Modeling surface layer convection in a rapidly rotating star

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Modelling convection

Solve the conservation laws

Mass

$$\partial\rho/\partial t \ = \ -\nabla\cdot\rho\mathbf{v}$$

Momentum

$$\partial\rho\mathbf{v}/\partial t \ = \ -\nabla\cdot\rho\mathbf{v}\mathbf{v} - \nabla p + \nabla\cdot\mathbf{\Sigma} - \rho g\mathbf{e}_{\mathbf{k}} - 2\rho\mathbf{\Omega}_{\mathbf{o}}\times\mathbf{v}$$

Energy

$$\partial E/\partial t \ = \ -\nabla\cdot\left(\frac{1}{\gamma-1}\rho T\mathbf{v} + p\mathbf{v} + (\rho v^2/2)\mathbf{v} - \mathbf{v}\cdot\mathbf{\Sigma} - \mathbf{f}\right) - \rho\mathbf{v}\cdot g\mathbf{e}_{\mathbf{k}}$$

$$\begin{aligned}
\Sigma_{ij} &= \mu(\partial v_i/\partial x_j + \partial v_j/\partial x_i) - 2\mu/3(\nabla\cdot\mathbf{v})\delta_{ij} \\
E &= \frac{1}{\gamma-1}\rho T + \rho v^2/2 \\
g &= g_{\mathrm{t}}/r^2
\end{aligned}$$

A Gas Law closes the system

Various types of Convection

1. Laminar convection (lab. experiments)

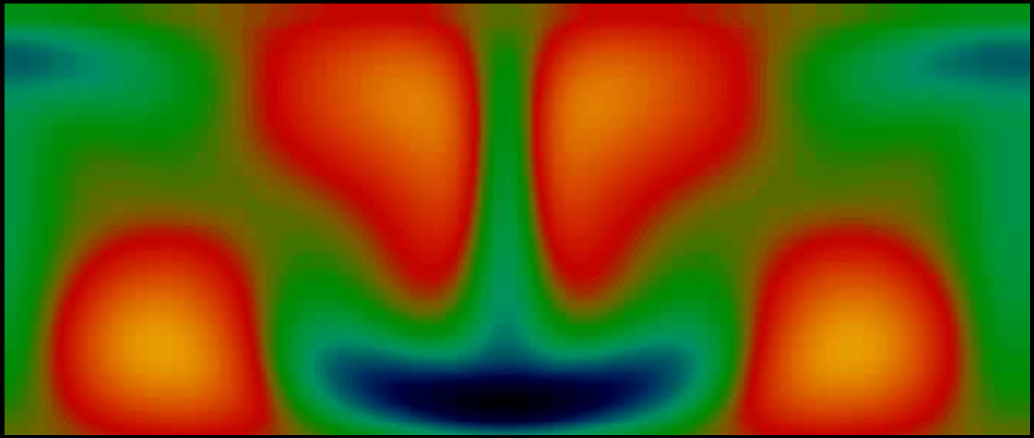
- Laminar (smooth) – only a few different length scales – motion is predictable, resolve all scales.; U~0.1cm/s, L~1cm

Simulate WATER CELL exps by
Gollub & Benson, 1980
'cross-sectional view of rolls'

- Hot blobs of fluid carried by the rolls

– oscillatory instability
– super critical Hopf- bifurcation

- Re = (vel. x length) / (viscosity)



(Robinson and Chan Phys. Fluids, 2003)

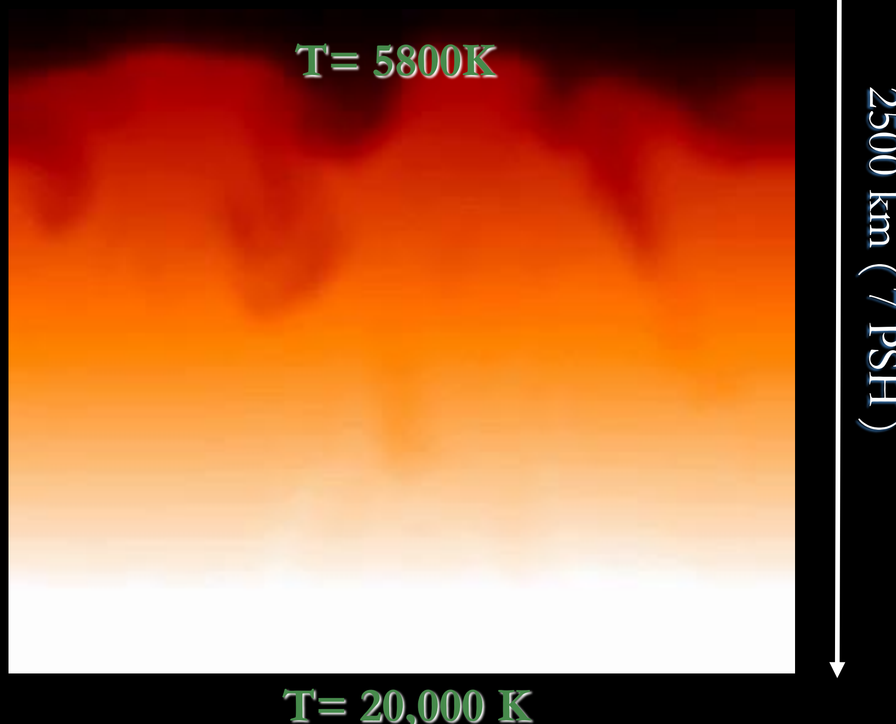
$$\mathrm{Re} = 20$$

Simulation Re = Actual Re
=> Direct Num. Sim. (DNS)

2. Convection near the surface of stars

3D Simulations seem to match observed solar granule size and reduce the discrepancy between observed and computed p-mode frequencies (Spada 2018)

Tanner 2016, Magic et al. 2013, Trampedach 2014



(Robinson et al., MNRAS, 2004)

A vertical cross-section of temperature

$$\mathrm{Re} = \frac{V\rho d}{\mu_{SGS}} \sim 2000$$

$$\mathrm{Re} = 10^{12}$$

Simulation Re <<< actual Re

=> Large Eddy Sim. (LES)

Stellar convection simulations

Global

- Spherical shell (entire Solar Convection Zone)
- Input flux >>> Stellar flux
- Idealised physics (PV=RT), radiation => diffusion approx. (local).
- Model global flow (e.g. differential rotation)

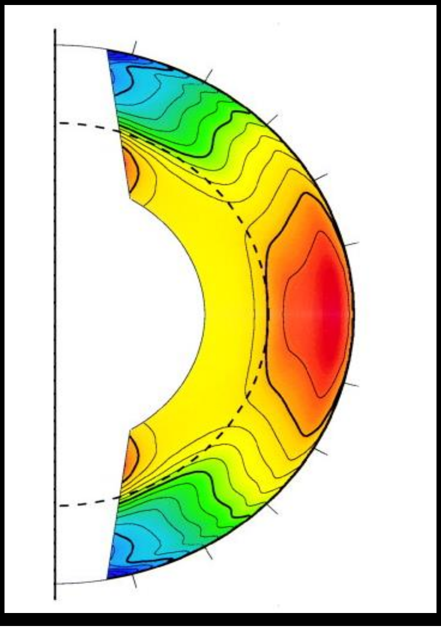
e.g. Robinson & Chan (2001), Miesch et al. (2006)

Local

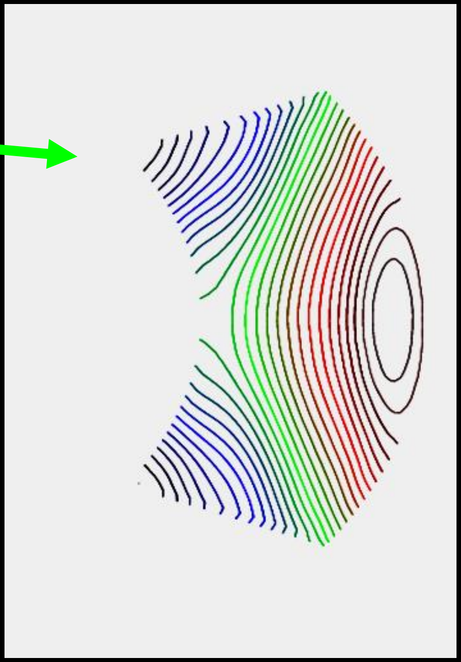
- Cartesian box at top of star (~1000km in depth)
- Input flux = Stellar flux
- Realistic physics (EOS tables), 3d radiative transfer (3d Eddington/ray integration)
- Model small scales (e.g granules))

e.g. Stein & Nordlund(2006), Robinson et al. (2006), Beeck et al. (2012), Tanner et al. (2016)

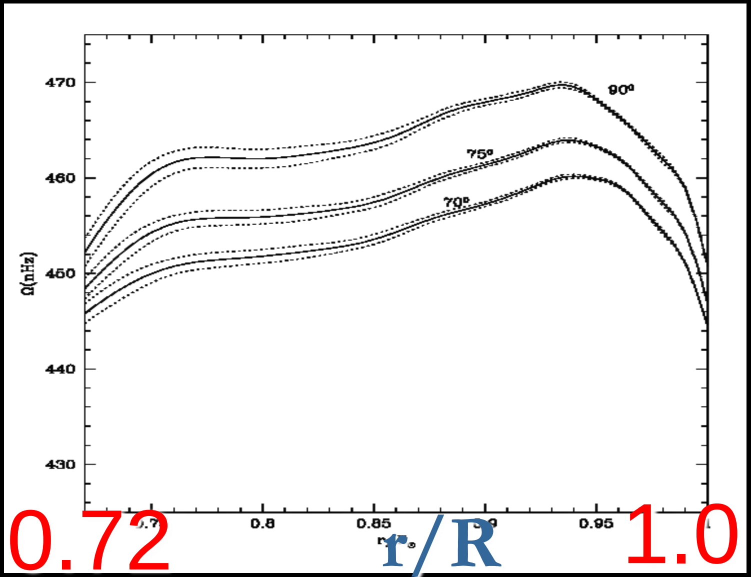
Observations vs global simulations

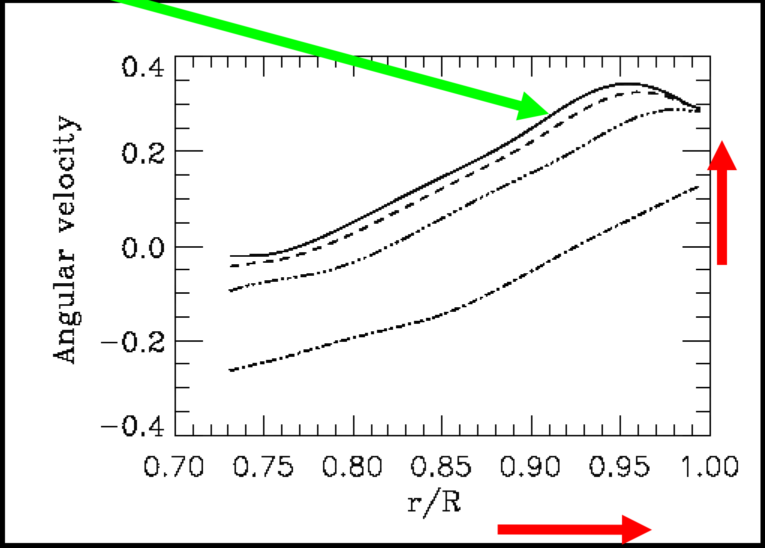


Figs. from Robinson & Chan, (2001) fully compressible shell



Bump





To the equator

To the surface

Realistic stellar surface convection (small box simulations)

- Standard Solar model, 1D Yale Stellar Evolution model (Guenther & Demarque 1997) used to compute initial stratification.

