Applying the Kuhfuss Convection Theory to Convective Envelopes

1D stellar evolution

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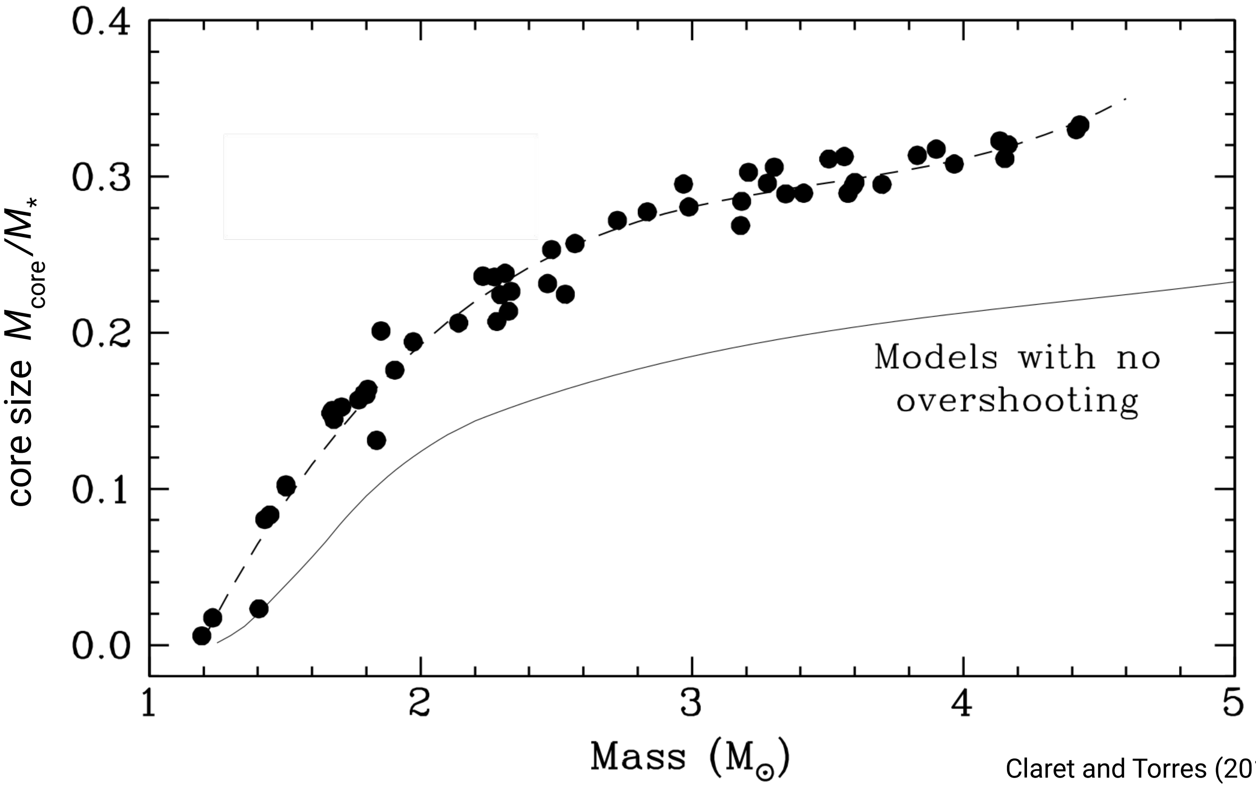
The Mixing Length Theory

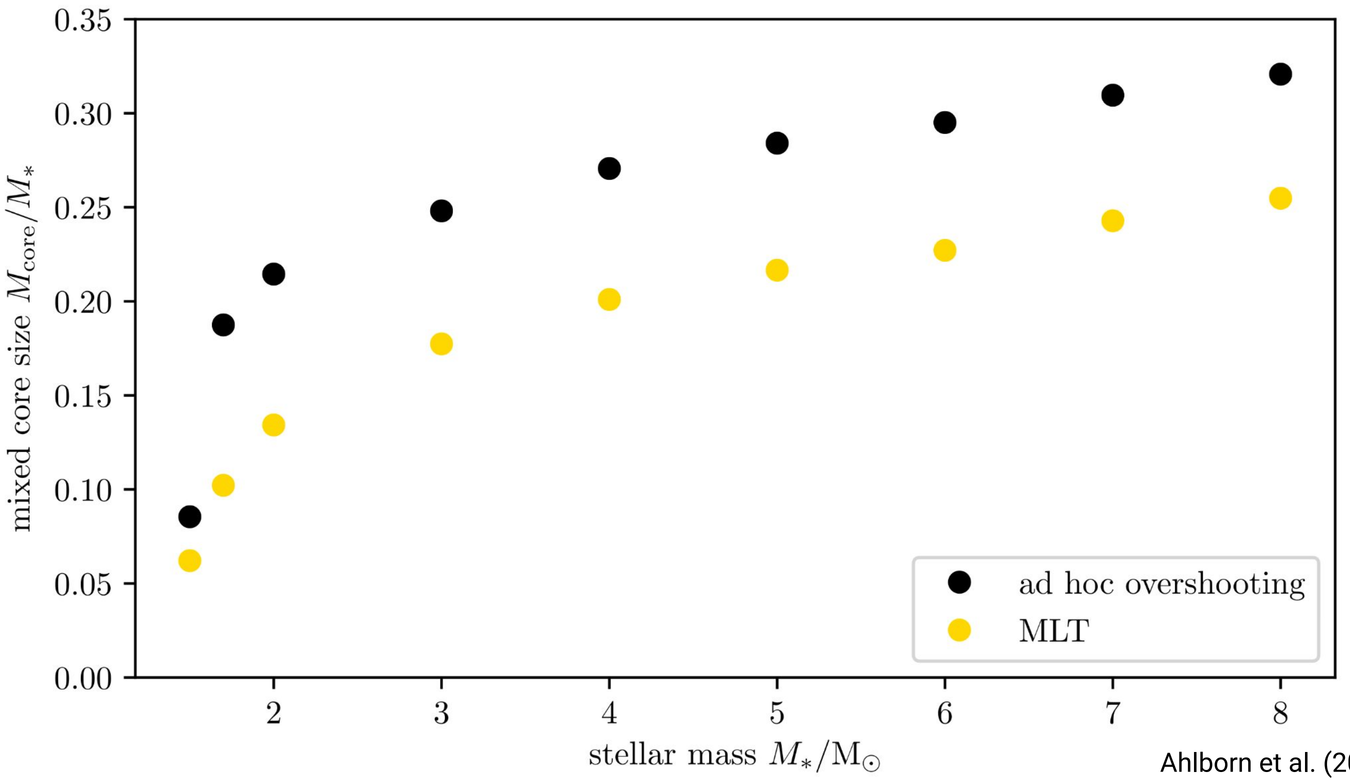
Prandtl (1925); Vitense (1952)



