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Resolving the complex dynamical evolution of supermassive black holes in cosmological simulations using the KETJU code

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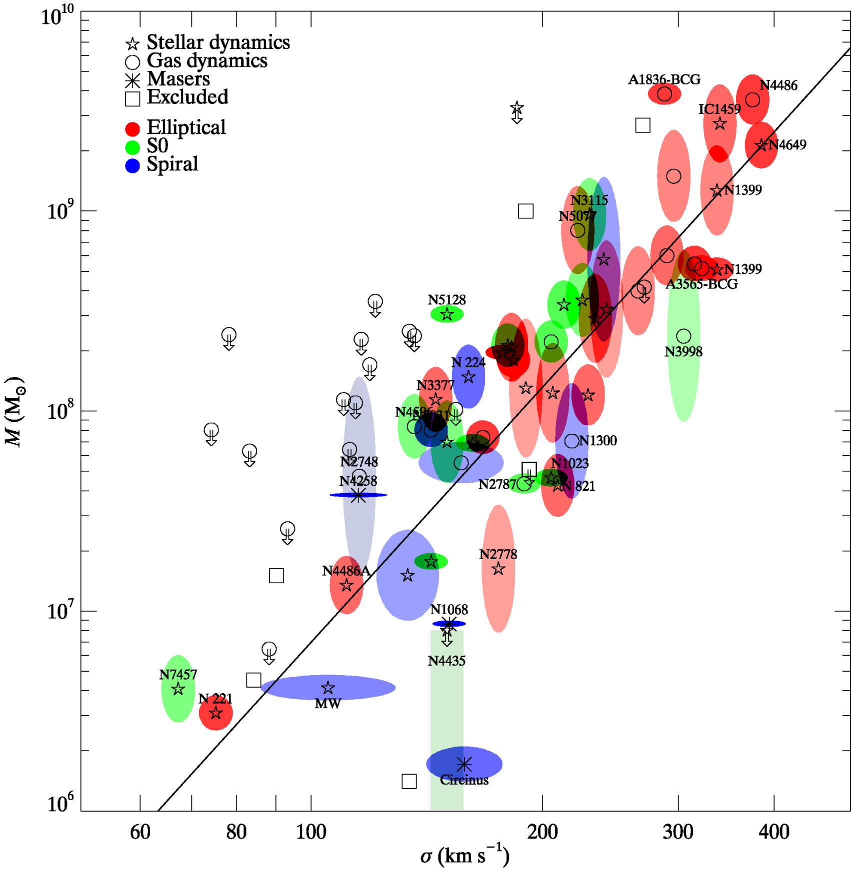
Mannerkoski, Rawlings, Johansson et al., 2023, 524, 4062
Liao, Johansson, Mannerkoski et al., 2023, MNRAS, 520, 4463
Mannerkoski, Johansson, Rantala, Naab, Liao, Rawlings, 2022, ApJ, 929, 167
Mannerkoski, Johansson, Rantala, Naab, Liao, 2021, ApJL, 912, 20
Rantala, Pihajoki, Mannerkoski, Johansson, Naab, 2020, MNRAS, 492, 4131
Mannerkoski, Johansson, Pihajoki, Rantala, Naab, 2019, ApJ, 857, 35
Rantala, Johansson, Naab, Thomas, Frigo, 2018, ApJ, 864, 113
Rantala, Pihjoki, Johansson, Naab, Lahén, Sawala, 2017, ApJ, 840, 53

1. Supermassive black holes

- Supermassive black holes (SMBH) are found in the centres of all massive galaxies.
- A strong correlation between the SMBH mass and the stellar mass in galaxies, implying co-evolution.

Gultekin et al. 2009, ApJ, 698, 198



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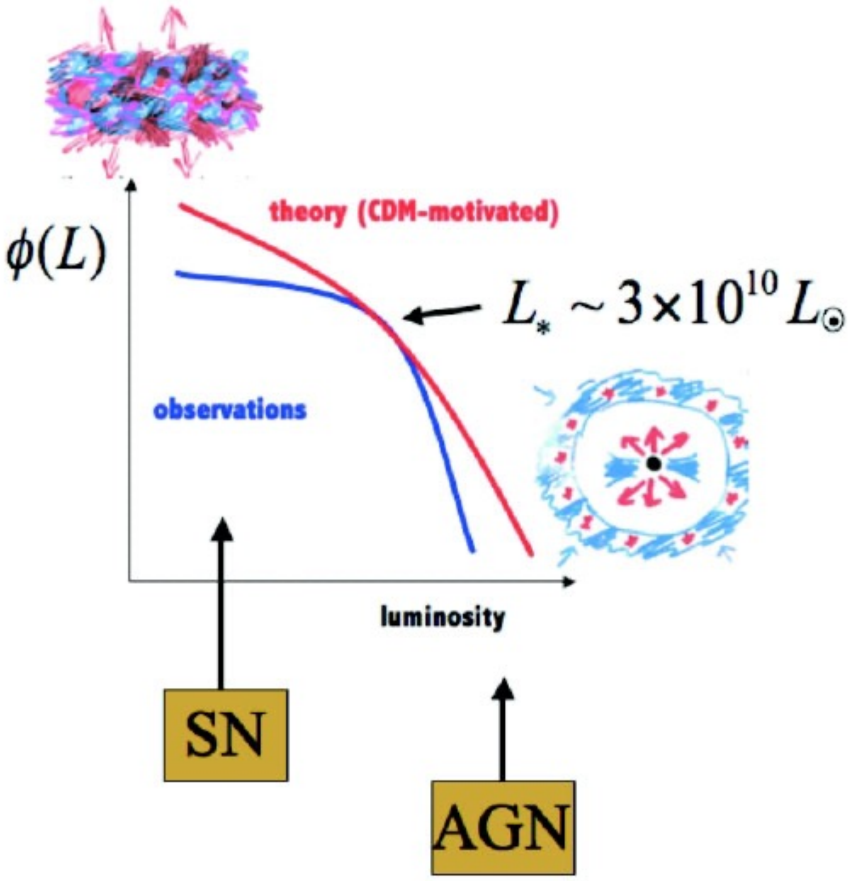

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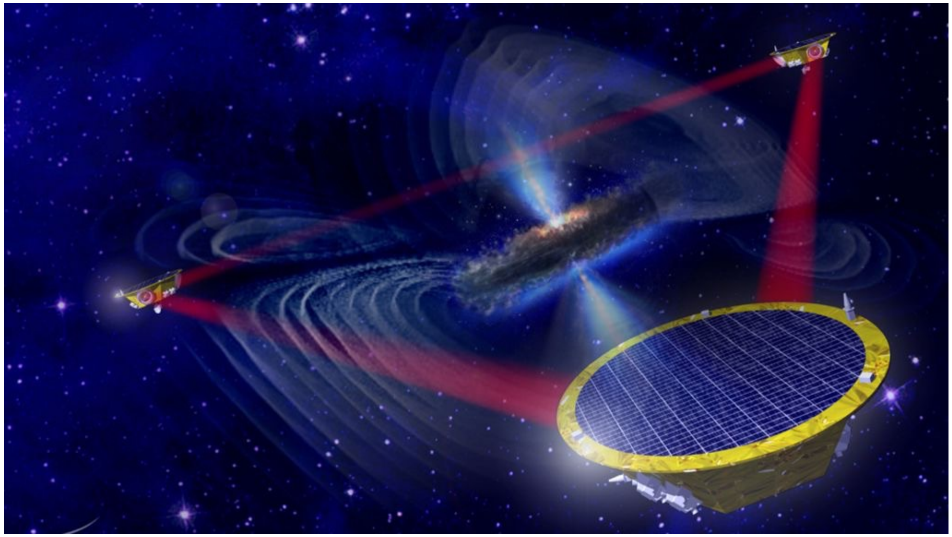
- A strong correlation between the SMBH mass and the stellar mass in galaxies, implying co-evolution.
- Energetic feedback from supermassive black holes might be responsible for setting the maximum mass of galaxies.
- In the standard ΛCDM model galaxies grow through mergers. Mergers of SMBHs could be detected by observing gravitational waves in the near future (LISA).



Numerical simulations

- The primary goal of numerical simulations is to calculate the positions, velocities and and accelerations of particles in a gravitational field using Newton's Law of gravity:

$$\frac{d}{dt}(m_\alpha \mathbf{v}_\alpha) = -\sum_{\beta,\alpha\neq\beta} \frac{G m_\alpha m_\beta}{|\mathbf{x}_\alpha - \mathbf{x}_\beta|^3}(\mathbf{x}_\alpha - \mathbf{x}_\beta)$$

- When we want to study large systems with a large number of particles we need to make some approximations.
- In the Milky Way there is about 200-400 billion stars, however in a typical simulation there is only some millions of particles.
- Each star particle represents ~10^5 stars and gravity must be softened.

$$\Phi_\alpha = -\sum_{\beta\neq\alpha} \frac{G m_\beta}{\sqrt{r^2 + \epsilon^2}}$$

Current state-of-the-art

- The dynamics of black holes have been traditionally studied with global hydrodynamical 10-100 million particle softened simulations (i.e. Gadget-3, AREPO, SWIFT, RAMSES).

- An alternative is to use a collisional direct N-body simulation, which are typically restricted to ~1 million particles (i.e. Nbody-7) and typically do not include gas.

- In KETJU the best aspects of a global softened code and an accurate N-body code are combined.

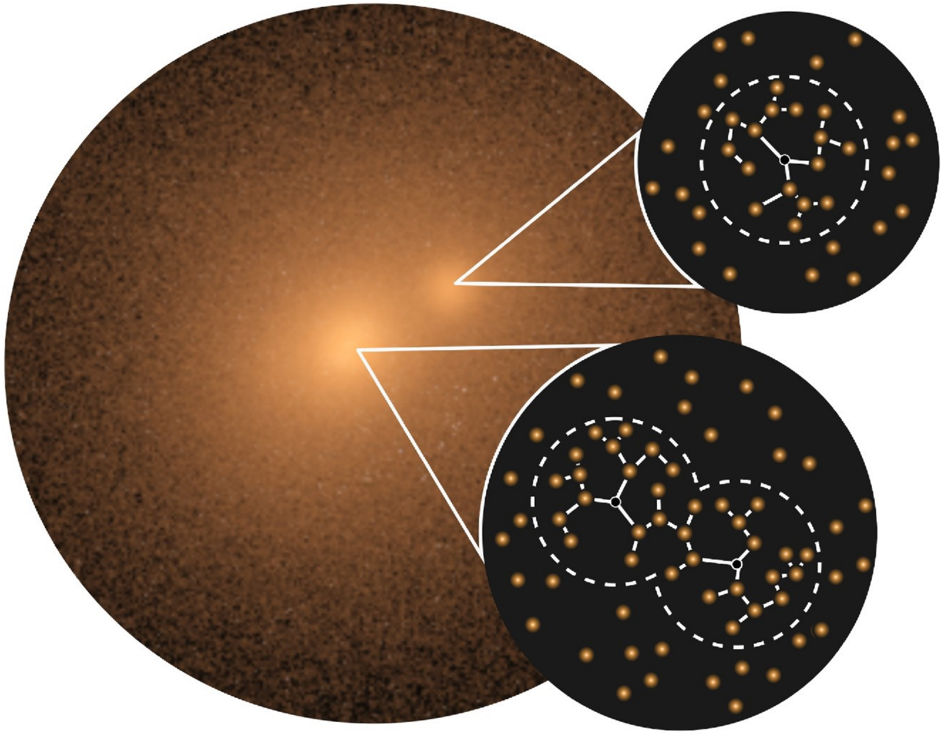
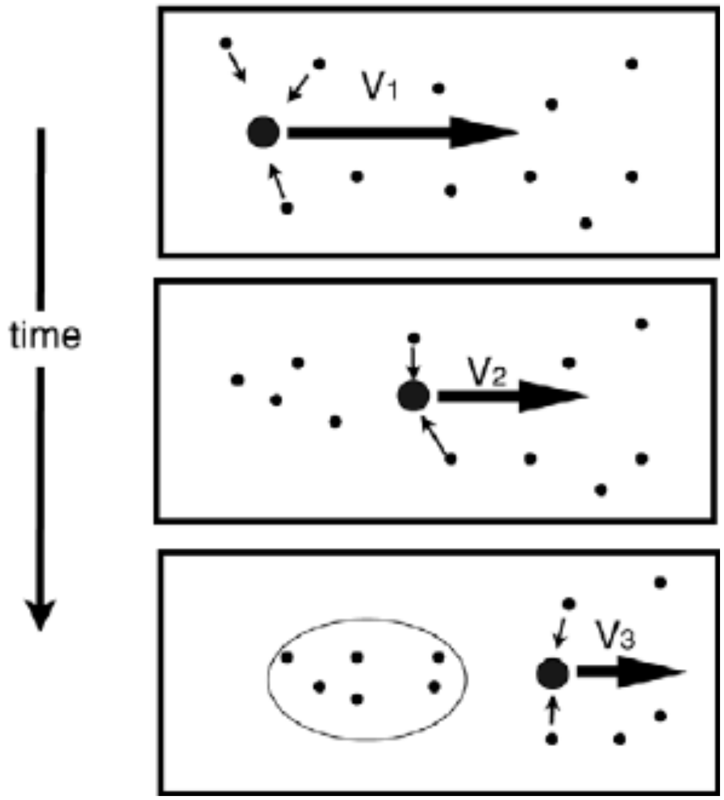


