Electroweak Sphaleron in a magnetic field

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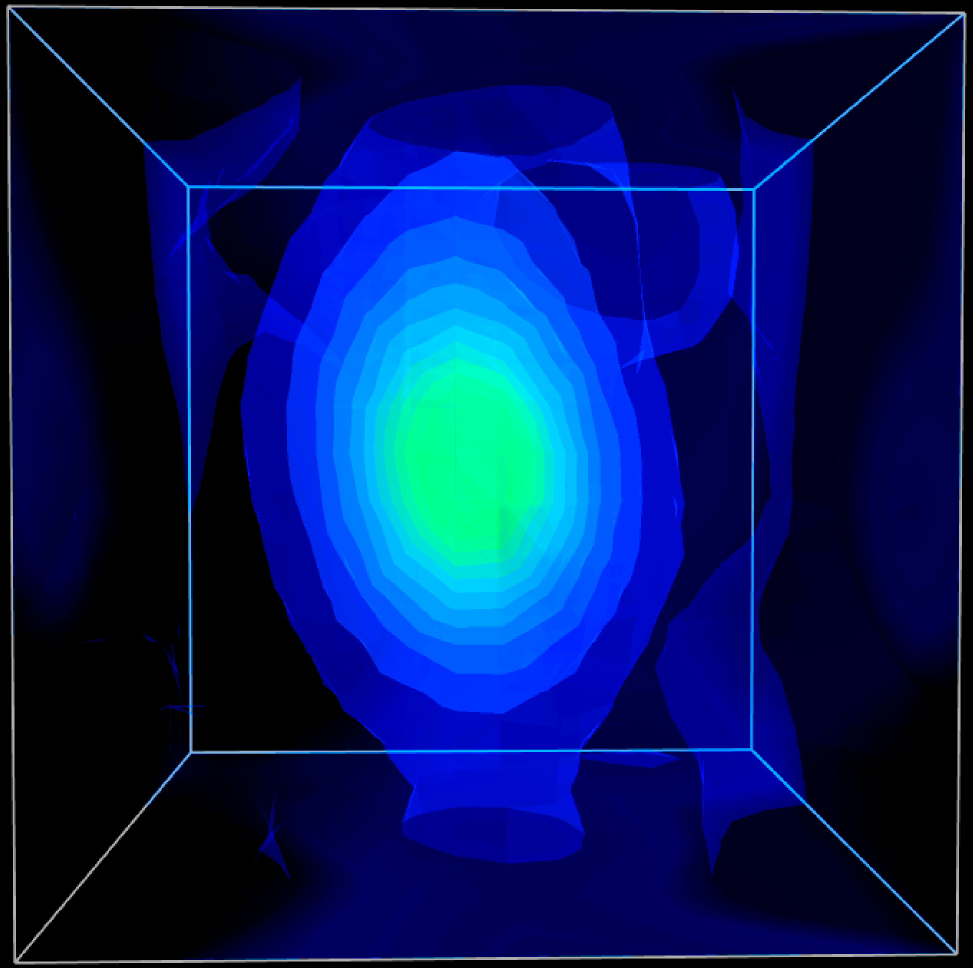


JA, Kari Rummukainen [arxiv:2301.08626]

Electroweak Sphaleron in a magnetic field

Contents

- Electroweak Sphaleron: what and why?
- What changes when there is an external (hyper)magnetic field?
    - Lattice simulation results

JA, Kari Rummukainen [arxiv:2301.08626]

