The University of Texas at Austin
Department of Physics
College of Natural Sciences

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THEORETICAL COSMOLOGY SEMINAR
CoPS division, the Oskar Klein Center, Department of Physics, Stockholm University

WARM INFLATION

Warm Natural inflation and other recent developments

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A BIT ABOUT MYSELF

You can find me in A5:1045 for the rest of the month!

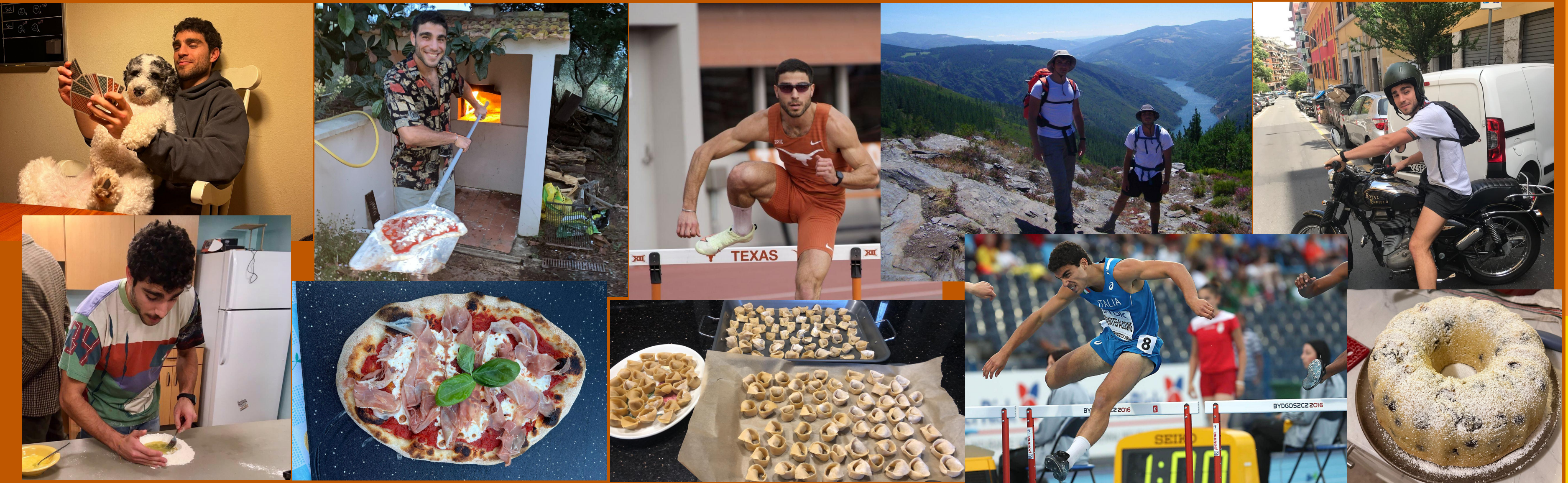
My Research Interests:

primordial cosmology

cosmological probes to constrain fundamental theories

the dark sector

Beyond Physics:



OUTLINE

1. **Standard (cold) Inflation:** What it is and the basics of how it works
2. **Warm Inflation:** What it is and how it differs from the standard picture
3. **Fine Tuning:** The conditions to realize a successful inflationary phase both in the cold and warm scenario
4. **Natural Inflation:**
    a. Its motivation in relation to the fine tuning problem
    b. Observations and model-building concerns in the standard scenario
5. **Warm Natural Inflation:**
    a. Intuitive picture of how a warm inflationary setting impacts the predictions of Natural inflation
    b. Results and Discussion

1. Standard (cold) Inflation

Motivations & Predictions

- A period of accelerated expansion in the early universe
- Explains the observed flatness, homogeneity, and the lack of relic monopoles
- Provides a mechanism for generating the inhomogeneities observed in the Cosmic Microwave Background (CMB)

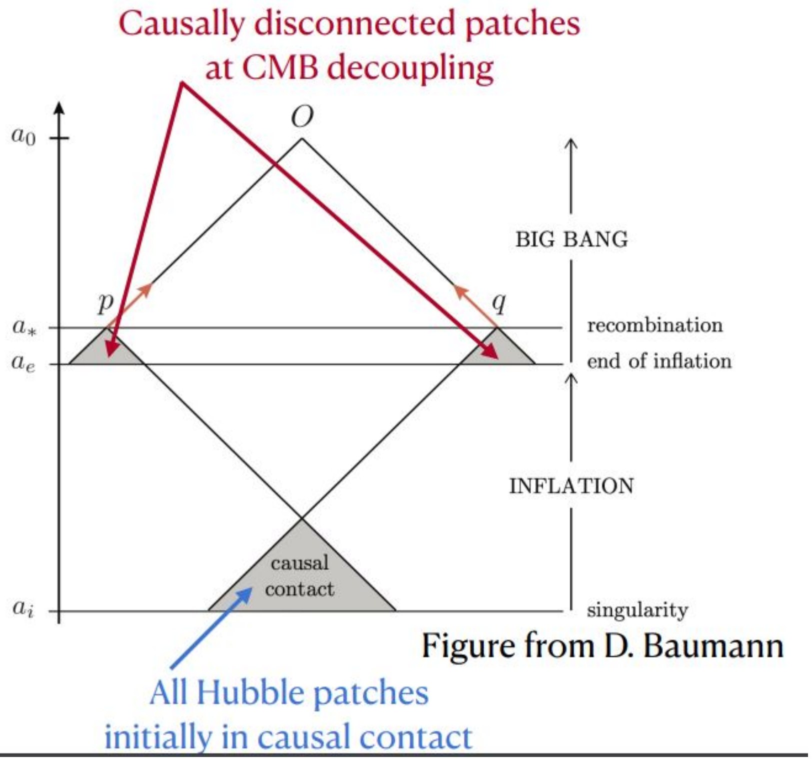


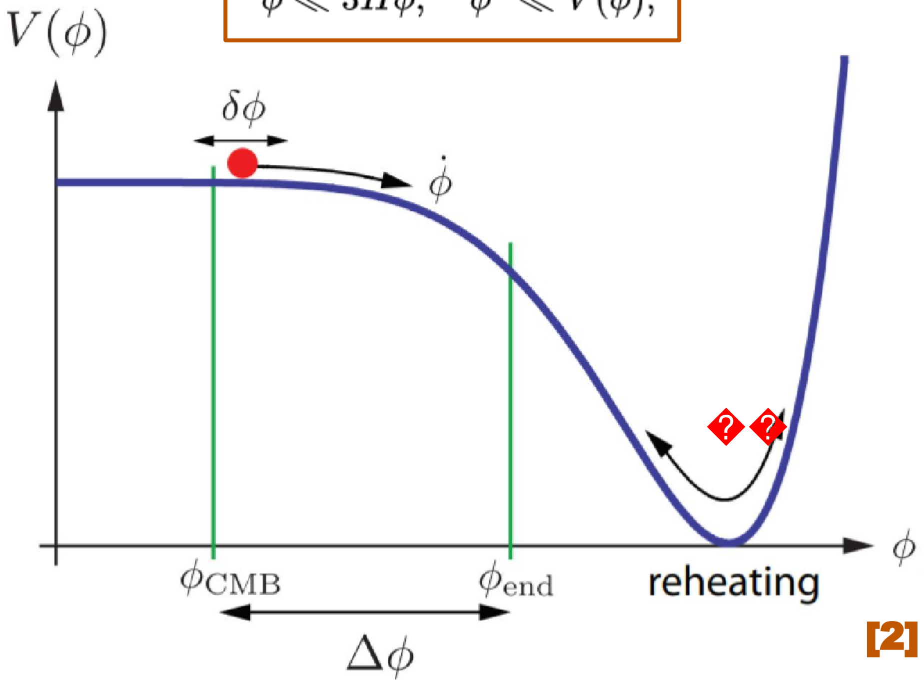
Figure from D. Baumann

[2] D. Baumann, doi:10.1142/9789814327183 0010 [arXiv:0907.5424 [hep-th]].

