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Optimisation and mitigation of spreading processes

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WINQ - dynamics and topology of complex network systems
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Outline

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- The relevance of epidemic spreading
- Inferring the spreading dynamics
- Optimizing use of (vaccination) budget
- Competitive and collaborative spreading
- Mitigating the spread in collaborative spreading
- Presymptomatic but infective state
- How effective are containment and mitigation measures?
- Effective mitigation on interacting networks
- Summary

A. Y. Lokhov and D. Saad, Proc. of the National Academy of Sci., **114** E8138 (2017).
H. Sun, D. Saad and A. Y. Lokhov, Phys. Rev. X, **11**, 011048 (2021).
B. Li and D. Saad, Phys. Rev. E **103**, 052303 (2021).
B. Li and D. Saad, Comm. Phys **7**, 144 (2024).

COVID-19 and other pandemics

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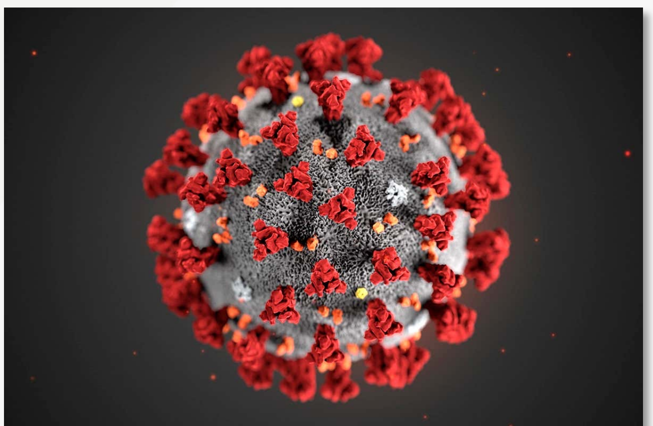
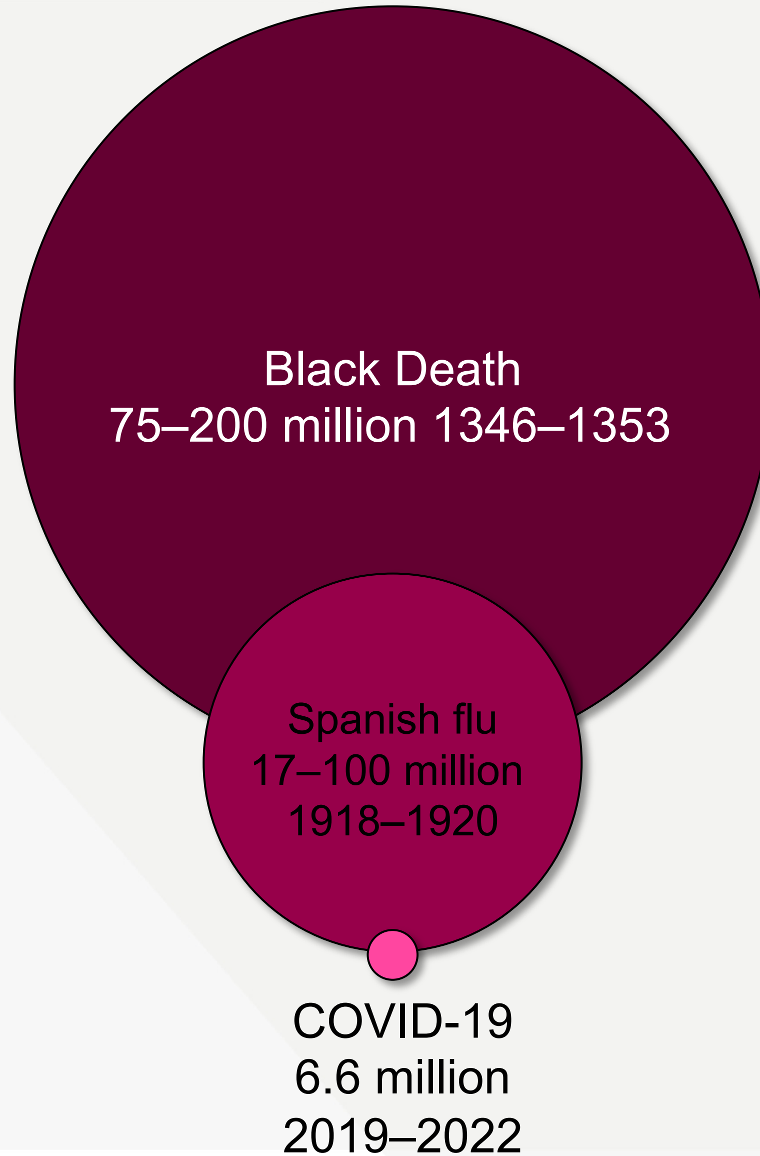
6.6 million deaths
$10trn in forgone GDP over 2020-21

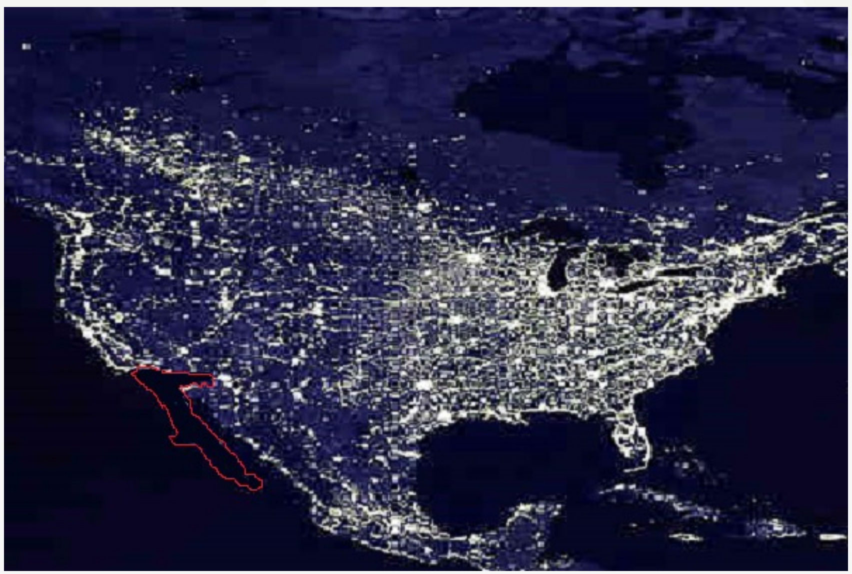




Other spreading processes?

2.7 million customers left without power after cascading outages in Arizona and California in 2011

U.S. economy losses from cascading bankruptcies during the 2008 crisis estimated at the level of $22 trillion



What about social networks?

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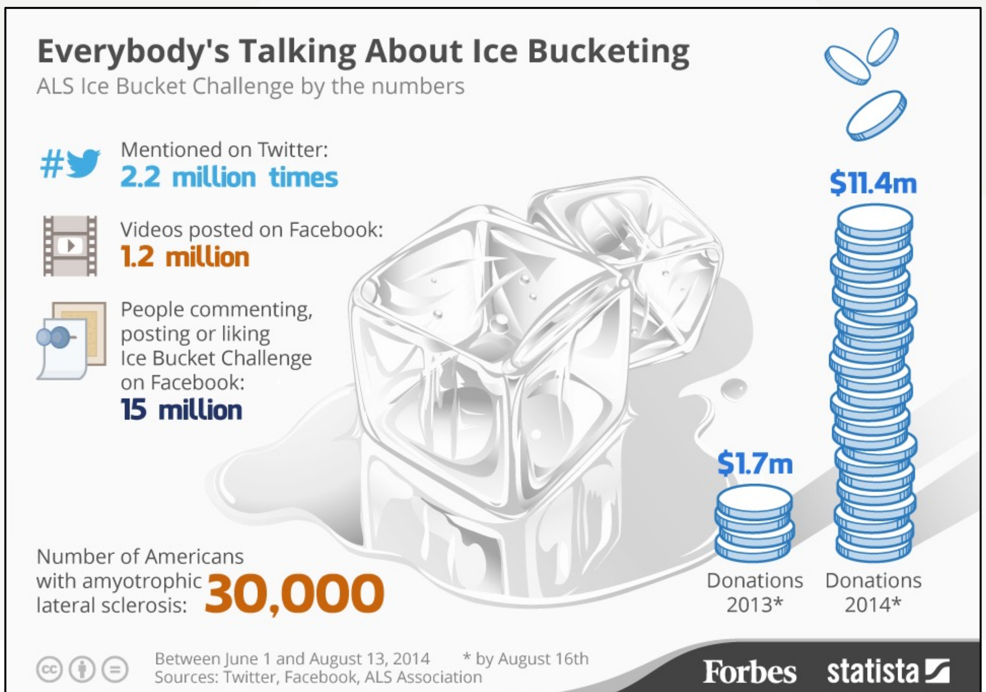
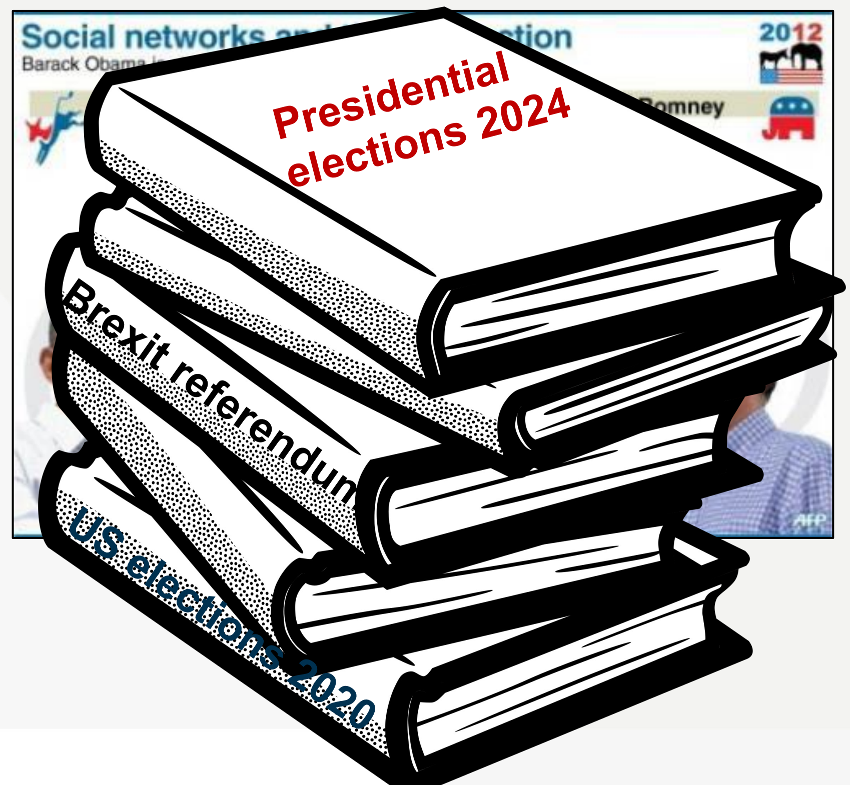
$115 million in donations generated by the ALS ice bucket challenge campaign in social networks



Win of the social media battle in 2012 presidential campaign in United States



What do we want to know?

