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Analytic Description of Beta-Decay Electron Thermalization in Kilonovae Ejecta

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Thermalization

- $t_{loss}(E, t)$ - energy-loss timescale of decay product.
- Initially for all decay products (excluding neutrinos) $t_{loss} \ll t \rightarrow \dot{Q}_{dep} = \sum_{\mu=\alpha,\beta,\gamma} \dot{Q}_{\mu}$
  - Electron that is emitted immediately loses all its energy $\equiv$ Efficient Thermalization
- But over time $t_{loss} \sim t \rightarrow \dot{Q}_{dep} < \sum_{\mu=\alpha,\beta,\gamma} \dot{Q}_{\mu}$
  - Electron gradually loses its energy $\equiv$ Inefficient Thermalization
- Interpreting kilonovae observations requires understanding the thermalization of decay products (for $t \gtrsim 1 - 2$ days , γ-particles mostly escape, leaving e, α-particles as main heating source. Fission?)

- Thermalization depends on:
  - ejecta mass, velocity (Metzger+2010 , Barnes+2016 , Kasen+2019)
  - Composition (Y_e, s_0)
  - nuclear physics inputs (Zhu+2021, Barnes+2021)

- <u>Research Goal:</u>

To derive an analytical description for electron thermalization that is valid for a wide range of ejecta parameters

- <u>What we found:</u>
  - t_e – inefficient thermalization timescales – are primarily dependent on ρt^3 of ejecta, with small corrections for different compositions (Y_e , s_0).
  - Largely robust to nuclear physics uncertainties.
  - Results can be used to constrain ejecta mass and velocity from measurements, “inverse problem” approach

Outline of Our Work

- Ran nuclear-reaction network SkyNet for different homologously expanding ejecta of uniform densities - discussed in next slide.
- Computed time-dependent energy released by electrons, including spectra (BetaShape, Mougeot 2017)
- Defined and calculated instantaneous energy deposition + full energy deposition of electrons (under assumption of electron confinement due to weak magnetic fields)
- Define and calculate t_e - inefficient thermalization timescales.
  - Find analytic description for $t_e(\rho t^3)$ with weak dependence on Y_e, s_0
  - Using these, provide analytical interpolating functions for deposition - $\dot{Q}_{dep}^{interp}$

Nucleosynthesis Calculations

- SkyNet calculates NSE for initial TD conditions of ejecta as input (either $\{Y_e, s_0, T_0\}$ or $\{Y_e, \rho_0, T_0\}$).
  - Evolves network using density history: $\rho(t) = \begin{cases} \rho_0 e^{-t/\tau} & \text{for } t \leq 3\tau \\ \rho_0 \left(\frac{3\tau}{et}\right)^3, & \text{otherwise} \end{cases}$
  - Altogether 4 parameters must be specified: $\{Y_e, s_0, T_0, \tau\}$
  - Lack of uniformity in the community (different $T_0,\ \tau$)
- We seek to examine influence of $\rho t^3, Y_e, s_0$ of ejecta on thermalization.
  - Initialize SkyNet with $Y_e,\ s_0,\ T_0 = 10\,\text{GK} \overset{EoS}{\rightarrow} \rho_0$
  - $\rho_0,\ \rho t^3 \rightarrow \tau$, using density history
  - Taking different T_0 would give different τ, but nucleosynthesis will be equivalent!
- Parameter Range ($T_0 = 10$ GK for all runs):
  - $1 \leq s_0 \leq 10^2 [k_b/\text{baryon}]$, semi-linearly spaced. (from simulations, $s_{0,avg} \approx 10 - 20\,[k_b/\text{baryon}]$)
  - $0.05 \leq Y_e \leq 0.45$, linearly spaced.
  - $10^{-3} \leq \rho t^3 \leq 10^2$ in units of $(\rho t^3)_0 = \dfrac{0.05 M_\odot}{4\pi (0.2c)^3}$, logarithmically spaced.