Claret and Torres (2016)

The Mixing Length Theory

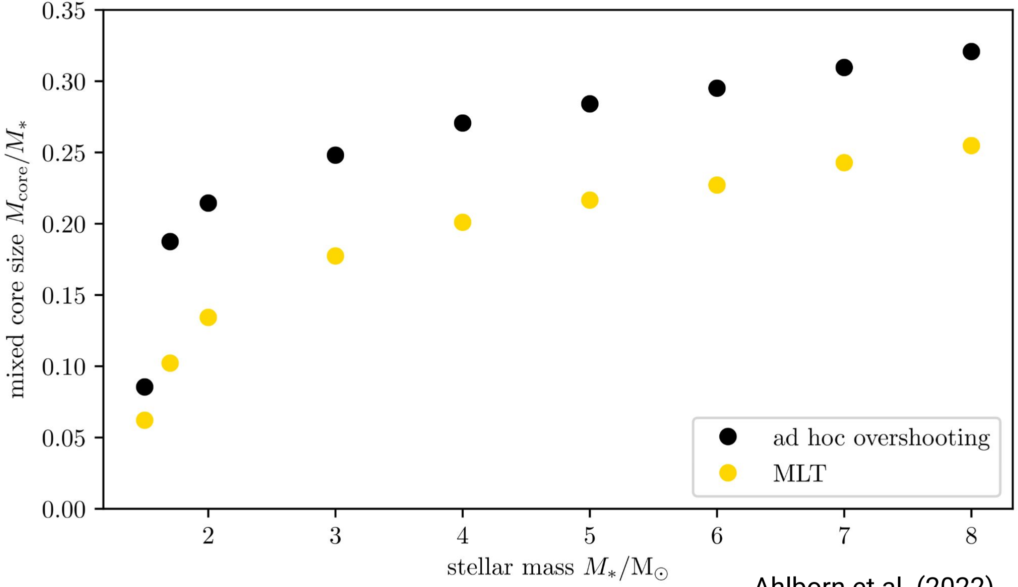
Prandtl (1925); Vitense (1952)



Ahlborn et al. (2022)

The Mixing Length Theory + Overshooting

Prandtl (1925); Vitense (1952)



Ahlborn et al. (2022)

Exponential overshoot:

$$D(z) = D_0 \exp \frac{-2z}{f_{\mathrm{ov}} H_P}$$

an "ad hoc" solution to an empirical problem

based on 2D-hydro simulations

Freytag et al. 1996

The Kuhfuss Turbulent Convection Model

Kuhfuss (1986, 1987)

- Reynolds averaged Navier-Stokes equations

$$\omega = \frac{1}{2}\overline{u'^2}$$

second order of the velocity fluctuations
→ turbulent kinetic energy

$$\Pi = \overline{u's'}$$

correlation of velocity and entropy fluctuations
→ convective flux

$$\Phi = \frac{1}{2}\overline{s'^2}$$

second order of the entropy fluctuations

The Kuhfuss Turbulent Convection Model

Kuhfuss (1986, 1987)

➔ **non-local and time dependent**

3-equation model

- equations for:
  - turbulent kinetic energy
  - convective flux
  - second order entropy fluctuations

implemented in GARSTEC (Schlattl and Weiss, 2008)

1-equation model

- diffusion approximation for the convective flux
- one equation left: turbulent kinetic energy

$$F_{\mathrm{conv}} \propto -\frac{\partial s}{\partial r}$$

The Kuhfuss 3-equation model

Kuhfuss (1986, 1987); Kupka et al. (2022); Ahlborn et al. (2022)

Buoyancy terms
viscous dissipation
potential terms
non-local terms
radiative losses

$$\frac{\partial \omega}{\partial t} = \frac{\nabla_{\rm ad} T}{H_{\rm p}} \Pi - \frac{C_{\rm D}}{\Lambda} \omega^{3/2} - \mathcal{F}_\omega$$

$$\frac{\partial \Pi}{\partial t} = \frac{2\nabla_{\rm ad} T}{H_{\rm p}} \Phi + \frac{2 c_{\rm p}}{3 H_{\rm p}} (\nabla - \nabla_{\rm ad}) \omega - \mathcal{F}_\Pi - \frac{1}{\tau_{\rm rad}} \Pi$$

$$\frac{\partial \Phi}{\partial t} = \frac{c_{\rm p}}{H_{\rm p}} (\nabla - \nabla_{\rm ad}) \Pi - \mathcal{F}_\Phi - \frac{2}{\tau_{\rm rad}} \Phi$$

\+ modification by Kupka et al. & Ahlborn et al. 2022: Including the dissipation by buoyancy waves

The Kuhfuss 1-equation model

$$\frac{\partial \omega}{\partial t} = \frac{\nabla_{\mathrm{ad}} T}{H_{\mathrm{p}}} \Pi - \frac{C_{\mathrm{D}}}{\Lambda} \omega^{3/2} - \mathcal{F}_{\omega}$$

$$\frac{\partial \Pi}{\partial t} = \frac{2\nabla_{\mathrm{ad}} T}{H_{\mathrm{p}}} \Phi + \frac{2c_{\mathrm{p}}}{3H_{\mathrm{p}}} (\nabla - \nabla_{\mathrm{ad}}) \omega - \mathcal{F}_{\Pi} - \frac{1}{\tau_{\mathrm{rad}}} \Pi$$

