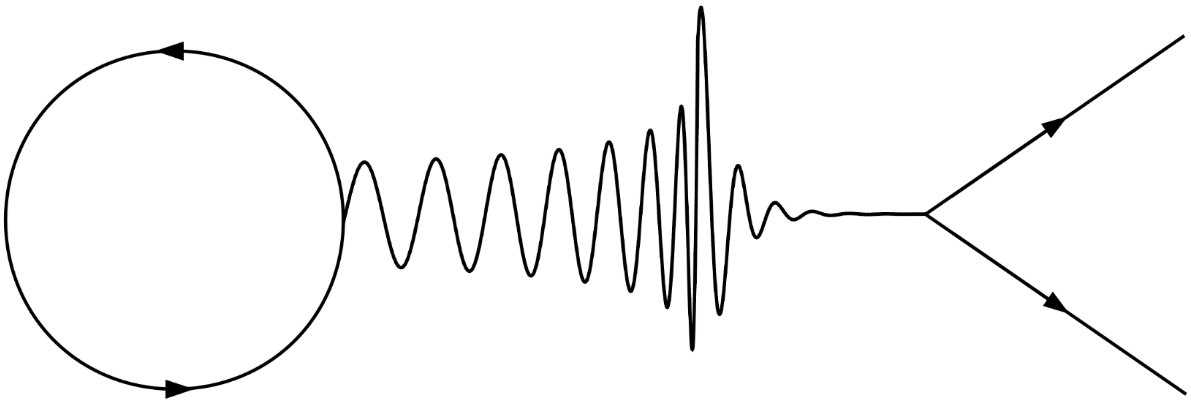
Testing General Relativity With Gravitational Waves



Vítor Cardoso

[Niels Bohr Institute & Técnico Lisbon]

Image: Cardoso & Pani CERN Courier (2016)

strong. niels bohr institute

grit. gravitation in técnico

KØBENHAVNS UNIVERSITET

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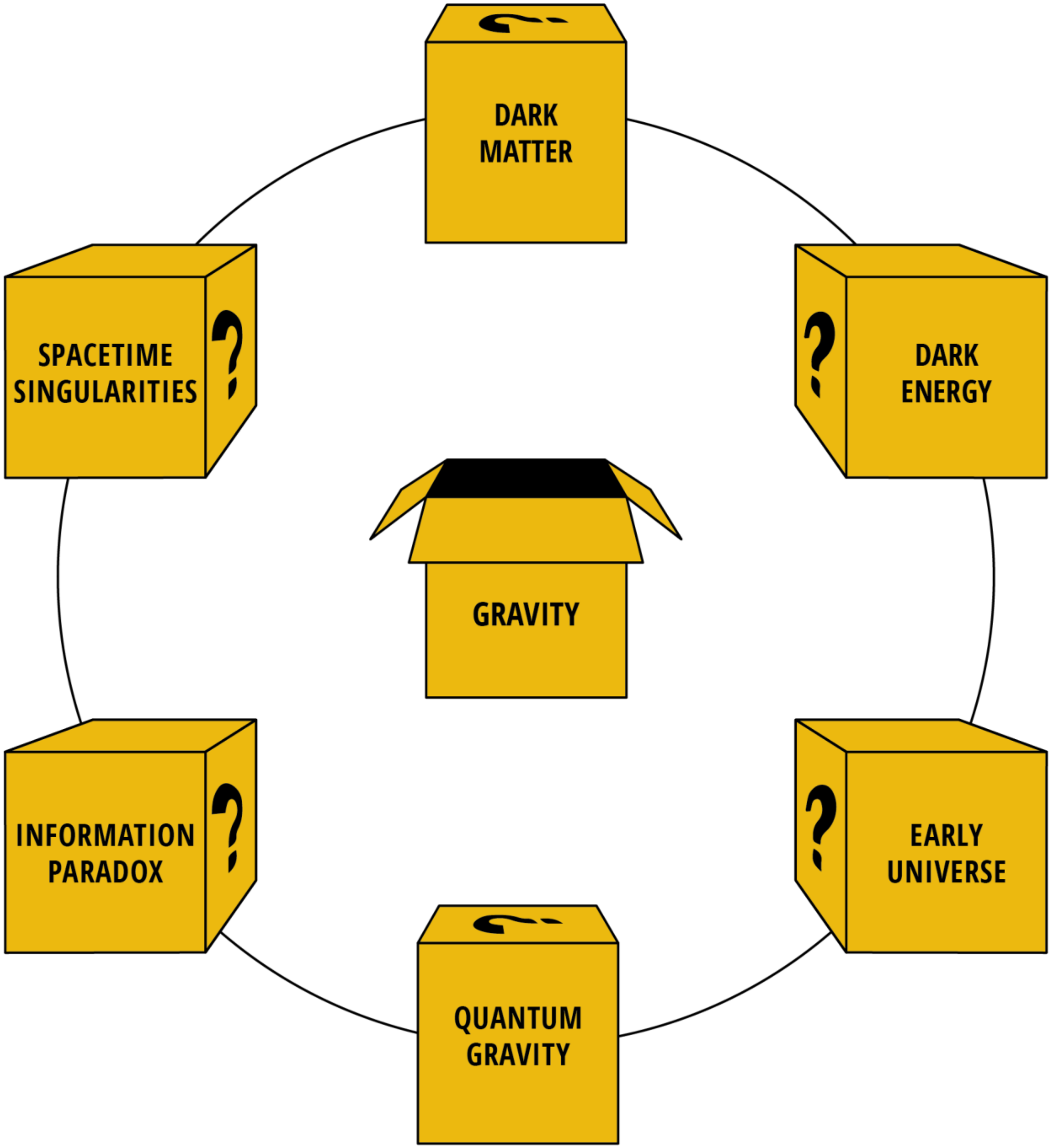

DARK MATTER
DARK ENERGY
SPACETIME SINGULARITIES
GRAVITY
INFORMATION PARADOX
EARLY UNIVERSE
QUANTUM GRAVITY

Black Holes

They hold the failure of the underlying theory

For “reasonable” matter, black hole formation results in “singularity,” where at least one of the following holds:

i. Einstein's equations are violated,
ii. the space-time manifold is incomplete *or*
iii. the concept of space-time loses its meaning at very high curvatures.

They are the simplest macroscopic objects

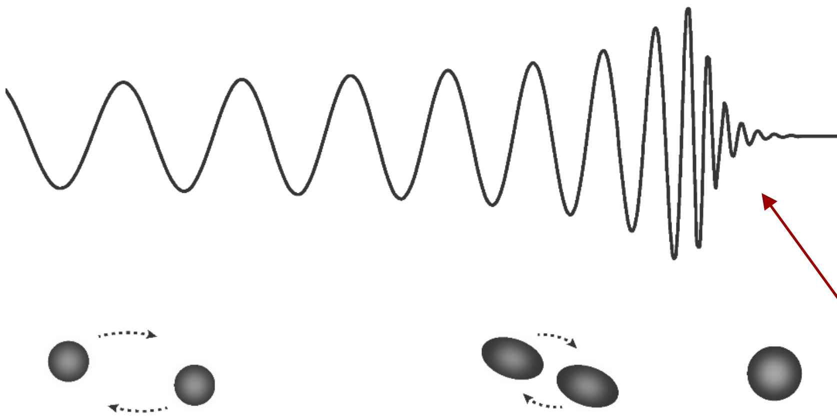
(Carter 1971; Robinson 1975; Chrusciel & Costa 2012):

Stationary, asymptotically flat, vacuum black holes belong to the Kerr family.

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

Black hole spectroscopy



Ringdown

$$\frac{\partial^2 \Psi}{\partial r_*^2} - \frac{\partial^2 \Psi}{\partial t^2} - V(r_*)\Psi = \mathrm{S} \qquad \sim e^{-i\omega t}$$

$$\left(\omega^2 - \mathcal{L}\right)\psi = s \rightarrow \psi = \left(\omega^2 - \mathcal{L}\right)^{-1} s$$

poles = QNMs

$$h = \sum_{n\ell m} A_{n\ell m} e^{-t/\tau_{n\ell m}} \sin(2\pi f_{n\ell m} t) Y_{\ell m}(\theta, \phi)$$

Leaver PRD34:384 (1986); Cardoso & Gualtieri CQG33: 174001 (2016); Baibhav + arXiv:2302.03050

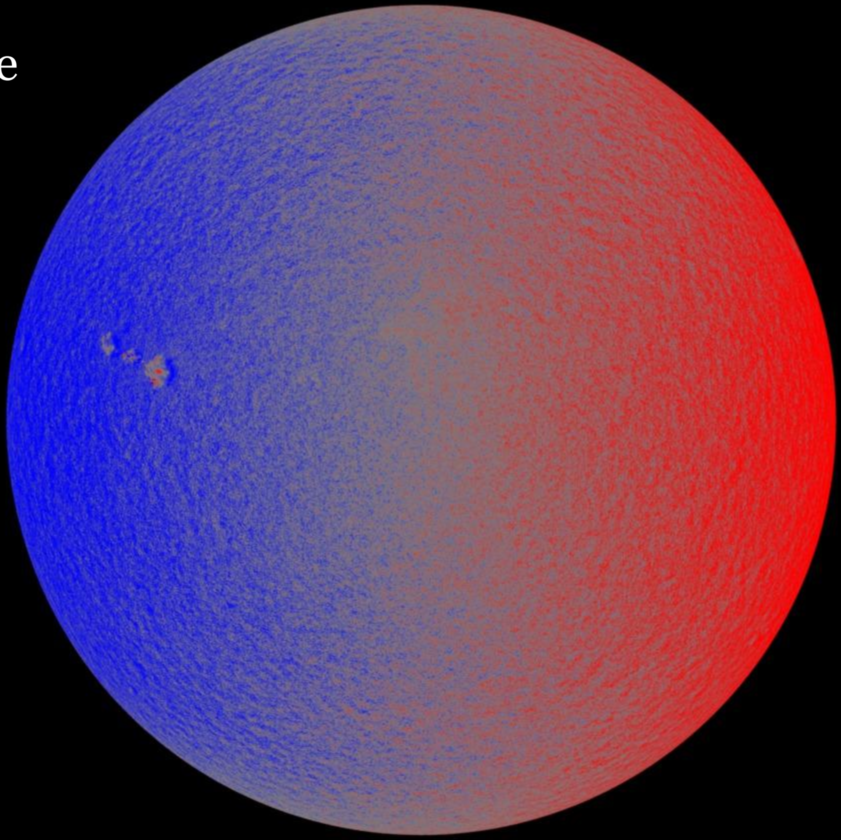
Helioseismology

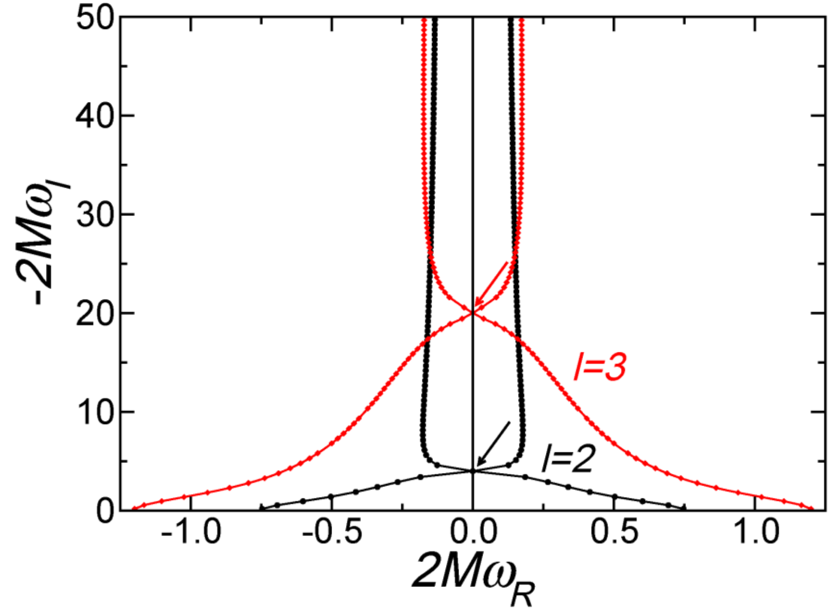
Solar spectrum sensitive to radial structure, opacity, temperature…