Illustration of three merging galaxies with three SMBHs and KETJU regions.

The three phases of black hole binary evolution

1. Dynamical friction from stars and gas reduces the semi-major axis of the BH binary to ~10 pc.

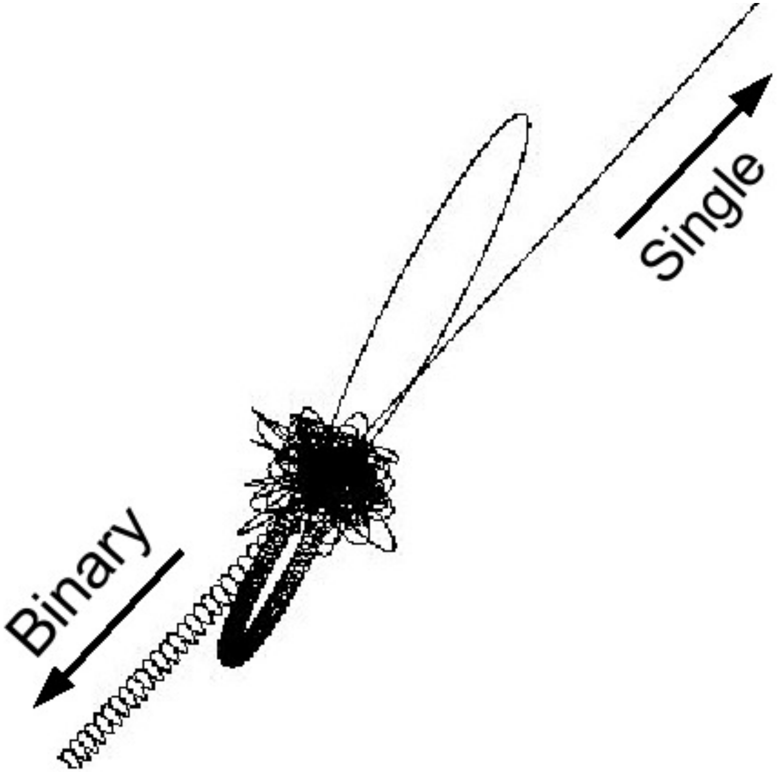


$$F_{\mathrm{DF}} \propto M^2$$

The three phases of black hole binary evolution

1. Dynamical friction from stars and gas reduces the semi-major axis of the BH binary to ~10 pc.

2. Next, the semi-major axis of the binary will shrink by kicking out stars in complex three-body interactions.



$$\frac{d}{dt}\left(\frac{1}{a}\right) \propto \frac{G\rho_\star}{\sigma_\star}$$

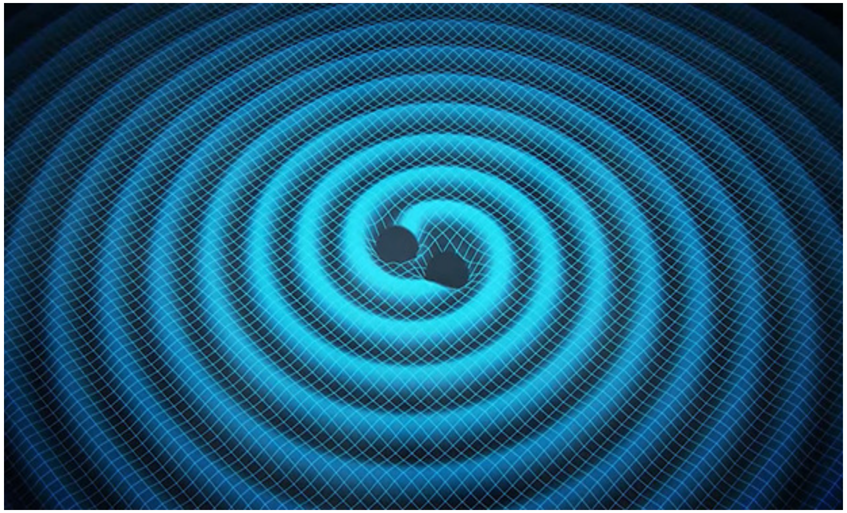
The three phases of black hole binary evolution

1. Dynamical friction from stars and gas reduces the semi-major axis of the BH binary to ~10 pc.

2. Next, the semi-major axis of the binary will shrink by kicking out stars in complex three-body interactions.

3. The emission of gravitational waves will eventually dominate the loss of orbital energy at very small ~0.01 pc binary separations.

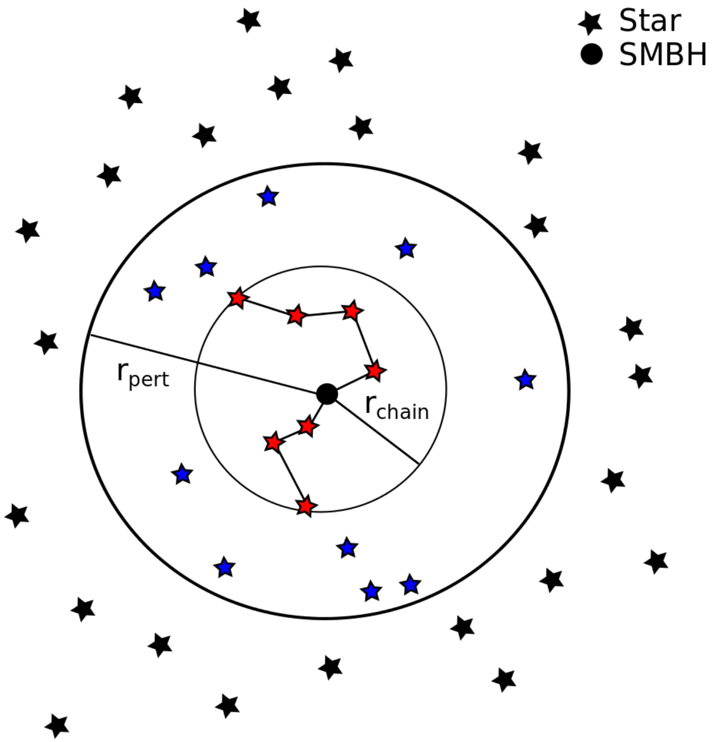
- Current simulation codes are unable to resolve the full BH merging process in a single simulation.



$$\left|\frac{da}{dt}\right| = \frac{64}{5}\frac{G^3 M_1 M_2 (M_1+M_2)}{c^5 a^3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

2. KETJU: Hybrid N-body+hydrodynamics

1. KETJU **(chain in Finnish)**: Built originally on the GADGET-3 code. Includes algorithmically regularised chain regions around every black hole.



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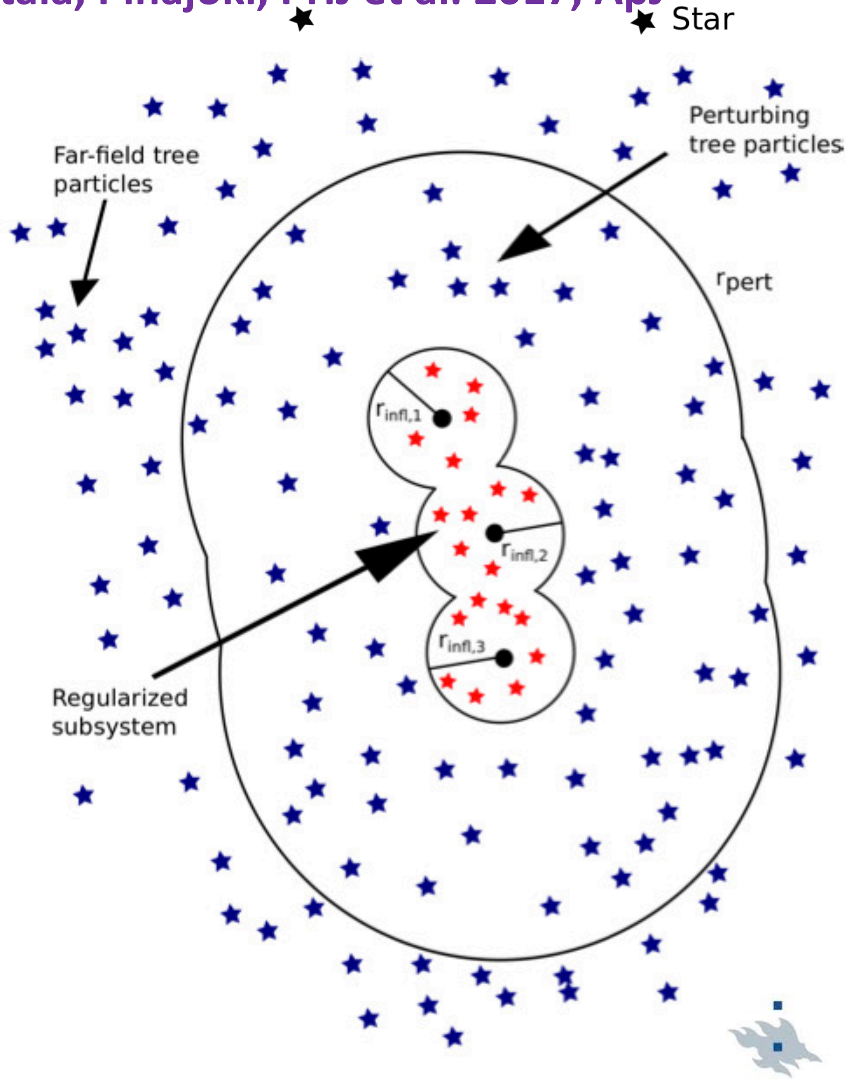
Rantala, Pihajoki, PHJ et al. 2017, ApJ