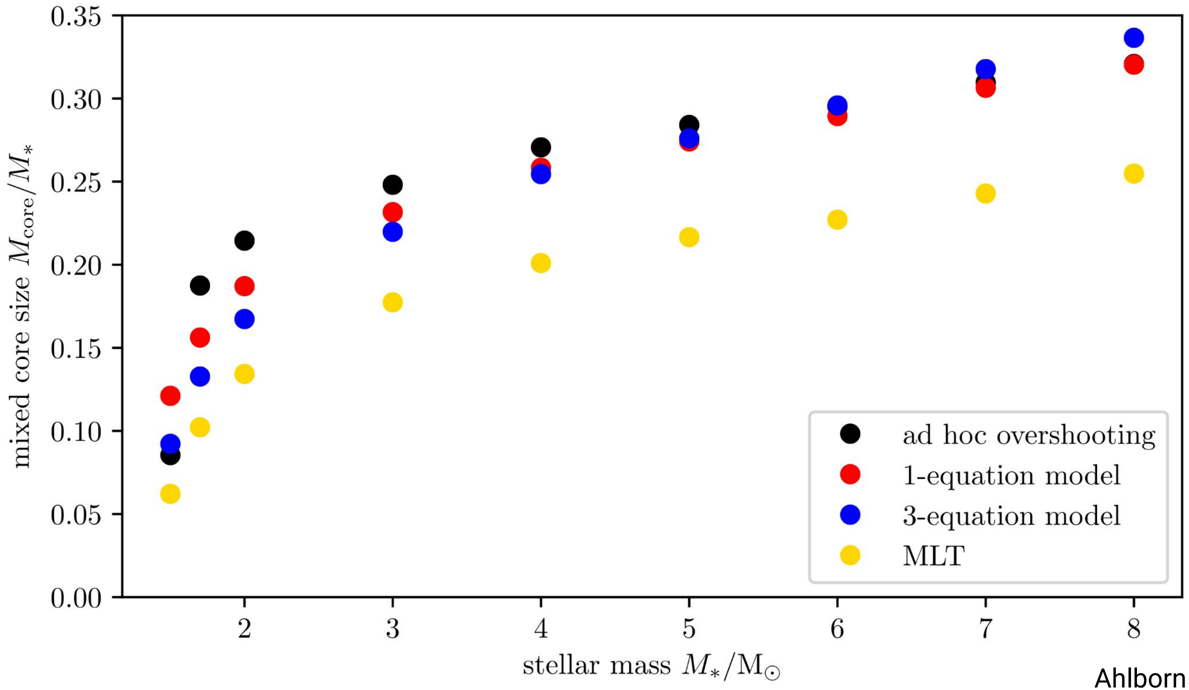
$$\frac{\partial \Phi}{\partial t} = \frac{c_{\mathrm{p}}}{H_{\mathrm{p}}} (\nabla - \nabla_{\mathrm{ad}}) \Pi - \mathcal{F}_{\Phi} - \frac{2}{\tau_{\mathrm{rad}}} \Phi$$

Downgradient approximation

$$\Pi = -\alpha_{\mathrm{s}} \Lambda \omega^{1/2} \frac{\partial s}{\partial r}$$

$$\frac{\partial s}{\partial r} = -\frac{c_{\mathrm{p}}}{H_{\mathrm{p}}} (\nabla - \nabla_{\mathrm{ad}})$$

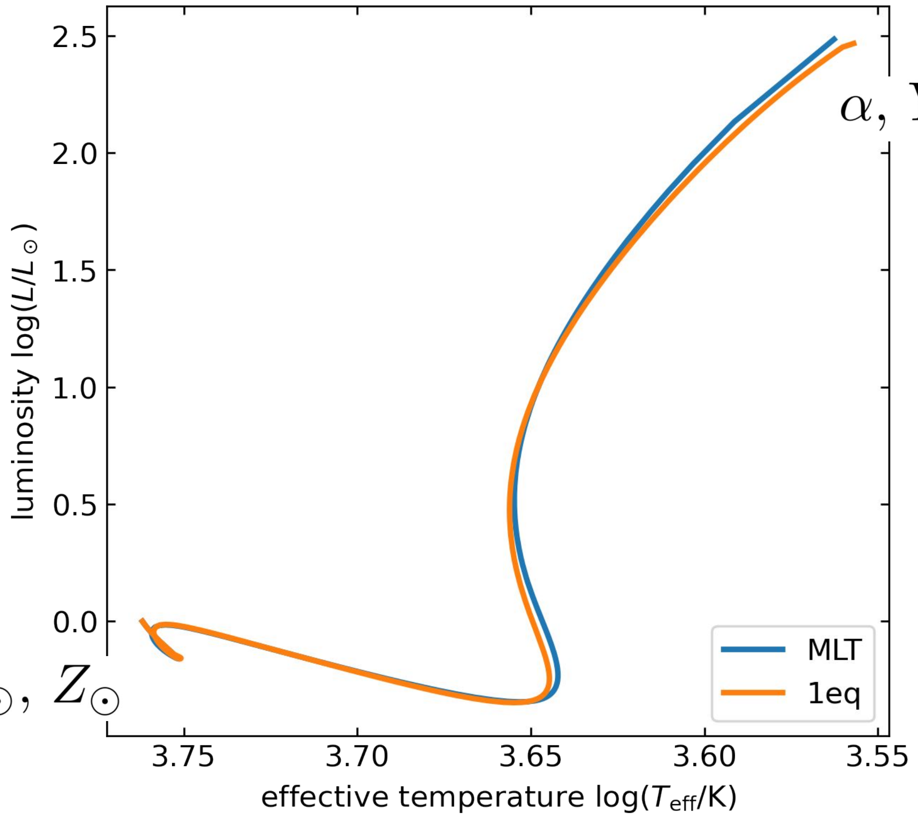
Convective Cores



Ahlborn et al. (2022)

Is the Kuhfuss TCM also applicable in convective envelopes?

