Forcing of convection vs. rotation

Flux-based Rayleigh number:

$$\mathrm{Ra_F} = \frac{gH^4F_{\rm tot}}{c_{\rm P}\rho T\nu\chi^2}.$$

Diffusion-free formulation (e.g. Christensen 2002):

$$\mathrm{Ra_F^\star} = \frac{\mathrm{Ra_F}}{\mathrm{Pr}^2\,\mathrm{Ta}^{3/2}} = \frac{gF_{\rm tot}}{8c_{\rm P}\rho T\Omega^3H^2} = \frac{F_{\rm tot}}{8\rho\Omega^3H^3}.$$

$$\mathrm{Pr} = \frac{\nu}{\chi},\ \ \mathrm{Ta} = \frac{4\Omega^2H^4}{\nu^2},\ \ H \equiv c_{\rm P}T/g.$$

Characterize the flux as a velocity and form a *flux Coriolis number*:

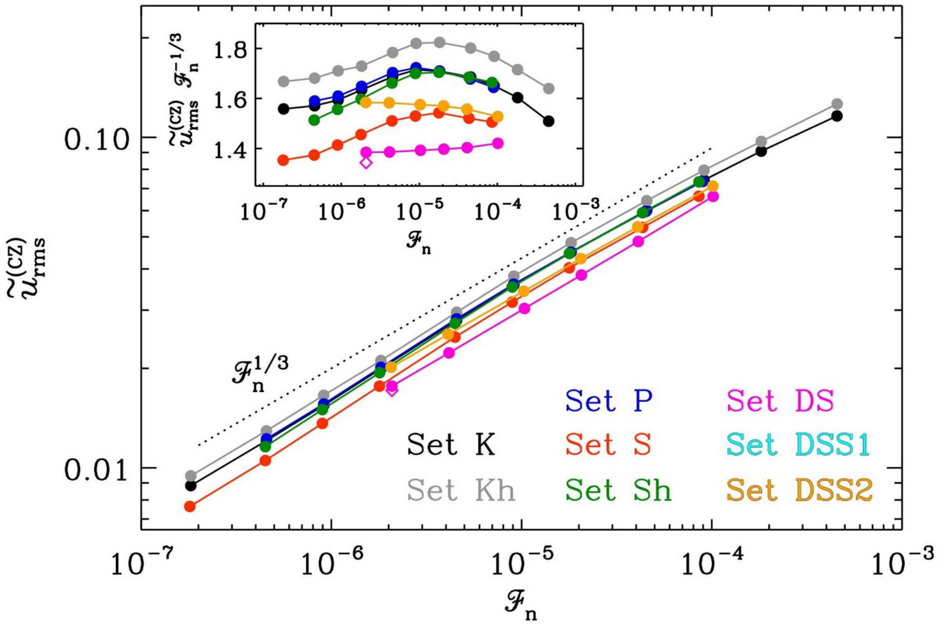
$$F_{\rm tot} = \rho u_\star^3,\quad u_\star = \left(\frac{F_{\rm tot}}{\rho}\right)^{1/3},\ \ \mathrm{Co_F} = \frac{2\Omega H}{u_\star} = 2\Omega H\left(\frac{\rho}{F_{\rm tot}}\right)^{1/3}.$$

Now we see that: $\mathrm{Co_F} = (\mathrm{Ra_F^\star})^{-1/3}$.