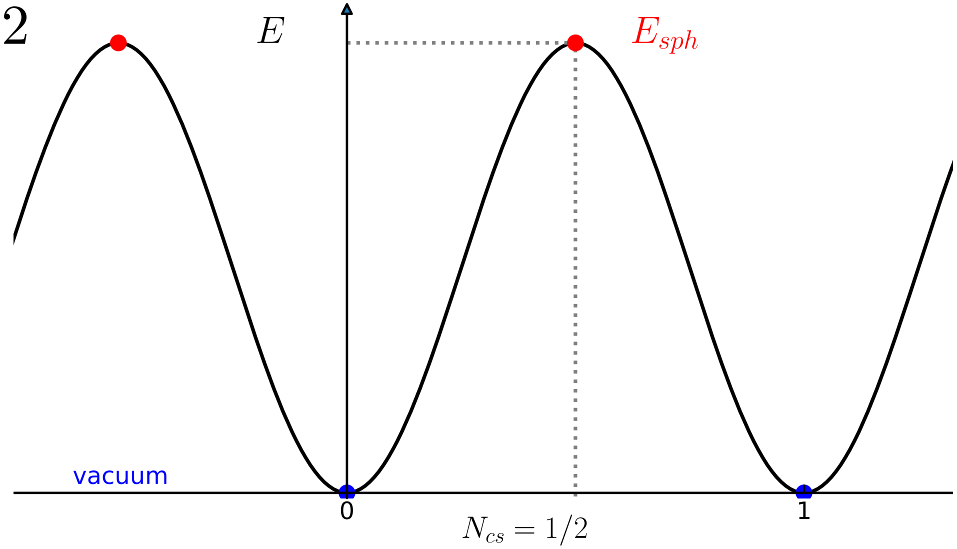
Electroweak Sphaleron

- EW chiral anomaly leads to non-conservation of baryon and lepton numbers: $3\Delta N_{CS} = \Delta B = \Delta L$
- Chern-Simons numbers: $N_{\rm CS}(t) \equiv N^W_{\rm CS}(t) - N^Y_{\rm CS}(t)$
- U(1): $$N^Y_{\rm CS}(t) \equiv \frac{g'^2}{32\pi^2}\int_0^t {\rm d}t \int {\rm d}^3x \epsilon_{\alpha\beta\gamma\delta} B^{\alpha\beta} B^{\gamma\delta}$$
  - Trivial in vacuum. Identical to zero. Non trivial in the presence of an external magnetic field.
- SU(2): $$N^W_{\rm CS}(t) \equiv \frac{g^2}{32\pi^2}\int_0^t {\rm d}t \int {\rm d}^3x \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$
  - Non-trivial due to the sphalerons.

Sphaleron arises from non-trivial topology of SU(2)

- Infinitely many classically equivalent but topologically different vacua.
- **Sphaleron**: finite energy solution of classical EoMs separating two topologically distinct vacua.
- In vacuum: $N_{\mathrm{CS}}^{W}(t) = \text{integer}$
- At Sphaleron: $N_{\mathrm{CS}}^{W}(t) = \text{integer} + \dfrac{1}{2}$

$$3\Delta N_{CS} = \Delta B = \Delta L$$



Sphaleron rate

$$\boxed{\Gamma = \lim_{V,t\to\infty} \frac{\langle N_{\mathrm{CS}}(t)^2 \rangle}{Vt}}$$

- How fast sphaleron transitions occur.
- How the rate behaves through the electroweak phase transition is important quantity for Baryo/Lepto-genesis scenarios.
- Studied extensively in the past. Usually U(1) has been ignored.

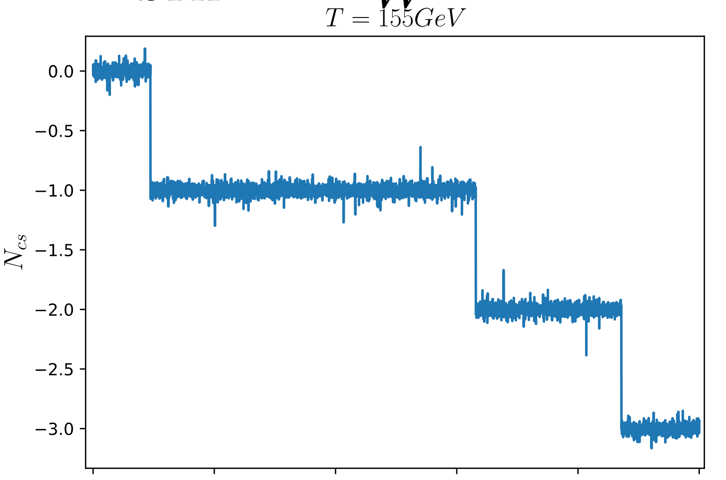
without U(1): [D'Onofrio et al. ArXiv:1207.0685, ArXiv:1404.3565 …]