More than 10^6 modes with periods between 4 and 6 minutes

Quality factors of order $10^6 - 10^9$

Driven by turbulence





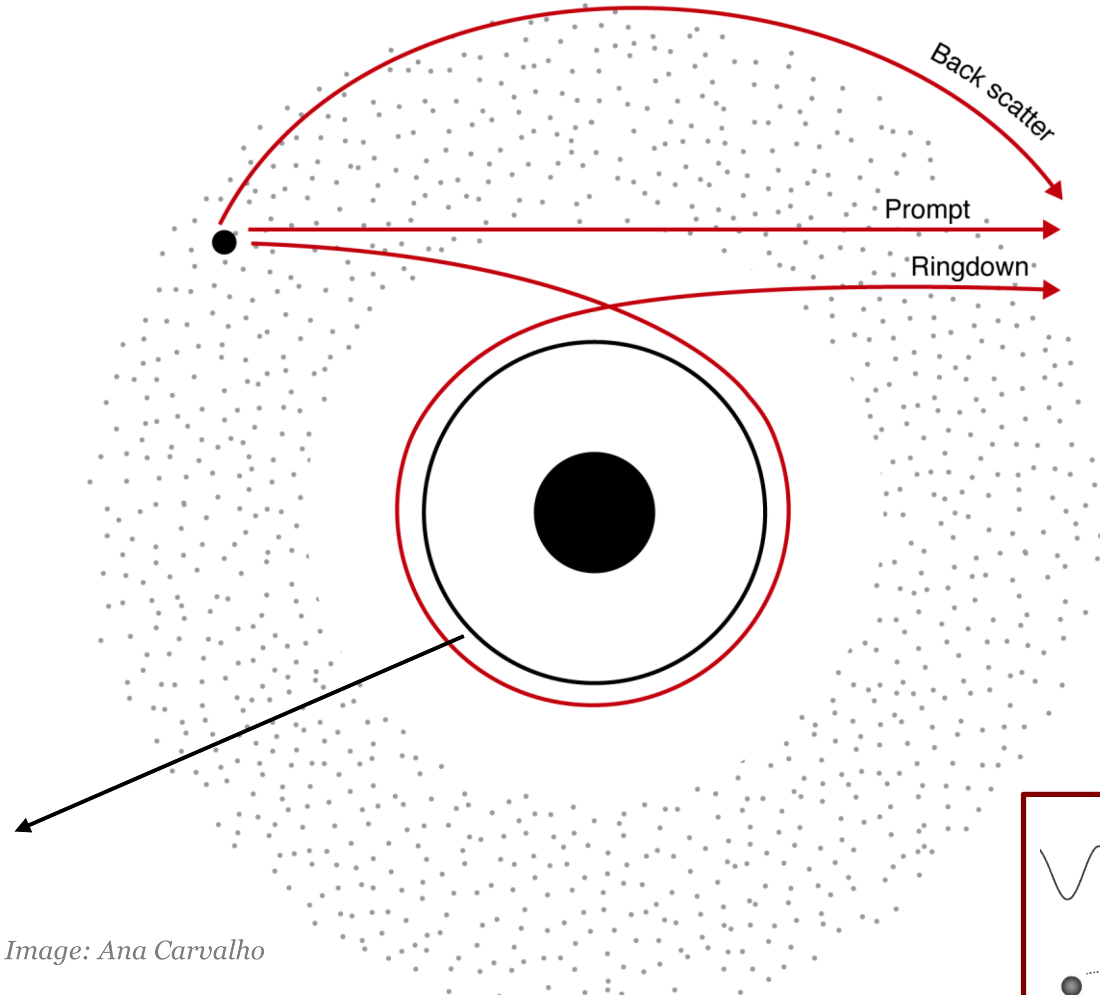
$$f = \omega_R/2\pi = 1.207 \left(\frac{10\ M_\odot}{M}\right) \mathrm{kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10\ M_\odot}\right) \mathrm{ms}$$

Quality factors of ~3. Black holes don’t ring, they relax with a thud.

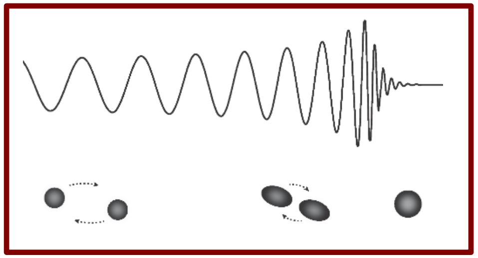
Berti, Cardoso and Starinets CQG 26: 163001 (2009); Baibhav + arXiv:2302.03050;

When is a *linear* ringdown description valid?



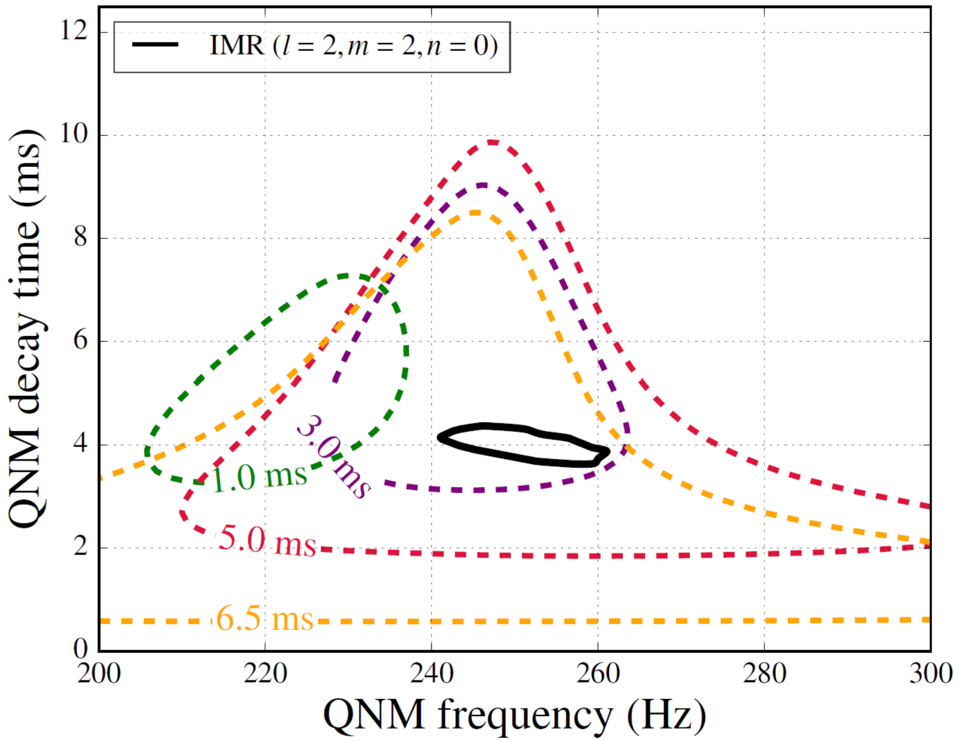
Light ring

Image: Ana Carvalho



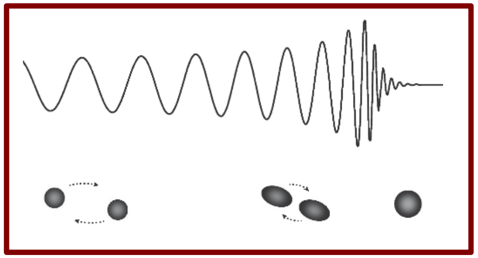
Leaver PRD34:384 (1986); Baibhav + arXiv:2302.03050; Nee+ PRD108:044032 (2023)

Let spectroscopy begin



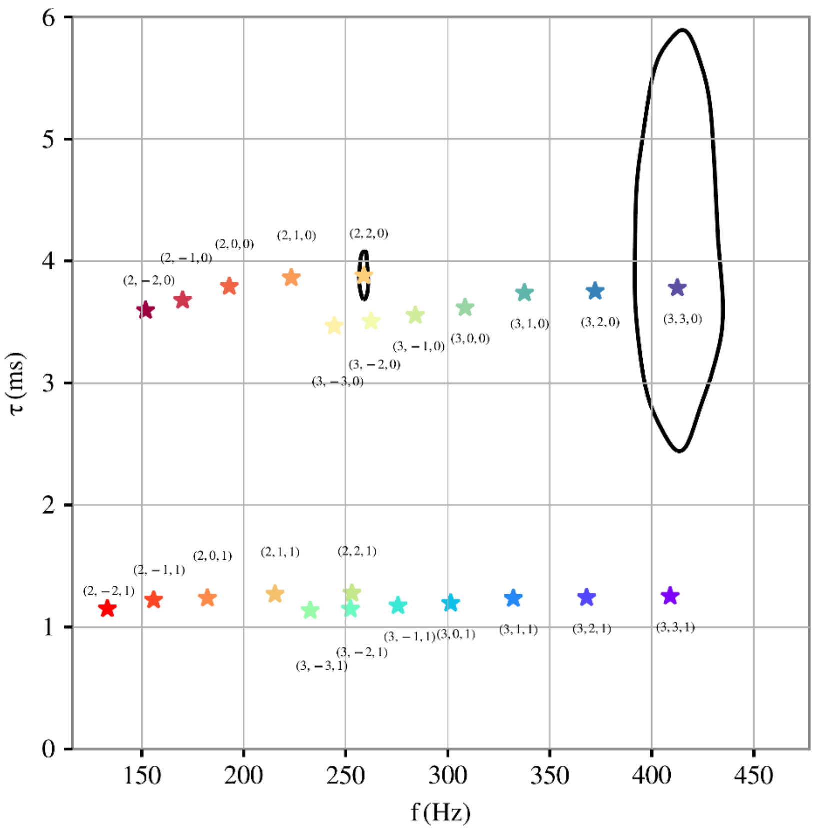
90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant



LSC PRL116:221101 (2016); arXiv:2010.14529;
For future detectors, Berti+ PRL117:10102 (2016)

Let spectroscopy begin



90% posterior distributions.

Black solid is 90% posterior of QNM from a future event with SNR=40 in ringdown.

LISA will see SNRs of thousands…

Courtesy of Gregorio Carullo
See also Berti+ PRL117:10102 (2016); Bhagwat+ arXiv:2304.02283

Nonlinearities

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$

$$R_{220\times220} = \frac{A_{220\times220}}{A_{220}^2}$$

For quasi-circular inspirals of non spinning binaries, from NR:

$$R_{220\times220} = 0.16$$

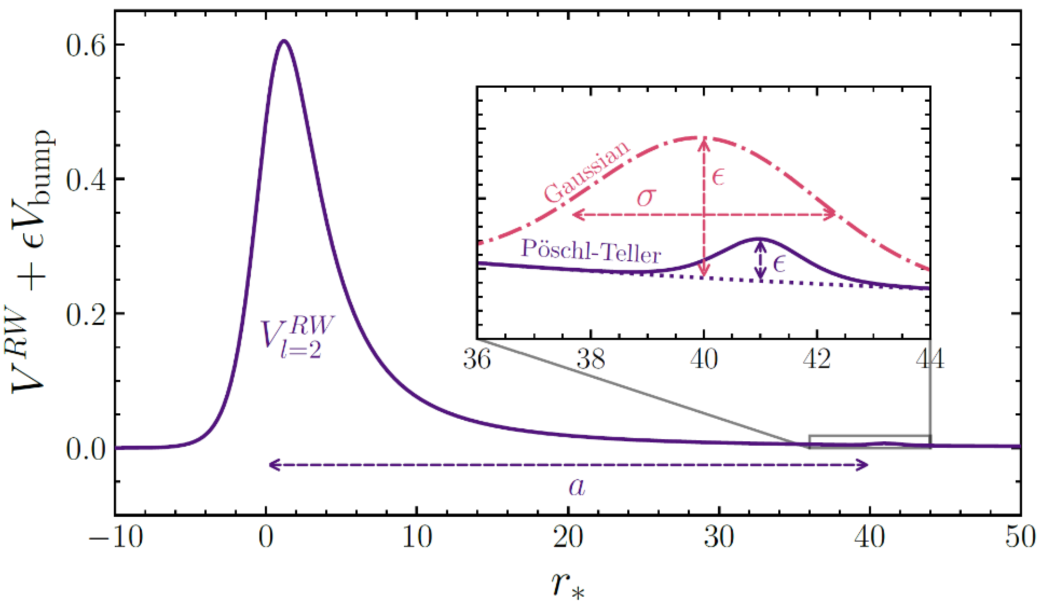
Not hopelessly small!