Electron Energy Losses

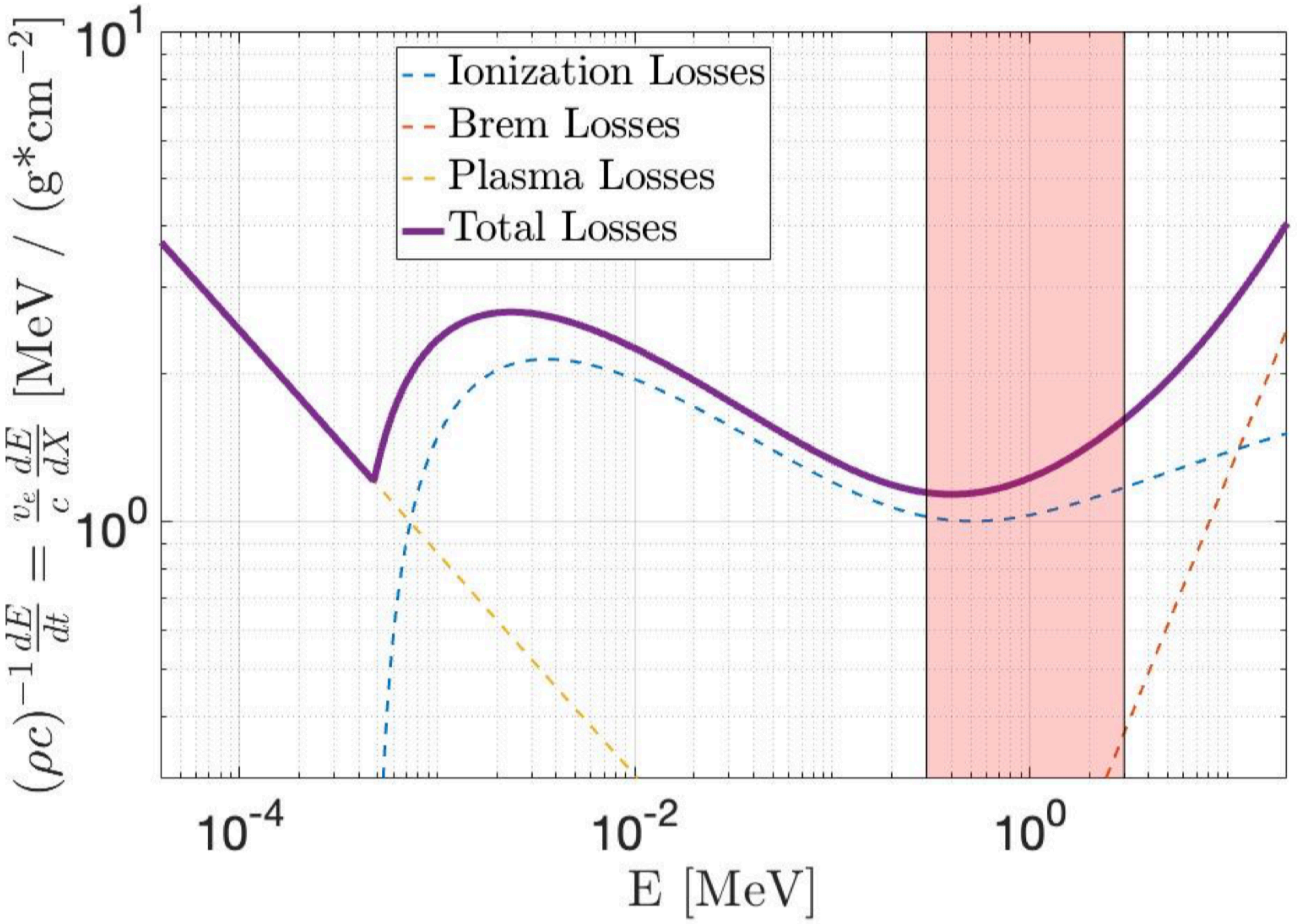


Figure 1: Energy loss rate of electrons propagating in a singly ionized $\chi_e = 1$ Xe plasma ($Z = 54$, $A = 131$). We take $\hbar\omega_p = 10^{-7} eV$. Shaded area shows typical average initial energies of β-decay electrons. For most relevant energies, ionization losses dominate.

- Time-dependent, mass-weighted composition:

$$\left(\frac{dE}{dX}\right)_{tot} = \sum_{iso} A_{iso} Y_{iso} \left(\frac{dE}{dX}\right)_{iso}$$

- Ionization Losses dominate the energy loss
  - Used Bethe-Bloch formula, with $\bar{I}$ from Segré 1977

Energy Deposition Description

Instantaneous Deposition

- Fraction of energy instantaneously deposited by electron with initial E_i at time t is approximated as:
  - $f_{dep}(E_i, t) = \begin{cases} 1 & \text{for } t_l \le t \\ \frac{t}{t_l} & \text{for } t_l \ge t \end{cases}$
  - Where $t_l(E_i, t) = E_i\left(\frac{dE}{dt}\right)^{-1}$ is the energy loss timescale, and $\frac{dE}{dt} = \rho v \frac{dE}{dX}$.
- Total instantaneous deposition calculated as:
  - $f^e_{tot}(t) = \frac{\dot{Q}_{inst}}{\dot{Q}_e} = \frac{1}{\dot{Q}_e}\int f_{dep}(E,t)\cdot E \frac{d\dot{N}_e(E,t)}{dE} dE$
- We define t_e as the time for which:
  - $f_{tot}(t_e) \equiv 1 - e^{-1}$

Delayed Deposition

- Also calculated full, delayed energy deposition :

