Gauge invariant and self-consistent phase transition calculations

Computing the gauge-invariant bubble nucleation rate in finite temperature effective field theory (JHEP, 2112.08912)
Nucleation at Finite Temperature: A Gauge-Invariant Perturbative Framework (PRL, 2112.05472),

Johan Löfgren, 2023-10-25 (about me)

Collaborators: Joonas Hirvonen, Michael J. Ramsey-Musolf, Philipp Schicho, Tuomas V.I. Tenkanen

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Abelian Higgs

Is there a 1st order PT?

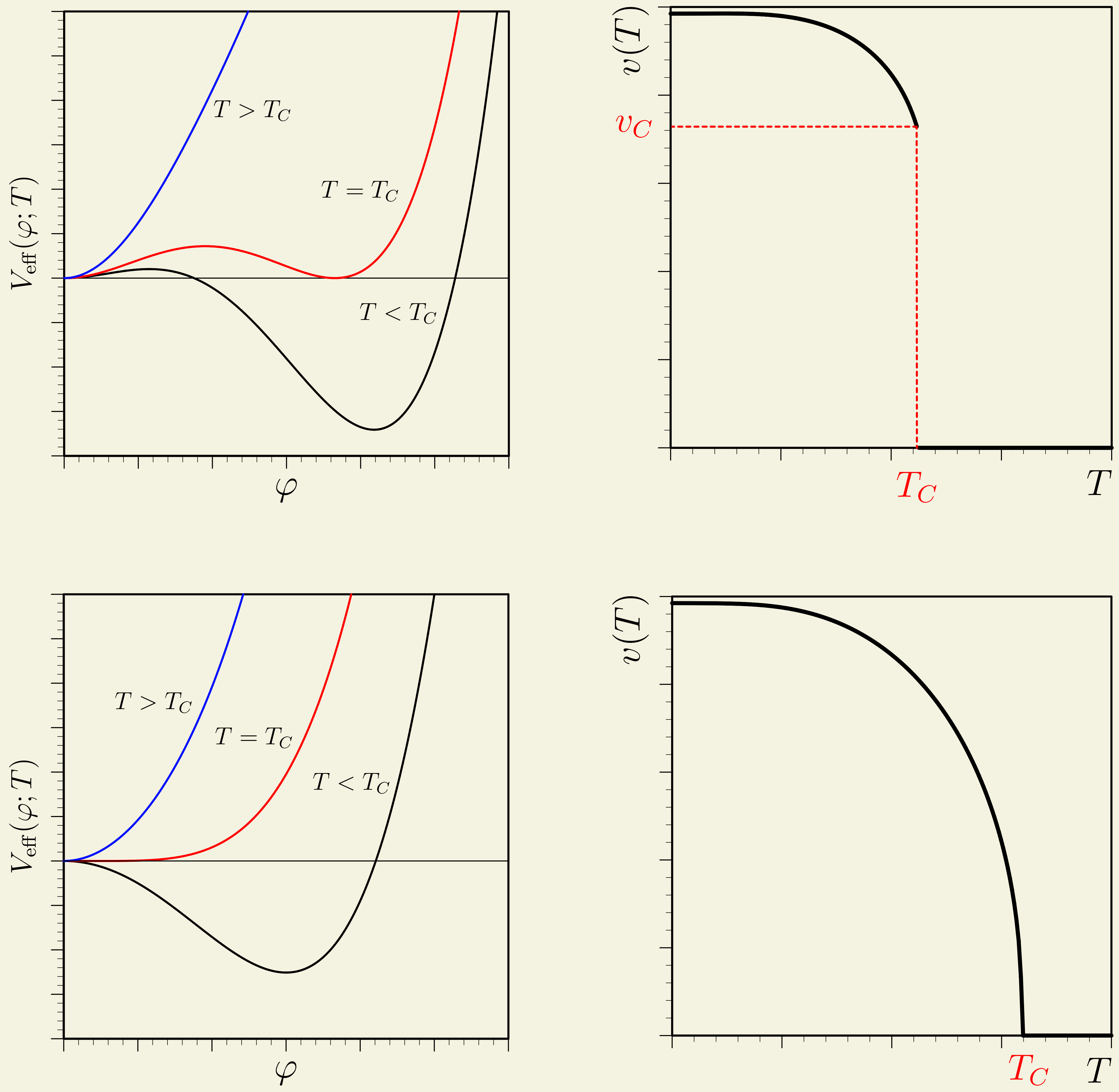
- Barrier?