1. Standard (cold) Inflation

Single field, slow-roll

$$\boxed{\ddot{\phi} \ll 3H\dot{\phi}, \quad \dot{\phi}^2 \ll V(\phi);}$$



[2]

Inflaton Dynamics:

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0$$

$$H^2 \simeq \frac{V}{3M_{\rm pl}^2}, \quad \Rightarrow \boxed{a(t) \sim e^{Ht},}$$

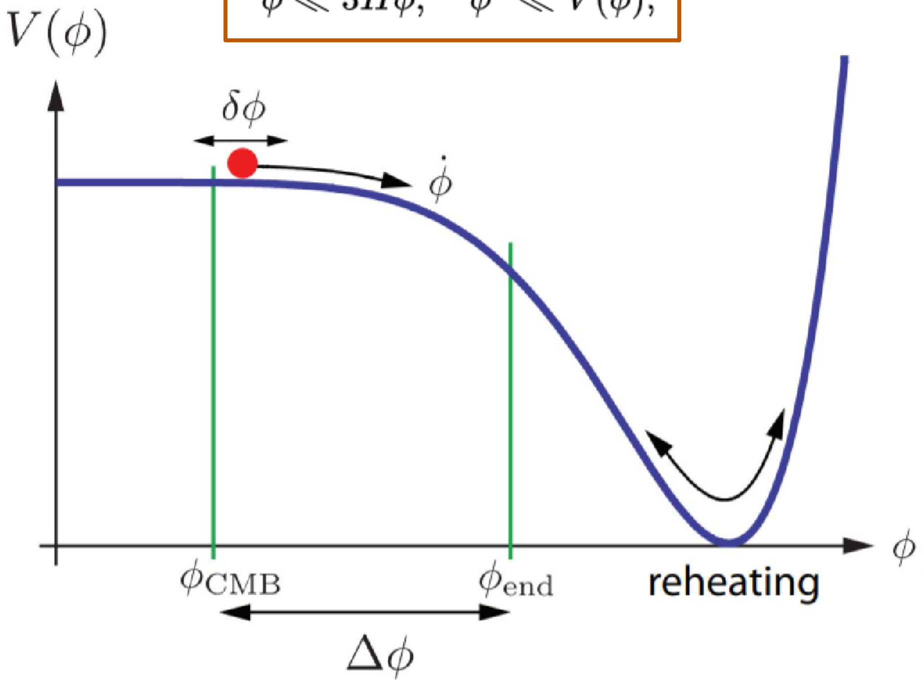
$$N_e \equiv \ln\left(\frac{a_{\rm end}}{a_k}\right) = \int_{t_k}^{t_{\rm end}} H dt;$$

- ➢ The energy density of the universe is dominated by **V($\boldsymbol{\phi}$)**
- ➢ Typically **N_e≅60**

1. Standard (cold) Inflation

Single field, slow-roll

$$\boxed{\ddot{\phi} \ll 3H\dot{\phi}, \quad \dot{\phi}^2 \ll V(\phi);}$$



Slow-roll parameters:

$$\epsilon_V \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta_V \equiv M_{\rm pl}^2\left(\frac{V_{,\phi\phi}}{V}\right);$$

$$\epsilon_V < 1 \quad \text{and} \quad |\eta_V| < 1.$$

- Inflation ends when $\boldsymbol{\varepsilon_V}$=1 or $|\eta_V|$ =1

1. Standard (cold) Inflation

From Perturbations to Cosmological Observables

$$\boxed{\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}}$$

- Quantum fluctuations are driven to cosmological scales via the expansion
  - Scalar perturbations associated with density perturbations
  - Tensor perturbations associated with primordial gravitational waves

$$\phi = \bar{\phi} + \delta\phi$$

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FRW}} + [\ldots] + h_{\mu\nu}$$

$$\Delta_s^2 = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2,$$

$$\sim \langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle$$

$$\Delta_t^2 = \frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{pl}}^2};$$

$$\sim \langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle$$

[3] Planck collaboration, Planck 2018 results. I., Astron. Astrophys.641 (2020)
[4] BICEP, Keck collaboration, Phys. Rev. Lett. 127 (2021) 151301

1. Standard (cold) Inflation

From Perturbations to Cosmological Observables

- The tensor to scalar ratio **r** is a measure of the magnitude of gravitational waves production during the inflationary phase
- The spectral index **n$_s$** is a measure of the scale variance of the scalar power spectrum

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V,$$

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} \simeq 2\eta_V - 4\epsilon_V; \qquad \left[\Delta_s^2 \simeq A_s\left(\frac{k}{k_*}\right)^{n_s-1}\right]$$

CMB constraints:

$n_s = 0.9649 \pm 0.0042, \quad \text{at } 68\% \text{ CL. (Planck2018)}$ **[3]** ➡ Nearly <u>scale-invariant</u> spectrum

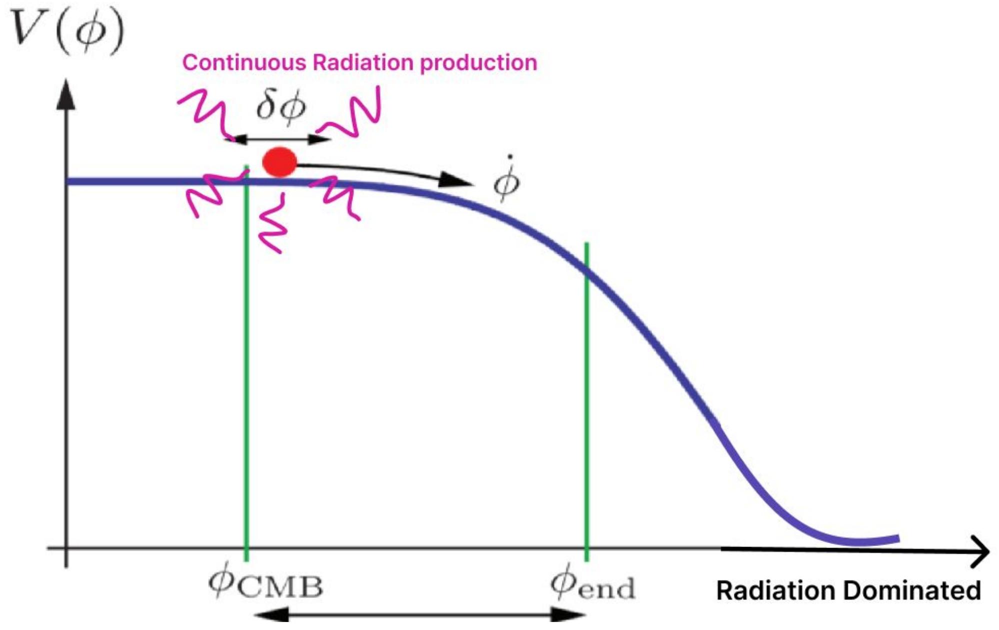
$r \lesssim 0.036, \quad \text{at } 95\% \text{ CL. (Plank2018+BAO+BK15)}$ **[4]** ➡ Relatively <u>small</u> tensor fluctuations