Microscopic

- Who is most at risk?
- Who should we vaccinate to mitigate the spread?
- How best to use the vaccination budget?
- Identify patient zero

Macroscopic

- What fraction of the population will be ill?
- Will the disease die out? or get out of control?
- Effective vaccination strategy
- Effectiveness of mitigation actions

Modelling epidemic spreading

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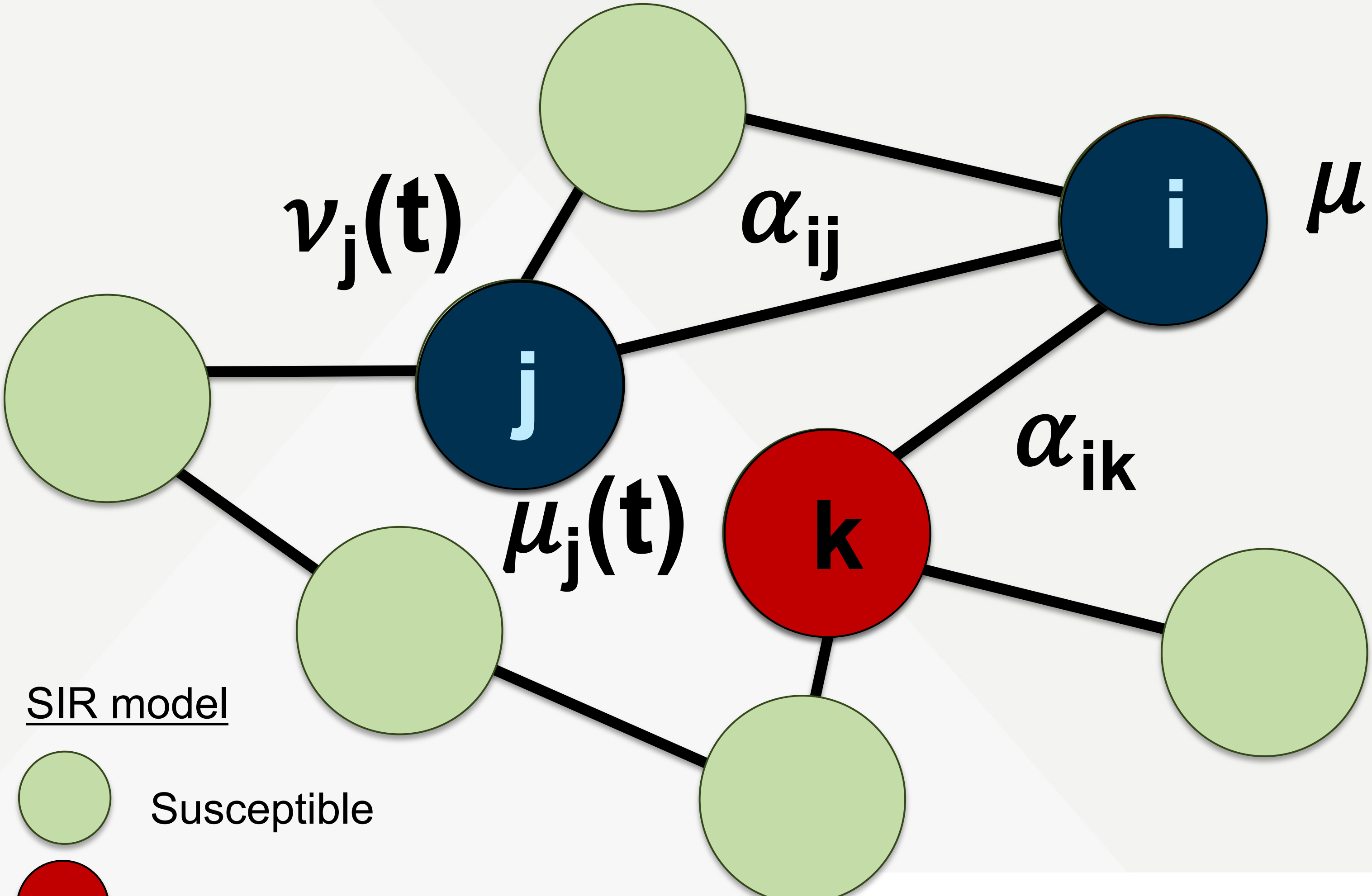

SIR model

- Susceptible
- Infected
- Recovered

$$S(i) + I(j) \xrightarrow{\alpha_{ji}} I(i) + I(j),$$

$$S(i) \xrightarrow{\nu_i(t)} I(i), \qquad S(i) \xrightarrow{\mu_i(t)} R(i).$$

So what do we do?

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Simulations:

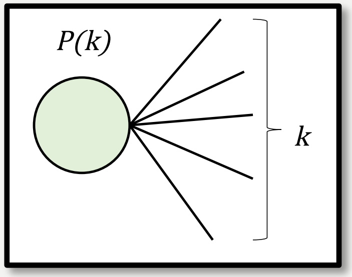
- Flexible, accommodate complex realistic rules ✓
- Volatile, large systems need high computing power for reliable results ✗

Continuous dynamics:

$$\begin{cases} \frac{dS}{dt} = f_S(S, I, R) \\ \frac{dI}{dt} = f_I(S, I, R) \end{cases}$$

- Network agnostic approach ✗
- Uniform dynamics ✗
- Easy to analyze ✓

Network approaches - degree distribution:



- Percolation or dynamics ✓
- Problematic for contact network ✗
- Ignore actual architecture, no results for individual nodes ✗

Message passing methods

Probabilistic approach:

- Principled ✓
- Models both statics and dynamics for individuals ✓
- Allows for specific decisions ✓
- Computationally efficient based on message passing

$$P(\sigma_i^t) = f(\overline{\sigma}_{\partial_i}^t)$$

- Exact on trees, approximate for loopy networks ✗
- Exact for unidirectional processes ✗

Dynamic Message Passing - main idea

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For any node *i* in time

SSSSSIIIIIIIIIIIIRRRRRRRRRRRRRRR

↑ t_{SI} ↑ t_{IR}

- It is sufficient to know the transition times
- Dynamics is irreversible! Exact on trees!
- Probabilities of neighboring nodes are interlinked
- **Aim:** calculate the probability of a node being infected/recovered $P^i_\sigma(t)$, $\sigma \in \{S, I, R\}$
- Algorithmic complexity $O(ET)$

A.Y. Lokhov, M. Mezard, L. Zdeborova, Phys. Review E (2015)
B. Karrer B, MEJ. Newman Phys Rev E (2010)
F. Altarelli, A. Braunstein, L. Dall'Asta, R. Zecchina J Stat Mech (2013)

Dynamic message passing

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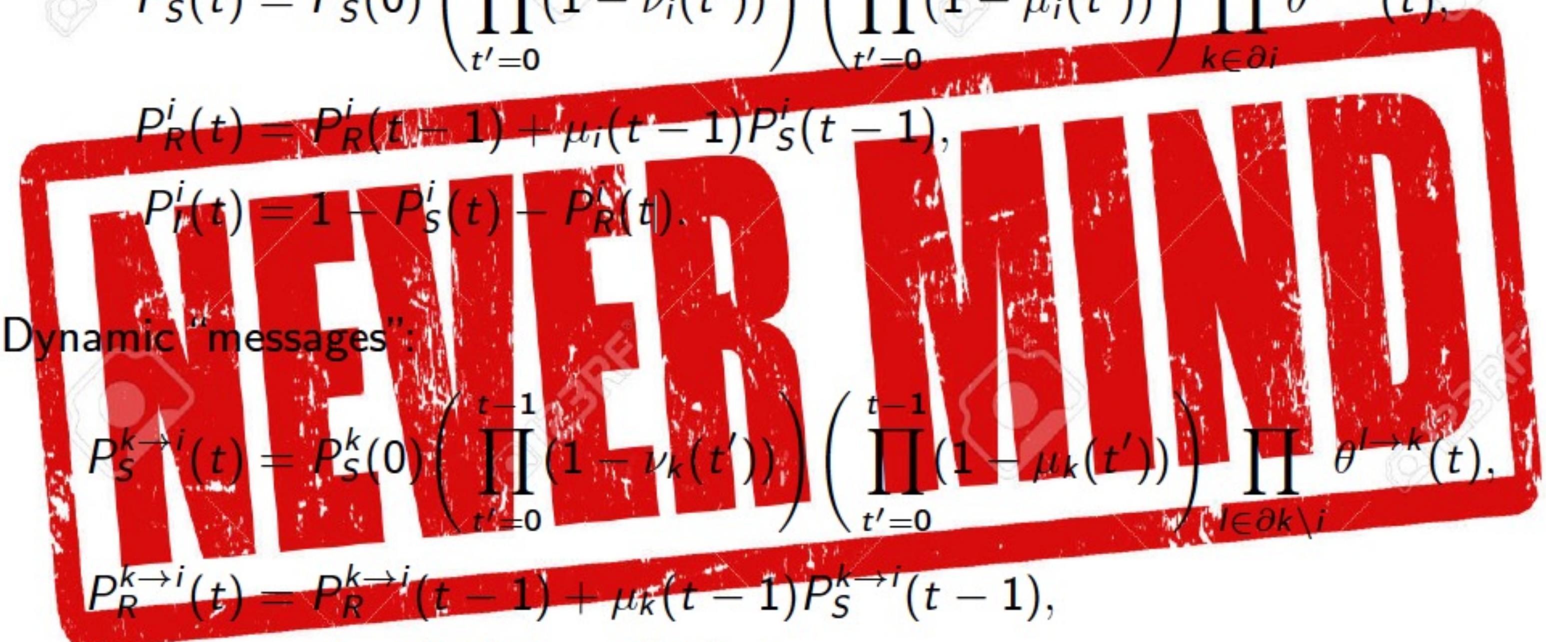
Marginal probabilities:

