Perturbative EFT expansions for cosmological phase transitions

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In collaboration with:
Oliver Gould [2104.04399, 2309.01672],
Andreas Ekstedt, Philipp Schicho [2205.08815]
Lauri Niemi, Michael Ramsey-Musolf and David Weir [2005.11332]
Also see talk by Johan Löfgren! [2112.05472, 2112.08912]

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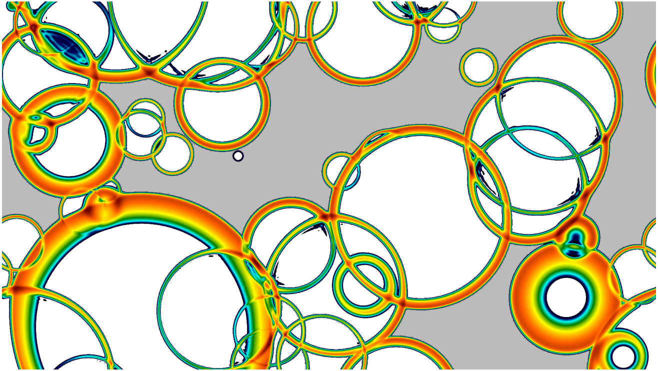
First Nordic Cosmology Meeting, Nordita
24.10.2023

Gravitational waves from 1st order cosmological phase transition



A looking glass to thermal history of our universe, but also to underlying quantum field theory $\rightarrow$ an opportunity to probe BSM physics!

Phase transition thermodynamics

At high T, the pressure, or effective potential in perturbation theory has formal expansion (g is the weak coupling)

$$p \simeq -V_{\text{eff}} \simeq T^4 \Bigg(\underbrace{\underbrace{1 + g^2 + g^3}_{\text{1-loop (with daisies)}}}_{V_{\text{tree}}+V_{\text{CW}}+V_{\text{T}}+V_{\text{daisy}}} + \underbrace{\underbrace{g^4}_{\text{2-loop}}}_{\text{next slide!}} + \underbrace{\underbrace{g^5}_{\text{3-loop}}}_{\text{upcoming!}} \Bigg) + \mathcal{O}(g^6 T^4). \tag{1}$$

Slow convergence and odd powers of g appear at high T due to high occupancy of bosonic modes, that need to be resummed. Linde's IR problem: $\mathcal{O}(g^6 T^4)$ non-perturbative, requires lattice QFT.

In practise, 1-loop thermal effective potential with daisy resummation is **not** accurate, but **2-loop corrections are important**! (Phase transition is generated by thermal loops, so 1-loop is *leading order*. First perturbative corrections come at 2-loops.)

2-loop thermal resummations automated in Dimensional Reduction algorithm (https://github.com/DR-algo/DRalgo)

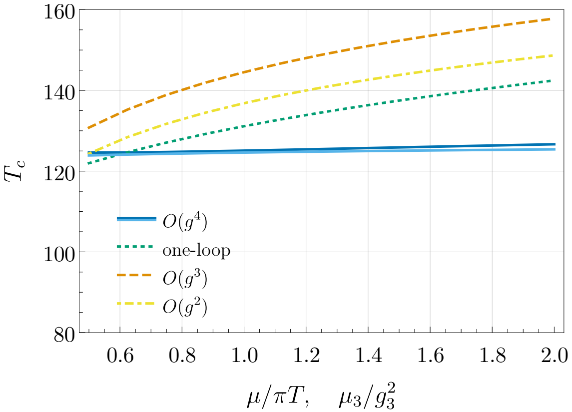
DRalgo: a package for effective field theory approach for thermal phase transitions

by Ekstedt, Schicho and Tenkanen, Comput.Phys.Commun. 288 (2023) 108725.

Automatic 3d EFT construction for generic models: upgrades daisy resummations to 2-loops, and computes 2-loop effective potential.