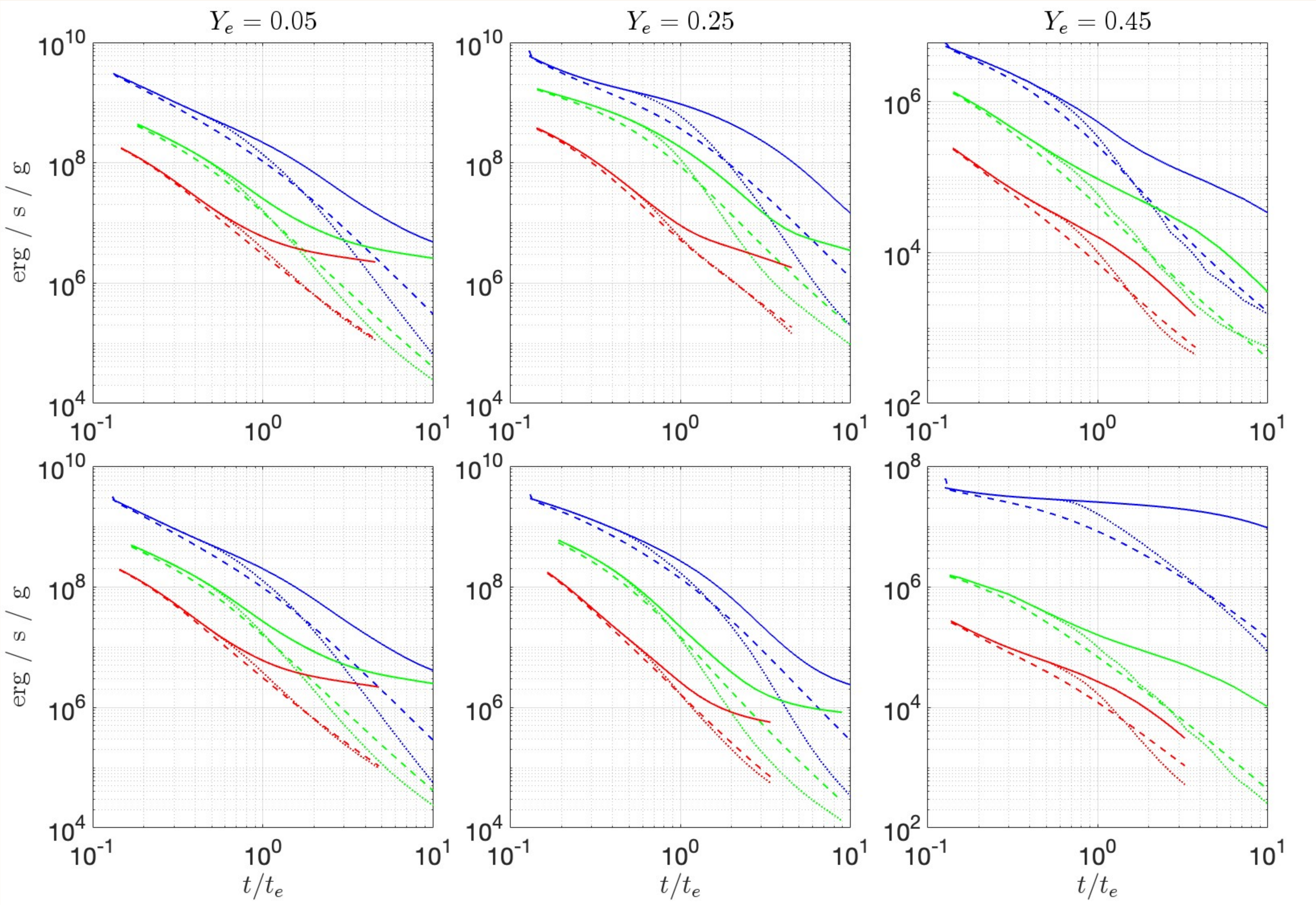
$$\dot{Q}_{dep}(t) = \int dE \frac{dE}{dt}(E,t) \times \frac{dN}{dE}(E,t)$$

- $\frac{dN}{dE}(E,t)$ is the electron distribution, dictated by: $\frac{\partial}{\partial t}\left(\frac{dN}{dE}\right) = -\nabla_E\left(\dot{E}\frac{dN}{dE}\right) + \dot{N}(E,t)$
- $\dot{E} = -x\frac{E}{t} - \rho v \frac{dE}{dX}$

(ad. losses + stopping-power) ,

$x = 1$,(2) for UR, (NR)