$$\mathcal{L}_{\text{4d}} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + (D_\mu\Phi)^*(D_\mu\Phi) + \mu^2\Phi^*\Phi + \lambda(\Phi^*\Phi)^2$$

- hep-ph/9212235, Arnold & Espinosa

- Yes, if:

$$\lambda \sim g^3\,, \qquad \mu^2_{\text{eff}} \sim g^3T^2\,, \qquad \phi \sim T \sim \frac{\mu}{g}$$



Source: 10.3390/sym12050733, Senaha

Gauge Invariance?

Radiative transitions

- Equilibrium quantities: **Yes!** (2006.12614, Andreas Ekstedt, **JL**)
- Tunneling rate (Zero T): **Yes!** (hep-ph/9507381, Metaxas, E. Weinberg)
- Bubble nucleation rate (Finite T): **No!** (1205.3392, Garny, Konstandin)

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- … But why would it work for zero T but not finite T?

Tunneling / Nucleation analogy

2201.07331, Andreas Ekstedt

2108.04377, Oliver Gould, Joonas Hirvonen

- The bounce extremizes the Euclidian action:

$$\delta S_{\text{eff}}[\phi_b] = 0$$

$$\nabla^2 \phi_b(x) = \left.\frac{\partial V^{\text{eff}}_{\text{LO}}}{\partial \phi}\right|_{\phi=\phi_b}, \quad \begin{cases} \phi_b(\infty) & = 0 \\ \phi_b'(0) & = 0 \end{cases}$$

Zero T (4D Field Theory):

$$\Gamma_{T=0} = 2\text{Im}e^{-S_{\text{eff}}}$$

Finite T (3D Field Theory):

$$\Gamma_{T\neq 0} = A_{\text{dyn}} e^{-\overline{S}_{\text{eff}}}$$

The Objection

The derivative expansion diverges

- An expansion in P^2/m_B^2 where $m_B^2 = g^2\phi^2$

- The expansion diverges when approaching the symmetric minimum!

$$Z = 1 + Z_{\text{NLO}} + \mathcal{O}(g^{\frac{3}{2}})\ ,$$

$$S_3 = \int \mathrm{d}^3x \Big[V^{\text{eff}}(\phi, T) + \frac{1}{2} Z(\phi, T)\, (\partial_i \phi)^2 + \dots\Big]$$

$$Z_{\text{NLO}}(\phi) = \frac{gT}{48\pi}\left[-\frac{22}{\phi} + \frac{\phi^2}{(\frac{1}{3}T^2 + \phi^2)^{\frac{3}{2}}}\right]$$

… But why would it work for zero T and not for finite T?

Similar divergence!

- An expansion in P^2/m_B^2 where $m_B^2 = g^2\phi^2$
- The expansion diverges when approaching the symmetric minimum!

$$\Gamma = Ae^{-(\mathcal{B}_0+\mathcal{B}_1+\ldots)}$$

$$\mathcal{B}_0 = \int \mathrm{d}^4x\left[V^{\mathrm{eff}}_{g^4}(\phi_b) + \frac{1}{2}\left(\partial_\mu\phi_b\right)^2\right],$$