Similar number recovered from near-horizon symmetries ("Kerr/CFT")

Cheung+ PRL130:8 (2023); Redondo-Yuste + arXiv:2308.14796

Spectral stability: the elephant and the flea

Spectrum is unstable: Nollert gr-qc/9602032; Barausse + PRD89:104059 (2014); Jaramillo+ PRX 11: 031003 (2021); Cheung+ PRL128:111103 (2022); PRD106:084011 (2022)



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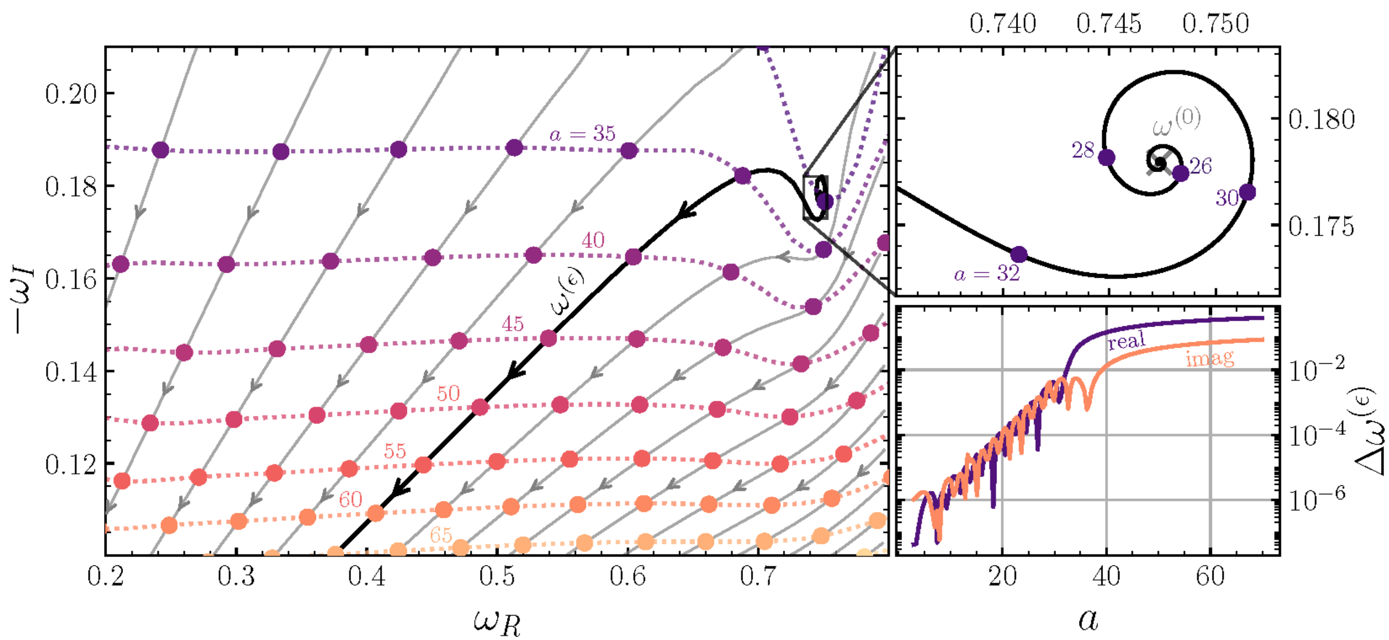

$\varepsilon = 10^{-6}$

Spectral stability: the elephant and the flea

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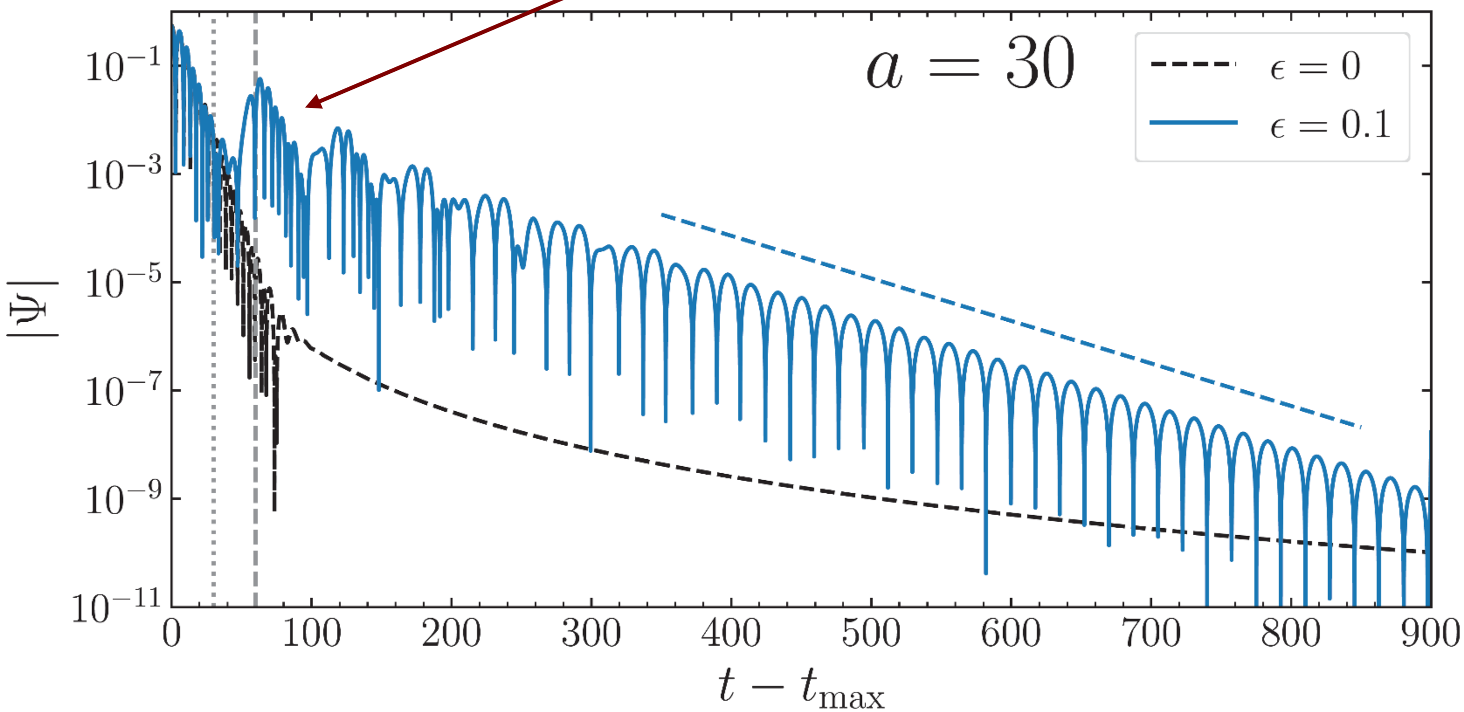
Amplitude scales like ε



Testing black hole nature

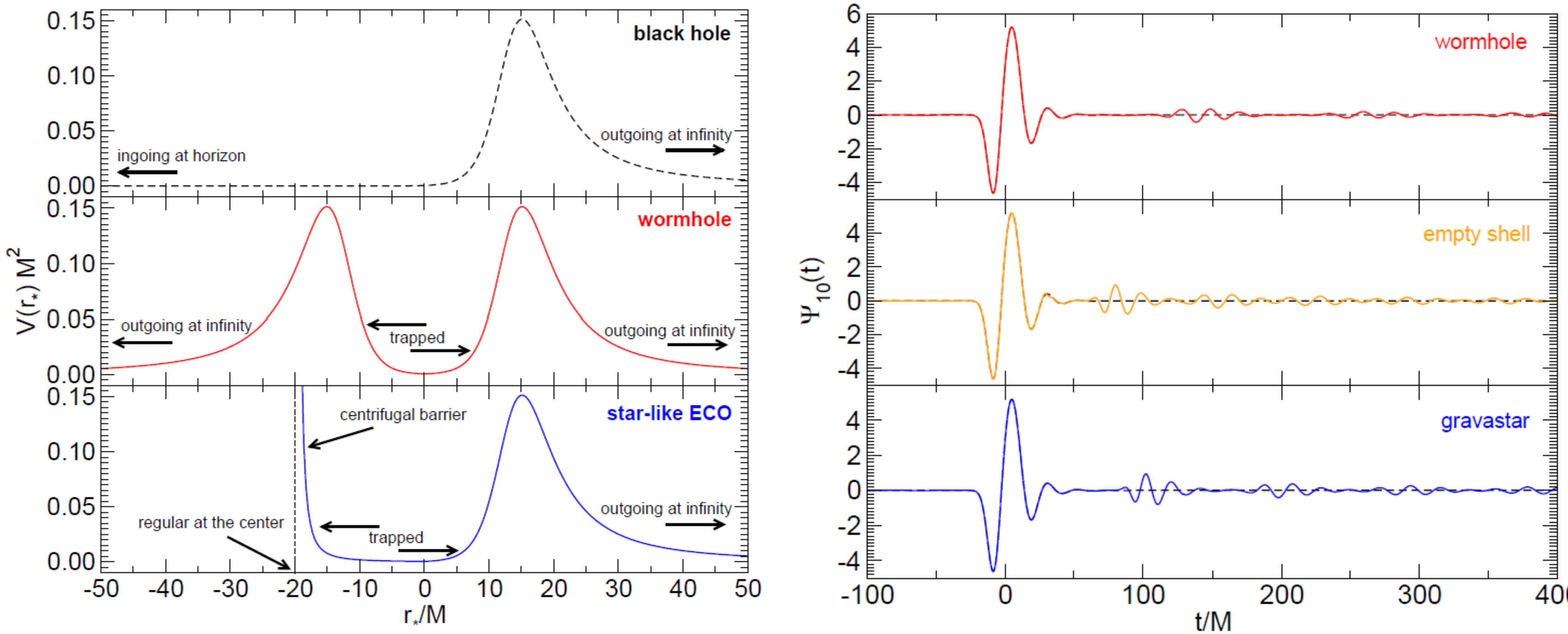
1. Black hole exterior is pathology-free, interior is not.

