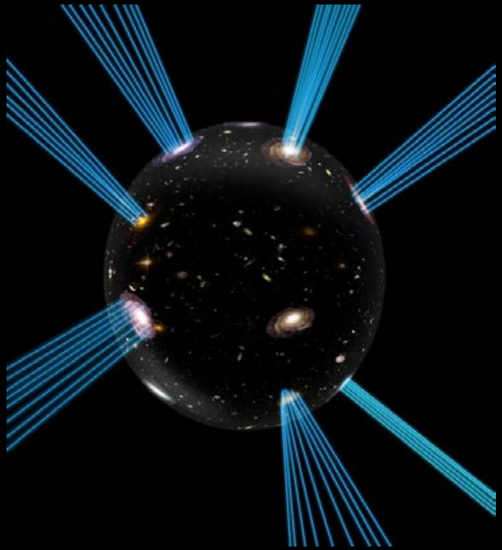
The dark bubble model

– how to get dark energy from string theory



Ulf Danielsson
Uppsala University
Sweden

Outline

Why doing this

Riding a dark bubble

The dark dimension

To get matter

The quantum origin

Outlook

Key papers

The basics of dark bubbles:

S. Banerjee, U.D, G. Dibitetto, S. Giri and M. Schillo, arXiv:1807.01570 (PRL), arXiv:1907.04268.

S. Banerjee, U.D and S. Giri, arXiv:2001.07433, arXiv:2009.01597, arXiv:2101.07433.

Review of dark bubbles:

S. Banerjee, U.D and S. Giri, arXiv:2103.17121, arXiv:2212.14004

Dark bubbles and quantum cosmology:

U.D, D. Panizo, R. Tielemans and T. Van Riet, arXiv2105.03253.

Gravitational waves:

U.D., S. Panizo, and R. Tielemans, arXiv:2202.00545.

Explicit realisation and scales:

U.D., O. Henriksson, and D. Panizo, arXiv:2211.10191.

U.D. and D. Panizo, arXiv:2311.xxxxx

Electromagnetism:

I. Basile, U.D.. S. Giri, and D. Panizo, arXiv: 2310.xxxxx

Holography:

S. Banerjee, U.D., and M. Zemsch, arXiv: 2310.xxxxx

Why doing this?

How was our universe created?

Why does it have the properties it has?

The “why” is explained by the string landscape and the anthropic principle.

The “how” is based on inflation – no need for a first moment in time. The multiverse has existed forever…

But…

Evidence accumulated over the last ten years suggest it might not be so easy to find dS in string theory…

U.D and T. Van Riet, arXiv:1804.01120

This culminated in the

The Swampland conjecture

G. Obied, H. Ooguri, L. Spodyneiko and C.Vafa, arXiv:1806.08362.

This is good news since it makes string theory more predictive (and more easy to falsify). The bigger the swampland the better it is…

What if there are no dS vauca at all? How to get dark energy?

Riding a dark bubble

Why insist on time independent vacuum? Instead of static coordinates…

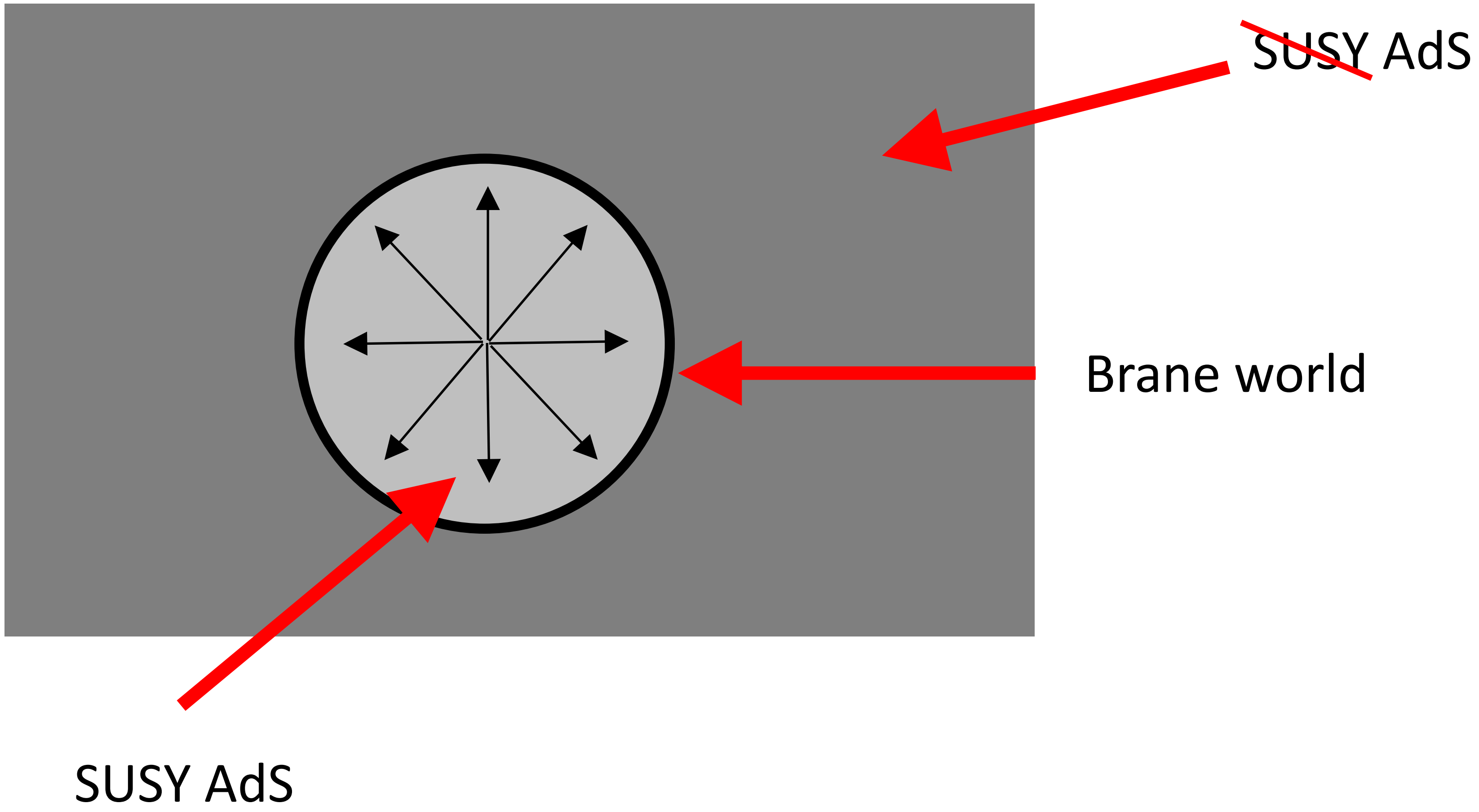
$$ds^2 = (1 - \Lambda r^2)\, dt^2 - \frac{dr^2}{1 - \Lambda r^2} - r^2 d\Omega_2^2$$