[5] A. Berera, Phys. Rev. Lett. 75 (1995) 3218

2. Warm Inflation

Basics (T>H) [5]

Γ is not negligible! $Q \equiv \Gamma/(3H)$ (Strength of dissipation)



Inflaton + Radiation bath Dynamics:

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0,$$

$$\dot{\rho_R} + 4H\rho_R = \Gamma\dot{\phi}^2,$$

$$\rho_R(T) = \alpha_1 T^4, \quad \text{with} \quad \alpha_1 = \frac{\pi^2}{30} g_*(T);$$

- ➢ The energy density of the universe is dominated by **V($\boldsymbol{\phi}$)**
- ➢ The inflaton continually sources the production of radiation during the accelerated expansion.
- ➢ Smooth transition to radiation dominated phase

2. Warm Inflation

Slow-roll (SR) conditions

$$\boxed{\ddot{\phi} \ll H\dot{\phi} \quad \dot{\rho}_R \ll H\rho_R \quad \text{and} \quad V(\phi) \gg \{\dot{\phi}^2, \rho_R\}}$$

$$\boxed{Q \equiv \Gamma/(3H)}$$

SR parameters:

$$\epsilon_w \equiv \frac{\epsilon_V}{1+Q} = \frac{M_{\rm pl}^2}{2(1+Q)}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad \eta_w \equiv \frac{\eta_V}{1+Q} = \frac{M_{\rm pl}^2}{(1+Q)}\left(\frac{V_{,\phi\phi}}{V}\right)$$

$$\boxed{\{\epsilon_V, |\eta_V|\} < 1+Q}$$

- Inflation ends when $\boldsymbol{\varepsilon_V}$=1+Q or $|\eta_V|$ =1+Q

Inflationary Dynamics:

$$H^2 \simeq \frac{V}{3M_{\rm pl}^2},$$

$$\dot{\phi} \simeq -\frac{V_\phi}{3H(1+Q)},$$

$$\rho_R \simeq \frac{3Q\dot{\phi}^2}{4}$$

For **Q ≫ 1**, the SR conditions are substantially relaxed and can in principle be satisfied by scalar field potentials that would otherwise violate the standard SR conditions in the cold inflation scenario.

[6] M. Bastero-Gil, A. Berera and R.O. Ramos, JCAP 07 (2011) 030
[7] C. Graham and I.G. Moss, JCAP 07 (2009) 013

3. Warm Inflation

Perturbation Spectra [6-7]

- The scalar power spectrum is enhanced by thermal effects;
- The tensor power spectrum is unaltered;
- **G(Q)** accounts for the direct coupling of the inflaton and radiation fluctuations due to a temperature dependent dissipative rate $\Gamma \propto T^c$
  - Approximated to a polynomial in **Q**
  - If **c>0** : spectrum is further enhanced;
  - If **c<0** : spectrum is suppressed;

$$\Delta_s^2 = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \left[\underbrace{1 + 2n_{BE}}_{\Delta_s^{2\,(\text{vac, warm})}} + \underbrace{\frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}\left(\frac{T}{H}\right)}_{\Delta_s^{2\,(\text{diss})}}\right] G(Q)$$

Recall:

$$\Delta_s^{2\,(\text{vac, cold})} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2$$

$$n_{\text{BE}} = 1/[\exp(H/T) - 1]$$

PRELIMINARY RESULTS
arxiv.2307(?)

3. Warm Inflation

The scalar dissipation function G(Q)

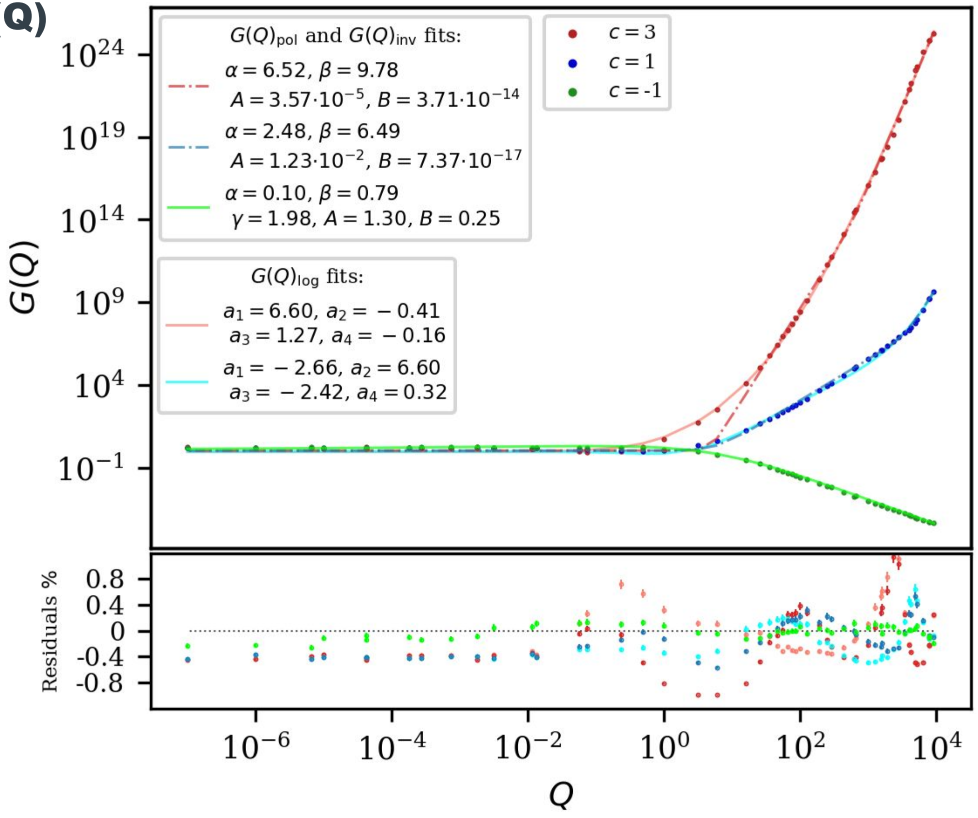
- G(Q) appears to be universal: only depends on **c**

- Github code to compute the perturbations in WI available soon

$$G(Q)_{\rm pol} = 1 + AQ^{\alpha} + BQ^{\beta},$$

$$G(Q)_{\rm log} = 10^{\sum_{n=1}^{4} a_n x^n}, \quad x \equiv \log_{10}(1+Q),$$

$$G(Q)_{\rm inv} = \frac{1+AQ^{\alpha}}{(1+BQ^{\beta})^{\gamma}}$$



[8] F.C. Adams, K. Freese and A.H. Guth, Phys. Rev. D 43 (1991) 965.