Instantaneous vs. Delayed Deposition



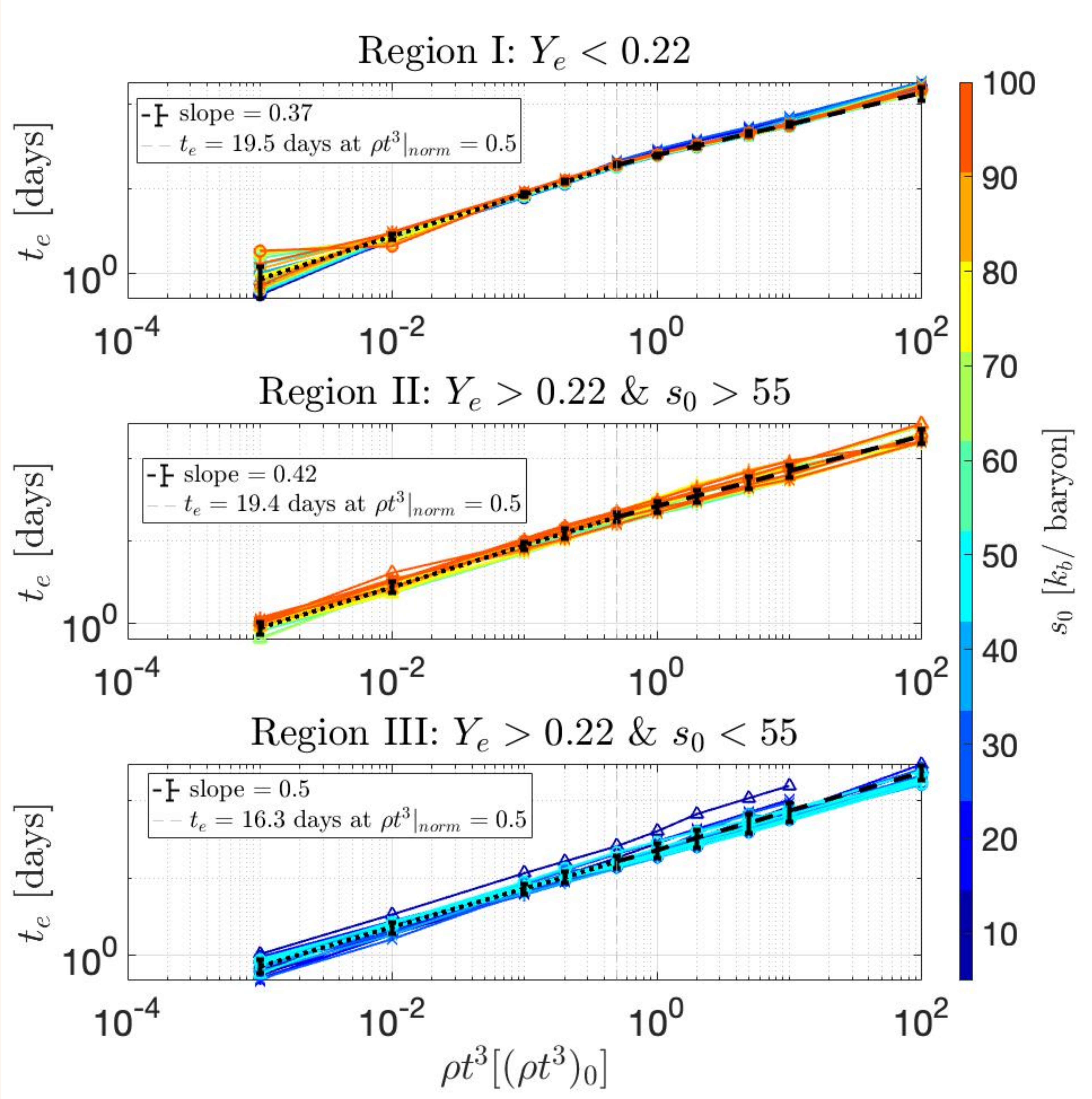
- Top (bottom) row - $s_0 = 20,\ (60)\ k_b$/baryon
- —— $\dot{Q}_\beta$ - $e^\pm$ energy release
- - - - - $\dot{Q}_{dep}$ - full deposition
- .... $\dot{Q}_{inst}$ - inst. deposition
- blue: $\rho t^3[(\rho t^3)_0] = 10^{-1}$
- green: $\rho t^3[(\rho t^3)_0] = 1$
- red: $\rho t^3[(\rho t^3)_0] = 10$

- $\dot{Q}_{inst} > \dot{Q}_{dep}$ for $t < t_e$ (ad. losses)
- $\dot{Q}_{dep} \sim t^{-2.85}$ for $t \gg t_e$ (Waxman et al. 2019)
  - adiabatic losses dominate
- t_e captures $\dot{Q}_{dep}$ transition to inefficiency (up to factor ~ 2)

$t_e(\rho t^3)$

- $t_e \propto \left((\rho t^3) E_i^{-1} \frac{v_e}{c} \frac{dE}{dX}\right)^{1/2}$, where E_i is the initial energy of beta electrons.
  - $< E^{-1/2} >^{-2}$ is correct char. energy of t_e, not $< E >$
  - Leading dependence: $t_e \propto (\rho t^3)^{1/2}$
  - If $< E_\beta > \propto t^{-k}$ as often assumed, then $\frac{d\log(t_e)}{d\log(\rho t^3)} \geq 1/2$

$t_e(\rho t^3)$, 3 Regions in $\{Y_e, s_0\}$ Space



- Broken power-law description:

$$t_e = t_0 \begin{cases} \left(\frac{\rho t^3}{0.5(\rho t^3)_0}\right)^{a_1} \text{days} & \text{for } \rho t^3 < (\rho t^3)_0 \\ \left(\frac{\rho t^3}{0.5(\rho t^3)_0}\right)^{a_2} \text{days} & \text{for } \rho t^3 > (\rho t^3)_0 \end{cases}$$