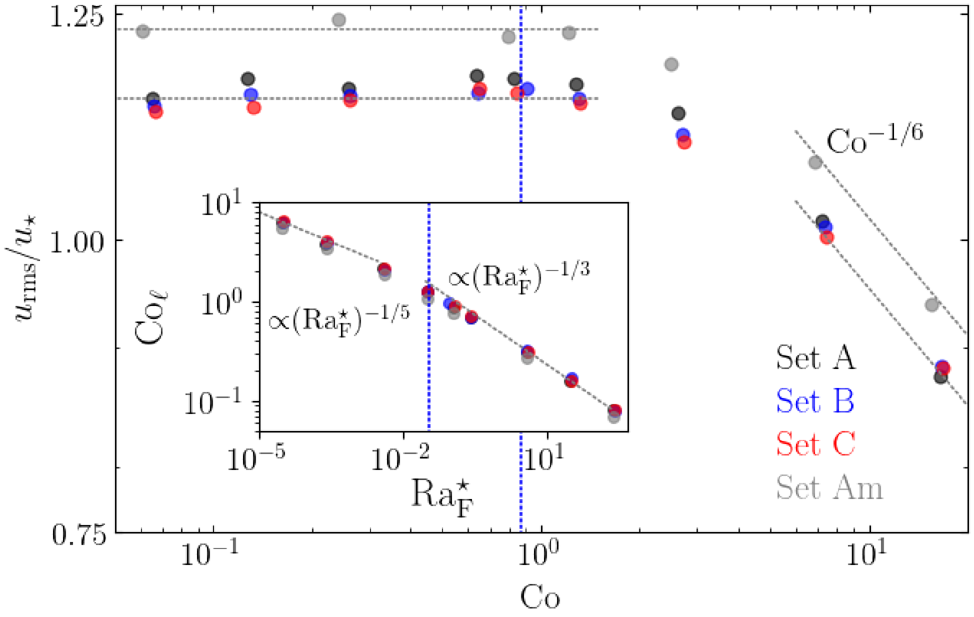
The length scale H can be chosen freely as long as it corresponds to the actual solar value.

For the Sun (base of CZ, with $H = H_{\rm p}$): $\mathrm{Co_F^\odot} \approx 3.1$.

Forcing of convection vs. rotation



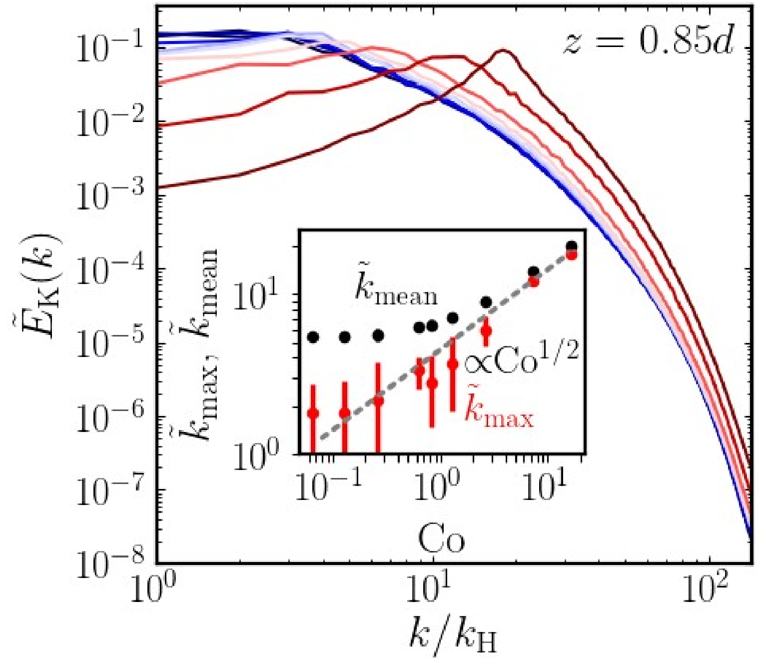
K. (2019), *Astron. Astrophys.*, **631**, 122

