THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

EFFECTS OF WAVE PACKET PROFILES ON NEUTRINO OSCILLATIONS

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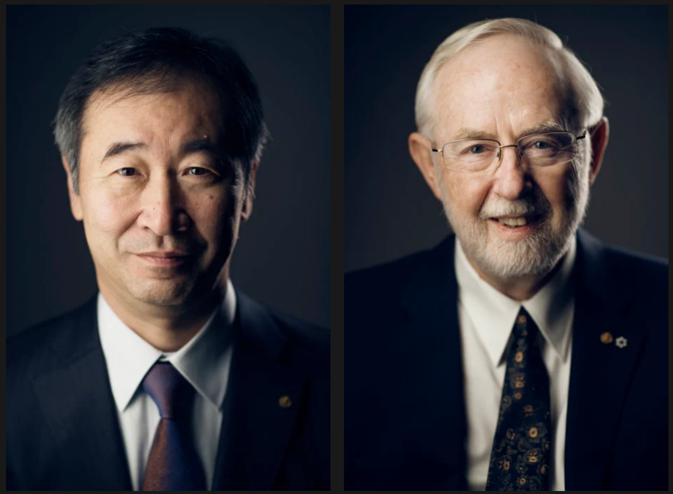

Takaaki Kajita & Arthur B. McDonald, Nobel Prize in Physics 2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

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where $T = t_d - t_p$ and $L = x_d - x_p$.

Transition probability found by

$$P_{\nu_\alpha \to \nu_\beta}(L) = \int dT \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \right|^2 .$$

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$$\psi_i(t,x) = \frac{1}{\sqrt{2\pi}} \int dp_i \, \widetilde{\psi}_i(p_i) e^{-i(E_i t - p_i x)} \ .$$

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Kayser [4] first to study oscillations with wave packets, and Giunti [5] first to obtain explicit results with Gaussians

[3] S. Nussinov, Phys. Lett. B **63**, 201 (1976).
[4] B. Kayser, Phys. Rev. D **24**, 110 (1981).
[5] C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D **44**, 3635 (1991).

Why a Gaussian?

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"A gaussian momentum distribution is the most convenient one for the calculation of several integrations …

Other distributions which are sharply peaked around an average momentum lead to the same results after their approximation with a gaussian …

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Expanding about minimum $p = P$, we require

$$\frac{1}{4!}\left|g^{(iv)}(P)\right| \ll \frac{1}{2}\left|g''(P)\right|^2$$

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Neutrinos in the context of the Mössbauer effect, described by a Lorentzian wave packet [8, 9]

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$$\tilde{\psi}(p;\bar{p},\gamma) = \mathcal{N}\left[\frac{\gamma}{(p-\bar{p})^2+\gamma^2}\right]$$

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Moments undefined. Cannot be approximated by a Gaussian!

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$$\tilde{\psi}(p_\mu; a_\mu) = \mathcal{N} \exp\left[-a_\mu p^\mu\right],$$

where $a_\mu = (\alpha, -\beta) \in \mathrm{C}^2$ and transforms as a vector.

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Gaussian minimises Heisenberg-Robertson uncertainty relation

$$\sigma_x\sigma_p = \left|\frac{1}{2i}\langle[\hat{x},\,\hat{p}]\rangle\right| = \frac{\hbar}{2}.$$

Assuming that position and momentum are independent

Robertson-Schrödinger uncertainty relation

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$$\tilde{\psi}(E_p, p; \alpha, \beta) = \mathcal{N} \exp[-\alpha E_p + \beta p] \,,$$

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RMU:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(T, L) \sim \sum_i U^*_{\alpha i} \frac{(T - i\alpha) m_i K_1\left(-m_i \sqrt{(T - i\alpha)^2 + (L - i\beta)^2} \right)}{\sqrt{(T - i\alpha)^2 + (L - i\beta)^2}} U_{\beta i}$$

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2. RMU wave packets follow semi-classical trajectories, so different mass eigenstates do not separate!

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Controversy in early 2000s whether group velocities can be equal [12-14]

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3. $E_1/E_2 \simeq 1$ in ultra-relativistic regime, not generally true for m_1/m_2

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Only agrees in semi-classical regime, when the wave packets are sufficient spread with respect to the Compton scale

Summary

- Neutrino wave packets could have a non-Gaussian profile
- If described by RMU wave packets, then neutrinos are highly localised, and decoherence is heavily suppressed
- Propagation at equal velocities should be taken seriously, and could be experimentally tested

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