“Plus un fait est extraordinaire, plus il a besoin d'être appuyé de fortes preuves; car, ceux qui l'attestent pouvant ou tromper ou avoir été trompés, ces deux causes son d'autant plus probables que la réalité du fait l'est moins en elle-même….”
Laplace, Essai philosophique sur les probabilities 1812

2. Dark matter exists, and interacts gravitationally. Are there compact DM clumps?

3. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

Echoes of new physics

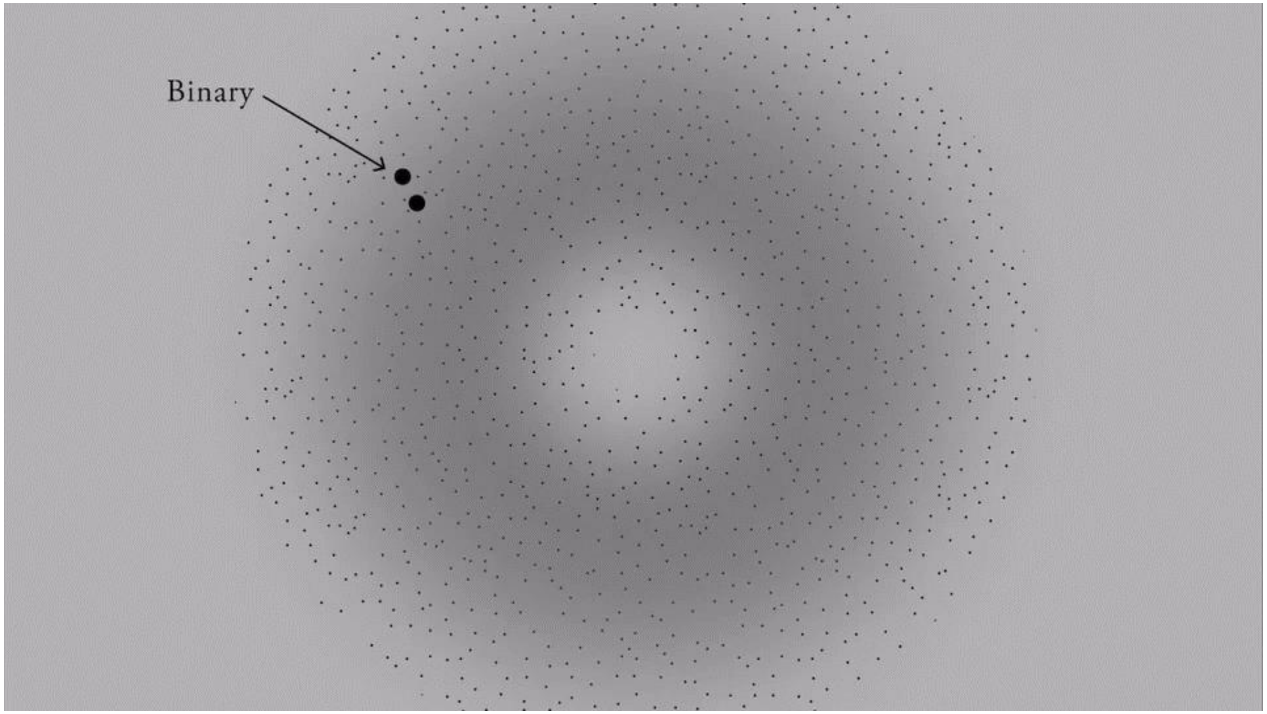


Cardoso + PRL116:171101 (2016); Nature Astronomy 1: 2017; Living Reviews in Relativity 22:1 (2019)
also Abedi+ PRD96:082004 (2017);
LIGO/Virgo Collaboration arXiv:2010:14529; arXiv:2112.06861

Environmental effects

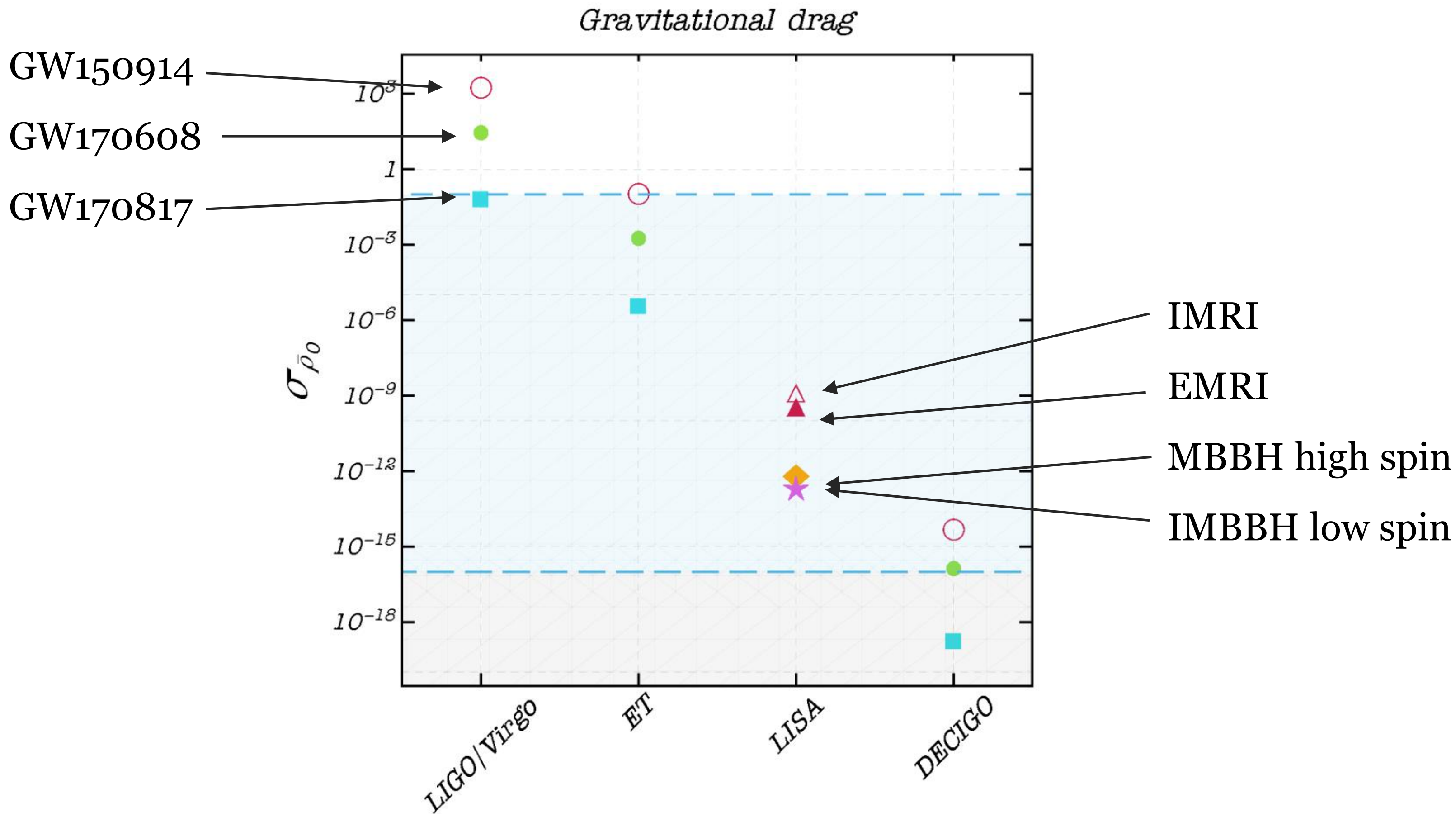
Inspiral occurs in DM-rich environment, within galaxies, and may modify the way inspiral proceeds, given dense-enough media

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Barausse+PRD89:104059 (2014); *Cardoso + AA644: A147 (2020) Kavanagh + arXiv 2002.12811; Annulli + PRD102: 063022 (2020);* Cardoso+PRDL105:104023 (2022)



Animation by Ana Carvalho

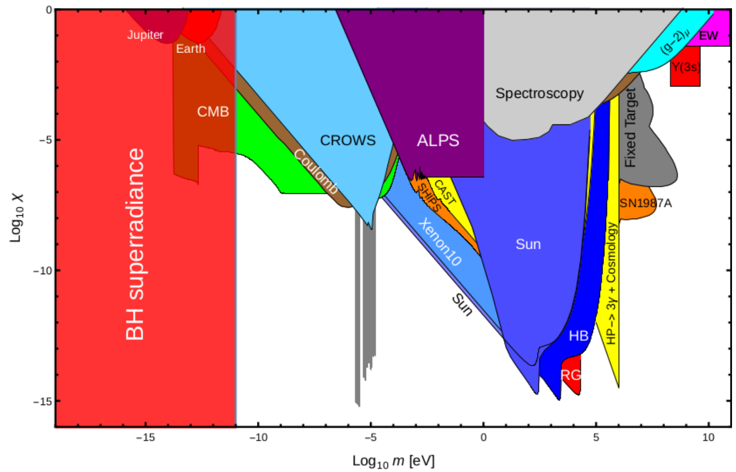
First constraints on environment



Effect is -5.5 PN on GW phase

Cardoso & Maselli AA644: A147 (2020); Santoro + arXiv:2309.05061
Also Eda + PRL 110 (2013) 221101; Macedo+ApJ774 (2013) 48; Annulli+ PRD102;063022 (2020)

Particle detectors in the sky: dark matter II



Cardoso+ 2018, adapted from Sigl (2017) and Jaeckel arXiv:1303.1821

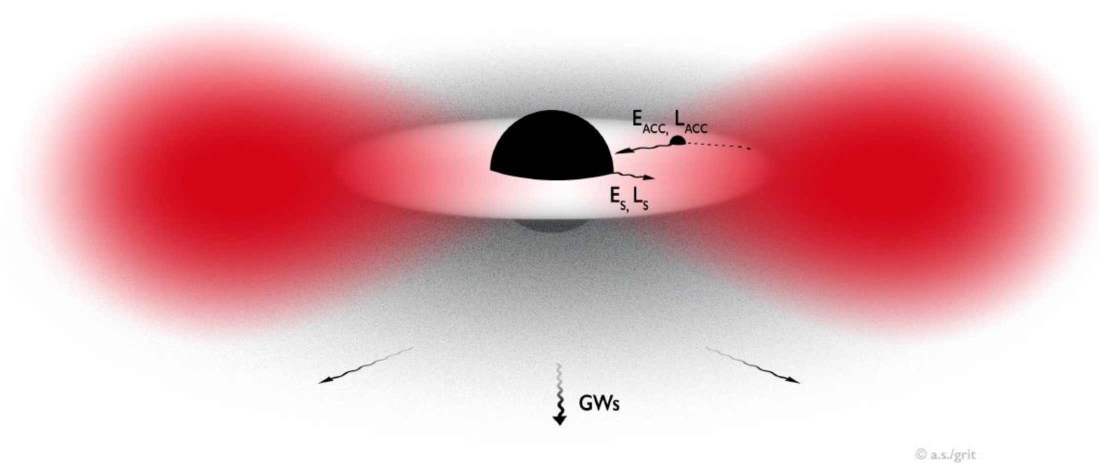
Interesting as effective description; proxy for more complex interactions; arise as interesting extensions of GR* *(BD or generic ST theories, f(R), etc)*

Bosons do exist (Higgs) and lighter versions may as well
Peccei-Quinn (interesting because not invented to solve DM problem), axiverse (moduli and coupling constants in string theory)

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - \frac{\mu_S^2}{2}\Psi\Psi - \frac{k_{\rm axion}}{2}\Psi\ {}^*F^{\mu\nu}F_{\mu\nu}$$

...and one or more could be a component of DM. *D. Marsh, Phys. Repts. 2016*

Fundamental fields: particle detectors in the sky



$$\nabla_\gamma \nabla^\gamma \Psi = \mu^2 \Psi\,, \quad \nabla_\gamma F^{\gamma\nu} = \mu^2 A^\nu\,, \quad \nabla_\gamma \nabla^\gamma h_{\mu\nu} = \mu^2 h_{\mu\nu}$$

$$\Psi \sim e^{-i\omega t} Y_{lm}$$

$$\omega \sim \mu + i(m\Omega_H - \mu)(M\mu)^{4l+5+S}$$

$$S = -s, -s+1..., s-1, s$$

$$\tau \sim 100 \left(\frac{10^6 M_\odot}{M}\right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu}\right)^9 \text{ seconds}$$

Wonderful sources of GWs

Brito, Cardoso, Pani, Lecture Notes Physics 971 (2020); also Spieksma + PRD108:063013 (2023)