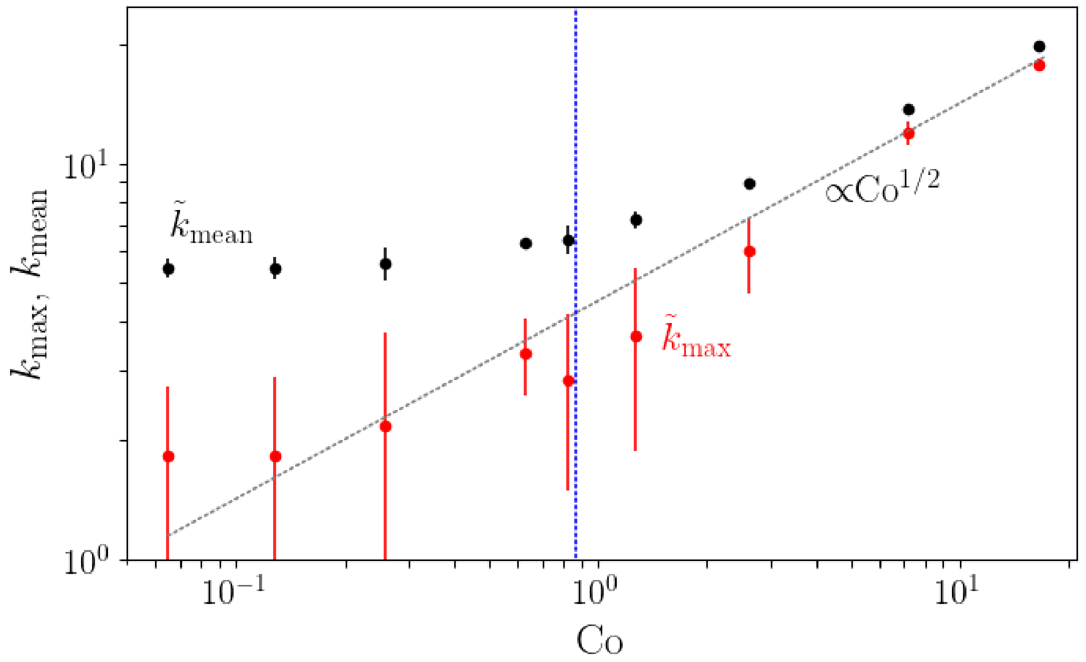


K. (2024), *Astron. Astrophys.*, **683**, 221

$$\mathrm{Co} = \frac{2\Omega}{u_{\rm rms} k_{\rm f}}, \; k_{\rm f} = \frac{2\pi}{d}.$$