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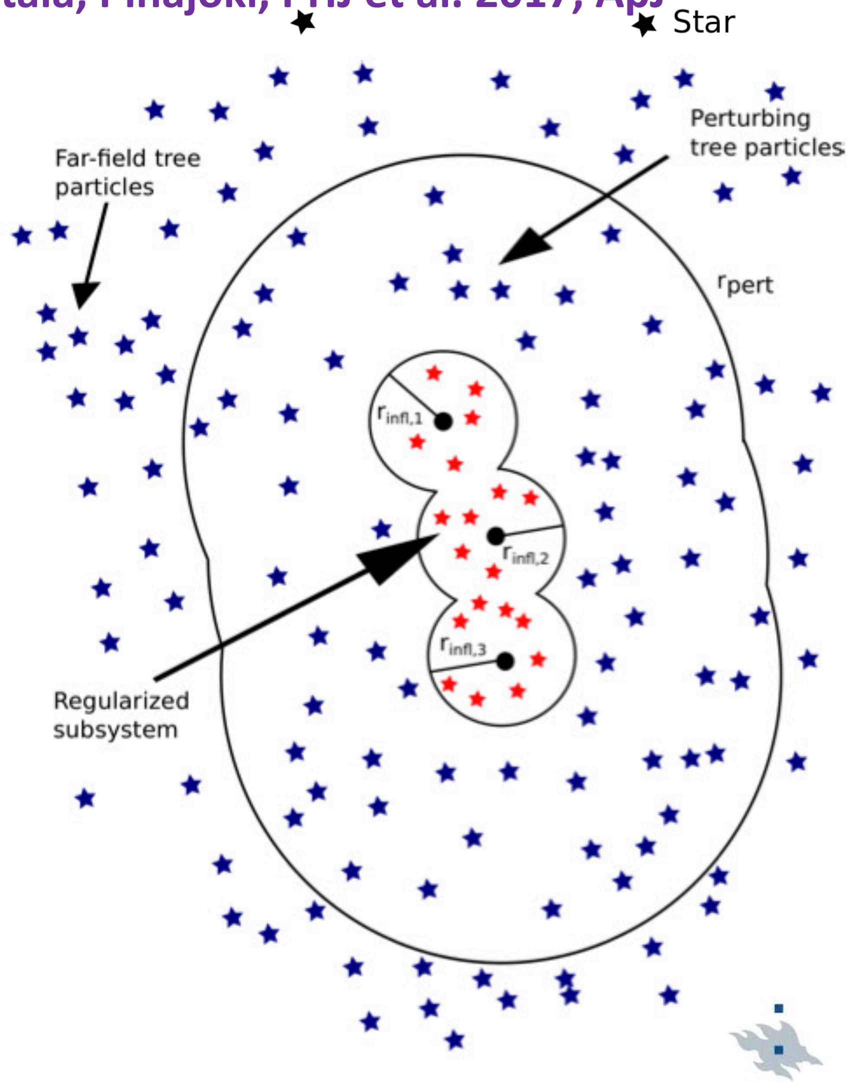
3. Includes Post-Newtonian correc-tions up to order 3.5 PN, valid down to ~10 Schwarzschild radii.

Rantala, Pihajoki, PHJ et al. 2017, ApJ



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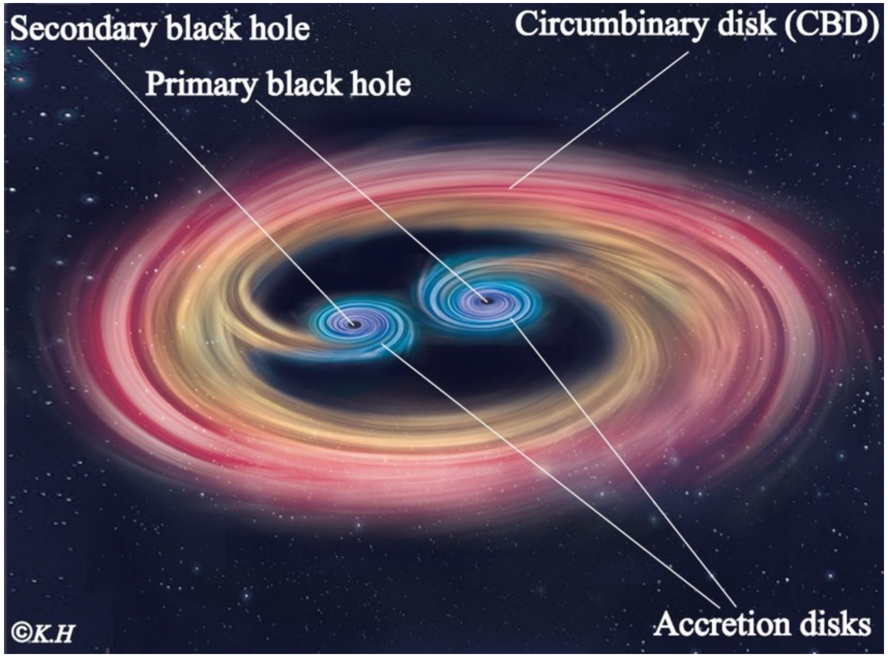
4. **The main novelty is that KETJU enables accurate BH dynamics simultaneously with gas physics.**



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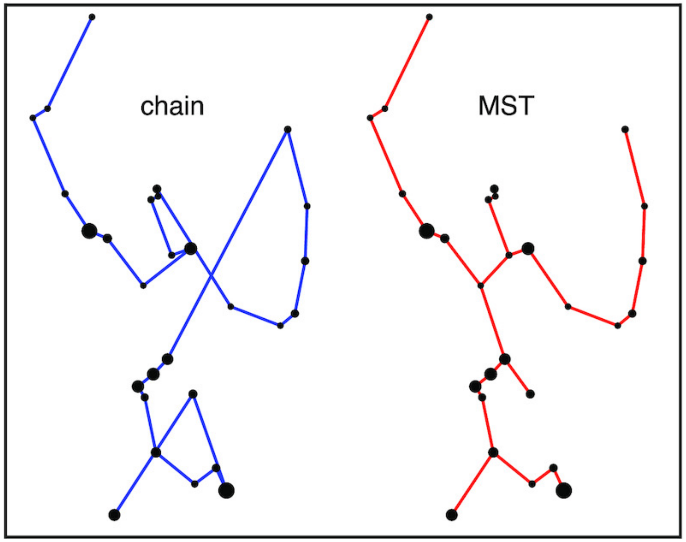
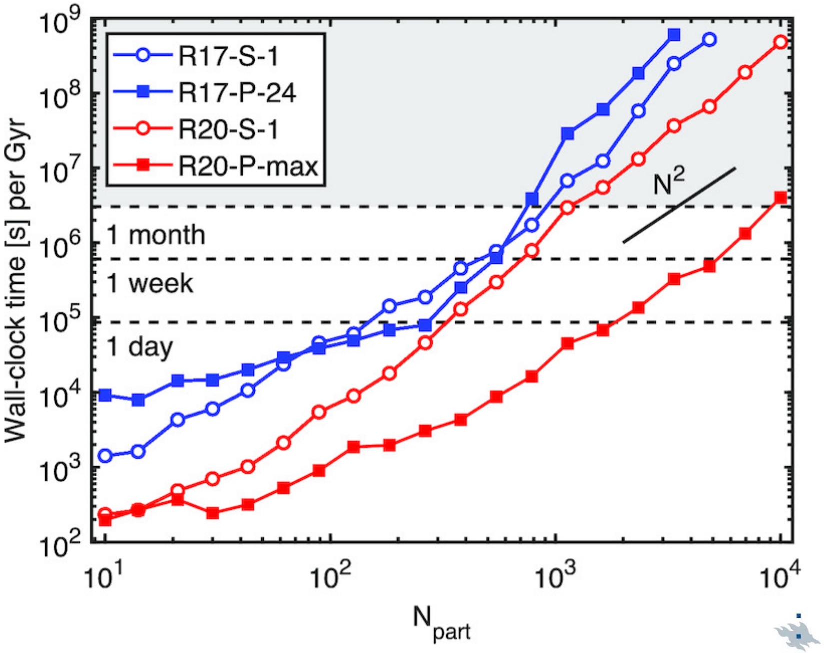
MSTAR – recent KETJU code improvements

- Recently the ARCHAIN integrator in KETJU has been replaced by the MSTAR integrator (Rantala et al., 2020).

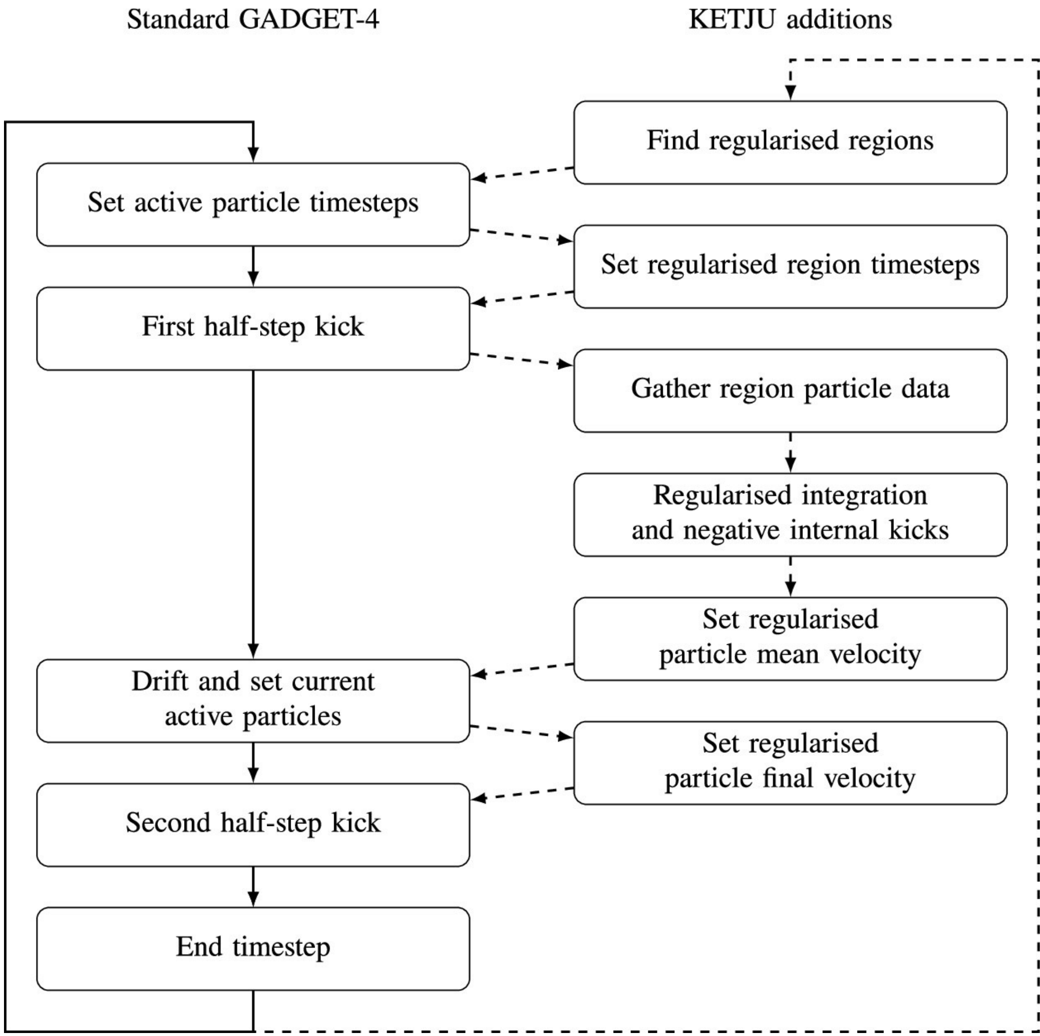
1. Uses a minimum spanning tree coordinates, instead of the chain.
2. Uses a more efficient node-based parallelisation technique for the Bulirsch-Stoer extrapolation.
3. Enables simulations with 10 000s particles in the "regularised region", as opposed to ~500 in original KETJU.
4. In addition ,we now include a parametrised model to account for BH spins and kicks (Zlochower & Lousto 2015).