Constraints on fundamental fields via superradiance

	excluded region (in eV)	source
*	$5.2\times10^{-13} < m_S < 6.5\times10^{-12}$	Direct bounds from absence of spin down in Cyg X-1.
*	$1.1\times10^{-13} < m_V < 8.2\times10^{-12}$	
*	$2.9\times10^{-13} < m_T < 9.8\times10^{-12}$	
	$6\times10^{-13} < m_S < 2\times10^{-11}$	Indirect bounds from BH mass-spin measurements.
	$7\times10^{-20} < m_S < 1\times10^{-16}$	
*	$2\times10^{-14} < m_V < 1\times10^{-11}$	
*	$1\times10^{-20} < m_V < 9\times10^{-17}$	
*	$6\times10^{-14} < m_T < 1\times10^{-11}$	
*	$3\times10^{-20} < m_T < 9\times10^{-17}$	
	$1.2\times10^{-13} < m_S < 1.8\times10^{-13}$	Null results from blind all-sky searches for continuous GW signals.
	$2.0\times10^{-13} < m_S < 2.5\times10^{-12}$	
	m_V: NA	
	m_T: NA	
	$5.8\times10^{-13} < m_S < 8.6\times10^{-13}$	Null results from searches for continuous GW signals from Cygnus X-1.
	m_V: NA	
	m_T: NA	
	$2.0\times10^{-13} < m_S < 3.8\times10^{-13}$	Negative searches for a GW background.
	m_V: NA	
	m_T: NA	
	$5\times10^{-13} < m_S < 3\times10^{-12}$	Bounds from pulsar timing.
	$m_V \sim 10^{-12}$	
	m_T: NA	
	$2.9\times10^{-21} < m_S < 4.6\times10^{-21}$	Bounds from mass and spin measurement of M87 with EHT.
	$8.5\times10^{-22} < m_V < 4.6\times10^{-21}$	
*	$1.0\times10^{-21} < m_T < 8.2\times10^{-21}$	

Brito, Cardoso & Pani, Superradiance, Lecture Notes Physics 971 (2020)

The next decimal place

“New realms will become visible by improving the accuracy of the numerical measurement of quantities with which one has long been familiar.“ [Maxwell, Scientific papers]

Test black hole & Kerr paradigm with precision & understand environments

Cardoso, Nature Reviews Physics (2019)
Cardoso and Pani, Living Reviews in Relativity 22: 1 (2019)

NEW HORIZONS FOR PSI

BLACK HOLES AND
FUNDAMENTAL FIELDS
SCHOOL & WORKSHOP
LISBON, 1-5 JULY 2024

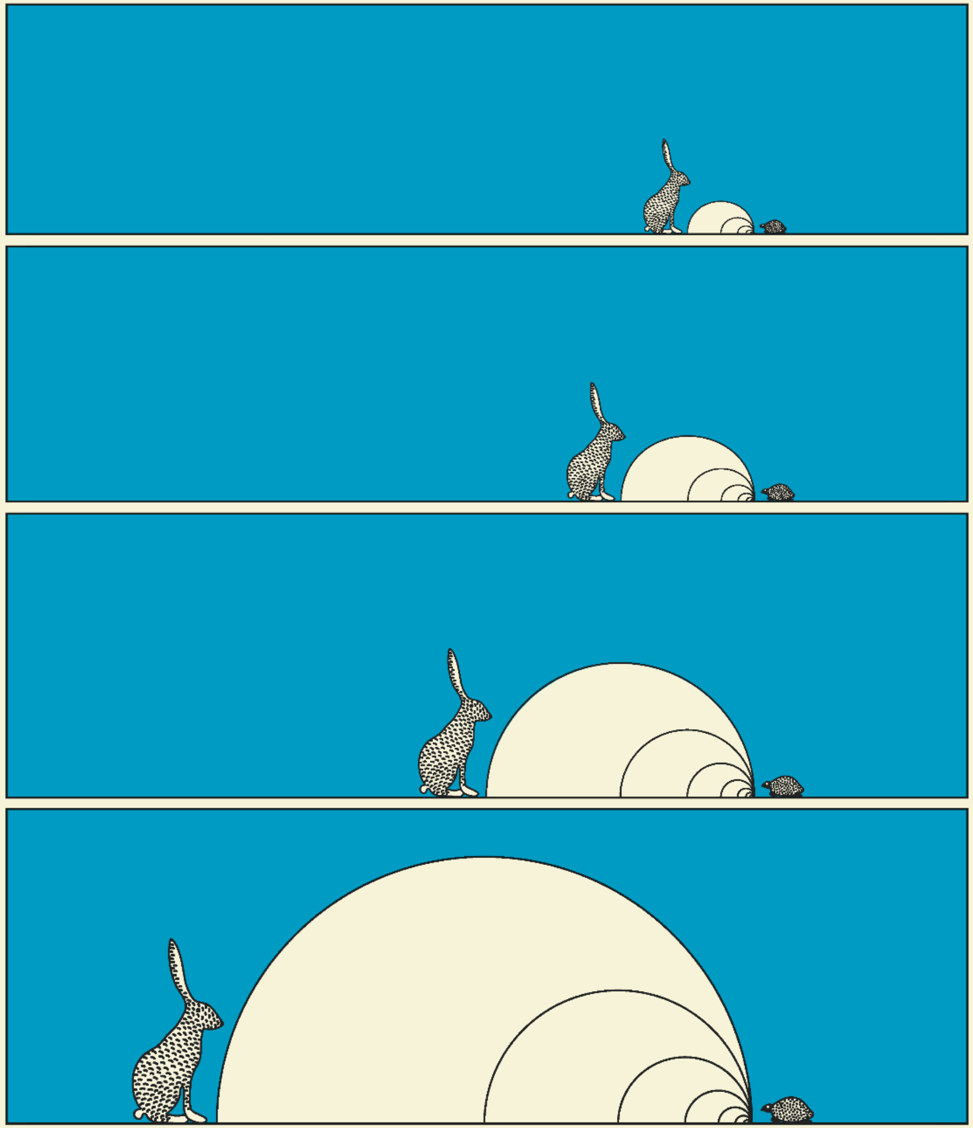


ILLUSTRATION & DESIGN: ANA CARVALHO

ORGANIZING COMMITTEE: RICHARD BRITO, ENRICO CANNIZZARO, VITOR CARDOSO, YIFAN CHEN, PAOLO PANI, THOMAS SPIEKSMA, XIAO XUE, RITA SOUSA. THE "NEW HORIZONS FOR PSI" SCHOOL HAS THE SUPPORT OF CENTRA, THE EUROPEAN RESEARCH COUNCIL AND THE VILLUM FONDEN.

BLACK HOLES INSIDE AND OUT

NIELS BOHR INSTITUTE

COPENHAGEN

26–30 AUGUST 2024

SOC: Luca Buoninfante, Vitor Cardoso, Astrid Eichorn, Raúl Carballo-Rubio, Francesco Di Filippo. LOC: Vitor Cardoso, Yifan Chen, Julie de Molade, Jose Ezquiaga, Takuya Katagiri, David Pereñiguez, Maarten van de Meent. This meeting is funded by the VILLUM FONDEN, the European Research Council and the Danish National Research Foundation.

Design by Ana Carvalho

Thank you

Mass for the graviton?

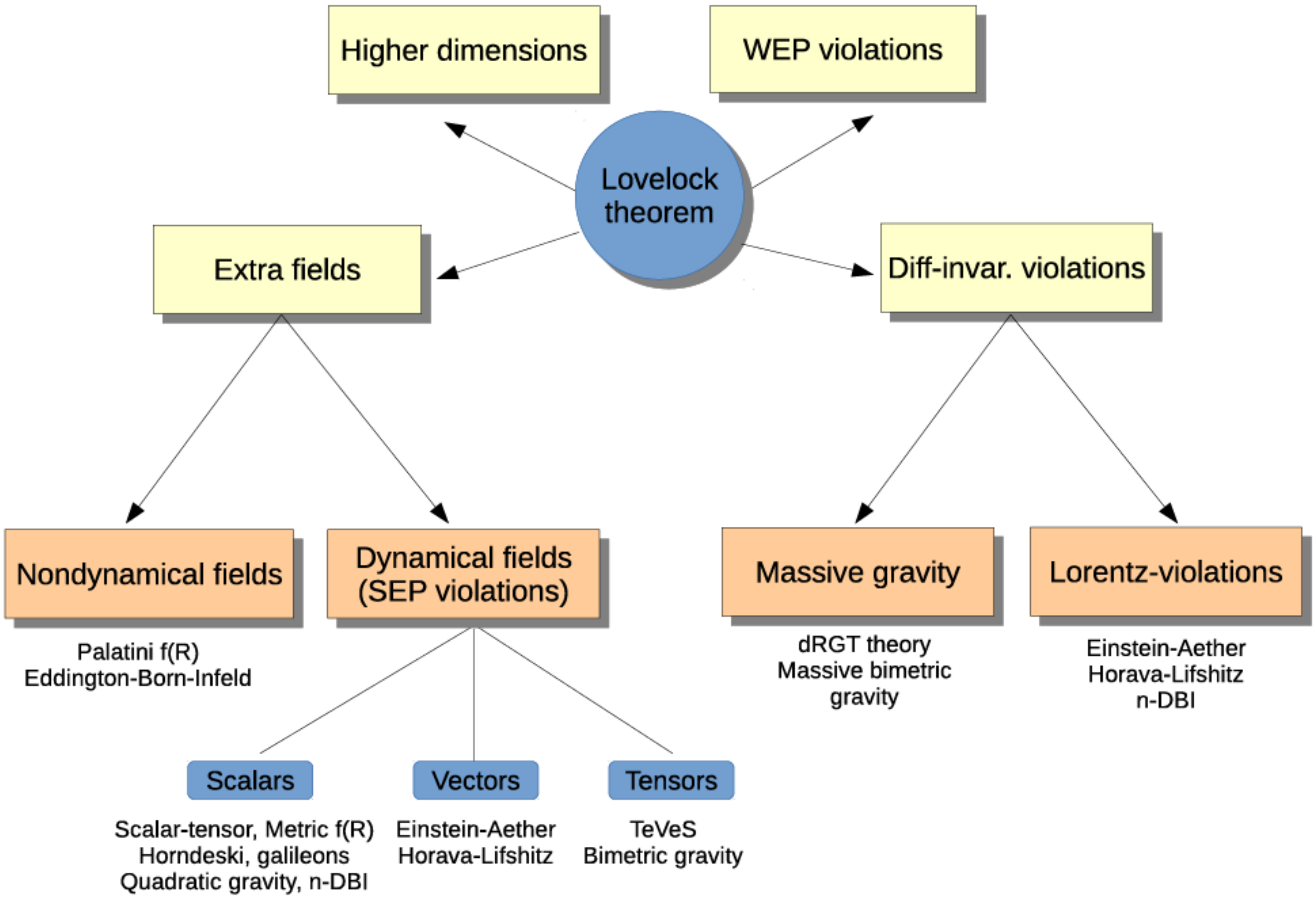
General Relativity is a beautiful theory, but cannot be final word

Want to test it, but limited by "uniqueness" property:

In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term (Lovelock 1971)

Must break one of the underlying assumptions

Mass for the graviton?



Berti + arXiv:1501.07274

Inspiralling compact objects

$$\text{Binding Energy}: E_b = -\frac{GM\mu}{2L} + \text{other interactions}$$

$$\text{Quadrupole emission}: \dot{E} = -\frac{32}{5}\frac{G\mu^2 L^4 \Omega^6}{c^5} + \text{other emission channels}$$

$$h(f, \text{pars}) = A(f, \text{pars}) e^{i\Psi(f,\text{pars})}$$

$$\Psi = \frac{3}{128}\left(G\mathcal{M}\pi f/c^3\right)^{-5/3}\left(... + \alpha_{-4PN} x^{-4} + ... + \alpha_{-1PN} x^{-1} + 1 + \alpha_{1PN} x + ...\right)$$

New physics or extra matter

$$x = (\pi M f)^{2/3}, \quad M = m_1 + m_2, \quad \nu = m_1 m_2 / M^2, \quad \mathcal{M} = \nu^{3/5} M$$

Variation of G

Dipole moment (electric charge...)

