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CONNECTING SCIENCES

Palatini $F(R, X)$: a new framework for inflationary attractors

Antonio Racioppi

National Institute of Chemical Physics and Biophysics, Tallinn, Estonia

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based on
arXiv:2307.02963
with C. Dioguardi (Taltech & NICPB)

Euroopa Liit
Euroopa Sotsiaalfond

Eesti tuleviku heaks

• Metric vs Palatini •

The properties of spacetime are essentially described by:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$ → parallel transport
- the metric tensor: $g_{\mu\nu}$ → distance

The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship. If they are to have any relationship, it must derive from

- additional constraints (metric formalism $\nabla_\alpha g_{\mu\nu} = 0$)
- EoM for both Γ and g (Palatini formalism)

If Einsteinian gravity ($\sim R$), EoM $\Rightarrow \nabla_\alpha g_{\mu\nu} = 0$ (i.e Palatini ≡ metric)

With non-minimal theories, metric and Palatini formalism generate different physical theories. (Koivisto & Kurki-Suonio: arXiv:0509422)

• Palatini $F(R) + \mathcal{L}(\phi)$ •

- We start with the following action in the Palatini formulation

$$S_J = \int \mathrm{d}^4x\sqrt{-g_J}\left[\frac{1}{2}F(R(\Gamma)) + \mathcal{L}(\phi)\right]$$

$$\mathcal{L}(\phi) = -\frac{1}{2}g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

- we rewrite the $F(R)$ term using the auxiliary field ζ, obtaining

$$S_J = \int \mathrm{d}^4x\sqrt{-g}_J\left[\frac{1}{2}\left(F(\zeta) + F'(\zeta)\left(R(\Gamma) - \zeta\right)\right) + \mathcal{L}(\phi)\right]$$

- we move to the Einstein frame: $g^E_{\mu\nu} = F' g^J_{\mu\nu}$ N.B. now $\boxed{\Gamma^E = \Gamma^J}$

$$S_E = \int \mathrm{d}^4x\sqrt{-g_E}\left[\frac{R}{2} - \frac{1}{2F'(\zeta)}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\chi,\zeta)\right]$$

$$U(\chi,\zeta) = \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$$

$$\frac{\partial\chi}{\partial\phi} = \frac{1}{\sqrt{F'(\zeta)}} \quad \text{(canonically normalized scalar)}$$

- no $-\frac{3}{2}\left(\frac{\partial F'}{F'}\right)^2$ like in metric gravity! ζ still auxiliary! still single field setup!

• EoM and problems •

- The full EoM for ζ is

$$G(\zeta) = \frac{1}{2}\,\partial^{\mu}\phi\partial_{\mu}\phi\,F'(\zeta) + V(\phi)$$

with

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right]$$

- The standard procedure would be now to solve the EoM and determine $\zeta(\phi, \partial^{\mu}\phi\partial_{\mu}\phi)$ and insert it back into the action $\rightarrow$ Problem!!!!

- Higher order scalar kinetic term because ζ depends also on $(\partial\phi)^2$

- manageable if $F \sim R^2$ (e.g. Enckell et al., 1810.05536)
- disastrous if $F \sim R^n$ and $n > 2$ (A.R. et al., 2212.11869)

- solution $\rightarrow$ $F(R - X)$, $X = g_J^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi$

• Beyond $F(R)$: $F(R,X)$ •

- We consider the following $F(R,X)$

$$S = \int d^4x\sqrt{-g^J}\left(\frac{1}{2}F(R_X) - V(\phi)\right)$$

where $R_X = R - X$, $X = g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ denotes the inflaton kinetic term

- again rewrite the action with the auxiliary field ζ:

$$S = \int d^4x\sqrt{-g^J}\left(\frac{1}{2}F(\zeta) + \frac{1}{2}F'(\zeta)(R - g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \zeta) - V(\phi)\right)$$

N.B. the only difference is the F' prefactor in front of $(\partial\phi)^2$

- the corresponding Einstein frame action:

$$S = \int d^4x\sqrt{-g^E}\left(\frac{R}{2} - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\zeta,\phi)\right)$$

- ϕ already canonically normalized! $\Rightarrow$ no disastrous dynamics
- U same as before!