KETJU-GADGET4 – Public version of the code

- A public version of the KETJU code based on GADGET-4 was published in June, 2023.
- Gravitational interactions on the GADGET side can be calculated using a one-sided tree-based multipole expansion.
- Alternatively, a fast multipole method (FMM), which is momentum conserving when combined with hierarchical time integration can also be used.



Download the KETJU-GADGET4 code:
https://www.mv.helsinki.fi/home/phjohans/group-website/research/ketju/

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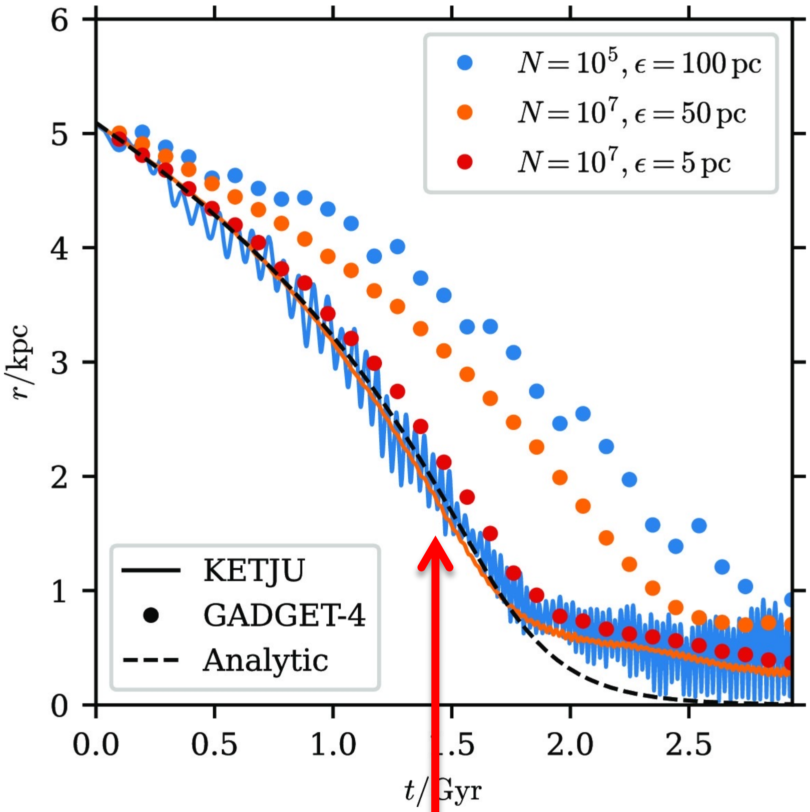
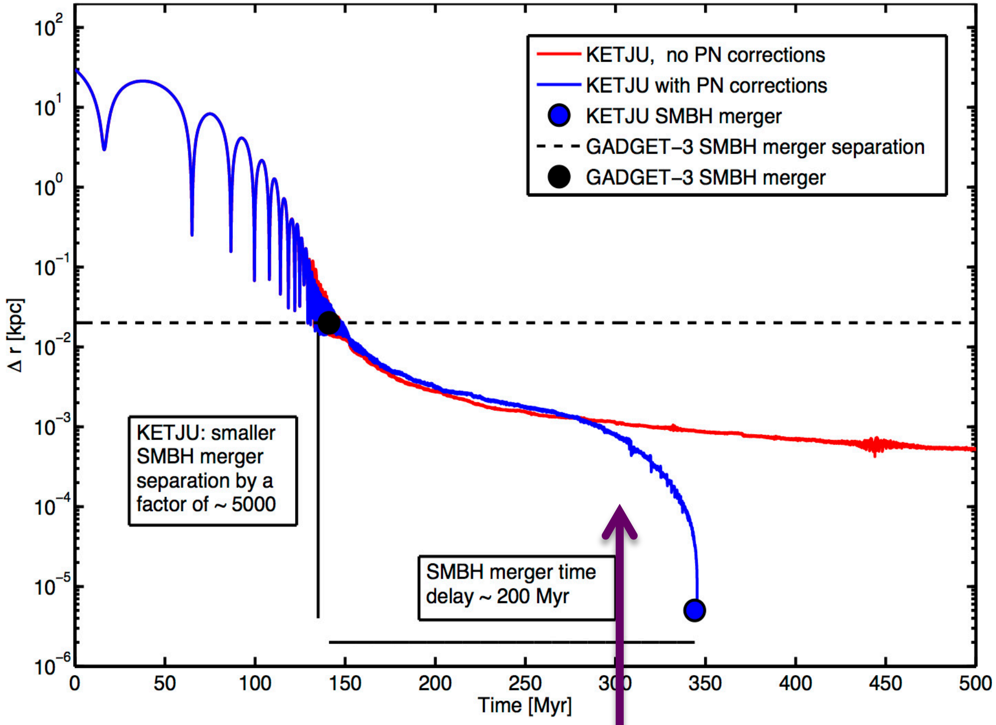
KETJU applications



Accurately resolving dynamical friction and the sinking of super-massive black holes.



KETJU: smaller SMBH merger separation by a factor of ~ 5000

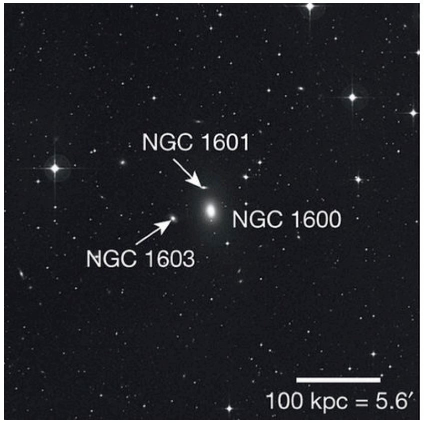
SMBH merger time delay ~ 200 Myr

Accurately resolving the dynamics of binary supermassive black holes up to ~10 Schwarzschild radii.

3. Formation of cored galaxies

Rantala, PHJ et al. 2018

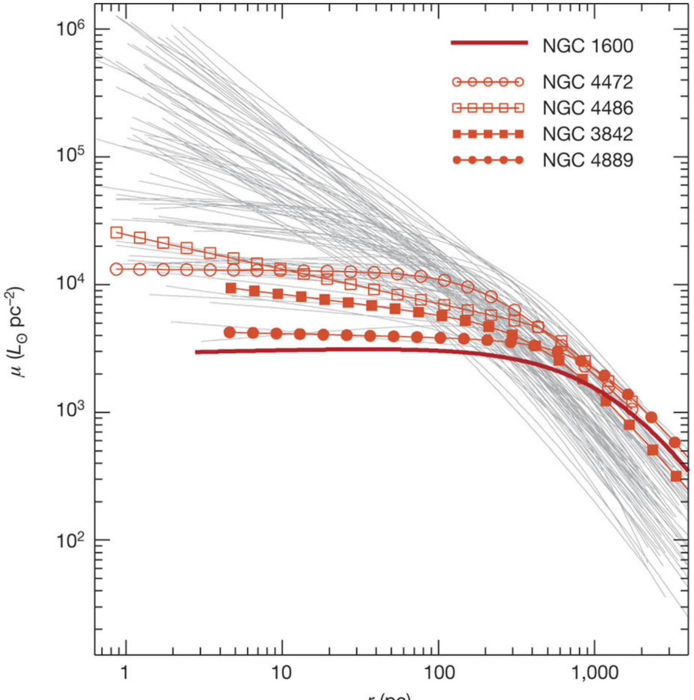
- Core ellipticals exhibit large cores with nearly constant surface brightness. Typically, very massive, slowly rotating and have boxy isophotes.



- NGC 1600 is an extreme example of a cored galaxy. (Thomas et al. 2016).

$$M_{\rm BH} = (1.7 \pm 0.15) \times 10^{10} M_\odot$$

- Probably formed through a dry (gas-poor) merger between two massive early-type galaxies and scouring of the core by the dynamical evolution of a SMBH binary.

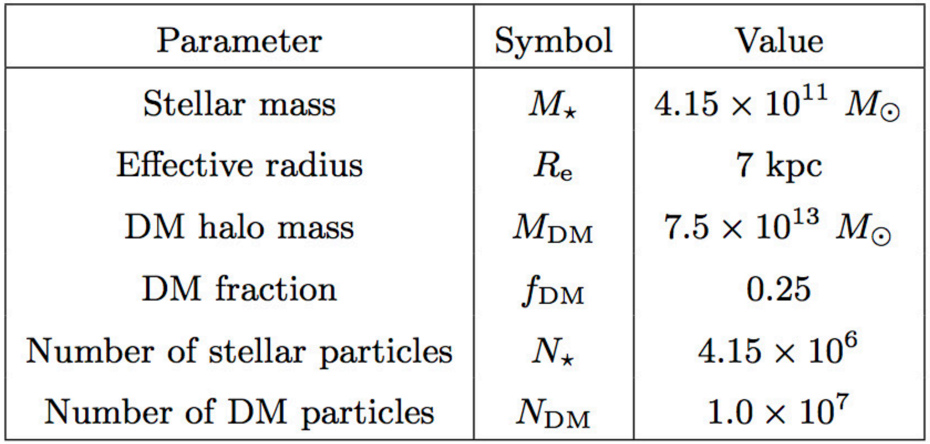


Initial conditions and simulations

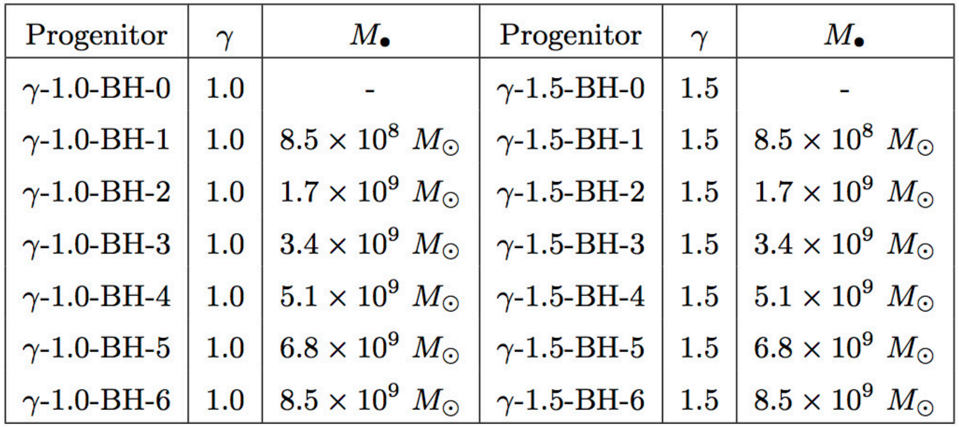
- Our collisionless initial conditions are modelled using isotropic Dehnen profiles (γ=1.5 or γ=1.0) for the stars and γ=1.0 for the dark matter, including a central SMBH.

$$\rho(r) = \frac{(3-\gamma)M}{4\pi}\frac{a}{r^{\gamma}(r+a)^{4-\gamma}}$$

- We simulate major mergers to describe the final dry major merger that NGC 1600 likely experienced.
- High numerical resolution for N-body type of simulation.