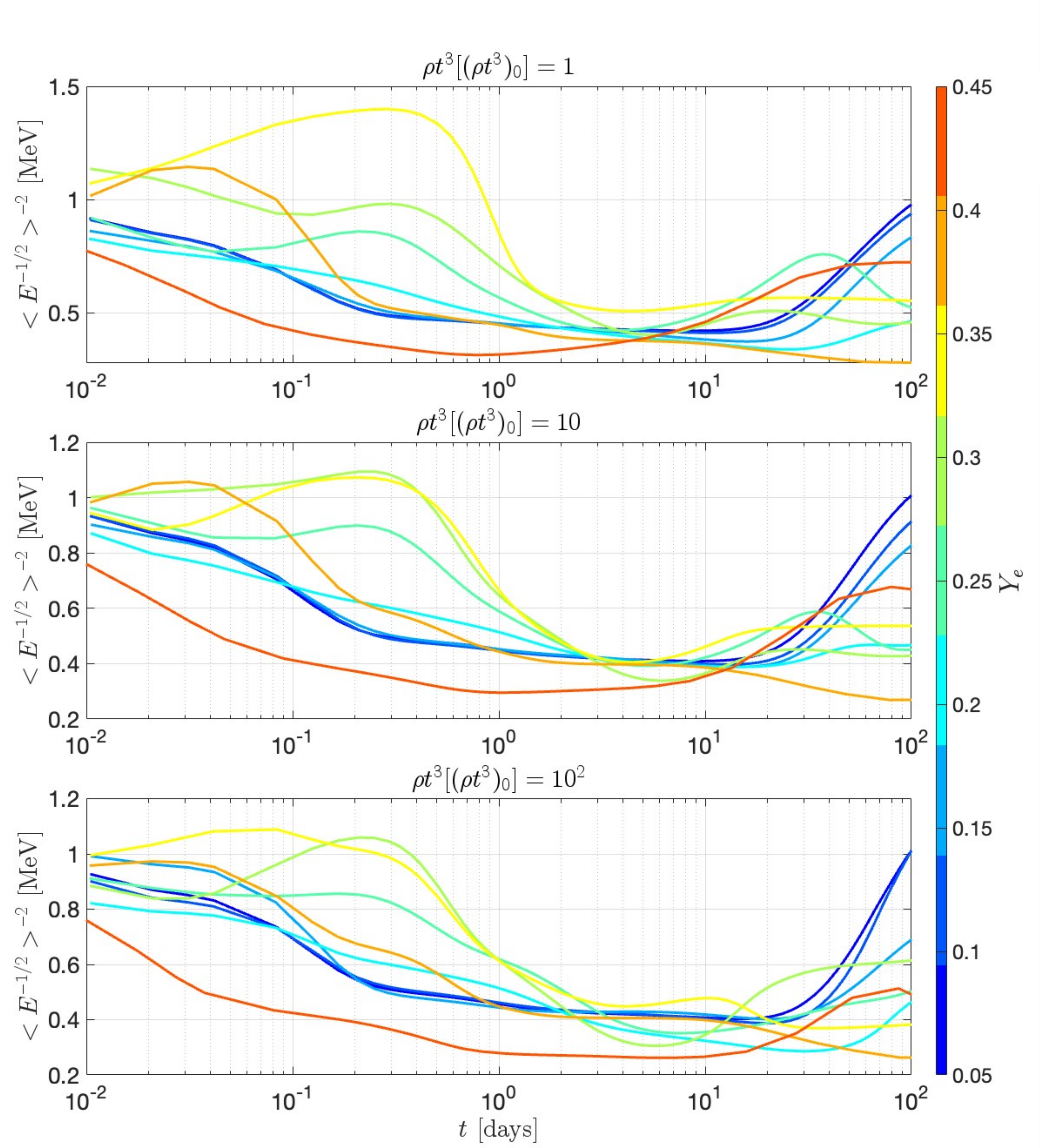
- Analytic estimate accurate to $\sim 20\%$, at worst

	Ejecta Parameters		Fitted Parameters		
	Y_e	s_0 [k_b/baryon]	a_1	a_2	$t_{e,0}$ [days]
Region I	< 0.22	$\forall s_0$	0.5	0.37	19.5
Region II	> 0.22	> 55	0.5	0.42	19.4
Region III	> 0.22	< 55	0.5	0.5	16.3

- Region I: Robust 3rd-peak
- Region II: mostly up to 2nd-peak
- Region III: some 1st-peak

Electron Characteristic Energy Release

- For $0.15 \leq Y_e$, $< E^{-1/2} >^{-2}$ rises at $t \gtrsim 15$ days
  - $^{94}\text{Os} \xrightarrow[<E>=0.03\text{MeV}]{t_{1/2}\approx 6\text{ yr}} {}^{94}\text{Ir} \xrightarrow[<E>=1.09\text{MeV}]{t_{1/2}\approx 20\text{ hr}} {}^{94}\text{Pt}$
  - Example of “inverted decay-chain”
- Other inverted chains active, $A = 140, 132, 106$, etc.
- Overall, 40 inverted chains with $\tau^d_{1/2} < 10^2 \times \tau^p_{1/2}$ of parent isotope



