BAYESIAN OR FREQUENTIST?
– THE IMPORTANCE OF A NUANCED ANALYSIS

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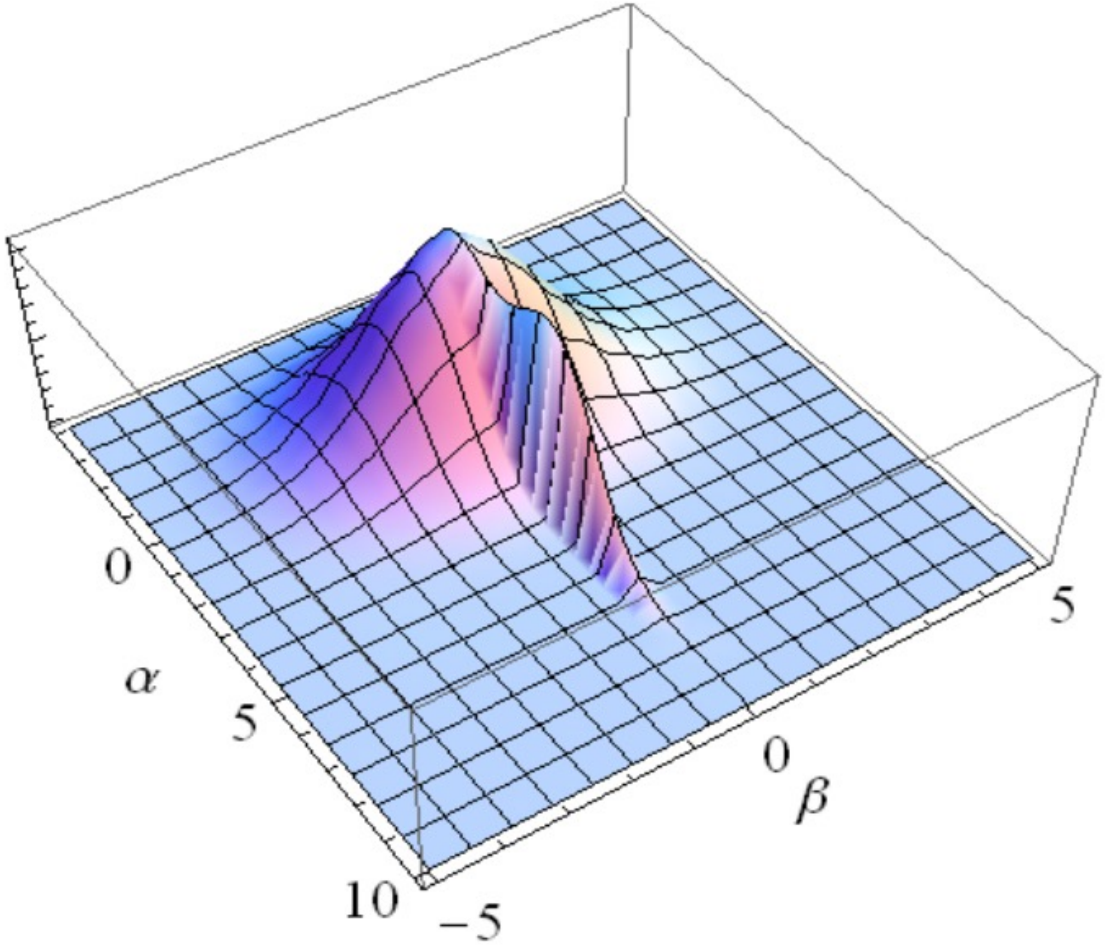
First Nordic Cosmology Meeting, Nordita, Stockholm

with supervisors: Thomas Tram Steen Hannestad

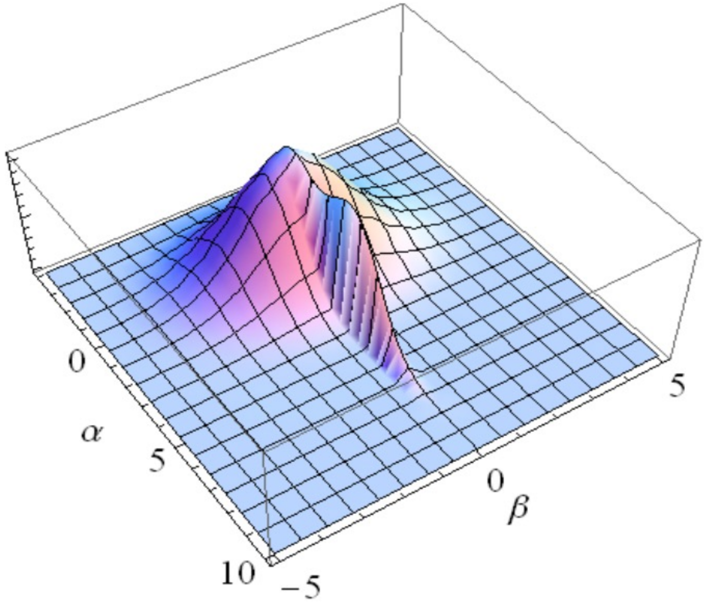
and collaborators: J. Cruz, E. Ferreira, L. Herold, F. Niedermann, A. Nygaard, V. Poulin, T. Simon, M. Sloth

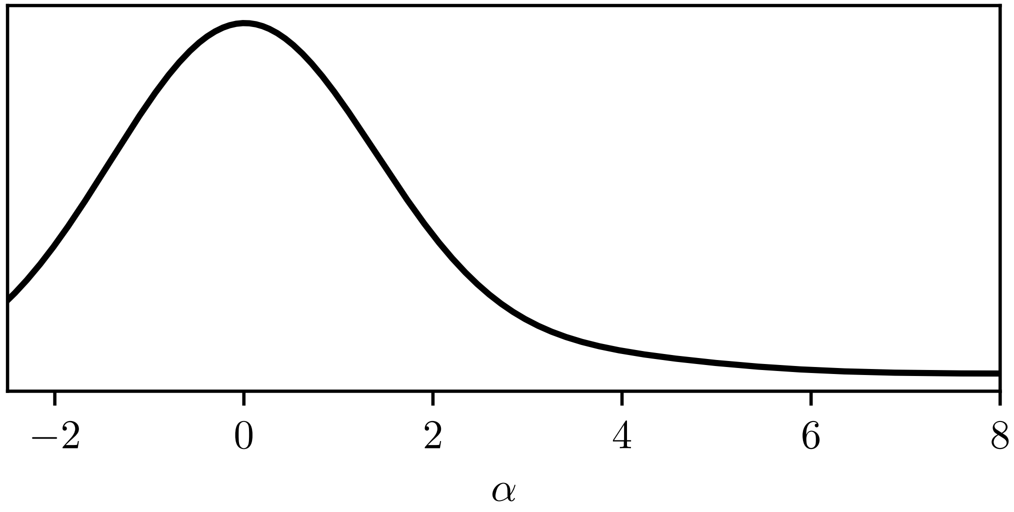
AARHUS UNIVERSITY

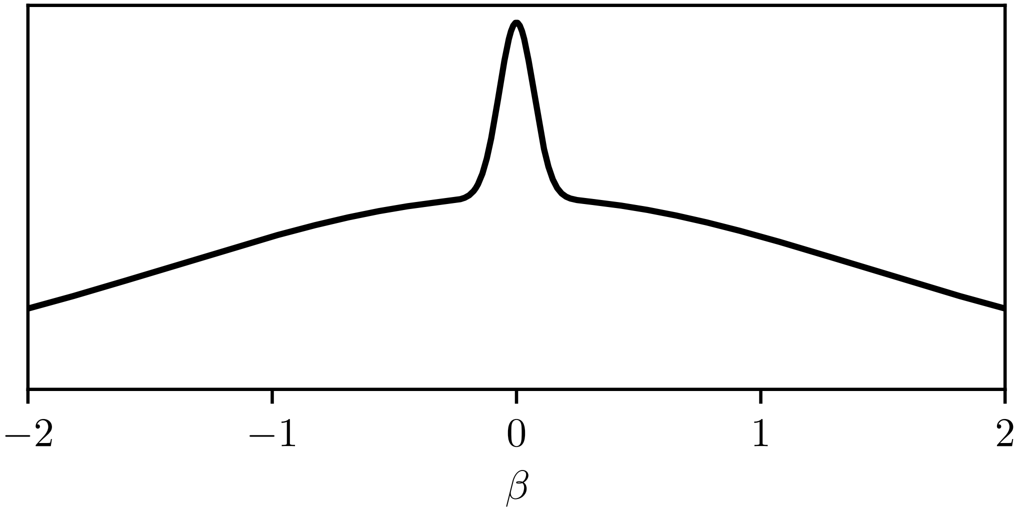


A. Gomez-Valent (arXiv:2203.16285)

$$\int \mathrm{d}\beta \; L(\alpha, \beta)$$







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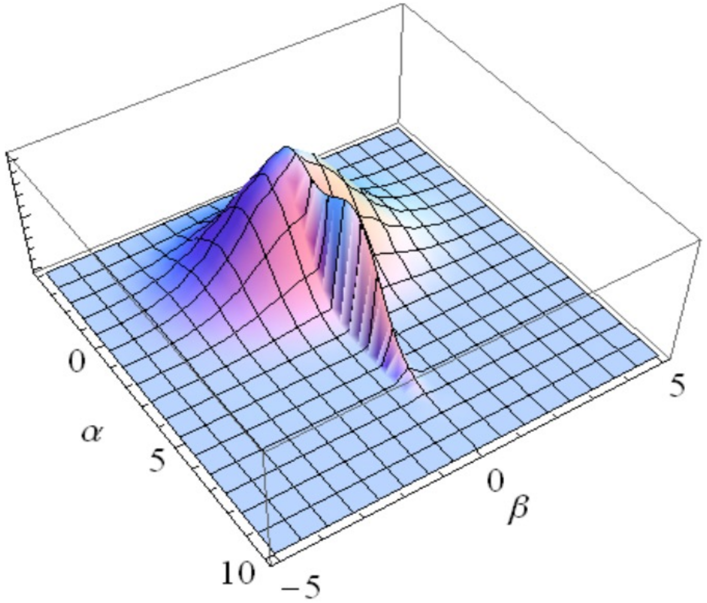

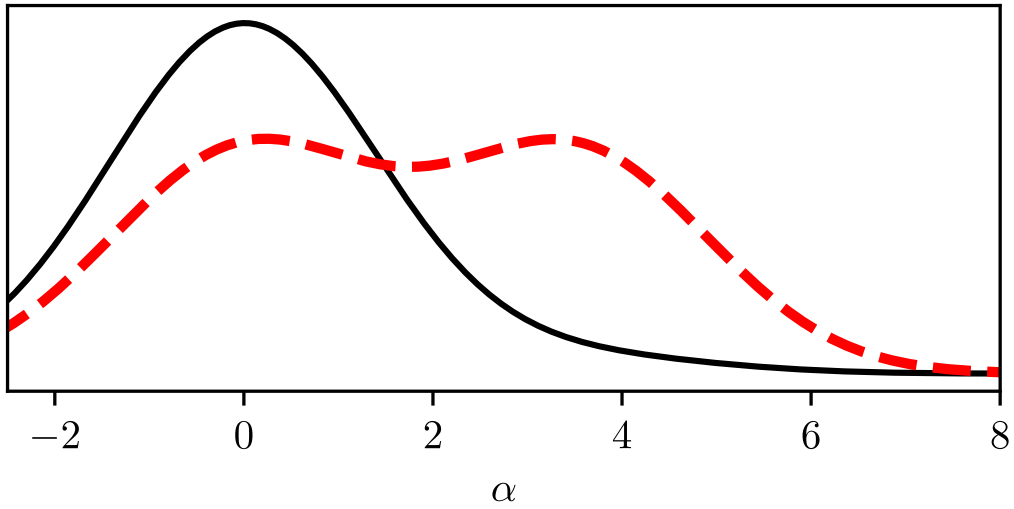

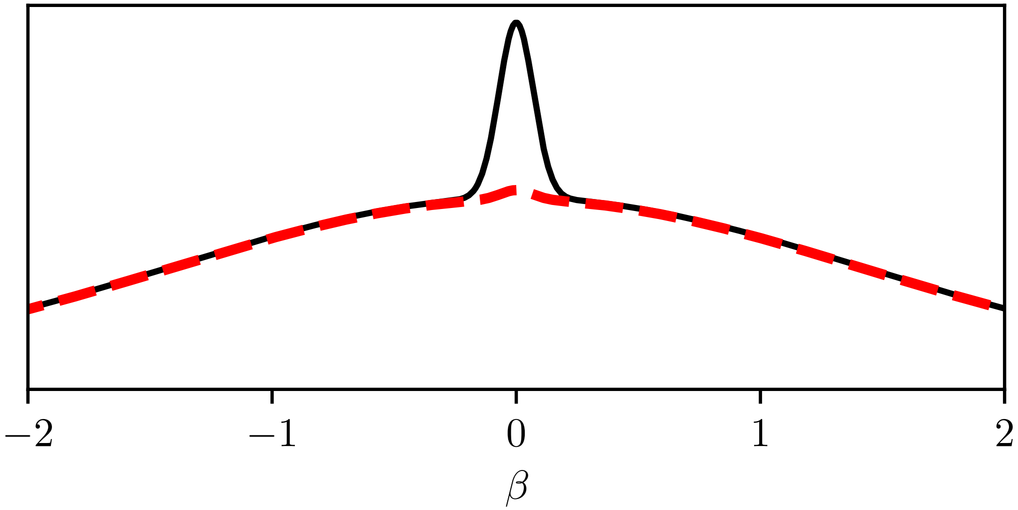

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Profile likelihoods

- Profile likelihood (PL):

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  - Reparametrization invariant
  - Prior independent

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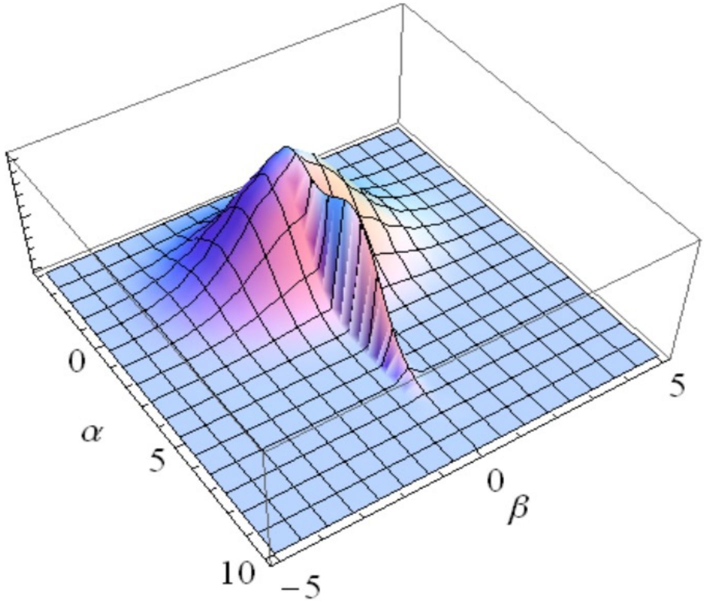