$$P_S^i(t) = P_S^i(0)\left(\prod_{t'=0}^{t-1}(1-\nu_i(t'))\right)\left(\prod_{t'=0}^{t-1}(1-\mu_i(t'))\right)\prod_{k\in\partial i}\theta^{k\to i}(t),$$

$$P_R^i(t) = P_R^i(t-1) + \mu_i(t-1)P_S^i(t-1),$$

$$P_I^i(t) = 1 - P_S^i(t) - P_R^i(t).$$



Dynamic "messages":

$$P_S^{k\to i}(t) = P_S^k(0)\left(\prod_{t'=0}^{t-1}(1-\nu_k(t'))\right)\left(\prod_{t'=0}^{t-1}(1-\mu_k(t'))\right)\prod_{l\in\partial k\setminus i}\theta^{l\to k}(t),$$

$$P_R^{k\to i}(t) = P_R^{k\to i}(t-1) + \mu_k(t-1)P_S^{k\to i}(t-1),$$

$$P_I^{k\to i}(t) = 1 - P_S^{k\to i}(t) - P_R^{k\to i}(t),$$

$$\theta^{k\to i}(t) = \theta^{k\to i}(t-1) - \alpha_{ki}\phi^{k\to i}(t-1),$$

$$\phi^{k\to i}(t) = (1-\alpha_{ki})\phi^{k\to i}(t-1) + P_I^{k\to i}(t) - P_I^{k\to i}(t-1).$$

Initial conditions:

$$\theta^{i\to j}(0) = 1, \qquad \phi^{i\to j}(0) = \delta_{\sigma_i^0, I} = P_I^i(0).$$

But what about optimization?

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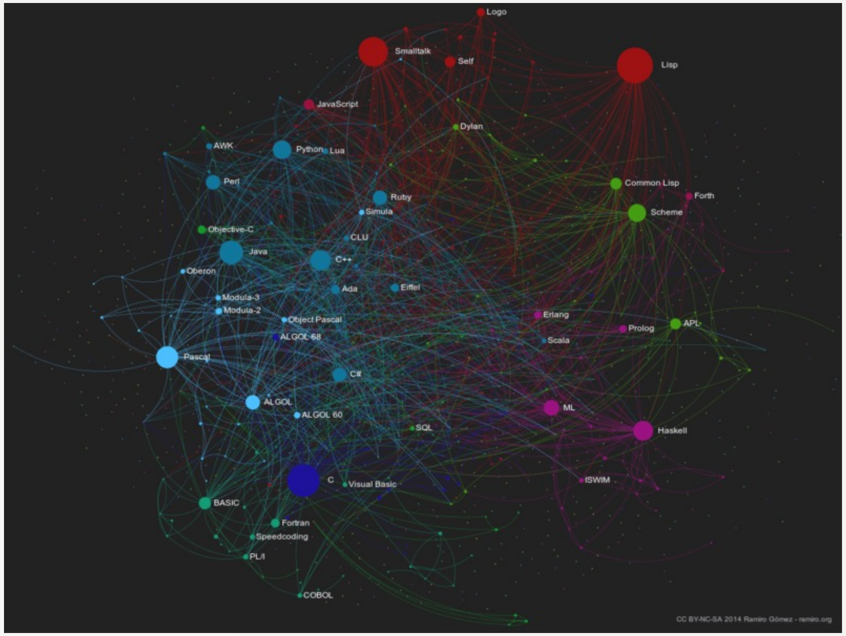
Simulations:

- Volatile
- Computationally difficult

Network approaches:

- High-degree nodes
- Betweenness centrality
- Random walk centralities
- k-shell decomposition
- Network decomposition



End-of-process optimization we are after is more difficult

Our approach

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- Constrained optimization
- Adopted from ***optimal control***

$$\mathcal{L} = \underbrace{\mathcal{O}}_{\text{objective}} + \underbrace{\mathcal{B} + \mathcal{P} + \mathcal{I} + \mathcal{D}}_{\text{constraints}}$$

$\mathcal{O}$ - Objective function (minimize/maximize)

$\mathcal{B}$ - infection/marketing budget

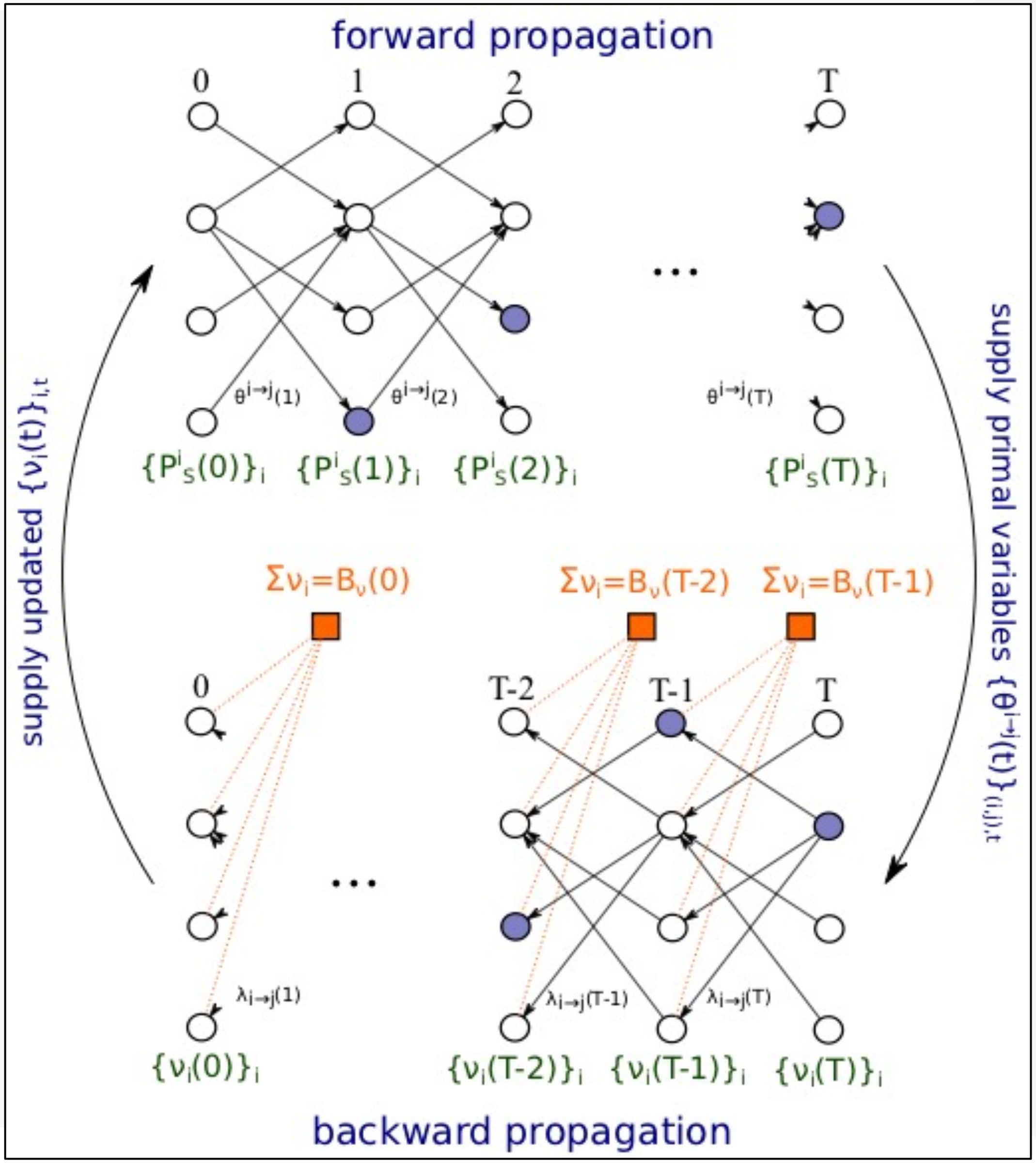
$\mathcal{P}$ - constraints on parameters

$\mathcal{I}$ - Initial conditions

$\mathcal{D}$ - Dynamics constraints

A.Y. Lokhov and D. Saad, Proc. Nat. Academy of Sci. USA, 114, E8138-E8146, (2017).