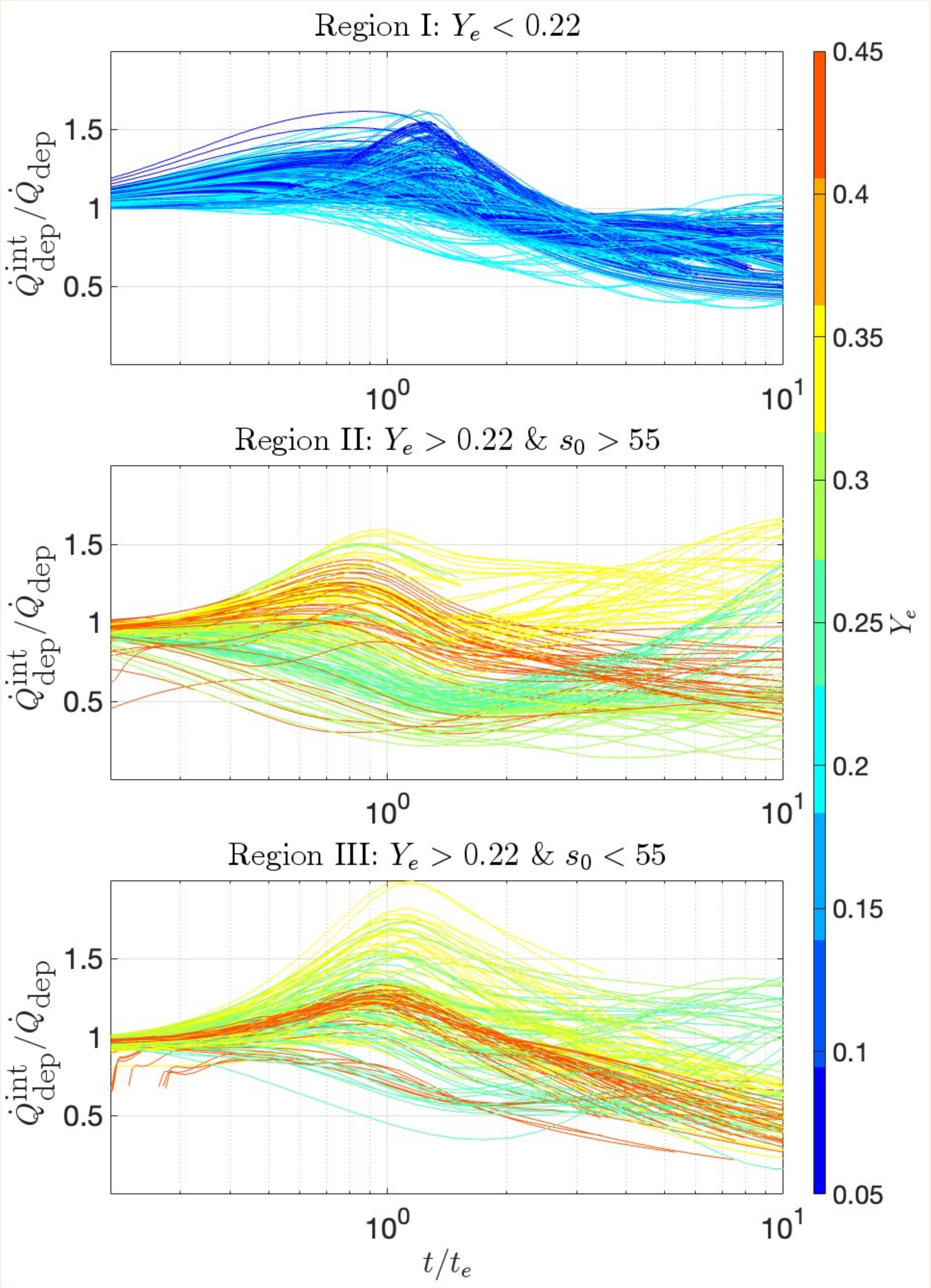
$\dot{Q}^{\rm int}_{\rm dep}(t)$ - Interpolating Function for Deposition

- $$\dot{Q}_{\rm early}(t) \equiv \dot{Q}_\beta(t) \times \left(1 - \exp\left[-\left(\frac{t}{t_e}\right)^{-n_1}\right]\right)$$

  $$\dot{Q}_{\rm late}(t) \equiv \dot{Q}_{\rm early}(t = t_D) \times \left(\left(\frac{t}{t_D}\right)^{-5\cdot 3} + \left(\frac{t}{t_D}\right)^{-3n_2}\right)^{1/3}$$