… it could make sense to consider a time dependent FRW to be more fundamental…

$$ds^2 = d\tau^2 - a(\tau)^2 dx_3^2$$

From a higher dimensional point of view there is a big difference…

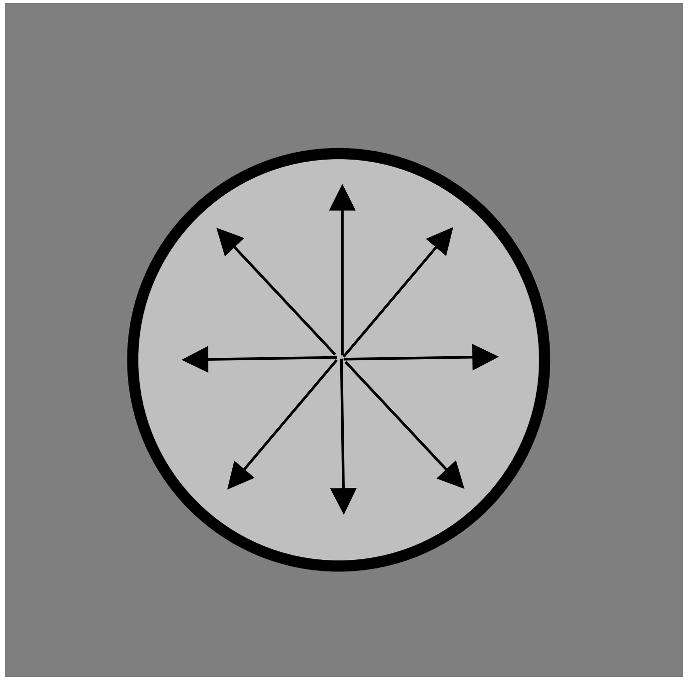
Our universe riding a bubble of true vacuum nucleating in an initial false vacuum…



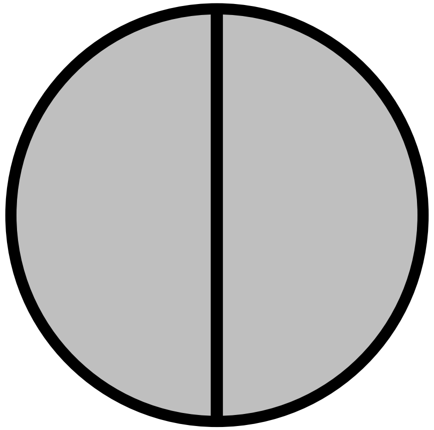
S. Banerjee, U.D, G. Dibitetto, S. Giri and M. Schillo, arXiv:1807.01570 (PRL).

Different from standard Randall-Sundrum, which makes use of a symmetric bubble with two insides, and naturally leads to AdS on the brane world for extremal branes. 4D-gravity with gravitational modes is localized close to the brane world.

The dark bubble has an inside and an outside, which automatically leads to time evolution and dS. Gravitational modes not localized, but still 4D-gravity.



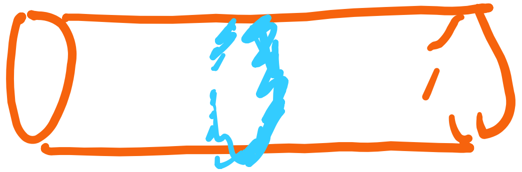
Dark bubble



RS

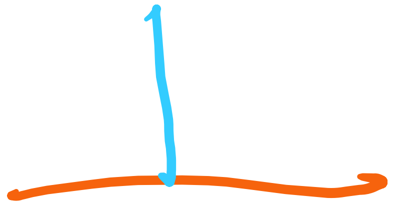
Different ways to hide extra dimensions

Compactification: Quantum smearing over extra dimension:



RS: Quantum localization on brane:

Dark bubble: Classically stretched over extra dimension:



The key is the junction conditions…

Einstein/Israel: across a thin shell, the extrinsic curvature jumps by the energy momentum tensor on the shell. Our brane is not a probe! Its backreaction on the background is what gives gravity.

Across the brane you get…

$$\sigma = \frac{3}{\kappa_5}\left(\sqrt{k_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{k_+^2 + \frac{1+\dot{a}^2}{a^2}}\right)$$

… where the cosmological constant outside/inside given by:

$$\Lambda_\pm = -6k_\pm^2$$

Note the sign!