Convective scale as a function of Ω

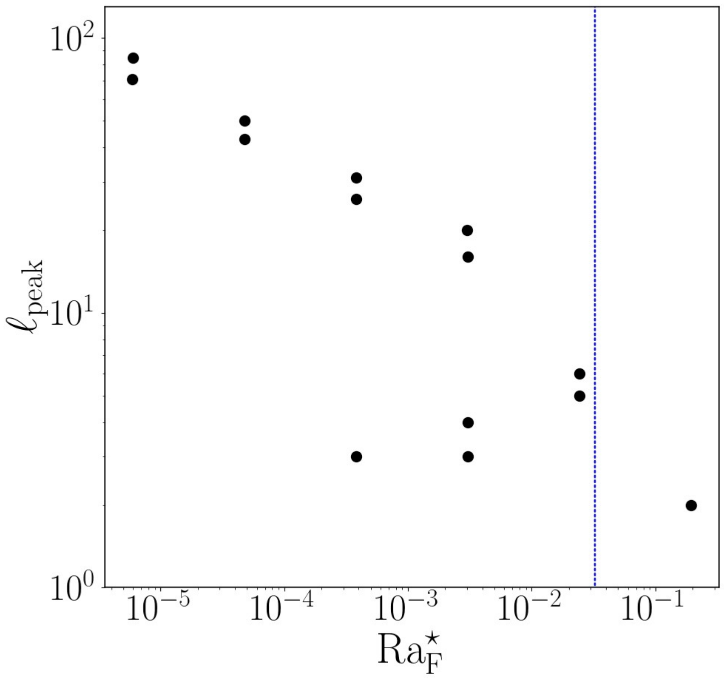


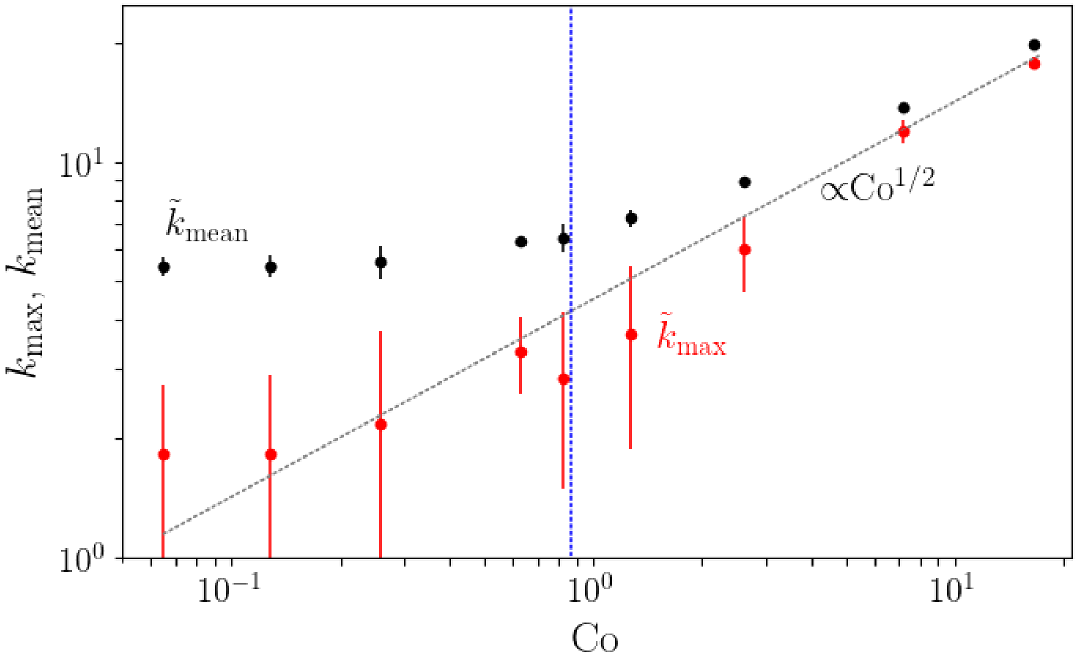


$$\ell_{\rm max} = \frac{2\pi}{k_{\rm max}}, \quad k_{\rm mean} = \frac{\int kE(k)dk}{\int E(k)dk}, \quad \ell_{\rm mean} = \frac{2\pi}{k_{\rm mean}}.$$

$$k_{\rm H} = 2\pi/L_{\rm H}, \quad H_{\rm p} \approx 0.49d \; (= 50 \ {\rm Mm}).$$

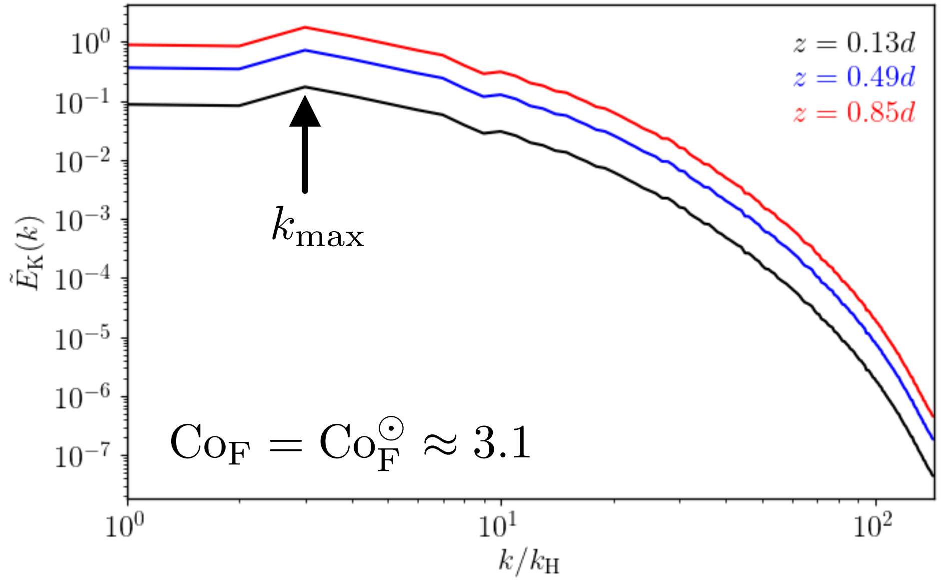
Convective scale as a function of Ω





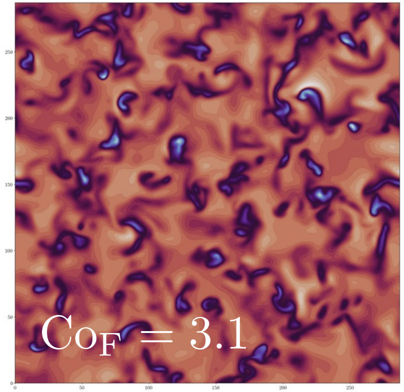
Featherstone & Hindman (2016), *Astrophys. J. Lett.*, **830**, L15