- Realistic Physics. Ferguson et al. (2005) low temperature opacities, OPAL opacities and OPAL 2005 Equation of State. Hydrogen and Helium ionizations zones included.

- LES of full Navier Stokes equations in a small box (g = constant and no rotation) located in the vicinity of the photosphere (Kim & Chan 1998). Use same opacities and equation of state as in 1D stellar model.

- Radiative energy transport modeled by diffusion approximation in deep layers and 3D Eddington approximation in shallow regions (we assume a gray atmosphere,
  [note: Tanner et al. (2012) compares 3d Eddington and ray integration methods]

- Vertical walls periodic. Horizontal walls free slip and impenetrable (closed box).

Prior to computing statistics, the simulations must be in equilibrium:

1. Thermally relaxed:

$$\tau > \frac{\int edz}{Input\ Flux}$$

2. Properly Mixed : (angled brackets denote instantaneous horizontal average) – condition met at every level

$$\langle \rho w \rangle \leq 0.001$$

How reliable are the small box simulation results?

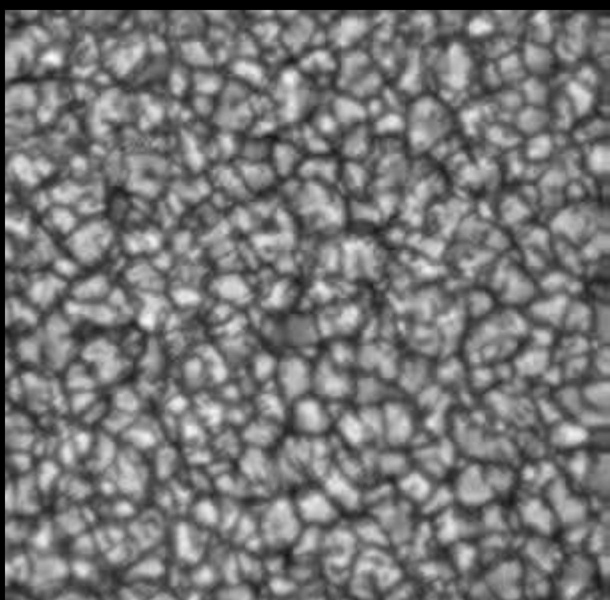
- Compare to observations
- Compare with other 3D Radiative Hydrodynamical models (Kupka, 2008)

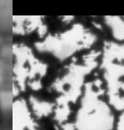
Observed and simulated solar granules

Intensity

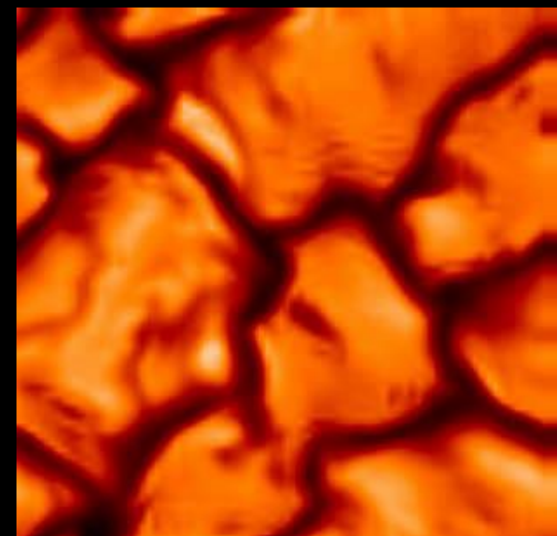
20,000 km

20,000 km





Surface Temperature (box width=4000km)



3800 km

Vertical velocity

(20 minutes or about 2 turnover times)

Compare RHD simulations

Simulation	Vertical boundary condition	RADIATION	Dx (km) Dz (km)	Size(Mm)	Grid
1. CKS-D	CLOSED	Gray	52 17.5	2.7x2.7x2.8	58x58x170
2. CKS-2007	CLOSED	Gray	35 15	4x4x3	117x117x190
3. C05BOLD-High Res	OPEN	Non-gray (5 bin)	28 12-28	11x11x 3	400x400x165
4. CO5BOLD DEEP	OPEN	Gray	56 21	11x11x 5	200x200x250
5. Nordlund & Stein	OPEN	Non-gray (4 bins)	40 20-40	6x6x3	150x150x150

1, Kurutz, GN93, 2 GS98, 3-5 GN98 abundances.
CKS=Chan,Kim,Sofia; C05BOLD=M. Steffen et al.