Standard Solar Models



$\alpha,\ Y_{\mathrm{init}},\ Z_{\mathrm{init}}$

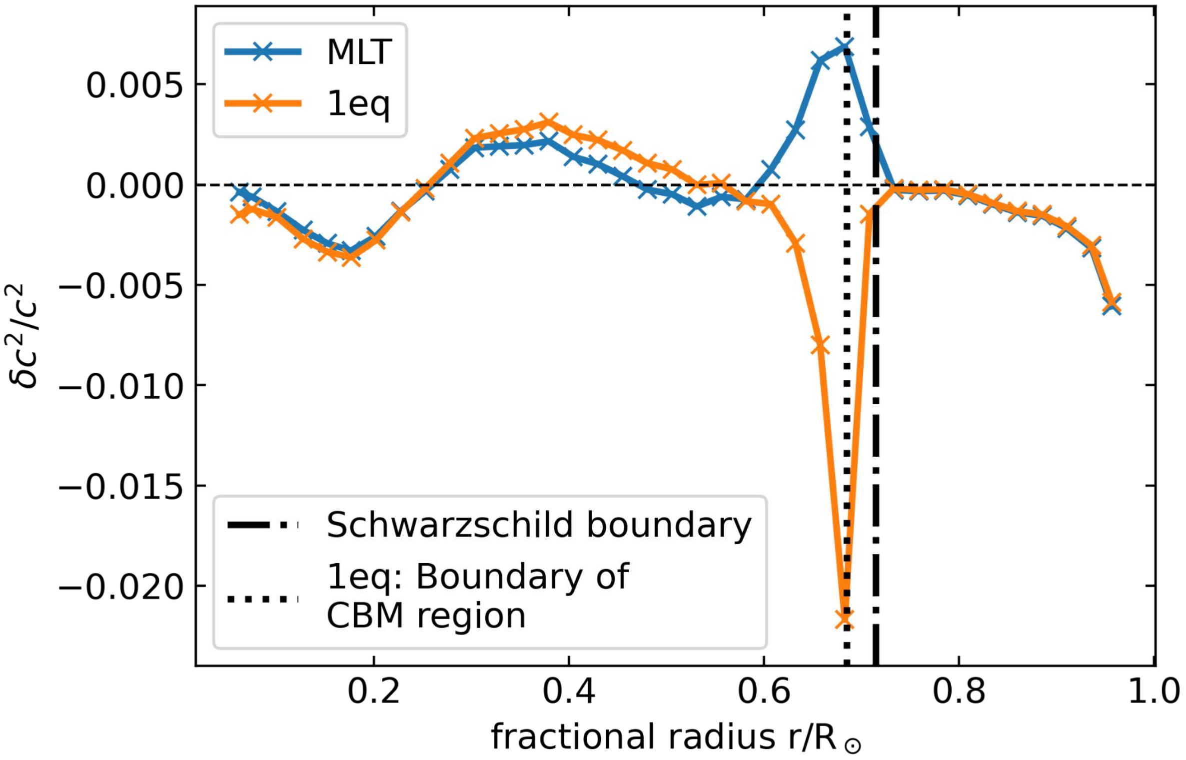
$R_{\odot},\ L_{\odot},\ Z_{\odot}$

The sound speed profile

Braun et al. (2024)

helioseismic measurement:
Basu et al. 2009

$$\frac{\delta c^2}{c^2} = \frac{c^2_{\mathrm{helio}} - c^2_{\mathrm{model}}}{c^2_{\mathrm{helio}}}$$

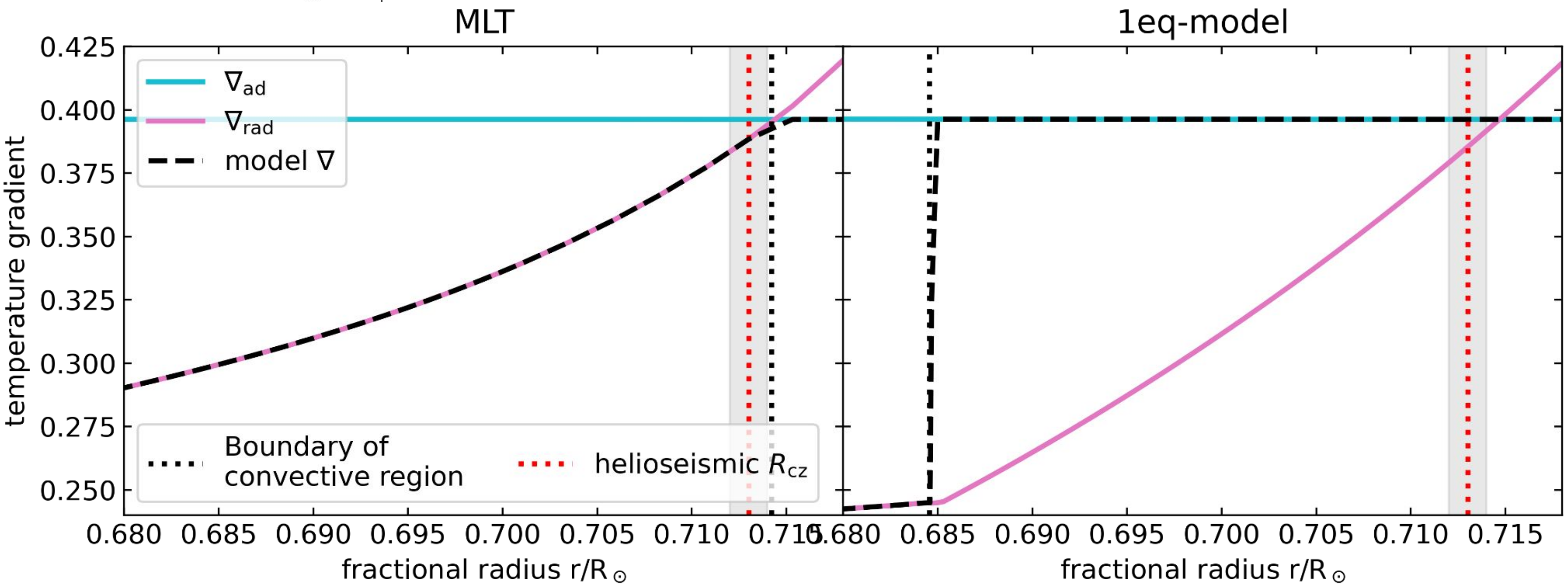


Abundances used:
Magg et al. (2022)