Obtaining a solution



Node targeting

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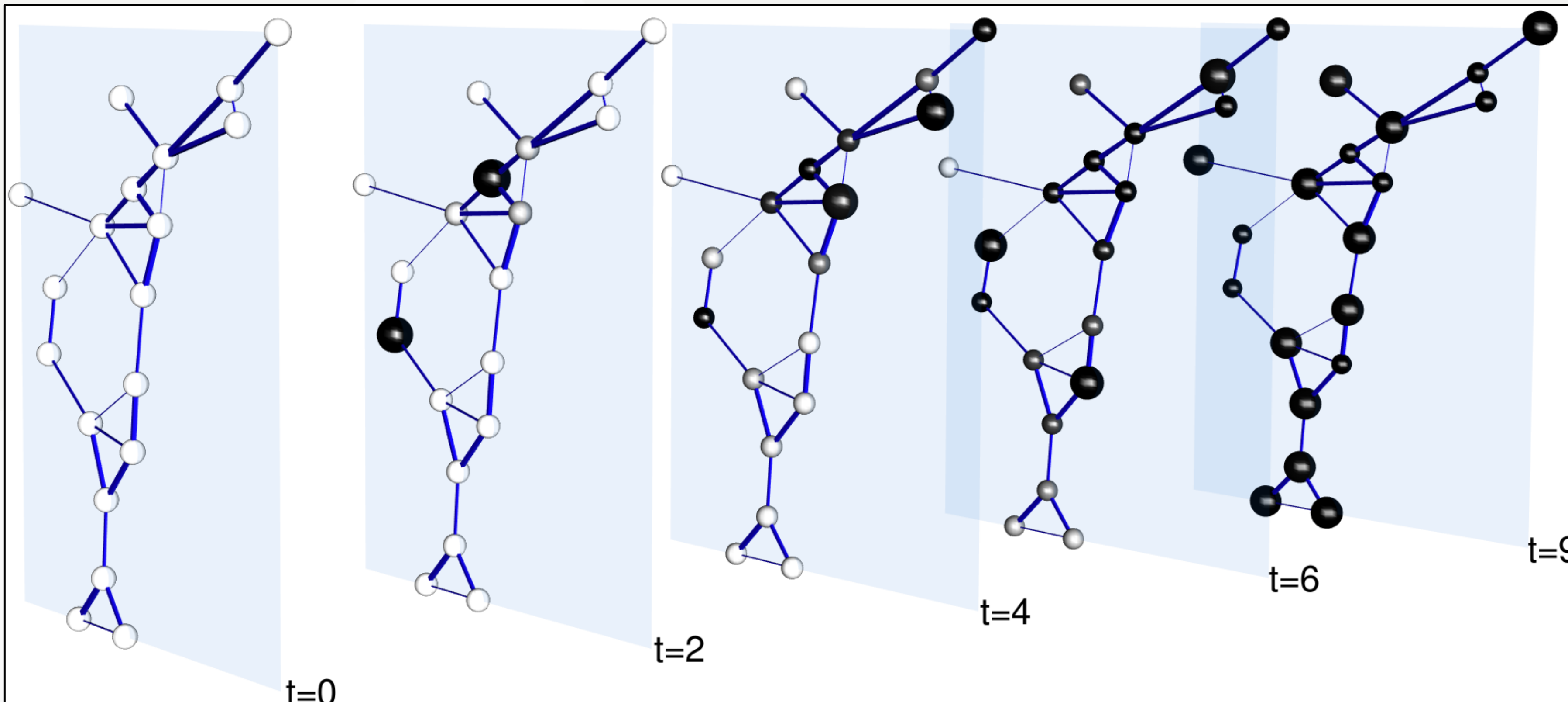
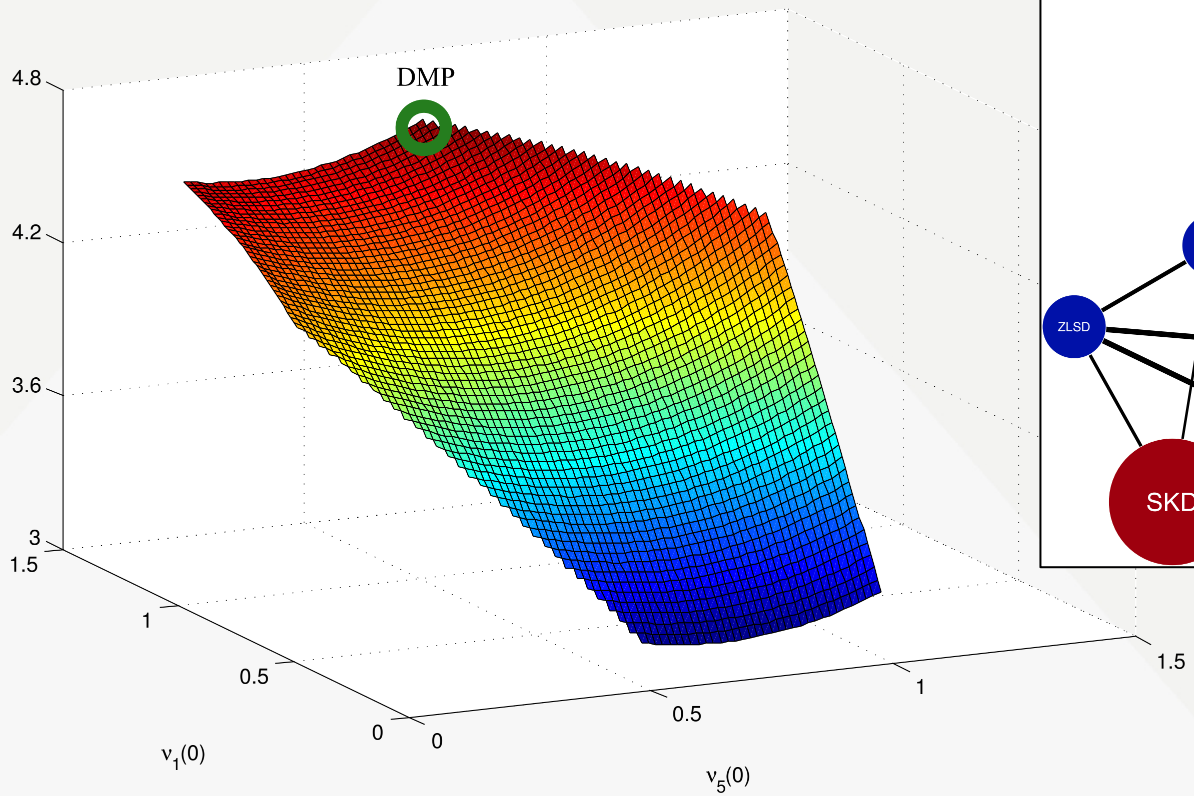
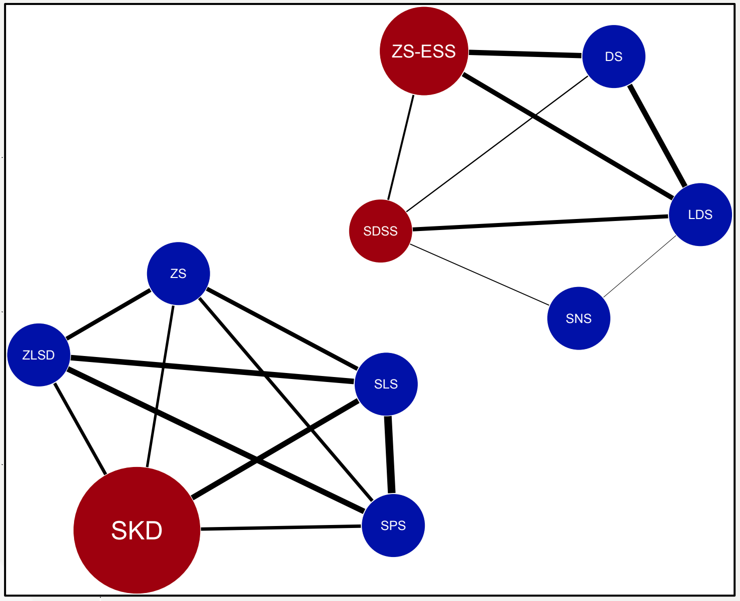
Dynamic resource allocation aims at targeting specific nodes at required times (larger-sized nodes). Color intensity indicates the probability value $N=19$, $B(t)=0.1N$, $\alpha_{ij} \in [0,1]$



Validation of solutions

Validation of the scheme in the seeding case on a small network (Slovene parliamentary political parties) with an explicit evaluation of the objective function, $B(0)=1.5$



