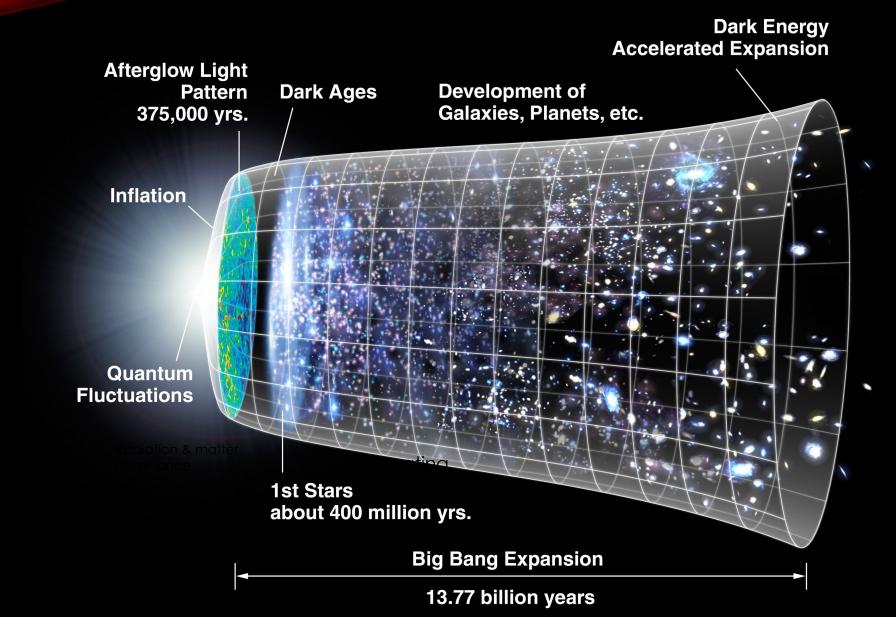
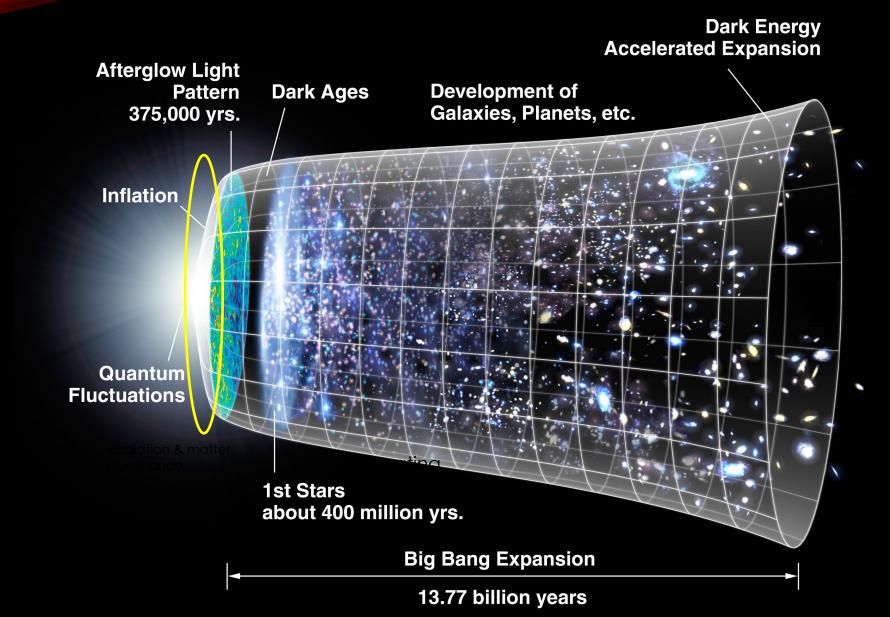
# Non-Gaussianity in rapid-turn multi-field inflation