3. Fine Tuning parameter

Standard (Cold) Inflation [8]

- To not overproduce density fluctuations, the potential for the slowly rolling field needs to be extremely FLAT.
- The height of the inflaton potential is set by the amplitude of the density perturbations
- The width of the inflaton potential is set by the number of e-folds

$$\frac{\Delta V}{M_{\rm pl}^4} \lesssim \delta^2, \quad \frac{\Delta\phi}{M_{\rm pl}} \sim N_e \qquad \Delta_s|_{\rm CMB} \leq \delta \approx 5 \times 10^{-5},$$

$$\lambda_{\rm ft} \equiv \frac{\Delta V}{(\Delta\phi)^4} \lesssim 10^{-12}$$

$$\lambda_{\rm ft} \simeq \lambda_q, \quad \text{where} \quad \mathcal{L} = [\ldots] - \frac{\lambda_q}{4!}\phi^4$$

[9] G. Montefalcone, V. Aragam, L. Visinelli and K. Freese, 2209.14908

3. Fine Tuning Parameter

Warm Inflation for Q≫ 1 [9]

- The field excursion Δϕ can be significantly reduced compared to the case of no dissipation.

- For $\Gamma \propto T^c$ and c≥0, the constraint on ΔV is more stringent than in cold inflation.
  To reproduce the observed density perturbations the scale of inflation must be reduced to counteract the large thermal enhancement factor in the power spectrum.

- Most warm inflationary models of physical interest require an even FLATTER potential than standard cold inflation.

$$\frac{\Delta\phi}{M_{\rm pl}} \sim \frac{N_e}{\sqrt{Q}}$$

$$\frac{\Delta V}{M_{\rm pl}^4} \lesssim \delta^{\frac{8}{3}} Q^{-2-\frac{4}{3}b_G}$$

$$\boxed{\lambda_{\rm ft} \lesssim 10^{-15} Q^{-\frac{4}{3}b_G}}$$

$$G(Q) \sim Q^{b_G},$$
$$b_g \geq 0 \quad \text{for} \quad c \geq 0,$$
$$b_g < 0 \quad \text{for} \quad c < 0,$$

See PhysRevD.107.063543
ArXiv 2209.14908

[10] K. Freese, J.A. Frieman and A.V. Olinto, Phys. Rev. Lett. 65 (1990) 3233.

4. Natural Inflation

Basics [10]

- Use of an axion as the inflaton to provide a *natural* explanation of the flat potential required for inflation.
- At the perturbative level, the axion field φ enjoys a continuous shift symmetry which is broken by nonperturbative effects to a discrete symmetry $\varphi \to \varphi + 2\pi/f$.
- The inflaton potential is protected against loop corrections by this shift symmetry, i.e. the inflaton may be a pseudo Nambu-Goldstone boson.

$$V(\phi) = \Lambda^4\left[1+\cos(\phi/f)\right], \longrightarrow \boxed{\lambda_q \sim \left(\frac{\Lambda}{f}\right)^4}$$

$\Lambda^4 = m^2_\phi f^2$ where **f** is the decay constant of the axion-like particle and represents the width of the effective potential.

[11] N.K. Stein and W.H. Kinney, JCAP 01 (2022) 022
[12] T. Banks, M. Dine, P.J. Fox and E. Gorbatov, JCAP 06 (2003) 001
[13] W. DeRocco, P.W. Graham and S. Kalia, JCAP 11 (2021) 011

4. Natural Inflation

Observational Constraints

- To match observations, it is generally required $\mathbf{f \gtrsim M_{pl}}$ **[11]**
- From a model-building prospective $\mathbf{f \gtrsim M_{pl}}$ is not desirable **[12]**
- Axions generically couple to gauge sectors and this can result in the generation of a thermal bath in significant parts of the axion-inflation parameter space. **[13]**

Can we avoid the trans-Planckian requirement of the decay constant **f** in the presence of the radiation bath of warm inflation?

See JCAP03(2023)002
ArXiv 2212.04482

[14] K.V. Berghaus, P.W. Graham, D.E. Kaplan, G.D. Moore and S. Rajendran, Phys. Rev. D 104 (2021) 083520

5. Warm Natural Inflation

Basics: dissipation rates Γ

- We assume the inflaton couples to a pure Yang-Mills gauge group through the Lagrangian term: $\mathcal{L}_{\rm int} \propto \frac{\phi}{f} \mathrm{Tr}\, \mathcal{G}\tilde{\mathcal{G}}\,,$
- We parameterize the dissipation rate as: $\boxed{\Gamma(T) = \gamma_c \left(\frac{T^c}{f^{c-1}} \right),}$ for **c={1,3}**.

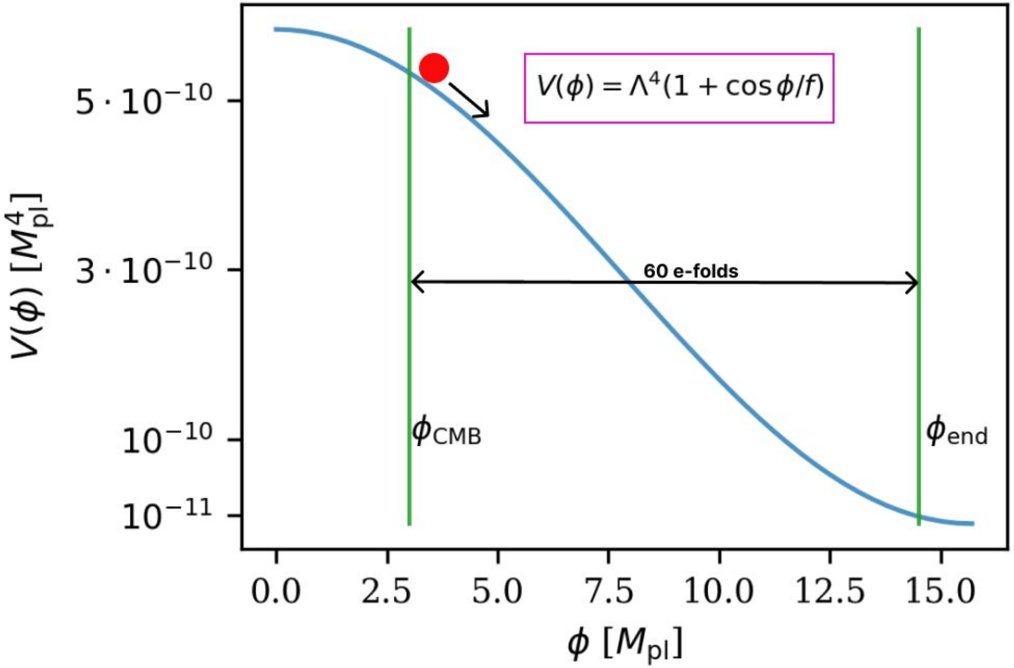
- This friction term arises from the sphaleron transition rate of physically motivated `axion-like interactions
  - Pure Yang-Mills: cubic dissipation rate **(c=3)**
  - Pure Yang-Mills + light fermion: linear dissipation rate **(c=1)**

[14]

5. Warm Natural Inflation

Basics: Computing Cosmological observables

1. We fix a priori the value of **f** and N_e
2. We determine **G(Q)** for **c={1,3}**
3. We compute ϕ_{end} : Impose ε_V=1+Q, $|\eta_V|$ =1+Q
4. We compute ϕ_{CMB}: via fixed N_e
5. We set m_ϕ by fixing the amplitude of the primordial power spectrum at the CMB pivot scale k_*=0.05 Mpc^{-1}
6. We compute **r** and n_s



5. Warm Natural Inflation

Building intuition: f vs Q

- The existence of a slowly-rolling regime in natural inflation generally depends on the value of the decay constant **f** and, in the context of warm inflation, on the dissipation strength **Q**.