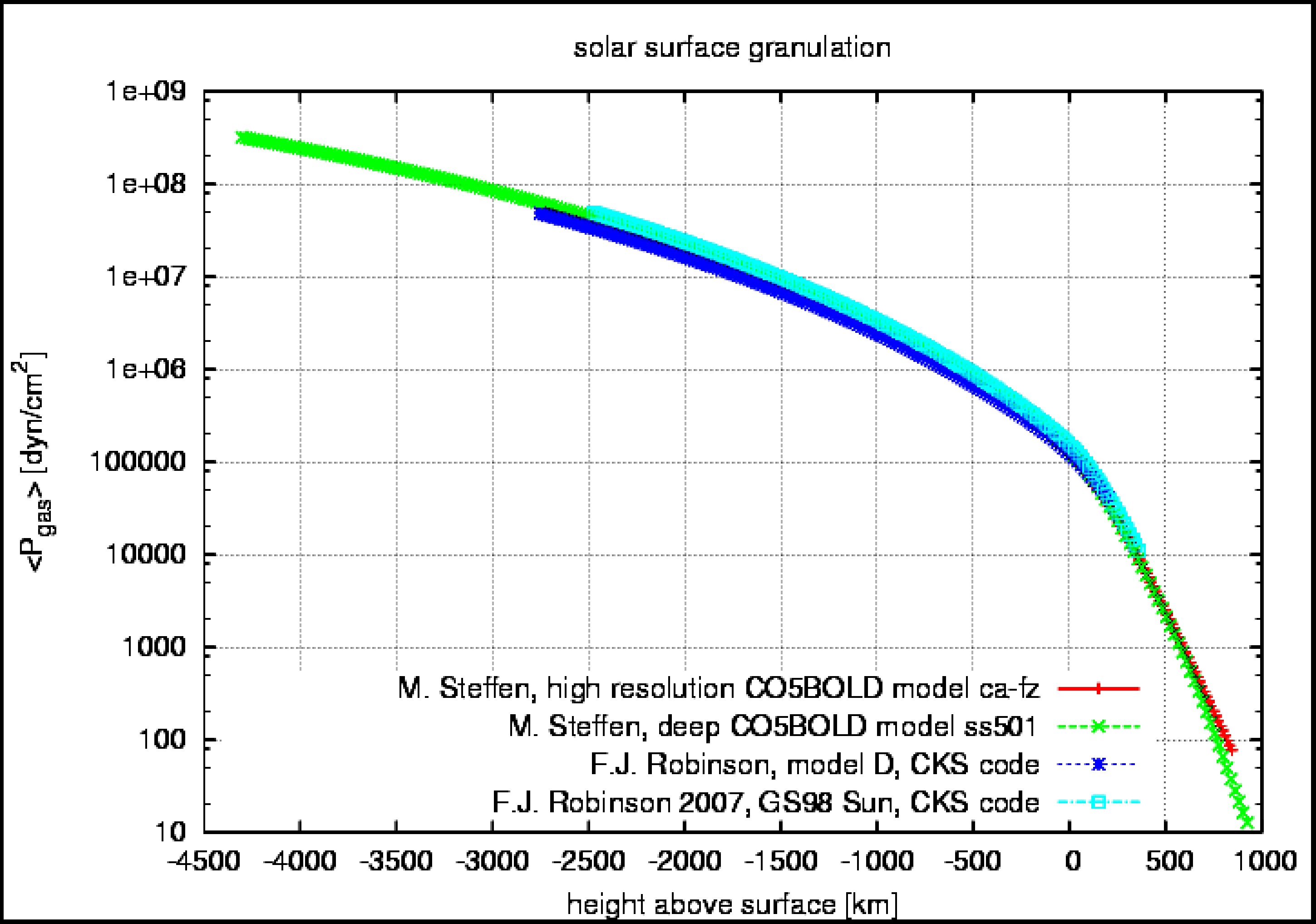


Figure courtesy of F. Kupka

Temperature fluctuation

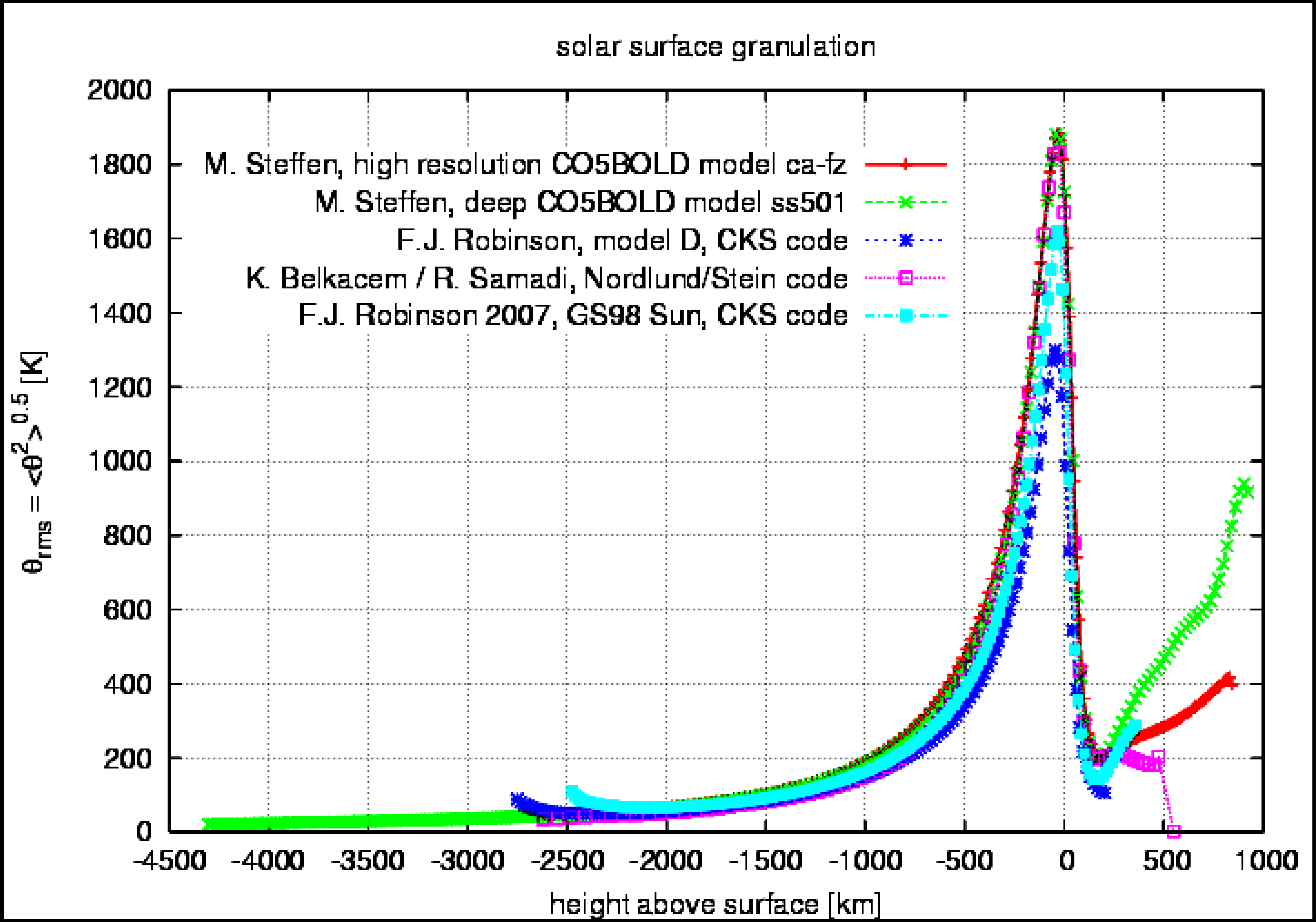


Figure courtesy of F. Kupka

r.m.s. vertical velocity

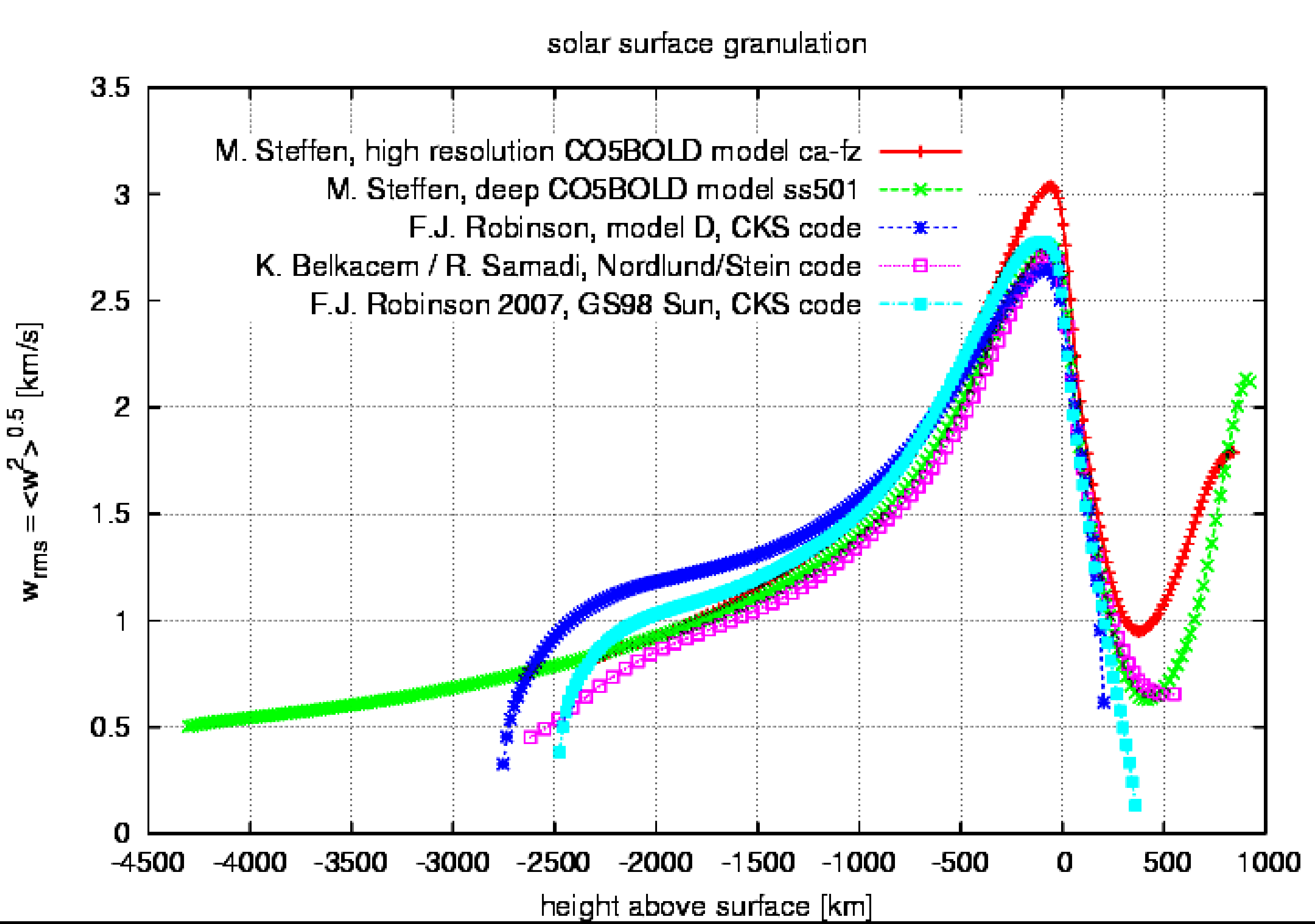


Figure courtesy of F. Kupka

Convective flux

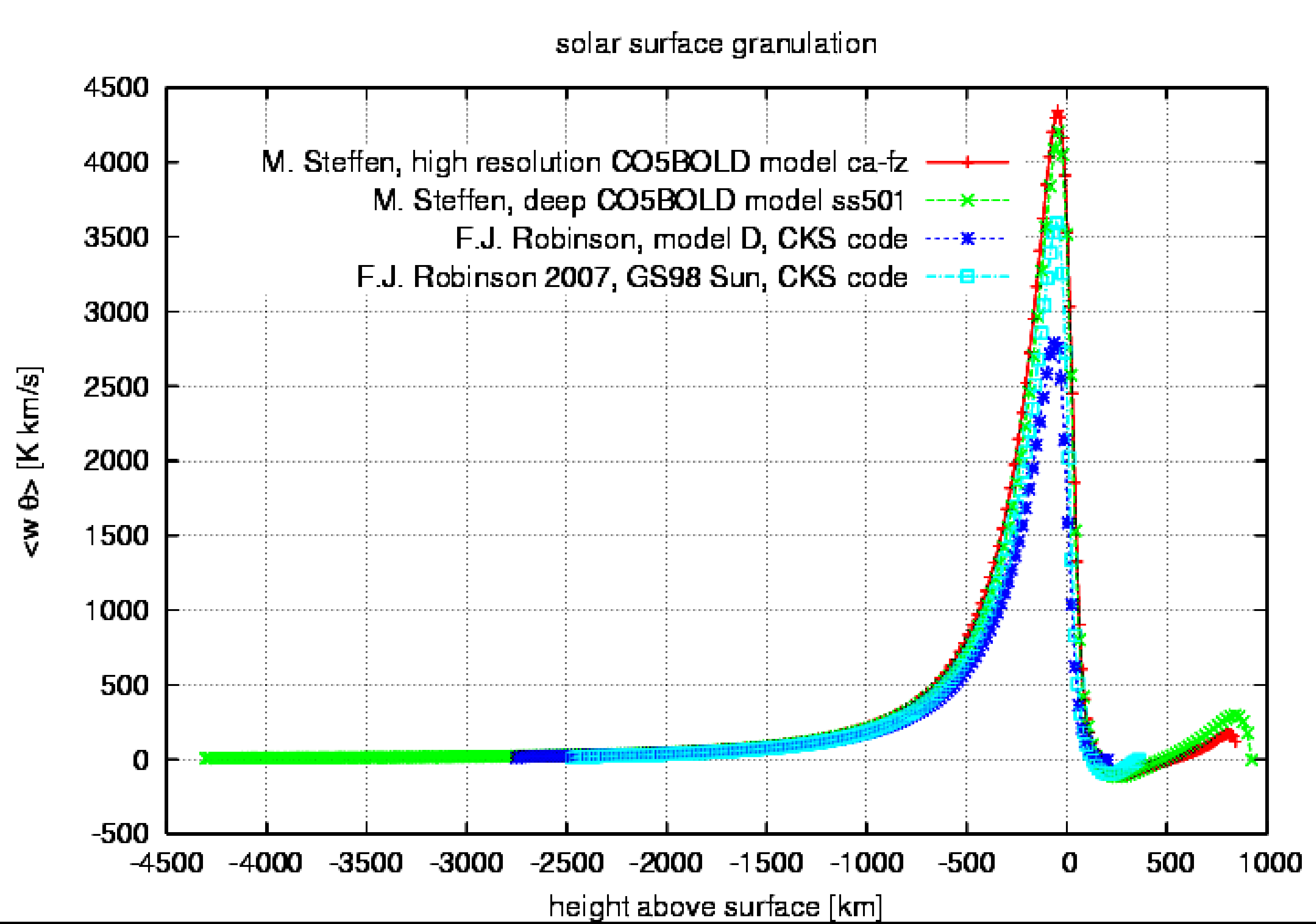
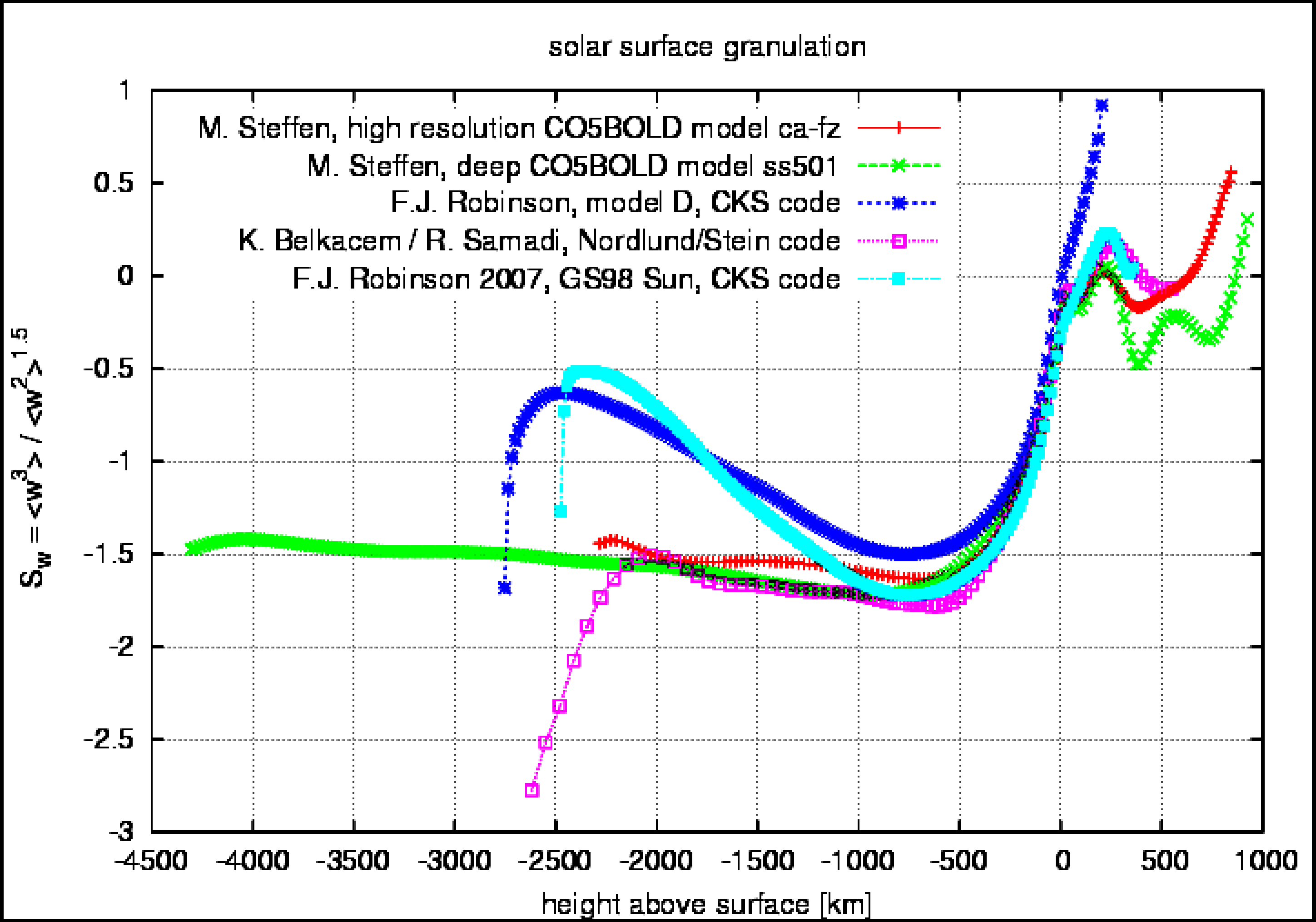


Figure courtesy of F. Kupka

Velocity Skewness



What useful information can be extracted from the simulations ?

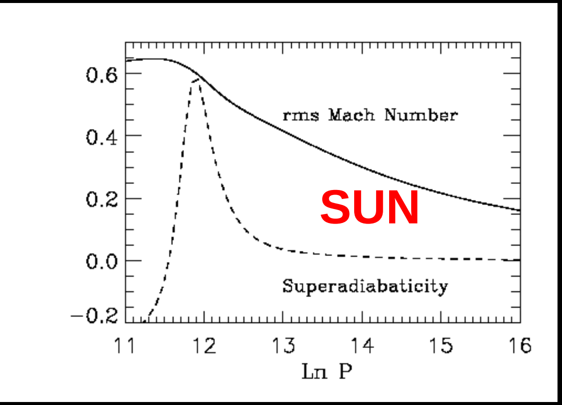
1. Improve Mixing Length theory in the surface layers of stars. (Tanner et al. 2016, Spada et al. 2018, Arnett 2018)
2. Improve models of tidal dissipation. (Penev et al. 2009, 2012, 2013)
3. Use simulations to test turbulent closures in stellar models. (Kupka and Robinson 2007, Kupka 2017)
4. Used as model atmospheres to determine stellar metalicities (Caffau 2008, Joergensen 2019)