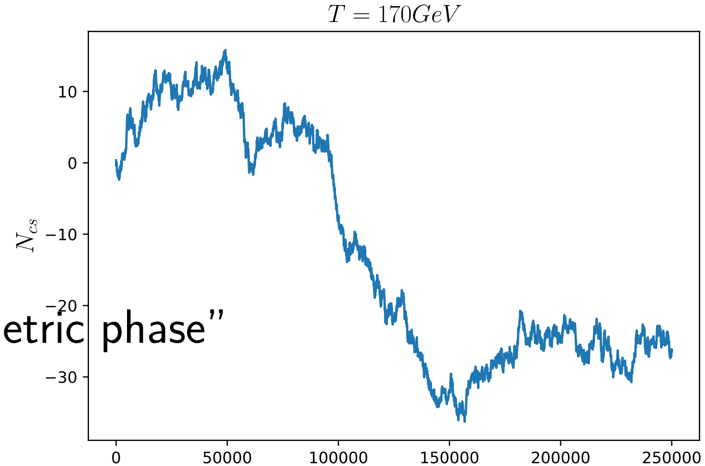
[Arnold et al. Phys. Rev. D 36, 581–595 (1987)]

$$\Gamma_{\text{brk}} \sim \alpha_W^4 T^4 e^{-E_{sph}/T}$$

[Arnold et al. arXiv:hepph/9609481]

$$\Gamma_{\text{sym}} \sim \alpha_W^5 \log(\alpha_W^{-1}) T^4$$

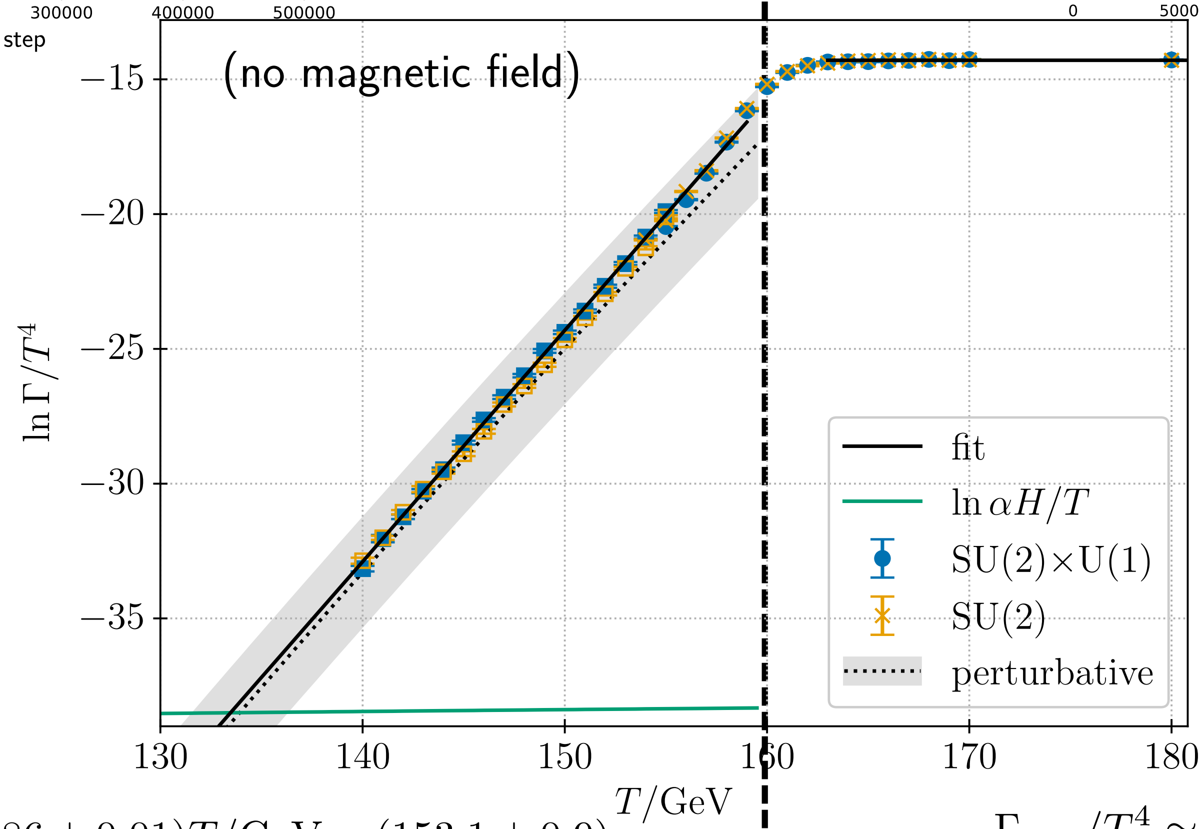




“Broken phase”