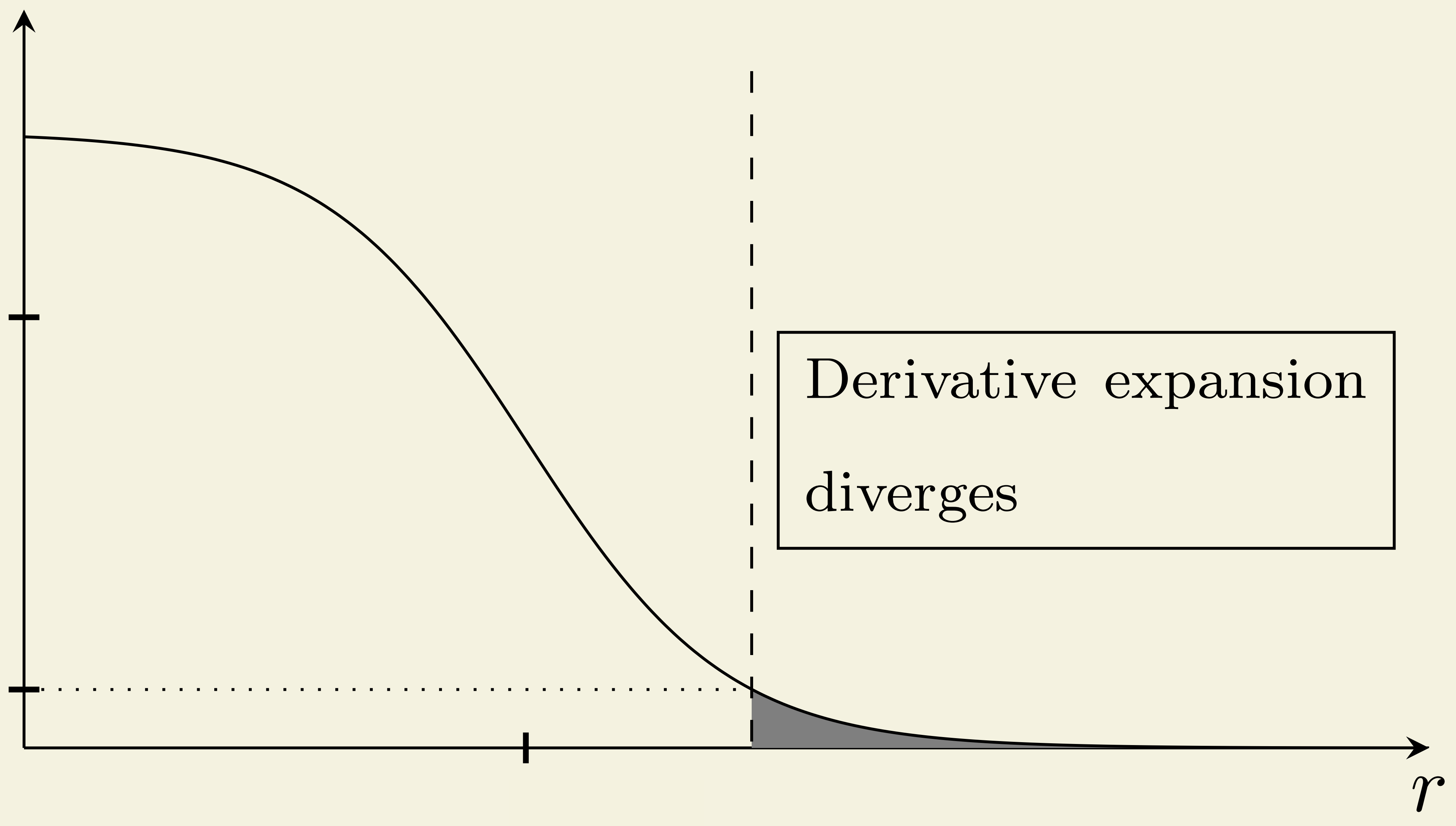
$$\mathcal{B}_1 = \int \mathrm{d}^4x\left[V^{\mathrm{eff}}_{g^6}(\phi_b) + \frac{1}{2}Z_{g^2}\left(\partial_\mu\phi_b\right)^2\right]$$

$$Z_{g^2} \sim \ln(\phi/\Lambda)$$

Asymptotics at zero T

$$\Box\phi_b \sim \mu^2\phi_b\,, \quad \phi_b(\infty) = 0$$

$$\implies \phi_b(r) \sim c\frac{e^{-\mu r}}{r^{3/2}} \qquad \text{as } r \to \infty$$



Asymptotics at zero T

Finite!