M. Maggiore, Gravitational waves, Volume I
N. Yunes, K. Yagi & F. Pretorius, PRD94:084002 (2016)

Parametrized tests

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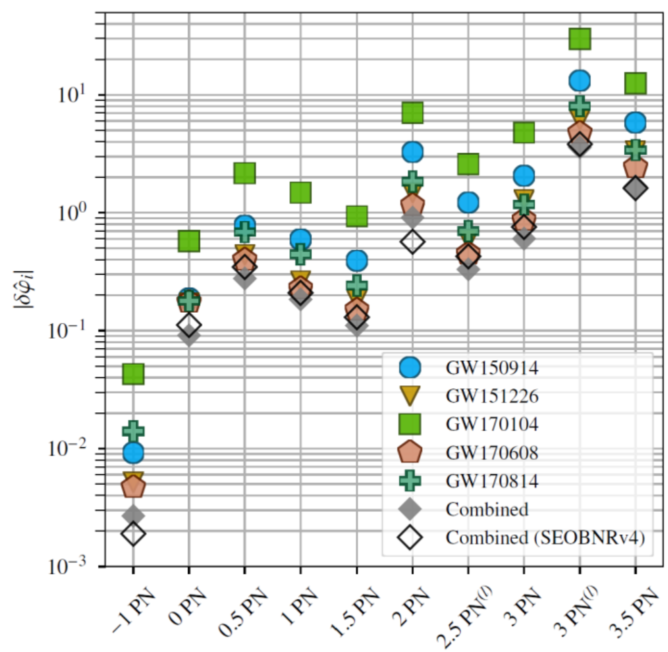

LVC arXiv:1903.04467

Dispersion tests of massive fields

$$\Box\varphi = \mu^2\varphi$$

$$\varphi \sim \exp\left(i\sqrt{\omega^2 - \mu^2}\, r - i\omega t\right)$$

$$v^2 = 1 - \frac{\mu^2}{\omega^2}$$

Group velocity depends on frequency,
phasing gets extra piece, at +1 PN order

C. M. Will, arXiv:9709011

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Variation of G

Dipole moment (electric charge...)

Graviton mass

M. Maggiore, Gravitational waves, Volume I

N. Yunes, K. Yagi & F. Pretorius, PRD94:084002 (2016)

Dispersion tests

Assume GR for emission

Assume GR for background

Assume two massive polarizations

Fluctuations of massive gravity

$$G^{(1)}_{\mu\nu}[h] + \frac{\mu^2}{2}\left(h_{\mu\nu} - h g_{\mu\nu}\right) = 8\pi T_{\mu\nu}$$

Unique equation of motion for spin-2 field in Ricci-flat background. Can think of this as dark new tensor field, or as massive graviton, all depends on couplings to matter

Caveat 1: Can depart from above, breaking Lorentz symmetry, allowing for additional dynamical metrics, etc

Caveat 2: higher order terms (in h) were dropped, their precise form depends on nonlinear theory. Some of these terms may contain inverse powers of graviton mass, such that convergence at small mass is an issue

See also Brito + arXiv:1304.6725 and Porrati arXiv:0011152

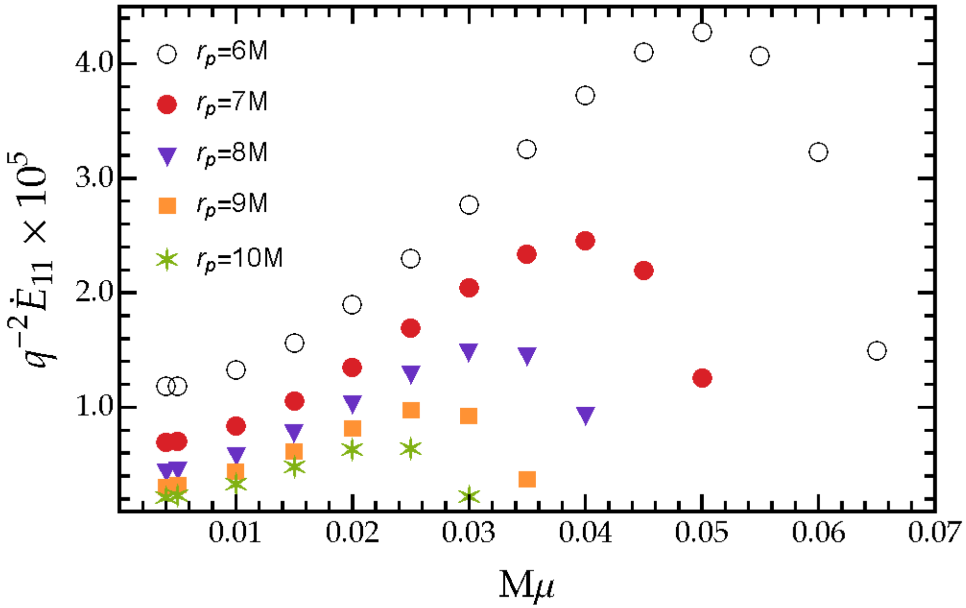
Dipolar emission

$$(\Box - \mathbf{V}) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \Sigma_X \\ \Sigma_Y \\ \Sigma_Z \end{pmatrix}$$

$$\dot{E} = \dot{E}_{\mathrm{N}} \left(1 + B \frac{r_p}{M}\right)$$

$$B = 2 \times 10^{-3} \Theta(\Omega - \mu)$$

Dipolar emission

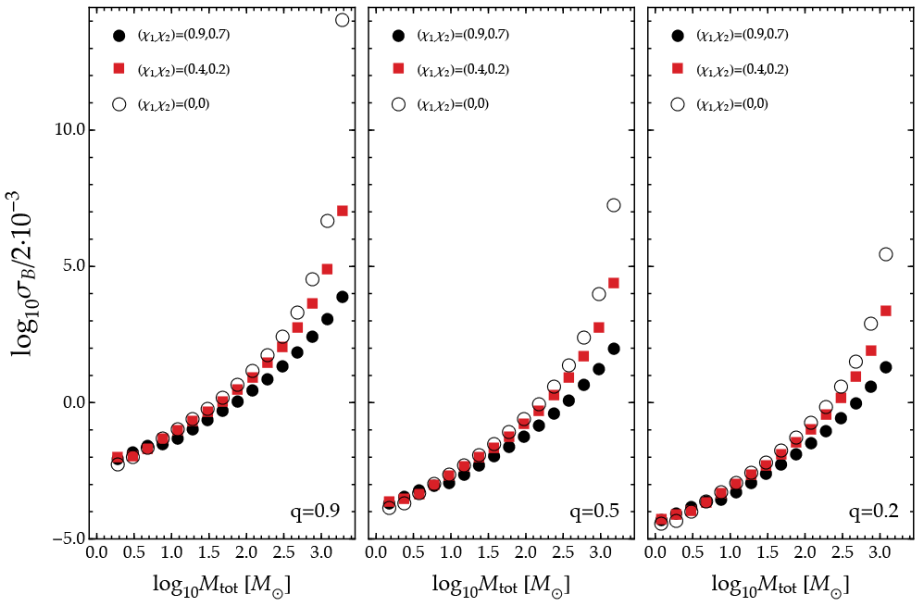


Only ways out:

A. Make graviton mass too large, therefore bracket mass: exclude theory

B. Make nonlinear terms important

Dipolar emission: extrapolation



1-σ uncertainty on the dipole parameter B inferred by ET observations of BH binaries with total detector-frame mass Mtot, at d = 500Mpc from the detector, assuming average orientation. Left, centre and right panels show results for mass ratio q = 0.9, 0.5 and 0.2, while colored dots represent binaries with different spin χ1,2 configurations. For all calculations we assumed two aligned L-shaped Einstein Telescope detectors in their ET-D configuration

Massive gravity

Extended theories of gravity are fascinating mathematically and provide interesting arenas to test GR

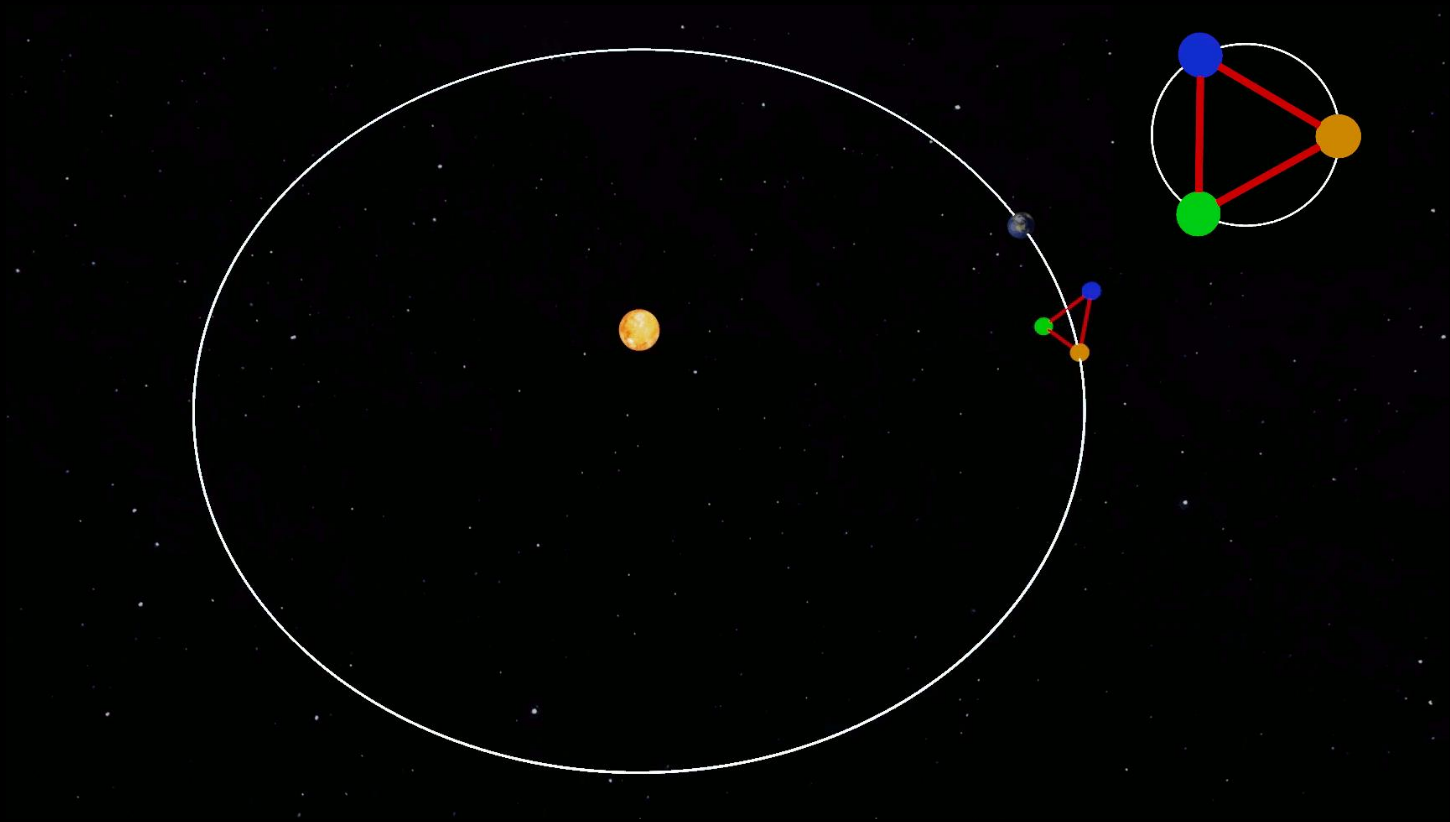
Massive gravity adds another large lengthscale to problem

Naïve, parametrized tests based on dispersion relation miss a substantial amount of the content of the theory

Hard work is necessary: calculate all degrees of freedom and dynamics of theory

Thank you

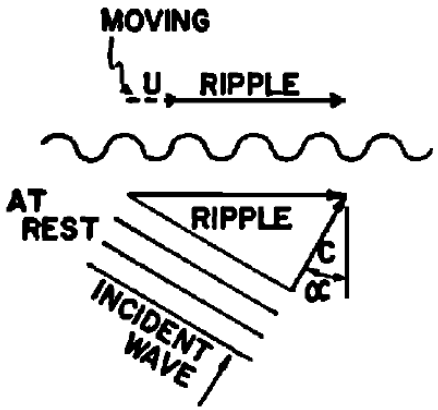
Observing from space with LISA
(Laser Interferometer Space Antenna)



The most amazing space mission ever conceived!