The full EoM for ζ now is

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = V(\phi)$$

- $V \to G$ in U:

$$\begin{aligned} U(\phi,\zeta) &= \frac{V(\phi)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\ &= \frac{G(\zeta)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\ &= \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} - \frac{\zeta}{4F'(\zeta)} - \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} + \frac{\zeta}{2F'(\zeta)} \\ &= \boxed{\frac{1}{4}\frac{\zeta}{F'(\zeta)} = U(\zeta)} \qquad \zeta = \zeta(\phi) \end{aligned}$$

- valid for any $F(R_X)$ and $V(\phi)$

• Quadratic $F(R_X)$ •

- Now we focus on the class of quadratic F's

$$F(R_X) = 2\Lambda + \omega R_X + \alpha R_X^2 \qquad \boxed{R_X = R - X}$$

- EoM for $\zeta \to \Lambda + \frac{\omega}{4}\zeta = V(\phi)$

- Einstein frame potential: *fractional attractors*

$$U(\phi) = \frac{\bar{V}(\phi)}{8\alpha\bar{V}(\phi) + \omega^2} \qquad \text{with } \bar{V}(\phi) = V(\phi) - \Lambda$$

N.B. $U \sim$ Enckell et al., 1810.05536 but no $\frac{\partial\chi}{\partial\phi}$
$\Rightarrow$ different predictions

Quadratic $F(R_X)$

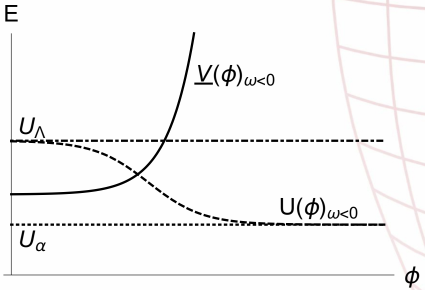
- $F(R_X) = 2\Lambda + \omega R_X + \alpha R_X^2$

$$\boxed{U(\phi) = \frac{\bar{V}(\phi)}{8\alpha\bar{V}(\phi) + \omega^2}} \qquad \text{with } \bar{V}(\phi) = V(\phi) - \Lambda$$

- Requiring $U(\phi) \geq 0$, $F'(\zeta) > 0$ allows only:
  1. $\omega > 0, \Lambda \leq 0, V(\phi) \geq 0 \Rightarrow \bar{V} > 0 \rightarrow$ *canonical* $\rightarrow$ Q&A
  2. $\omega < 0, \Lambda > 0, V(\phi) \leq 0 \Rightarrow \bar{V} < 0 \rightarrow$ *tailed*

- In both cases $\alpha > \frac{\omega^2}{8\Lambda}$ for $\Lambda \neq 0$

Quadratic $F(R_X)$: $\omega < 0$



- $U(\phi) = \frac{\bar{V}(\phi)}{8\alpha\bar{V}(\phi)+\omega^2} = \frac{\underline{V}(\phi)}{8\alpha\underline{V}(\phi)-\omega^2}$

  with $\bar{V}(\phi) = V(\phi) - \Lambda = -\underline{V}(\phi)$

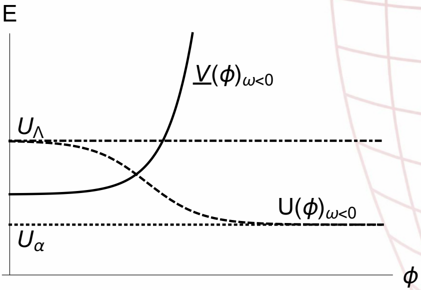
- $V = 0 \;\Rightarrow\; \zeta_0 = -\frac{\Lambda}{4\omega},\; U_\Lambda = \frac{\Lambda}{8\alpha\Lambda-\omega^2}$

