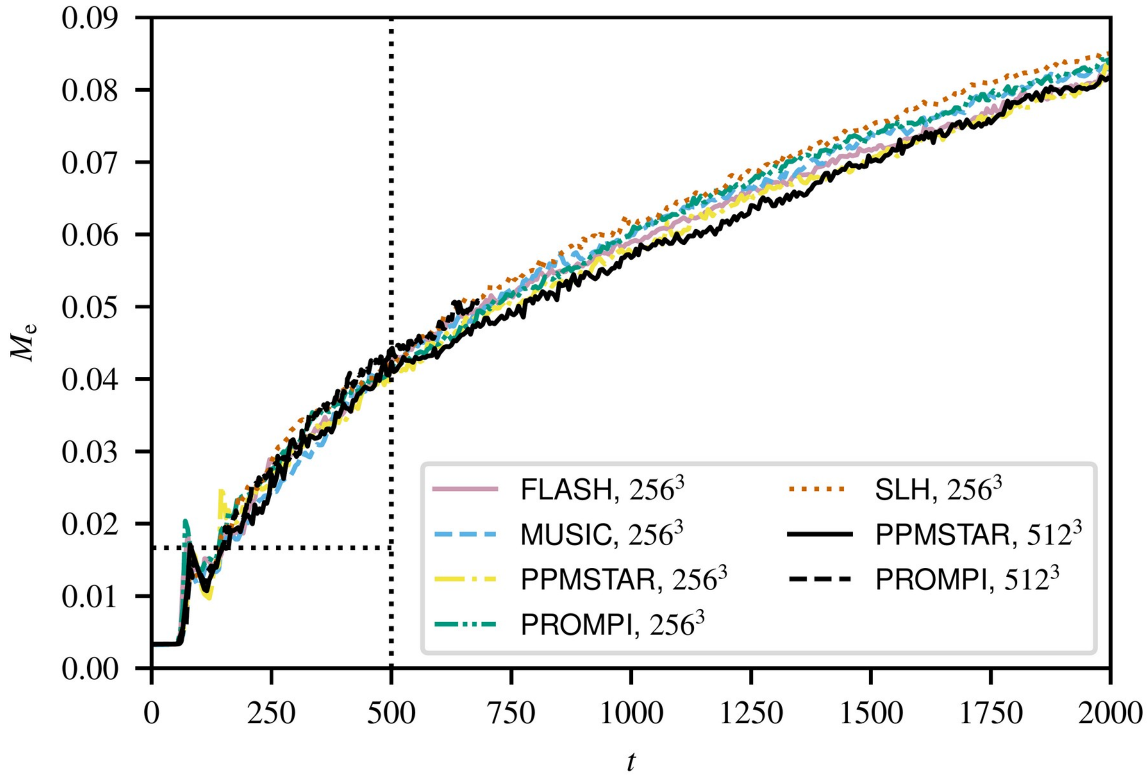
Warning: Measurements often
include thermal expansion velocity.

- Examples: Meakin & Arnett (2007), Cristini+ (2019), Horst+ (2021)

Hydrodynamic entrainment

- Entrainment rates directly computable shortly before core collapse.
- Five different codes, the same entrainment rate (Andrassy+ 2022):





Convective penetration

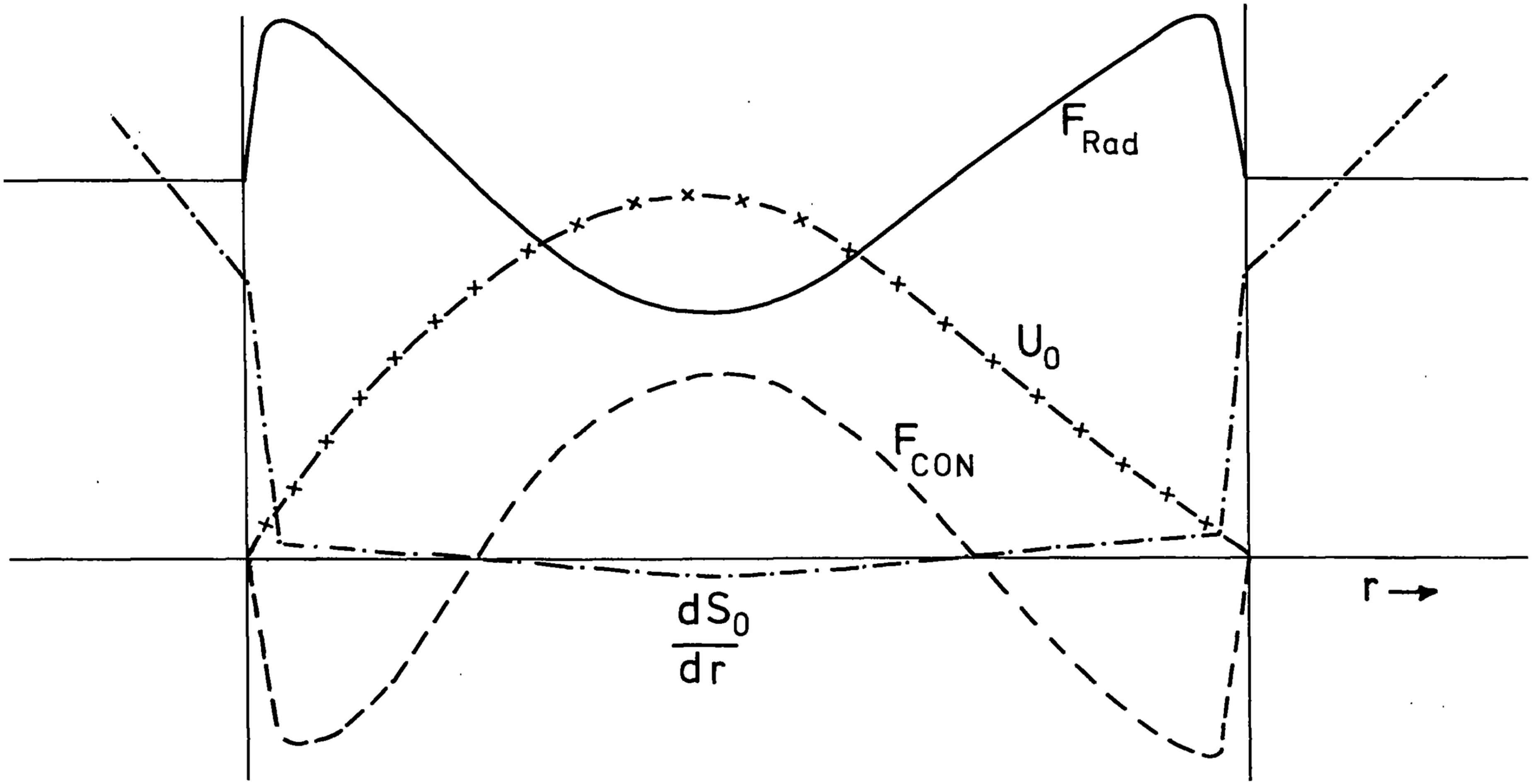
- Radiative diffusion modifies the entropy profile.
- Thermal timescale phenomenon.
- Relevant during core H and He burning.
- Equilibrium between advective and diffusive heat transport.
- Roxburgh (1978, 1989, 1992):

$$\int_0^{r_\mathrm{c}} (L_\mathrm{rad} - L_\mathrm{nuc}) \frac{1}{T^2} \frac{\mathrm{d}T}{\mathrm{d}r} \mathrm{d}r = \int_0^{r_\mathrm{c}} \frac{\Phi}{T} 4\pi r^2 \mathrm{d}r > 0$$