Adding radiation through AdS-Schwarzschild and expand in large k…

$$H^2 = -\frac{1}{a} + \frac{\kappa_4}{3}\left(\rho_4 + \frac{3}{4\pi^2}\left(\frac{M_+}{k_+} - \frac{M_-}{k_-}\right)\frac{1}{a^4}\right)$$

4D radiation comes from geometry…

$$\rho_{\Lambda_4} \equiv \frac{3}{\kappa_4} R^{-2} = \frac{3\left(k_- - k_+\right)}{\kappa_5} - \sigma > 0$$

$$\kappa_4 = \frac{2k_-k_+}{k_- - k_+}\kappa_5$$

If bubble can form then dS …

The cosmological horizon as a Rindler horizon

Contrary to a black hole horizon, the cosmological horizon is observer dependent.
Just like a Rindler horizon?

... but with acceleration in the fifth dimension.

What about the Unruh temperature?

$$T = \frac{1}{2\pi}\sqrt{a^\mu a_\mu - k^2}$$

$$T = \frac{1}{2\pi}\sqrt{\frac{\left(f'(r)/2 + \ddot{r}\right)^2}{f(r) + \dot{r}^2} - k^2} = \frac{H}{2\pi} + \mathcal{O}(\mathrm{e}^{-2H\tau})$$

… which agrees with the expected dS temperature!

The cosmological horizon is a higher dimensional Rindler horizon.

The dark dimension of the dark bubble

Consider the geometry of a deformed version of $AdS_5 \times S^5$ generated by stack N rotating D3-branes with angular momentum along the internal sphere.

The angular momentum becomes a charge in 5D and the solution becomes that of a charged black hole in AdS_5.

The creation of the universe corresponds to the nucleation of a single brane in this background.

U.D., O. Henriksson, and D. Panizo, arXiv:2211.10191

Use standard relations…

In 10D… $\frac{1}{l_{10}^8} \sim \frac{1}{g_s^2 l_s^8}$ → $\frac{1}{l_5^3} \sim \frac{L^5}{l_{10}^8}$ ….in 5D.

In $AdS_5 \times S^5$ … $L^3 \sim N_c^2 l_5^3$ …and $L^4 \sim g_c N_c l_s^4$

…this leads to a hierarchy of scales:

$$L \sim N_c^{2/3} l_5 \gg l_{10} \sim N_c^{5/12} l_5 \gg l_5$$

Check junction conditions…

$$T_3 = \frac{3\Delta k}{8\pi G_5} = -\frac{3}{8\pi G_5}\frac{\Delta L}{L^2}.$$

With constants… $G_5 = \frac{\pi L^3}{2N_c^2}$ and… $L^4 = 4\pi g_s N_c l_s^4$

Across the junction we need to keep G_5 constant, so… $\frac{\Delta L}{L} = \frac{2}{3}\frac{\Delta N_c}{N_c},$

… leading to…

$$T_3 = -\frac{1}{(2\pi)^3 g_s l_s^4}\Delta N_c = -\Delta N_c T_{D3}$$

Introduce 4D gravity…

We have…

$$l_4^2 \sim \frac{k_- k_+}{k_- - k_+} l_5^3 \sim \frac{\frac{1}{L^2}}{\frac{1}{N_C L}} l_5^3 \sim \frac{N_c}{L} l_5^3$$

… from which we get…

$$L \sim N_c^{1/2} l_4$$

… and the hierarchy…

$$L \sim N_c^{1/2} l_4 \gg l_{10} \sim N_c^{1/4} l_4 \gg \; l_4 \gg l_5 \sim N_c^{-1/6} l_4$$

What is N_c?

We expect corrections that decrease the tension of the strings in line with WGC.

The single dark bubble brane in the background of N_c branes suggest correction of order $1/N_c$...

... stringy corrections to the brane tension:

$$\sim -\tau \times \alpha'^2 \Omega^2 \sim -\tau \frac{l_s^4}{L^4} \sim -\tau \frac{1}{N_c} \sim -\frac{1}{g_s L^4}$$

U.D. and D. Panizo, arXiv:2311.xxxxx

We can now determine N_c through...

$$\frac{1}{g_s L^4} \sim \frac{1}{l_4^2 R_H^2} \rightarrow R_H \sim g_s^{1/2} N_c l_4$$

... this suggests

$$N_c \sim 10^{60}$$

$L \sim 10^{-5} m$ $\quad 1/l_{10} \sim 10\ TeV$ $\quad l_5 \sim 10^{-10} l_4$

How to get matter

4D Einstein from Gauss (-Codazzi)

Is gravity really Einstein 4D?

According to Gauss (-Codazzi) we have

$$R^{(5)}_{\alpha\beta\gamma\delta}\, e^{\alpha}_{c} e^{\beta}_{a} e^{\gamma}_{d} e^{\delta}_{b} = R^{(4)}_{cadb} + (K_{ad}K_{cb} - K_{cd}K_{ab})$$

We want to solve for the extrinsic curvature and set the jump across the shell equal to the 4D energy-momentum…

S. Banerjee, U.D, G. Dibitetto, S. Giri and M. Schillo, arXiv:1907.04268

We do this by expanding according to...

$$K_{ab} = kh_{ab} + \tau_{ab}$$

This leads to...

$$G^{(4)}_{ab} = h_{ab}\left[16\pi G_5\sigma\left(\frac{k_+k_-}{k_- - k_+}\right) - 3k_+k_-\right] + \left(\frac{k_+k_-}{k_- - k_+}\right)\left[\left(\frac{\mathcal{J}^+_{ab}}{k_+} - \frac{\mathcal{J}^-_{ab}}{k_-}\right) - \frac{1}{2}h_{ab}\left(\frac{\mathcal{J}^+}{k_+} - \frac{\mathcal{J}^-}{k_-}\right)\right]$$

... where...