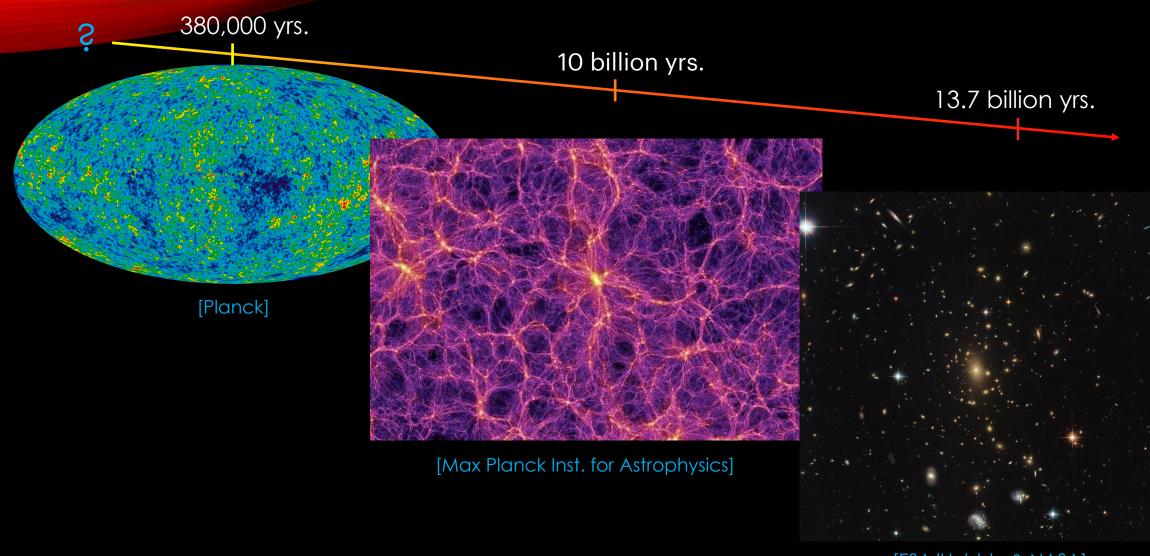
Oksana larygina

OKC @ 15, Stockholm 18 October 2023

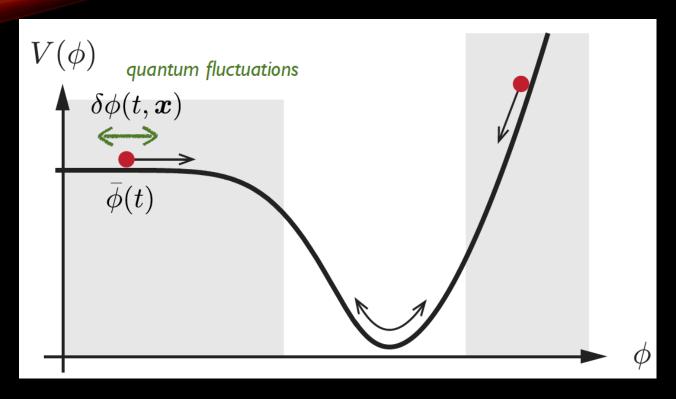








[ESA/Hubble & NASA]



[Baumann]

$$\delta\phi(x)$$

 $\delta \rho(x)$ 

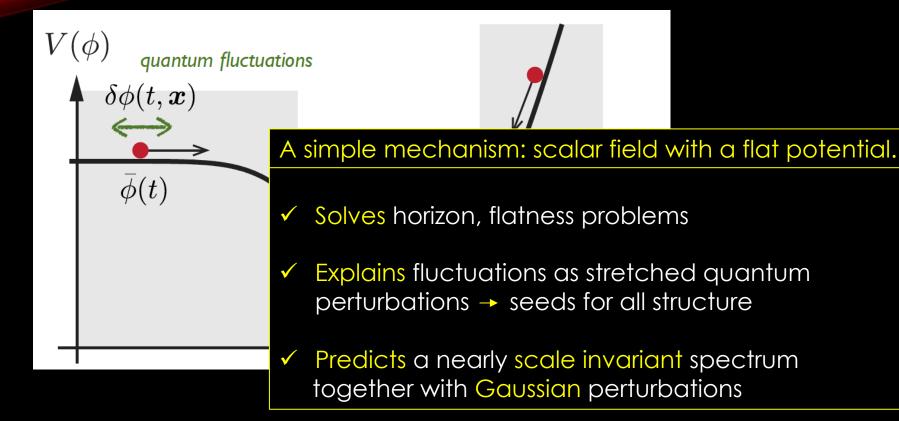


 $\delta T(x)$ 

Quantum vacuum fluctuations around the inflaton vev

...translate into classical density fluctuations after inflation

...which become the CMB anisotropies.