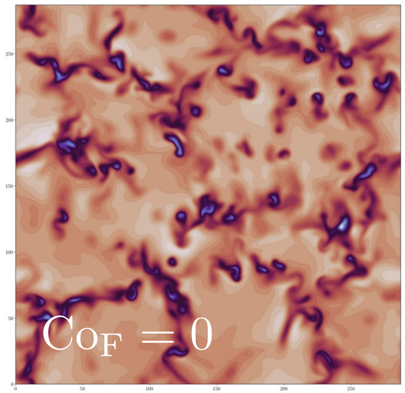
So what about the Sun?



For the solar analogue with $\mathrm{Co_{Flux}} = 3.1$:

$$k_{\max} = 3k_{\rm H}, \quad \ell_{\max} = \frac{4}{3}d \approx 2.7H_{\rm p} \approx 135\ \mathrm{Mm}.$$

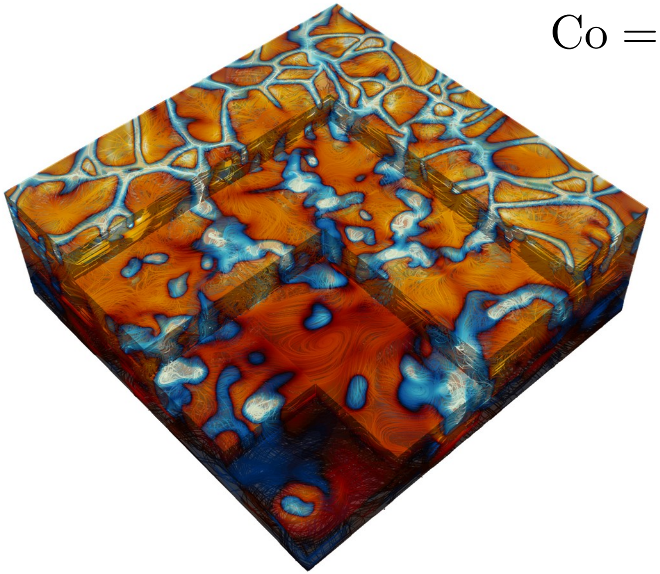


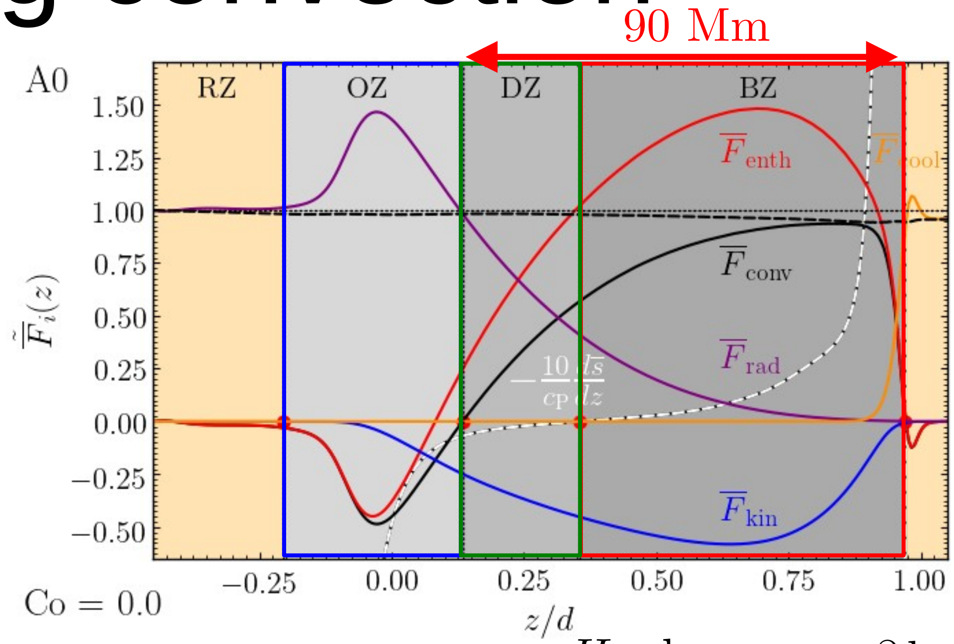


Convective scale in the Sun is affected by rotation but only mildly even in the deep convection zone.