$$\begin{aligned}\mathcal{J}_{ab} &= R^{(5)}_{\alpha\beta\gamma\delta}\, e^\alpha_c e^\beta_a e^\gamma_d e^\delta_b\, h^{cd} \\ &= e^\beta_a e^\delta_b \left(R^{(5)}_{\alpha\beta\gamma\delta}\, e^\alpha_c e^\gamma_d h^{cd}\right) \\ &= e^\beta_a e^\delta_b \left(R^{(5)}_{\beta\delta} - R^{(5)}_{\mu\beta\nu\delta} n^\mu n^\nu\right)\end{aligned}$$

Dark energy

Radiation

$$\left(G^{(4)}\right)^a_{\ b} = \underbrace{-\,2k_+k_-\left(3-\frac{\kappa_5^2}{k_- - k_+}\sigma\right)}_{\equiv\,\kappa_4^2(\sigma_{\rm crit}-\sigma)\,\equiv\,\kappa_4^2\Lambda_4}\delta^a_b - \frac{\kappa_5^2}{\pi^2 a(\tau)^4}\left(\frac{M_+k_- - M_-k_+}{k_- - k_+}\right)\left(\delta^a_0\delta^0_b - \frac{1}{3}\sum_{i=1}^{3}\delta^a_i\delta^i_b\right)$$

$$-\,\frac{3\kappa_5^2}{4\pi a(\tau)^3}\left(\frac{\alpha_+k_- - \alpha_-k_+}{k_- - k_+}\right)\delta^a_0\delta^0_b,$$

Dust

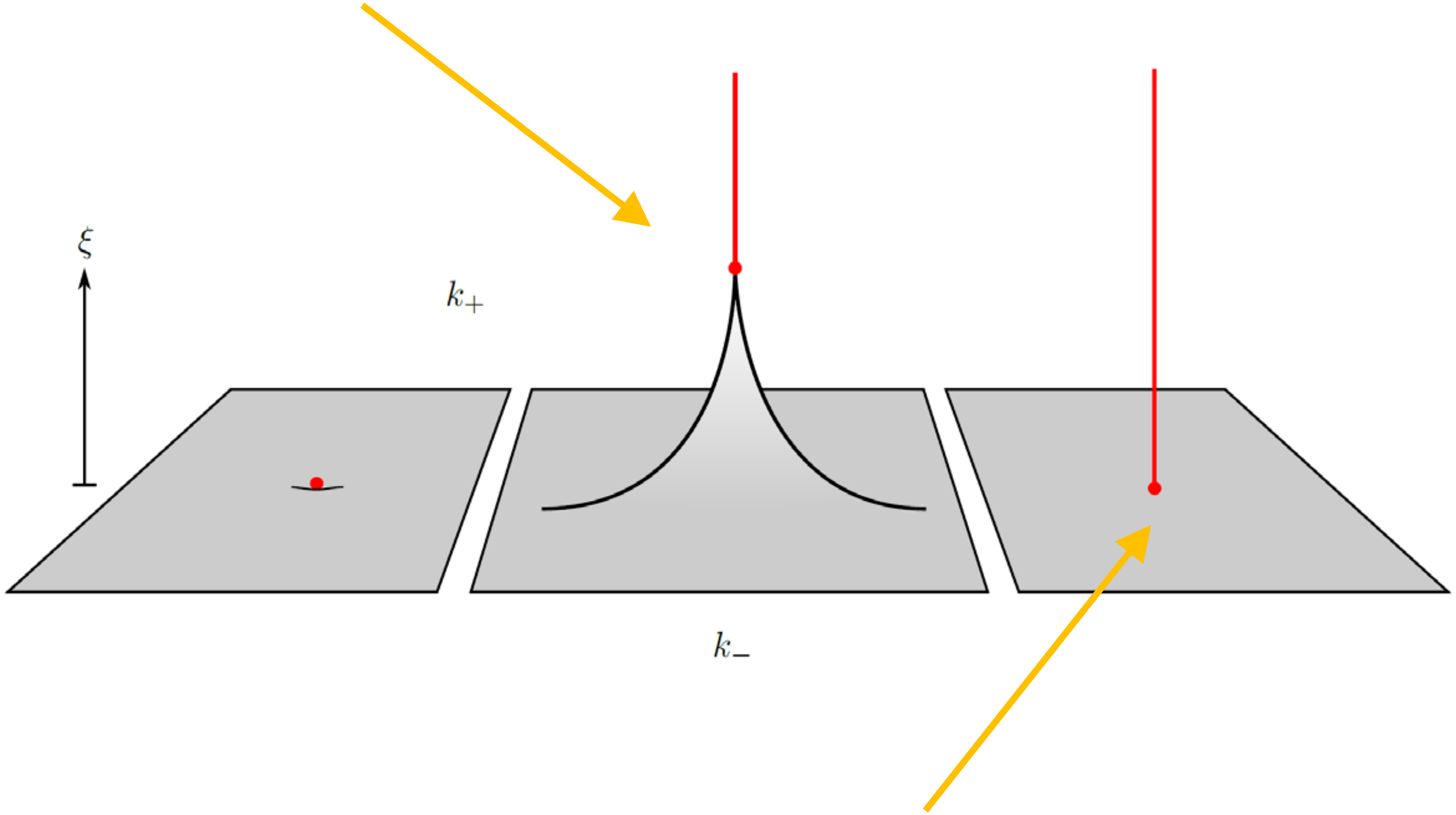
Dust

Matter is NOT localized on the shell. To get dust you need strings ending on the brane world…

The point is that you need to compensate for the redshift of radiation through….

$$m \sim r \to \frac{m}{r^2} \sim \frac{1}{r} \to \frac{m}{a^4} \sim \frac{1}{a^3}$$