Parameter	Symbol	Value
Stellar mass	$M_\star$	$4.15 \times 10^{11}\ M_\odot$
Effective radius	R_e	7 kpc
DM halo mass	$M_{\rm DM}$	$7.5 \times 10^{13}\ M_\odot$
DM fraction	$f_{\rm DM}$	0.25
Number of stellar particles	$N_\star$	4.15×10^{6}
Number of DM particles	$N_{\rm DM}$	1.0×10^{7}

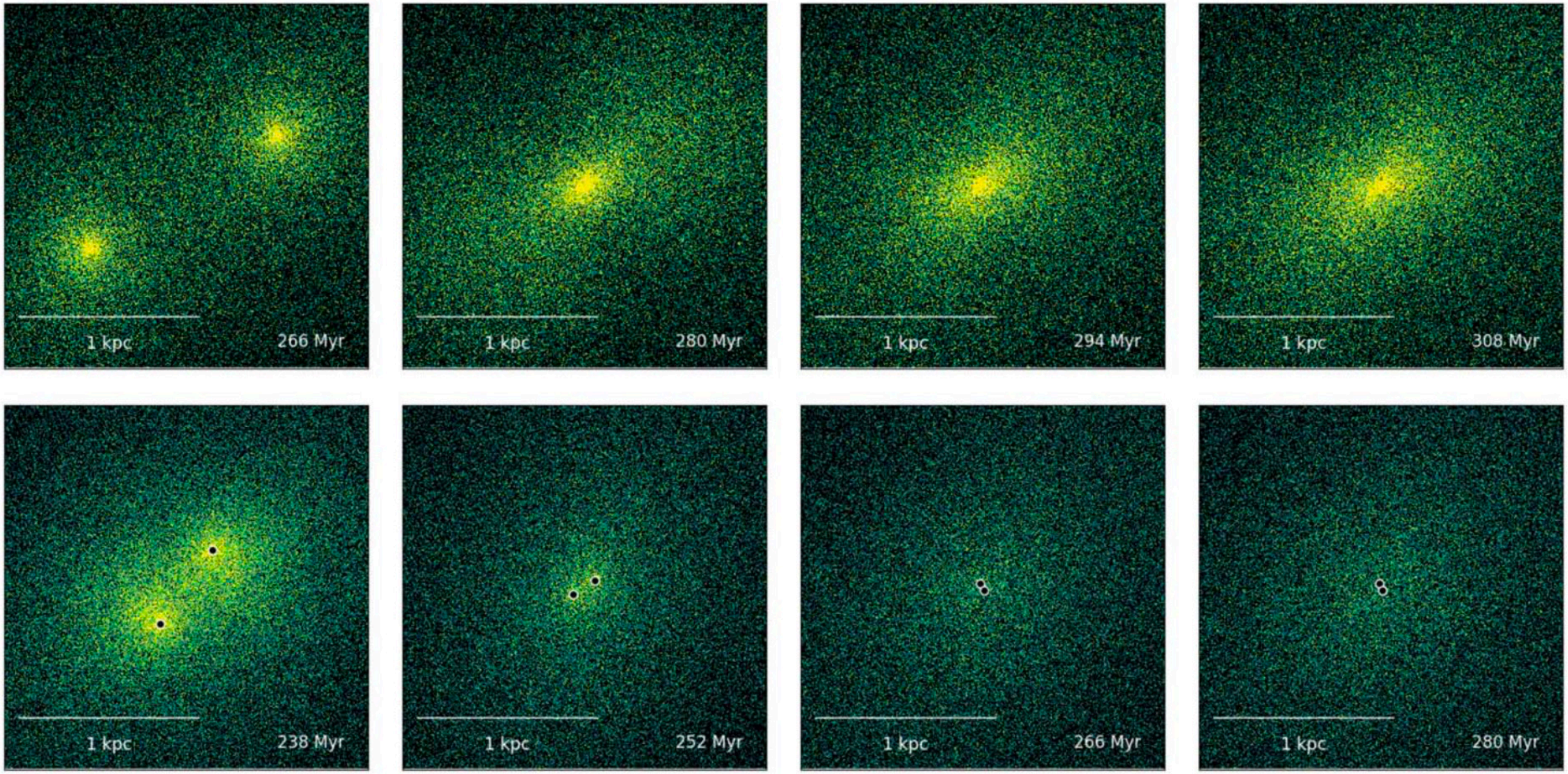


Progenitor	γ	$M_\bullet$	Progenitor	γ	$M_\bullet$
γ-1.0-BH-0	1.0	-	γ-1.5-BH-0	1.5	-
γ-1.0-BH-1	1.0	$8.5 \times 10^{8}\ M_\odot$	γ-1.5-BH-1	1.5	$8.5 \times 10^{8}\ M_\odot$
γ-1.0-BH-2	1.0	$1.7 \times 10^{9}\ M_\odot$	γ-1.5-BH-2	1.5	$1.7 \times 10^{9}\ M_\odot$
γ-1.0-BH-3	1.0	$3.4 \times 10^{9}\ M_\odot$	γ-1.5-BH-3	1.5	$3.4 \times 10^{9}\ M_\odot$
γ-1.0-BH-4	1.0	$5.1 \times 10^{9}\ M_\odot$	γ-1.5-BH-4	1.5	$5.1 \times 10^{9}\ M_\odot$
γ-1.0-BH-5	1.0	$6.8 \times 10^{9}\ M_\odot$	γ-1.5-BH-5	1.5	$6.8 \times 10^{9}\ M_\odot$
γ-1.0-BH-6	1.0	$8.5 \times 10^{9}\ M_\odot$	γ-1.5-BH-6	1.5	$8.5 \times 10^{9}\ M_\odot$

BH-0: no SMBHs and BH-6: The observed SMBH in NGC 1600.

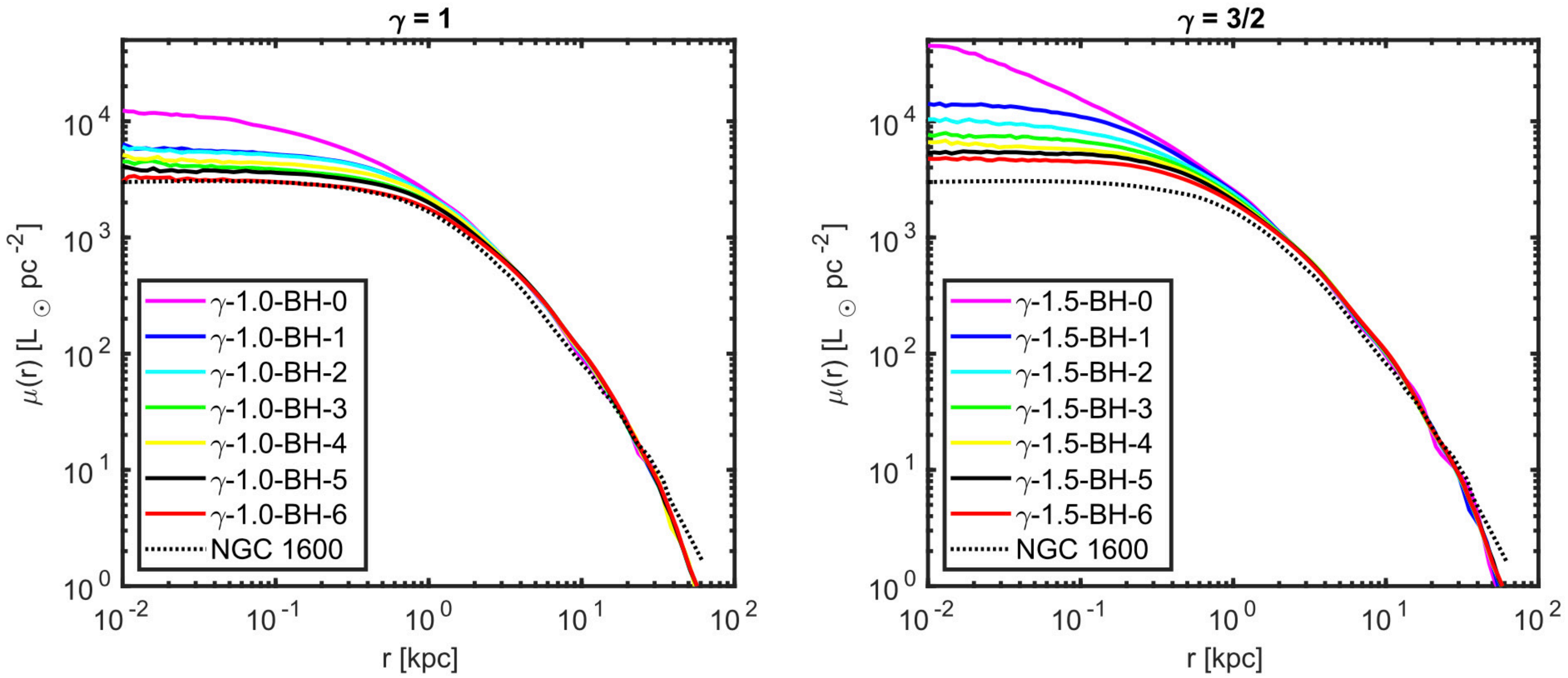
Stellar surface densities

Rantala, PHJ et al. 2018



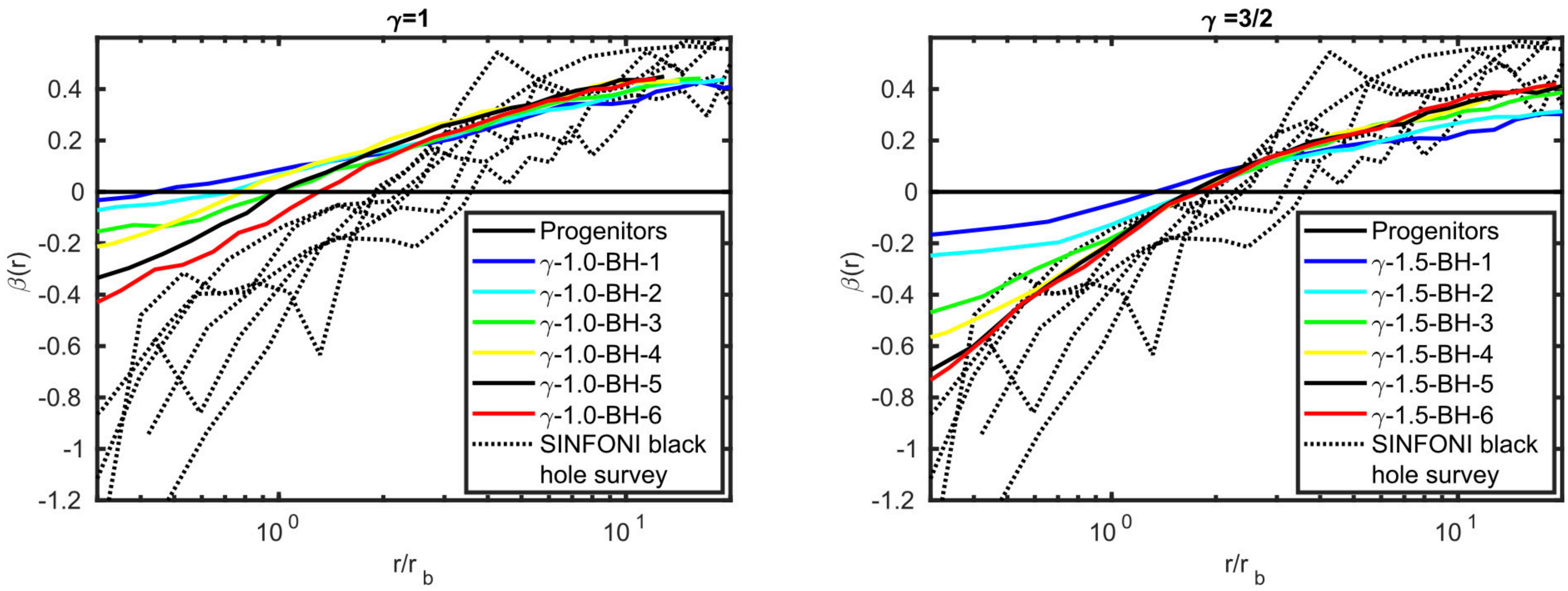
- Top: Merger without SMBHs. Bottom: Merger with massive SMBHs.
- The effect of core scouring by the SMBHs can clearly be seen in the surface density plot.