 $\delta\phi(x)$ 

Quantum vacuum fluctuations around the inflaton vev

 $\delta \rho(x)$ 



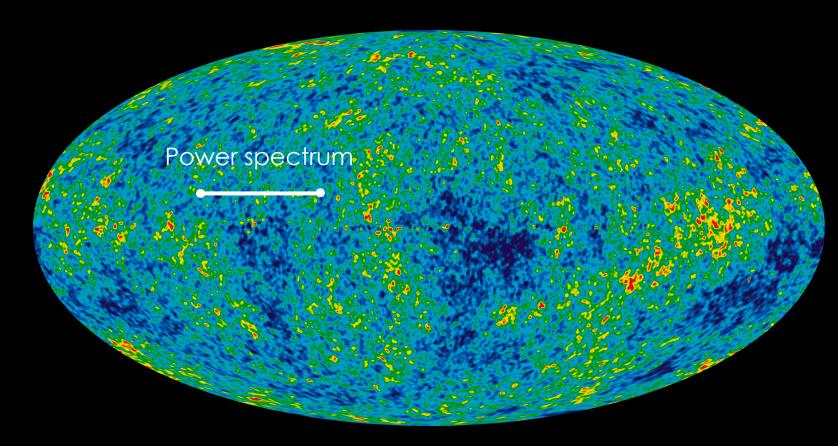
 $\delta T(x)$ 

...translate into classical density fluctuations after inflation

...which become the CMB anisotropies.

$$P_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\bar{\phi}}}\right)^2 P_{\delta\phi}(k)$$

$$\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = (2\pi)^3 \delta^{(3)}(k+k') P_{\mathcal{R}}(k)$$



$$\Delta_{\mathcal{R}}^{2}(k) = \frac{k^{3}}{2\pi^{2}} P_{\mathcal{R}}(k) = A_{s}(k_{*}) \left(\frac{k}{k_{*}}\right)^{n_{s}-1}$$

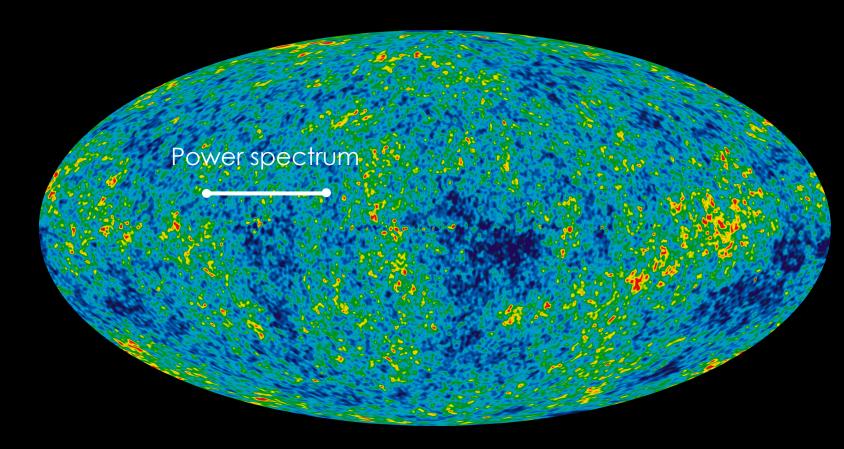
amplitude of the scalar power spectrum

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2(k)}{d \ln k}$$

scalar spectral index

$$P_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\bar{\phi}}}\right)^2 P_{\delta\phi}(k)$$

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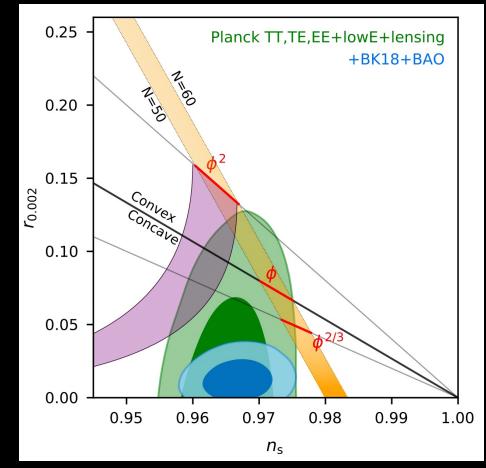
amplitude of the scalar power spectrum