A single string pulling on the brane induces the Schwarzschild metric…



… you can also go to a flat gauge, where a mass, which is pulling down, is induced at the end point of the string in accordance with:

$$\nabla_\mu T^{\mu\nu} = 0$$

Shaking the bubble

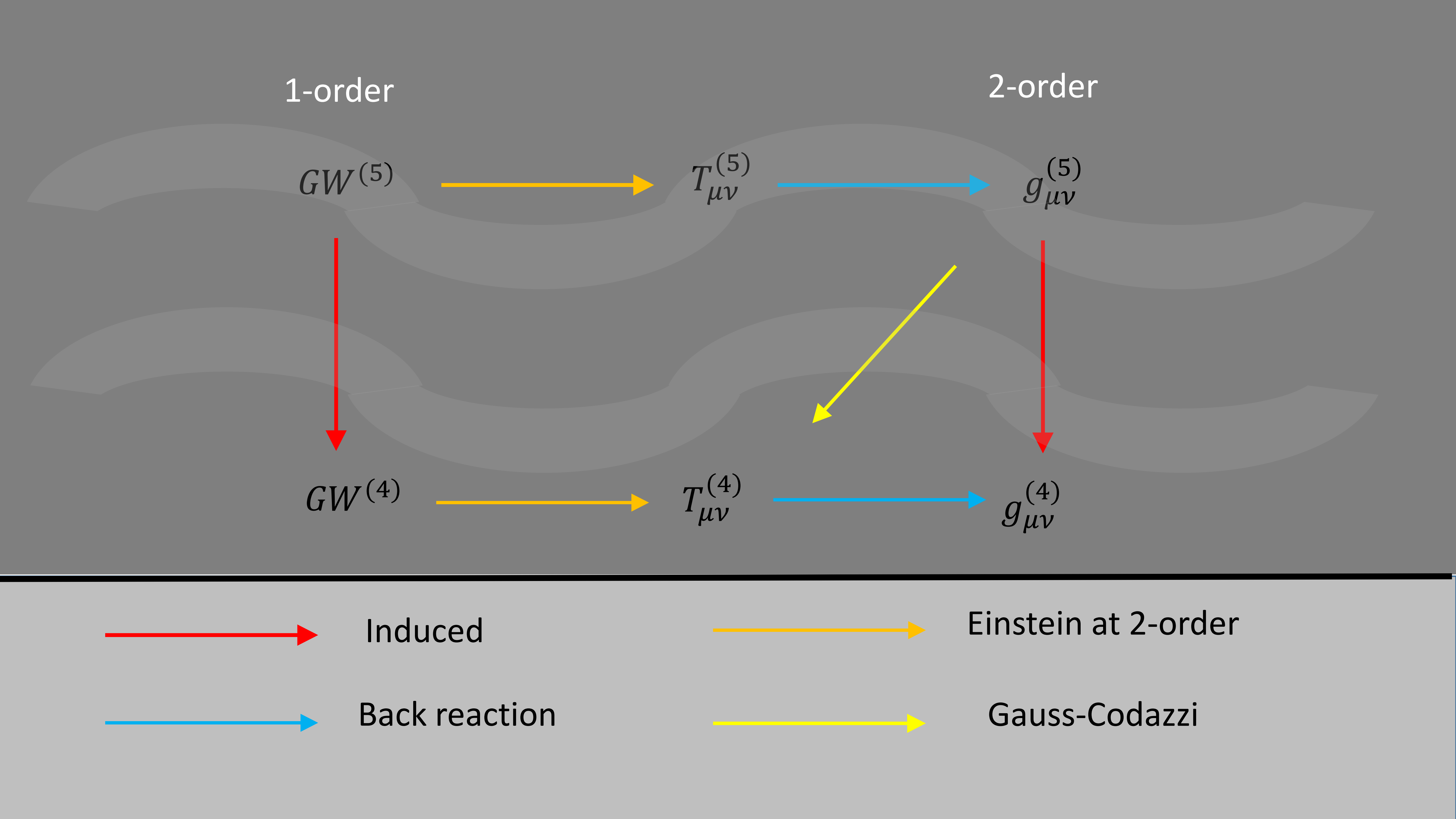
Standard procedure…

Einstein in vacuum solved to first order to get GW.

Evaluate Einstein at second order and identify the failure to solve as the energy-momentum tensor, which, in turn, will generate a second order back reaction.

In the process, average over several wave-lengths.

U.D., S. Panizo, and R. Tielemans, arXiv:2202.00545


1-order
2-order
$GW^{(5)}$
$T_{\mu\nu}^{(5)}$
$g_{\mu\nu}^{(5)}$
$GW^{(4)}$
$T_{\mu\nu}^{(4)}$
$g_{\mu\nu}^{(4)}$
Induced
Einstein at 2-order
Back reaction
Gauss-Codazzi