⇓

upper limit on r_c

Convective penetration

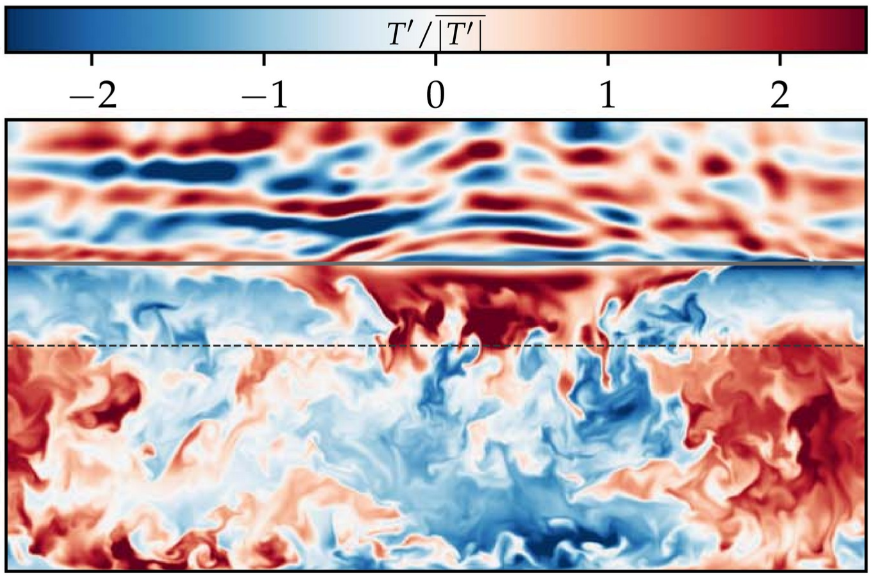


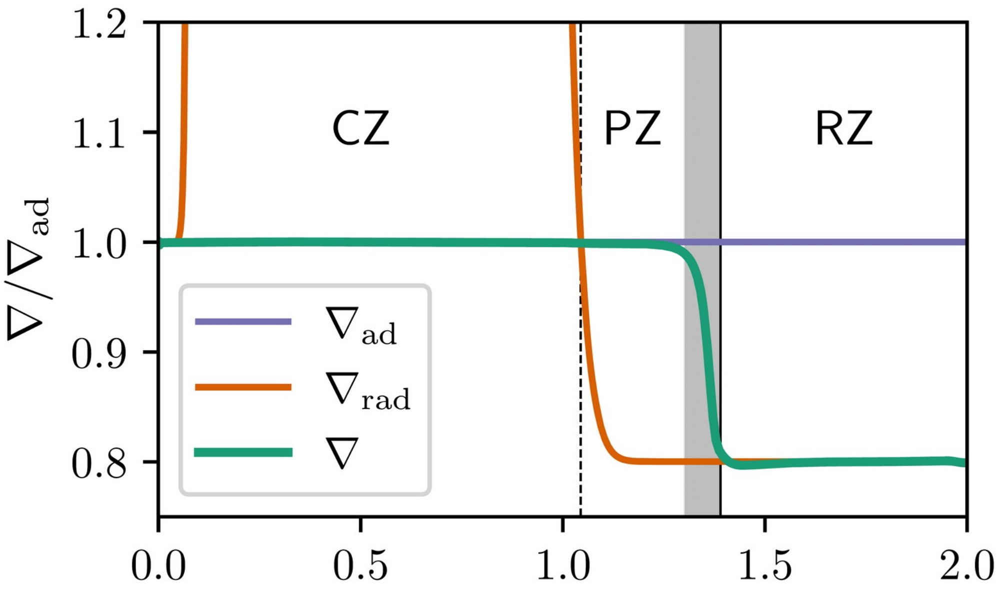
Schematic Model of a Convective Zone

Roxburgh (1978, i.e. no simulations!)

Convective penetration

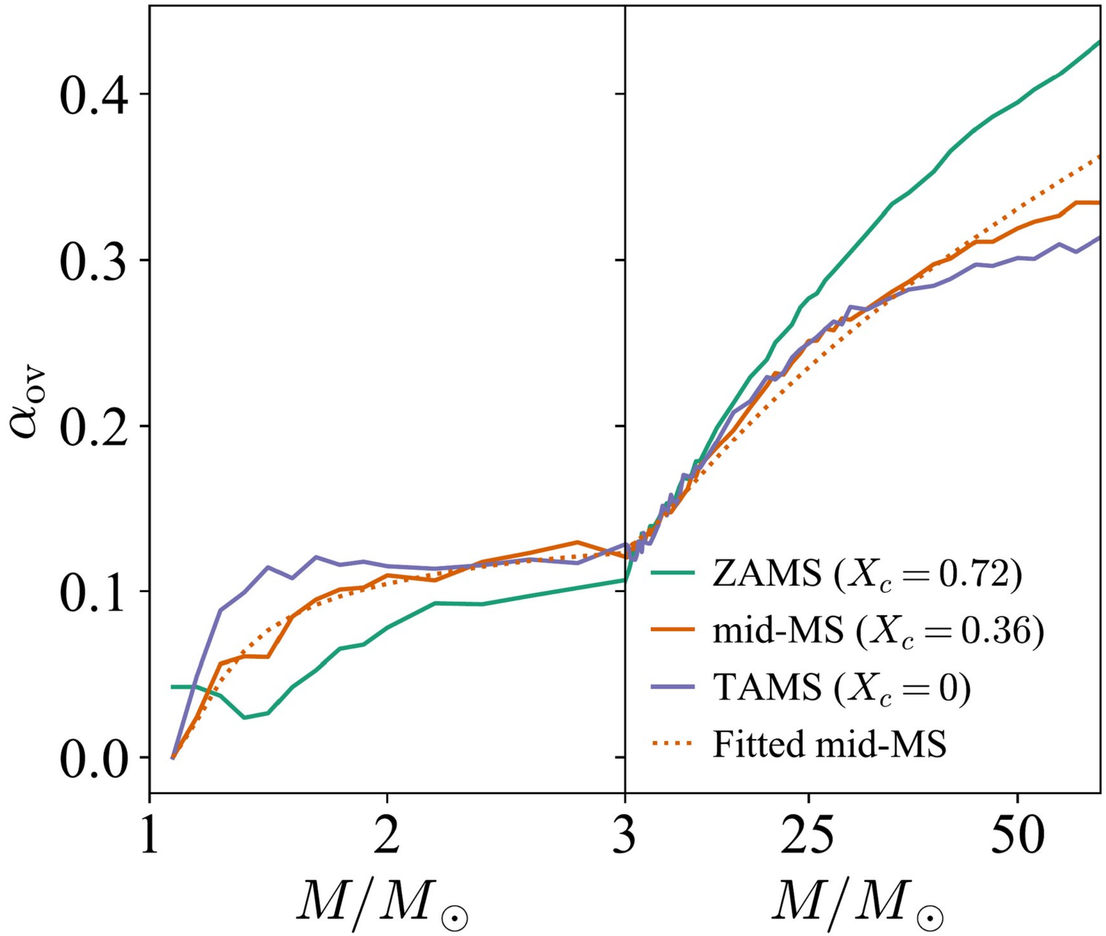
- Anders+ (2022a): Generalisation of models of Roxburgh (1989) and Zahn (1991) calibrated using 3D simulations.
- Main limitations:
  - Fixed profile of radiative conductivity.
  - Boussinesq approximation.
  - Plane-parallel geometry.