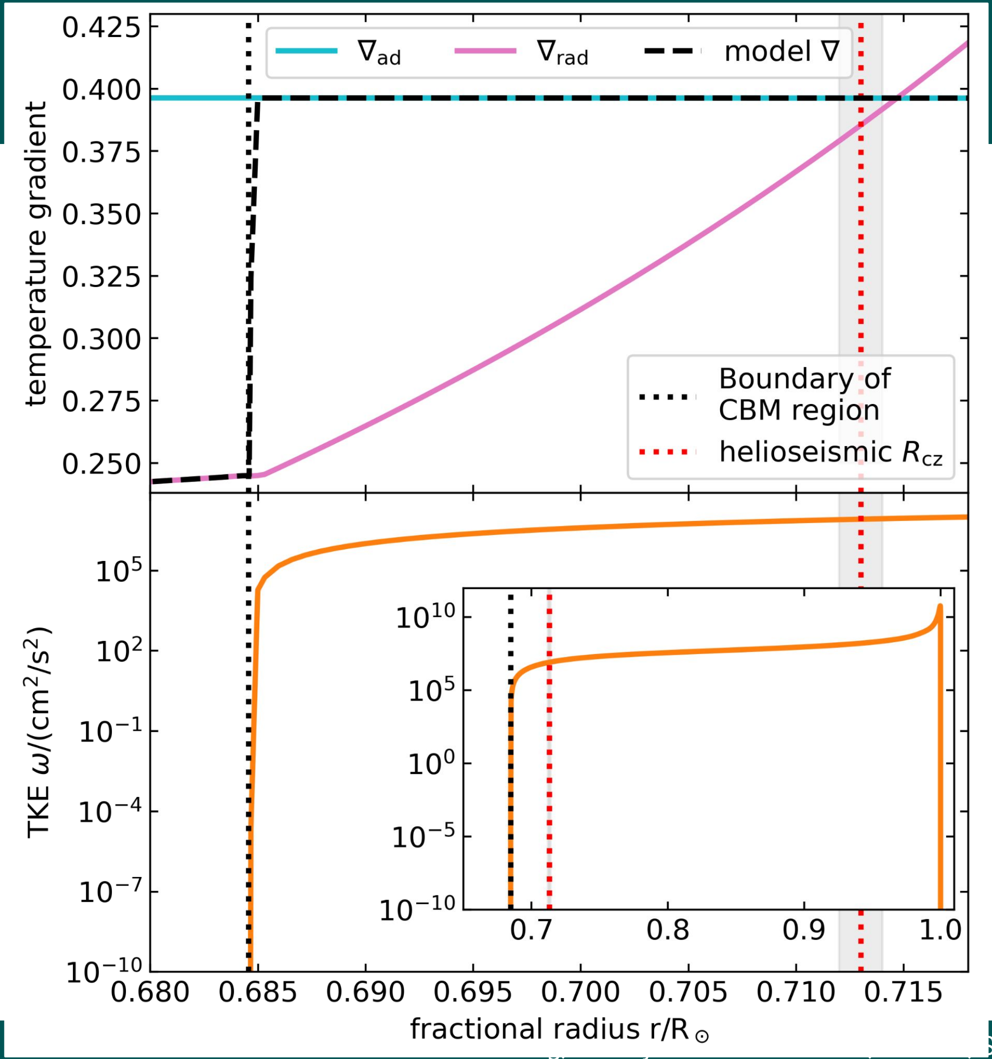
The location of the convective boundary

Braun et al. (2024)

	MLT	1-eq.	Measurement	
R_{cz} [$R_\odot$]	0.7144	0.6845	0.713±0.001	Basu & Antia 1997
Y_{cz}	0.2423	0.2456	0.2485±0.0034	Basu & Antia 2004



Braun et al. (2024)



$$\nabla - \nabla_{ad} = \frac{\nabla_{rad} - \nabla_{ad}}{1 + \frac{\rho c_p \alpha_s \Lambda \sqrt{\omega}}{k_{rad}}}$$

due to the diffusion approximation

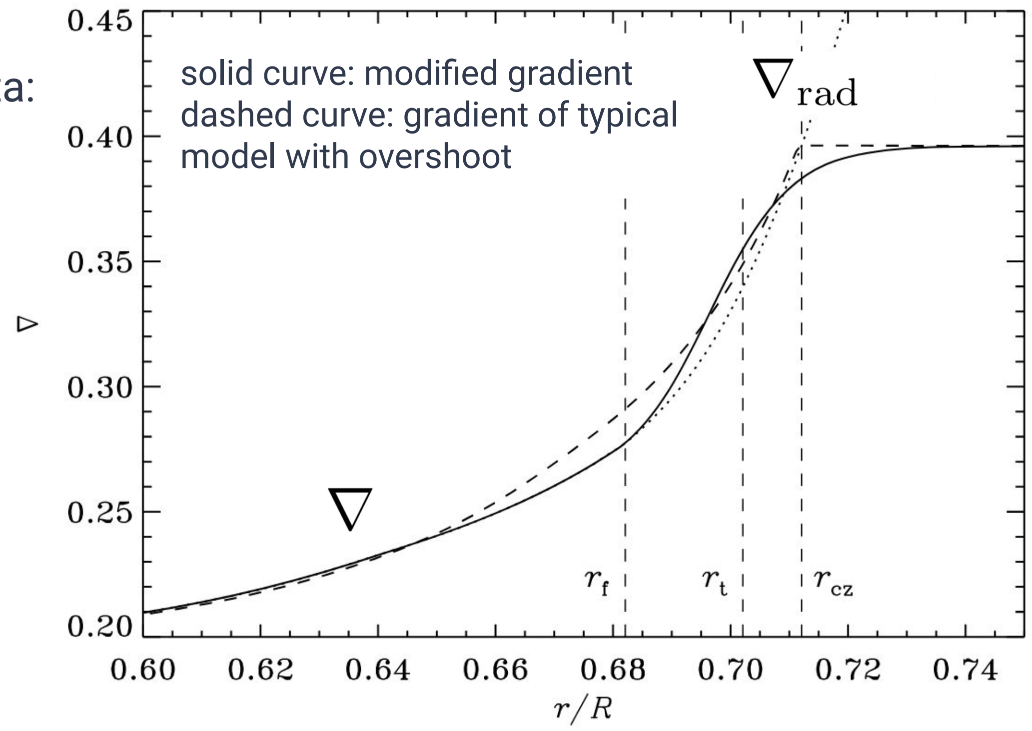
$$F_{conv} \propto -\frac{\partial s}{\partial r}$$

What kind of temp. gradient do we expect?

Christensen-Dalsgaard et al. (2011)
Model which best fits the helioseismic data:

- some overshoot necessary
- smoother than 'classic' overshoot
- lower part of CZ subadiabatic

also:
Xiong & Deng (2001) - TCM
Baraffe et al. (2021), (2022) - 2D hydro
Käpylä et al. (2017) - 3D hydro
measurements of subadiabaticity:
Bekki (2024)



Christensen-Dalsgaard et al. (2011)

Why should the 3-equation model be better?