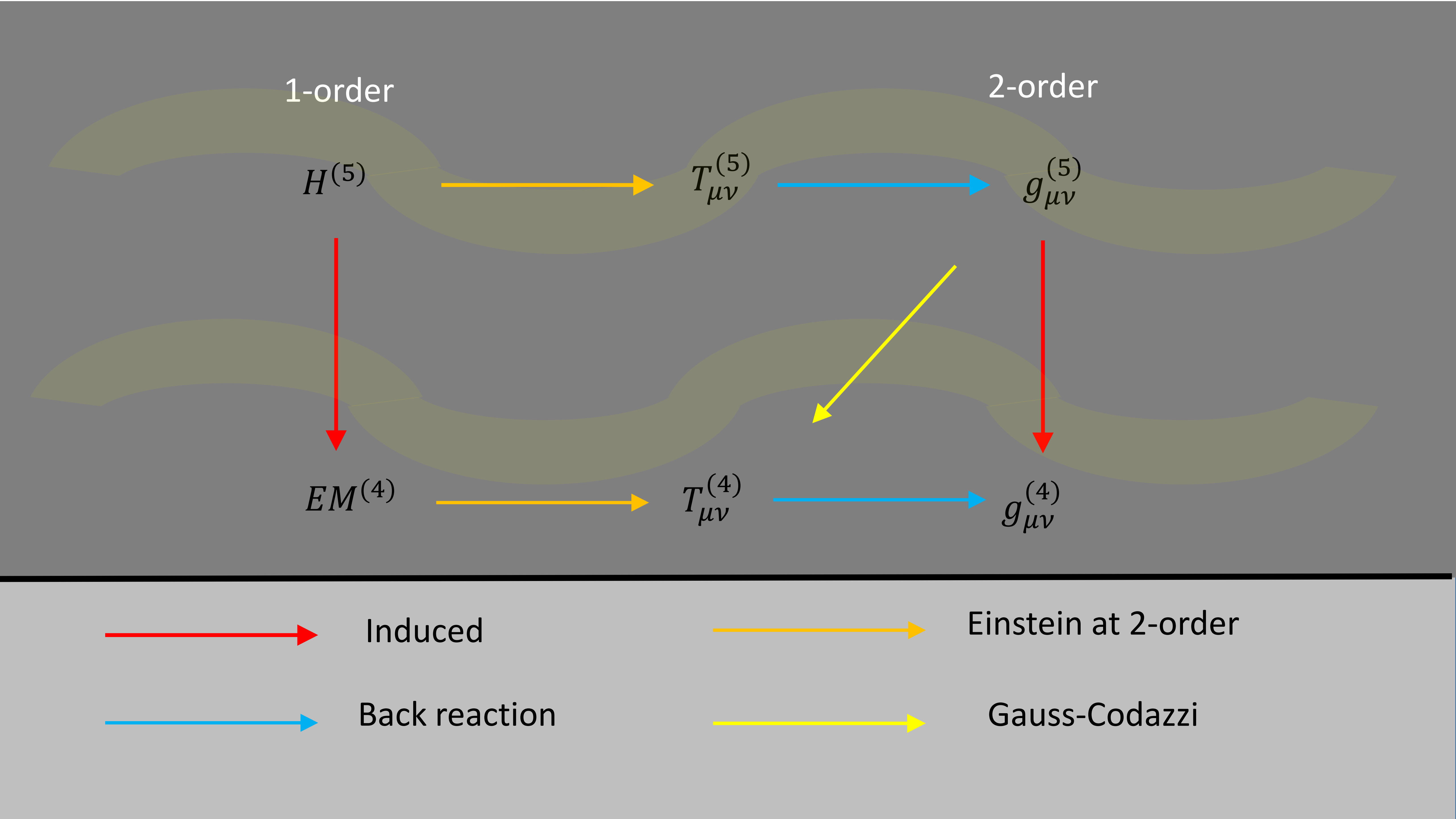
Turning on the light

Electromagnetism is identified with the gauge field carried by the DBI-action…

$$S_5 = \frac{1}{2\kappa_5}\int d^5x\sqrt{-\det\left(g_5\right)}\left(R-\frac{1}{12}H^2\right) - T_3\int d^5x\,\delta(r-a[\eta])\,\sqrt{-\det\left(g_4+\tau\mathscr{F}_{\mu\nu}\right)}$$

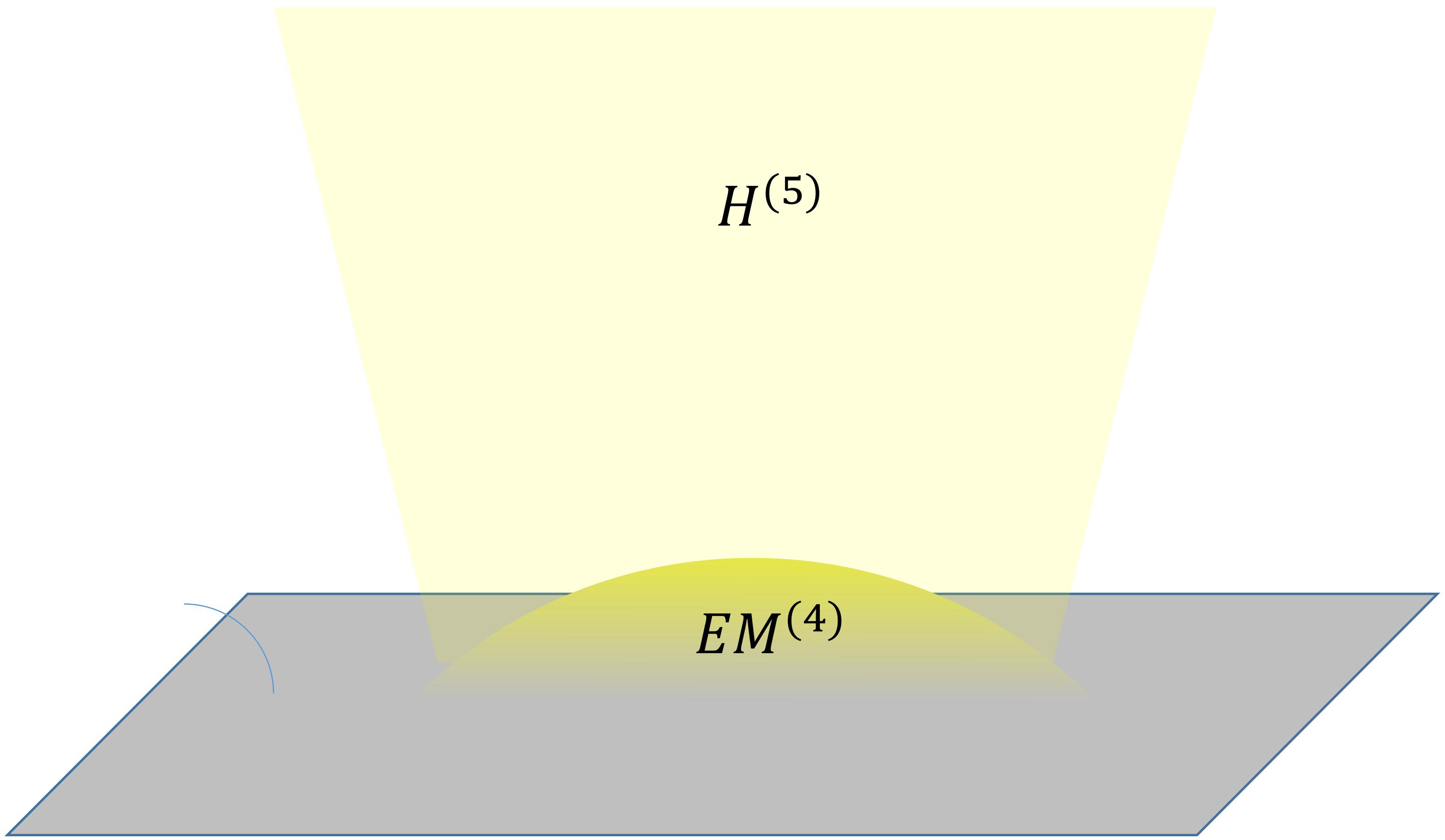
… which will act as a source for the bulk H-field…