- $V(\phi) \to \pm\infty \;\Rightarrow\; U \to U_\alpha = \frac{1}{8\alpha}$

- $V(\phi) \sim -\lambda_k \phi^k$
- $\alpha \to +\infty$

$$\left.\begin{matrix} V(\phi) \sim -\lambda_k \phi^k \\ \alpha \to +\infty \end{matrix}\right\} \Rightarrow k\text{-hilltop inflation} \Rightarrow \begin{cases} r \sim \frac{2}{3\pi^2 A_s}\frac{\Lambda}{8\Lambda\alpha-\omega^2} \sim 0 \\ n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e} \end{cases}$$

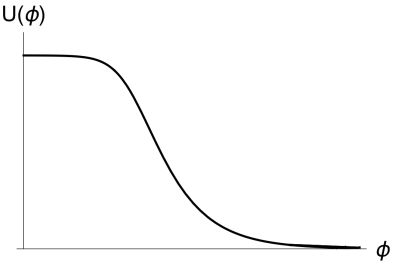
• Quadratic $F(R_X)$: $\omega < 0$ •



- $U_\Lambda = \frac{\Lambda}{8\alpha\Lambda - \omega^2} \rightarrow$ inflation
- $U_\alpha = \frac{1}{8\alpha} \rightarrow$ CC
- α gigantic $\Rightarrow$ usually U_Λ too small

With an extreme tuning of Λ and α it is possible to keep U_Λ well separated from U_α $\rightarrow$ confirmation (not a solution!) of the problem

• General behavior of $F_{>2}(R_X)$ •



- A.R. et al., 2212.11869, 2307.02963
- $F_{>2}(R_X)$ i.e. $R + R_X^3,\ e^{R_X},\ \ldots$
- $U(\phi) = \frac{\zeta(\phi)}{4F'(\zeta(\phi))}$
- inflation around plateau $\leftrightarrow \zeta_0$
- tail $\to 0 \Rightarrow$ CC solved?

- expand $F_{>2}(\zeta)$ in Taylor series around ζ_0 up to the 2nd order:

$$F_2(\zeta) = \cdots = \boxed{2\Lambda + \omega\zeta + \alpha\zeta^2}$$

$$\Lambda = -\zeta_0 G'(\zeta_0) > 0\,, \quad \omega = 4G'(\zeta_0) < 0\,, \quad 2\alpha = F''(\zeta_0) > 0$$

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right]$$

- same inflationary predictions as before

Conclusions

- We studied single field inflation within Palatini $F(R_X)$ gravity
- $F(R_X)$ solves the problem of troublesome $(\partial\chi)^2$
- $F(R_X) = 2\Lambda + \omega R_X + \alpha R_X^2$
  - asymptotically flat U's

  $\omega < 0$ $\rightarrow$ quintessential inflation(?) but overtuned
- overtuning could be solved by $F_{>2}(R - X)$
- More studies coming soon (or later ☺)

Grazie! - Thank you! - Aitäh!

• Conclusions •

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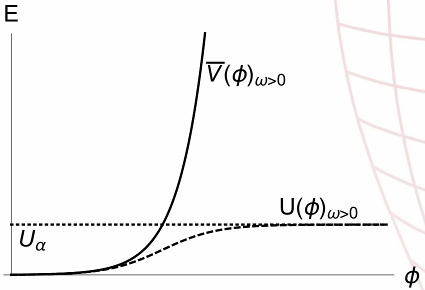
BACKUP SLIDES