Convective penetration

- Jermyn+ (2022): Application of the calibrated penetration model to 1D stellar evolution models.



Convective penetration

- Andrassy+ (2024): compressible simulations of penetration in a 15 $M_\odot$ star.
- SLH code.
- Special methods (see Edelmann+ 2021):
  - Implicit time stepping.
  - Low-Mach fluxes.
  - Separate treatment of HSE.
- Modified parameters:

$$L \to bL$$

$$\kappa \to \kappa/b$$

$$10^3 \leq b \leq 10^6$$

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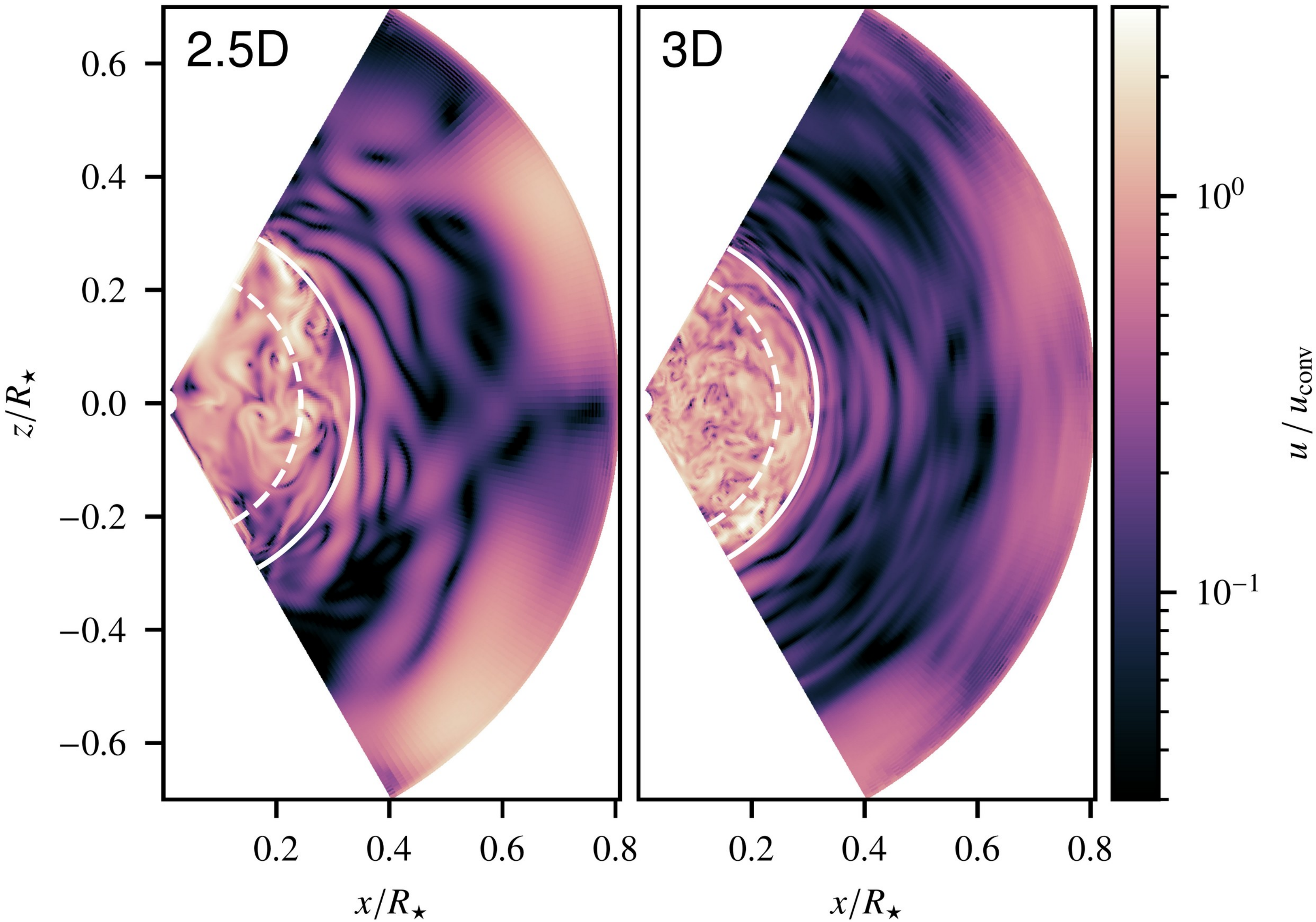

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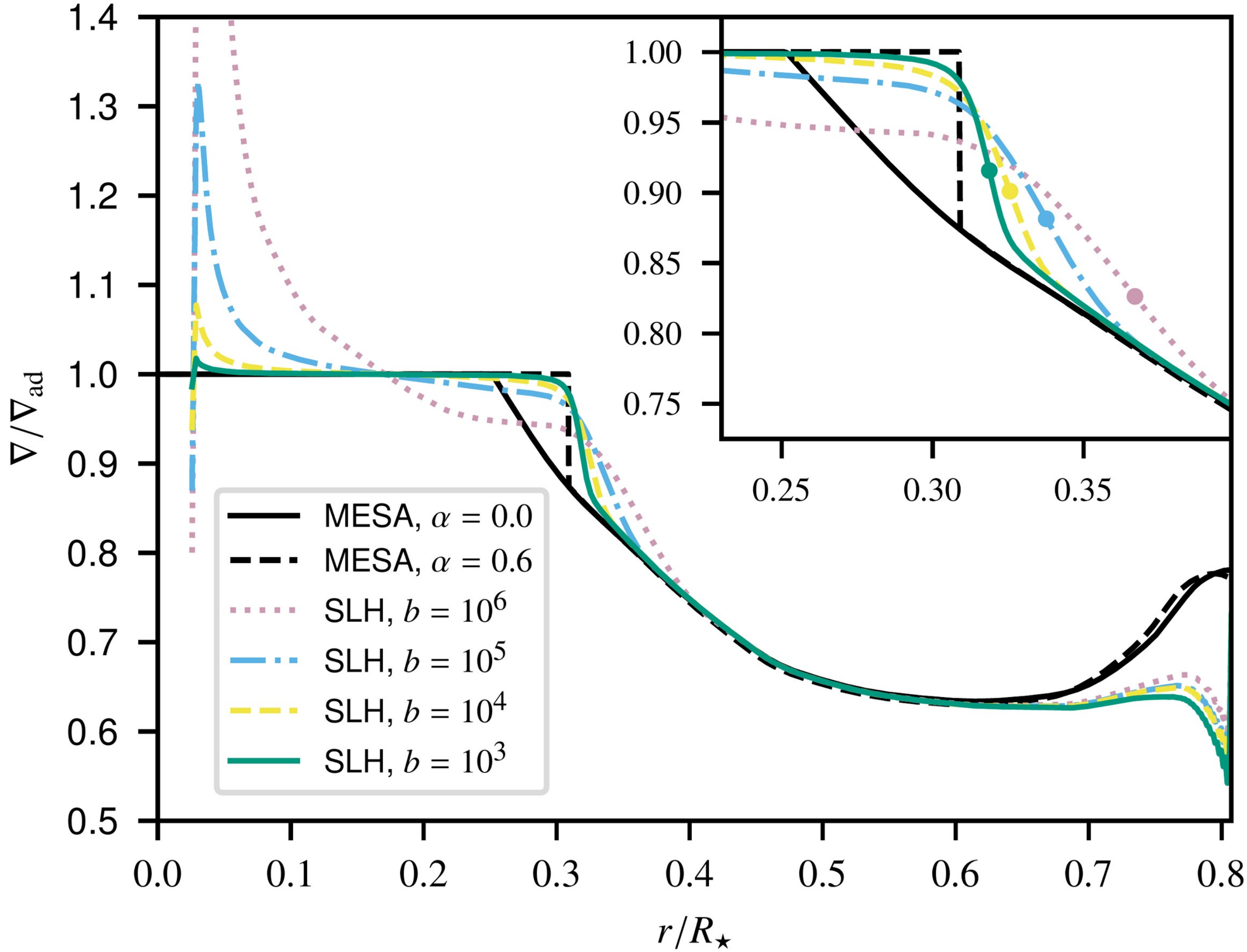

