Bimetric and multimetric theories of gravity

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Juri Smirnov

Disclaimer: Many more people have contributed to this field

Outline of the talk

Motivating multi spin-2 theories

Historical timeline

Ghost-free Bimetric theory

Uniqueness and the local structure of spacetime

Ghost-free multi spin-2 theories

Discussion

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What kind of theories do we consider in this talk?

General relativity:

The gravitational metric $g_{\mu\nu}(x)$ is a field of $spin = 2$ and $mass = 0$

Bimetric & multimetric theories:

Gravity ($g_{\mu\nu}$) coupled to other spin-2 fields, say ($f_{1\mu\nu}, f_{2\mu\nu}, \cdots$)

Spectrum:

A massless spin-2 state + massive spin-2 states

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** Are these theories useful?*

(dark matter, dark energy candidates)

** Why are they interesting?*
** What are the challenges?*
** What is the progress?*

Recall: Ghost instabilities in field theory

Ghost: A field with negative kinetic energy

Example:

$$\mathcal{L} = T - V = (\partial_t \phi)^2 \cdots \qquad (\textit{healthy})$$

But

$$\mathcal{L} = T - V = -(\partial_t \phi)^2 \cdots \qquad (\textit{ghostly})$$

Consequences:

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Consequences:

- Classical instability: unlimited energy transfer from ghost to other fields possible
- Negative quantum probabilities, violation of unitarity in quantum theory

Higher spin and the ghost problem

Number of propagating d.o.f. (n_{dof}) for a spin s field:

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But, Lorentz invariance (general covariance) requires a field with $s \geq 1$ to have more than $2s + 1$ components. Examples:

$s = 1 : n_{dof} = 2 \ or \ 3 < the \ \ 4$ components of A_μ

$s = 2 : n_{dof} = 2 \ or \ 5 < the \ \ 10$ components of $g_{\mu\nu}$

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The extra components contain ghost fields. Need to be eliminated by symmetries+constraints.

(Are there enough of these?)

Ex: The Boulware-Deser ghost (1972) of massive spin-2 fields $n_{dof} = 5 + 1$:

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- $s = 0$: $(\Box + m^2)\phi = 0$ *Klein-Gordon*
- $s = \frac{1}{2}$: $(i\gamma^\mu \partial_\mu - m)\psi = 0$ *Dirac*
- $s = 1$: $D_\mu F^{\mu\nu} = 0$ *Maxwell* (+ *Yang-Mills*)
- $s = 2$: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$ *Einstein*
- String theory

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Standard Model: multiplets of $s = 0, \frac{1}{2}, 1$ + *intricate structures*

General Relativity: The *simplest possible* theory of $s = 2$

Beyond GR: What are the possibilities?

Recall: Spin based classification of theories

- $s < 2$: Well known field theories *(e.g. in Standard Model)*
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  (*The spin-2 equivalent of* $\Box\phi = 0$ *&* $\partial_\mu F^{\mu\nu} = 0$)

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  $\phi \rightarrow$ *Higgs multiplet,*
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Do theories of *multiple* spin-2 fields exist? Or, is GR unique?

(Unexplored corner of the theory space)

Recap: why are multiple spin-2 theories interesting?

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- Uncharted corner of the space of local field theories, difficult to probe.
- Features relevant to gravity, dark matter, dark energy, inflation, etc.
- Not demanded by experiment, but motivated by experience!

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Historical timeline

- Einstein (GR and linearized gravity) (1915-17)
- Fierz and Pauli (linearized massive gravity) (1939)
- van Dam, Veltman, Zakharov (1970)
- Vainshtain (1972)
- Boulware, Deser (1972)
- Isham, Salam, Strathdee (1971-79)
- Creminelli, Nicolis, Papucci, Trincherini (2005)
- de Rham, Gabadadze (2010)

Outline of the talk

GR + a generic spin-2 field

A dynamical theory of the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{|g|} R - \sqrt{|g|}\, V(g^{-1} f) +$$

Digression:

Non dynamical $f_{\mu\nu} = \eta_{\mu\nu}$: Massive Gravity

describes a massive spin-2 (5 helicities) + a ghost (1 helicity)

A very special $V(g^{-1}\eta) \Rightarrow$ ghost-free massive gravity:

[Creminelli, Nicolis, Papucci, Trincherini, (2005)]

[de Rham, Gabadadze (2010); de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]

GR with a generic spin-2 field

A dynamical theory of the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2\sqrt{|g|}R - \sqrt{|g|}\,V(g^{-1}f) + \mathcal{L}(f, \nabla f)$$