“Symmetric phase”

$$T_c \simeq 160\,\text{GeV}$$



$$\ln(\Gamma_{\text{brk}}/T^4) = (0.86 \pm 0.01)T/\text{GeV} - (153.1 \pm 0.9)$$

$$\Gamma_{\text{sym}}/T^4 \simeq (13.9 \pm 0.1)\alpha_W^5$$

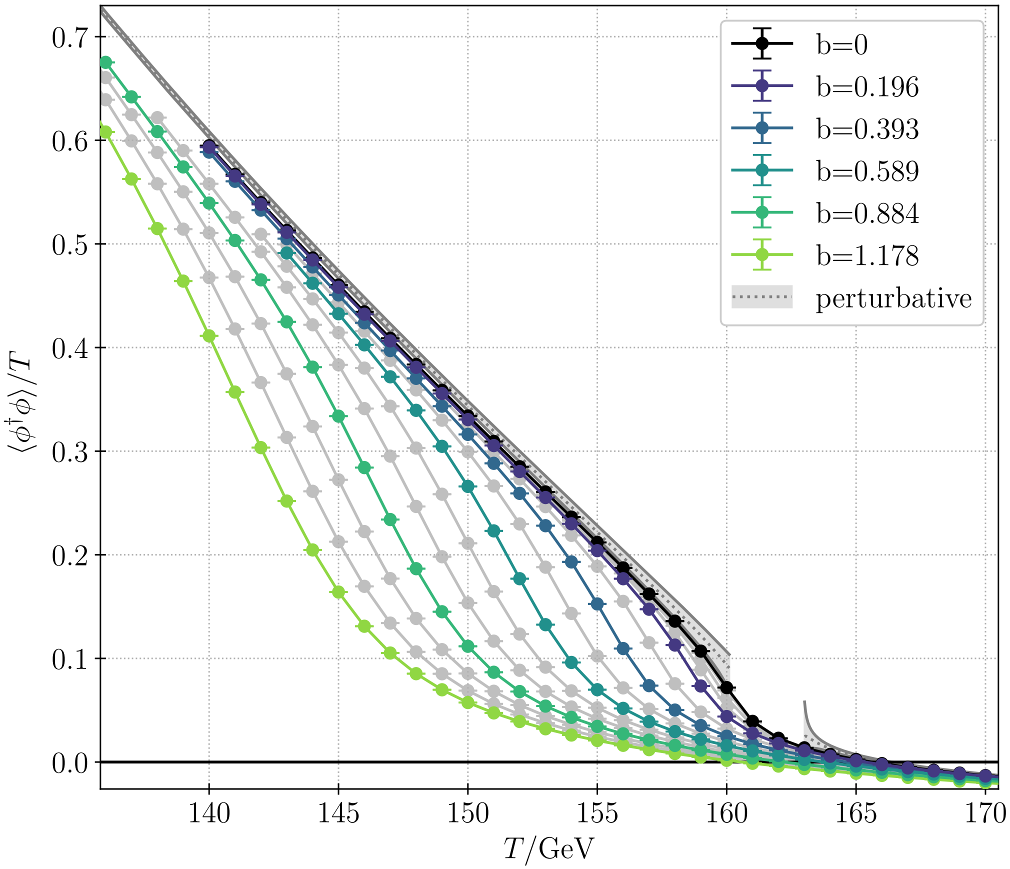
Why study the sphaleron in a (hyper)magnetic field?