SM + singlet example: critical temperature as function of RG scale μ

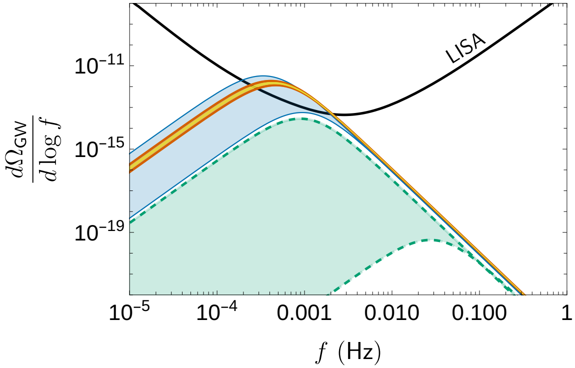
Gould and Tenkanen, JHEP 06 (2021) 069:



First possible RG improvement at $\mathcal{O}(g^4)$ (2-loop thermal mass, 2-loop 3d EFT $V_{\rm eff}$, in contrast to $T = 0$ case where 1-loop is enough).

Multiple orders-of-magnitude uncertainty in the peak GW amplitude

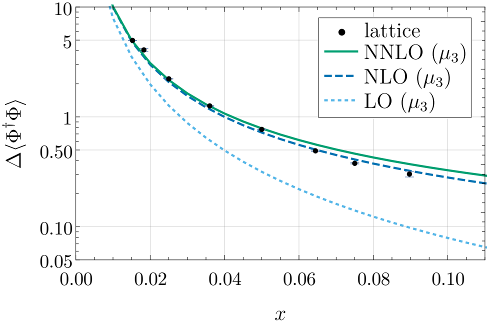
Croon et al, JHEP 04 (2021) 055, Gould and Tenkanen, JHEP 06 (2021) 069:



Green: one-loop. Blue: $\mathcal{O}(g^4)$ (2-loop) for equilibrium thermodynamics. Yellow (estimate!): $\mathcal{O}(g^4)$ for bubble nucleation rate.

SU(2) + Higgs gauge theory

Ekstedt, Gould and Löfgren, Phys.Rev.D 106 (2022) 3, 036012, Gould, Güyer and Rummukainen, Phys.Rev.D 106 (2022) 11, 114507:



An EFT expansion: after integrating out hard thermal scale (πT), gauge field in the broken phase is heavier (soft, gT) than transitioning Higgs field (supersoft, $g^{\frac{3}{2}}T$): integrating out gauge field leads to strict expansion in $x \equiv \frac{\lambda_3}{g_3^2}$.

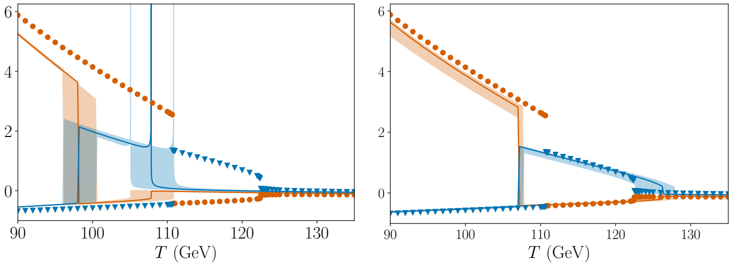
EFT expansions for BSM theories (Gould and Tenkanen [2309.01672])

- 1. Integrate out UV modes and stop at the mass scale of the transitioning fields.
- 2. This mass scale can be identified by power counting, applied to the tree-level potential of the EFT. If apparent 2nd order transition, more modes must be integrated out. Often the final EFT lives at the *supersoft* scale ($g^{\frac{3}{2}}T$), in-between soft (gT) and *non-perturbative* ultrasoft scales (g^2T).
- 3. Carry out strict perturbative expansions within the final EFT for the transitioning field.

$\rightarrow$ resolves lots of theoretical issues, see "The Laundry List" in the talk by Johan Löfgren (and [2301.05197]).

Supersoft EFT expansions for SM + triplet 2-step transition

Gauge invariant order parameters as function of T, for triplet (blue) and Higgs (orange); lattice (scatter points) vs 2-loop perturbation theory (solid lines).