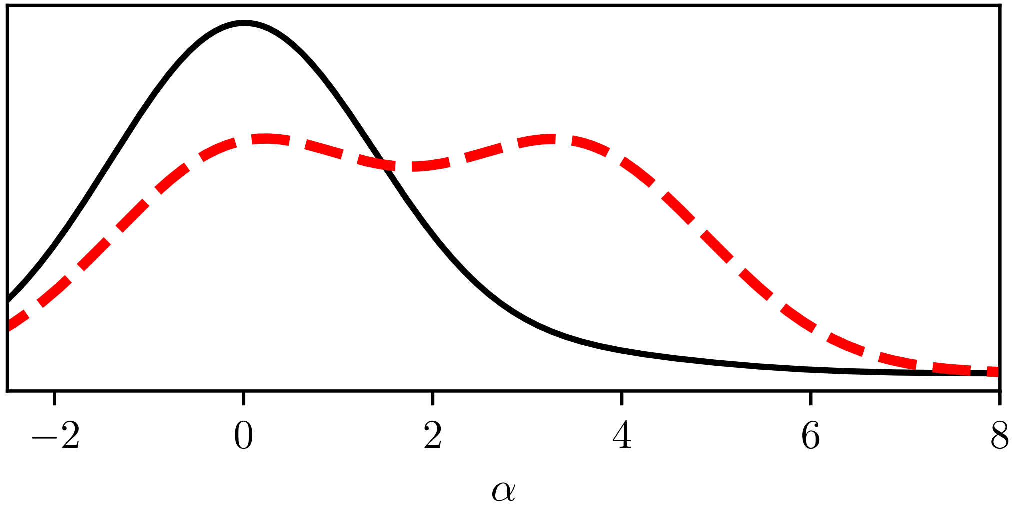

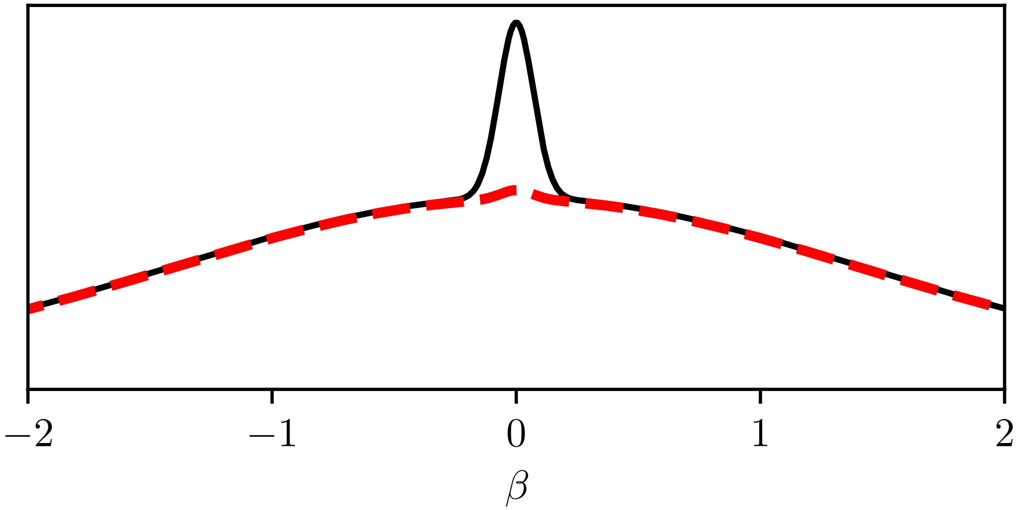

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Volume and prior effects occur due to...

Cosmological parameters

↓

New Early Dark Energy

Decaying dark matter

Interacting dark sectors

...

Nuisance parameters

↓

EFTofLSS nuisance parameters

Correlations btw. parameters

Unconstrained parameters

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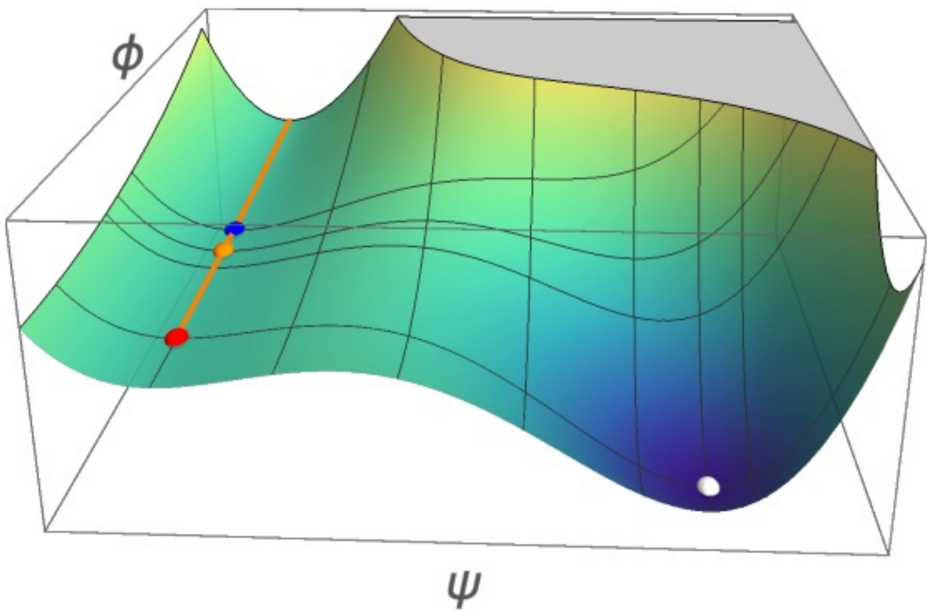
Cold New Early Dark Energy (NEDE)

Triggered phase transition:

$$V(\psi,\phi)=\frac{\lambda}{4}\psi^4+\frac{1}{2}M^2\psi^2-\frac{1}{3}\alpha M\psi^3+\frac{1}{2}m^2\phi^2+\frac{1}{2}\tilde{\lambda}\phi^2\psi^2$$



F. Niedermann, M. Sloth (arXiv:1910.10739, arXiv:2006.06686)

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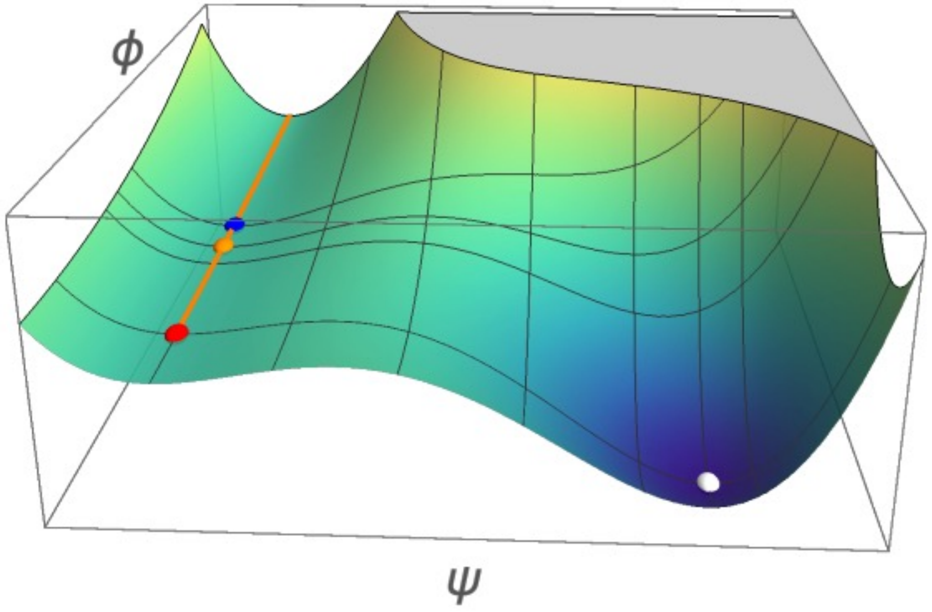

After bubble collisions:

$$w_{\text{NEDE}}(t) = \begin{cases} -1, & z > z_{\text{decay}} \\ w^*_{\text{NEDE}} & \text{else} \end{cases}$$

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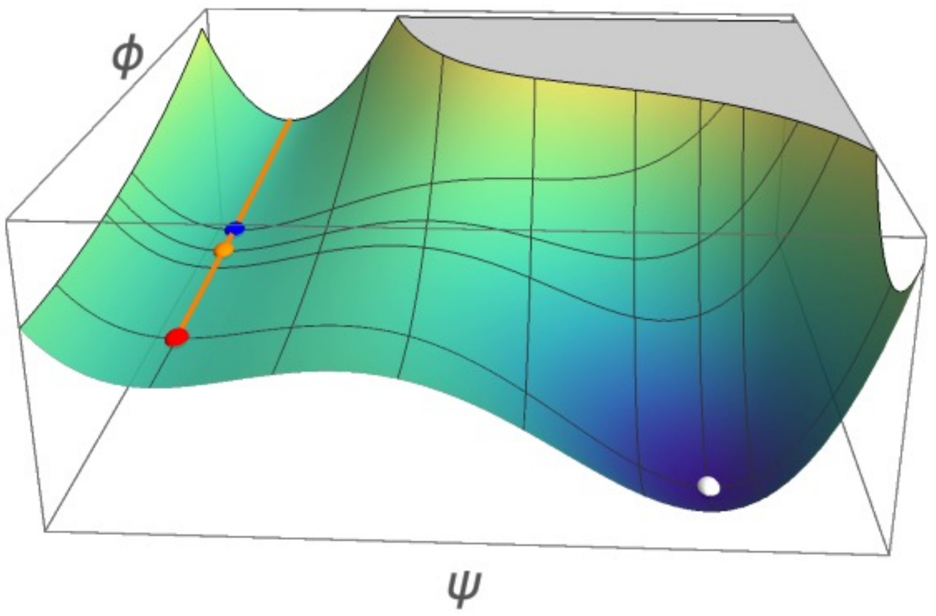

$$\{f_{\mathrm{NEDE}}, z_{\mathrm{decay}}, w^*_{\mathrm{NEDE}}\}$$

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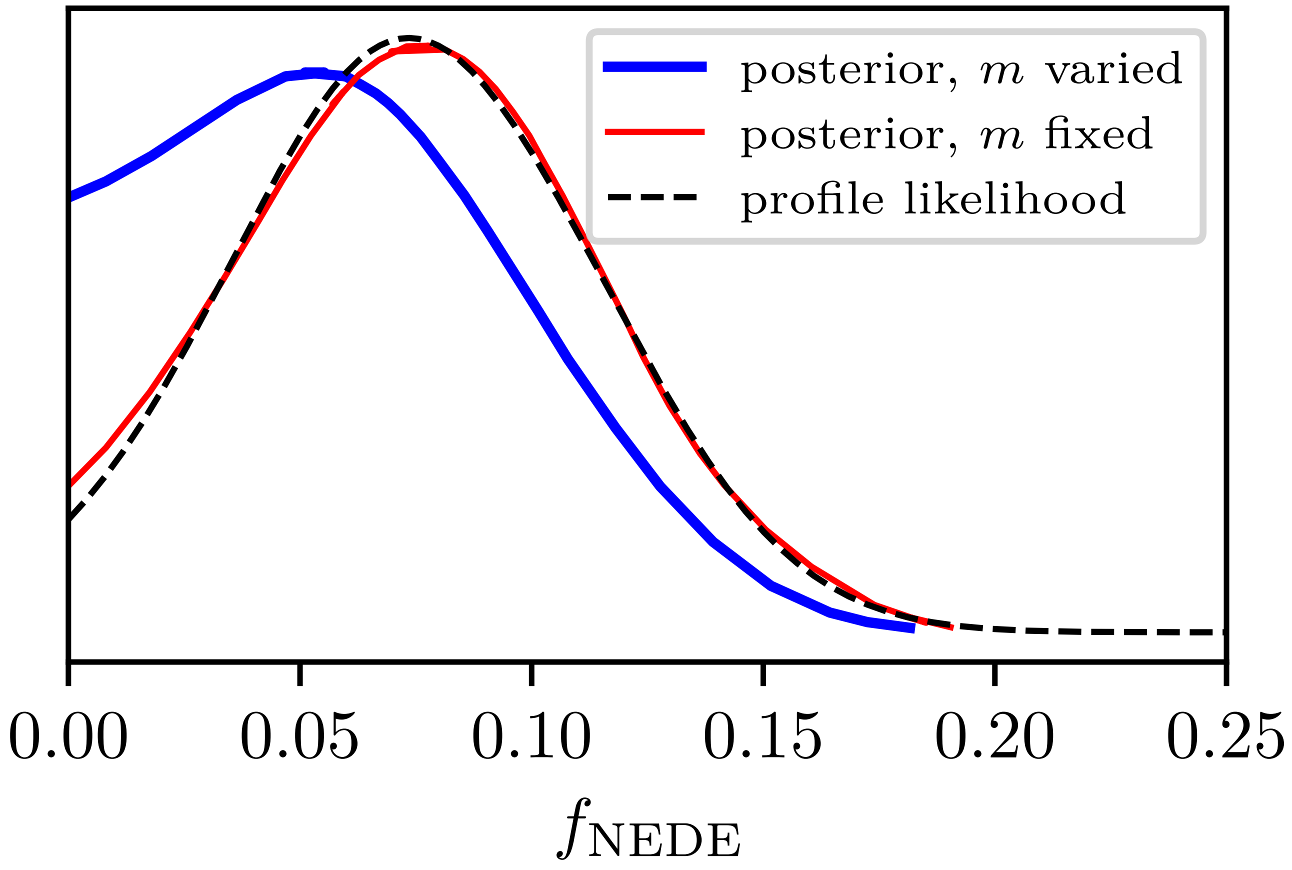
Volume effect in NEDE



J. Cruz, S. Hannestad, **EBH**, F. Niedermann, M. Sloth, T. Tram (arXiv:2302.07934)

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MCMC:

$f_{\mathrm{NEDE}} < 0.076$

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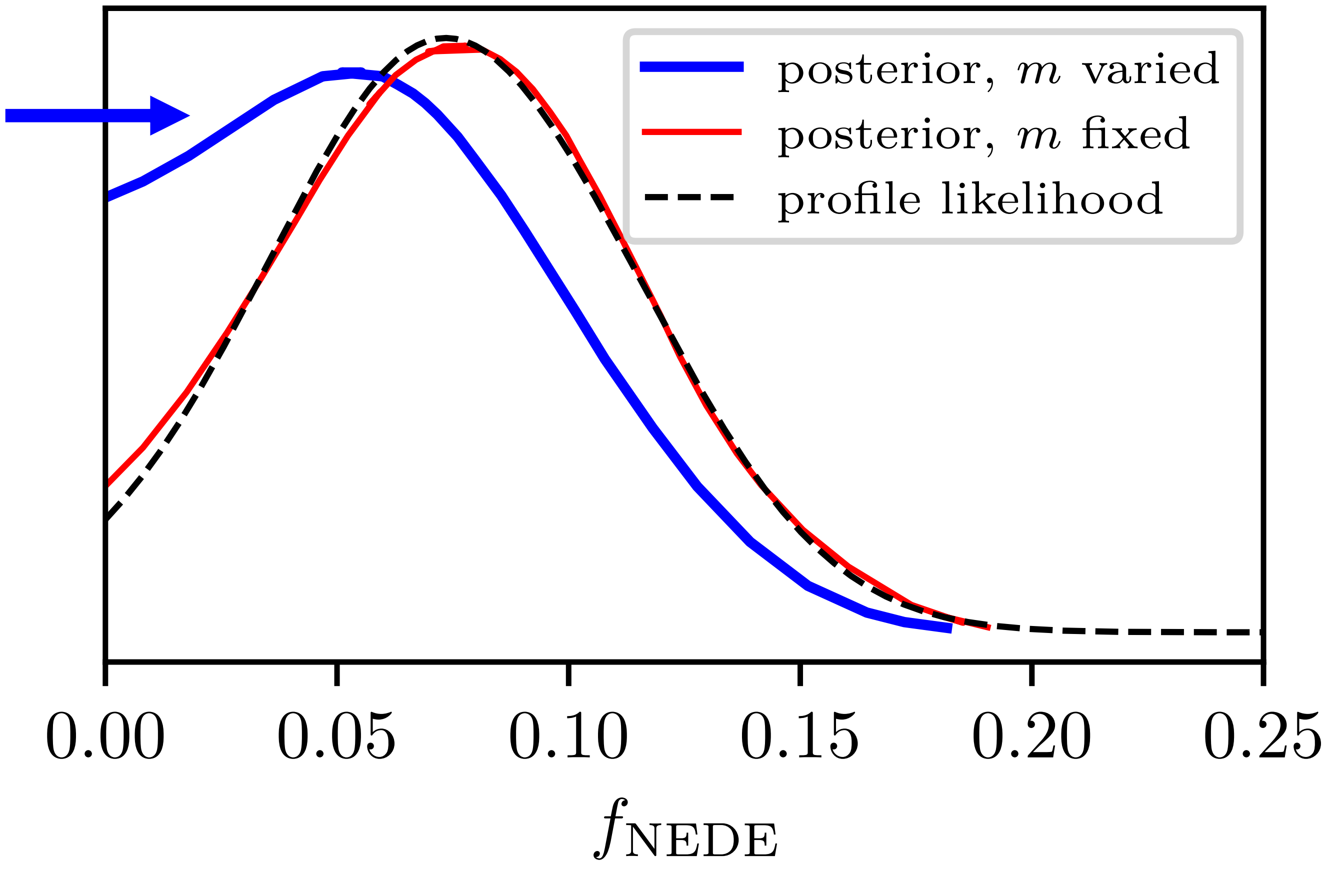

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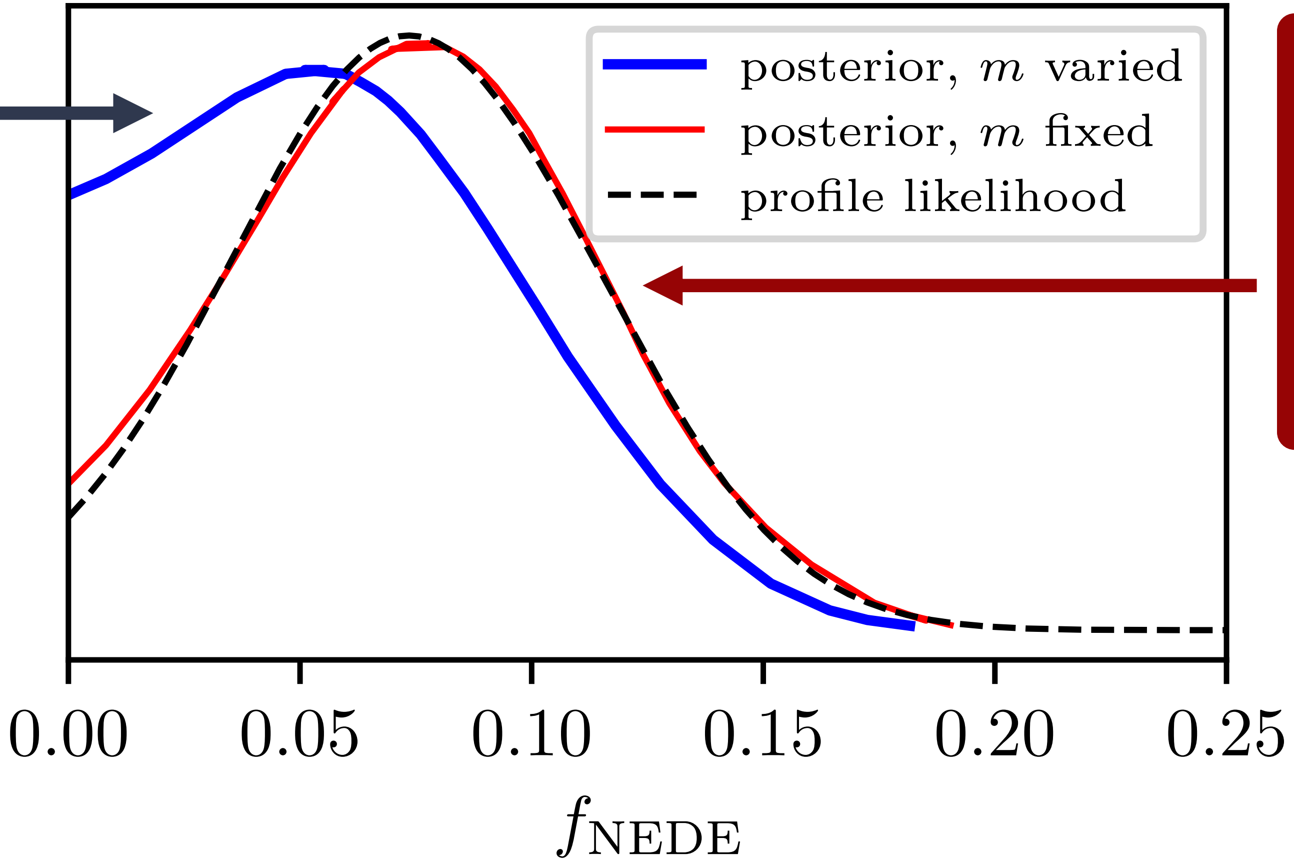

Profile likelihood:

f_{NEDE}

$= 0.076^{+0.040}_{-0.035}$

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Volume effects in ΛCDM extensions

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- Expect volume effects in ΛCDM extensions with **abundances** or **coupling constants**

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Effective field theory of large scale structure **(EFTofLSS)**

Idea:

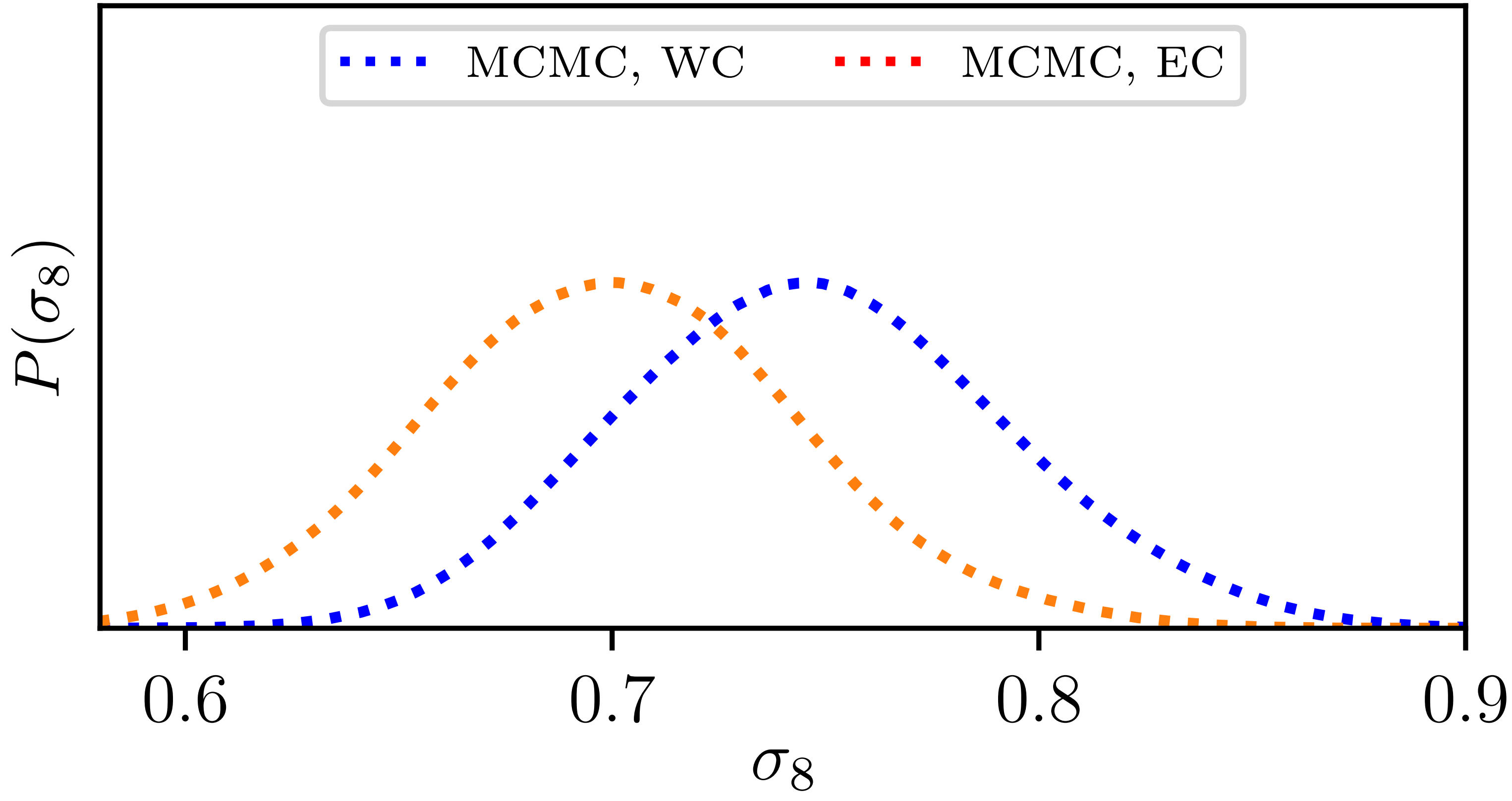
- Parametrization of nonlinear matter power spectrum
- Unknown coefficients → EFT nuisance parameters

Issues:

- Different parametrizations
- Priors on EFT parameters

e.g. M. Ivanov (arXiv:2212.08488v1)

EFTofLSS nuisance parameters



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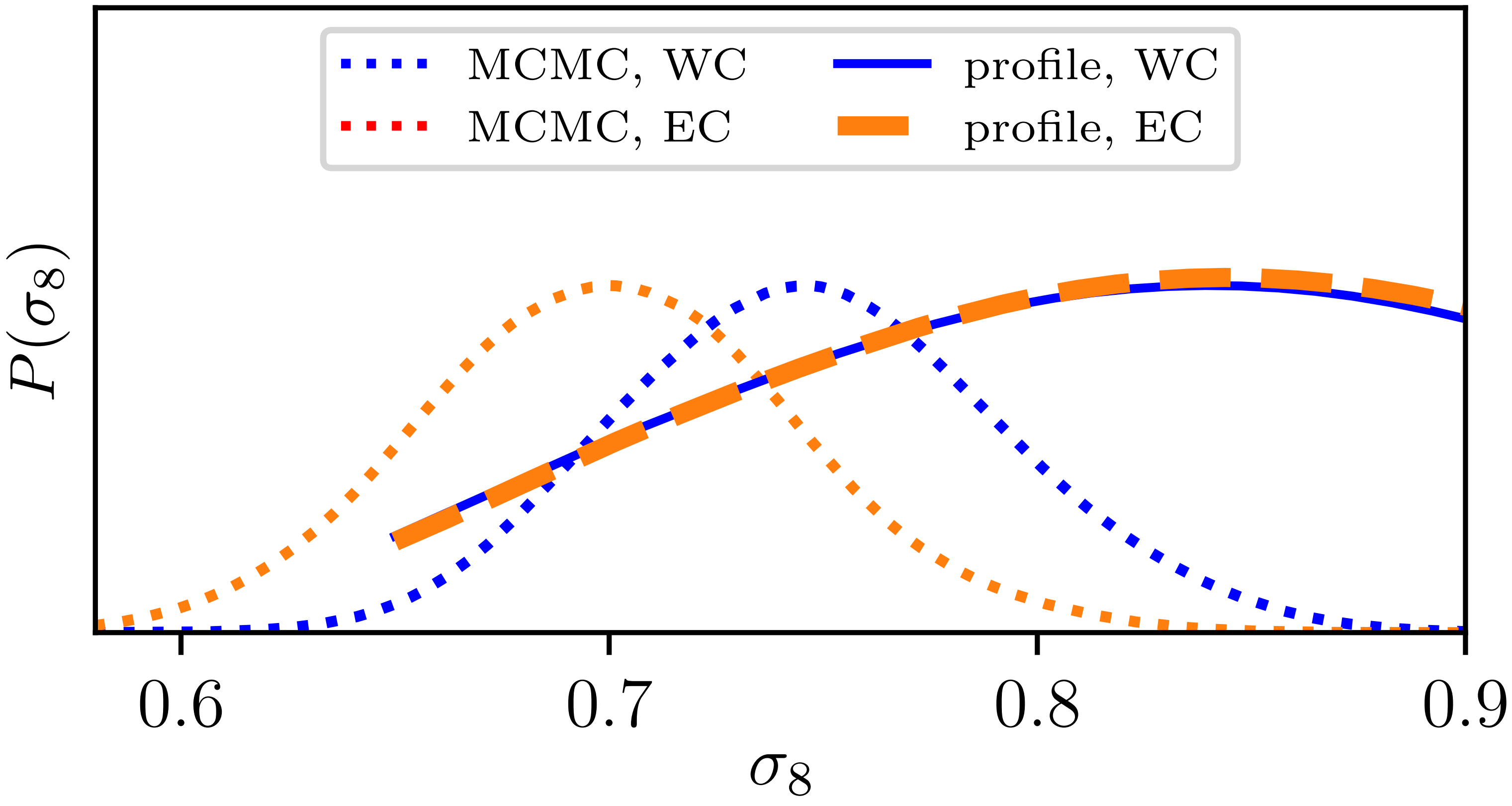