1. Electroweak crossover changes

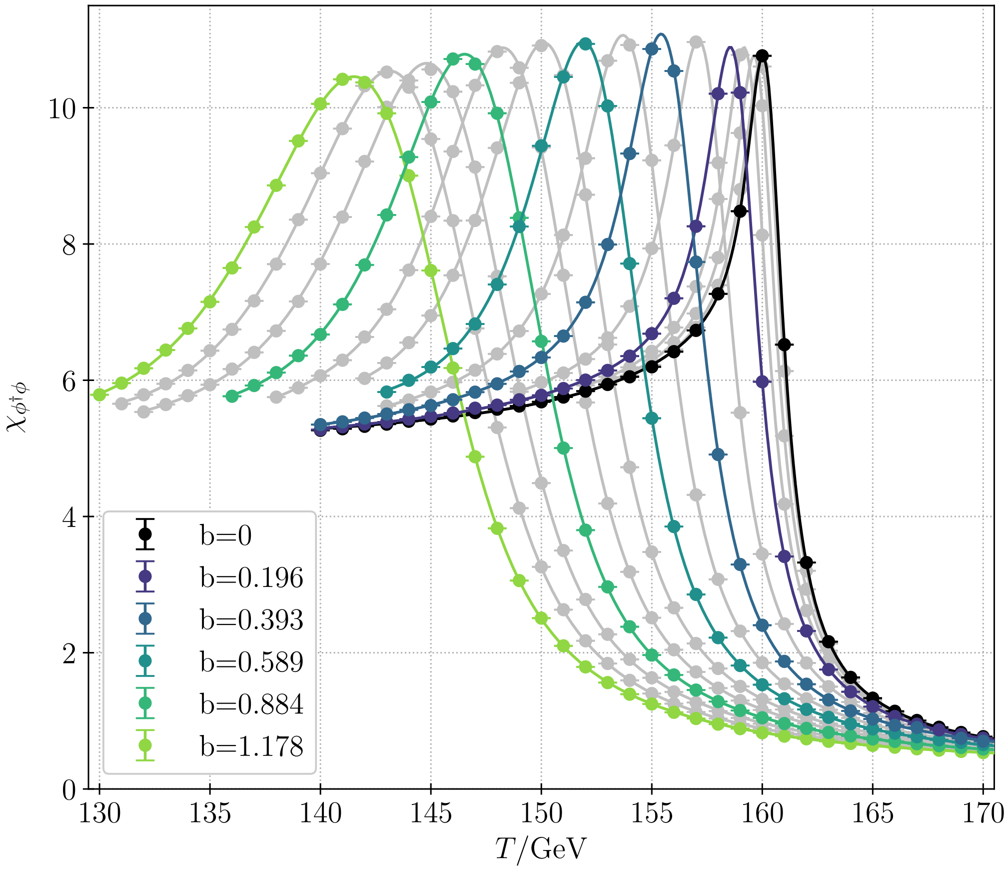
- With increased magnetic field the EW transition shifts to lower temperatures and the transition gets “wider”.

$$B_Y^{4d} \simeq 2bT^2$$

Higgs expectation value



Higgs susceptibility



2. Sphaleron has a magnetic dipole moment

- In a magnetic field the energy of the sphaleron can be lowered.
  - For small magnetic fields a dipole interaction is expected:

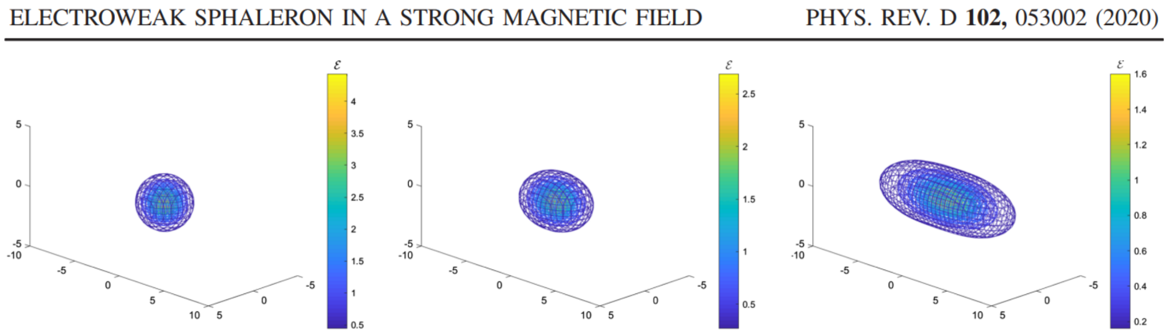
$$\Delta E_{\mathrm{sph}} = -B_{\mathrm{ext}} \cdot \mu_{\mathrm{sph}}$$

[arXiv:hepph/9903227]