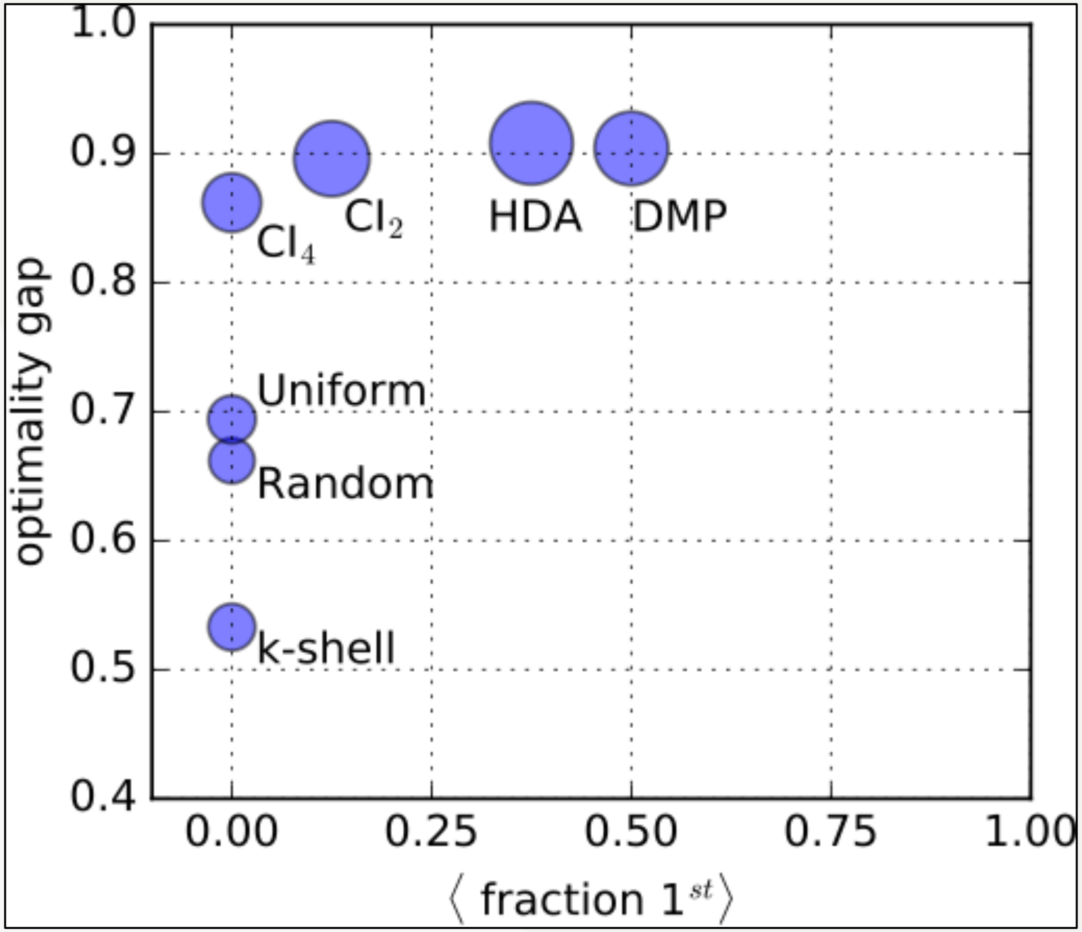


Comparison with other methods

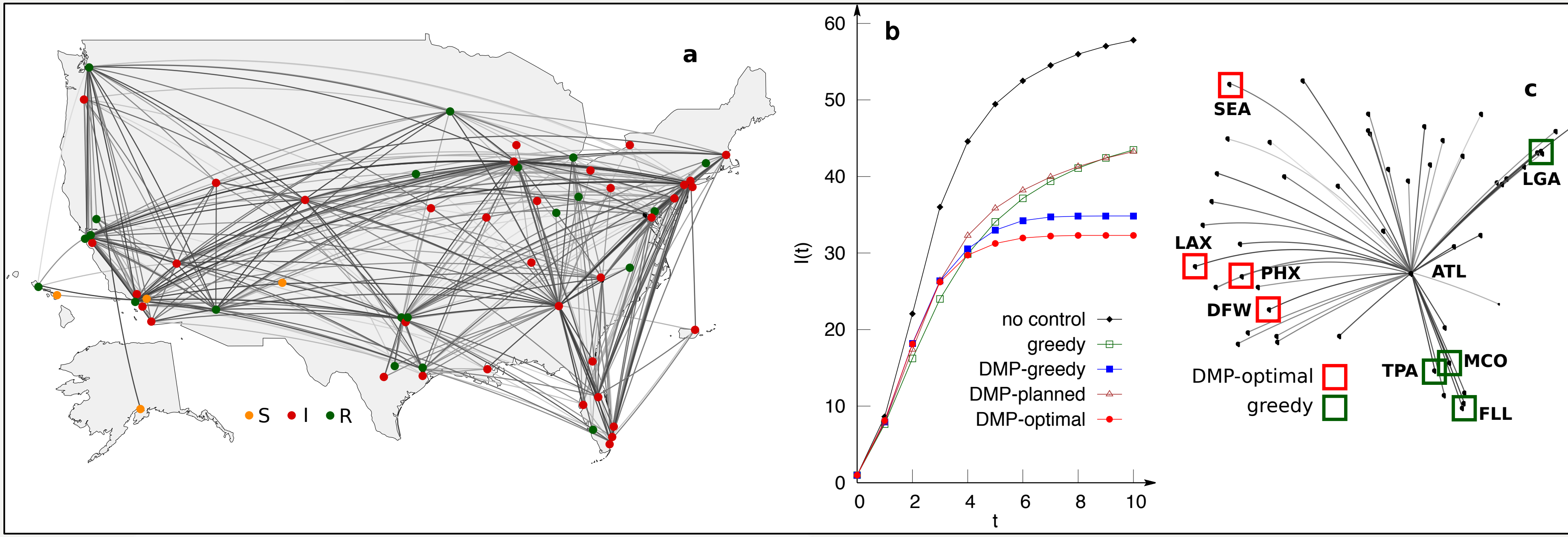
Existing algorithms: Random, HDA, k-shell, CI2 and CI4

Networks: Road EU, Protein, US Power Grid, GR Collaborations, Internet, Web-sk, Scale-Free, Erdos-Renyi

Results: optimal DMP not always best but better overall



Mitigating an epidemic



$N=61,\ E=383,\ B(t)=0.05N,\ T=10,\ \alpha_{ij}$ according to the number of passengers

- DMP-planned (offline resource allocation with T-horizon)
- Greedy: vaccination of nodes at “high risk”
- DMP-greedy (optimization at one-time step only)
- DMP-optimal: repeated re-evaluation of T-horizon problem based on feedback from current realization of dynamics

Competitive/collaborative processes

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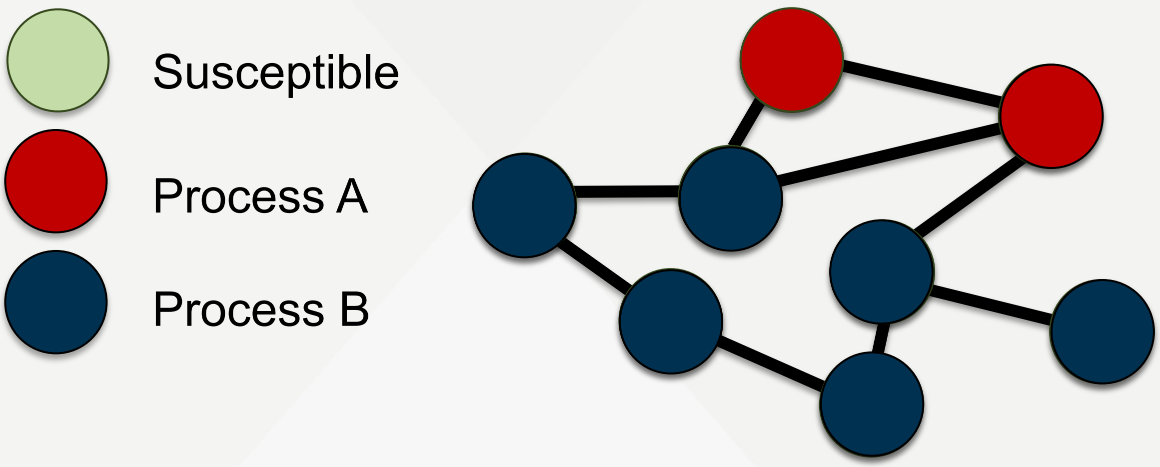
Competitive process

- Multiple processes
- States are mutually exclusive
- Get there first
- Block opponent

Collaborative process

- Multiple processes
- Infected nodes more/less susceptible to other processes
- Immune one to reduce spread of the other
- **Example:** HIV/Tuberculosis
- Exploit one to increase spread of another
- **Example:** Supporters of certain parties less likely to believe in climate change

SI competitive/collaborative processes



$$S(i) + A(j) \xrightarrow{\alpha^A_{ji}} A(i) + A(j),$$

$$S(i) + B(j) \xrightarrow{\alpha^B_{ji}} B(i) + B(j),$$

$$A(i) + B(j) \xrightarrow{\alpha^{BA}_{ji}} AB(i) + B(j),$$

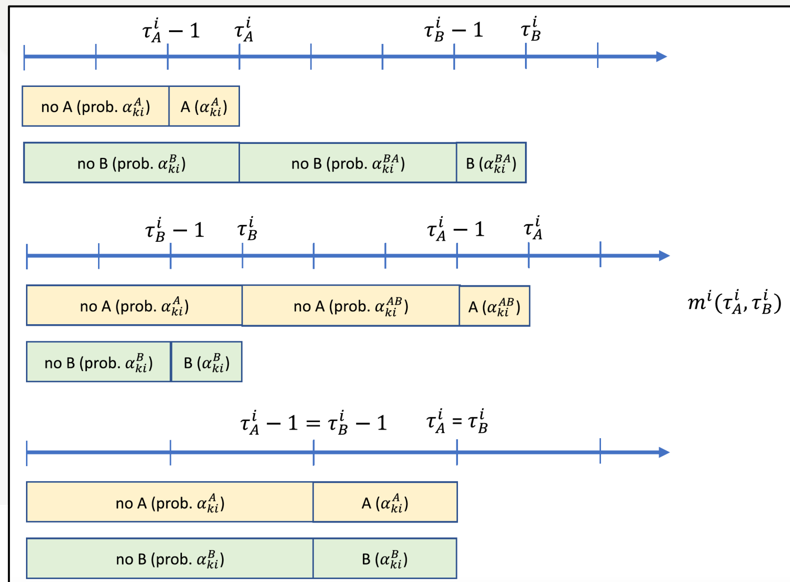
$$A(i) + B(j) \xrightarrow{\alpha^{AB}_{ij}} A(i) + AB(j).$$