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2(k)}{d \ln k}$$

scalar spectral index



tensor-toscalar ratio



[BICEP/Keck]

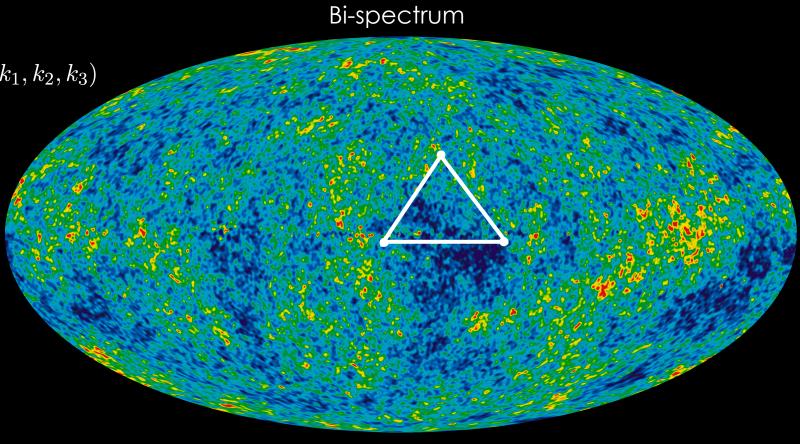
$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2(k)}{d \ln k}$$

scalar spectral index

$$n_s = 0.9603 \pm 0.0073$$

 $\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3)$ 

Result of non-linear evolution of initially Gaussian fluctuations.

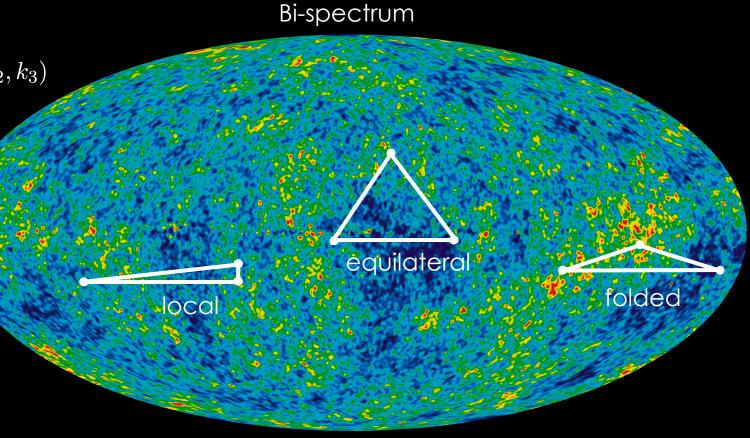


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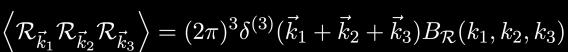
## $\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3)$

Result of non-linear evolution of initially Gaussian fluctuations.

$$B_{\mathcal{R}}(k_1, k_2, k_3) \propto \sum_{\mathrm{type}} f_{\mathrm{NL}}^{\mathrm{type}} S_{\mathrm{type}}(k_1, k_2, k_3)$$

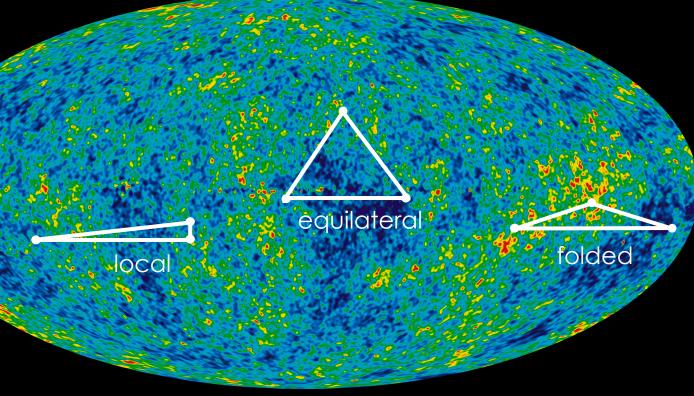


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 $B_{\mathcal{R}}(k_1,k_2,k_3) \propto \sum_{ ext{type}} f_{ ext{NL}}^{ ext{type}} S_{ ext{type}}(k_1,k_2,k_3)$ 