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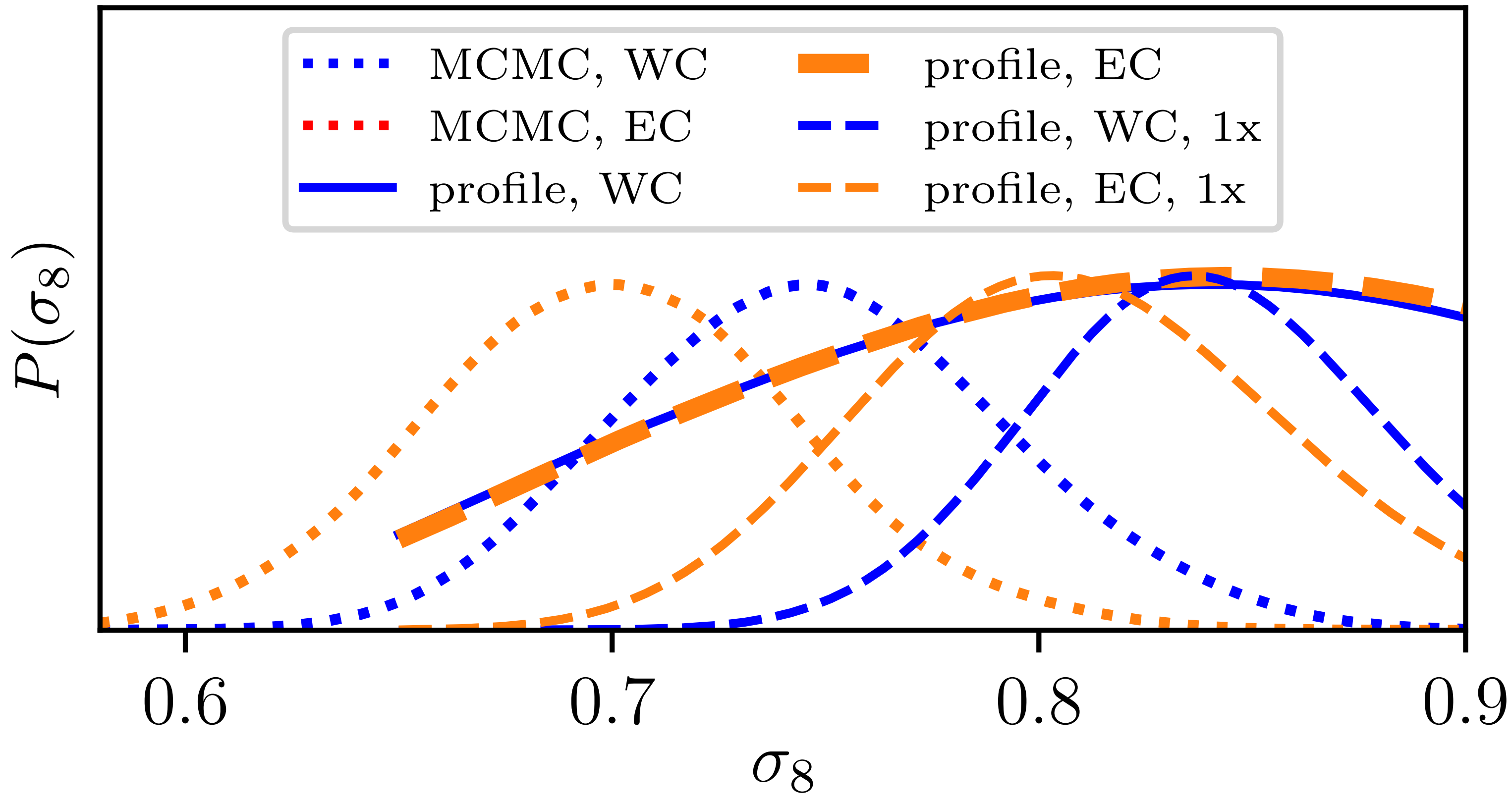

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Conclusion: Two ways to do EFTofLSS analysis:

- With priors → but then priors are informative
- Without priors → but then model is unphysical

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Look to future data to solve these issues!

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Takeaway :

- Profile likelihoods independent of
  1. model parametrization
  2. priors

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PROSPECT: An easy-to-use profile likelihood code

PROSPECT: A profile likelihood code for efficient frequentist cosmological parameter inference

Emil Brinch Holm, Andreas Nygaard, Jeppe Dakin, Steen Hannestad, Thomas Tram

Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark

EBH, A. Nygaard, J. Dakin, S. Hannestad, T. Tram (in prep.)

PROSPECT: An easy-to-use profile likelihood code

PROSPECT

- Efficient optimization algorithm
- Plug-and-play interfacing with MontePython and cobaya
- Dynamic and modular to work with

EBH, A. Nygaard, J. Dakin, S. Hannestad, T. Tram (in prep.)

Outlook

- Expect volume effects in:

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  - ΛCDM extensions with **abundances** or **coupling constants**

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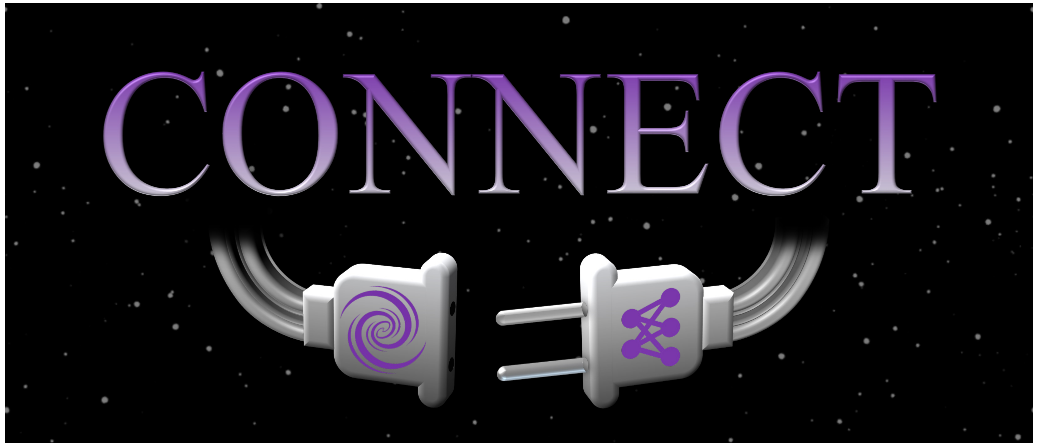
  - Future: Full triangle plots w. emulators and gradient-based opt.

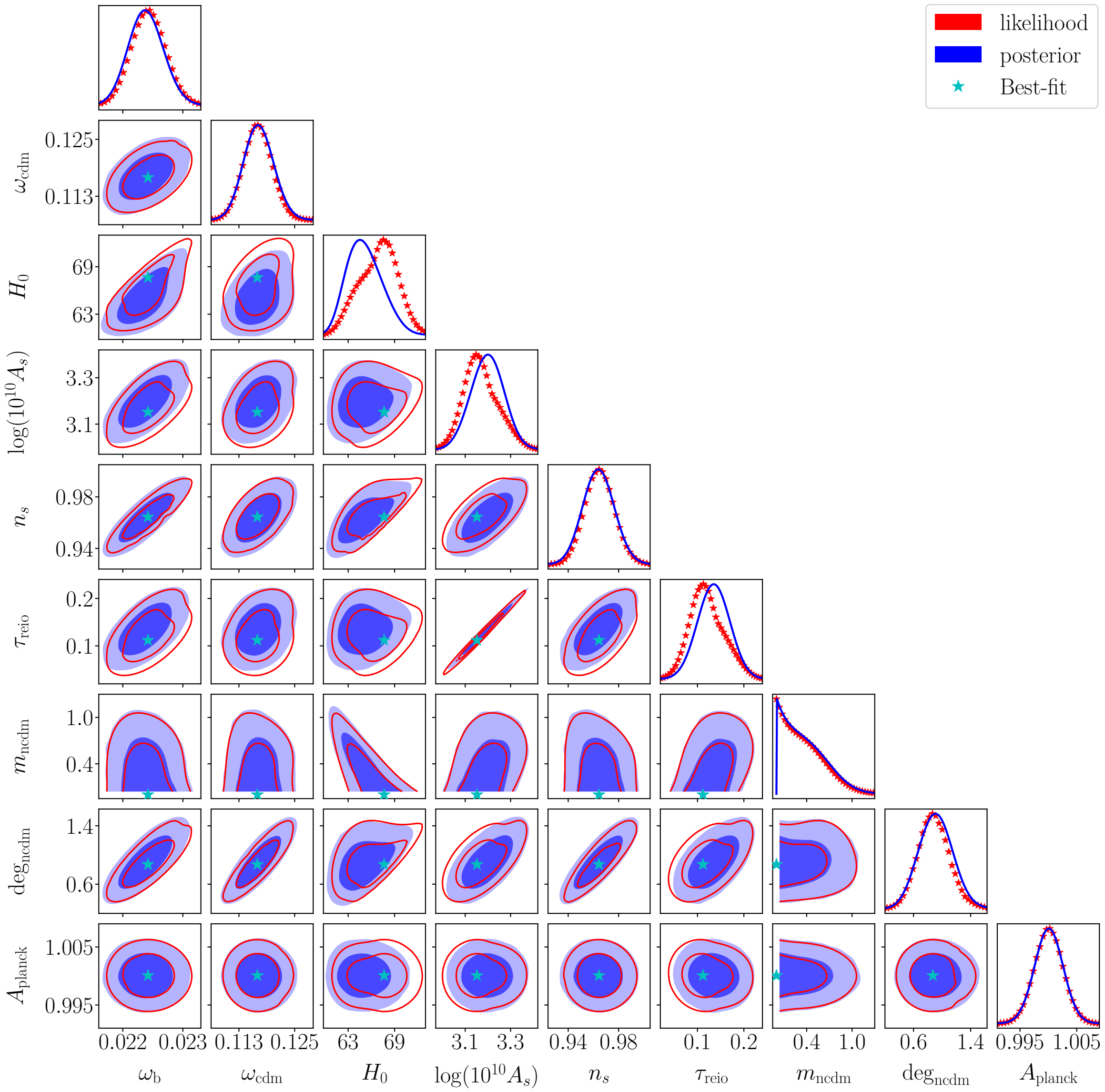
The future is now!



A. Nygaard, **EBH**, S. Hannestad, T. Tram
(arXiv:2308.06379, arXiv:2205.15726)

Emulate your favourite CLASS model:

https://github.com/AarhusCosmology/connect_public



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- Both MCMC and PL are “correct”

  → Use together!

PROSPECT: A profile likelihood code for efficient frequentist cosmological parameter inference

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Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark

PROSPECT now available at:

https://github.com/AarhusCosmology/prospect_public

(paper coming soon™!)

EBH, A. Nygaard, J. Dakin, S. Hannestad, T. Tram (in prep.)

ADDITIONAL SLIDES

Profile likelihoods

- Profile likelihood (PL):

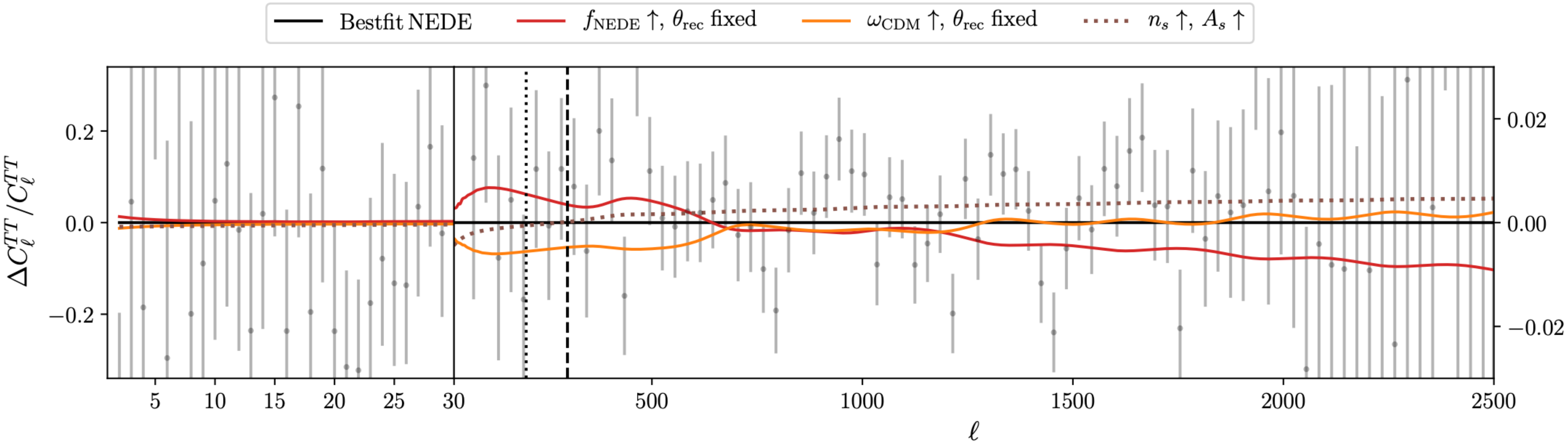
$$L(\theta_j) = \max_{\theta_i, j \neq i} L(\theta_1, \dots, \theta_N)$$

$$\Delta\chi^2(\theta_j) = -2 \ln\left(\frac{L(\theta_j)}{L_{\max}}\right) \qquad \sim \chi^2(1 \text{ DoF})$$

- Gaussian PL:
  - 68 % CI: $\Delta\chi^2(\theta_j) < 1$
  - 95 % CI: $\Delta\chi^2(\theta_j) < 3.84$