1-equation model

$$\nabla - \nabla_{\rm ad} = \frac{\nabla_{\rm rad} - \nabla_{\rm ad}}{1 + \frac{\rho c_{\rm p} \alpha_{\rm s} \Lambda \sqrt{\omega}}{k_{\rm rad}}}$$

The 1-equation model does not model the temperature stratification well

due to the diffusion approximation

$$F_{\rm conv} \propto -\frac{\partial s}{\partial r}$$

3-equation model

$$\nabla = \nabla_{\rm rad} - \frac{H_p \rho}{k_{\rm rad}} \Pi$$

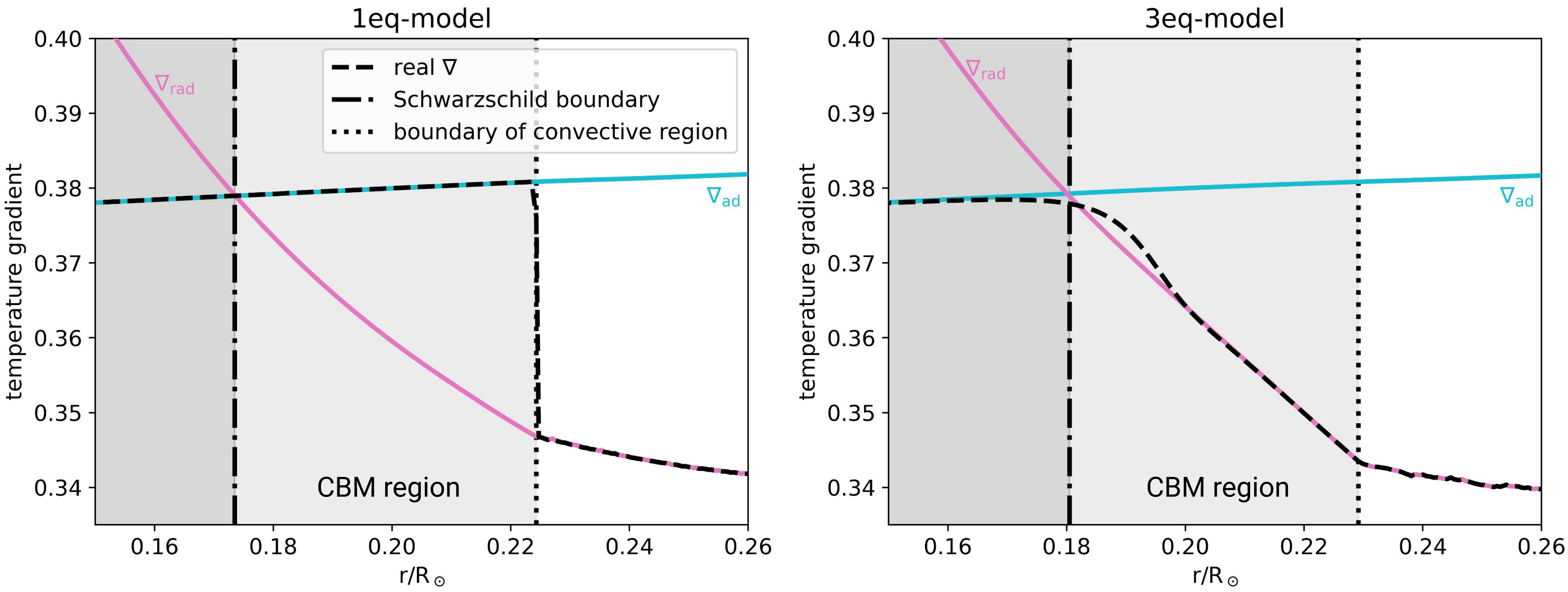
convective flux variable,
full equation is solved

Unfortunately: No success yet in calculating a solar model with the 3-equation model → work in progress

3-equation model for cores

Ahlborn et al. (2022)

core overshooting region of a 5 $M_\odot$ model

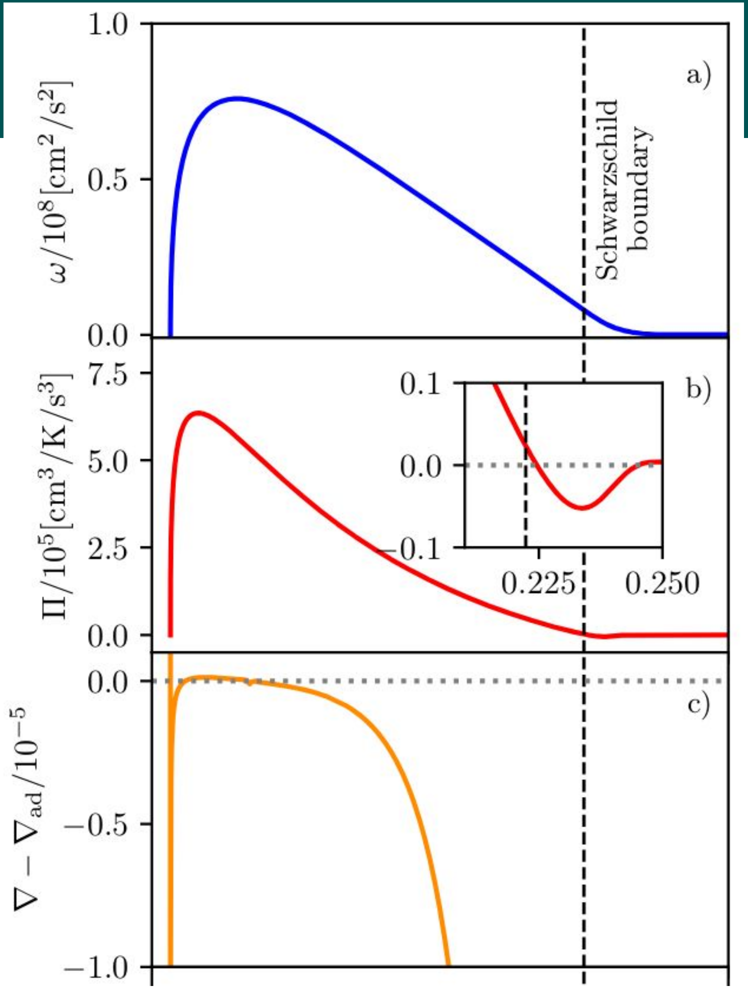


3-equation model for cores

Ahlborn et al. (2022)

In convective cores, we saw:

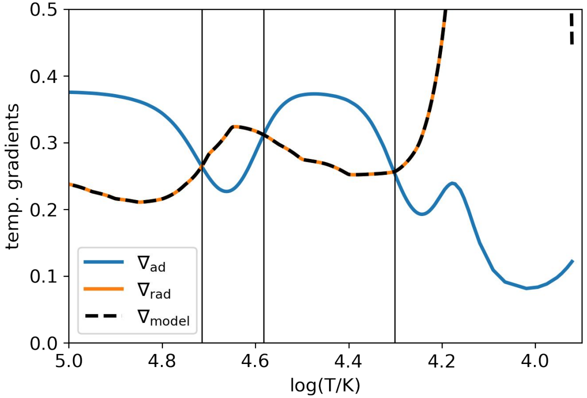
- gradual change from adiabatic to radiative gradient
- subadiabatic gradient well within the Schwarzschild radius