The amount of non-Gaussianity is quantified by the parameter



$$-\frac{6}{5}f_{NL} = \frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (\vec{k} \text{ cyclic perms})}$$

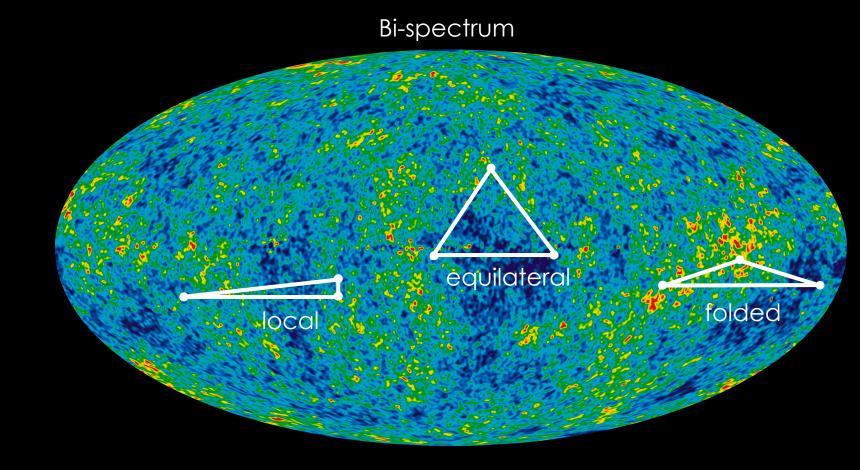
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[Maldacena, 2002]

$$f_{\rm NL}^{
m loc} = 0$$

[Tanaka & Urakawa, 2011] [Pajer, Schmidt, Zaldarriaga, 2013]



Single-field models of inflation most strongly couple momenta of similar wavelengths and result in bispectra that are highly suppressed in the 'squeezed limit' where one long-wavelength-mode couple to two short-wavelength-modes.

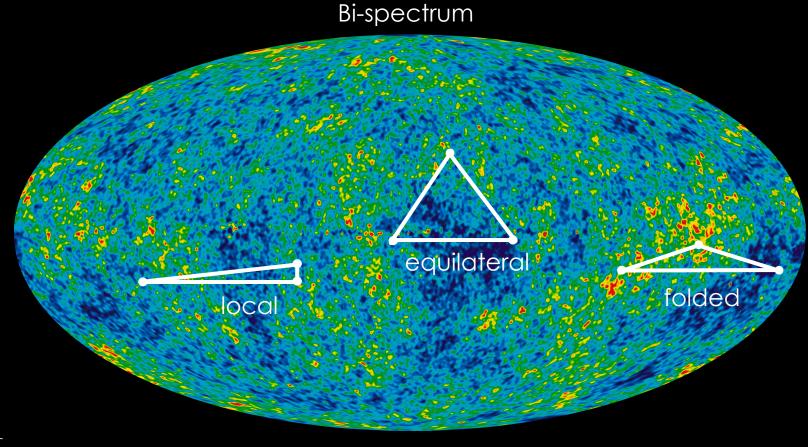


[Maldacena, 2002]

$$f_{\rm NL}^{
m loc} = 0$$

[Tanaka & Urakawa, 2011] [Pajer, Schmidt, Zaldarriaga, 2013]

CMB constraint:  $f_{
m NL}^{
m loc} = -0.9 \pm 5.1$ 



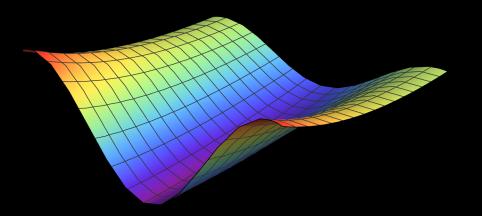
Detection of  $f_{
m NL}^{
m loc}\simeq\mathcal{O}\left(1
ight)$  would rule out all attractor models of single-field inflation!