(but generally, the $\Delta\chi^2(\theta_j)$ distribution requires data simulation…)

Cold New Early Dark Energy (NEDE)



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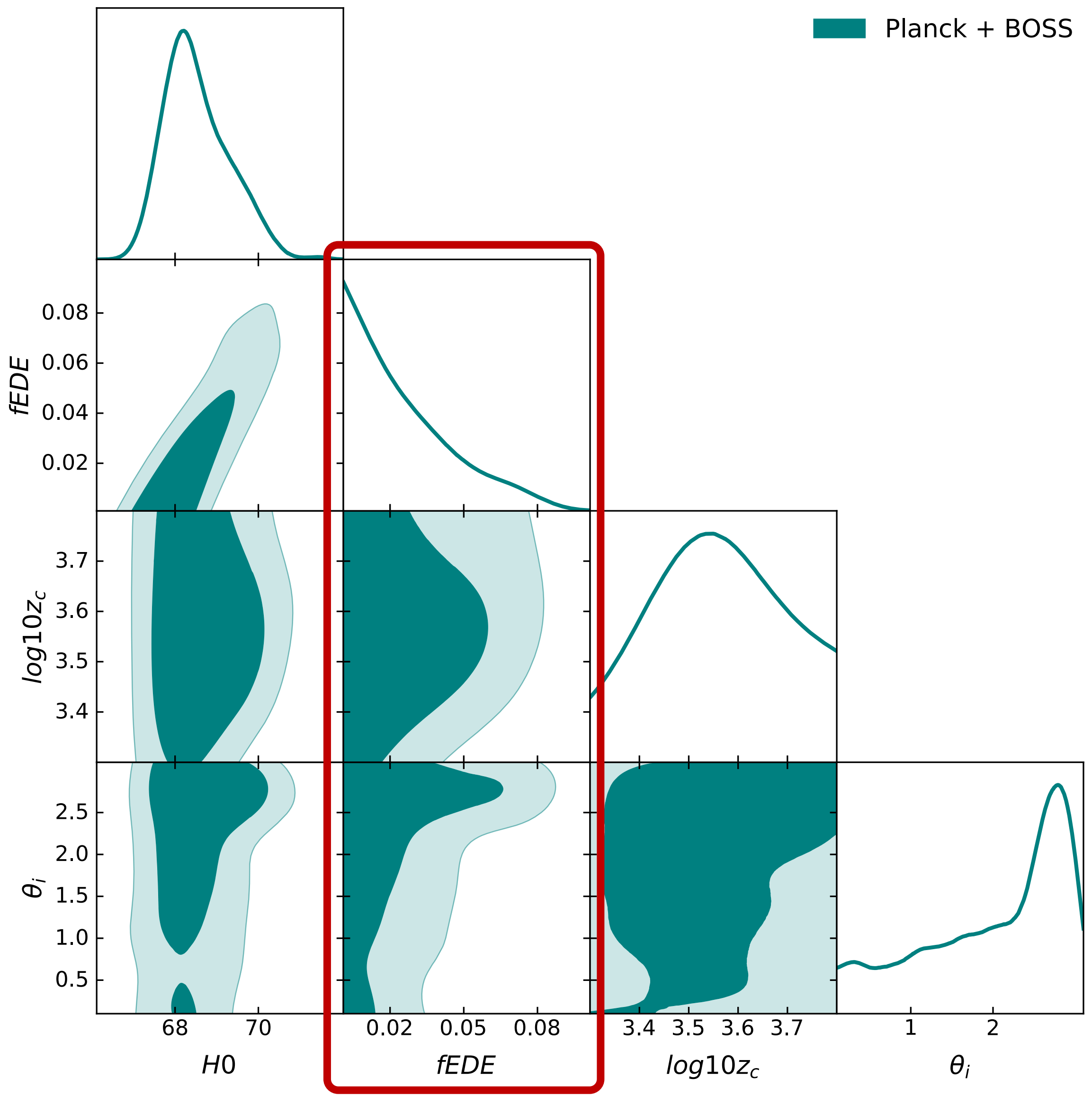
Volume effects in (N)EDE



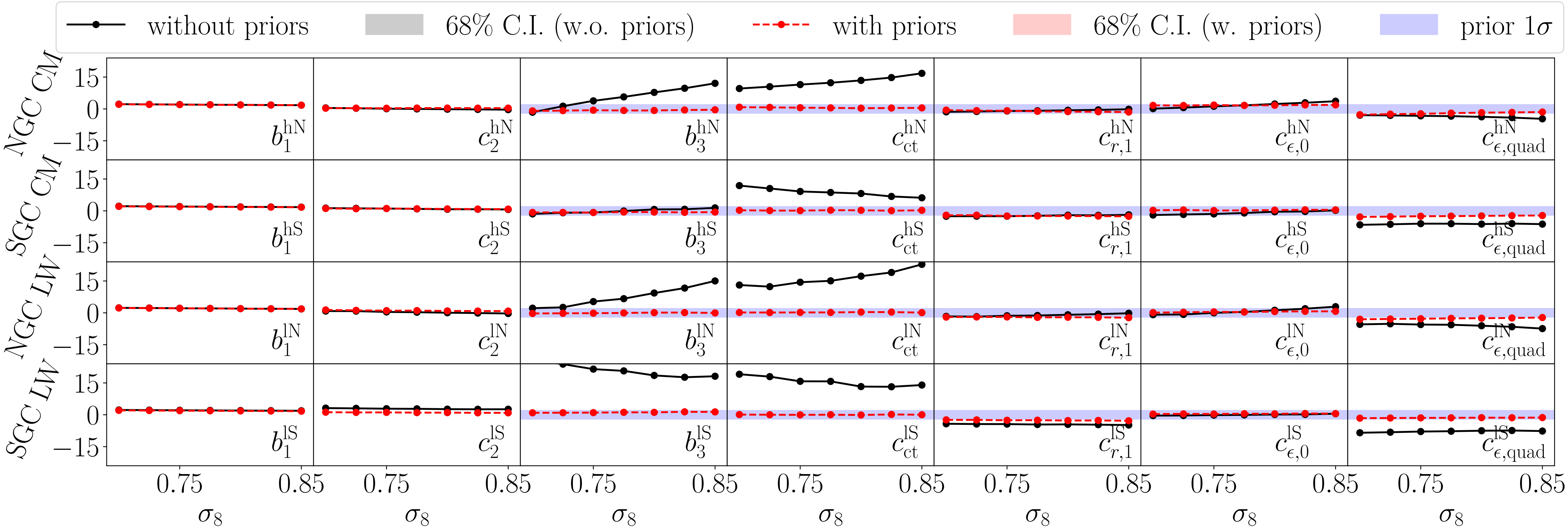
Laura Herold 2022

Volume effects in ΛCDM extensions

Model	$\Delta N_{\rm param}$	M_B	Gaussian Tension	$Q_{\rm DMAP}$ Tension		$\Delta\chi^2$	ΔAIC		Finalist
ΛCDM	0	-19.416 ± 0.012	4.4σ	4.5σ	✗	0.00	0.00	✗	✗
$\Delta N_{\rm ur}$	1	-19.395 ± 0.019	3.6σ	3.8σ	✗	−6.10	−4.10	✗	✗
SIDR	1	-19.385 ± 0.024	3.2σ	3.3σ	✗	−9.57	−7.57	✓	✓ (3)
mixed DR	2	-19.413 ± 0.036	3.3σ	3.4σ	✗	−8.83	−4.83	✗	✗
DR-DM	2	-19.388 ± 0.026	3.2σ	3.1σ	✗	−8.92	−4.92	✗	✗
SIν+DR	3	$-19.440^{+0.037}_{-0.039}$	3.8σ	3.9σ	✗	−4.98	1.02	✗	✗
Majoron	3	$-19.380^{+0.027}_{-0.021}$	3.0σ	2.9σ	✓	−15.49	−9.49	✓	✓ (2)
primordial B	1	$-19.390^{+0.018}_{-0.024}$	3.5σ	3.5σ	✗	−11.42	−9.42	✓	✓ (3)
varying m_e	1	-19.391 ± 0.034	2.9σ	2.9σ	✓	−12.27	−10.27	✓	✓ (1)
varying m_e+Ω_k	2	-19.368 ± 0.048	2.0σ	1.9σ	✓	−17.26	−13.26	✓	✓ (1)
EDE	3	$-19.390^{+0.016}_{-0.035}$	3.6σ	1.6σ	✓	−21.98	−15.98	✓	✓ (2)
NEDE	3	$-19.380^{+0.023}_{-0.040}$	3.1σ	1.9σ	✓	−18.93	−12.93	✓	✓ (2)
EMG	3	$-19.397^{+0.017}_{-0.023}$	3.7σ	2.3σ	✓	−18.56	−12.56	✓	✓ (2)
CPL	2	-19.400 ± 0.020	3.7σ	4.1σ	✗	−4.94	−0.94	✗	✗
PEDE	0	-19.349 ± 0.013	2.7σ	2.8σ	✓	2.24	2.24	✗	✗
GPEDE	1	-19.400 ± 0.022	3.6σ	4.6σ	✗	−0.45	1.55	✗	✗
DM → DR+WDM	2	-19.420 ± 0.012	4.5σ	4.5σ	✗	−0.19	3.81	✗	✗
DM → DR	2	-19.410 ± 0.011	4.3σ	4.5σ	✗	−0.53	3.47	✗	✗

CITATION!

EFTofLSS nuisance parameters



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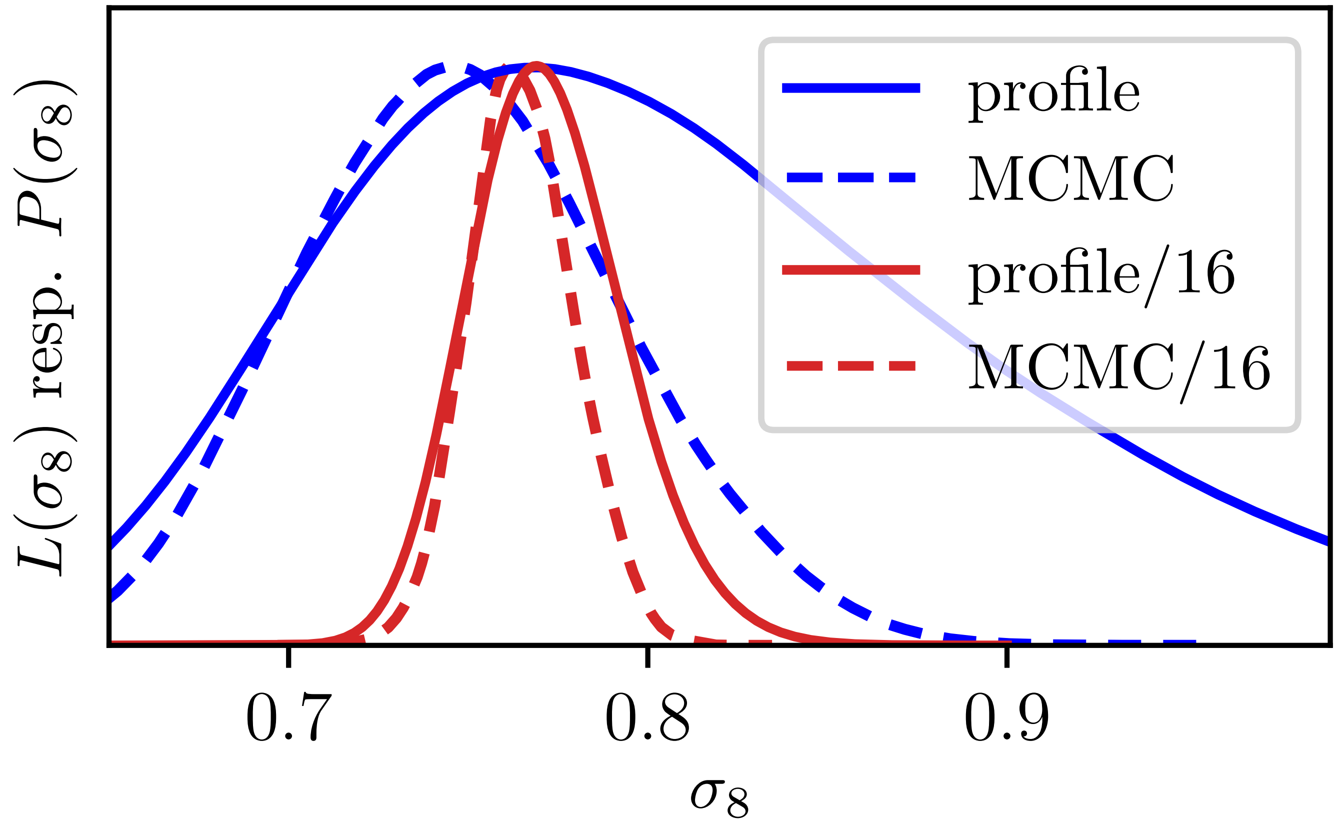