5. Examine effect of f-plane rotation on convection in fast rotators (more recent work)

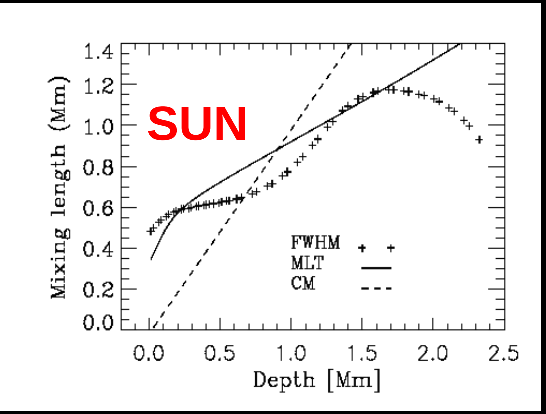
1. Testing MLT

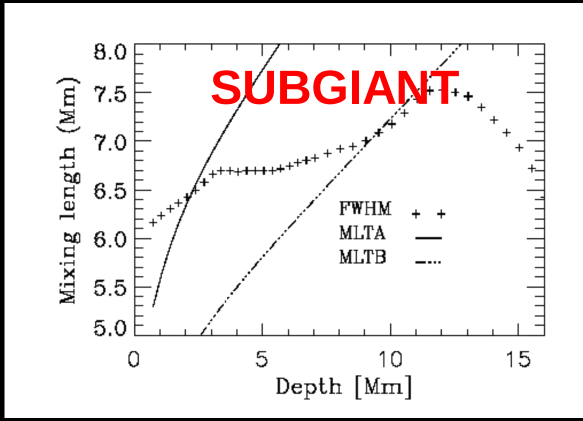
First two frames are for present Sun, lowermost frame is for the Sun at 11.6 billion years.

- We use the FWHM of C[w' w'] as the simulation mixing length.
- MLT is the mixing length as a constant multiple of the local pressure scale height.
- CM prescribes mixing length as distance to convection surface.
- Trampadech and Magic results

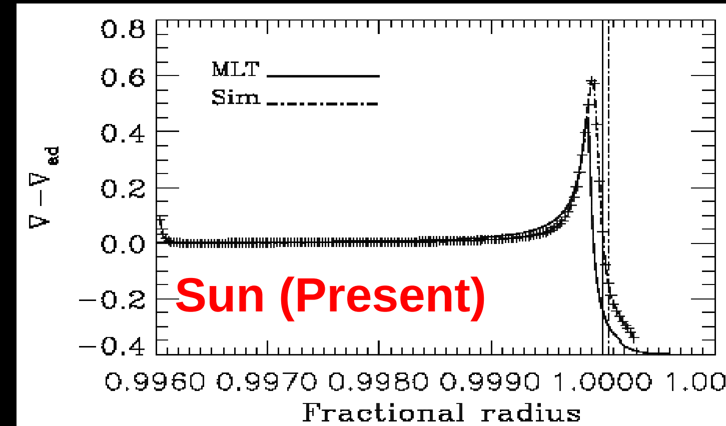
- **Simulations suggest MLT is a poor approximation in the SAL – particularly in more evolved models**

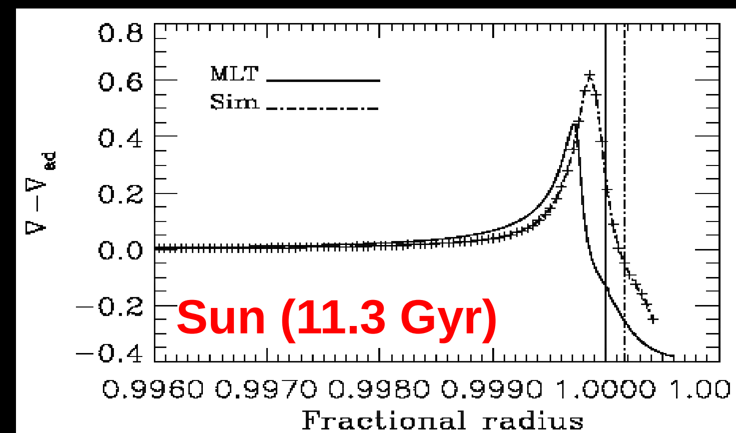


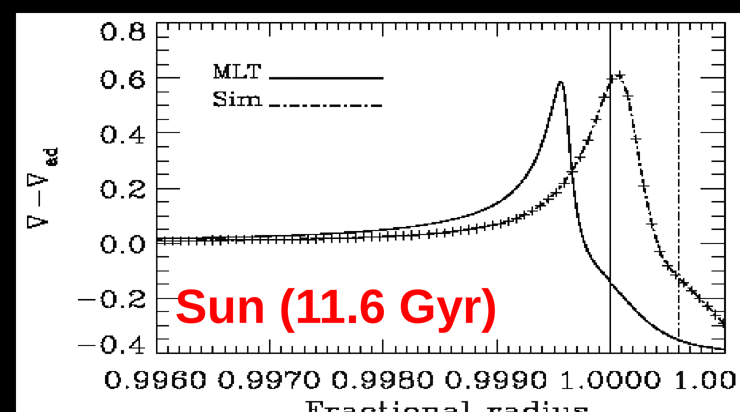




Model	Age (Gyr)	$\log T_{eff}$	$\log g$	Size (Mm)	Grid
S	4.55	3.761	4.44	$5.4^2 \times 2.8$	$114^2 \times 170$
SG1	11.3	3.704	3.75	$13.6^2 \times 9$	$58^2 \times 120$
SG2	11.6	3.698	3.37	$46^2 \times 23$	$58^2 \times 140$