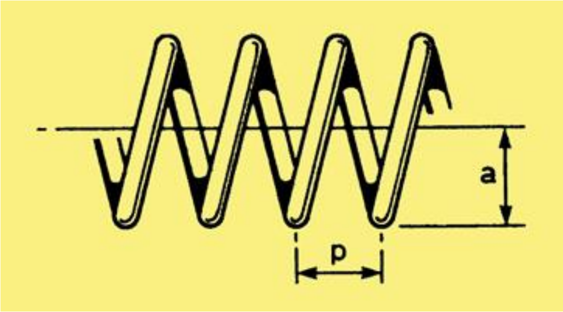
Superradiance

Zel'dovich JETP Lett. 14:180 (1971)
review of all known results in Brito+ Lect. Notes Phys. 971 (2020)

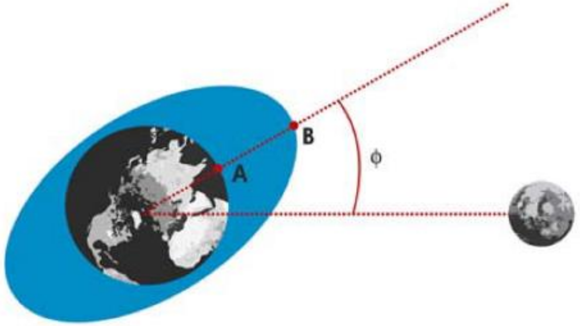


Ribner, J. Acous. Soc. Amer. 29 (1957)

Tamm & Frank, Doklady AN SSSR 14 (1937)



Pierce (& Kompfner), Bell Lab Series (1947)
Ginzburg, anomalous Doppler year



G. H. Darwin, Philos. Trans.
R. Soc. London 171 (1880)

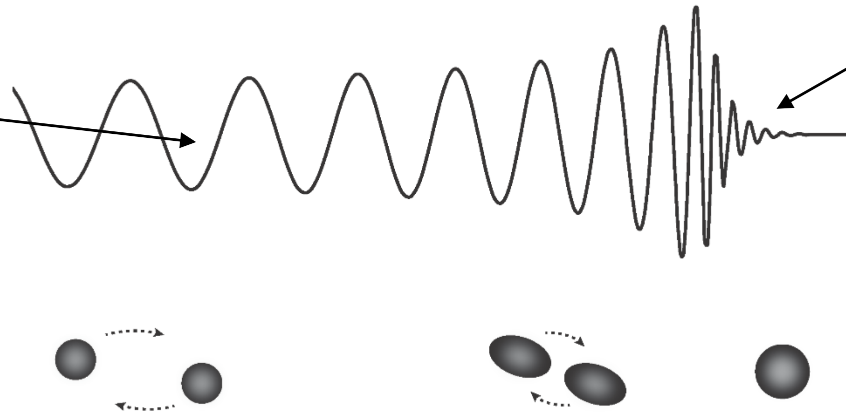
Black holes as perfect laboratories

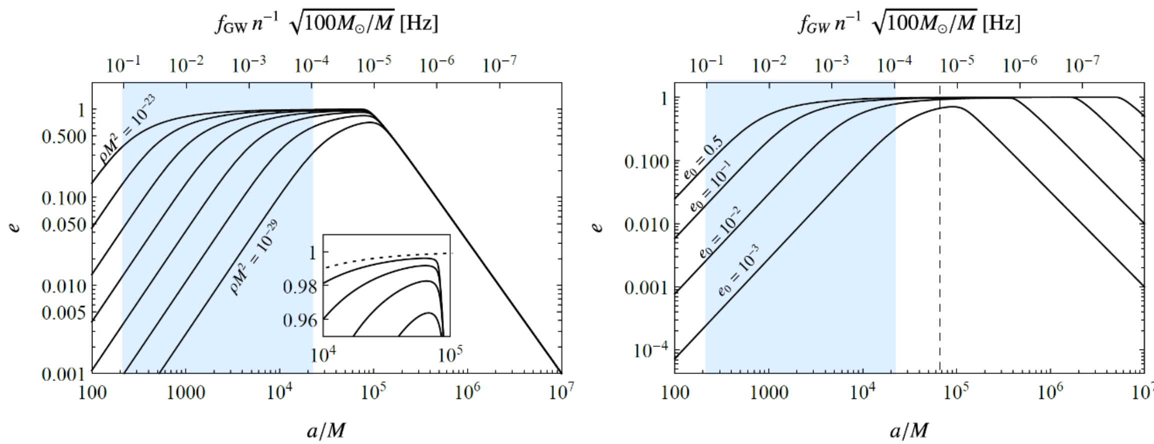
Simple spin-induced multipoles (uniqueness)

Tidal heating

Tidal Love number (BHs don't polarize)

Simple relaxation of final state (uniqueness)

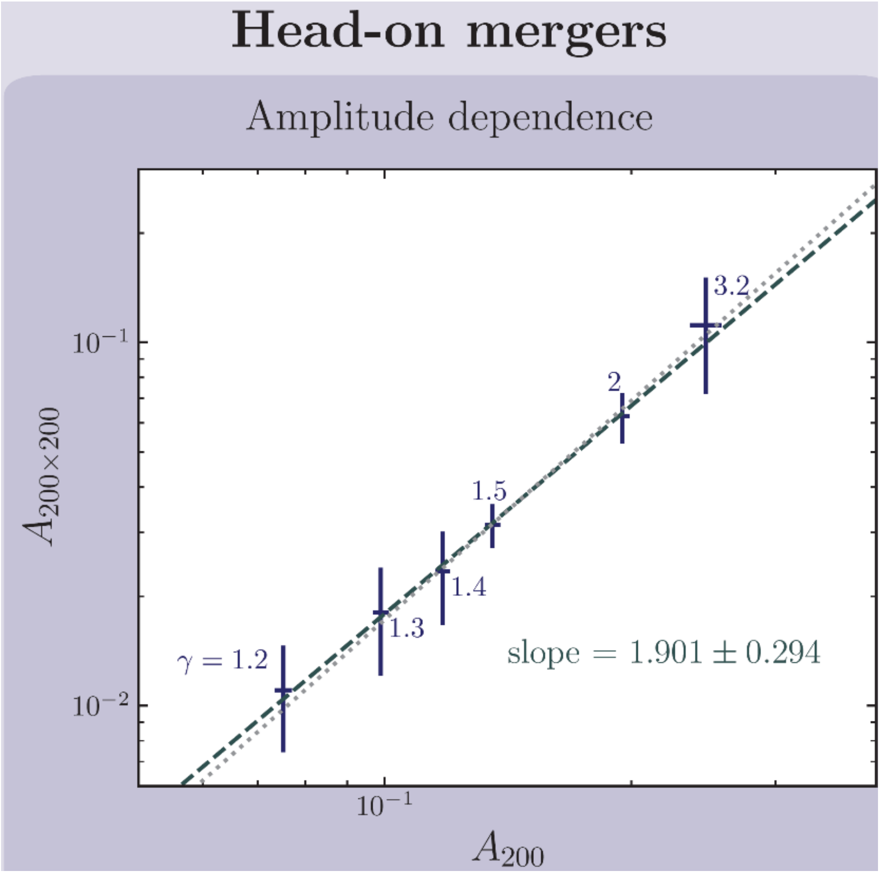




Peters PR136: B1224 (1964);
Cardoso & Pani, Nature Astronomy 1: 586 (2017); Living Reviews in Relativity 22: 1 (2019)
Cardoso, Macedo & Vicente PRD103: 023015 (2021)

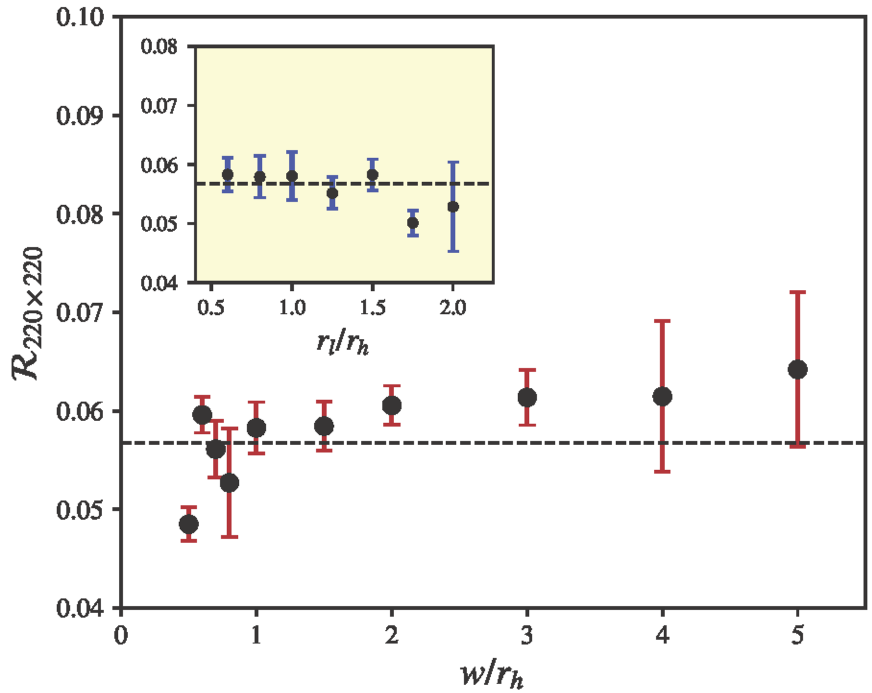
Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$



Cheung + PRL130:8 (2023)

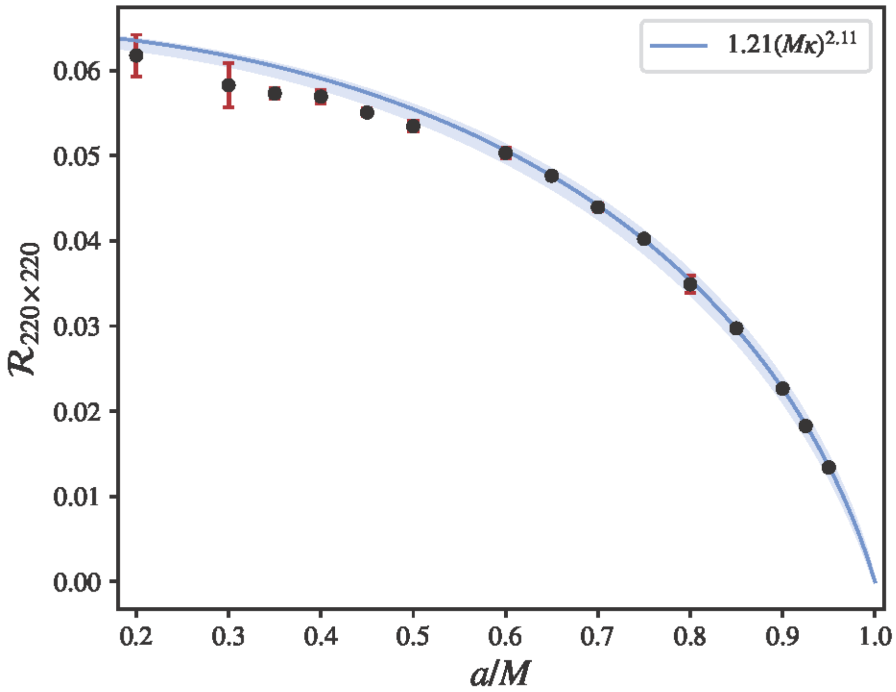
Nonlinearities in ringdown: second-order