Competitive: for any node i in time

$SSSSSI_AI_AI_A$ or $SSSSSI_BI_BI_B$

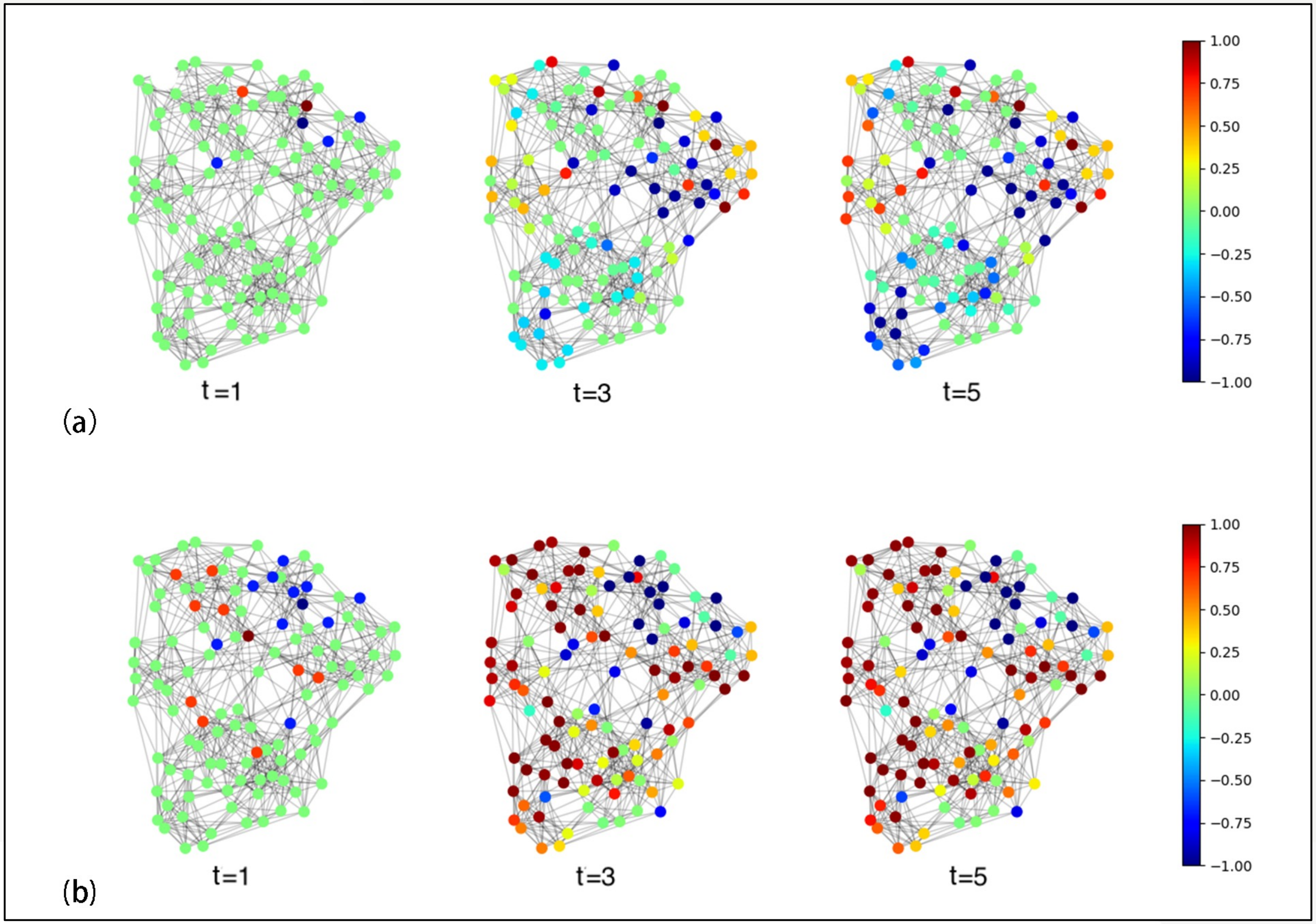
Collaborative: more complex

$S, I_A, I_B, I_AI_B, I_BI_A$



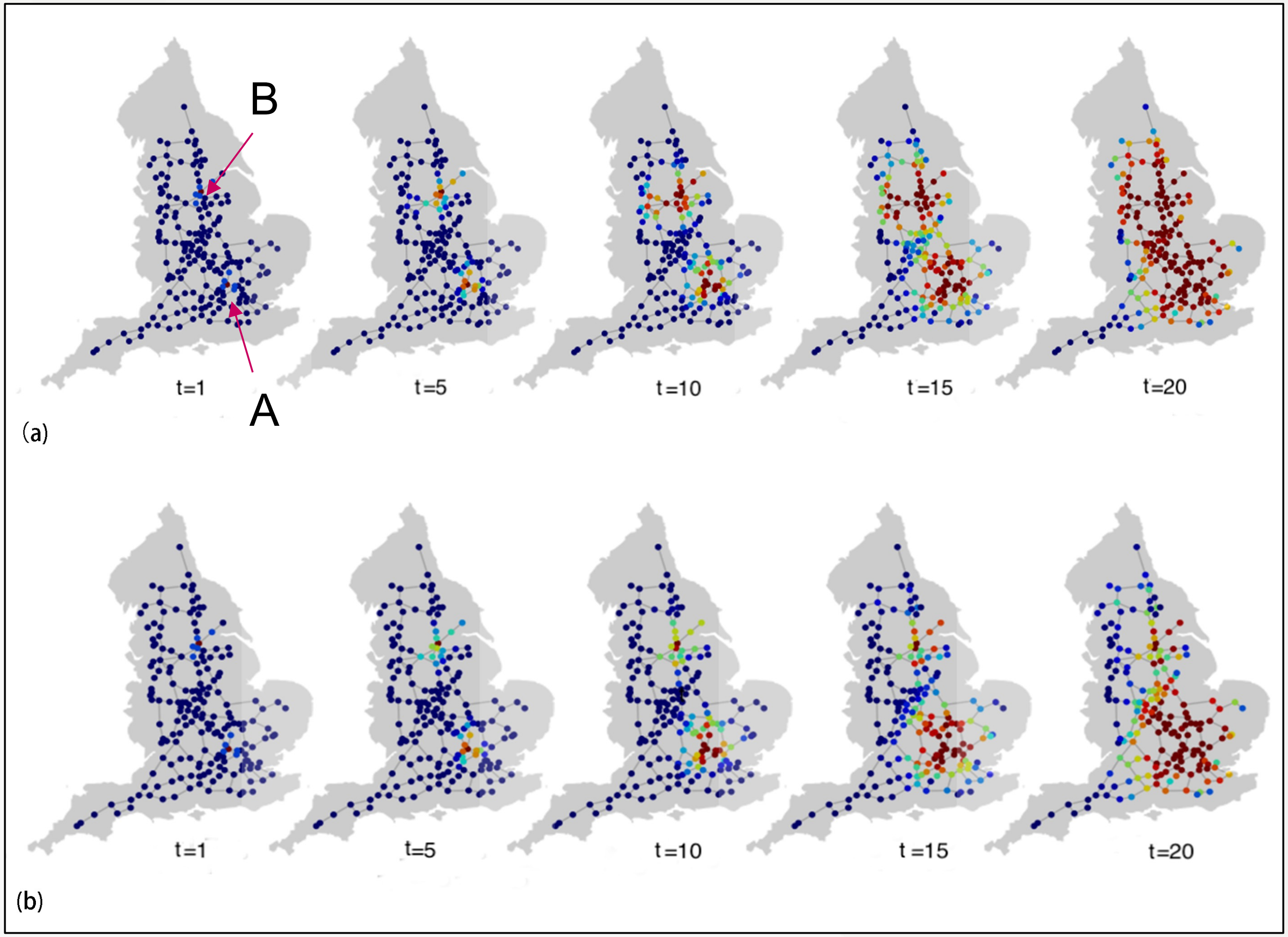
Competitive processes in time

Football net (115 nodes), p_A=p_B =0.7, budget for B(t=0) =1 and for A is 1 per time step; A process is optimized: (a) DMP-greedy; (b) DMP-optimal; heat bar – $P_i^A(t) - P_i^B(t)$



Containment optimisation

- Infecting London (A -red) and Leeds (B -blue); $p_A = p_B = 0.2$, $p_{AB} = p_{BA} = 0.99$;
- Process B supports A
- vaccination budget against B - one unit per time step;
- Color $1 - P_i^s(t)$
- (a) Free spread; (b) DMP-optimal



Presymptomatic but infectious

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- For COVID-19, infectiousness is 2.3 days *prior to symptoms*
- Consider model SEIR, Exposed is a presymptomatic but infectious state
- Exposed infection rate - α, Infected rate - β

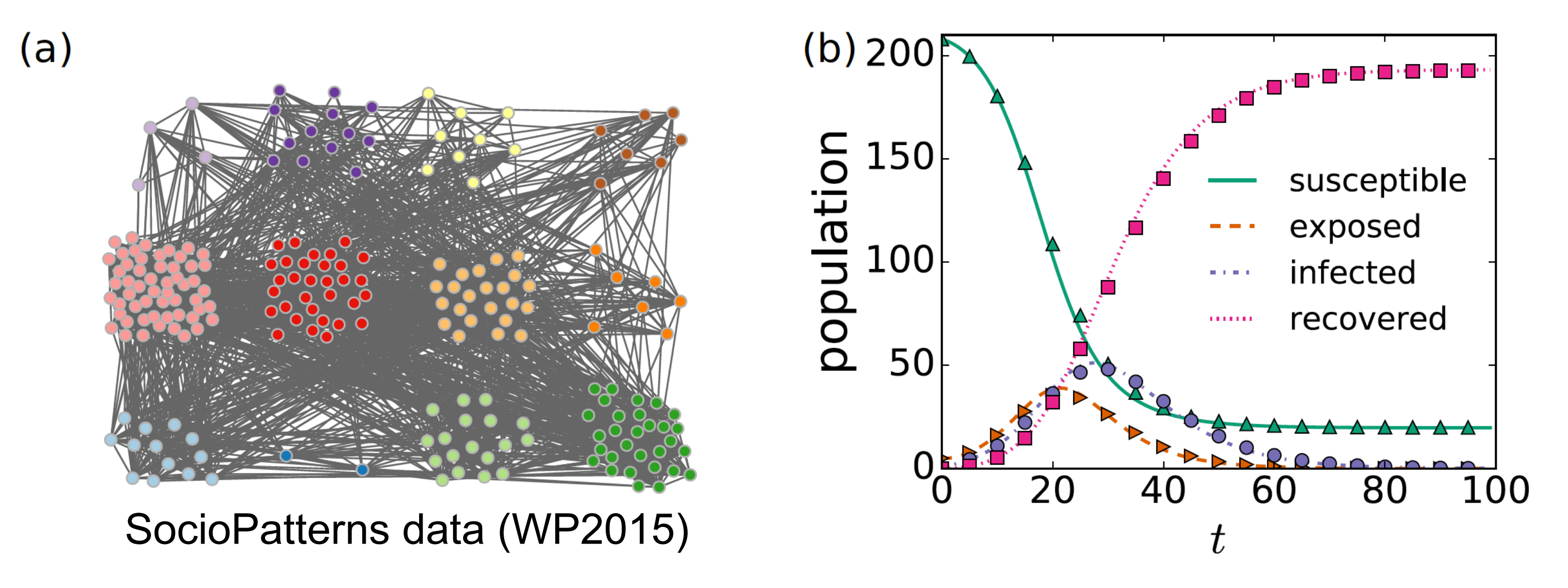
$$S(i) + E(j) \xrightarrow{\alpha_{ji}} E(i) + E(j),$$