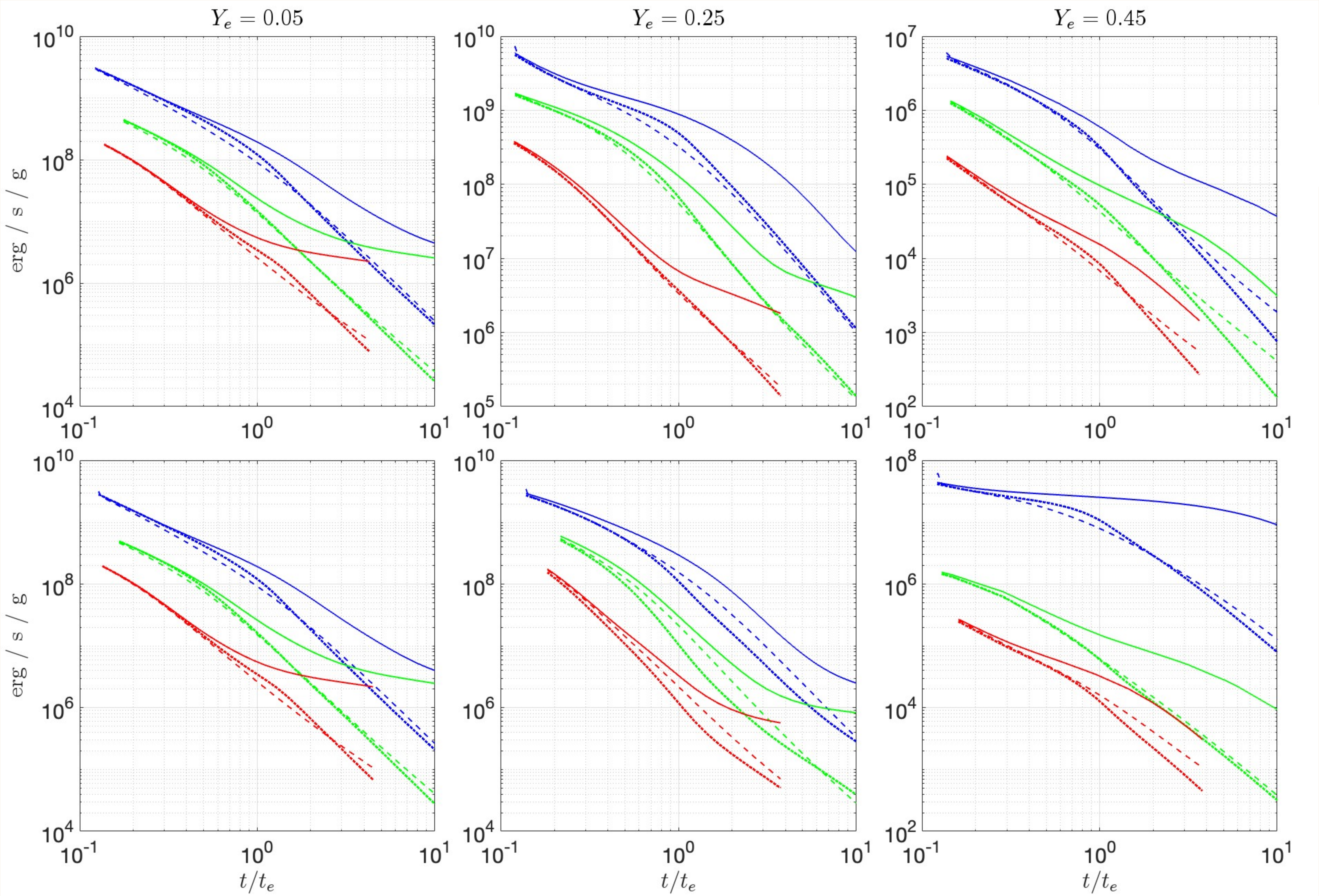
  $$\dot{Q}^{\rm int}_{\rm dep}(t) = \left(\dot{Q}^m_{\rm early} + \dot{Q}^m_{\rm late}\right)^{1/m}$$

	Ejecta Parameters		**Interpolation Parameters**			
	Y_e	s_0 $[k_b/\text{baryon}]$	m	n_1	n_2	$t_D\,[t_e]$
Region I	< 0.22	$\forall s_0$	4.5	1.1	2.8	1.3
Region II	> 0.22	> 55	0.8	0.5	2.5	1.3
Region III	> 0.22	< 55	1.5	0.5	2.8	1.3



- Accurate to within factor ~2 over 4 orders-of-magnitude change in $\dot{Q}_\beta$.