- Solve system $\boldsymbol{\varepsilon_V < 1+Q}$, $\boldsymbol{|\eta_V| < 1+Q}$

$$
\begin{aligned}
\epsilon_w &\equiv \frac{\epsilon_V}{1+Q} = \frac{1}{2(1+Q)} \frac{M_{\rm pl}^2}{f^2} \frac{\sin^2 \phi/f}{(1+\cos\phi/f)^2}, \\
\eta_w &\equiv \frac{\eta_V}{1+Q} = -\frac{1}{(1+Q)} \frac{M_{\rm pl}^2}{f^2} \frac{\cos\phi/f}{1+\cos\phi/f},
\end{aligned}
$$

[15] K. Freese and W.H. Kinney, Phys. Rev. D 70 (2004) 083512

5. Warm Natural Inflation

Building intuition: f vs Q

$$\boxed{\tilde{f} \equiv f/M_{\rm pl}}$$

Cold Inflation Case: Q=0

- ➢ $\tilde{f} \geq \frac{1}{\sqrt{2}}$: Broad SR regime, only ε_V bound plays a role
- ➢ $\sqrt{\frac{\sqrt{2}-1}{2}} \leq \tilde{f} \leq \frac{1}{\sqrt{2}}$: SR shrinks, ε_V and $|\eta_V|$ bounds push in opposite directions
- ➢ $\tilde{f} \leq \sqrt{\frac{\sqrt{2}-1}{2}}$: No SR regime

$$\text{ESRC:} \quad \tilde{f} > \sqrt{\frac{\sqrt{2}-1}{2}} \approx 0.5$$

$$\text{BSRC:} \quad \tilde{f} > \frac{1}{\sqrt{2}} \approx 0.7$$

[15]

5. Warm Natural Inflation

Building intuition: f vs Q

$$\boxed{\tilde{f} \equiv f/M_{\rm pl}}$$

Warm Inflation Case: Q>0

- By taking **Q=**const., we can interpret $\sqrt{1+Q\tilde{f}}$ as an effective decay constant $\tilde{f}_w$ and recover the same constraints from cold inflation.

$$\text{ESRC:} \quad \tilde{f} > \sqrt{\frac{\sqrt{2}-1}{2(1+Q)}},$$

$$\text{BSRC:} \quad \tilde{f} > \frac{1}{\sqrt{2(1+Q)}}.$$

$\Rightarrow$

$$\text{ESRC:} \quad \text{for } \tilde{f} < (\sqrt{2}-1)/2, \quad Q > \frac{\sqrt{2}-1}{2\tilde{f}^2} - 1,$$

$$\text{BSRC:} \quad \text{for } \tilde{f} < 1/\sqrt{2}, \quad Q > \frac{1}{2\tilde{f}^2} - 1.$$

We can achieve <u>sub-Planckian values</u> of **f** but only in a strongly dissipative regime **Q≫1**.

i.e. for **f~10^{-1} M_{pl}, Q~50**; for **f~10^{-3} M_{pl}, Q~10^6**

5. Warm Natural Inflation

Building intuition: n_s and r for Q≫1

➢ For Q≫1, the spectral tilt $\mathbf{n_s}$ is blue-shifted while the tensor-to-scalar ratio **r** is strongly suppressed compared to the cold inflation case.

$$r_{\text{warm}} \sim r_{\text{cold}}/Q^{\frac{5}{2}+b_G}$$

Linear case: $$n_{s,1} \approx 1 + \frac{7.37\epsilon_V - 2.46\eta_V}{5Q} > 1,$$

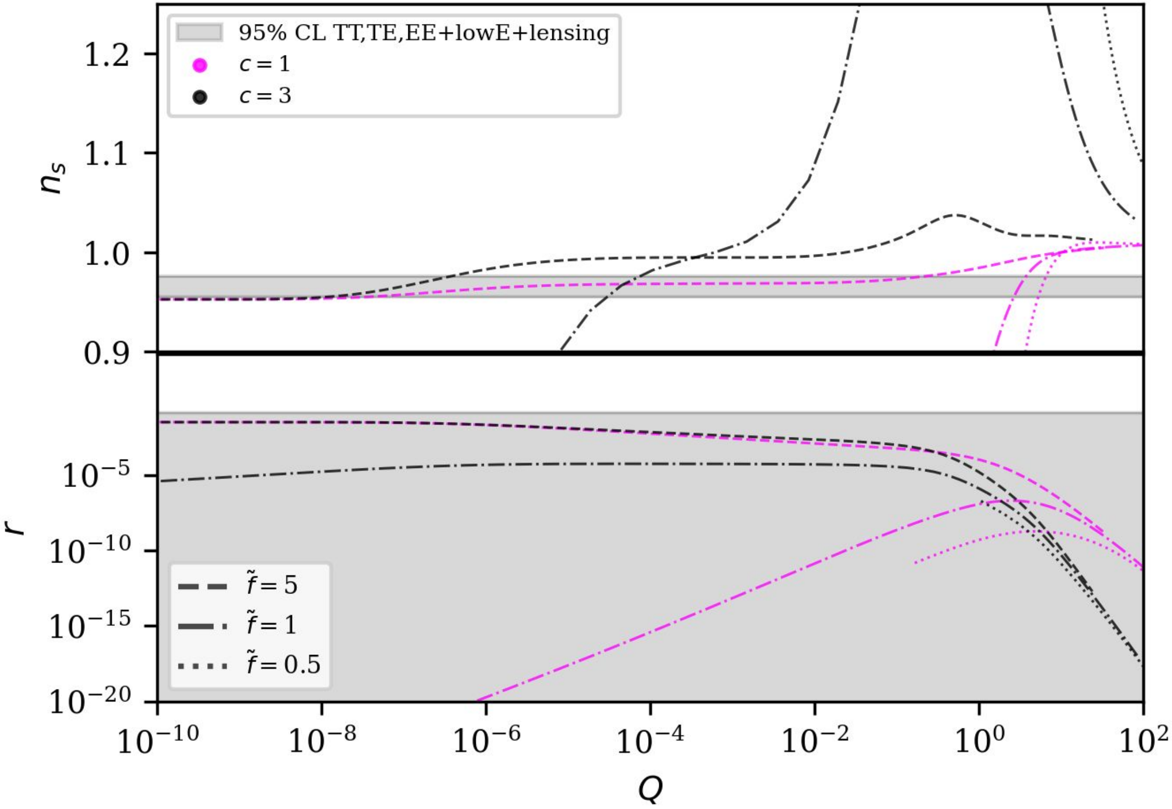
Cubic case: $$n_{s,3} \approx 1 + \frac{48.21\epsilon_V - 37.33\eta_V}{7Q} > 1,$$

CMB measurements set a limit on the maximum allowed value of **Q**: **Q≲50 for c=1** and **Q≲15 for c=3**