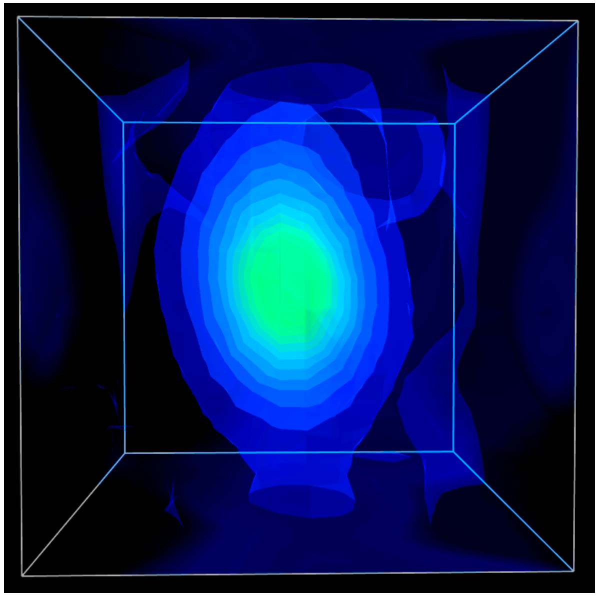
- Sphaleron gets elongated along the magnetic field.

Change to sphaleron energy at large fields, Non dynamical simulations at zero temperature:

[L.-J. Ho & Rajantie, 2005.03125 . . .]



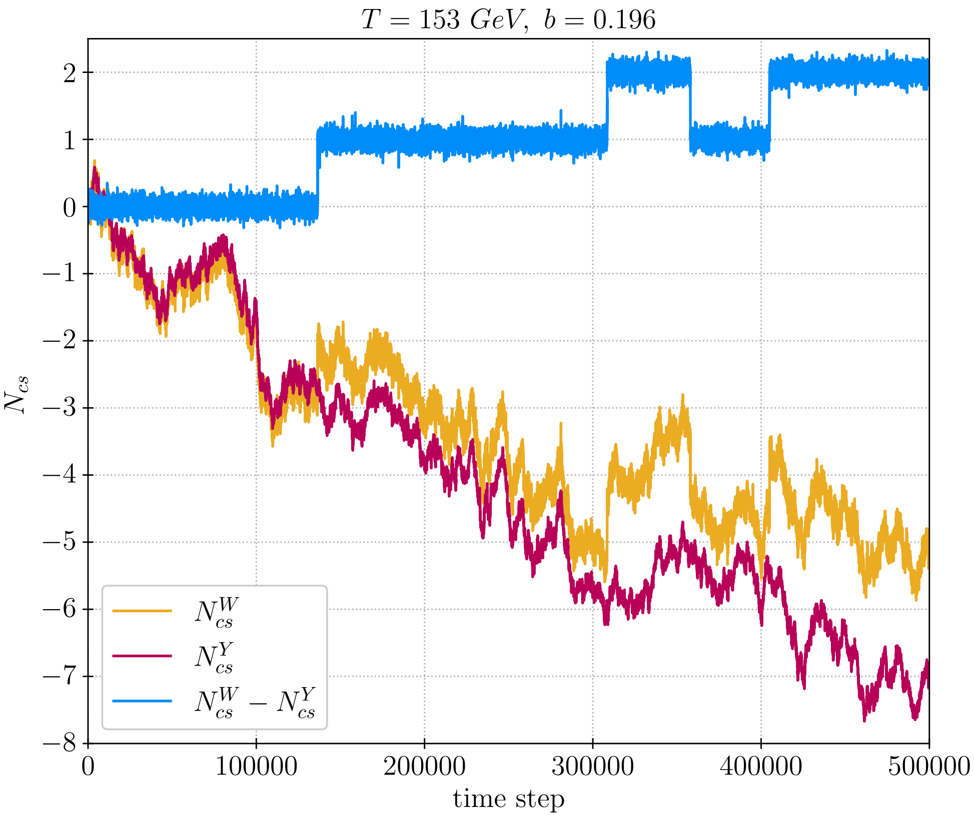
ELECTROWEAK SPHALERON IN A STRONG MAGNETIC FIELD PHYS. REV. D **102,** 053002 (2020)