Second order amplitude only weakly dependent on ID

Redondo-Yuste + arXiv:2308.14796

Nonlinearities in ringdown: second-order



But dependent on spin...raising doubts about Kerr/CFT applications

Redondo-Yuste + arXiv:2308.14796

Wonderful sources for different detectors

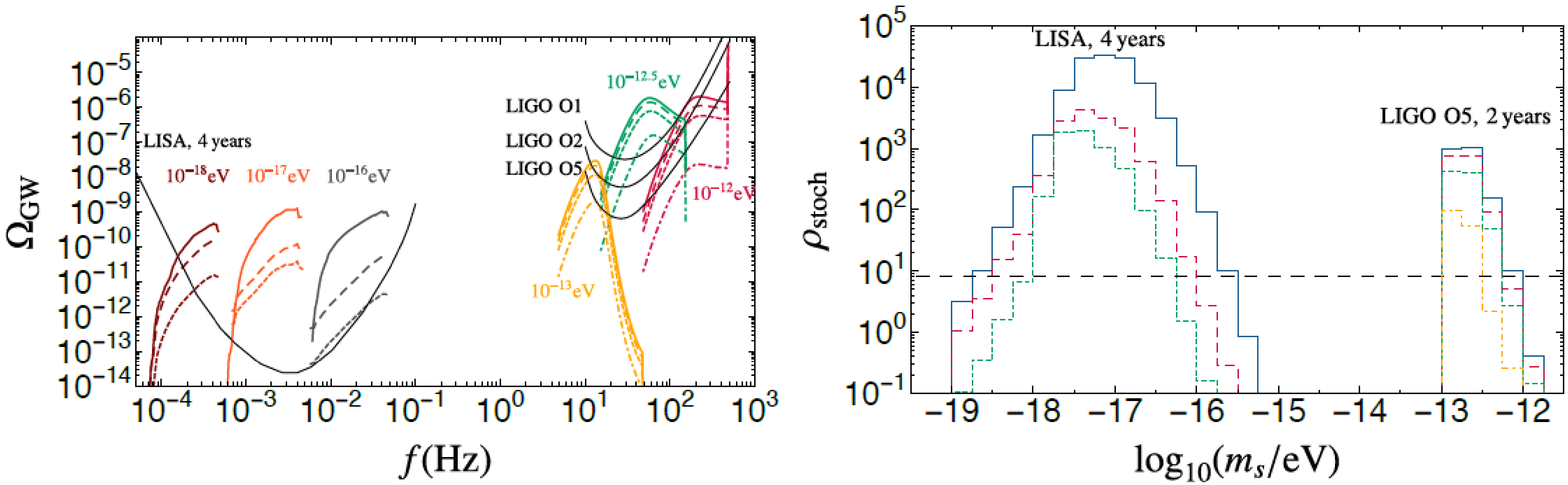
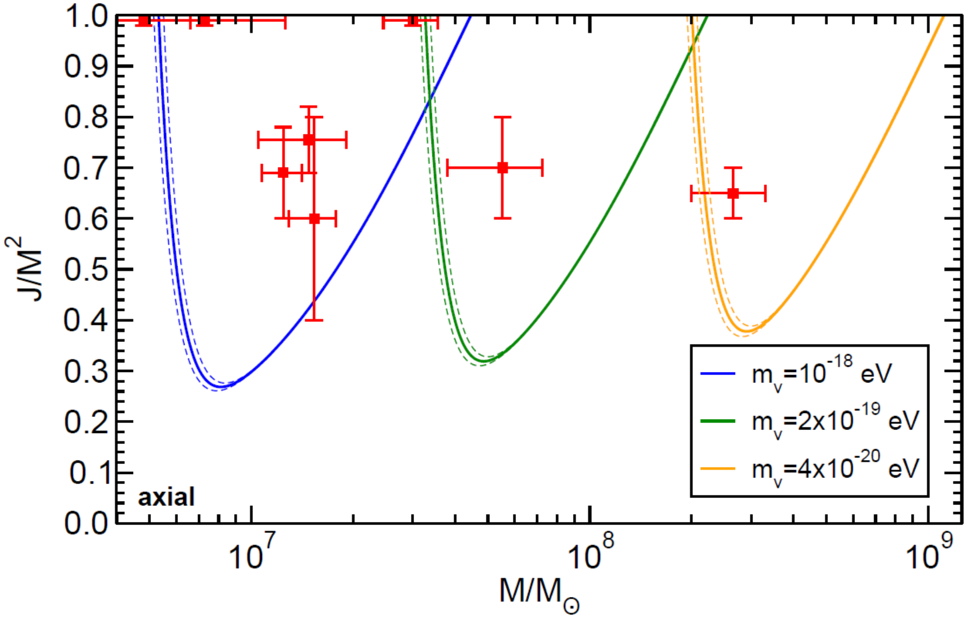


FIG. 2. Left panel: stochastic background in the LIGO and LISA bands. For LISA, the three different signals correspond to the "optimistic" (top), "less optimistic" (middle) and "pessimistic" (bottom) astrophysical models. For LIGO, the different spectra for each scalar field mass correspond to a uniform spin distribution with (from top to bottom) $\chi_i \in [0.8, 1], [0.5, 1], [0, 1]$ and $[0, 0.5]$. The black lines are the power-law integrated curves of Ref. [61], computed using noise PSDs for LISA [9], LIGO's first two observing runs (O1 and O2), and LIGO at design sensitivity (O5) [62]. By definition, $\rho_{\rm stoch} \geq 1$ when a power-law spectrum intersects one of the power-law integrated curves. Right panel: $\rho_{\rm stoch}$ for the backgrounds shown in the left panel. We assumed $T_{\rm obs} = 2$ yr for LIGO and $T_{\rm obs} = 4$ yr for LISA.

Brito + PRL119: 131101 (2017); Tsukada + arXiv: 2011.06995; Yuan + arXiv:2106.00021

Bounding the boson mass with EM observations

Pani + PRL109, 131102 (2012)



Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important... but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23}\,\mathrm{eV}$$

Brito + PRD88:023514 (2013); Review of Particle Physics 2014

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting.

Black holes remain the most outstanding object in the universe. Black-hole spectroscopy will allow to test GR and provide strong evidence for the presence of horizons...Black holes can play the role of perfect laboratories for particle physics, or high energy physics.

"But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future."

S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 Sept. 1986

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting. Black holes respond in simple way to external perturbations, and may serve as detectors for nontrivial environments.

Black holes remain the most outstanding object in the universe.
Spectroscopy will allow to test GR and provide strong evidence for the presence of horizons… improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators.

Singularities & Cosmic Censorship

Theorem (Penrose 1965; 1969):

For "reasonable" matter, trapped surface formation results in "singularity," where at least one of the following holds:

a. Negative local energy occurs.
b. Einstein's equations are violated.
c. The space-time manifold is incomplete.
d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)
General Relativity is deterministic (strong CCC)

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel, Costa & Heusler 2012):
A stationary, asymptotically flat, *vacuum* BH solution must be Kerr

$$\begin{aligned} ds^2 &= \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta)\sin^2\theta}{\Sigma} dt d\phi \\ &- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \end{aligned}$$

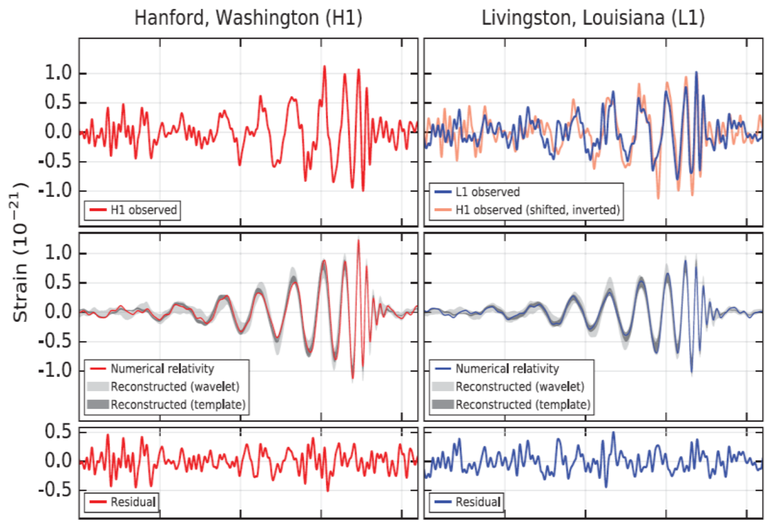
$$\Sigma = r^2 + a^2 \cos^2\theta\,, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

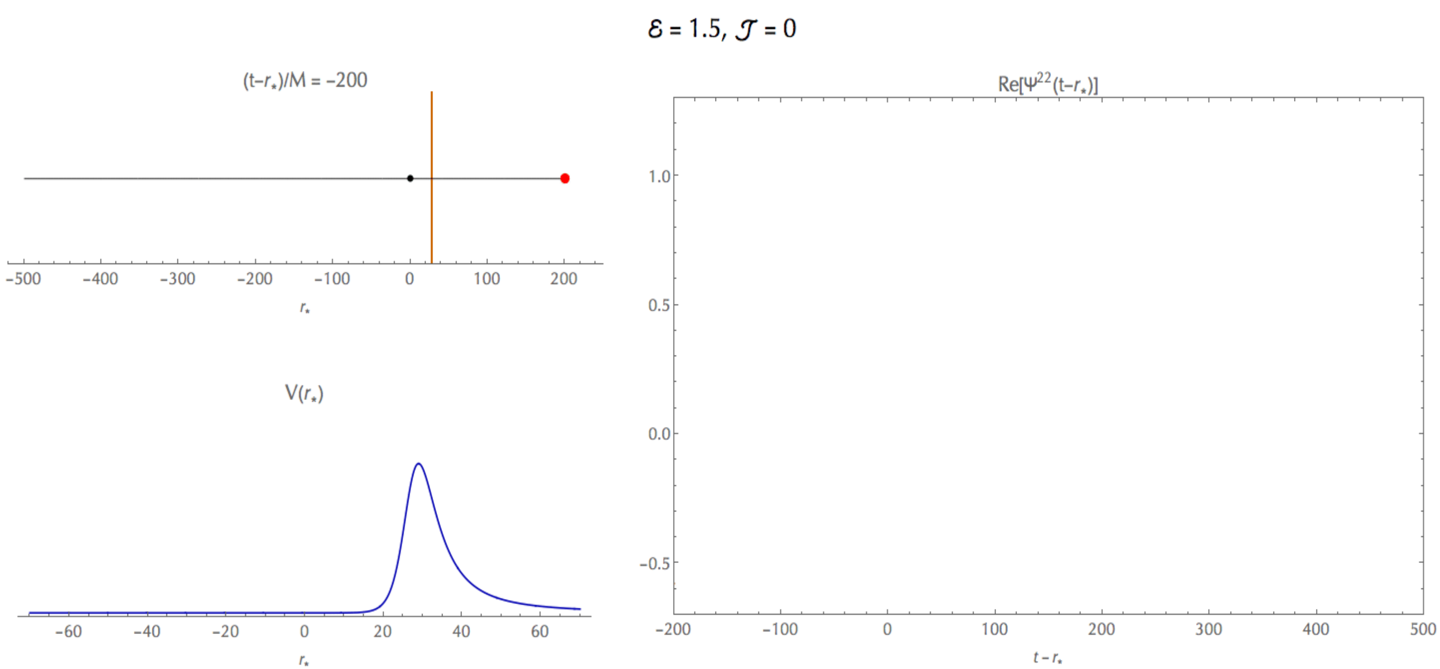
"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

Post-merger



Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual



$\mathcal{E} = 1.5$, $\mathcal{J} = 0$
$(t-r_*)/M = -200$
r_*
$V(r_*)$
$\mathrm{Re}[\Psi^{22}(t-r_*)]$
$t - r_*$