Delayed Deposition with Interpolation



- Top (bottom) row - $s_0 = 20,\ (60)\ k_b$/baryon
- —— $\dot{Q}_\beta$ - $e^\pm$ energy release
- - - - - $\dot{Q}_{dep}$ - full deposition
- .... $\dot{Q}^{interp}_{dep}$ - interpolating dep.
- blue: $\rho t^3[(\rho t^3)_0] = 10^{-1}$
- green: $\rho t^3[(\rho t^3)_0] = 1$
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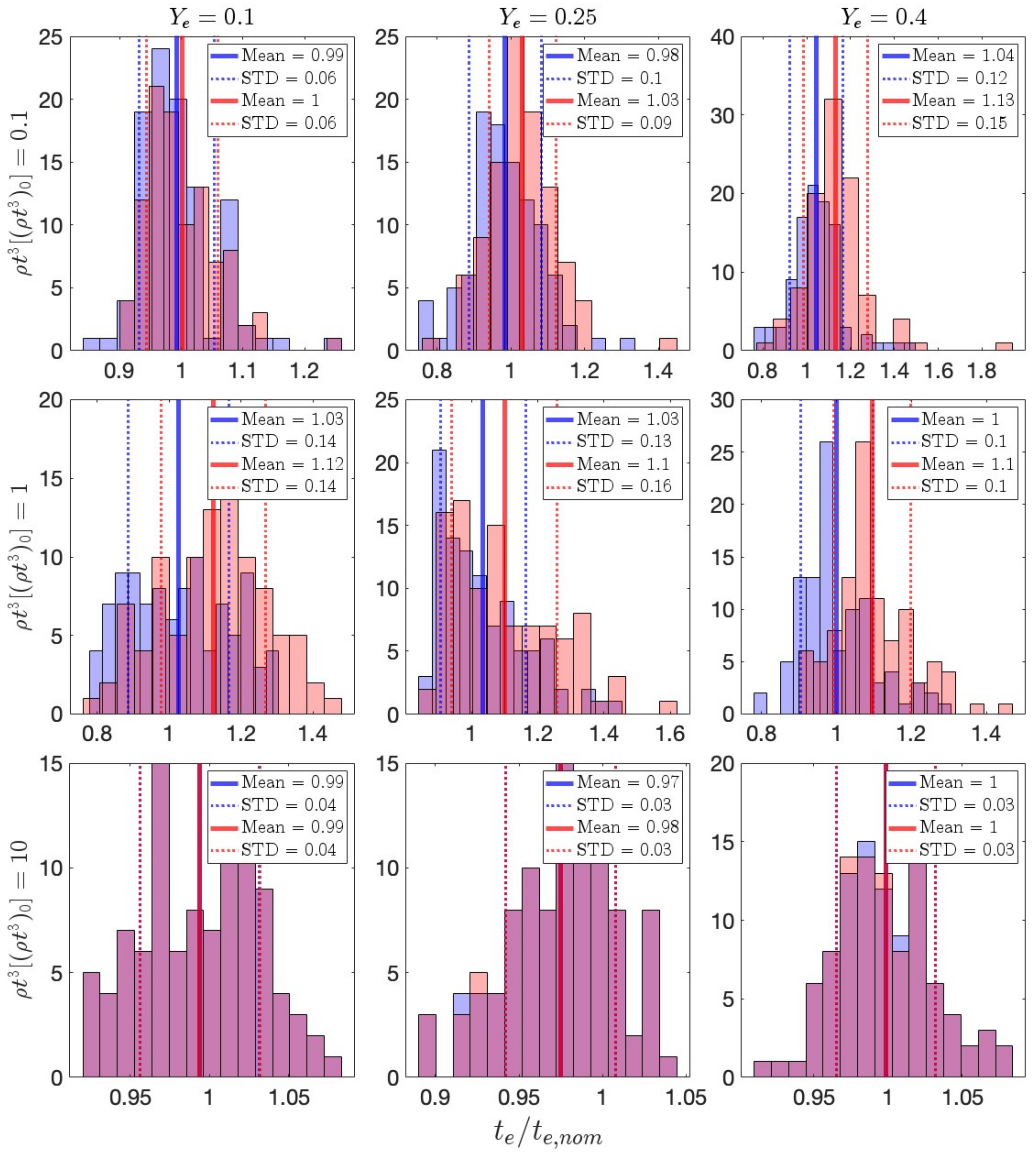
Dependence on Nuclear Physics Uncertainties

- Different nuclear mass models may result in orders-of-magnitude differences in final ejecta composition (esp. for low Y_e)
- Vary nuclear physics inputs:
  - Every theoretical rate $\lambda \to C\lambda$, where $C \in [10^{-2}, 10^{2}]$ (~70,000 rates, ~90%)
  - Rerun nuclear networks ~100 times
  - Check for FRDM and UNEDF1 mass-models.
- t_e remains robust



FRDM
UNEDF1

Key Takeaways

- $t_e(\rho t^3)$ broken power-law , small corrections for different compositions ($Y_e,\ s_0$).
  - Robust to nuclear physics uncertainties
  - Interpolating function for deposition $\dot{Q}_{dep}^{interp}$ - easy implementation in kilonovae calculations for wide range of ejecta parameters

- Understanding thermalization in future measurements mainly constrains ρt^3
  - Semi-analytic approach enables us infer ejecta parameters from future measurements - "inverse problem approach"
- $< E_\beta >$ does not steadily decline over time - "inverted decay-chains"
  - For $t \gtrsim 10 - 15$ days, β-emission is not dominated by isotopes with $\tau_{1/2} \sim t$ (for most Y_e values)
- Will be on the arXiv soon!

Thank You!