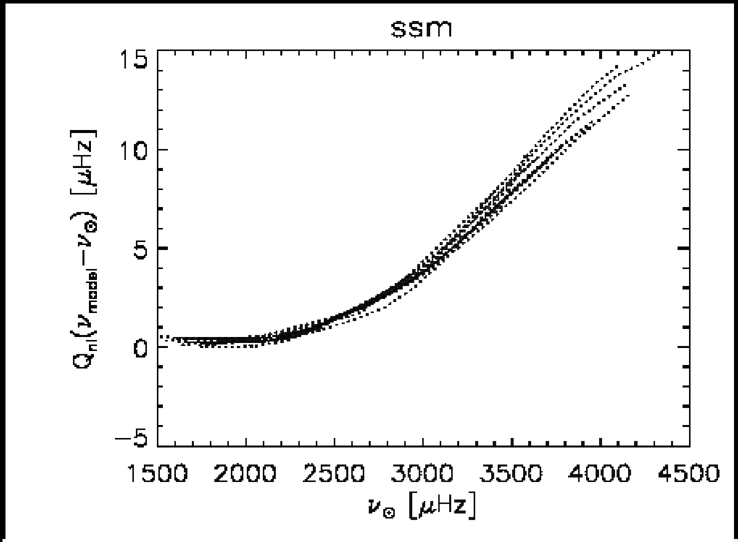


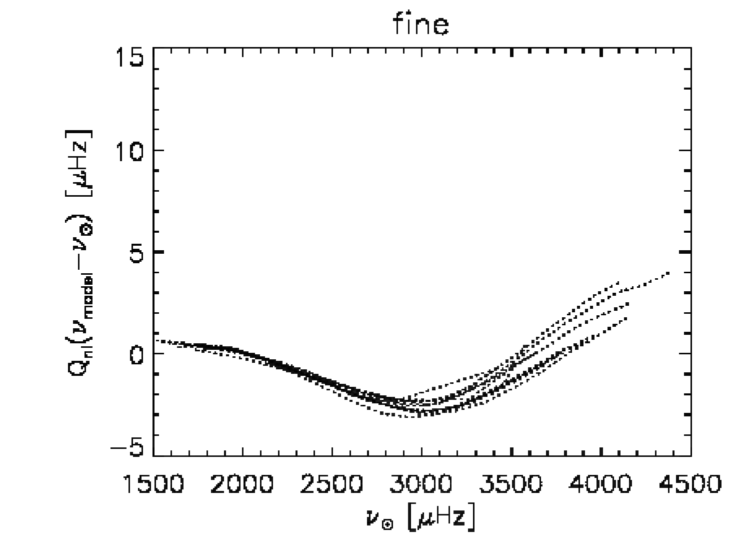


- **Superadiabaticity from Mixing Length Theory (MLT) compared to convection simulations.**
- **Agreement between MLT and simulation is worse in the more evolved models.**
- **Vertical lines mark position of the photosphere.**

Incorporating turbulence into stellar models (Li et al., ApJ, 2002)

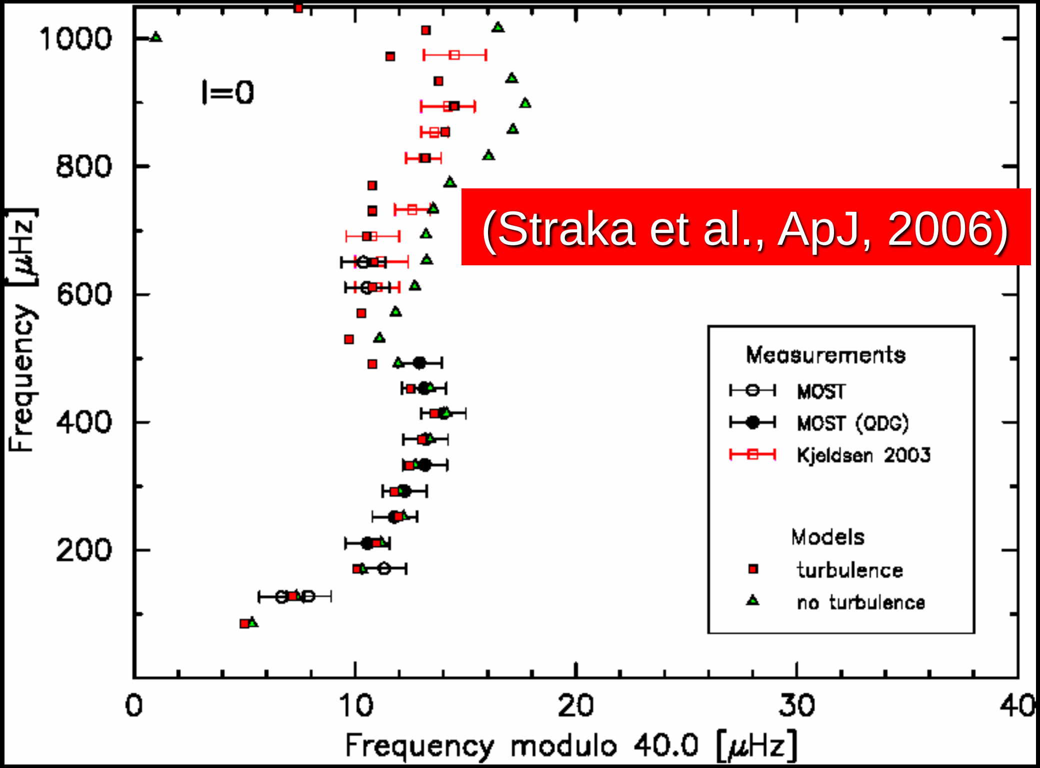
- Observed and computed solar p-modes (l=0-100) tend to disagree near the surface (for the highest frequencies).
- By inserting simulation data (TKE and P_turb) back in to the original stellar models and re-computing the frequencies, we found the discrepancy was reduced by up to a factor of 10.





Application to eta-Bootis

Similarly insert TKE and turbulent pressure into stellar model of eta-Bootis



(Straka et al., ApJ, 2006)

2. Tidal Dissipation in Stars

Models of eddy viscosity in dissipation in stars tested with simulation data

Penev, Sasselov, Robinson & Demarque, (2007, 2009)

Zahn, Ann. d'Astrophys. 1966

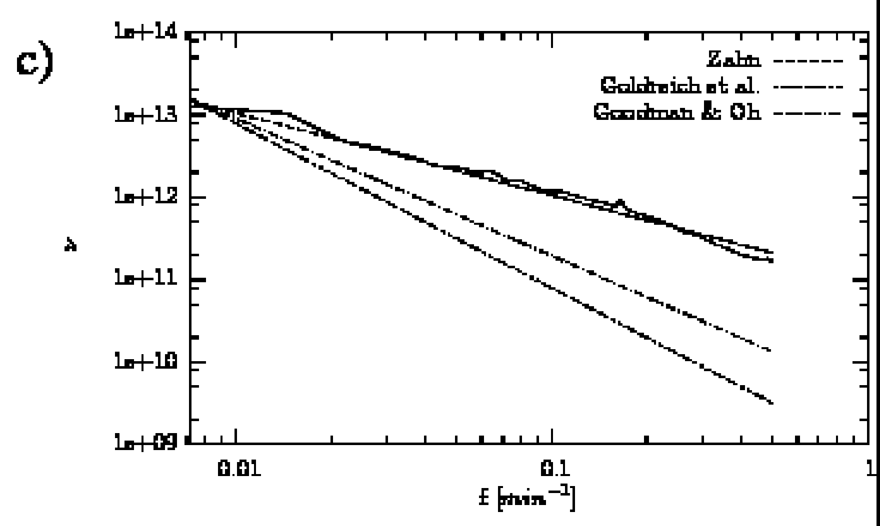
$$\nu = \nu_{\max} \max\left[\frac{T}{2\tau}, 1\right]$$

Goldreich & Keely, ApJ, 1977.

$$\nu = \nu_{\max} \max\left[\left\{\frac{T}{2\pi\tau}\right\}^2, 1\right]$$

T=perturbation period, tau=eddy turn over time

FFT of V(x,y,z,t) from simulations suggest Zahn's linear scaling law may be more appropriate for modeling dissipation in stars



3. Testing turbulence closures in non-local stellar models

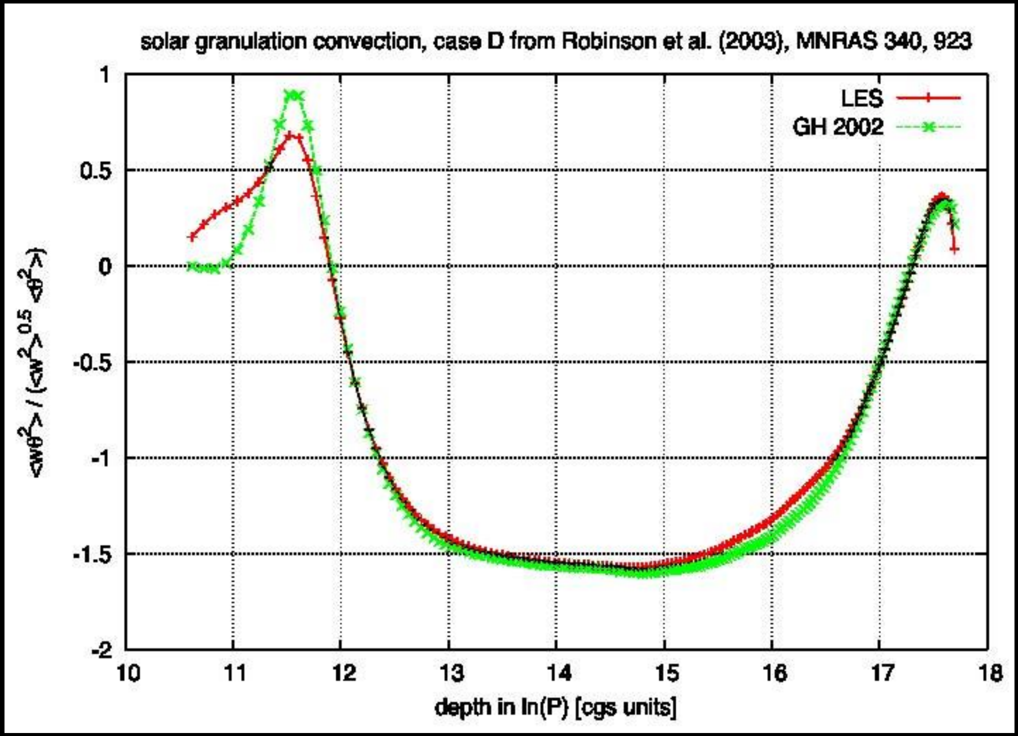
(Fig. taken from Kupka and Robinson MNRAS, 2007)