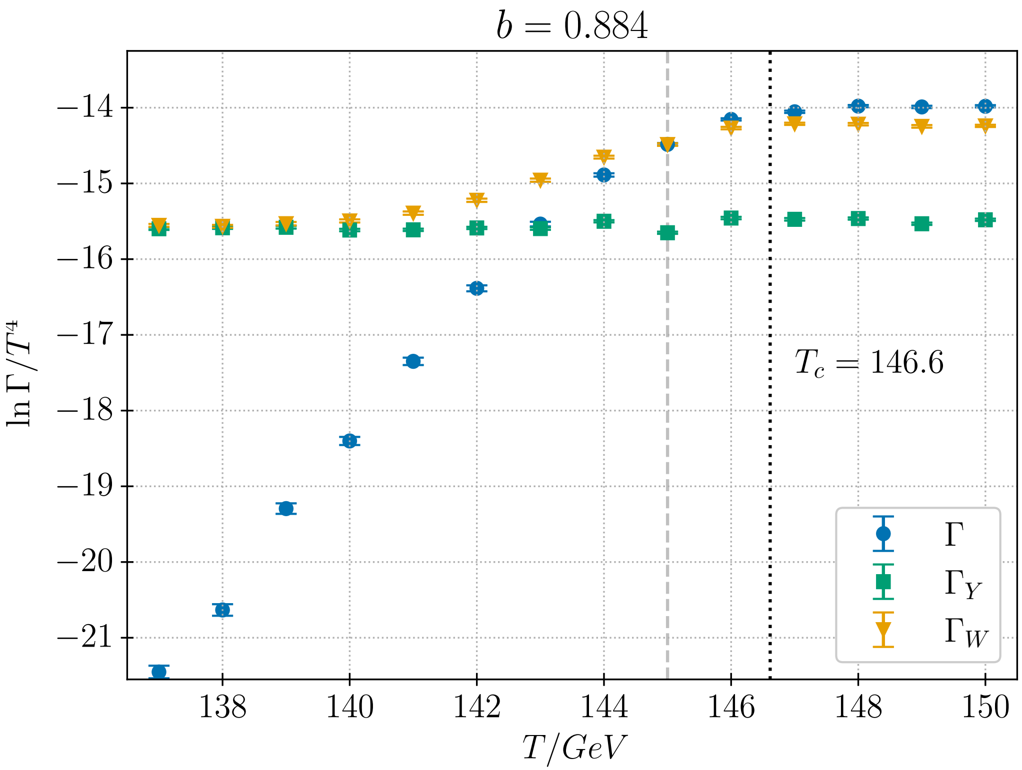


In the broken phase SU(2) and U(1) N_{CS} are not independent

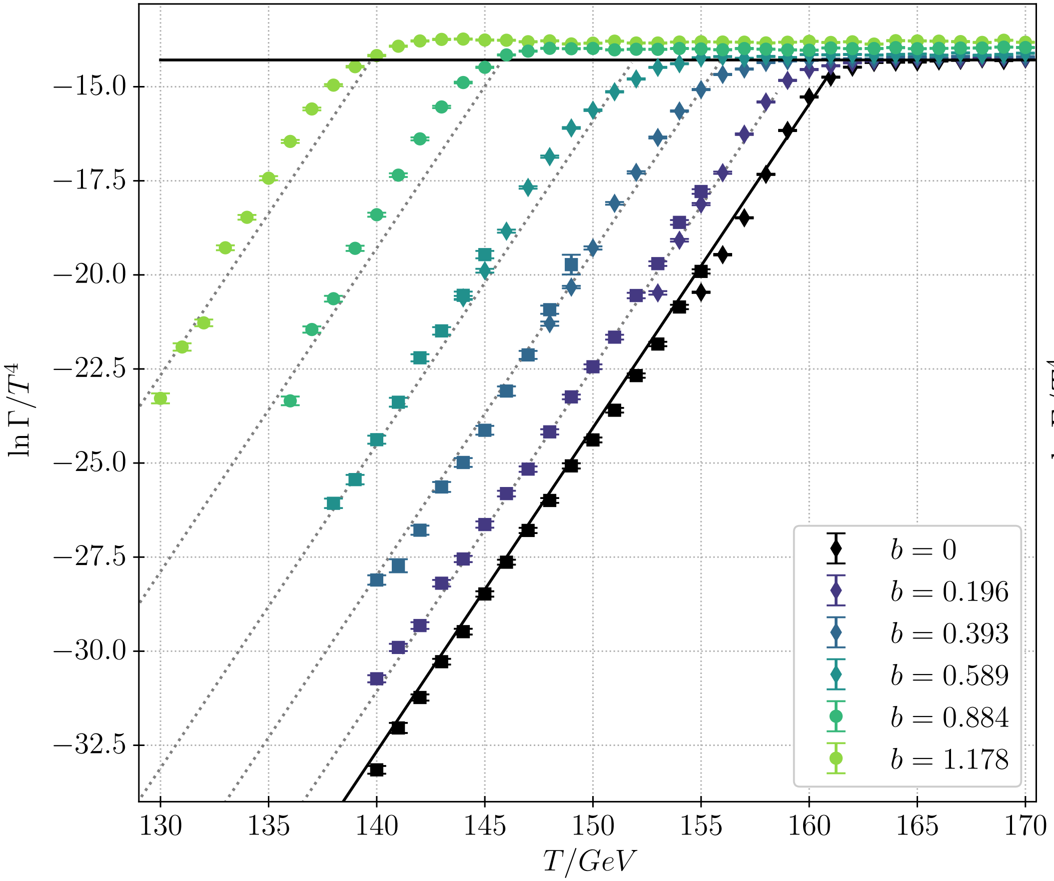
- In the broken SU(2) and U(1) CS numbers are highly correlated, only the physical difference of the two gets suppressed, with non-zero magnetic field present.