#### Multi-field inflation and turning trajectory

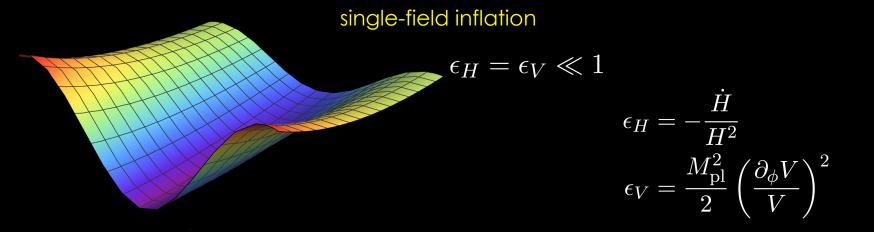
$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} G_{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^a) \right]$$
 field-space metric multi-field potential

#### Multi-field inflation and turning trajectory

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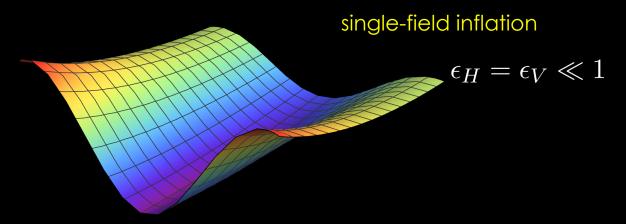


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### Multi-field inflation and turning trajectory

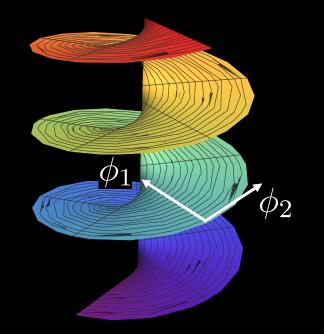
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 field-space metric multi-field potential



generic potentials:  $\epsilon_V \sim \mathcal{O}(1)$ 

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 field-space metric multi-field potential



In multi-field inflation

$$\epsilon_H \ll 1$$

 $\epsilon_V \sim \mathcal{O}(1)$ 

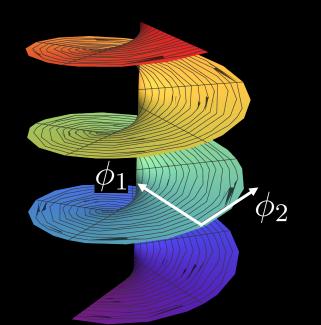
can coexist when  $\eta_{\perp}\gg 1$ 

$$\eta_{\perp} \gg 1$$

$$\epsilon_V = \epsilon_H \left( 1 + \eta_\perp^2 / 9 \right)$$

[Achucarro at al, arXiv: 1807.04390]

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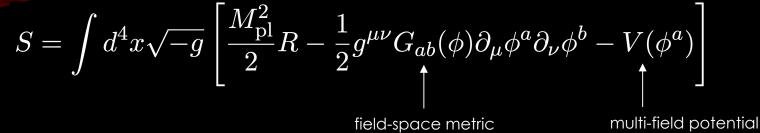
$$\eta_{\perp} \gg 1$$

$$\epsilon_V \sim \mathcal{O}(1)$$

$$\epsilon_V = \epsilon_H \left( 1 + \eta_\perp^2 / 9 \right)$$

[Achucarro at al, arXiv: 1807.04390]

Rapid-turn inflation!





Turn-rate:  $\eta_{\perp}$ 

$$D_N e^a_{\parallel} = \eta_{\perp} e^a_{\perp}$$

Trajectory turns couple the fluctuations and modify their dispersion relations and correlators.