Convective penetration

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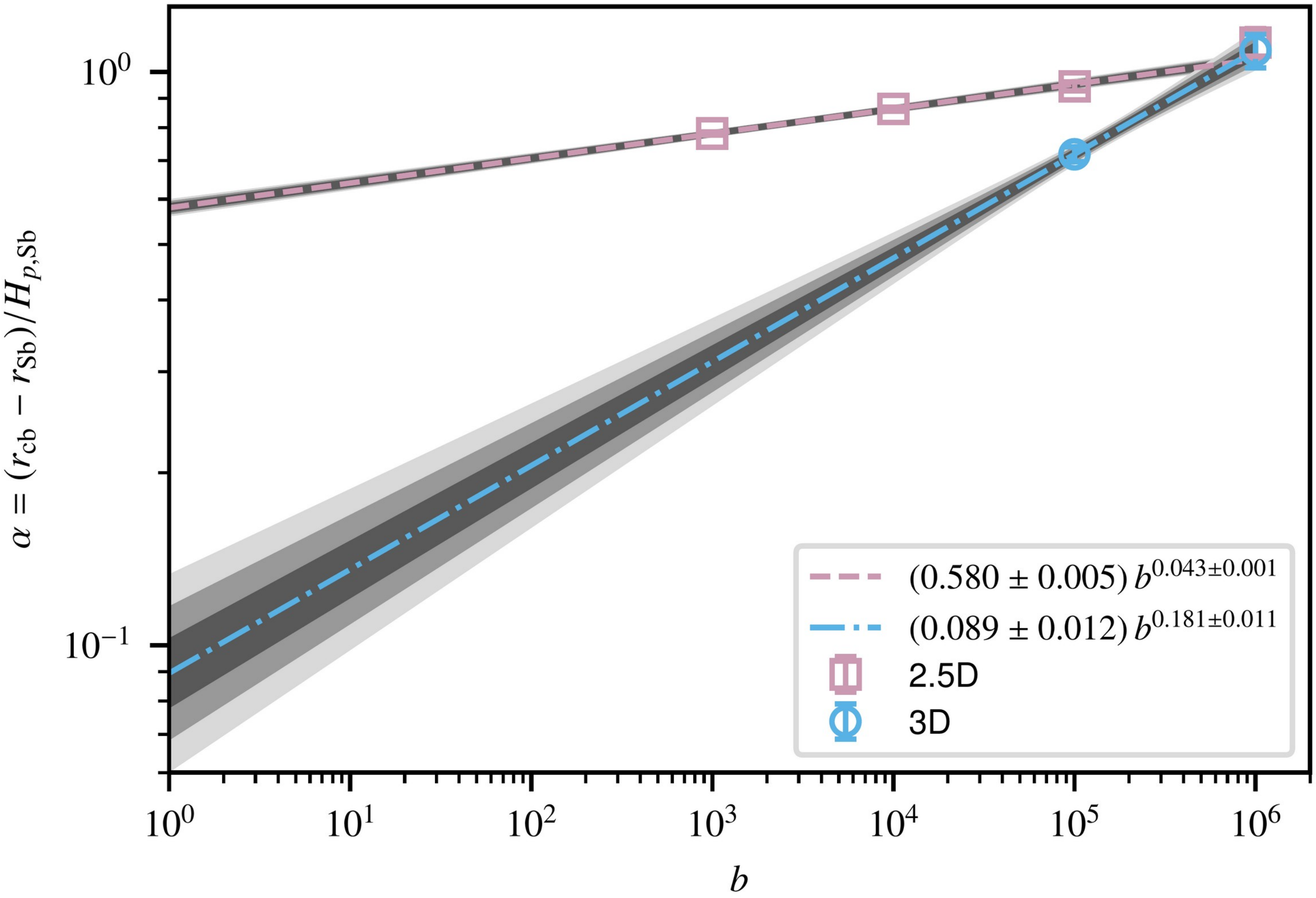
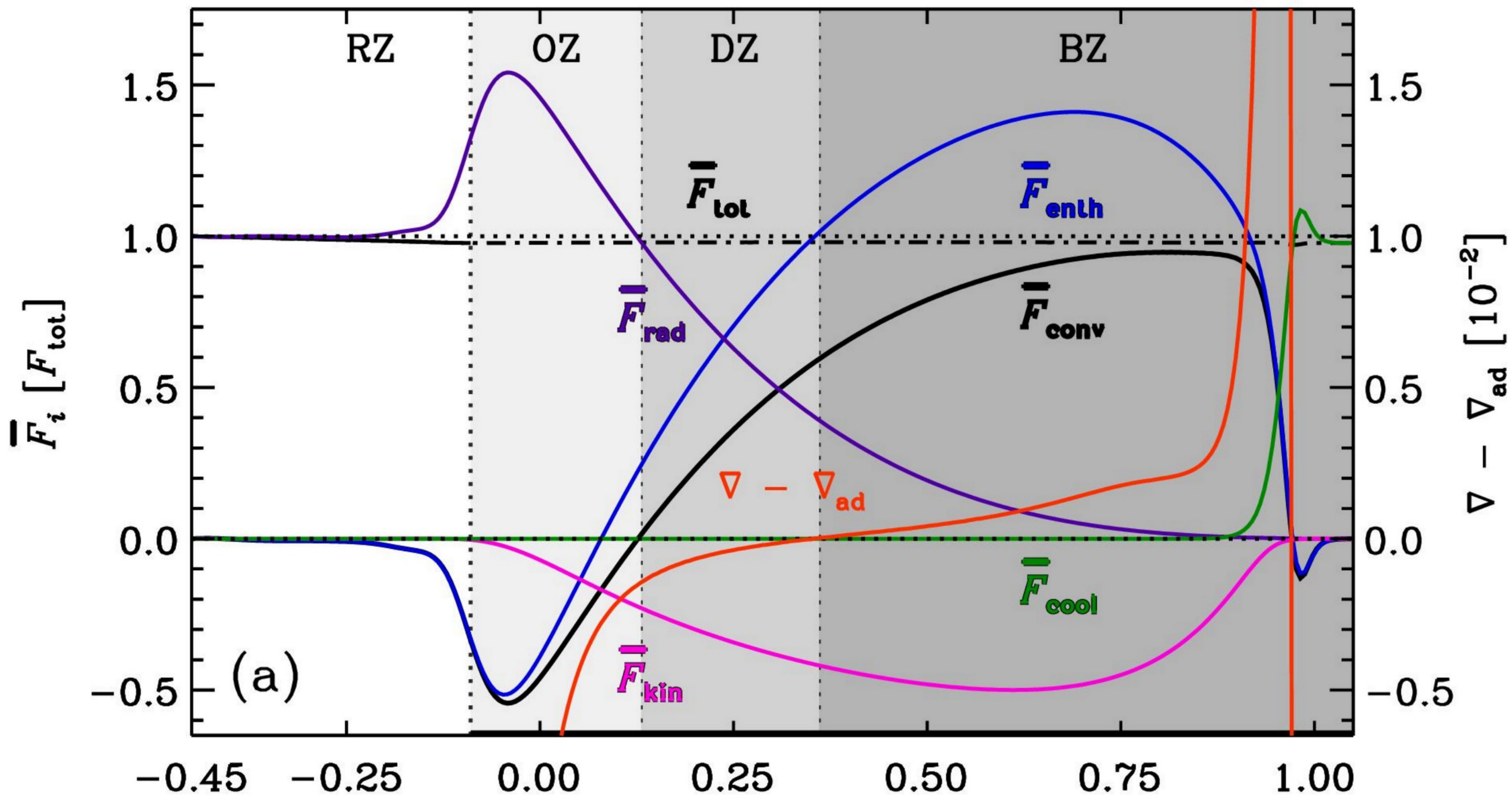
- Detailed comparison with analytical models in progress.