Gryanik et al., JAS, 2005

$$\overline{w\vartheta^2} = \frac{\overline{\vartheta^3}}{\overline{\vartheta^2}}\,\overline{w\vartheta}$$

Compute both LHS and RHS from 3d solar simulation (average over time and x-y space)

Almost perfect agreement



Fourth order moments

Compute both LHS and RHS from simulations – plot LHS/RHS

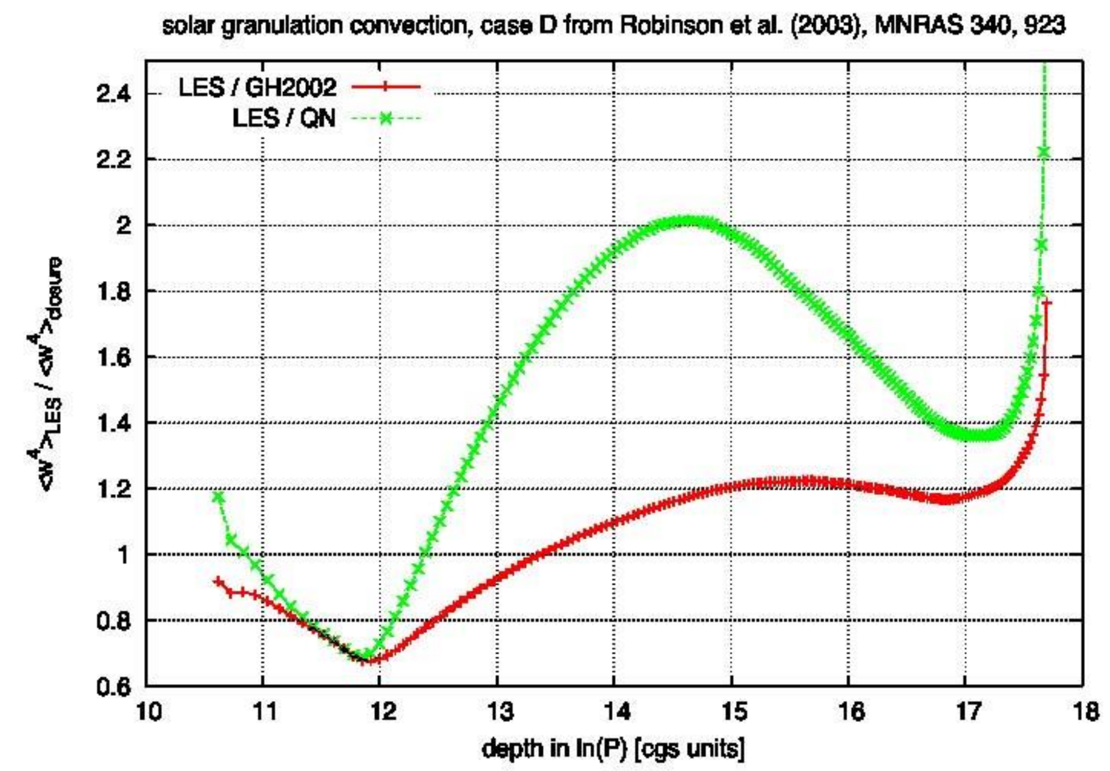
$$\overline{w^4} = 3\overline{w^2}^2$$

Green

$$\overline{w^4} = S_w{}^2\,\overline{w^2}^2$$

Red

$$S_w = \frac{\overline{w^3}}{\left(\overline{w^2}\right)^{\frac{3}{2}}}$$



Gryanik et al. (GH) model is a significant improvement over quasi-normal (QN) approximation

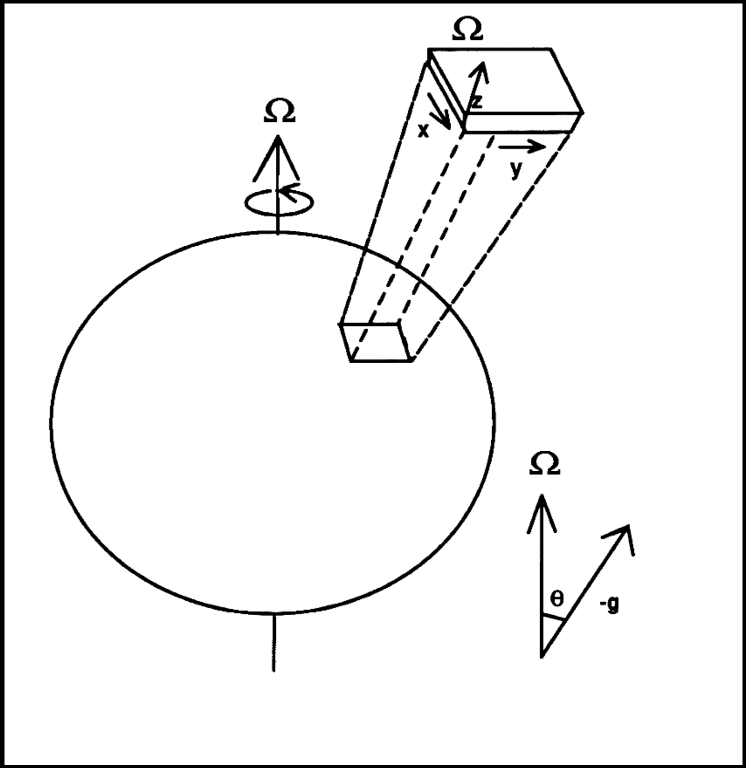
5. Box simulations of δScuti

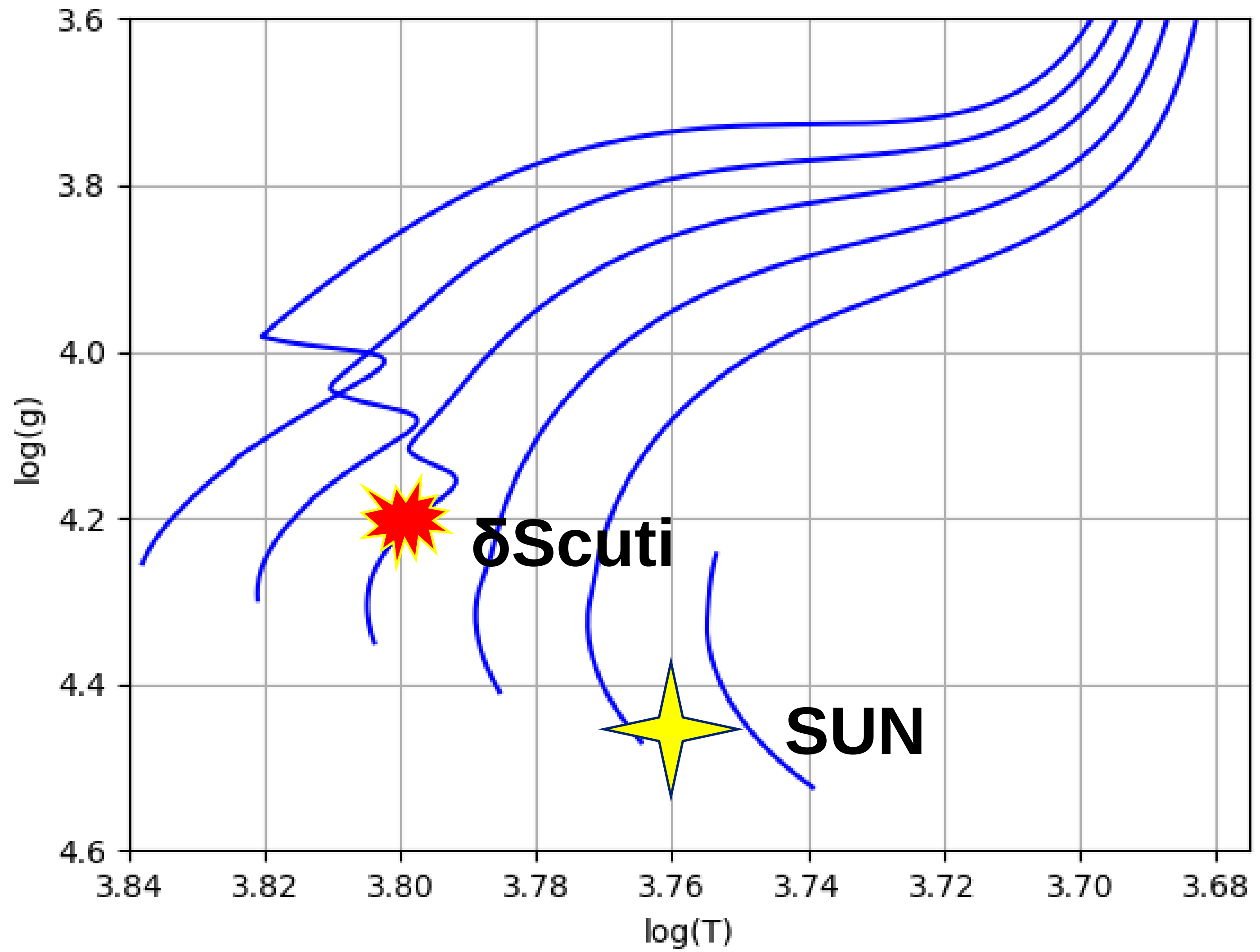
F-plane approximation

- Omega has constant size and direction throughout the box (consistent with periodic bc).