 $\phi_1$ 

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} G_{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^a) \right]$$
 field-space metric multi-field potential

 $\phi_2$ 



Two types of field perturbations:

- Adiabatic (curvature) -- along trajectory  ${\cal R}$ 

Non-Adiabatic (isocurvature) --- orthogonal to trajectory  ${\mathcal S}$ 

 $\phi_1$ 

#### Classification of perturbations

[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000]

[L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002]

[D. Wands, N. Bartolo, S. Matarrese, A. Riotto,2002]



$$\begin{cases} \dot{\mathcal{R}} & \simeq \alpha H \mathcal{S} \\ \dot{\mathcal{S}} & \simeq \beta H \mathcal{S} \end{cases}$$

$$\alpha = 2 \, \eta_{\perp}$$

$$eta = -2\epsilon - rac{\mathcal{M}_{\perp\perp}}{V} + rac{\mathcal{M}_{\parallel\parallel}}{V} - rac{4}{3}\left(\eta_{\perp}
ight)^2$$

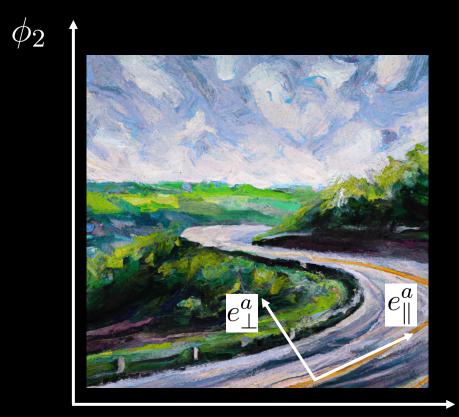
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$$\phi_1$$

The sourcing of curvature perturbations by isocurvature perturbations is proportional to the turn-rate!

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#### Classification of perturbations

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$$\begin{cases} \dot{\mathcal{R}} & \simeq \alpha H \mathcal{S} \\ \dot{\mathcal{S}} & \simeq \beta H \mathcal{S} \end{cases}$$

$$lpha = 2 \, \eta_{\perp}$$
 $eta = -2\epsilon - \underbrace{\left( \frac{\mathcal{M}_{\perp \perp}}{V} \right)}_{V} \underbrace{\left( \frac{\mathcal{M}_{\parallel \parallel}}{V} \right)}_{2} \frac{4}{3} \left( \eta_{\perp} \right)^{2}$ 

$$\mathcal{M}^a{}_b = G^{ac} \nabla_b \nabla_c V - R^a_{dfb} \dot{\phi}^d \dot{\phi}^f$$

Determined by potential & geometry of field-space.

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[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000]

[L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002]

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$$\begin{cases} \dot{\mathcal{R}} & \simeq \alpha H \mathcal{S} \\ \dot{\mathcal{S}} & \simeq \beta H \mathcal{S} \end{cases}$$

$$\alpha = 2 \eta_{\perp}$$

$$\beta = -2\epsilon - \frac{\mathcal{M}_{\perp \perp}}{V} + \frac{\mathcal{M}_{\parallel \parallel}}{V} - \frac{4}{3} (\eta_{\perp})^2$$

isocurvature perturbations.

$$\begin{split} \langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\parallel \vec{k}_1} \delta \phi_{\parallel \vec{k}_2} \rangle \ \Rightarrow \ P_{\mathcal{R}} & \text{Power spectrum of curvature perturbations,} \\ \langle \mathcal{R}_{\vec{k}_1} \mathcal{S}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\parallel \vec{k}_1} \delta \phi_{\perp \vec{k}_2} \rangle \ \Rightarrow \ C_{\mathcal{RS}} & \text{cross-correlation,} \\ \langle \mathcal{S}_{\vec{k}_1} \mathcal{S}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\perp \vec{k}_1} \delta \phi_{\perp \vec{k}_2} \rangle \ \Rightarrow \ P_{\mathcal{S}} & \text{isocurvature perturbatio} \end{split}$$

18 October 2023 Oksana larygina [C. Peterson, M. Tegmark, 2011]

Slow-turn:  $\eta_{\perp} \ll 1$ 

$$f_{
m NL}^{
m loc} \supset rac{5}{6} \sqrt{rac{r}{8}} \left(rac{T_{\mathcal{RS}}}{\sqrt{1+T_{\mathcal{RS}}^2}}
ight)^3 \partial_{\perp *} \ln T_{\mathcal{RS}}$$

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{R}\mathcal{S}} \\ 0 & T_{\mathcal{S}\mathcal{S}} \end{pmatrix} \begin{pmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{pmatrix}$$

[C. Peterson, M. Tegmark, 2011]