- what is $V(g^{-1}f)$?
- what is $\mathcal{L}(f, \nabla f)$?
- proof of absence of the Boulware-Deser ghost

Recall: Elementary symmetric polynomials $e_n(S)$

For a 4×4 matrix S with eigenvalues $\lambda_1, \cdots, \lambda_4$,

$$
\begin{aligned}
e_1(S) &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\,, \\
e_2(S) &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4, \\
e_3(S) &= \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\,, \\
e_4(S) &= \lambda_1\lambda_2\lambda_3\lambda_4\,, \qquad e_{n>4}(S) = 0\,.
\end{aligned}
$$

$$
\begin{aligned}
e_0(S) &= 1\,, \\
e_1(S) &= \mathrm{Tr}(S) \equiv [S]\,, \\
e_2(S) &= \tfrac{1}{2}([S]^2 - [S^2]), \\
e_3(S) &= \tfrac{1}{6}([S]^3 - 3[S][S^2] + 2[S^3])\,, \\
e_4(S) &= \det(S)\,, \qquad e_{n>4}(S) = 0\,.
\end{aligned}
$$

$$
\boxed{\det(\mathbb{1} + S) = \textstyle\sum_{n=0}^{4} e_n(S)}
$$

The interaction potential:

$$\det(\mathbb{1} + S) = \sum_{n=0}^{4} e_n(S)$$

$$V(S) = \sum_{n=0}^{4} \beta_n \, e_n(S)$$

Where:

$$S^\mu_{\ \nu} = \left(\sqrt{g^{-1} f}\right)^\mu_{\ \nu}$$

(“a” square root of the matrix $g^{\mu\lambda} f_{\lambda\nu}$. More on this later $\cdots$)

[de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]

Ghost-free "bi-metric" theory

[SFH, Rosen (1109.3515,1111.2070)]

Ghost-free combination of *kinetic* and *potential* terms:

$$\mathcal{L} = m_g^2 \sqrt{|g|} R_g - \sqrt{|g|} \sum_{n=0}^{4} \beta_n \, e_n \left(\sqrt{g^{-1} f} \right) + m_f^2 \sqrt{|f|} \, R_f$$

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- Bimetric structure
- $7 = 2 + 5$ nonlinear propagating modes, no BD ghost!
- No ghost $\Rightarrow$ minimal matter couplings:

$$\mathcal{L}_{min}(g, \psi) + \mathcal{L}_{min}(f, \psi')$$

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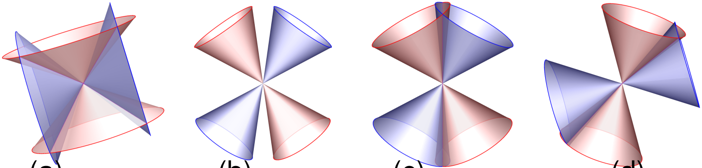
Potential consistency problems and their solutions

A Potential problem: Incompatible spacetimes

$g_{\mu\nu}$ & $f_{\mu\nu}$ may not admit compatible notions of *space* and *time*
(3+1 splits)



(a) (b) (c) (d)

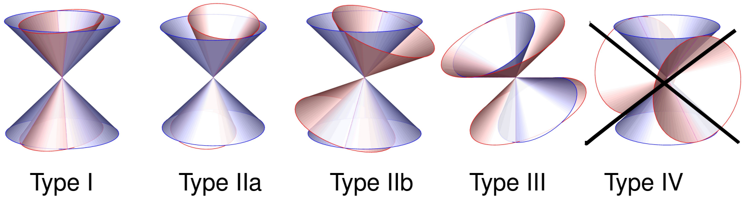
Then:

No consistent time evolution, no Hamiltonian formulation

Uniqueness and the local structure of spacetime

[SFH, M. Kocic (arXiv:1706.07806)]

The only allowed configurations: are when the null cones of $g_{\mu\nu}$ and $f_{\mu\nu}$ intersect:



(Implication for accausality arguments in the literature)

GR limit and applications

* Example of cosmological solution in the GR limit
$(m_g = M_P \, , m_f / m_g \to 0)$

$$3H^2 = \frac{\rho}{M_{Pl}^2} - \frac{2}{3}\frac{\beta_1^2}{\beta_2} m^2 - \alpha^2 \frac{\beta_1^2}{3\beta_2^2} H^2 + \mathcal{O}(\alpha^4)$$

[Akrami, SFH,Konnig,Schmidt-May,Solomon (arXiv:1503.07521)]

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* More on bimetric cosmology: *(Mörtsell, Högas, Enander)*

* Gravitational waves *(Smirnov et al.)*

* Numerical methods *(Mikica, Torsello)*

* Massive spin-2 particle as a dark matter candidate

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Beyond two spin-2 fields: do they exist?