$$S(i) + I(j) \xrightarrow{\beta_{ji}} E(i) + I(j),$$

$$E(i) \xrightarrow{\nu_i} I(i),$$

$$I(i) \xrightarrow{\mu_i} R(i),$$

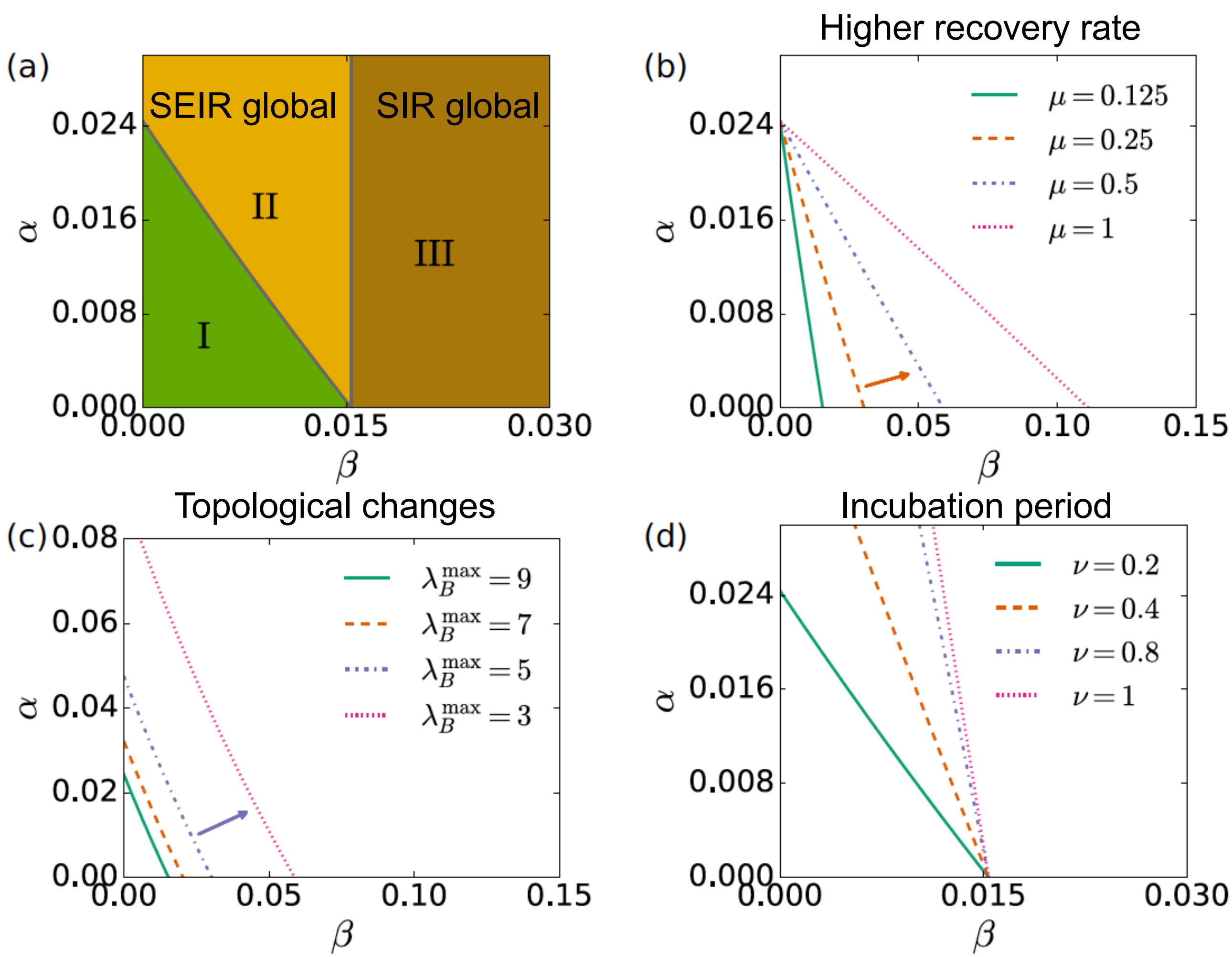
Phase diagram



- Analyze effectiveness of mitigation measures
  - Reduce transmission rate (face masks, social distancing)
  - Topological changes (self-isolation, working from home)
- Spreading measure relating the dynamical properties to the epidemiological parameters and network structure (***not*** R_0 nor $R(t)$)

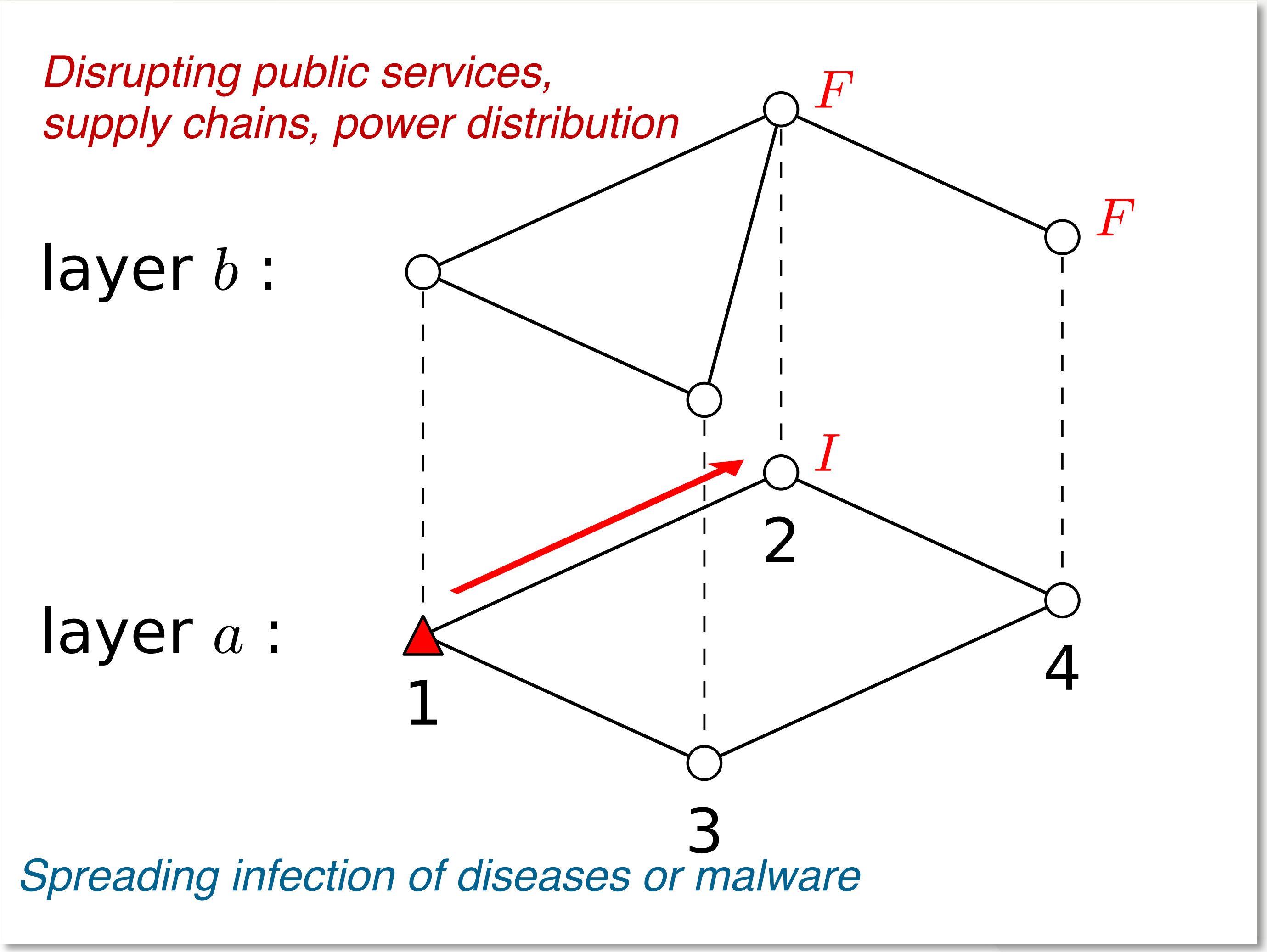
Phase diagram

Critical line separating the parameter regions of localized infections and global outbreaks



Interacting systems

Where spreading processes in two separate systems are interlinked and can be addressed by one of them, for instance