Current results only at EQUATOR

(Robinson, Tanner and Basu, MNRAS, 2020)





3.6
3.8
4.0
4.2
4.4
4.6
log(g)
δScuti
SUN
3.84
3.82
3.80
3.78
3.76
3.74
3.72
3.70
3.68
log(T)

Model	Log T	Log g	W''max km/s	L (km)	Pturb/Pgas % (Max)
Sun [S]	3.761	4.44	3.0	1000	15
δScuti A	3.81	4.21	4.5	10,000	29
δScuti E	3.81	4.21	4.3	10,000	27

Simulation values

d = model depth (box thickness)

timescale, t = d/(sound speed at the top)

Sun: t ~ 160s, d ~ 1Mm (granule turnover time ~8min)

2π/Ω ~ 25 days >> 8 min [ignore Ω effects]

δ – Scuti : t ~ 520s, d ~ 10 Mm

(granule turnover time ~ 30 mins

2π/Ω ~ 6 hours [can we ignore Ω?}

(Solano & Fernley (1997), Molenda- Zakowicz et al. (2008))

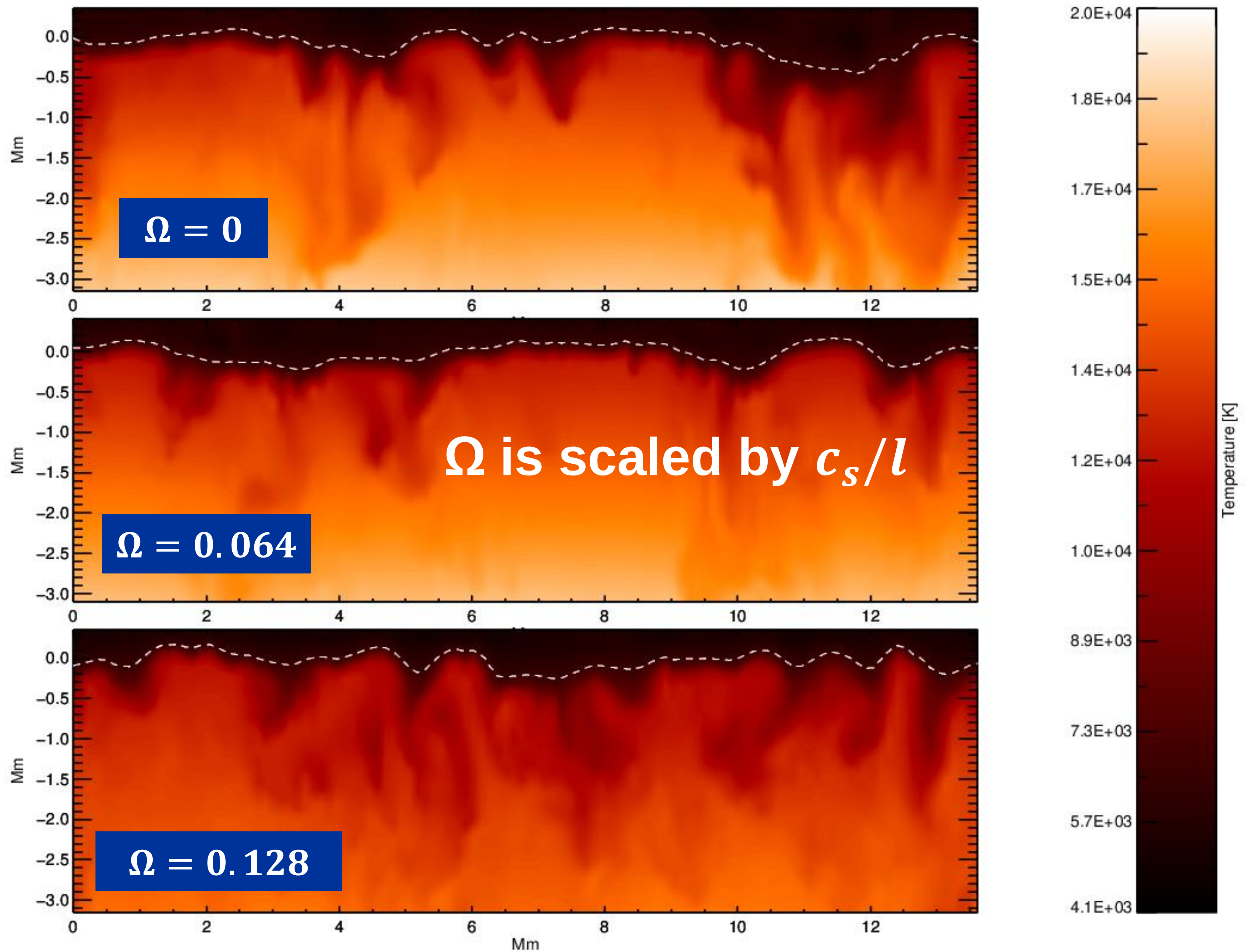
‘δ –Scuti’ **Model**	**V_rot (km/s)**	**Period (hrs)**	**<v”> km/s**	**Co**	**Re**
A	**0**	**-**	**4.7**	**0.0**	**2070**
B*	**153**	**12**	**4.4**	**0.2**	**1760**
C	**153**	**12**	**4.3**	**0.2**	**1800**
D	**184**	**10**	**4.2**	**0.3**	**1870**
E	**307**	**6**	**4.0**	**0.5**	**1890**

$$Co = \frac{\Omega d}{v''} \qquad \mathrm{Re} = \frac{v'' \rho d}{\mu_{SGS}}$$

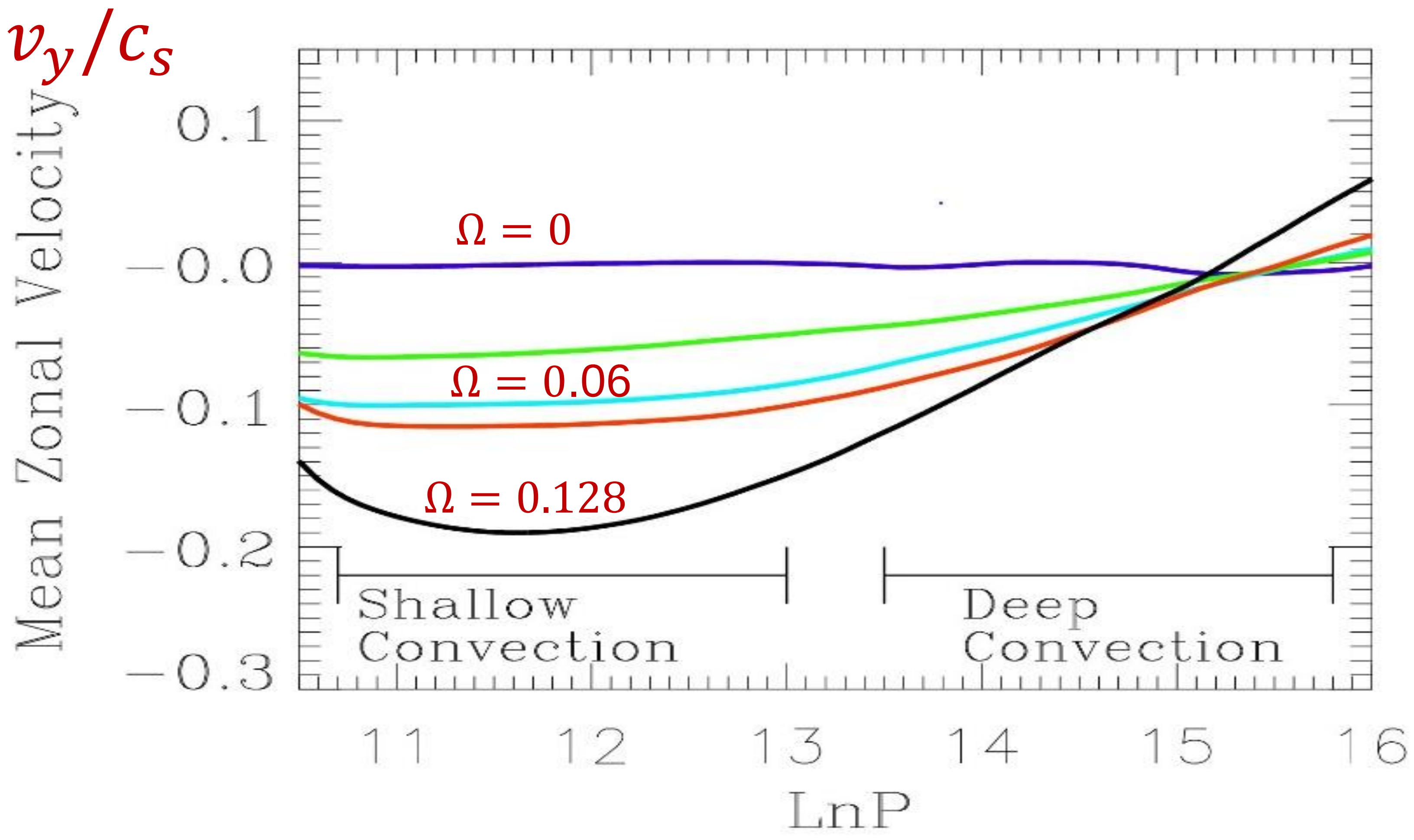
$$\mathrm{V\_rot} = R\Omega$$

B* Excludes cent. force

v'' = turbulent velocity
R = stellar radius
d = box depth
Ω = rotation rate
Co = Coriolis number
Re = Reynolds number



Ω = 0
Ω = 0.064
Ω = 0.128
Ω is scaled by c_s/l
Mm
Temperature [K]
2.0E+04
1.8E+04
1.7E+04
1.5E+04
1.4E+04
1.2E+04
1.0E+04
8.9E+03
7.3E+03
5.7E+03
4.1E+03



Why does zonal velocity vary this way with depth?