5. Warm Natural Inflation

Results

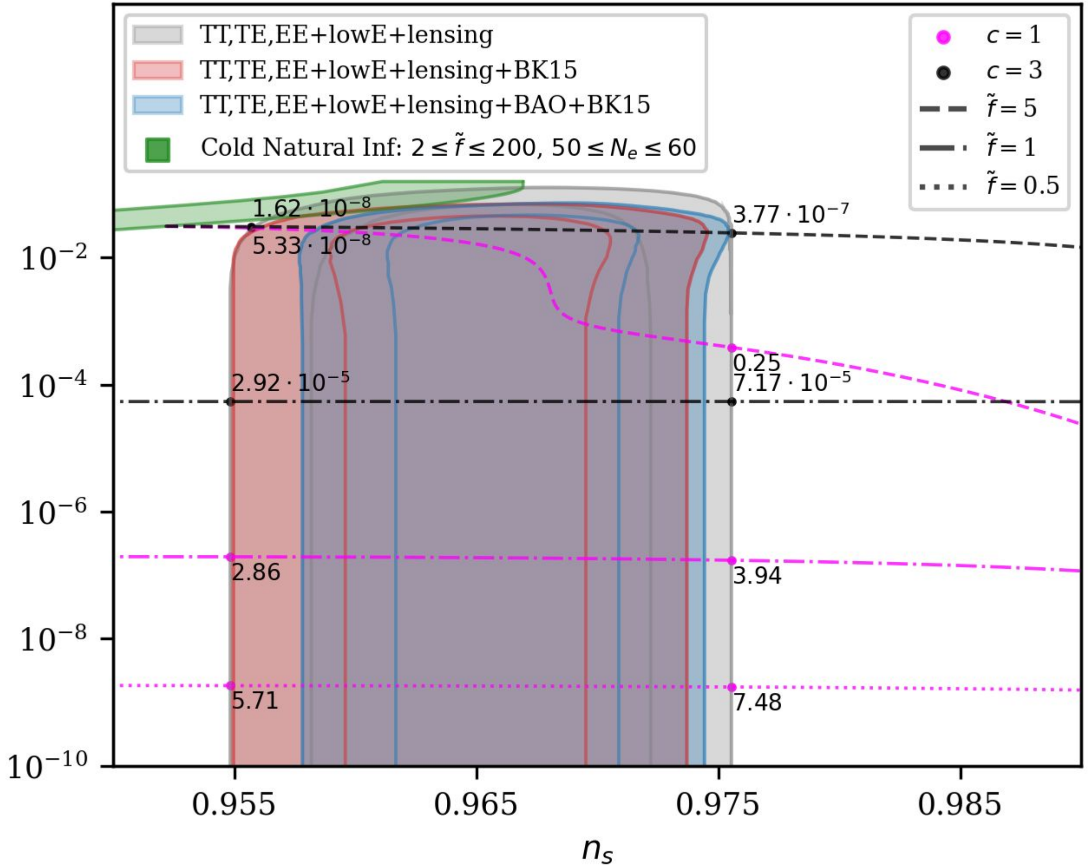
- **r** is within the observational constraints at the 2**σ** level for all values of **Q** and decreases rapidly for **Q≳1**.
- As **f** decreases, the region where $\mathbf{n_s}$ is within the observational constraints moves to higher values of **Q** and shrinks in size.
- For a given value of **f**, $\mathbf{n_s}$ becomes blue-shifted at smaller values of **Q** for **c=3** compared to **c=1**.



5. Warm Natural Inflation

Results

- For both **c={1,3}** and **f≥M_{pl}**, WNI is consistent with observations at the 1σ level exists.
- For **f=5 M_{pl}**, both cases **c={1,3}** reduce to the cold natural inflation result. This occurs precisely when **$Q_* \leq 9.5 \times 10^{-11}$ (c=1)** and **$Q_* \leq 3.7 \times 10^{-10}$ (c=3)**.
- We found that for **c=1: f_{min}**=0.3**M_{pl}** and for **c=3**: **f_{min}**=0.8**M_{pl}**



SUMMARY

Constraints on the scalar-field potential in warm inflation (ArXiv:2209.14908)

For most warm inflationary models of physical interest the requirements on the flatness of the scalar field potential are very stringent and significantly more severe than those in the cold inflationary scenario.

Observational Constraints on Warm Natural Inflation (ArXiv:2212.04482)

1. We found that, in contrast with the standard cold inflation scenario, for **$f \geq M_{pl}$** warm natural inflation is consistent with observational constraints on **r** and **n_s** at the 1σ level, respectively in a weak (moderate) dissipative regime for **c=3** (**c=1**).

2. As **f** is lowered, the dissipation strength **Q** must increase in order to maintain the existence of a (broad) slowly-rolling phase.
   However, a larger **Q** leads to a larger scalar spectral index **n_s** such that
   f can at most be marginally sub-Planckian without resulting in **$n_s \geq 1$.**

PRESENT AND FUTURE WORK

1. Generalized code to compute the scalar dissipation function **G(Q)** [IN PREPARATION!]
2. Conditions for Eternal inflation in Warm inflation [FUTURE WORK]

GRAZIE PER L'ATTENZIONE!

GABRIELE MONTEFALCONE

BACK-UP SLIDES

A. Standard Cold Inflation

Motivations & Predictions

I HORIZON PROBLEM

The uniformity of the CMB implies that the universe at decoupling was in *thermal equilibrium*. Oddly, the comoving horizon right before photons decoupled was significantly *smaller* than the corresponding horizon observed today.

II FLATNESS PROBLEM

Refers to the necessity of an extreme *fine tuning* of the initial value of Ω.
Present observations suggest that $|\Omega_0-1|\lesssim 10^{-3}$, this implies $|\Omega-1|\lesssim 10^{-16}$ at nucleosynthesis epoch, and $|\Omega-1|\lesssim 10^{-64}$ at Planck epoch.

III MONOPOLE PROBLEM

All GUT predict the existence of magnetic monopoles, extremely heavy particles with net magnetic charge.
If these particles exist in the early universe, they could be the *dominant* materials in the universe.

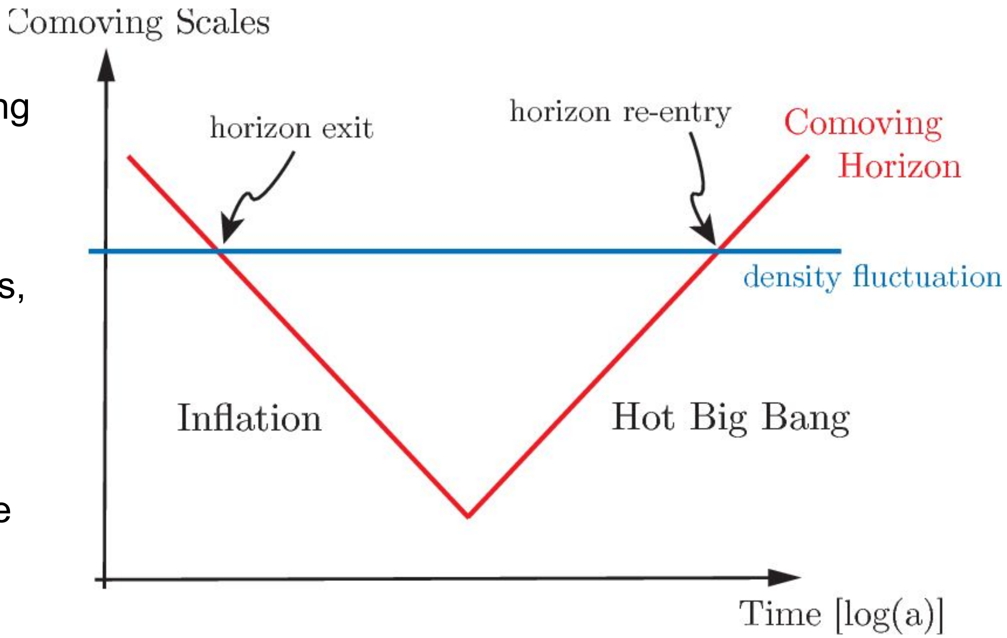
I CMB ANISOTROPIES

The CMB presents small temperature anisotropies with $\Delta T/T \sim 10^{-5}$ a characteristic angular scale of about 1 degree.