Convective penetration

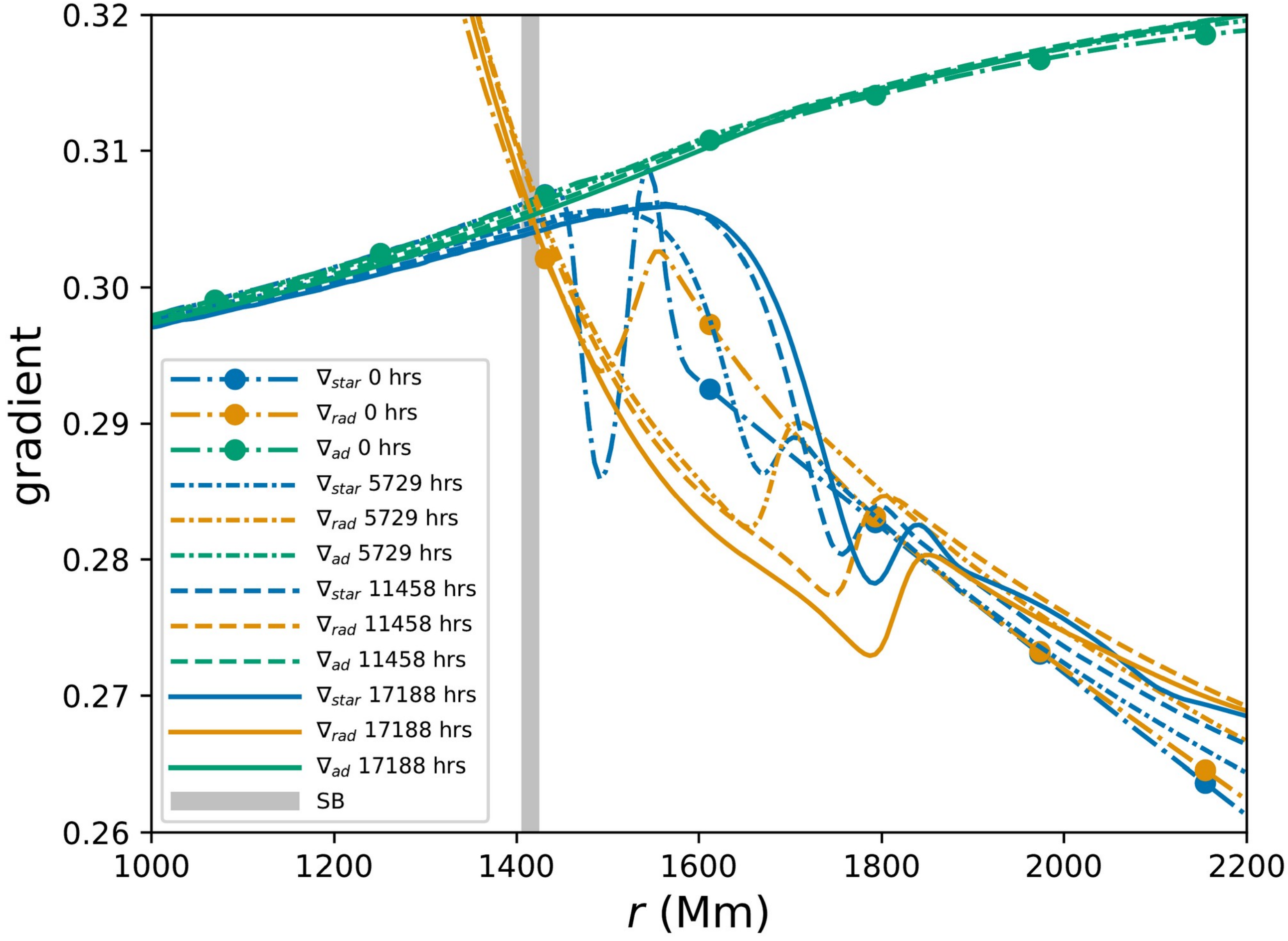
- Several groups actively working on the problem.



Käpylä (2019)

Convective penetration

- Several groups actively working on the problem.
- Complicating factor: composition (but see Anders+ 2022b).