I. Basile, U.D.. S. Giri, and D. Panizo, arXiv: 2310.xxxxx



1-order
2-order
$H^{(5)}$
$T^{(5)}_{\mu\nu}$
$g^{(5)}_{\mu\nu}$
$EM^{(4)}$
$T^{(4)}_{\mu\nu}$
$g^{(4)}_{\mu\nu}$
Induced
Einstein at 2-order
Back reaction
Gauss-Codazzi

The B- field lifts the brane much in the same way as the pulling string does…



Matter on brane governed by equations of motion and Bianchi identities with a conserved energy momentum tensor.

Back reaction from bulk.

$$G = \underbrace{-T_{brane} + T_{extrinsic}}_{T_{4D}}$$

Effective energy momentum tensor in 4D.

If $T_{4D} = T_{brane}$ we only have a **visible** sector.

If $T_{4D} \neq T_{brane}$ we have a **dark** sector.

The quantum origin

The “how” is quantum nucleation out of nothing. Natural to consider this as the first moment in time.

The “why” would be explained by a peaked wavefunction…

Let us consider a metric of FRW form

$$ds^2 = -N^2(\tau)d\tau^2 + a(\tau)^2 d\Omega_3^2$$

With a positive cosmological constant we obtain the action

$$S = \frac{6\pi^2}{\kappa_4} \int d\tau N \left(-\frac{a\dot{a}^2}{N^2} + a - \frac{a^3}{R^2} \right) \qquad R^{-2} = \frac{\kappa_4}{3} \rho_{\Lambda_4} = \Lambda_4$$

… and the classical equations of motion: $\dot{a}^2 = -1 + \frac{a^2}{R^2}$

The conjugate momentum is given by $$p = -\frac{12\pi^2 a\dot{a}}{N}$$

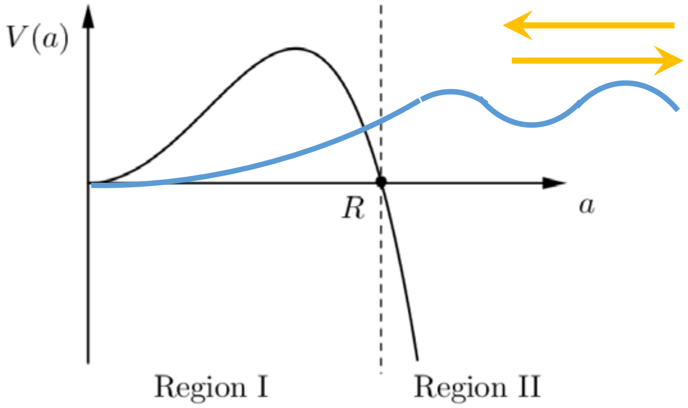
Using canonical quantization $$p \rightarrow i\frac{d}{da}$$

… we find:

$$\frac{N}{a}\left(-\frac{1}{24\pi^2}\frac{d^2}{da^2} + 6\pi^2 V(a)\right)\Psi(a) = 0 \qquad V(a) = a^2 - \frac{a^4}{R^2}$$

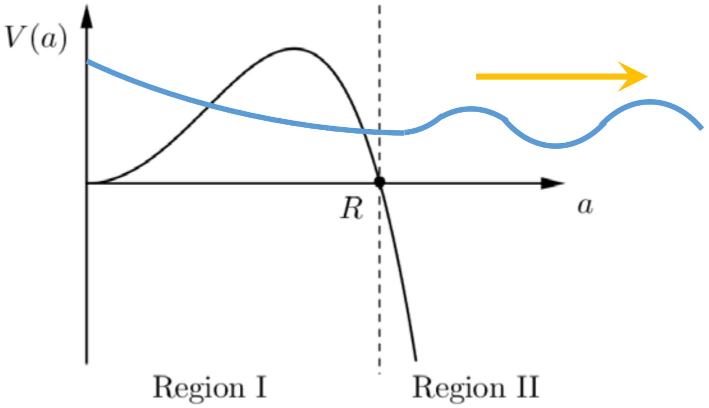
Hartle-Hawking (HH) = no-boundary

$$S(a, a_i) \equiv \frac{12\pi^2}{\kappa_4} \int_{a_i}^{a} \sqrt{|V(a')|}\, da' \,, \qquad S_0 \equiv S(R,0) = \frac{4\pi^2 R^2}{\kappa_4}$$



$$\Psi_{\mathrm{HH}}(a) = \frac{1}{|2V(a)|^{1/4}} \begin{cases} e^{S(a,0)} & \text{Region I} \\ 2e^{S_0} \cos\left(S(a,R) - \frac{\pi}{4}\right) & \text{Region II} \end{cases}$$