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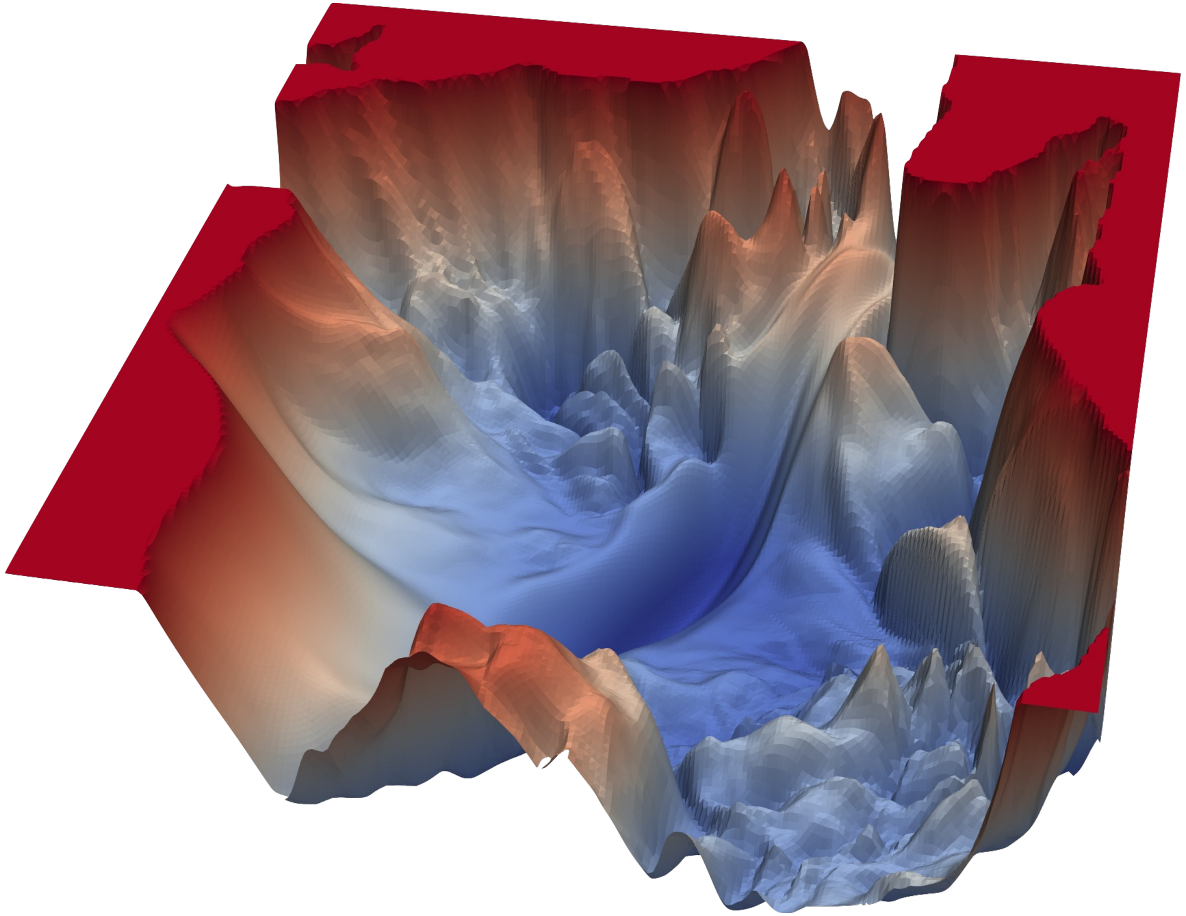

https://www.cs.umd.edu/~tomg/projects/landscapes/

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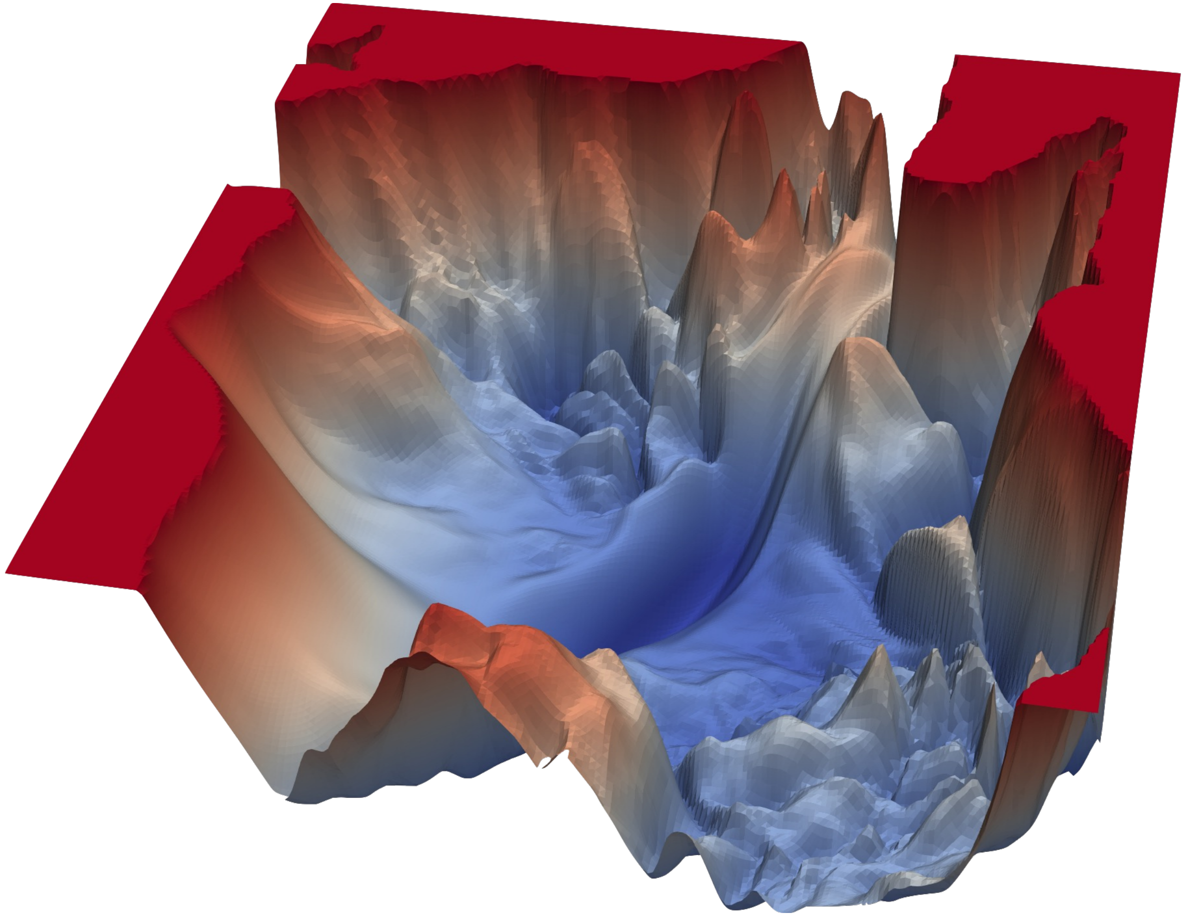
→ Simulated annealing



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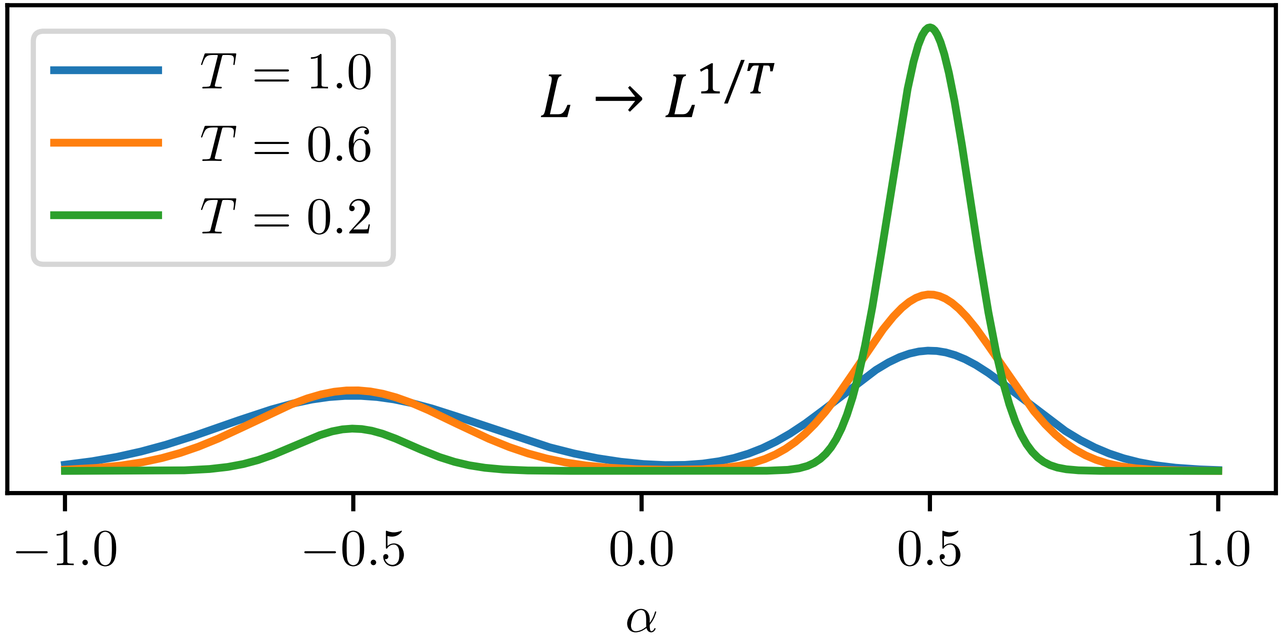

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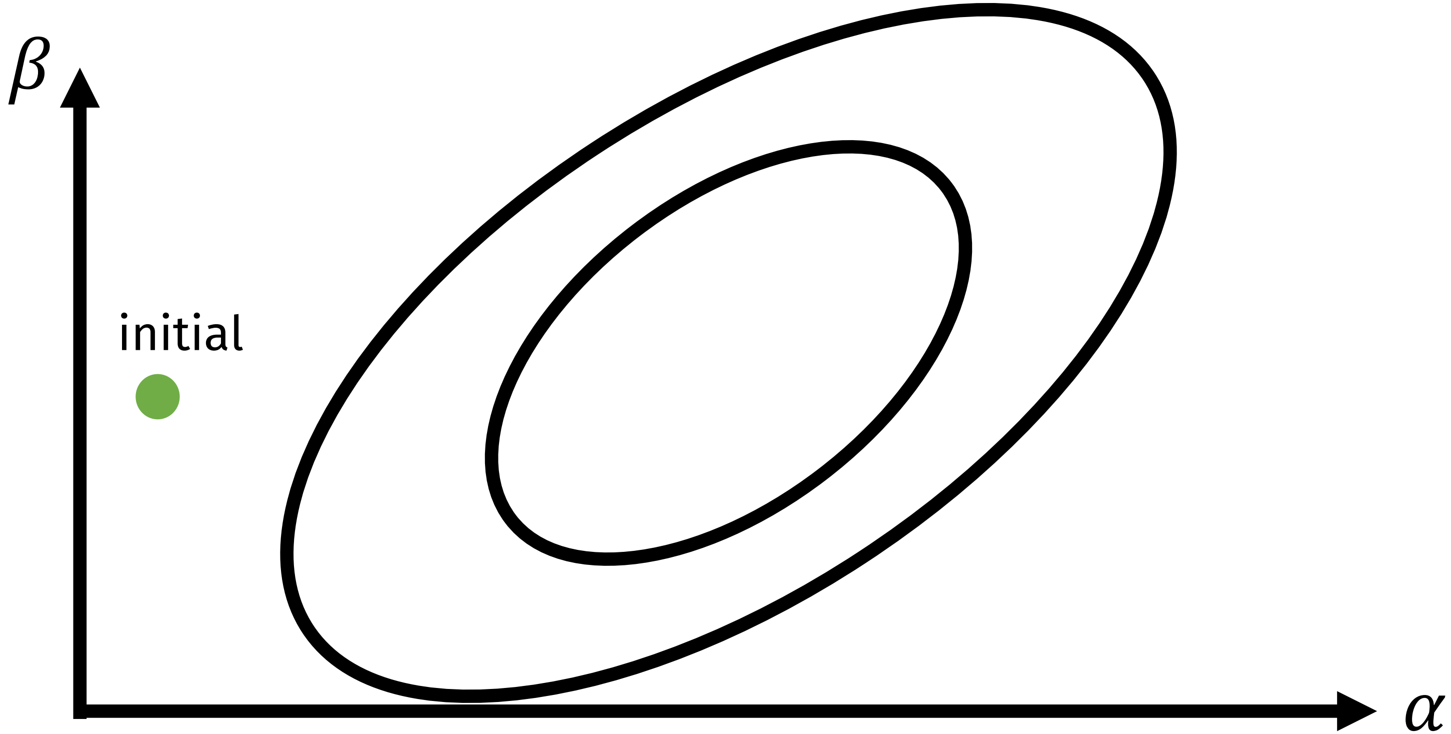
Simulated annealing