$$\Box\phi_b \sim \mu^2\phi_b\,, \quad \phi_b(\infty) = 0$$

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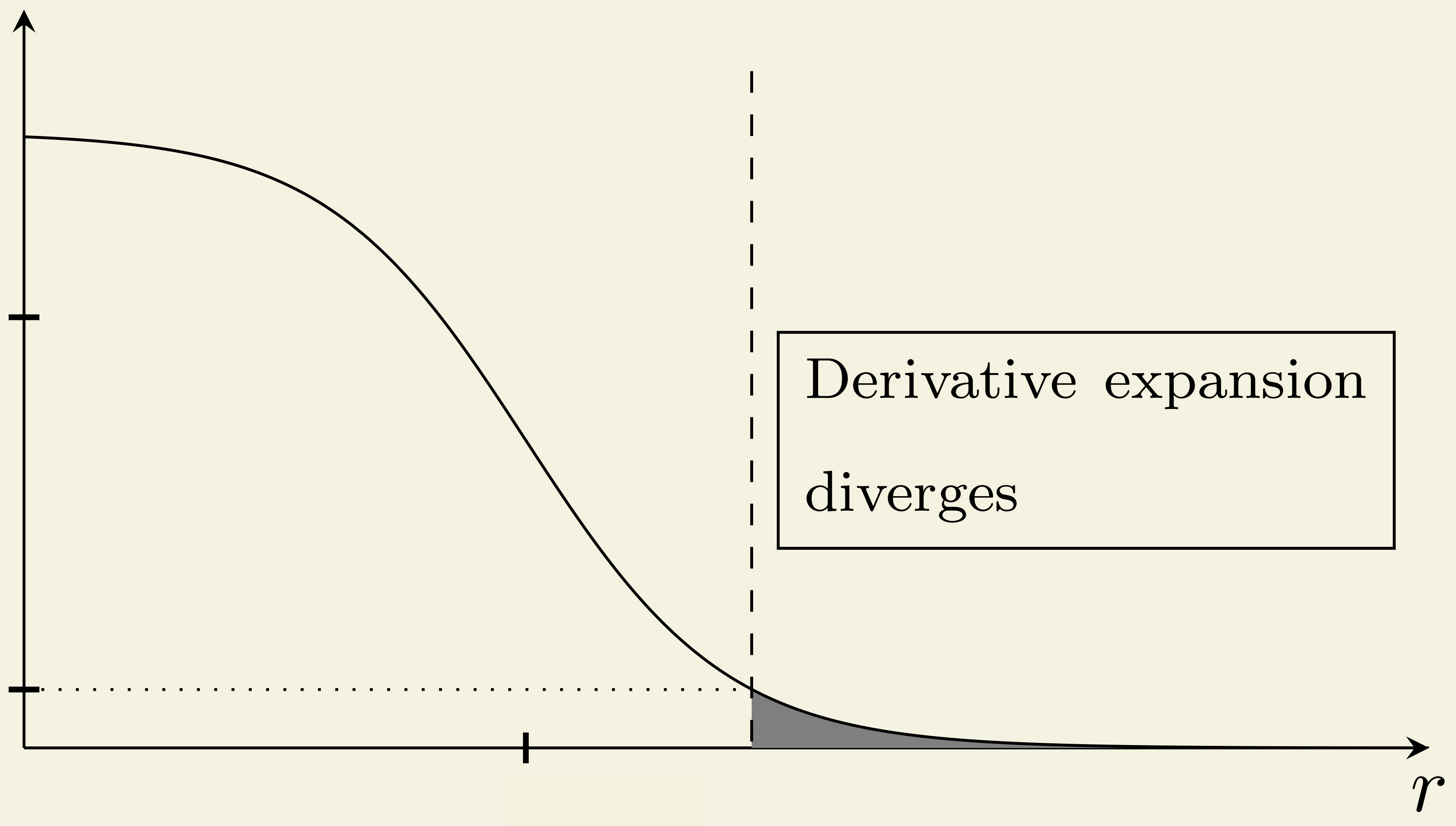
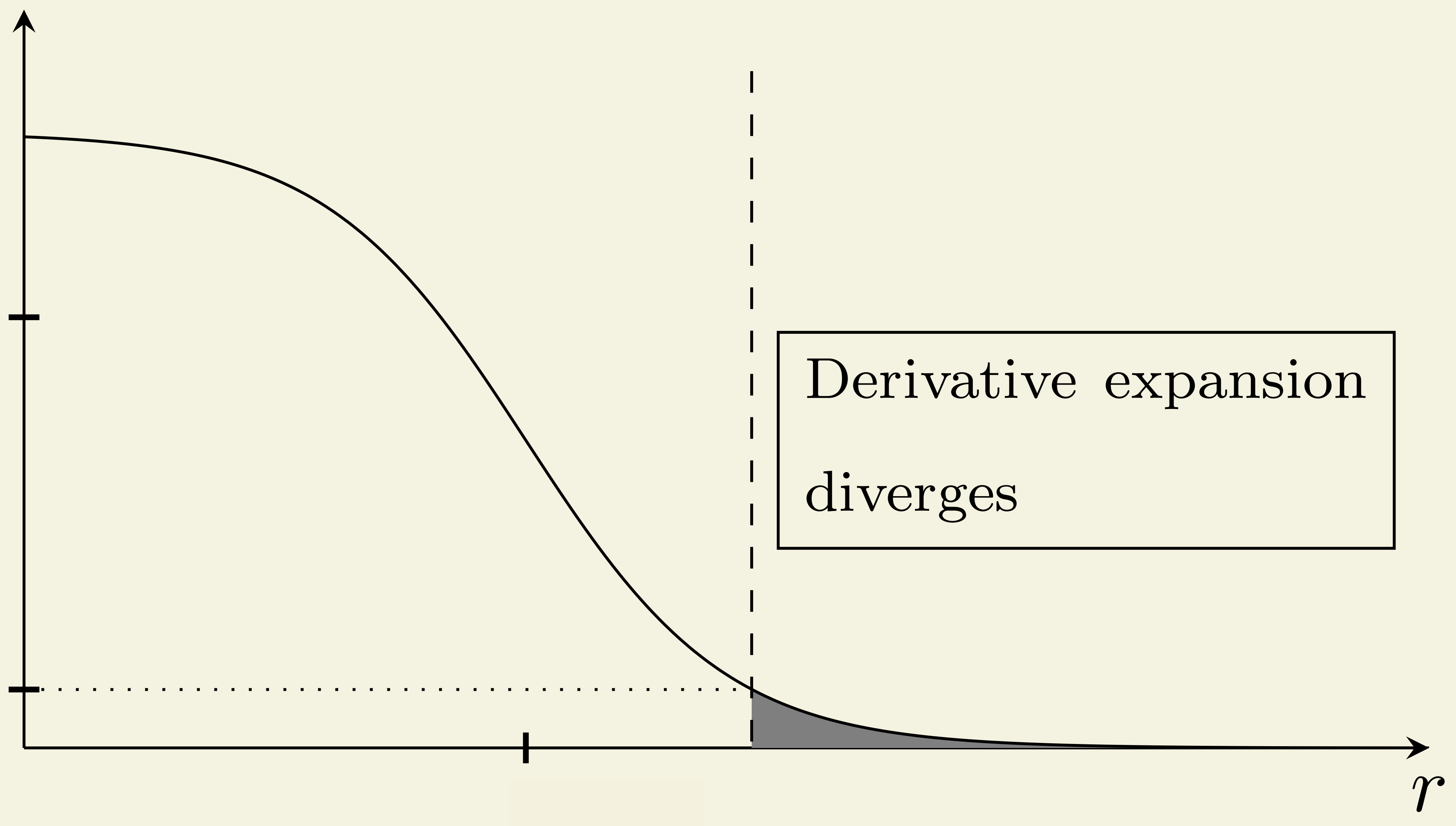