Vilenkin (V) = tunneling



$$\Psi_{\mathrm{V}}(a) \approx \frac{1}{|2V(a)|^{1/4}} \begin{cases} e^{-S(a,0)+i\frac{\pi}{4}} & \text{Region I} \\ e^{-S_0} e^{-iS(a,R)} & \text{Region II} \end{cases}$$

A quantum bubble

Let us consider the quantum nucleation of a dark bubble.

The classical equation is the junction condition. This can be viewed as a Hamiltonian constraint of the form

$$H = 2\pi^2 N a^3 \left(\sigma - \frac{3\left(\beta_- - \beta_+\right)}{\kappa_5 a} \right) = 0$$

$$\beta_\pm = \sqrt{A_\pm N^2 + \dot{a}^2}$$

$$A_\pm(r) = 1 - \frac{\Lambda_\pm}{6} r^2$$

It can be obtained from the action...

$$S = \frac{1}{2\kappa_5} \int d^5x \sqrt{g} \left(R^{(5)} - 2\Lambda \right) - \sigma \int d^4\zeta \sqrt{\eta} + \frac{1}{\kappa_5} \oint d^4x \sqrt{h} K$$

The Hamiltonian constraint can be obtained from a Lagrangian…

$$L = \frac{6\pi^2}{\kappa_5}\left[-a^2\dot{a}\tanh^{-1}\frac{\dot{a}}{\beta} + a^2\beta\right]_+^- - 2\pi^2 a^3 \sigma N$$

This can be used to calculate the nucleation probability given by e^{-B}. In the limit where k is large, the calculation coincides with that of Vilenkin, yielding:

$$B = \frac{24\pi^2}{\kappa_4}\int_0^R da\sqrt{a^2 - \frac{a^4}{R^2}} = \frac{8\pi^2 R^2}{\kappa_4}$$

An equivalent way to obtain the same result, is to calculate the Euclidean action of the O(5) invariant instanton of Coleman de Luccia (see also Brown Teitelboim)…

We get:

$$B = \sigma A_4 + \frac{1}{\kappa_5}\left[4k^2 V_5\left(R,k\right) - \frac{4}{R}\beta A_4\right]_{+}^{-}$$

$$A_4 \;=\; 8\pi^2 R^4/3 \qquad\qquad dV_5/dR \;=\; A_4/\beta$$

This gives exactly the same result in the limit.

We can also obtain the Wheeler de Witt equation from the constraint…

$$H = -\frac{6\pi^2}{\kappa_5}\left(A_- + A_+ - 2\sqrt{A_- A_+}\cosh\left(\frac{\kappa_5 p}{6\pi^2 a^2}\right)\right)^{1/2} = 0$$

where:

$$\cosh\left(\frac{\kappa_5 p}{6\pi^2 a^2}\right) = \frac{\beta_-\beta_+ - \dot{a}^2}{N^2\sqrt{A_- A_+}}$$

Canonical quantization using the correct measure…

$$p \to \frac{i}{a^{3/2}}\frac{d}{da}a^{3/2}$$

… yields a difference equation.

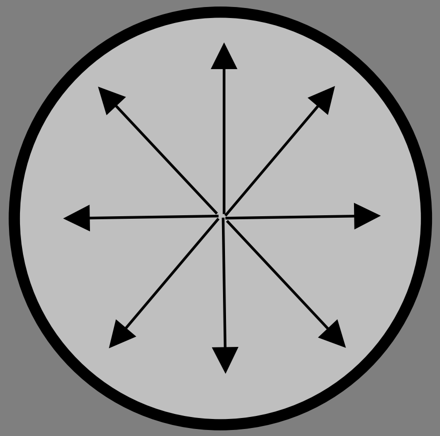
In the limit we find:

$$\left(-\frac{1}{24\pi^2}\frac{1}{a^{3/2}}\frac{d^2}{da^2}(a^{3/2}\psi)+6\pi^2V(a)\right)\psi=0\,,\qquad V(a)=a^2-\frac{a^4}{R^2}$$

… which with correct normalization coincides with Vilenkin.

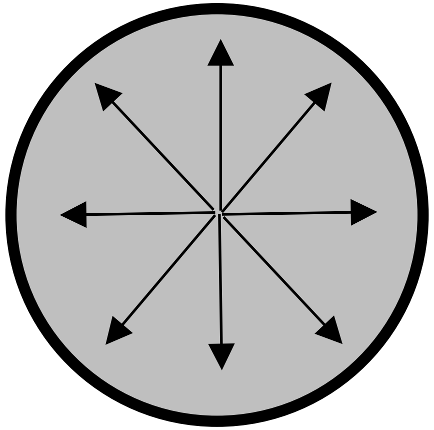
The nucleation of a bubble in AdS5 coincides with Vilenkin’s tunneling wave function!

Something



Dark bubble => Vilenkin

Nothing



RS => Hartle-Hawking

Outlook

Fit the early universe. Physics of nucleation?

Make contact with standard model. New physics at 10 TeV?

Dark matter from the dark sector?

Quantum effects on the effective cc. Swampland?

Need to incorporate black holes. Is it all bubbles?

(U.D. and S. Giri, arXiv:2310.12148)