Surface brightness profiles



- Similarly to the Thomas et al. (2016) observations we assume a constant mass-to-light ratio of $M_*/L=4.0$.
- As expected we find a systematic decrease in the surface brightness as a function of increasing BH mass (e.g. Merritt 2006).

Velocity anisotropy profiles



- We find a monotonic decrease in the central β-parameter, meaning an increasingly more tangentially biased stellar population in the core region.
- More massive BHs have larger spheres of influence -> more negative β.

$$\beta = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_\mathrm{r}^2} = 1 - \frac{\sigma_\mathrm{t}^2}{\sigma_\mathrm{r}^2}.$$

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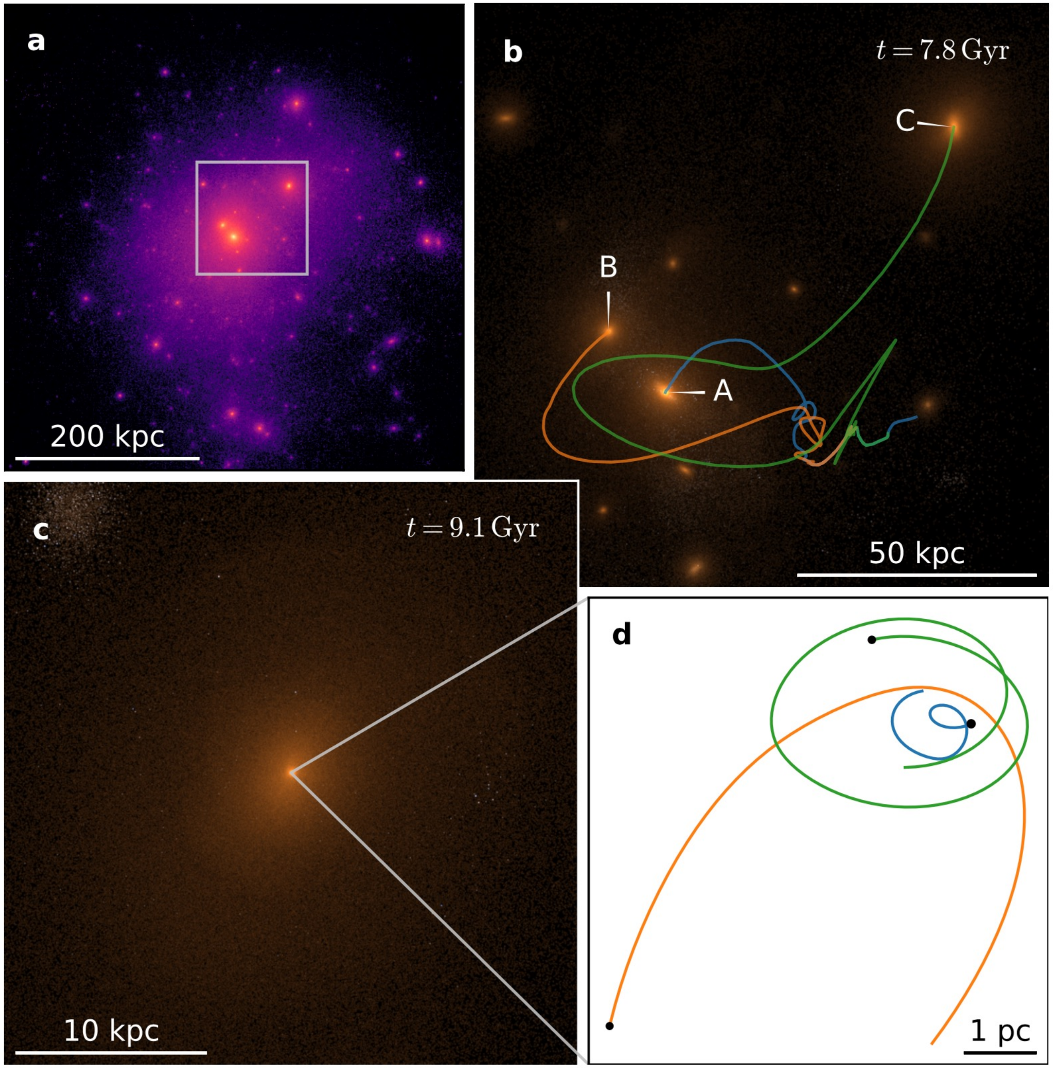
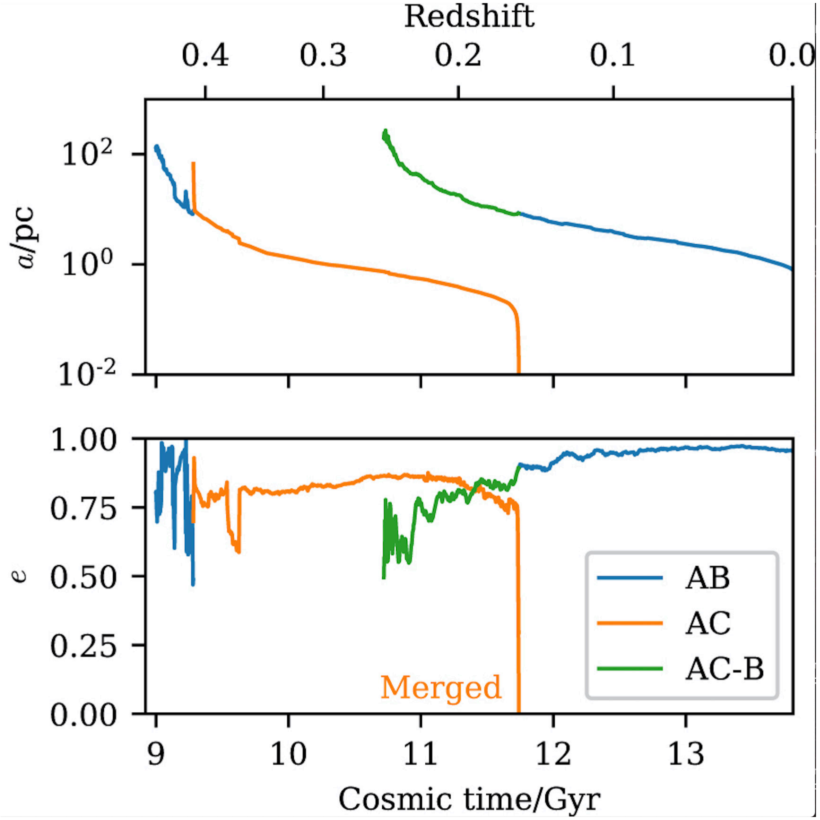
4. Cosmological KETJU zoom-in simulations

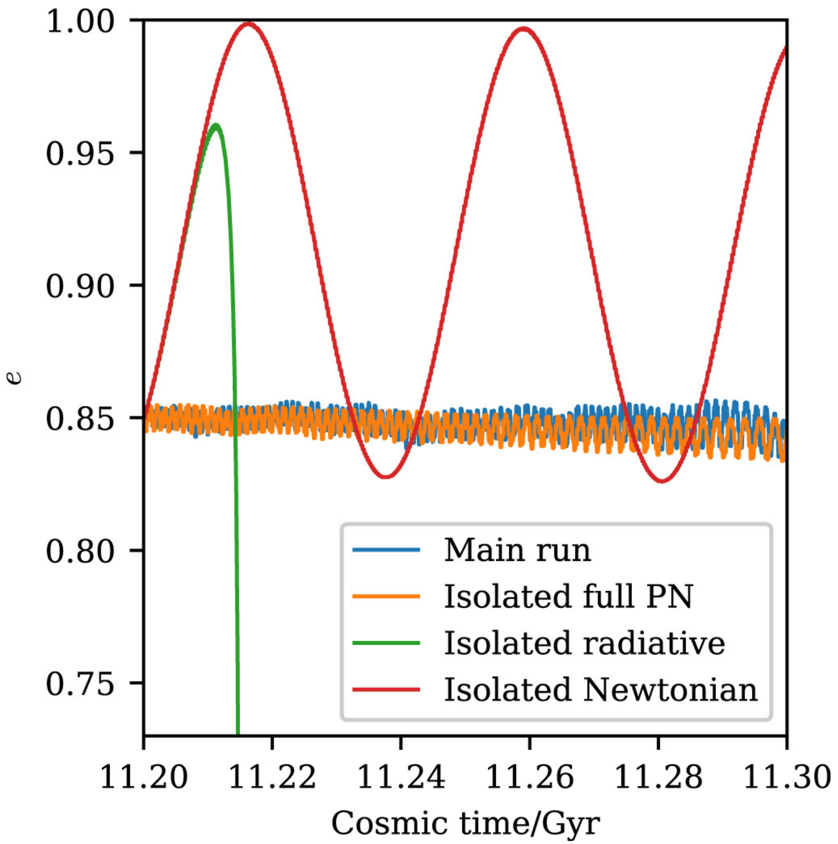
- The simulations include gas cooling, star formation, and both supernova and thermal black hole feedback.
- Black holes are seeded in massive dark matter haloes and evolved using standard BH models (similar to e.g. Illustris & EAGLE).
- KETJU is switched on at z~0.6, when the mass ratio between the massive SMBHs and the stellar particles is sufficiently large (m_{BH}/m_{*}~500-1000).
- Gravitational softening: $\varepsilon=40h^{-1}$ pc. Radii of the regularised KETJU regions: $r=120h^{-1}$ pc.