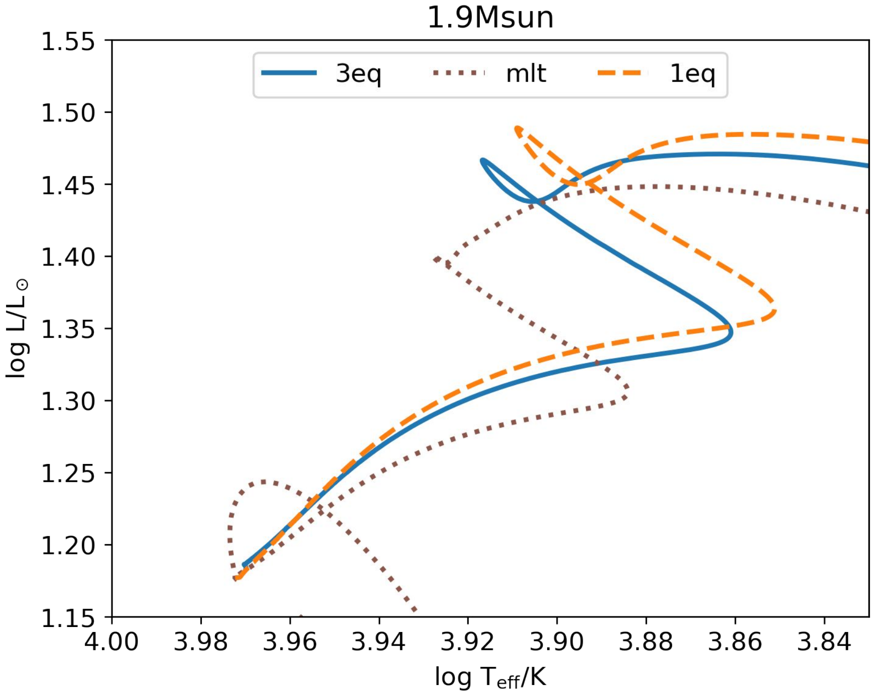


Convective shells in A-type stars

work in progress!

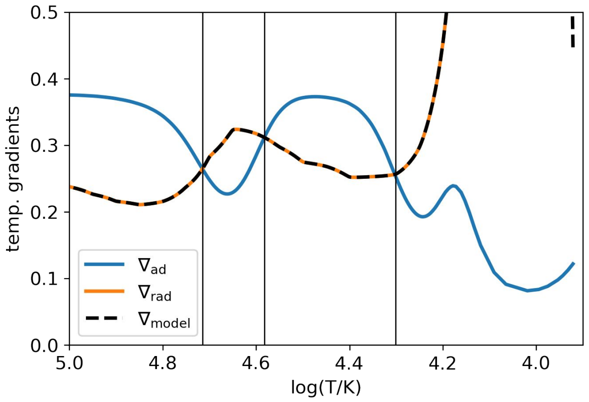
- A-type stars:
  - Teff: 7,300 to 10,000 K
  - Stellar masses: 1.4 to 2.1 Msun
  - Two thin convective shells: Ionization zones of H and He II





MLT & the 1-equation model

work in progress!



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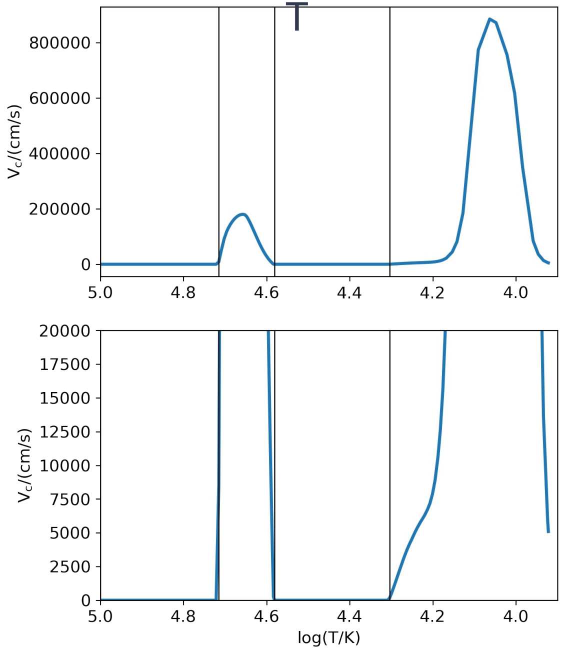
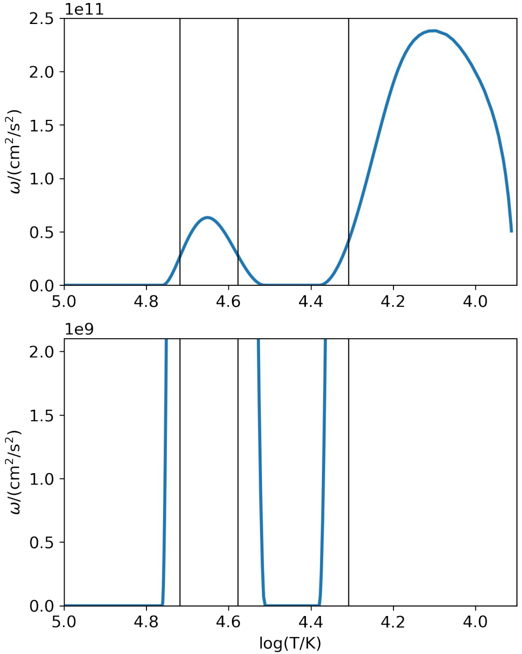
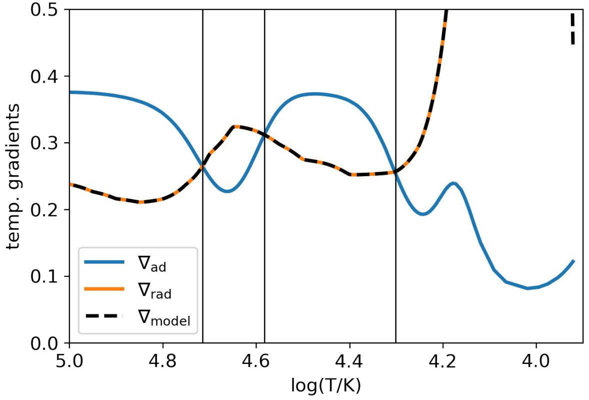