Non-rotating convection

Co = 0.





$$\overline{F}_{\rm conv} = \overline{F}_{\rm kin} + \overline{F}_{\rm enth} = \overline{\tfrac{1}{2}\rho \boldsymbol{u}^2 u_z} + c_{\rm P}\overline{(\rho u_z)'T'}, \quad \Delta\nabla = \nabla - \nabla_{\rm ad} = -\frac{H_{\rm P}}{c_{\rm P}}\frac{{\rm d}s}{{\rm d}z}, \quad \nabla = \frac{\partial \ln T}{\partial \ln p}.$$

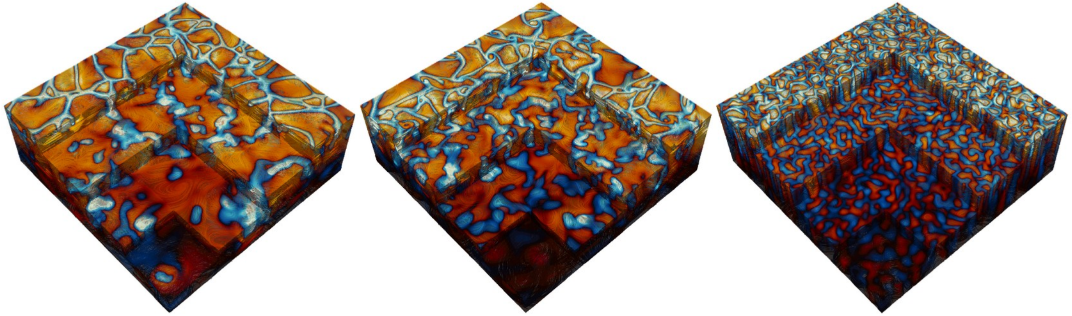
Buoyancy zone (BZ): convective energy flux positive, superadiabatic temperature gradient.

Deardorff zone (DZ): convective energy flux positive, subadiabatic temperature gradient.

Overshoot zone (OZ): convective energy flux negative, subadiabatic temperature gradient.

Numerical simulations

Cartesian setup with the PENCIL CODE; fully compressible equations in rotating frame:



https://github.com/pencil-code

The Pencil Code Collaboration (2021), *J. Open Source Softw.*, **6**, 2807

$$
\begin{aligned}
\frac{D\ln\rho}{Dt} &= -\boldsymbol{\nabla}\cdot\boldsymbol{u},\\
\frac{D\boldsymbol{u}}{Dt} &= \boldsymbol{g} - \frac{1}{\rho}(\boldsymbol{\nabla}p - \boldsymbol{\nabla}\cdot 2\nu\rho\mathbf{S}) - 2\boldsymbol{\Omega}\times\boldsymbol{u},\\
T\frac{Ds}{Dt} &= -\frac{1}{\rho}\left[\boldsymbol{\nabla}\cdot(\boldsymbol{F}_{\rm rad}+\boldsymbol{F}_{\rm SGS}) - \mathcal{C}\right] + 2\nu\mathbf{S}^2,
\end{aligned}
$$

$$\boldsymbol{F}^{\rm rad} = -K\boldsymbol{\nabla}T, \quad K = K_0\rho^{-2}T^{6.5}.$$

$$\boldsymbol{F}^{\rm SGS} = -\chi_{\rm SGS}\rho\boldsymbol{\nabla}s'.$$

$$\mathrm{Re} = \frac{u_{\rm rms}\ell}{\nu} \approx 40,\ \ \mathrm{Pe} = \mathrm{Pr}\mathrm{Re},\quad \mathrm{Pr} = \frac{\nu}{\chi_{\rm SGS}} = 1,\ \ \ell = k_1^{-1}.\quad \mathrm{Co} = \frac{2\Omega\ell}{u_{\rm conv}} = 0\ldots 17.$$