$$\int \mathrm{d}^4x\, \ln(\phi)\,(\partial_\mu\phi_b)^2 \approx (\text{contribution from } r \leq R) - 4\pi^2c^2\mu^3 \int_{r\geq R} \mathrm{d}r\, re^{-2\mu r},$$

Asymptotics at finite T

$$\Box\phi_b \sim \mu^2\phi_b \ , \quad \phi_b(\infty) = 0 \ ,$$

$$\implies \phi_b(r) \sim c\frac{e^{-\mu r}}{r} \qquad \text{as } r \to \infty$$

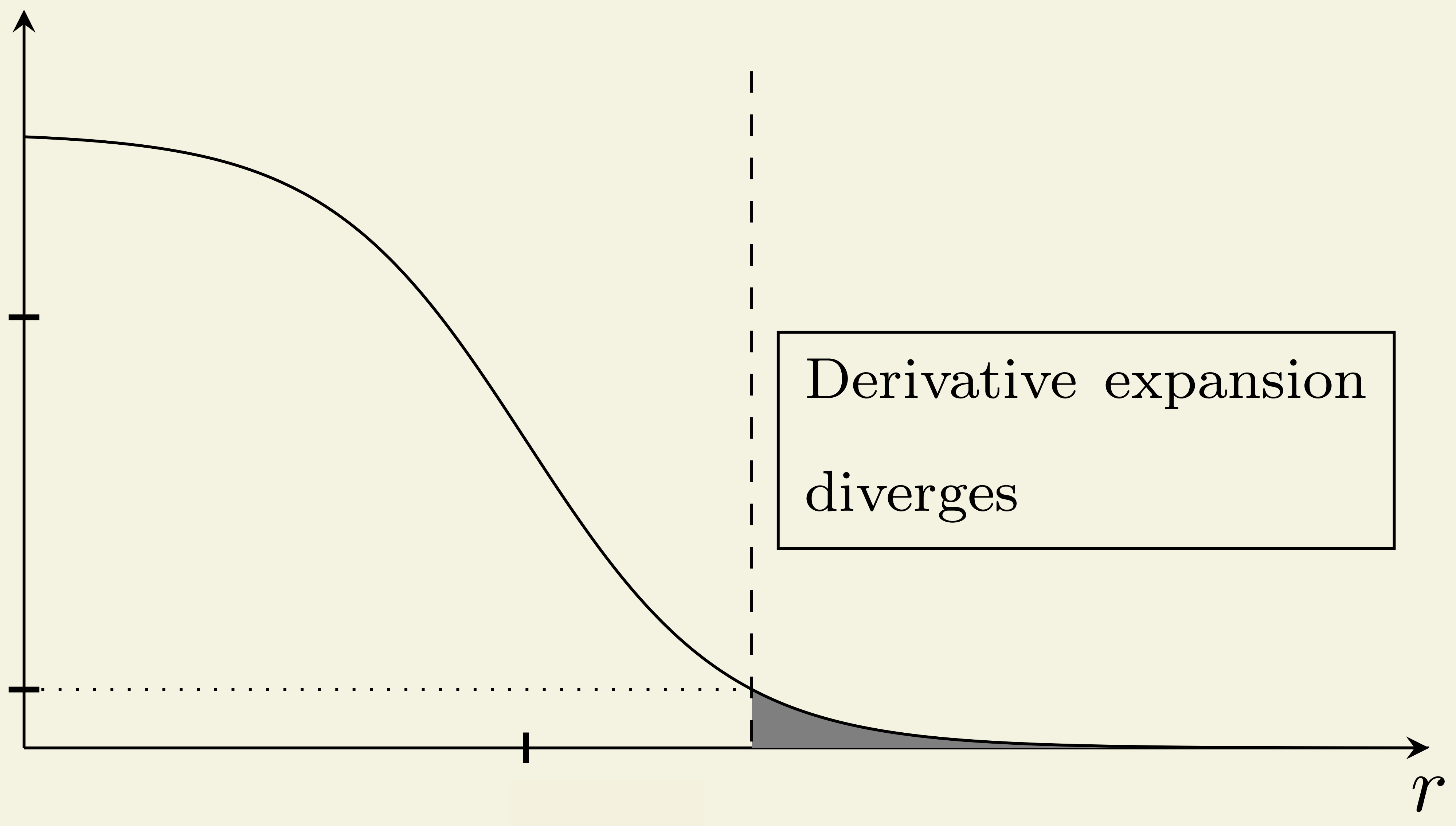


Asymptotics at finite T

Also finite!

$$\Box\phi_b \sim \mu^2\phi_b\ , \quad \phi_b(\infty) = 0\ ,$$

$$\implies \phi_b(r) \sim c\frac{e^{-\mu r}}{r} \qquad \text{as } r \to \infty$$



$$\int \mathrm{d}^3x \frac{(\partial_\mu\phi_b)^2}{\phi_b} \approx (\text{contribution from } r \leq R) - 4\pi c\mu^2 \int_{r\geq R} \mathrm{d}r\, r e^{-\mu r}$$

Protected by a Hierarchy of Scales

$$\underbrace{\pi T}_{\text{thermal scale}} \overset{\text{Step 1}}{\gg} \underbrace{gT}_{\text{intermediate scale}} \overset{\text{Step 2}}{\gg} \underbrace{\frac{g^{\frac{3}{2}}}{\sqrt{\pi}}T}_{\text{nucleation scale}} \gg \underbrace{\frac{g^2}{\pi}T}_{\text{ultrasoft scale}}$$