Mannerkoski, PHJ et al. 2021, ApJL, 912, 20

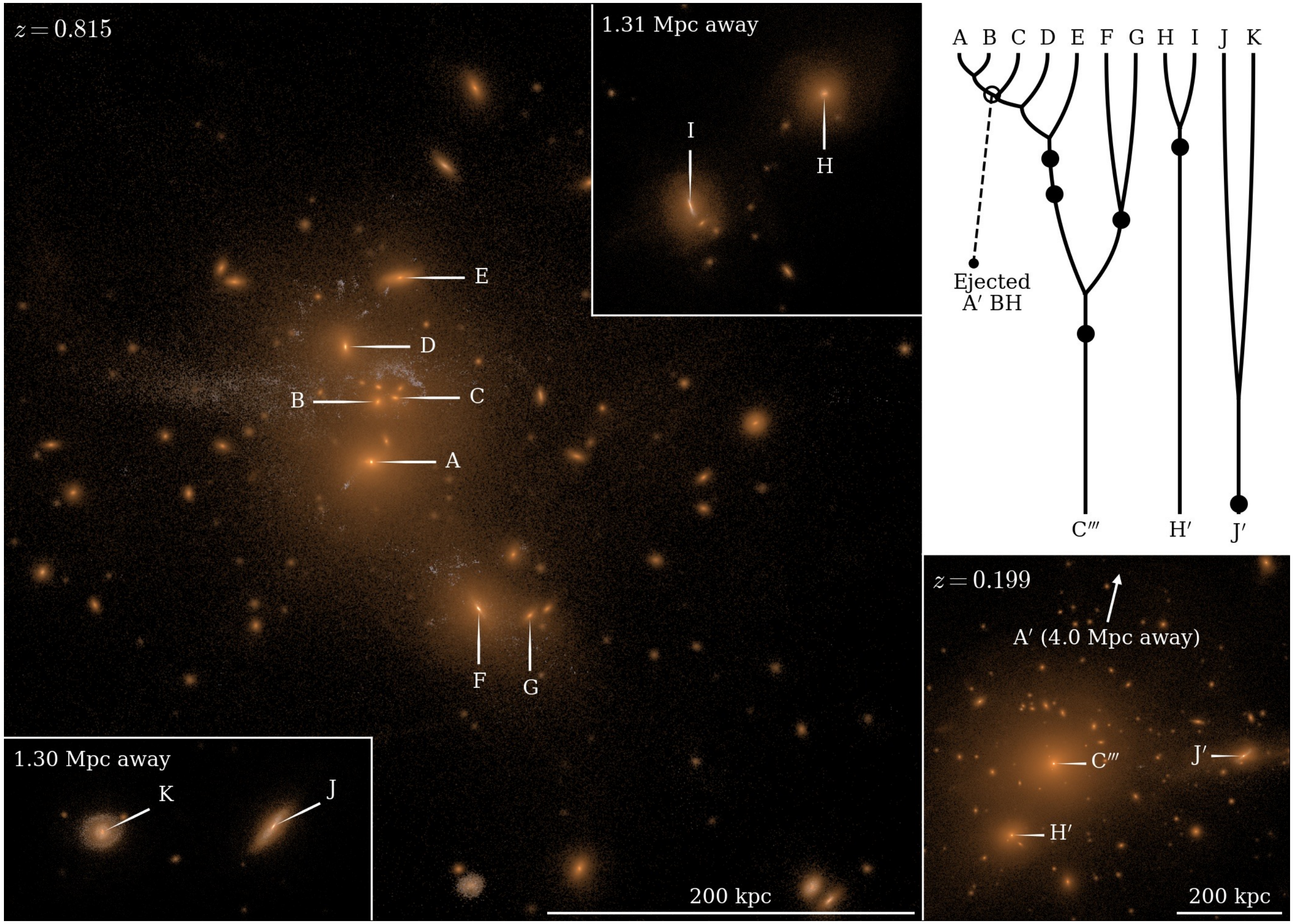
Resolving a complex triple SMBH encounter





- The simulation is able to resolve a triple encounter between three merging SMBHs. The hardening of the AB-SMBH binary is interrupted by the incoming C-SMBH, which ejects the B-SMBH to a wide orbit.
- The inner AC-SMBH binary shows supressed Lidov-Kozai oscillations due to the relativistic precession of the orbit on a short timescale.

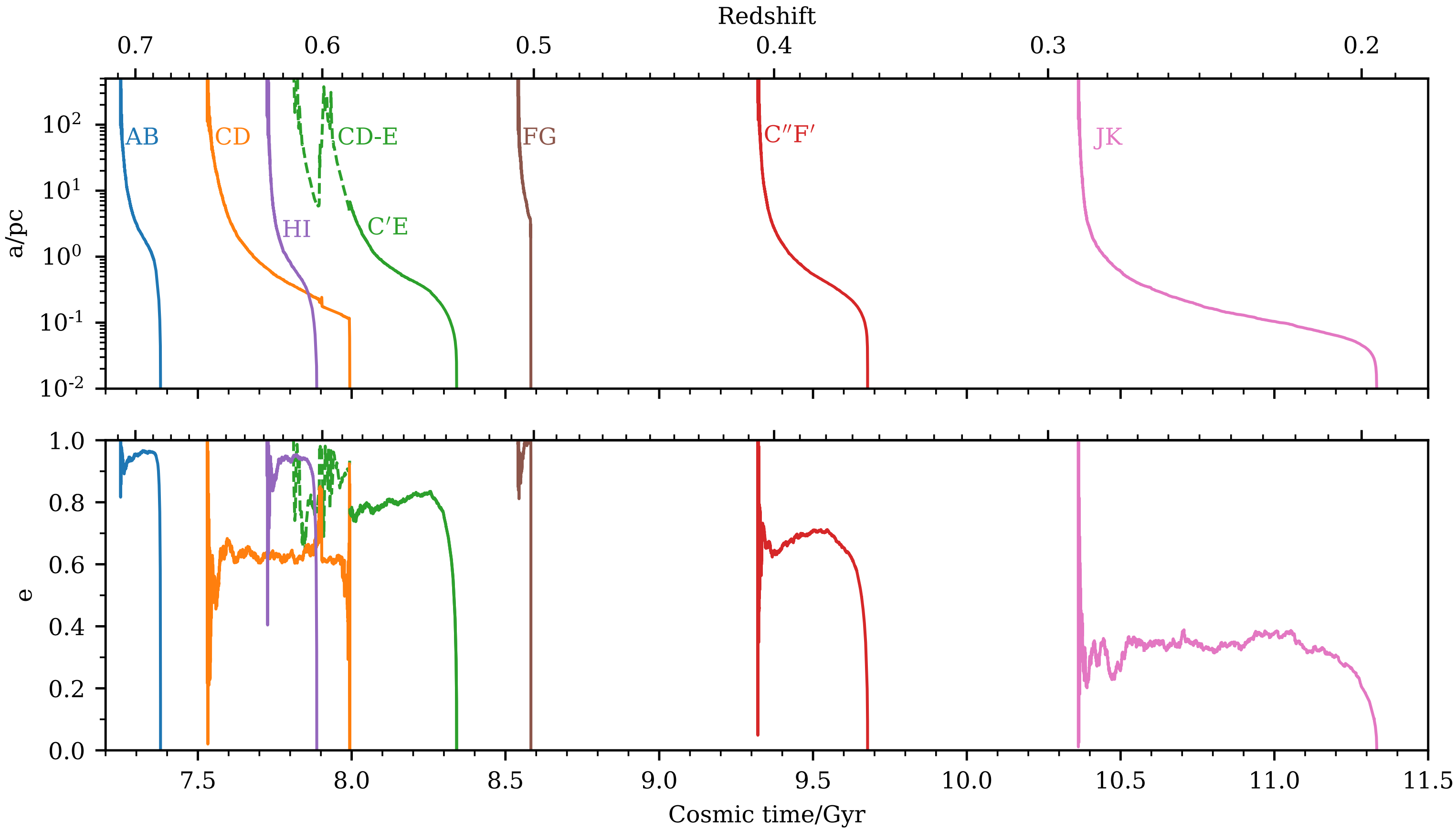
KETJU simulations of larger group volumes



Mannerkoski, PHJ et al. 2022

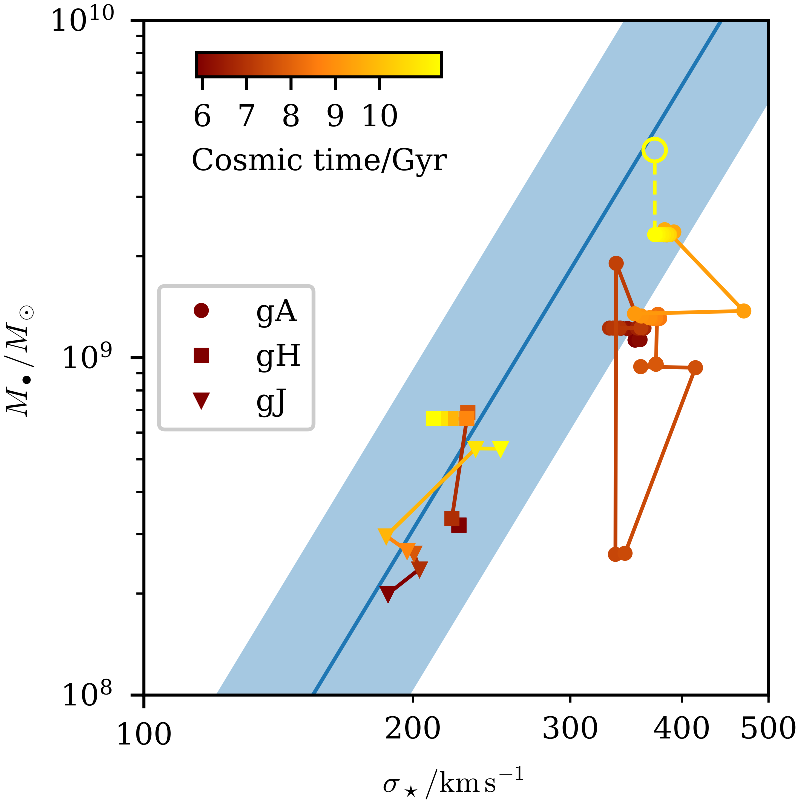
- Simulate a large group volume containing 11 galaxies with KETJU SMBHs ($M>7.5\times10^7\ M_\odot$) involved in SMBH mergers.
- Most of the binaries have high eccentricities e=0.6-0.95, and in one case (AB) the remnant SMBH is even kicked out of its host galaxy.

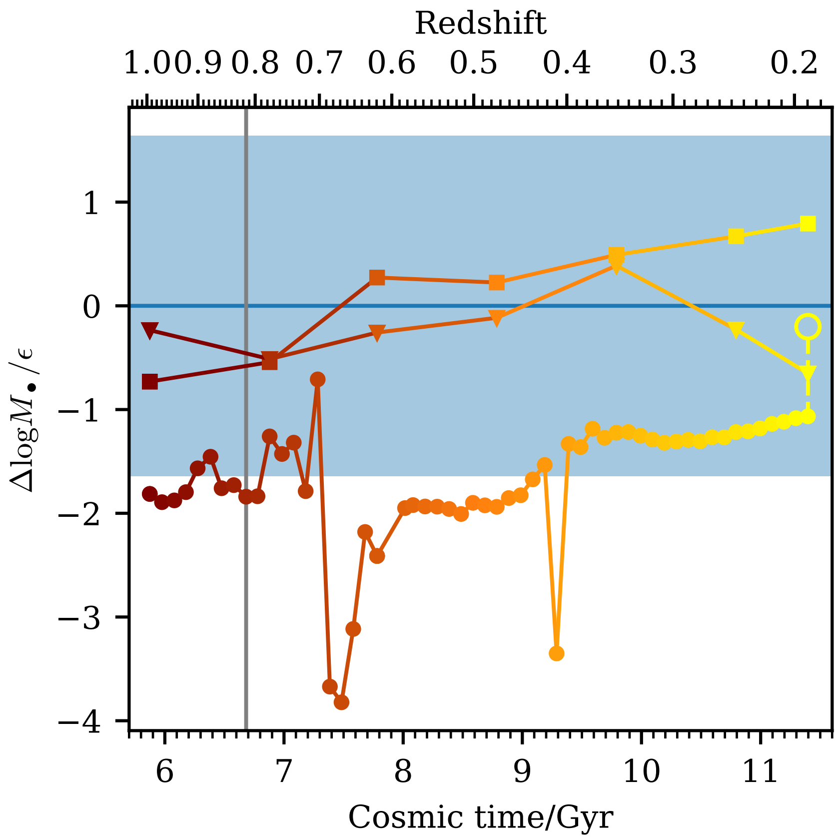
SMBH binary parameters



- The evolution of the semi-major axis (top) and eccentricity (e) of the simulated SMBH binaries. Binaries form with e~0.3-0.9, most merge fairly rapidly.
- Again, we resolve a triple system (CD-E), but now the outer orbital period is shorter than the relativistic precession period of the inner binary -> Lidov-Kozai oscillations.