Mao+ (2023)

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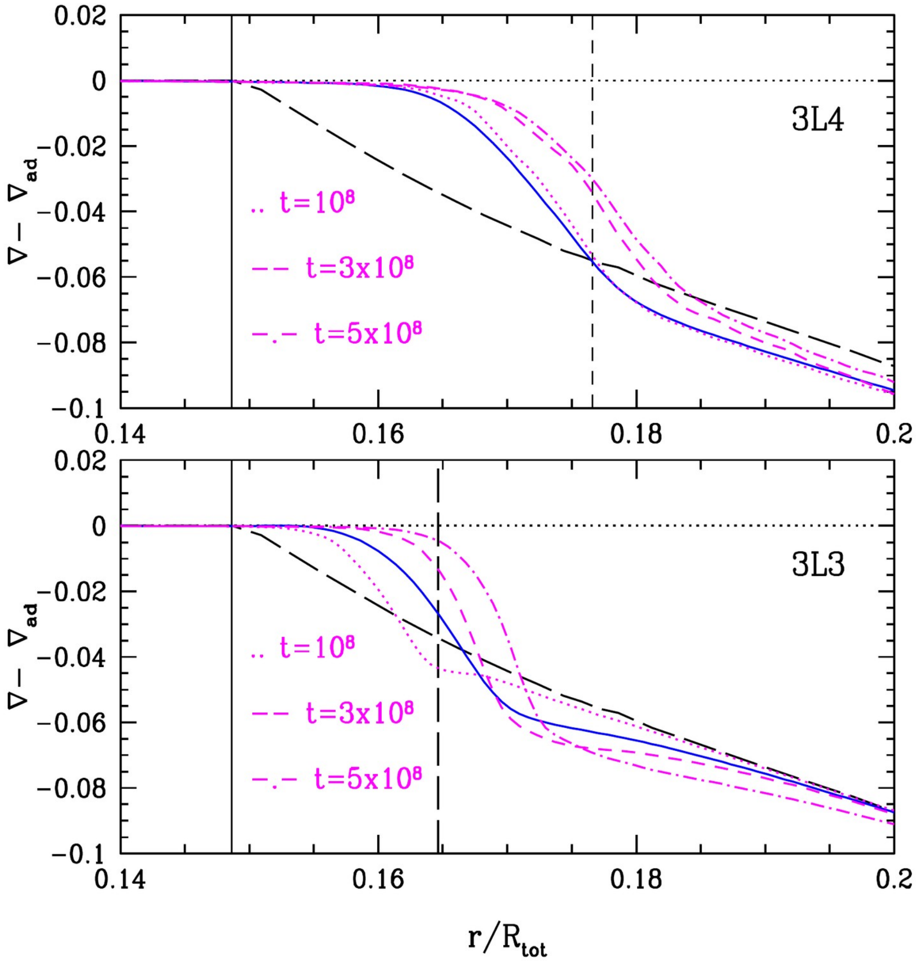
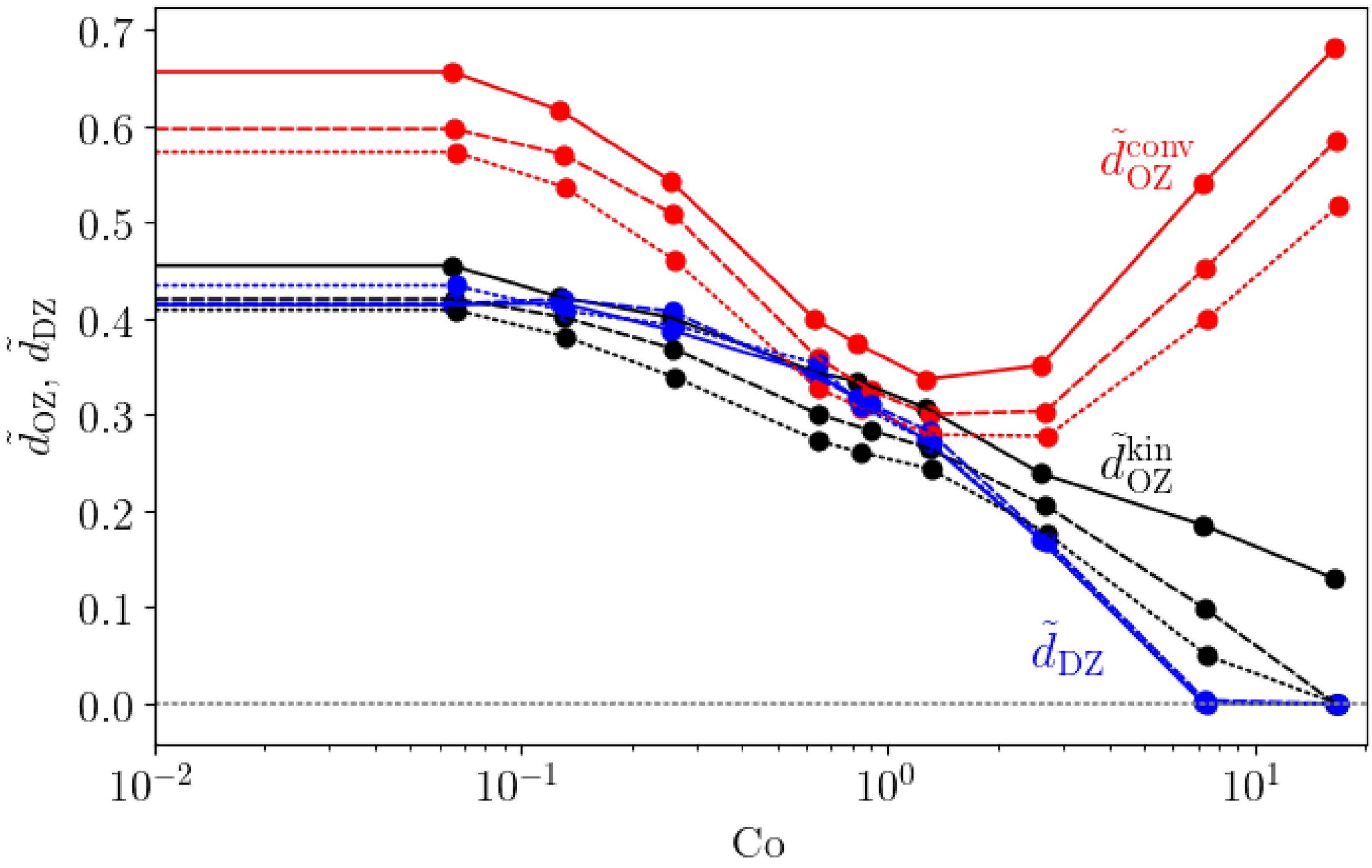


Baraffe+ (2023)