The structure is not fully known. Easier to investigate in terms of vielbeins $e^A{}_\mu$. Recall

$$g_{\mu\nu} = \eta_{AB}\, e^A{}_\mu e^B{}_\nu$$

$g_{\mu\nu}$: 10 components

$e^A{}_\mu$: 16 components

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$e^A{}_\mu$: 16 components

A useful parametrization:

$$e^A{}_\mu = L^A_B\, \hat{e}^B{}_\mu$$

L^A_B: a local Lorentz transformation with 6 parameters (3 Lorentz boosts + 3 rotations)

$\hat{e}^B{}_\mu$: A "gauge fixed" vielbein fully parameterized by the 10 parameters of $g_{\mu\nu}$

Ghost-free multi spin-2 theories

[SFH, Angnis Schmidt-May (arXiv:1804.09723)]

[SFH, Joakim Flinckman (to appear)]

Certain genuine multi spin-2 interactions for $(e_I)^A{}_\mu$ can be constructed. E.g.,

$$\mathcal{L} = \sum_{I=1}^{N} m_I^2 \sqrt{|g_I|} R(g_I) - 2M^4 \det\left(\beta^1 e_1 + \beta^2 e_2 + \cdots + \beta^N e_N\right)$$

- Has the correct number of constraints to eliminate the ghosts.

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- Has the correct number of constraints to eliminate the ghosts.
- Certain generalizations exist

Vielbein EoM's and antisymmetrization conditions

$$\mathcal{L} = \sum_{I=1}^{N} m_I^2 \sqrt{|g_I|}\,(R(g_I) - 2\lambda_I) - 2M^4 \det(\sum_{I=1}^{N} \beta^I e_I)$$

Vielbein EoMs:

$$R_{I\mu\nu} - \frac{1}{2} g_{I\mu\nu} R_I + V^I_{(\mu\nu)} + V^I_{[\mu\nu]} = 0 \quad \Rightarrow V^I_{[\mu\nu]} = 0$$

$$N = 2 : \quad (e_1)^A_{\ [\mu} \eta_{AB} (e_2)^B_{\ \nu]} = 0 \quad \Longleftrightarrow \quad \text{evaluation of } \sqrt{g^{-1} f}$$

$$N \quad : \qquad (e_1)^A_{\ [\mu} \eta_{AB} (e_2 + e_3 + \cdots + e_N)^B_{\ \nu]} = 0\,, \quad etc.$$

Implications:

* The structure of null cones (3+1decompositions)
* Absence of ghosts
* More general vielbein interactions

Mass matrix

Mass eigenstates exist around proportional backgrounds

$$(\bar{e}_I)^A{}_\mu = c_I\,\bar{e}^A{}_\mu \qquad (Einstein\;\; spacetimes)$$

Cosmological constant: $\Lambda = c_I^2 \lambda_I + M^4 \frac{\beta_I}{m_I^2 c_I} (\sum_J^N c_J \beta_J)^3$

(*these determine the* c_I)

Parametrization of fluctuations (computable to all orders):

$$(e_I)^A{}_\mu = L^A_{IB}\,(\hat{e}_I)^A{}_\mu = (\eta + A_I)^{-1}(\eta - A_I)\left(c_I\,\bar{e}^A{}_\mu + E^A_{I\mu}(\delta g_I)\right)$$

$$Mass\;\; matrix: \qquad M_{IJ} = \frac{1}{4} M^4 k^2 \left(k \frac{\beta_I}{m_I^2 c_I} \delta_{IJ} - \frac{\beta_I \beta_J}{m_I m_J} \right)$$

Easy to see the mass = 0 eigenstate.

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The beginnings of understanding spin-2 fields beyond General Relativity

- Causality
- Superluminality? (yes, not necessarily harmful)
- Unavoidable mixings of mass eigenstates (unlike neutrino mixings)
- Systematics of multispin-2 interactions? Though certain "basic" extensions can be constructed and argued to be ghost free. Is there a formulation purely in terms of metrics?
- Extra symmetries $\Rightarrow$ Modified kinetic terms? MacDowell-Mansouri type theories. More interesting but less understood.

Thank you!