Iteration 1



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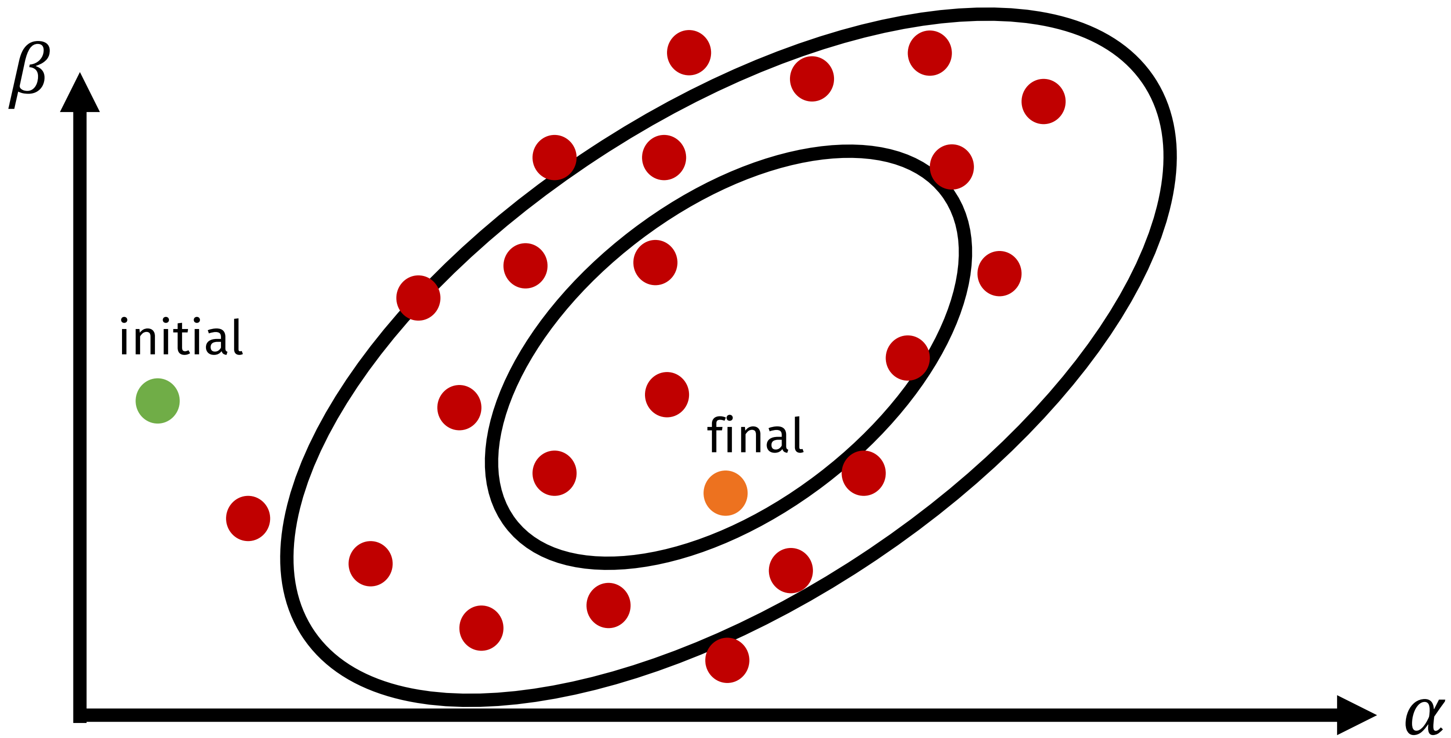
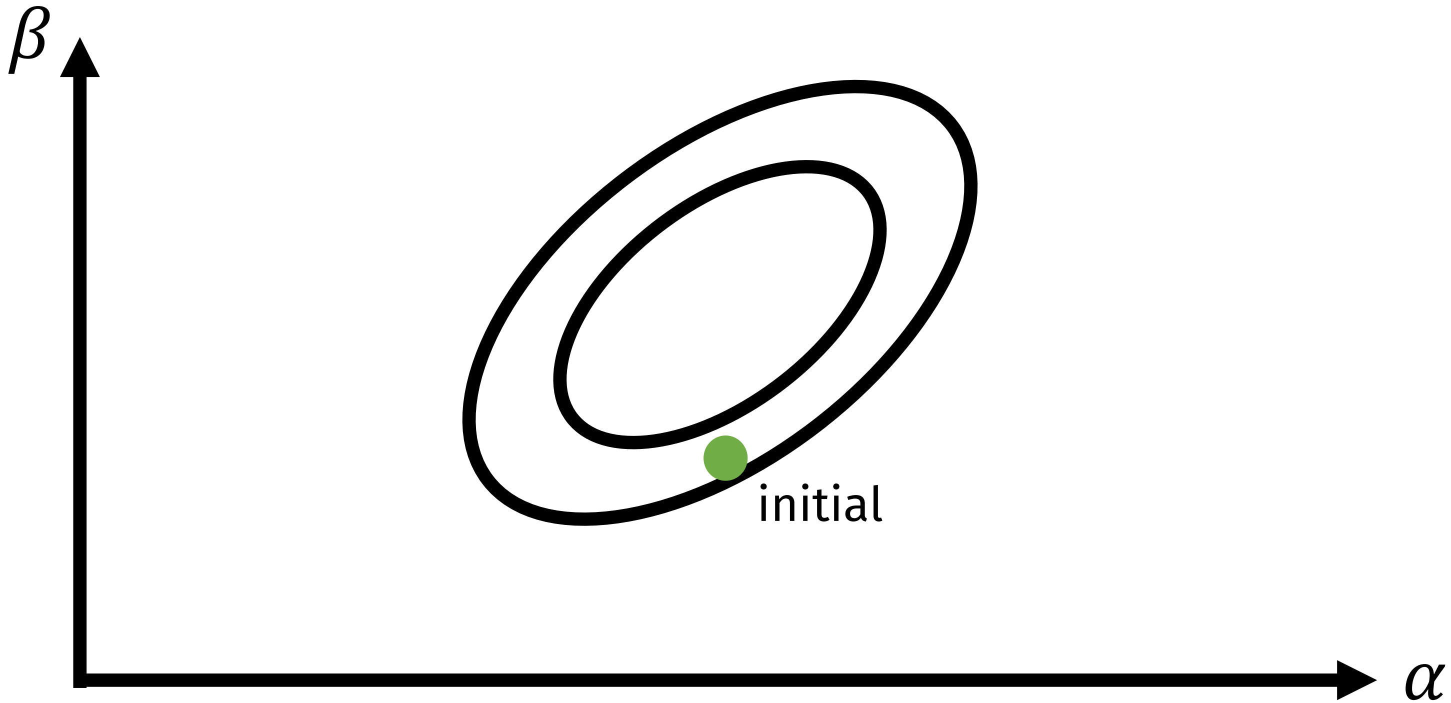


Simulated annealing

Iteration 2



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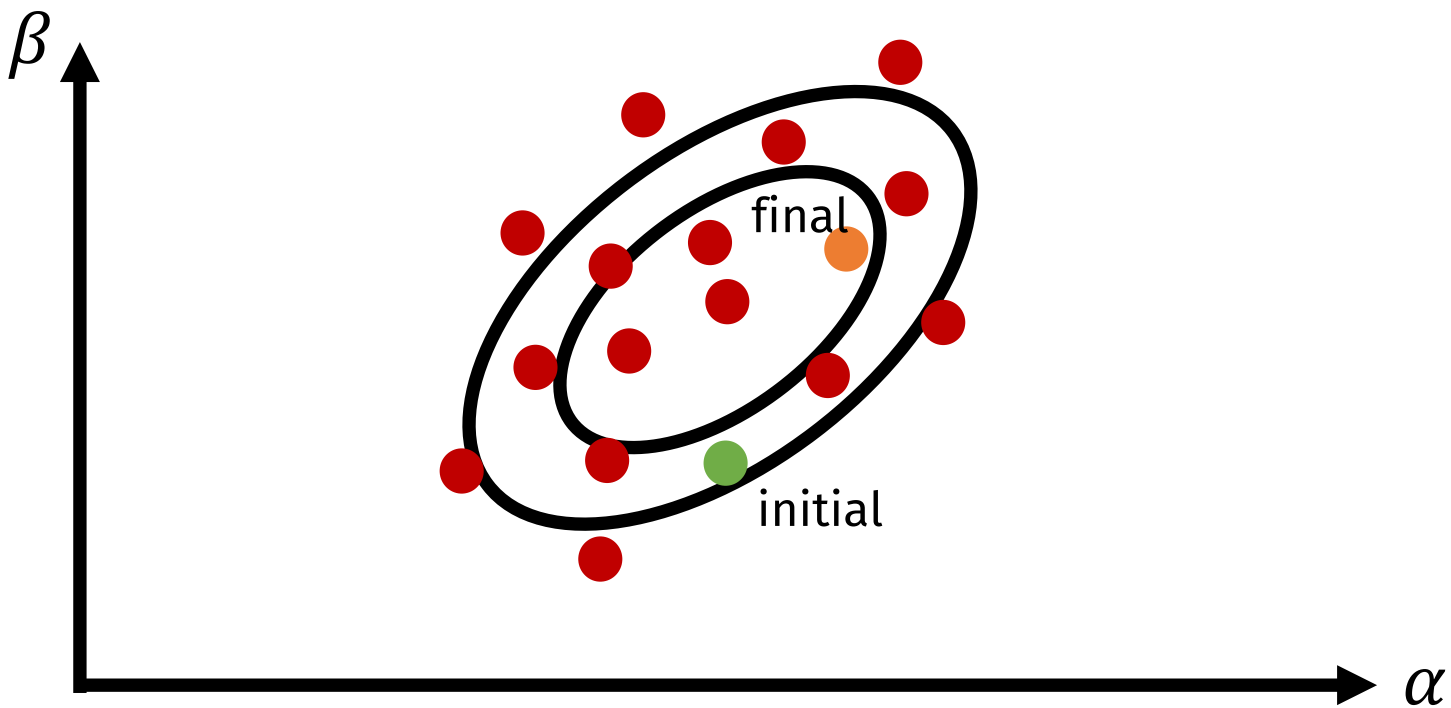

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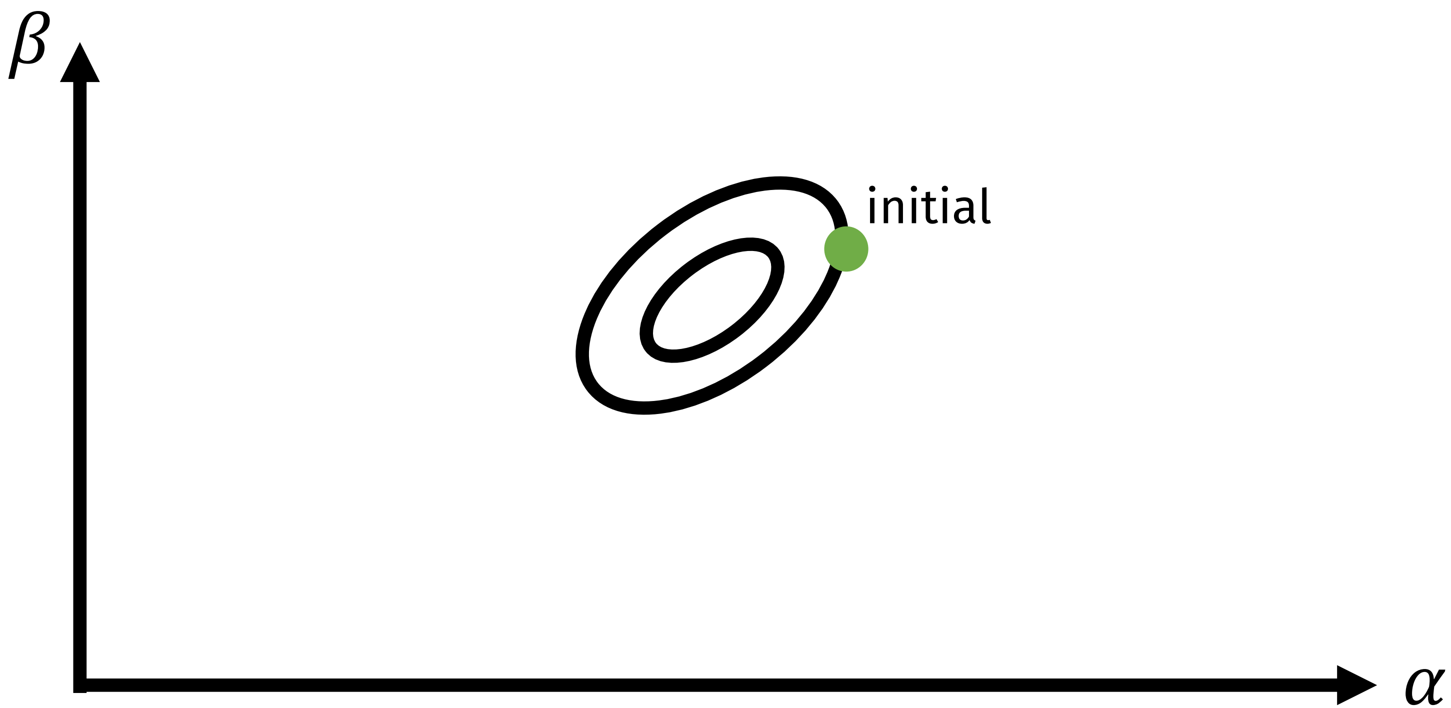

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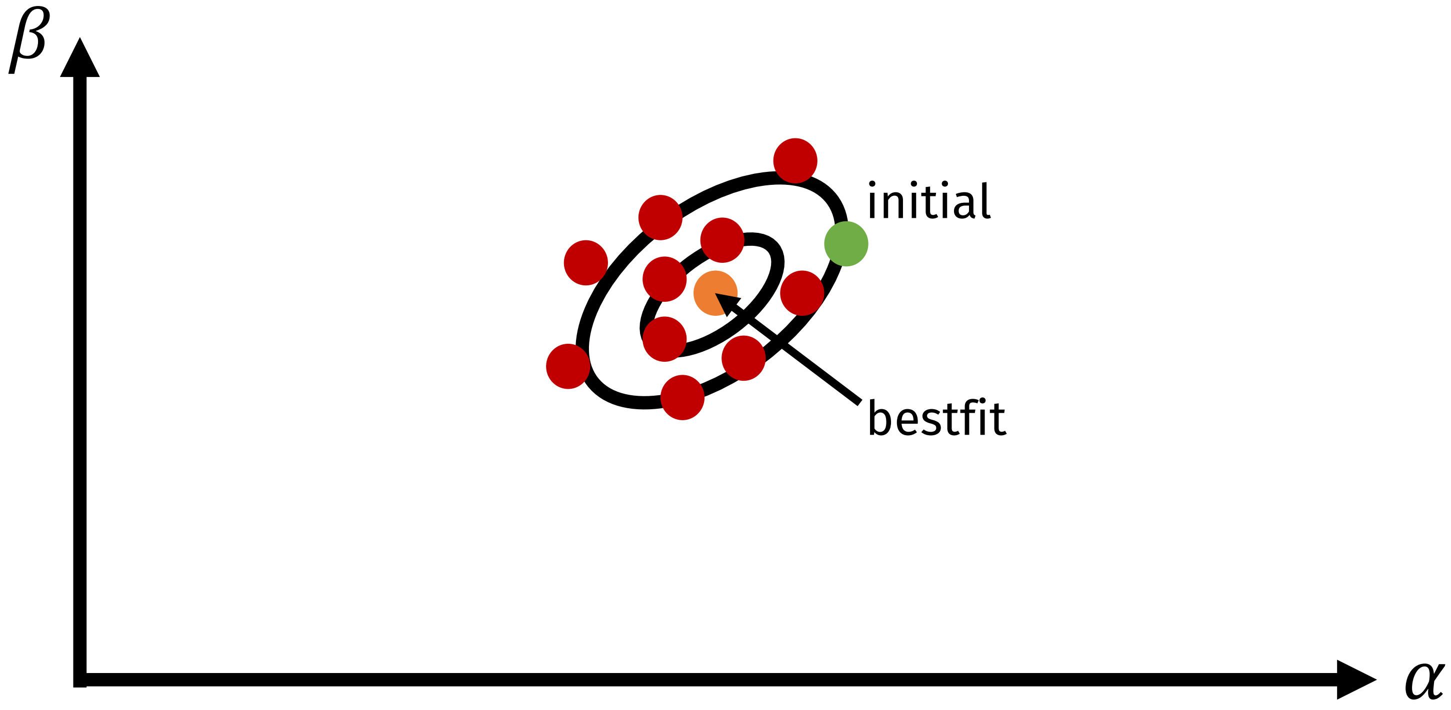