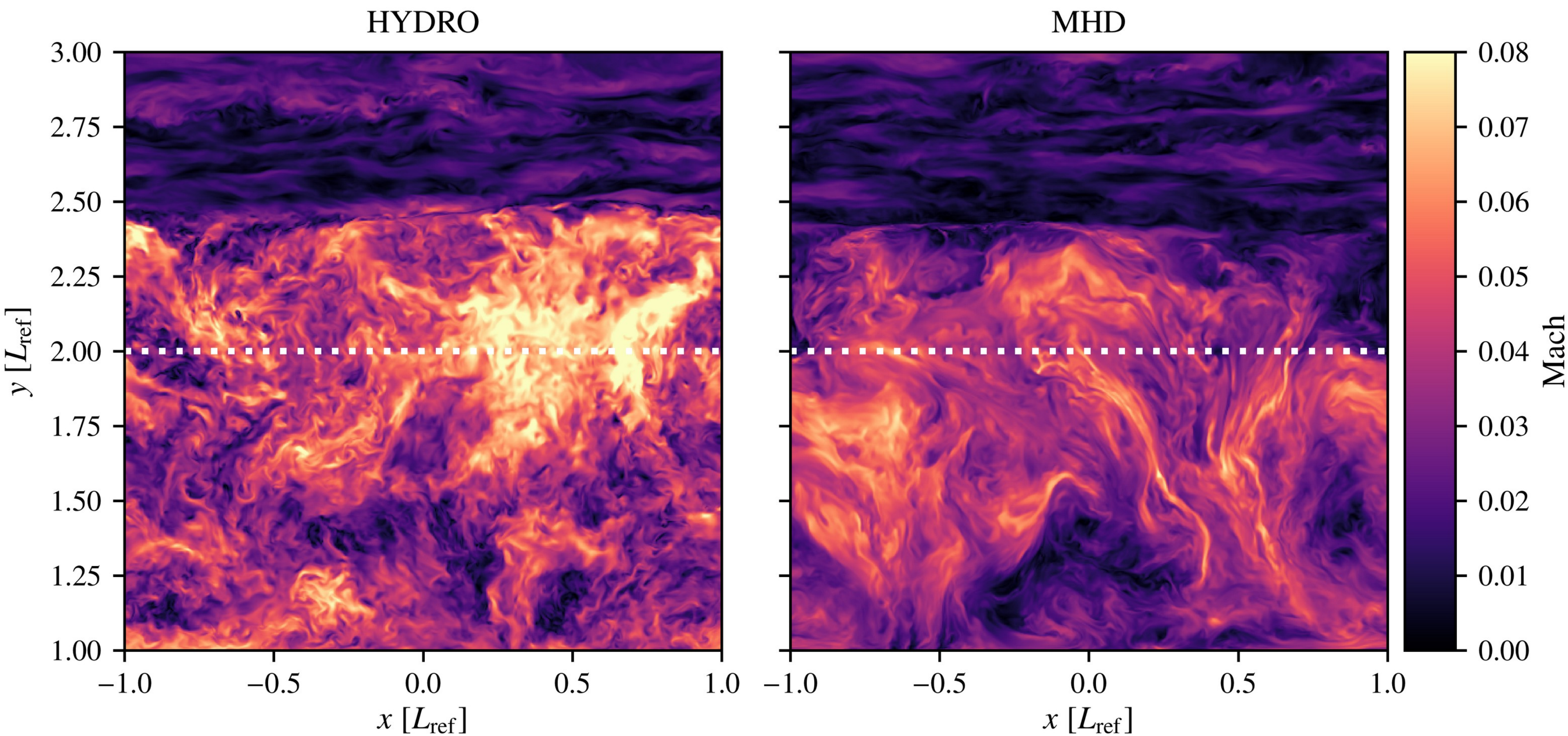
Influence of rotation on convective penetration

- Käpylä (2024): rotation decreases penetration depth.



Influence of magnetic fields on mass entrainment

- Leidi+ (2023): SLH code, statification similar to an O-burning shell.
- Flow patterns very different.



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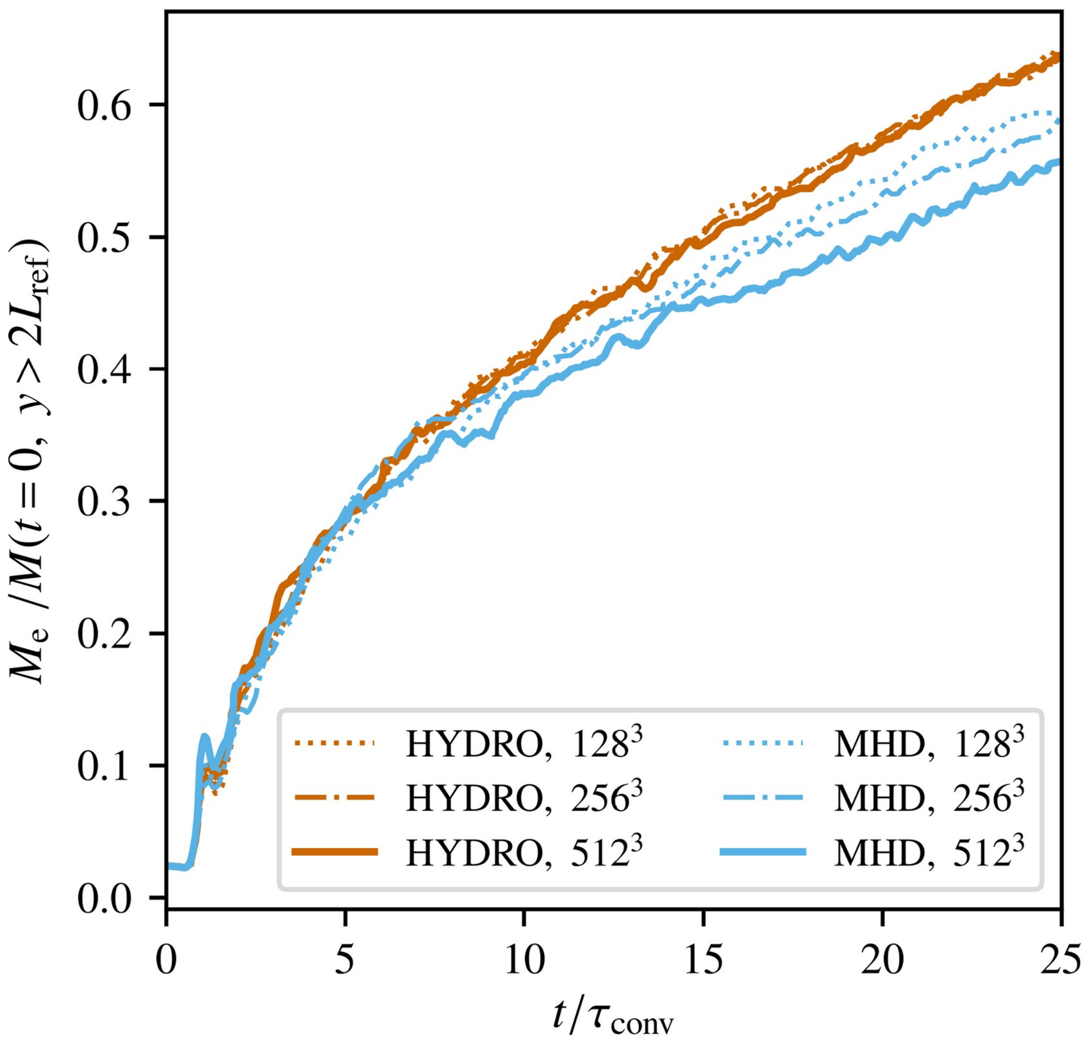
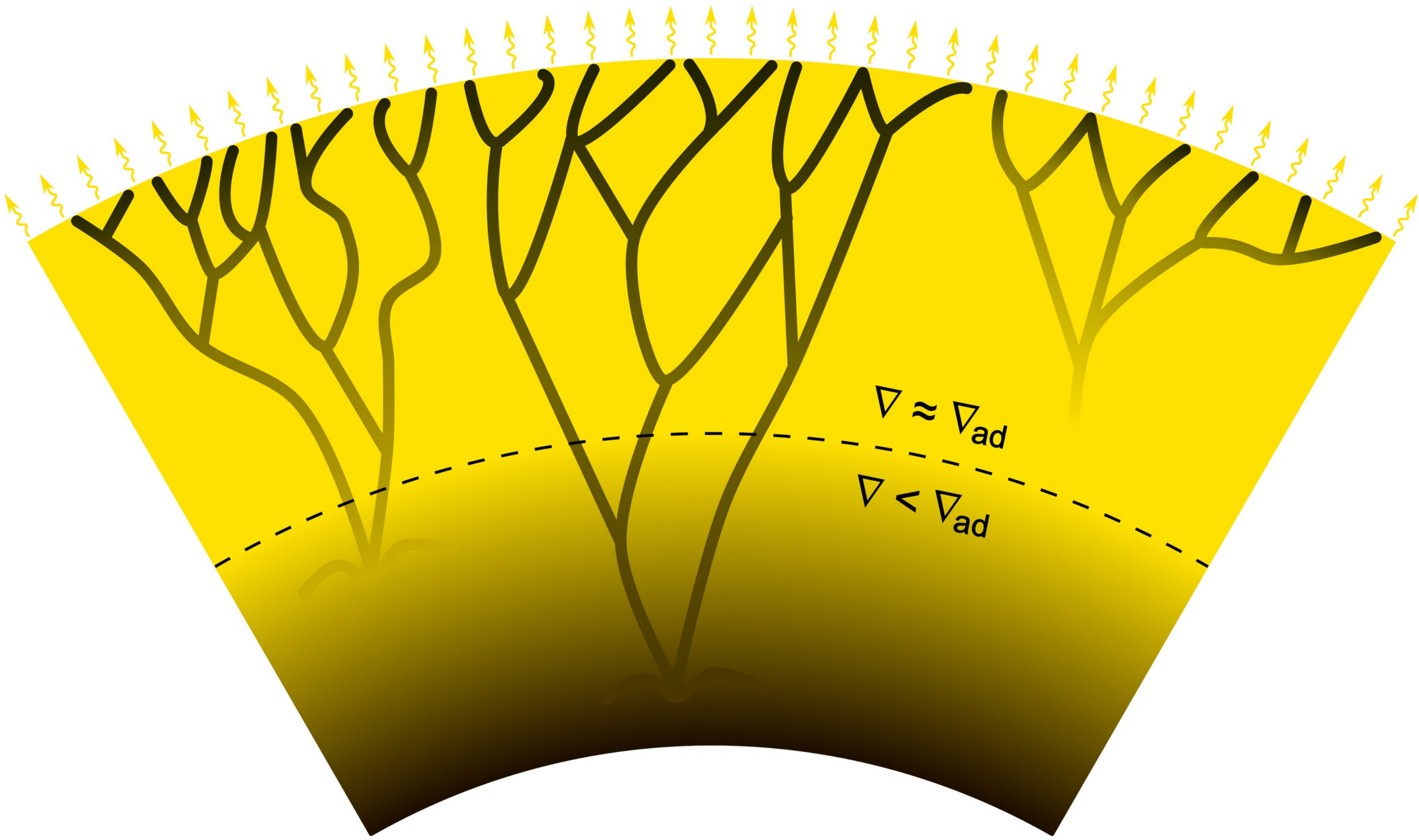