Scale	**Validity**	**Dimension**	**Action**	**Fields**
Intermediate	gT	d	$S_3(\phi_3)$	$B_{3,i}, B_0, H_3, \chi_3, c_3$
		$\downarrow$ Step 2: *Matching 1PI actions.*		
Nucleation	$g^{\frac{3}{2}}T$	d	$S_{\text{nucl}}(\hat{\phi}_3)$	$\hat{H}_3$

$$Z = \int \mathcal{D}\Phi^{\text{IR}} e^{-S_{\text{eff}}[\Phi^{\text{IR}}]},$$

$$S_{\text{eff}}[\Phi^{\text{IR}}] = -\log \int \mathcal{D}\Phi^{\text{UV}} e^{-S[\Phi^{\text{IR}}+\Phi^{\text{UV}}]}.$$

See also:

2205.02687 , Joonas Hirvonen

2108.04377, Oliver Gould, Joonas Hirvonen

The strict expansion

$$S_{\text{eff}}[\phi] = S_{\text{LO}}[\phi] + xS_{\text{NLO}}[\phi] + \dots$$

$$\left.\frac{\delta S_{\text{LO}}[\phi]}{\delta\phi(x)}\right|_{\phi=\phi_{\text{LO}}} = 0$$

$$\phi_{\text{min}} = \phi_{\text{LO}} + x\phi_{\text{NLO}} + \dots$$

$$\Sigma = Ae^{-(\mathcal{B}_0+\mathcal{B}_1)} \, ,$$

$$\mathcal{B}_0 = \beta \int \mathrm{d}^3x \left[V^{\text{eff}}_{g^3}(\phi_b) + \frac{1}{2}\left(\partial_i\phi_b\right)^2\right] \, ,$$

$$\mathcal{B}_1 = \beta \int \mathrm{d}^3x \left[V^{\text{eff}}_{g^4}(\phi_b) + \frac{1}{2}Z_g\left(\partial_i\phi_b\right)^2\right]$$

Gauge invariance

$$\xi \frac{\partial}{\partial \xi} V^{\text{eff}}_{g^4} = -C_g \frac{\partial}{\partial \phi} V^{\text{eff}}_{g^3} \ ,$$

$$\xi \frac{\partial}{\partial \xi} Z_g = -2 \frac{\partial}{\partial \phi} C_g \ ,$$

$$Z_{\text{NLO}}(\phi) = \frac{gT}{48\pi}\left[-\frac{22}{\phi} + \frac{\phi^2}{(\frac{1}{3}T^2 + \phi^2)^{\frac{3}{2}}}\right]$$

$$C_{\text{LO}} = T\frac{\sqrt{\xi}}{16\pi} g$$

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$$C_{\text{LO}} = T \frac{\sqrt{\xi}}{16\pi} g$$

$$\begin{aligned}
\xi \frac{\partial}{\partial \xi} \mathcal{B}_1 &= \xi \frac{\partial}{\partial \xi} \beta \int \mathrm{d}^3 x \left[V^{\text{eff}}_{\text{NLO}}(\phi_b) + \frac{1}{2} Z_{\text{NLO}} \left(\partial_\mu \phi_b\right)^2 \right] \\
&\overset{(A)}{=} \beta \int \mathrm{d}^3 x \left[-C_{\text{LO}} \frac{\partial}{\partial \phi} V^{\text{eff}}_{\text{LO}}(\phi_b) \right] \\
&\overset{(B)}{=} -C_{\text{LO}}\, \beta \int \mathrm{d}^3 x \left[\Box \phi_b \right] \\
&\overset{(C)}{=} -C_{\text{LO}}\, \beta \int \mathrm{d}^2 S \cdot (\partial \phi_b) \\
&\overset{(D)}{=} 0 \ .
\end{aligned}$$

Recommendations

- Establish that an LO potential with correct behaviour exists
- When calculating nucleation rates, pay close attention to
  1. The hierarchy of scales
  2. How to generate a barrier
- Use EFTs + **strict** expansions! (Do not mix orders in the perturbative expansion.)
- Go forth and count powers!

Thank you!

Bonus: Future Work

- Apply the methods to phenomenological models (2HDMs, SMEFT, …)
- Extend the methods to higher orders to probe general behaviour of perturbation theory
- “What does a ghost weigh?” — I.e. what does it mean to integrate out a non-physical field?