A. Standard Cold Inflation

From Quantum to Large-scale perturbations

- ➢ The comoving Hubble radius <u>shrinks</u> during inflation, so eventually all fluctuations <u>exit</u> the horizon
- ➢ After inflation, the comoving horizon grows, so eventually all fluctuations will re-enter the horizon.
- ➢ Adiabatic curvature perturbations <u>freeze</u> when they exit the horizon: their amplitude is not affected by the physics shortly after inflation



B. Warm Natural Inflation

Dynamics in Warm Natural Inflation

$$Q^{4-c}(1+Q)^{2c} = \frac{M_{\rm pl}^{2(2+c)} m_\phi^{2(c-2)} \gamma_c^4}{9 f^{4c} \alpha_1^c} \frac{[\sin^2(\phi/f)]^{2c}}{[1+\cos(\phi/f)]^{2+c}}$$

$$\equiv \frac{\xi}{\tilde{f}^{4c}} \frac{[\sin^2(\tilde{\phi})]^{2c}}{[1+\cos\left(\tilde{\phi}\right)]^{2+c}},$$

$$\boxed{\xi \equiv \frac{\gamma_c^4}{9\alpha_1^c}\left(\frac{m_\phi}{M_{\rm Pl}}\right)^{2(c-2)},}$$

$$\tilde{f} \equiv f/M_{\rm pl}\ , \quad \tilde{\phi} \equiv \phi/f,$$

$$\epsilon_w \equiv \frac{\epsilon_V}{1+Q} = \frac{1}{2(1+Q)} \frac{M_{\rm pl}^2}{f^2} \frac{\sin^2\phi/f}{(1+\cos\phi/f)^2},$$

$$\eta_w \equiv \frac{\eta_V}{1+Q} = -\frac{1}{(1+Q)} \frac{M_{\rm pl}^2}{f^2} \frac{\cos\phi/f}{1+\cos\phi/f},$$

$$N_e = \tilde{f}^2 \int_{\tilde{\phi}_{\rm CMB}}^{\tilde{\phi}_{\rm end}} (1+Q) \frac{1+\cos\tilde{\phi}}{\sin\tilde{\phi}} d\tilde{\phi}$$

B. Warm Natural Inflation

Dynamics in Warm Natural Inflation

$$H = \frac{m_\phi f}{M_{\rm pl}} \sqrt{\frac{1+\cos(\phi/f)}{3}},$$

$$T = \left[\frac{Q}{(1+Q)^2} \frac{1}{4\alpha_1} \frac{9 M_{\rm pl}^6}{f^4 m_\phi^2} \frac{\sin^2(\phi/f)}{[1+\cos(\phi/f)]^3} \right]^{1/4},$$

$$G_{\rm linear}(Q) \simeq 1 + 0.189\, Q^{1.642} + 0.0028\, Q^{2.729},$$

$$G_{\rm cubic}(Q) \simeq 1 + 3.703\, Q^{2.613} + 0.0011\, Q^{5.721}.$$

- $$n_s - 1 = 4\frac{d\ln H}{dN_e} - 2\frac{d\ln\dot\phi}{dN_e} + \left(1 + 2n_{\rm BE} + \frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}\frac{T}{H}\right)^{-1} \left\{ 2n_{\rm BE}^2 e^{\frac{H}{T}} \frac{H}{T}\left(\frac{d\ln T}{dN_e} - \frac{d\ln H}{dN_e}\right) + \frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}\frac{T}{H}\left[\left(\frac{3+2\pi Q}{3+4\pi Q}\right)\frac{d\ln Q}{dN_e} + \frac{d\ln T}{dN_e} - \frac{d\ln H}{dN_e}\right]\right\} + \frac{G'(Q)}{G(Q)} Q \frac{d\ln Q}{dN_e},$$

- $$r = \frac{16\epsilon_w}{1+Q}\left(1 + 2n_{\rm BE} + \frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}\frac{T}{H}\right)^{-1} \frac{1}{G(Q)},$$

B. Warm Natural Inflation

Slow-roll in Natural Inflation

$$(\epsilon_V < 1): \quad \tilde{\phi} < \arccos\left(\frac{1-2\tilde{f}^2}{1+\tilde{f}^2}\right) \equiv \tilde{\phi}_\epsilon;$$

$$(|\eta_V| < 1): \begin{cases} \tilde{\phi} < \arccos\left(\frac{-\tilde{f}^2}{1+\tilde{f}^2}\right) \equiv \tilde{\phi}_{\eta,1} & \text{for } \tilde{f} \geq \frac{1}{\sqrt{2}}, \\ \tilde{\phi} > \arccos\left(\frac{\tilde{f}^2}{1+\tilde{f}^2}\right) \equiv \tilde{\phi}_{\eta,2} & \text{otherwise.} \end{cases}$$

for $\tilde{f} \geq 1/\sqrt{2}$, broad SR regime: $\quad \Rightarrow \quad \phi \in (0, \tilde{\phi}_\epsilon)$.

for $\sqrt{(\sqrt{2}-1)/2} \leq \tilde{f} < 1/\sqrt{2}$, SR regime: $\quad \Rightarrow \quad \phi \in (\phi_{\eta,2}, \tilde{\phi}_\epsilon)$.

for $\tilde{f} < \sqrt{(\sqrt{2}-1)/2}$, no SR regime.

$$\boxed{\begin{aligned} \text{ESRC:} \quad & \tilde{f} > \sqrt{\frac{\sqrt{2}-1}{2}}, \\ \text{BSRC:} \quad & \tilde{f} > \frac{1}{\sqrt{2}}. \end{aligned}}$$

B. Warm Natural Inflation

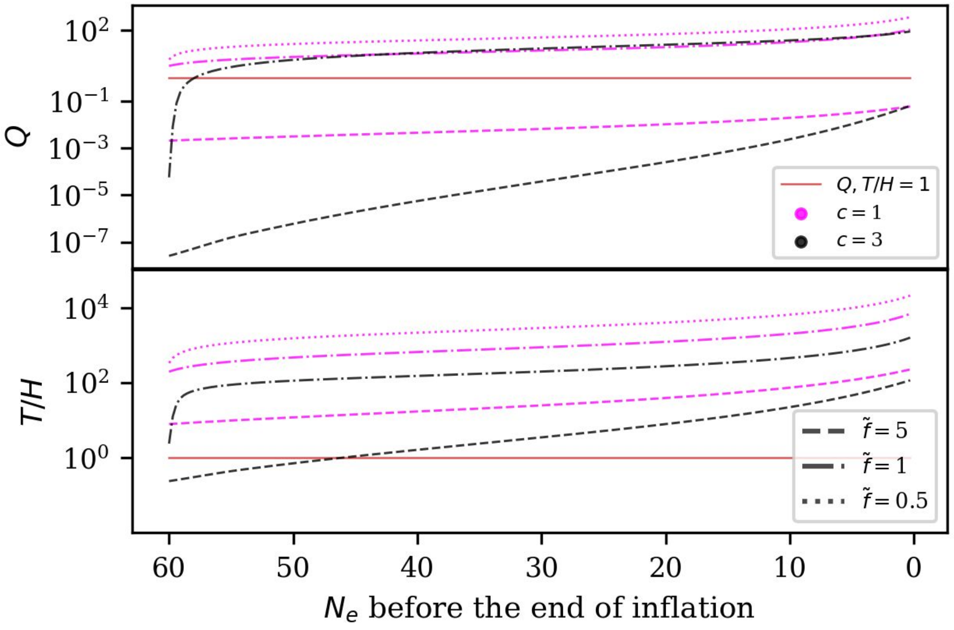
Dynamics during the inflationary period

- both **Q**and **T/H** increase during inflation.