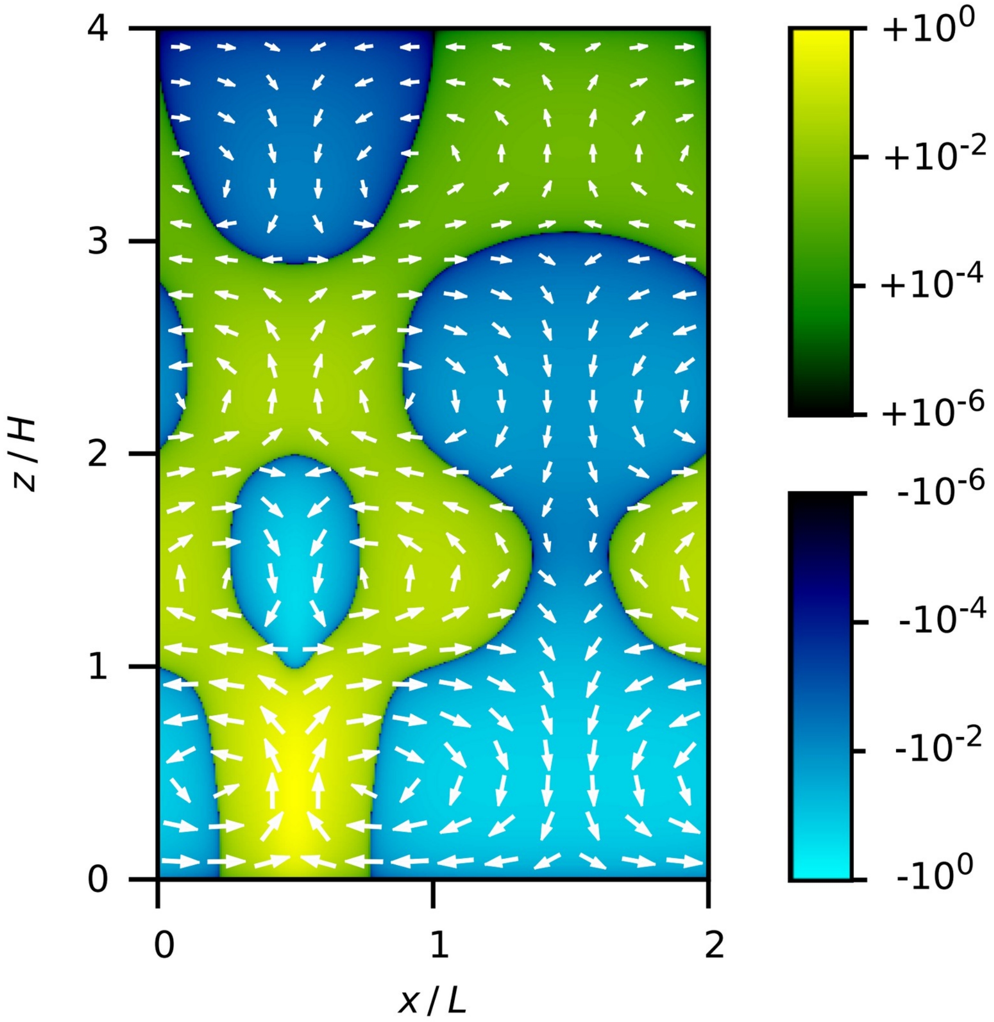
Other processes: convective settling

- Surface-driven convection: entropy rain (Stein & Nordlund 1989, 1998, Spruit 1997).
- Model to explain Li and Be depletion: Andrassy & Spruit (2013, 2015b).



Other processes: differential heating

- Radiation transports temperature fluctuations into a stable stratification.
- Resulting flow studied by Andrassy & Spruit (2015a).



Summary

- Late evolutionary stages:
  - Entrainment regime.
  - Relatively easy to simulate.
- Early evolutionary stages:
  - Penetrative regime.
  - Difficult to simulate.
- Less explored processes:
  - Convective settling.
  - Differential heating.
  - Wave-induced mixing.