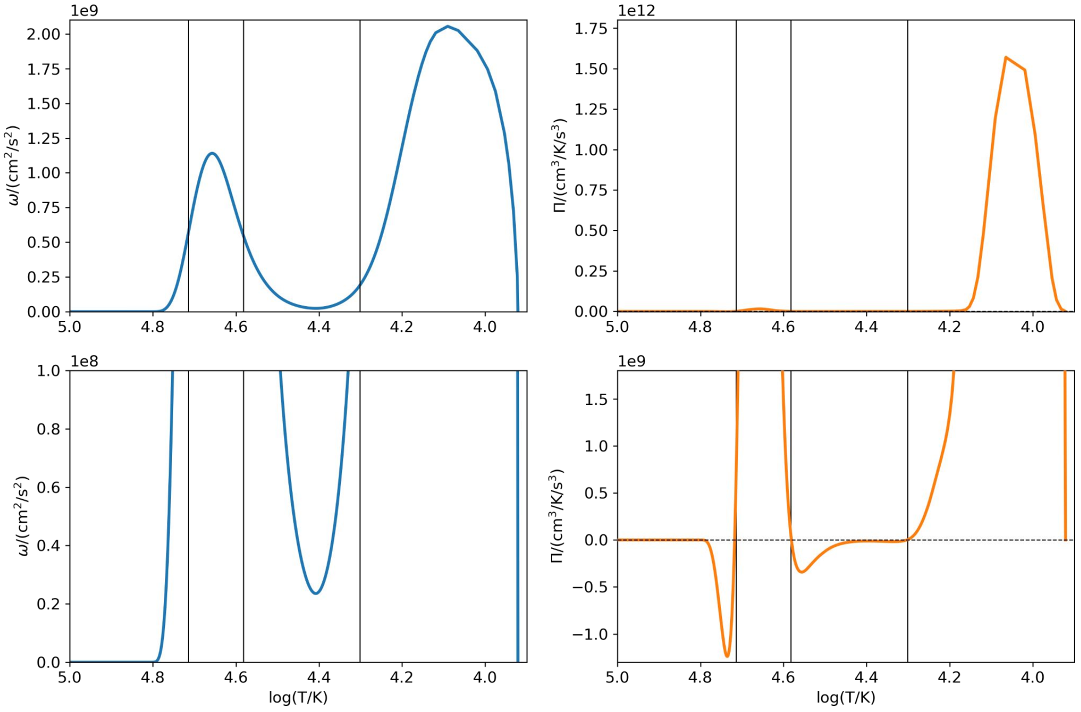
1eq



The 3-equation model

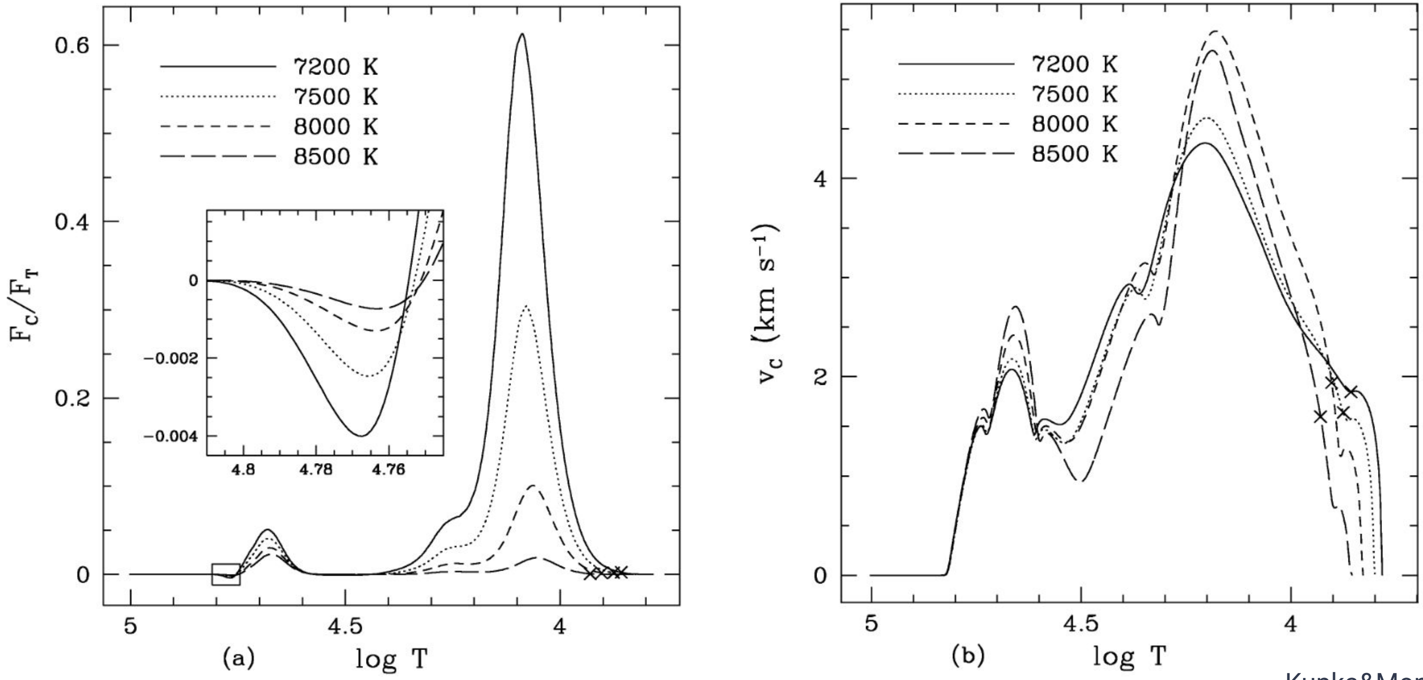
work in progress!





Comparison to other work

work in progress!



Kupka&Montgomery 2002

The Kuhfuss TCM

- Two versions:
  - 1-equation version: gives a temperature profile like penetrative convection
  - 3-equation version: Subadiabatic region within the Schwarzschild radius
- For convective cores: good results from both versions
- For convective envelopes:
  - Sun: 1-eq: convective envelope is too deep; change to radiative temperature gradient seems too sharp
    3-eq: work in progress
  - A-type stars: First successes in calculating a full 3-equation stellar model

- Processes lacking in the model: Anisotropy, rotation, …