- For **c=3**, **f=5 M$_{pl}$**, inflation starts in a cold scenario (**T/H<1**) and evolves in the warm scenario (**T/H>1**) via the coupling to the radiation.

- For **f=5 M$_{pl}$**, we have **Q<1** during all the inflation period.

- For **f={1,0.5} M$_{pl}$**, inflation only starts with a **Q~O(1)** which quickly increases to values **Q>1**, through most of the inflationary period.



B. Warm Natural Inflation

The Dissipation rate in axion-like interactions

➢ Gauge group **SU(N$_c$)** with **N$_f$** fermions in a representation **R** of dimension **d$_R$** and with trace normalization **T$_R$** normalization

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \bar{\Psi}\,(\not{D} + m_f)\Psi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{\varphi}{f}\frac{\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}}{16\pi^2} - V(\phi)$$

$$\partial_\mu\partial^\mu\varphi = V_{,\phi} + \frac{\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}}{16\pi^2 f}$$

$$\left\langle \frac{\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}}{16\pi^2} \right\rangle = \Gamma(T)\dot{\varphi};$$

$$\boxed{\Gamma(T) = \frac{\Gamma_{\mathrm{sph}}}{2Tf^2}\left(1 + \frac{24T_R^2}{d_R T^3}\frac{\Gamma_{\mathrm{sph}}}{\Gamma_{\mathrm{ch}}}\right)^{-1},}$$

$$\Gamma_{\mathrm{sph}} \equiv \tilde{\kappa}(\alpha, N_c, N_f)\alpha^5 T^4\,,$$

$$\Gamma_{\mathrm{ch}} \equiv \frac{\kappa\, N_c\, \alpha\, m_f^2}{T}$$

B. Warm Natural Inflation

The Dissipation rate in axion-like interactions

- The role of light fermions is to allow chirality-violating processes that diminish the friction associated with sphaleron transitions.

- The estimation of this dissipation rate is only known to be valid for $m_\varphi < \alpha^2 T$.

Cubic case: $m_f \to \infty$: $\Gamma(T) \simeq \left(\frac{\tilde{\kappa}\alpha^5}{2f^2}\right) T^3;$

Linear case: $m_f \lesssim (N_c^2 \alpha^2) T$: $\Gamma(T) \simeq \left(\frac{d_R N_c \alpha m_f^2}{48 f^2 T_R^2}\right) T;$

B. Warm Natural Inflation

The Dissipation rate in axion-like interactions

- **α** is bounded from perturbativity and the inflaton thermalization which respectively require **$\alpha \lesssim 0.1$** and **$\alpha < 10^{-2}\sqrt{Q}$**
- The entirety of the viable parameter space that we obtained in this work strongly violates the theoretical bounds on the cubic and linear axion-like interaction terms.

Cubic case:

$$\gamma_3 = \frac{\tilde{\kappa}\alpha^5}{2} \sim \mathcal{O}(10^2)\cdot\alpha^5,$$

$$\Rightarrow \boxed{\alpha \sim \left(\frac{\gamma_3}{10^2}\right)^{\frac{1}{5}};}$$

Linear case:

$$\gamma_1 = \frac{d_R N_c \alpha m_f^2}{48 f^2 T_R^2} \lesssim \frac{d_R N_c^5 \alpha^5 T^2}{48 f^2 T_R^2} \sim \mathcal{O}(1)\cdot\frac{\alpha^5 T^2}{f^2},$$

$$\Rightarrow \boxed{\alpha \gtrsim \left(\frac{f^2\gamma_1}{T^2}\right)^{\frac{1}{5}}.}$$

C. Warm Inflation

Effective Langevin-like EOM

- The effective equation of motion for the inflation φ becomes of Langevin-like type when the microphysical dynamics determining Γ and sourced by a stochastic noise term ξ_T operates at time scales much faster than that of the macroscopic motion of the φ field and the expansion scale of the Universe.

$$-\partial_\mu \partial^\mu \varphi + \Gamma \dot{\varphi} + V_{,\varphi} = \xi_T(\vec{x}, t)$$

$$\Gamma = \int d^4x' \Sigma_R(, x')(t' - t)$$

$$= -\lim_{\omega \to 0} \frac{\mathrm{Im}\Sigma_R(\vec{k} = 0, \omega)}{\omega},$$

$$\langle \xi_T(\vec{x}, t) \xi_T(\vec{x}', t') \rangle = 2\Gamma T a^{-3} \delta(t - t') \delta(\vec{x} - \vec{x}'),$$

C. Warm Inflation

An attractor solution

- We assume there is a well-defined initial temperature of the bath at the onset of inflation T_0. T_{eq} is the steady-state equilibrium temperature.
- For a dissipation rate $\Gamma \propto T^c$ and **|c|<4**, it takes less than one Hubble time to reach the equilibrium temperature for warm inflation.

$$\dot{\rho}_R \approx \Gamma(T)\dot{\phi}^2,$$

$$T^{3-c}\dot{T} \sim \frac{\dot{\phi}^2}{4},$$

$$\frac{T_{\rm eq}^{4-c} - T_0^{4-c}}{4-c} \gtrsim \frac{\dot{\phi}_{\rm eq}^2}{4} t_{\rm eq}$$

$$\Rightarrow \boxed{t_{\rm eq} < \frac{1}{(4-c)H}}$$

Slide made by Dr. Luca Visinelli

C. Warm Inflation

The scalar perturbations

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V''\right) + \ldots = \boxed{\zeta_q} + \boxed{\zeta_T}$$

$$\langle \zeta_q(\mathbf{k},t), \zeta_q(\mathbf{k}',t') \rangle \simeq \frac{H^2}{\pi a^3}(2\pi)^2 \delta^{(3)}(\mathbf{k}-\mathbf{k}')\delta(t-t')$$

Quantum fluctuations

$$\langle \zeta_T(\mathbf{k},t), \zeta_T(\mathbf{k}',t') \rangle \simeq \frac{2\Gamma T}{a^3}(2\pi)^2 \delta^{(3)}(\mathbf{k}-\mathbf{k}')\delta(t-t')$$

Thermal fluctuations

- Scalar power spectrum from the ensemble average of the noise realizations ⟶ $$\Delta^2_{\mathcal{R}}(k) = \frac{1}{2\pi^2}\langle |\mathcal{R}|^2 \rangle$$

- **G(Q)** is defined with respect to the analytical estimate of the scalar power spectrum ⟶ $$G(Q) \equiv \frac{\Delta^2_{\mathcal{R},\mathrm{numerical}}\big|_{c\neq 0}}{\Delta^2_{\mathcal{R},\mathrm{analytic}}}$$

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