• $\omega > 0$ configuration •



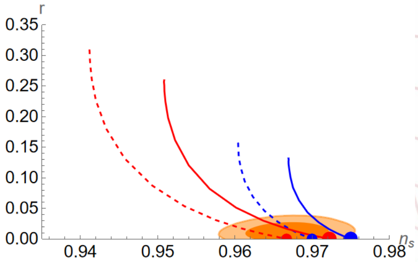
In this case we have $U_\alpha > U_\Lambda$

Inflation happens at large ϕ close to the U_α plateau

In this region the potential shape can be approximated by $U(\phi) \sim U_\alpha(1 - \frac{\omega^2 U_\alpha}{V(\phi)})$ which generalizes the polynomial α-attractors

Since for $\alpha\bar{V} >> \omega^2$ we generate asymptotically flat potentials → **canonical fractional attractors**

• $\omega > 0$ configuration •



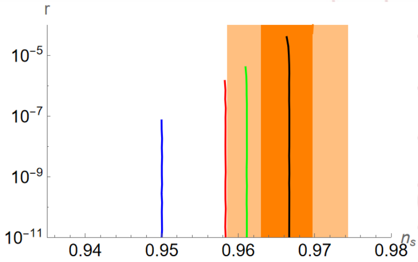
By choosing a monomial potential $V(\phi) = \frac{\lambda}{k!}\phi^k$ and taking the strong coupling limit we get the polynomial α-attractors prediction

$$r \sim 0$$

$$n_s = 1 - \frac{k+1}{k+2}\frac{2}{N_e}$$

The plot above shows the results for $V(\phi) = \frac{m^2}{2}\phi^2(blue)$ and $V(\phi) = \frac{\lambda}{4!}\phi^4(red)$

• Quadratic $F(R_X)$: $\omega < 0$ •



By choosing a monomial potential $V(\phi) = -\frac{\lambda}{k!}\phi^k$ and taking the strong coupling limit we get the small r hilltop prediction

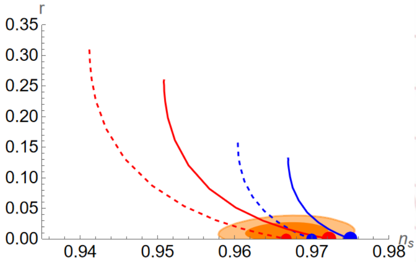
$$r \sim 0$$

$$n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e}$$

The plot above shows the results for $k = 4(blue), 6(red), 8(green)$ and $V(\phi) = -e^{\lambda\phi}(black)$

Not every α is allowed in this case, $\alpha > \alpha_{min}$ model dependent, but always such that $r < 10^{-5}$

• $\omega > 0$ configuration •



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• CC tuning •

It is convenient to introduce the parameter δ so that $\alpha\Lambda = \frac{\omega^2}{8}(1+\delta)$. We can prove that

$$r \approx \frac{1}{12\pi^2 A_s \alpha}\frac{1+\delta}{\delta} \qquad \text{when } \alpha \gg 1 \text{ and } \delta \sim O(1)$$

It is immediate to check that

$$U_\Lambda = \frac{\Lambda}{8\alpha\Lambda - \omega^2} = \frac{1}{8\alpha}\frac{1+\delta}{\delta} \gtrsim \frac{1}{8\alpha}\,.$$

Therefore, adjusting $U_\alpha = \frac{1}{8\alpha}$ to the observed value of the vacuum energy density also lowers the inflationary plateau making its value too low to be phenomenologically consistent with the evolution of the universe. An alternative option is to take $\delta \ll 1$, but $\alpha\delta A_s \gg 1$ so that we still get a small value

$$r \sim \frac{1}{12\pi^2 \alpha\delta A_s}\,.$$

For instance, if we consider $U_\alpha = \frac{1}{8\alpha} \sim 10^{-47}\text{GeV}^4$ we have that $\alpha \sim 10^{122}$. In order to get a value of $r \sim 10^{-6}$ (which still corresponds to a high enough energy scale for inflation around 10^{15}GeV) we would need an extremely fine-tuned $\delta \sim 10^{-110}$.