Bonus: “The Laundry List”

*Stop comparing resummation methods, **JL**, 2301.05197*

- Gauge dependence
- Strong renormalization scale dependence
- The Goldstone boson catastrophe
- IR divergences
- Imaginary potentials
- Mirages
- Perturbative breakdown
- Resummation method dependence
- Linear terms

Bonus: "The Laundry List" — Dissolved

*Stop comparing resummation methods, **JL**, 2301.05197*

- ~~Gauge dependence~~
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- ~~Imaginary potentials~~
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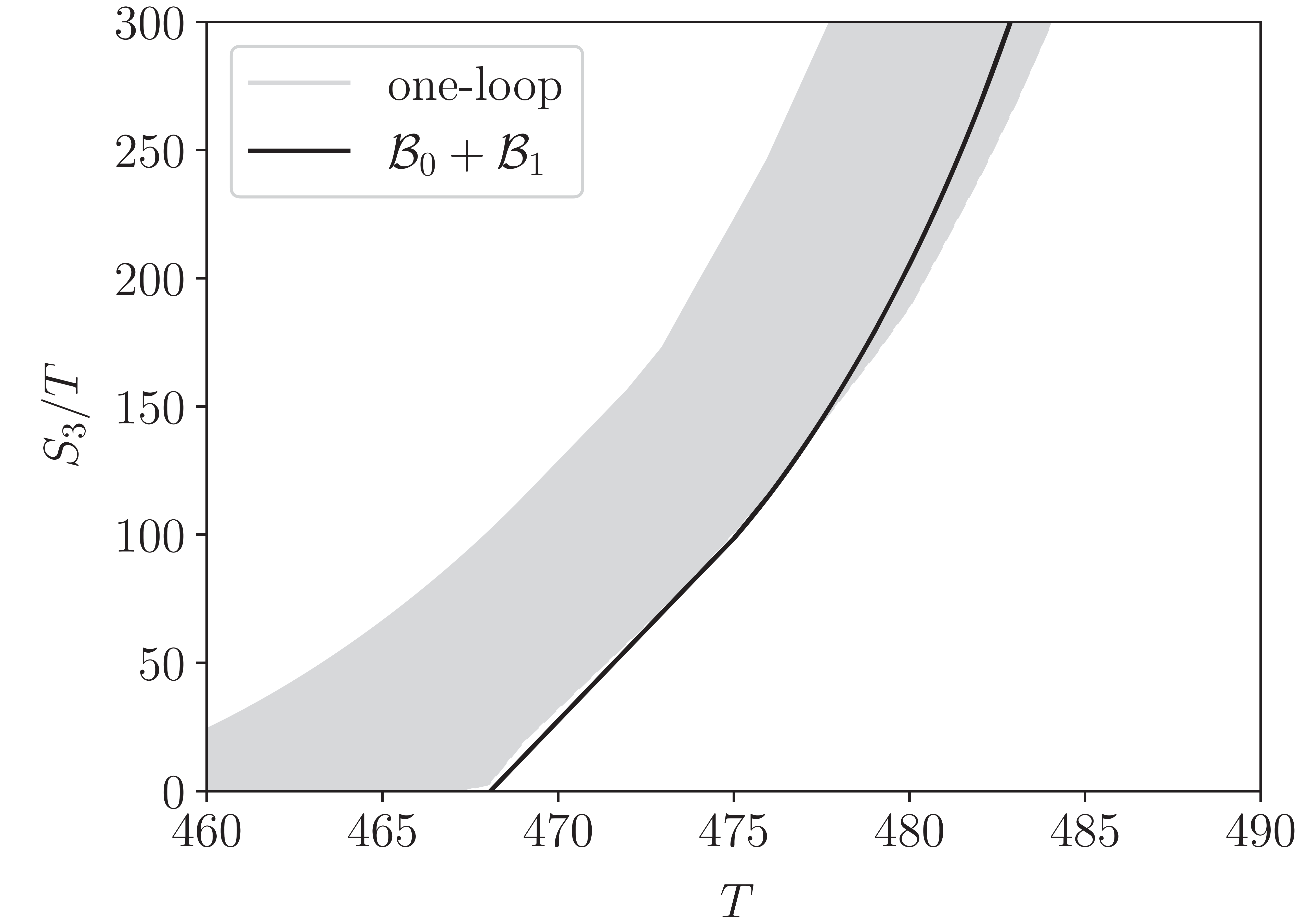
- Perturbative breakdown
- ~~Resummation method dependence~~
- ~~Linear terms~~

- Use EFTs together with strict expansions!
- Take perturbation theory seriously
- Use hierarchies of scales to guide your thinking

Bonus: Numerical Results



$$\xi = [0, 4]$$