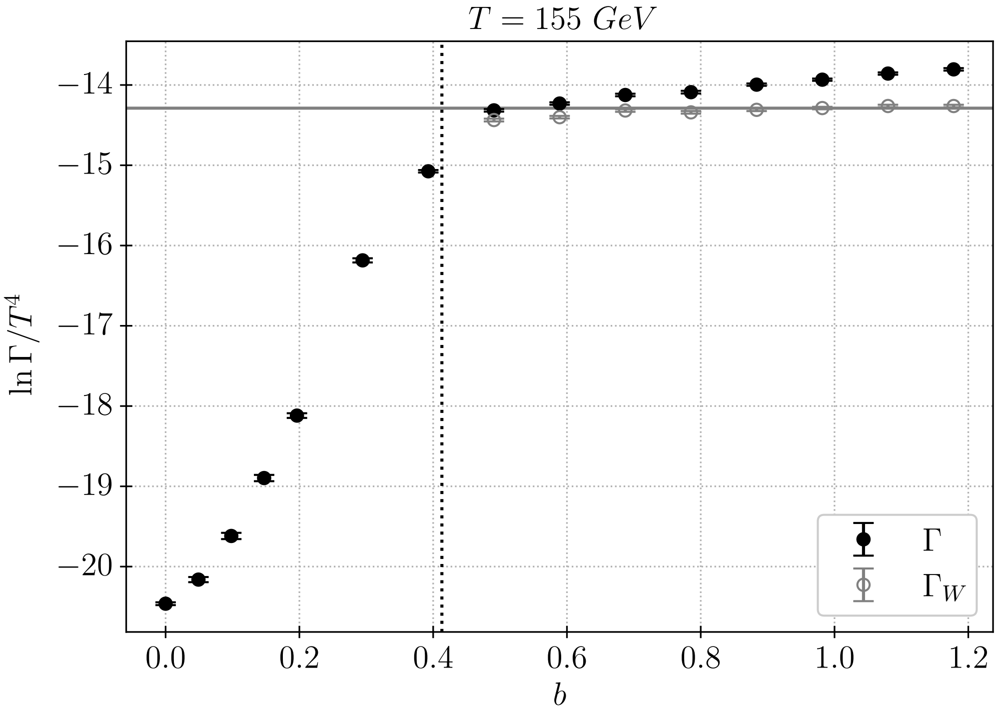
- SU(2) Ncs on its own is not a good ‘order parameter’





Sphaleron rate in a magnetic field





- Black solid lines: b=0 fits
- Grey dotted: b=0 fit shifted according to the shift of the pseudo critical temperature.

- For small external fields the sphaleron dipole moment has bigger effect compared to the changing Higgs expectation value.

Conclusions

- We performed first dynamical lattice simulations investigating effects of external magnetic field on the sphaleron rate.
- Verified that U(1) does not change the result when magnetic field is zero.
- Sphaleron has a magnetic dipole moment and its energy can be lowered in a magnetic field.
  - For small fields the dipole moment gives the biggest effect. Ultimately the shifting of the pseudocritical temperature dominates.
- Electroweak transition shifts to lower temperatures with increased external magnetic field $B_Y = 0 ... 2T^2 : \quad T_c = 160 ... 145 \text{ GeV}$

Thank you!

JA, Kari Rummukainen [arxiv:2301.08626]

EW transition shifts to lower temperature

- With increased magnetic field the EW transition shifts to lower temperatures and the transition gets “wider”.
- $T_c =$ peak of the Higgs susceptibility