Left: conventional expansions do not work well (Niemi et al, Phys.Rev.Lett. 126 (2021) 17, 171802)

Right: EFT expansions resolve these issues, and improves agreement with lattice (Gould and Tenkanen, [2309.01672])

Summary

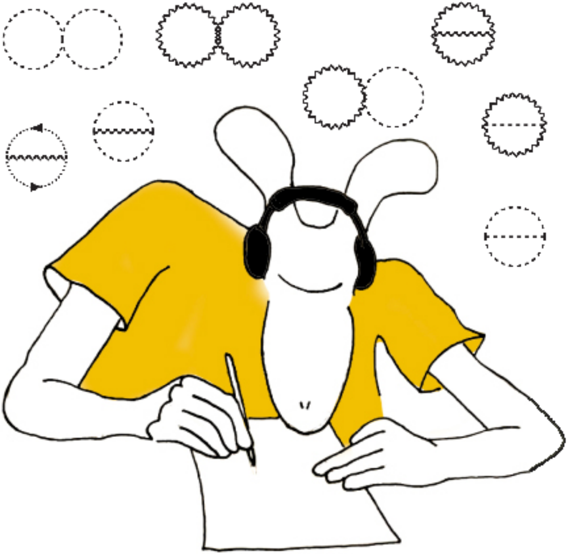
- In past few years: major theoretical developments for perturbative determination of cosmological phase transition thermodynamics, using 3d EFT techniques. (Also see talks by Ekstedt and Lögren tomorrow!)

- It is crucial to go beyond 1-loop approximations in perturbation theory. At higher orders perturbation theory describes strong transitions rather reliably.

- However, lattice simulations still required to separate weak transitions from crossovers.

- Challenge: determination of cosmological phase transition thermodynamics up to the maximal order ($\sim$ 3-loops) in perturbation theory for generic models $\rightarrow$ sets a gold-standard which cannot be exceeded without non-perturbative lattice simulations.

Thanks!



Backup: Gauge invariant condensates

$$\langle \phi^\dagger \phi \rangle \equiv \frac{\partial V_{\text{eff}}}{\partial m^2_{\phi,3}}. \tag{2}$$

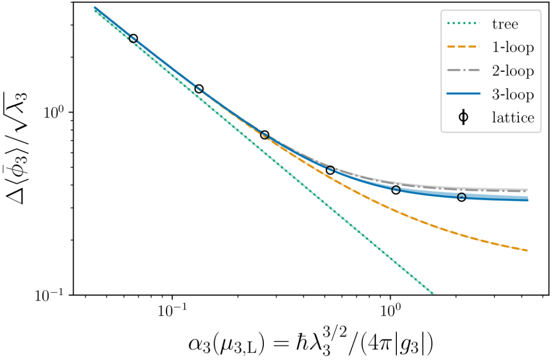
A discontinuity in the condensate signals a first-order phase transition:

$$\begin{aligned} p'(T_c) &= -T \frac{d}{dT} V_{\text{eff}}(\kappa_i) = -\sum_i T \frac{d\kappa_i}{dT} \frac{\partial V_{\text{eff}}}{\partial \kappa_i} \\ &= \eta(m^2_{\phi,3}) \langle \phi^\dagger \phi \rangle + \eta(\lambda_{\phi,3}) \langle (\phi^\dagger \phi)^2 \rangle + \dots \ , \end{aligned} \tag{3}$$

$\eta(\kappa_i) \equiv T d\kappa_i / dT$.
The η-functions (UV) are continuous in T, and condensates (IR) have discontinuities at phase transitions.
V_{eff} is evaluated at its minima, which quarantees gauge invariance.

Backup: Closer look to strict perturbative expansion for real scalar theory

Gould, JHEP 04 (2021) 057:



Strict expansion – expansion around LO minima – matches lattice result extremely well! There is single expansion parameter which is dimensionless ratio of 3d EFT parameters.