Consider Coriolis force on vertically moving parcels at equator

$$\frac{dv_y}{dt} \sim -2\Omega\, v_z$$

=> $$\frac{\Delta v_y}{\Delta t} \sim -\frac{2\Omega\Delta z}{\Delta t}$$

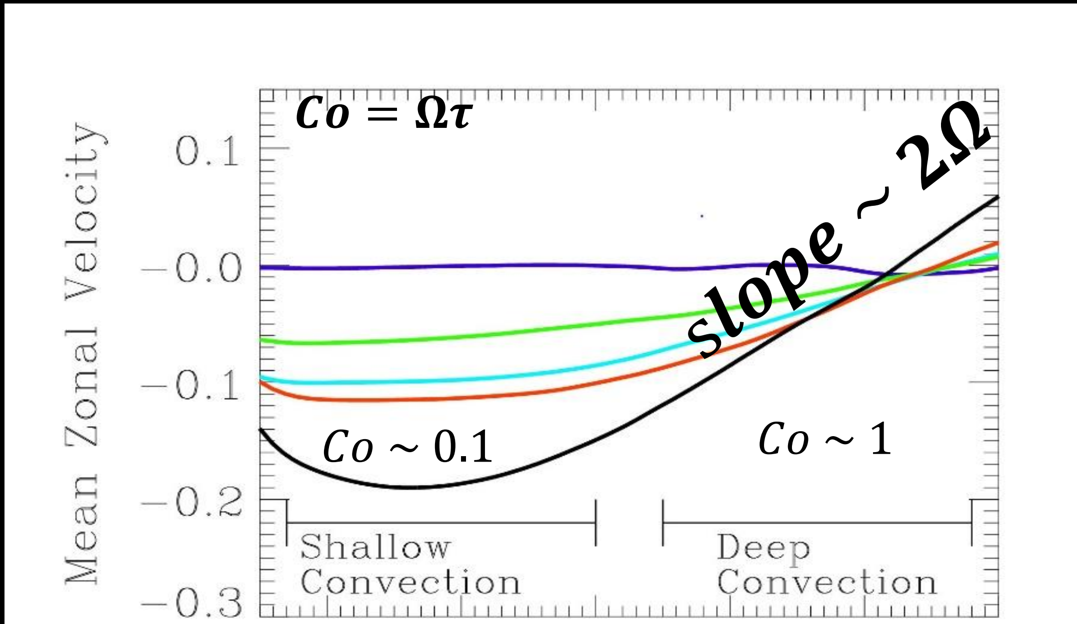
For deep convection:

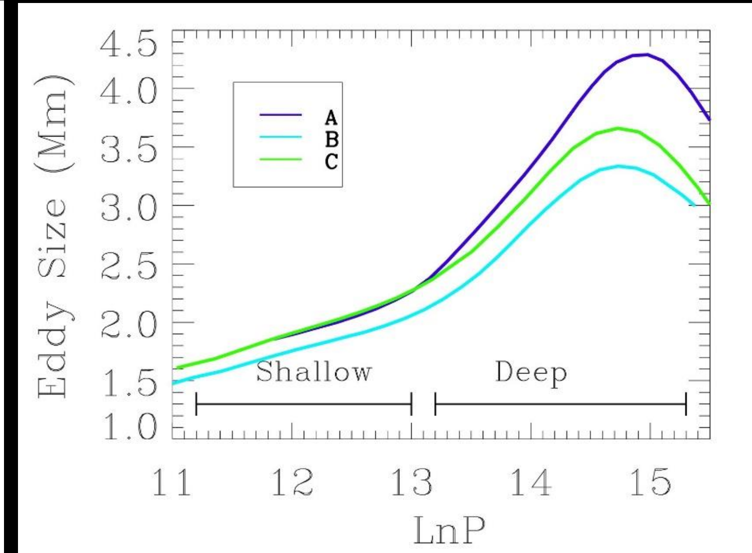
$$\frac{\Delta v_y}{\Delta z} \sim -2\Omega$$

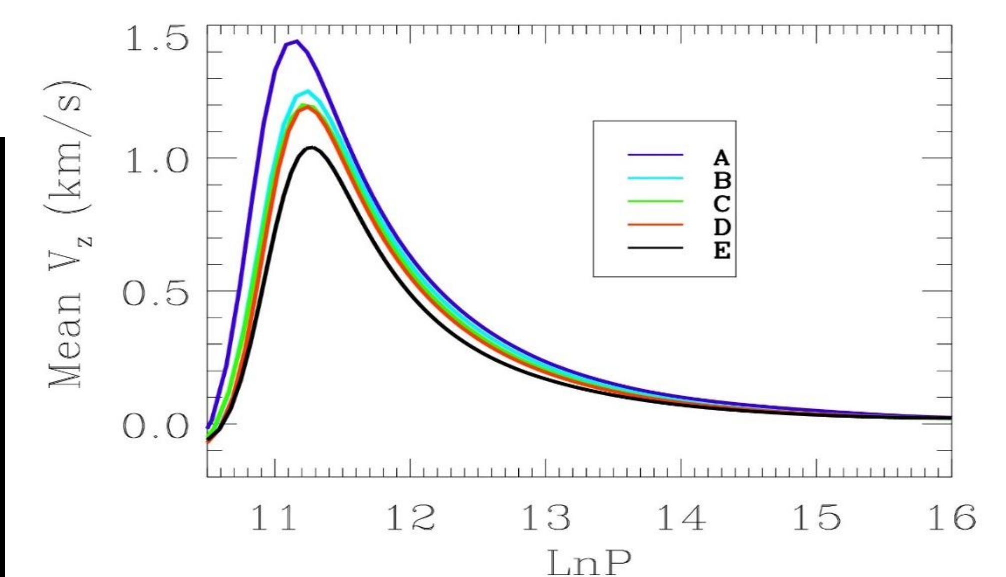
But for shallow convection, we find that,

$$\frac{\Delta v_y}{\Delta z} \sim 0$$

Why is $v_y \sim$ constant near the surface?







Shallow:

$\tau \sim \frac{l_z}{v_z} \sim 25$ mins << $2\pi/\Omega$

Flow too fast to feel Ω

Deep:

$\tau \sim \frac{l_z}{v_z} \sim 6$ hours ~ $2\pi/\Omega$

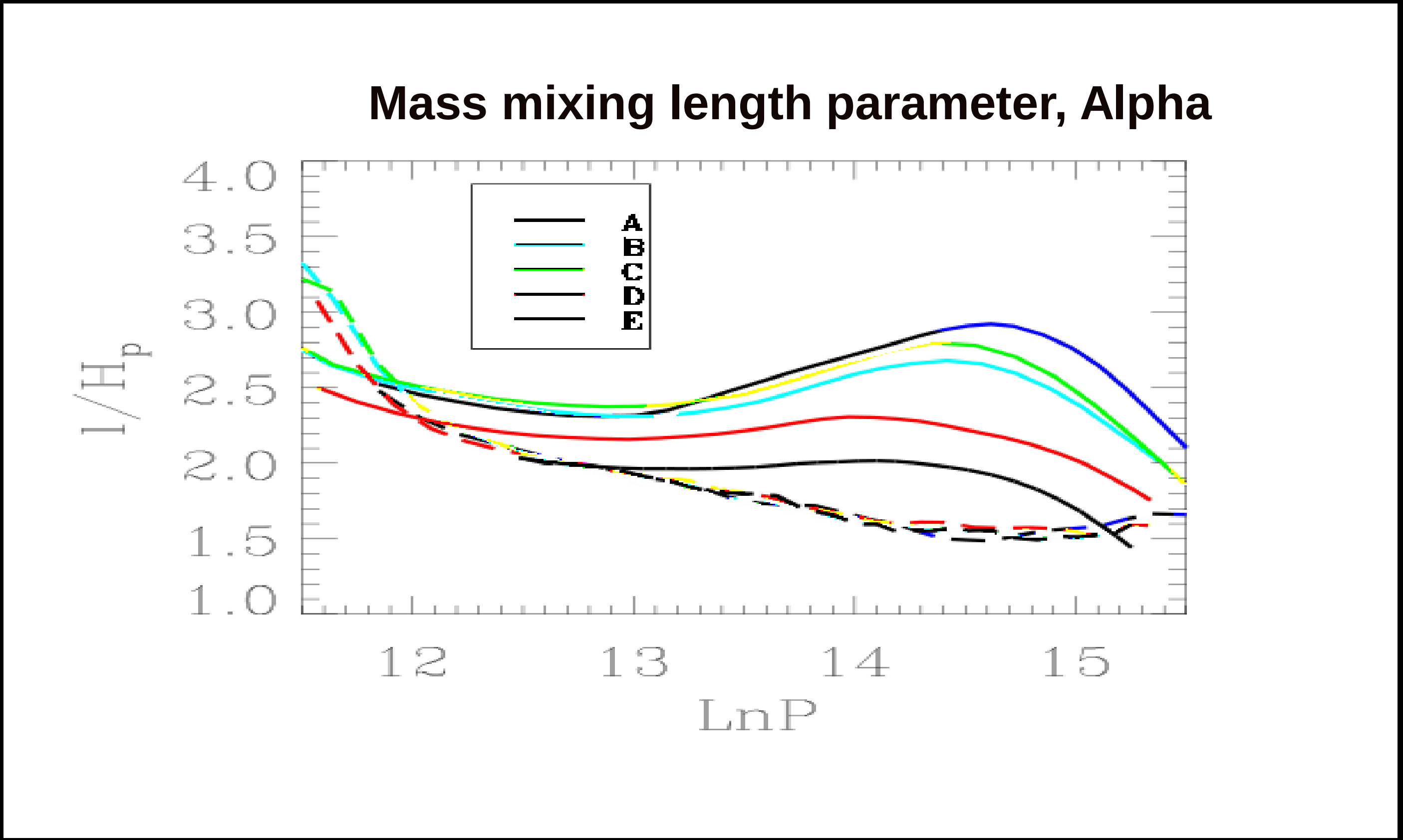
Flow is slow and feels Ω

Effects of rapid rotation on shallow vs. deep convection

- Eddies/granules in the SAL don't feel rotation!
- Rapid upflow/downflow in SAL region (granules) create a constant zonal velocity (flat profile) near the top of the star

- In the deeper regions, beyond the reach of granulation, rotation controls zonal velocity

$l = \ l_{eddy} \ or \ l_m$



l_{eddy} = dist over which $C[w', w']$ drops below 0.5 (solid lines)

$l_m = 1/ \left| \frac{d \ ln\rho(z)}{dz} + \frac{d \ \ln w(z)}{dz} + \frac{d \ \ln A(z)}{dz} \right|$ (dashed lines) +> stellar mixing length

taken from Trampedach & Stein 2011

$\rho(z), w(z),$ A (z) are time and horizontal average values of density, vertical velocity and area of up-flows

Change in Eddy (solid lines) size due to rotation don't seem to be accounted in stellar mixing length parameter (dashed lines)

Summary Points

Local f-plane models might be useful for looking at fast rotators such as δScuti

But, need to use spherical shell simulation to model rotation properly (to include meridional circulation/Reynolds stress).

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