Evolution on the M_{BH}-σ plane

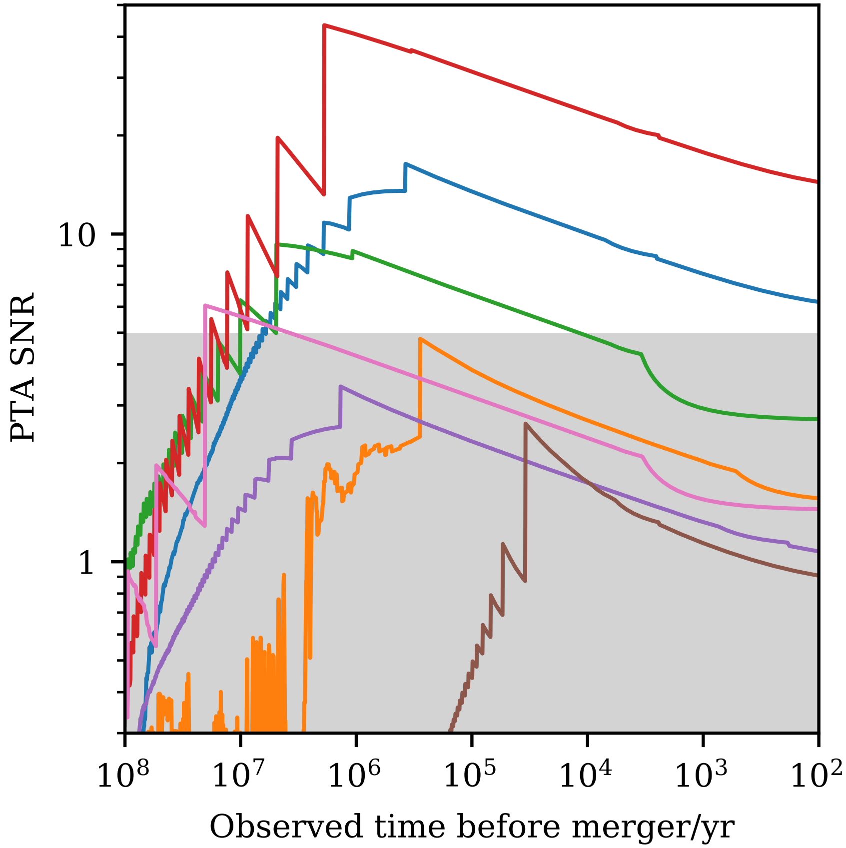




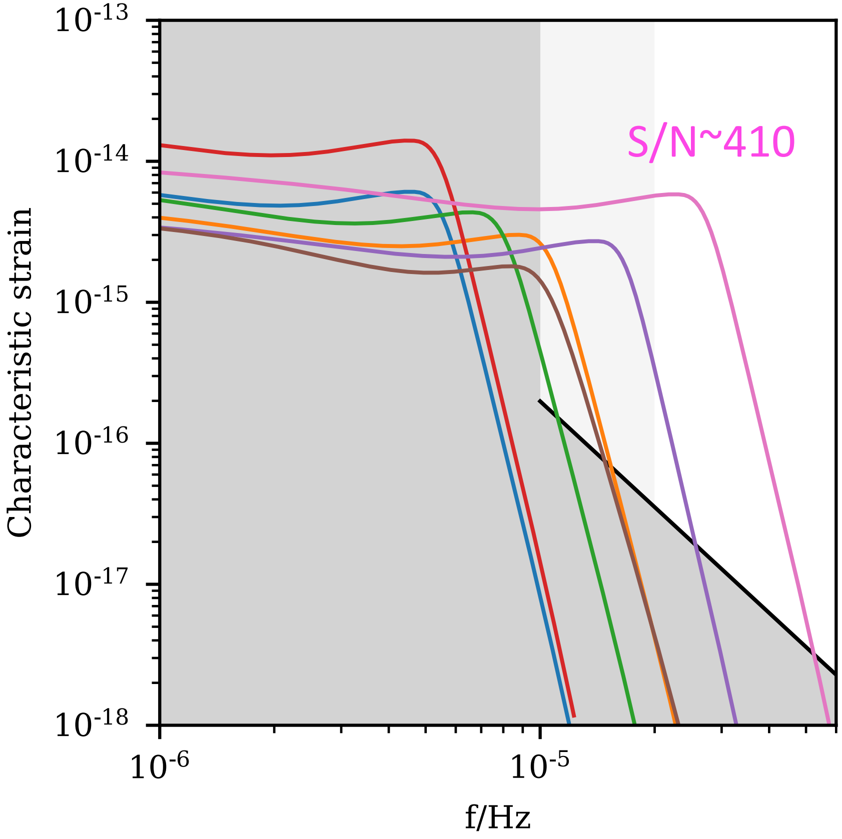
- Evolution of the main SMBHs in the galaxies on the M_{BH}-σ plane (left) and the relative offset from the centre of the Kormendy & Ho (2013) relation (right). Shaded region: 90% prediction interval for the intrinsic scatter
- Galaxy gA evolves initially significantly off the plane, when SMBH A’ is ejected, but recovers later when new SMBHs are brought in.

Gravitational waves from merging SMBHs

Pulsar timing array detections T_{obs}=25 yrs



LISA GW detections, predicted sensitivity curve.



- The GW signal can be calculated from KETJU simulations using 1) Semi-analytic orbit averaged formulae at large separations ($a>100\ R_S$) and a 2) direct discrete Fourier transform of the waveform ($a<100\ R_S$). The final unresolved merger at ($a<10\ R_S$) calculated using the 3) PhenomD waveform (Khan et al. 2016).

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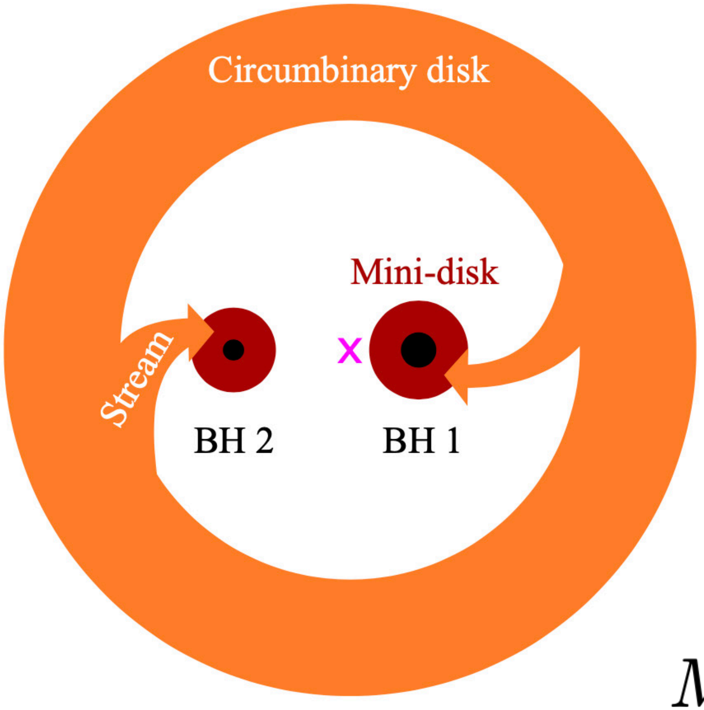
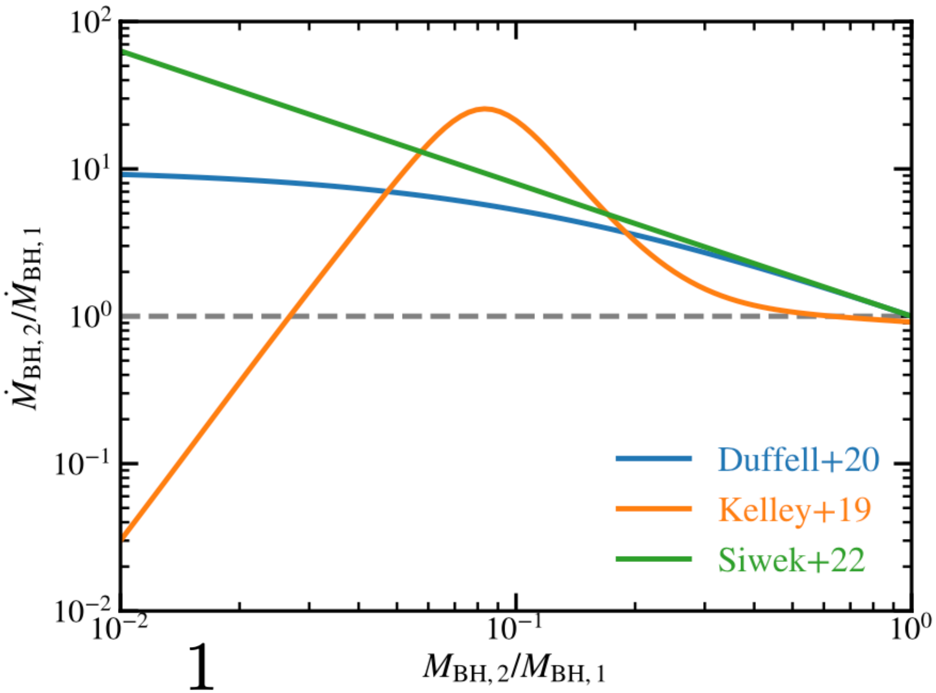
KETJU BH binary feedback model





$$\frac{\dot{M}_2}{\dot{M}_1} = \frac{1}{0.1 + 0.9q}$$

Liao, PHJ et al. 2023

- The current simulations have used a simplified Bondi-Hoyle accretion model and we are now developing binary black hole feedback models based on using fitting formulae derived from detailed simulations of circumbinary discs (Duffell et al. 2020).
- Typically, the accretion drives the binary towards more equal SMBH masses. To be implemented in cosmological simulations.

5. Summary & Outlook

1. The KETJU code is a version of Gadget that includes an algorithmically regularised region around each SMBH, enabling accurate small-scale dynamics.
2. A new version of the KETJU code based on GADGET4 has just been publicly released and it is ready to be downloaded at: **https://www.mv.helsinki.fi/home/phjohans/group-website/research/ketju/**
3. Cores form rapidly on the order of the crossing timescale by SMBH binary evolution, and the velocity distribution becomes increasingly more tangential over a longer timescale.
4. KETJU can be used to model the complex interactions of SMBHs in full hydrodynamical cosmological simulations and can also provide direct predictions of the expected GW signal.
5. LISA will be most sensitive to GW signals from SMBHs with masses in range 10^6-10^7 $M_\odot$, thus modelling the accurate small-scale dynamics simultaneously with the gas physics will be important.