Interacting systems - model

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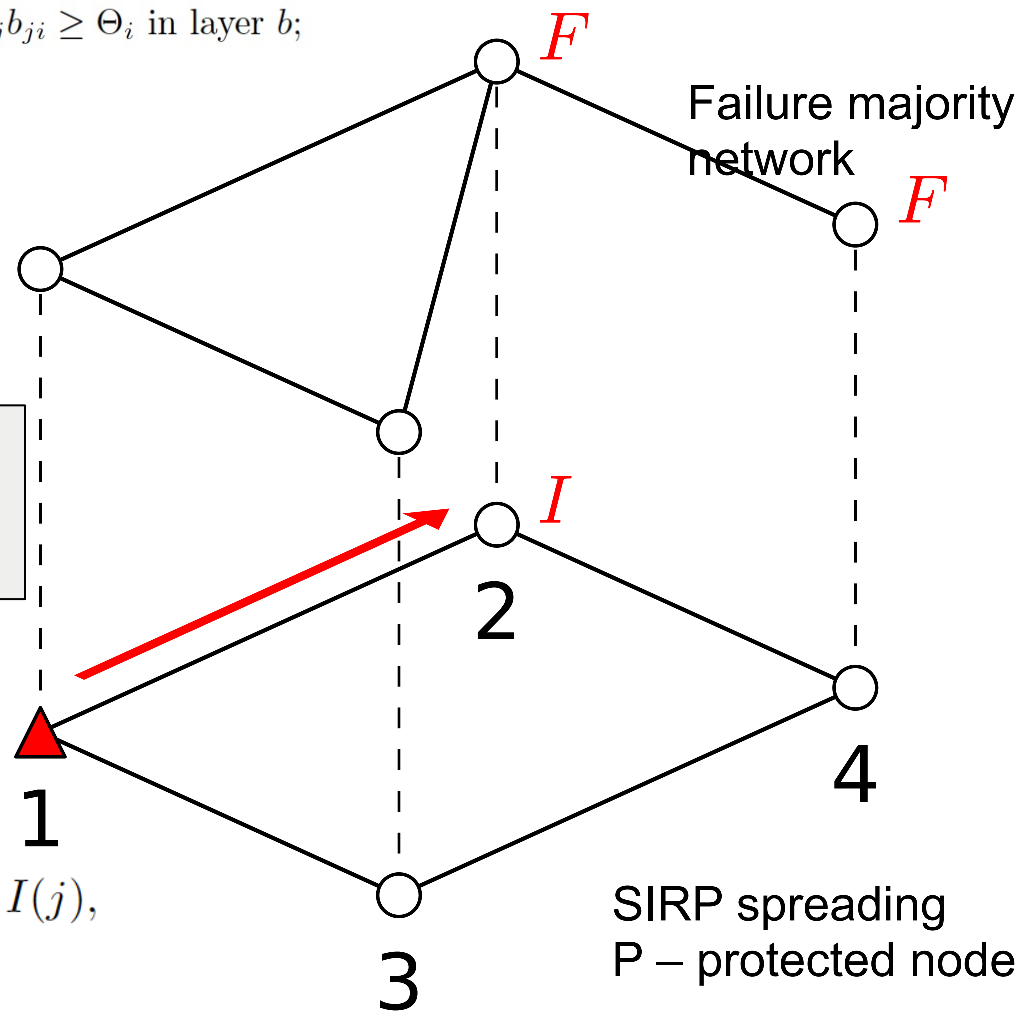
$$x_i = \begin{cases} 1, & \text{either (i) node } i \text{ in state } I \text{ or } R \text{ in layer } a, \\ & \quad \text{or (ii) } \sum_{j \in \partial_i^b} x_j b_{ji} \geq \Theta_i \text{ in layer } b; \\ 0, & \text{otherwise.} \end{cases}$$

layer b :

b_{ij} – weight between nodes in layer b
Θ_i – Failure threshold

layer a :



SIRP spreading
P – protected node

$$S(i) + I(j) \xrightarrow{\beta_{ji}} I(i) + I(j),$$
$$I(i) \xrightarrow{\mu_i} R(i),$$
$$S(i) \xrightarrow{\gamma_i(t)} P(i),$$

Experiments - asymptotics

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Failure measures:

$$\rho_I(t) + \rho_R(t) = \frac{1}{N} \sum_{i \in V_a} P_I^i(t) + \frac{1}{N} \sum_{i \in V_a} P_R^i(t).$$

$$\rho_F(t) = \frac{1}{N} \sum_{i \in V_b} P_F^i(t).$$

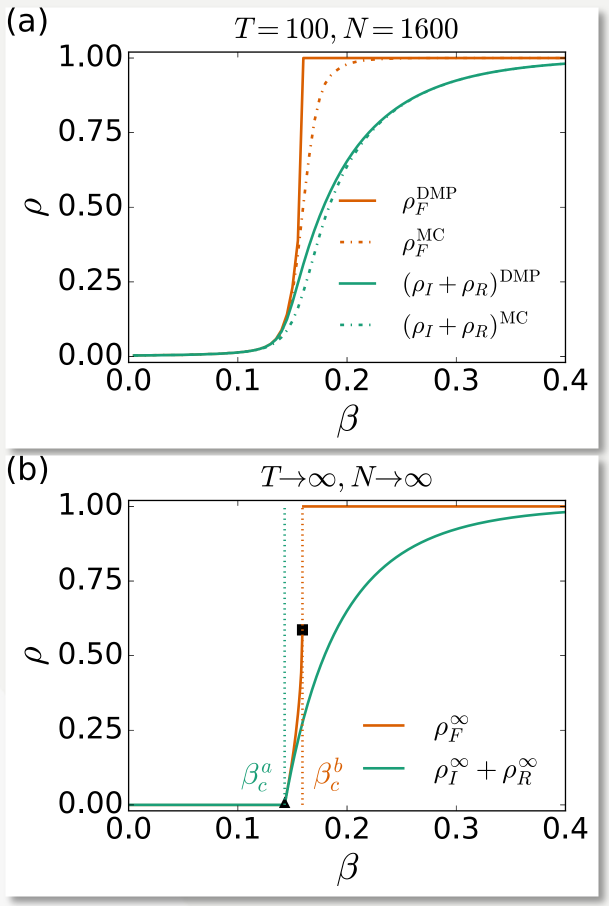
Protection -

$$\min_{\gamma} \quad \mathcal{O}(\gamma) := \rho_F(T) = \frac{1}{N} \sum_{i \in V_b} P_F^i(T),$$

$$\text{s. t.} \quad 0 \leq \gamma_i(t) \leq 1 \quad \forall i, t,$$

$$\sum_{i \in V_b} \sum_{t=0}^{T-1} \gamma_i(t) \leq \gamma^{\text{tot}},$$

Cascading failure
Rand-Reg K=5 N=1600



Experiments – optimal protection

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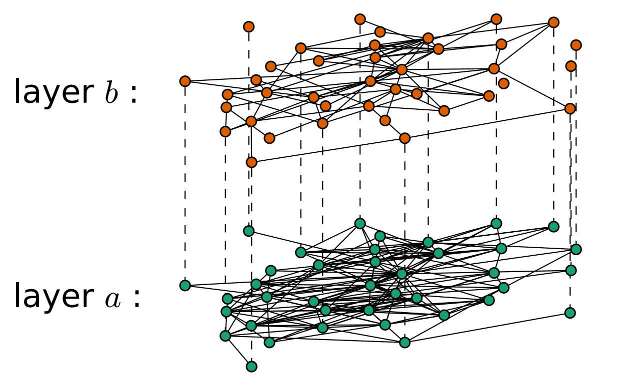
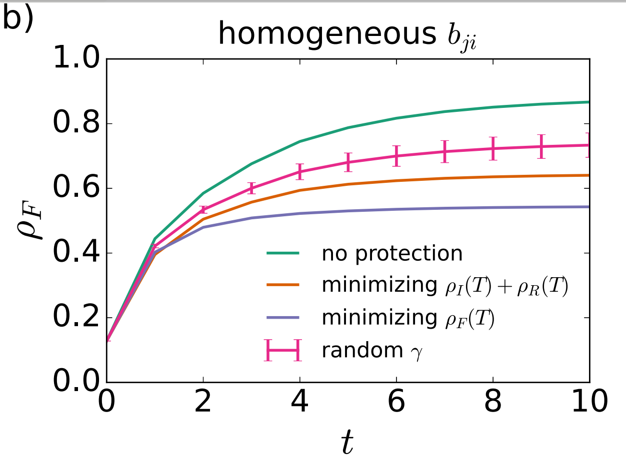
Kapferer's tailor shop N=39

(a)

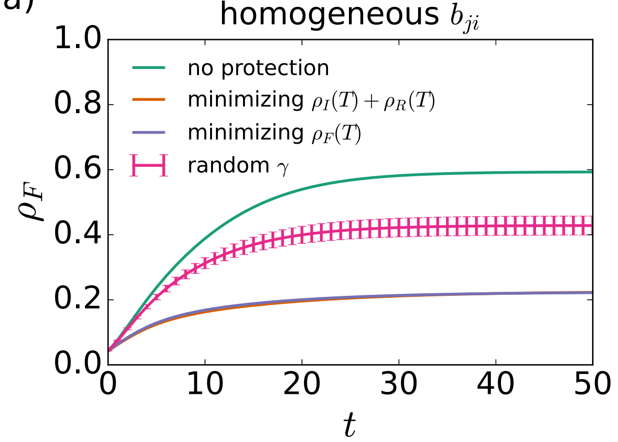


(b)



Communication network N=118, $\Theta_{\mathrm{i}} - 60\%$ of neighbours

(a)



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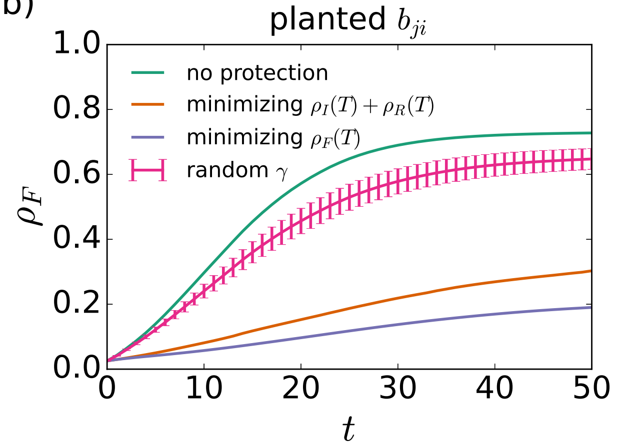
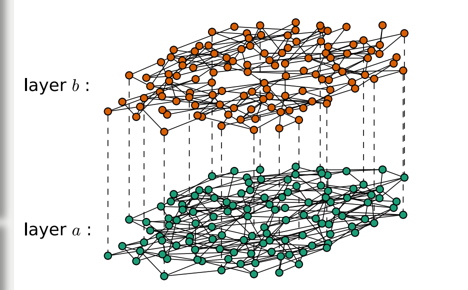


(a) Small world network, Rand Reg K=4 rewiring prob. 0.3.

(b) IEEE 118-bus



What else can we do?

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- **Timely resource allocation** to maximize impact at given time (maximize impact ahead of crucial votes)
- Consider **accessibility of nodes** (not all villages infected with Ebola are accessible)
- Address **dynamically changing** parameters/topology
- **Identify patient zero** from measurements
- **Optimal deployment of sensors**

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