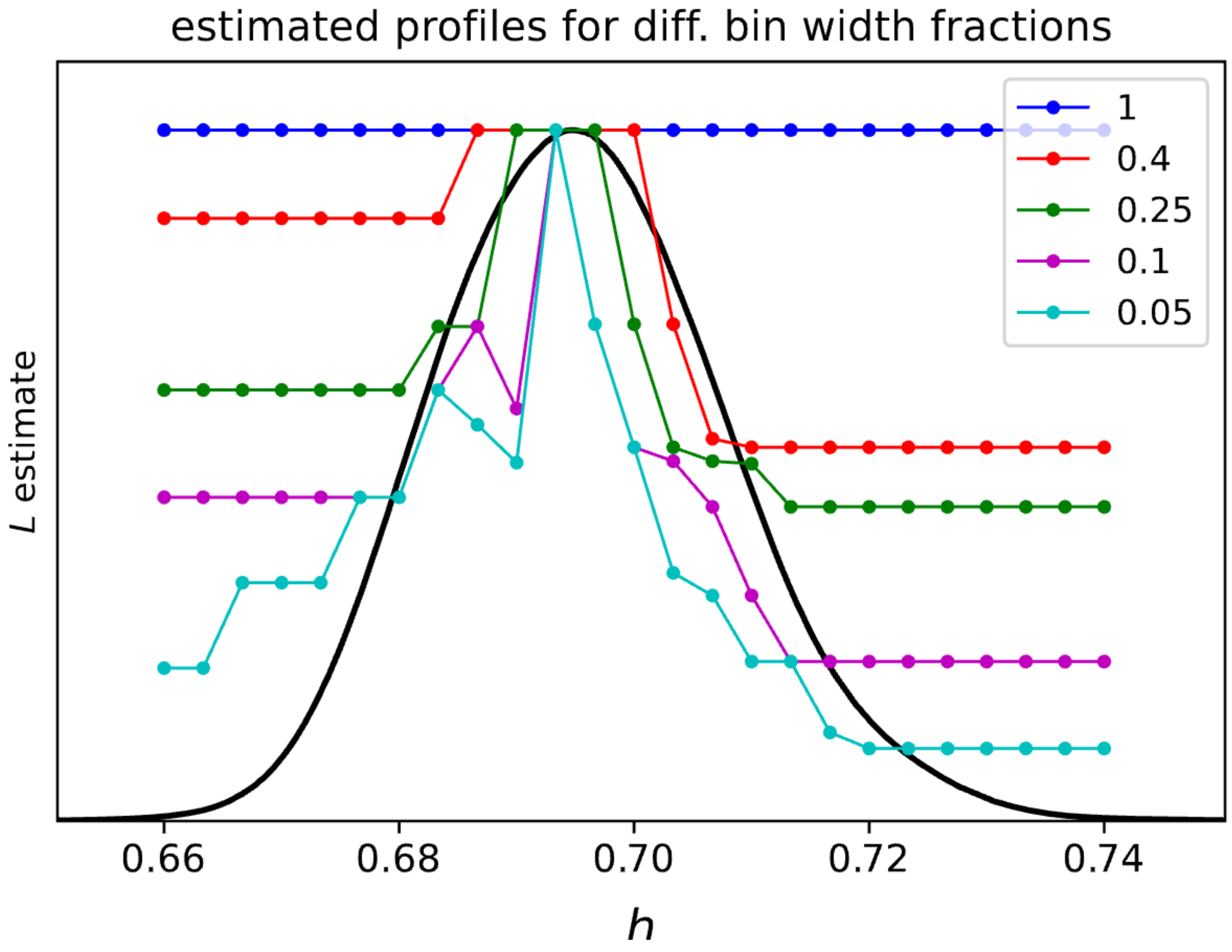
The PROSPECT optimization algorithm

Initialization from MCMC:

- Binned MCMC ≈ profile likelihood

EBH, A. Nygaard, J. Dakin, S. Hannestad, T. Tram (in prep.)

The PROSPECT optimization algorithm



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The PROSPECT optimization algorithm

Initialization from MCMC:

- Binned MCMC ≈ profile likelihood
- Binned covariance matrices

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The PROSPECT optimization algorithm

Hyperparameters:

- Temperature schedule: Exponential or logarithmic?
- Step size schedule: Adjusted to match acceptance rate ≈ 0.2

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```yaml
io:
    jobname: 'lcdm_H0'
    dir: 'output/lcdm_H0'
    write: True
    overwrite_dir: False
    snapshot_interval: 1000.0

run:
    jobtype: 'profile'
    # 'mpi', 'threaded' or 'serial'
    mode: 'mpi'

kernel:
    type: 'cobaya'
    param: 'lcdm_H0.yaml'

profile:
    parameter: 'H0'
    values: 'np.linspace(67.0, 73.0, 10)'

    optimiser: 'simulated annealing'

    temperature_schedule: 'exponential'
    temperature_range: [0.05, 0.001]

    step_size_schedule: 'exponential'
    step_size_range: [0.5, 0.01]

    steps_per_iteration: 1000
    max_iterations: 15
    repetitions: 3

    start_from_mcmc: '/projects/example_mcmc/'
    start_bin_fraction: 0.1

    plot_profile: True
    detailed_plot: True
    plot_Delta_chi2: False
    plot_schedule: True

    chi2_tolerance: 0.01
```

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```yaml
kernel:
    type: 'cobaya'
    param: 'lcdm_H0.yaml'
```

lcdm_H0.yaml:

```yaml
theory:
  classy:
    path: '/home/ebholm/class_public'
    extra_args:
      ...

likelihood:
  planck_2018_highl_plik.TTTEEE:
    clik_file: /home/ebholm/my_clik.clik
    ...

params:
  omega_b:
    prior:
      min: 0.005
      max: 0.08
  ...

sampler:
  mcmc:
  ...
```

```yaml
kernel:
    type: 'montepython'
    param: 'lcdm.param'
    conf: 'class_public.conf'
```

lcdm.param:

```
data.experiments=['Planck_highl_TTTEEE', ...]
data.over_sampling=[1, 5, 5]

data.parameters['omega_b']      = [  2.2377,   None, None,      0.015, 0.01, 'cosmo']
data.parameters['omega_cdm']    = [ 0.12010,   None, None,     0.0013,    1, 'cosmo']
data.parameters['H0']           = [    67.6,   40.0,100.0,        0.5,    1, 'cosmo']
data.parameters['ln10^10A_s']   = [  3.0447,   None, None,      0.015,    1, 'cosmo']
data.parameters['n_s']          = [  0.9659,   None, None,     0.0042,    1, 'cosmo']
data.parameters['tau_reio']     = [  0.0543,  0.004, None,      0.008,    1, 'cosmo']
...
```

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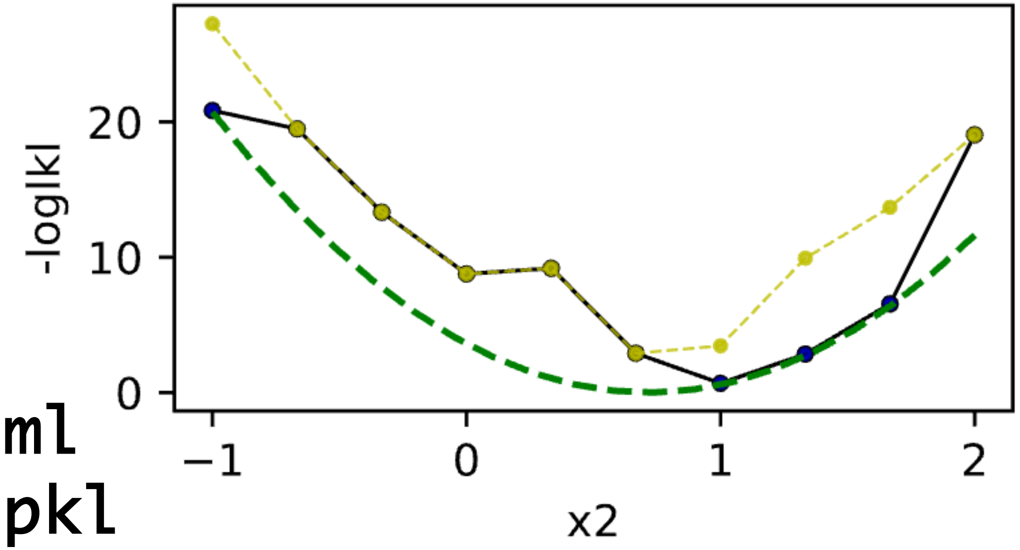
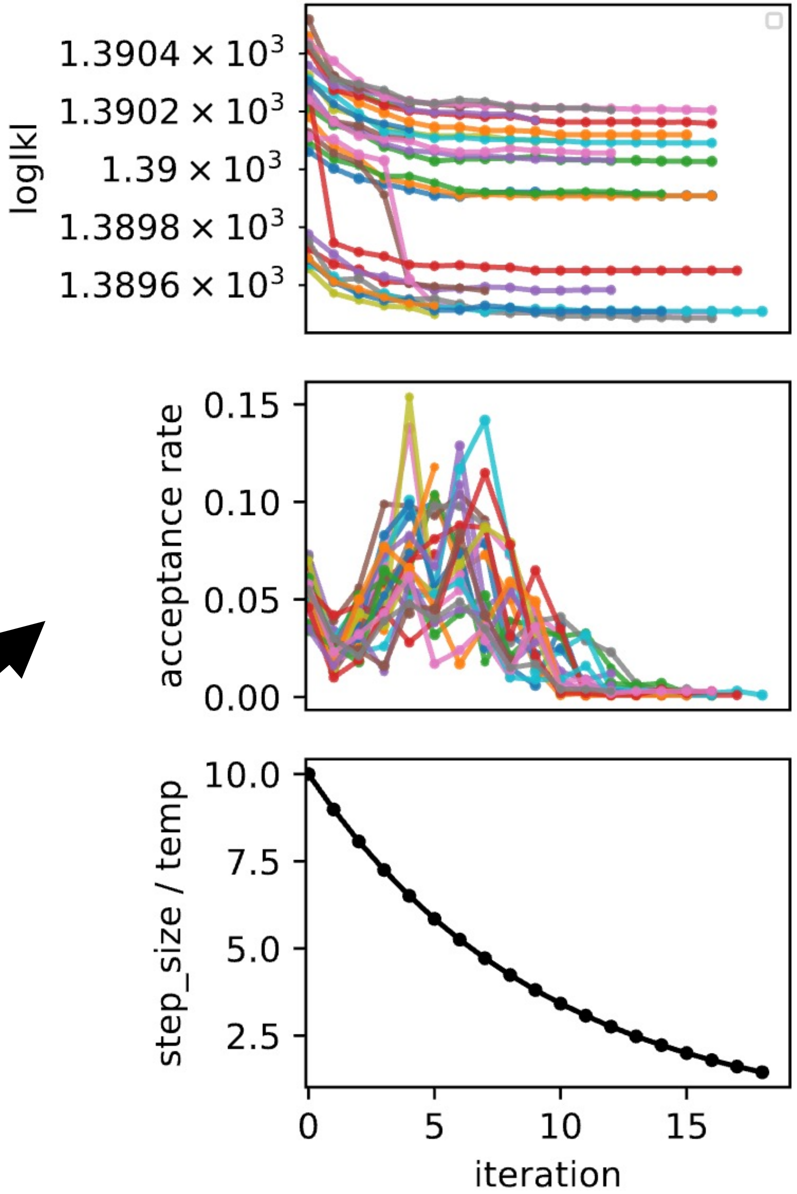
The PROSPECT output

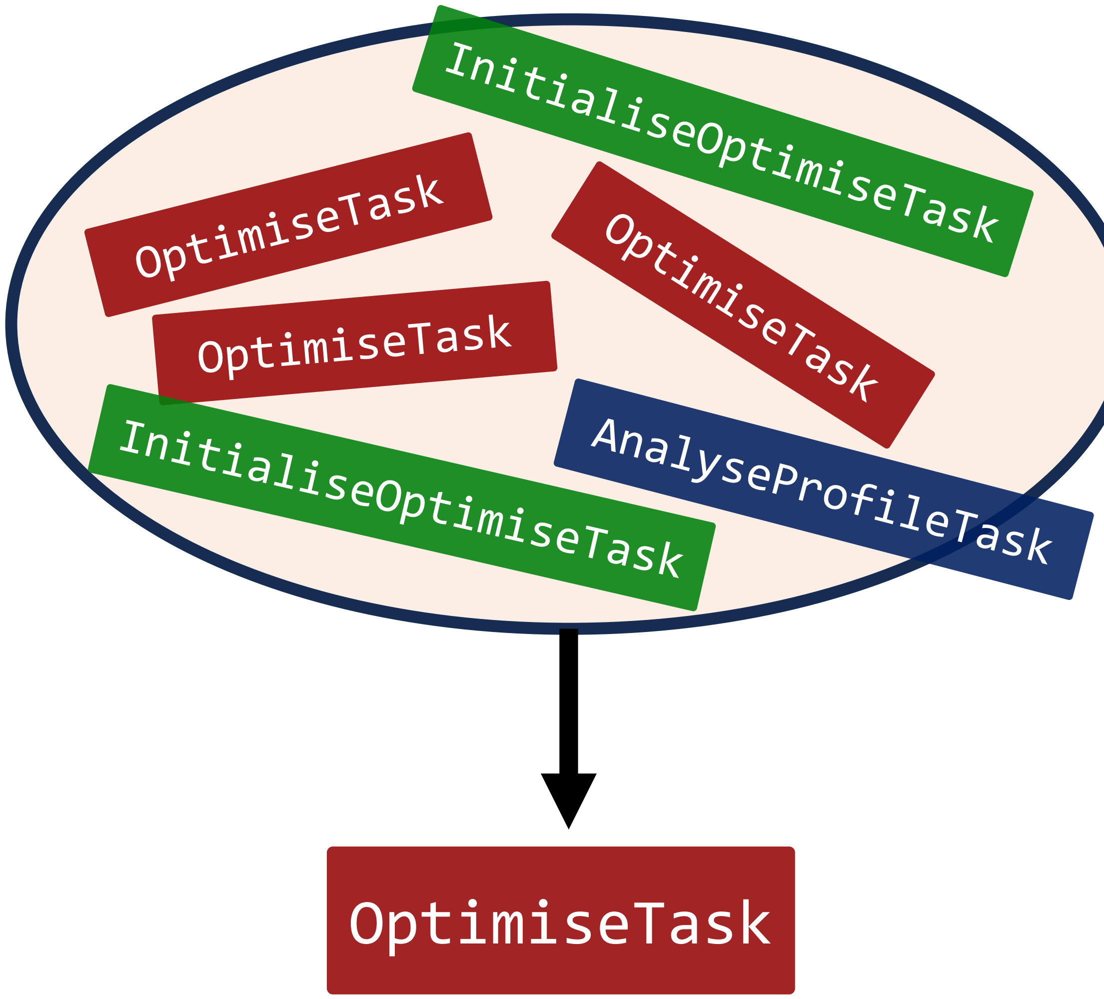
- mnu/
  - log.yaml
  - state.pkl
  - profile/
    - mnu.txt
    - mnu.pdf
    - mnu_schedule.pdf





The PROSPECT dynamical task system

- efficient parallellization
- on-the-go analyses
- interactive load-and-inspect
- continuing cancelled runs
- re-optimizing existing runs



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