Slow-turn:  $\eta_{\perp} \ll 1$ 

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What is different in the rapid-turn regime?  $\eta_{\perp}\gg 1$ 







M.C. David Marsh

Gustavo Salinas

[arXiv:2303.14156]

#### Non-Gaussianity in rapid-turn multi-field inflation

Oksana Iarygina<sup>1,2</sup>, M.C. David Marsh<sup>2</sup>, Gustavo Salinas<sup>2\*</sup>

<sup>1</sup>Nordita, KTH Royal Institute of Technology and Stockholm University,

Hannes Alfvéns väg 12, SE-106 91 Stockholm, Sweden

<sup>2</sup> The Oskar Klein Centre, Department of Physics,

Stockholm University, Stockholm 106 91, Sweden

Slow-turn: 
$$\eta_{\perp} \ll 1$$

$$\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P^*_{\phi}(k_1) \delta^{ab}$$

Slow-turn: 
$$\eta_{\perp} \ll 1$$

$$\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P_\phi^* (k) \delta^{ab}$$
$$P_{\mathcal{R}*} = P_{\mathcal{S}*}, \quad C_{\mathcal{R}\mathcal{S}*} = 0$$

Slow-turn: 
$$\eta_{\perp} \ll 1$$

$$\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P^*_{\phi}(k_1) \delta^{ab}$$

$$P_{\mathcal{R}*} = P_{\mathcal{S}*}, \quad C_{\mathcal{R}\mathcal{S}*} = 0$$

Rapid-turn:  $\eta_{\perp} \gg 1$ 

$$\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2 (P_{\phi}^{*ab}(k_1)))$$

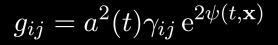
$$P_{\mathcal{R}*} \neq P_{\mathcal{S}*} \neq C_{\mathcal{R}\mathcal{S}*} \neq 0$$

#### δN -formalism

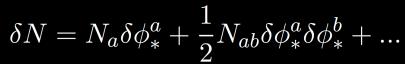
[Sasaki and Stewart,(1996)] [Wands, Malik, Lyth, Liddle (2000)]

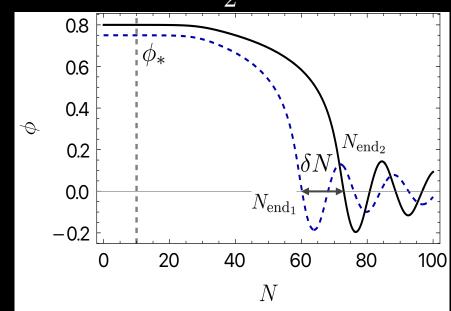
uniform density hypersurface

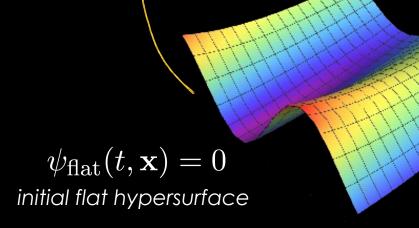
 $\overline{|\psi_{
m UD}(t,{f x})}\equiv \zeta(t,{f x})^{\dagger}$ 



$$\zeta(t, \mathbf{x}) = \delta N = N(t, \mathbf{x}) - N_0(t)$$







[D.H. Lyth and Y. Rodriguez, (2005)]

Slow-turn:  $\eta_{\perp} \ll 1$ 

$$-\frac{6}{5}f_{\rm NL} = \frac{N_a N_b N^{ab}}{(N_c N^c)^2}$$

[OI, D. Marsh, G. Salinas, 2023]

Rapid-turn:  $\eta_{\perp} \gg 1$ 

$$-\frac{6}{5}f_{\rm NL} = \frac{N_a N_b N_{cd} \left[ P_{\phi}^{*ac} (k_1) P_{\phi}^{*bd} (k_2) + (\vec{k} \text{ cyclic perms}) \right]}{N_e N_f N_g N_h \left[ P_{\phi}^{*ef} (k_1) P_{\phi}^{*gh} (k_2) + (\vec{k} \text{ cyclic perms}) \right]}$$

Rapid-turn:  $\eta_{\perp}\gg 1$ 

[OI, D. Marsh, G. Salinas, 2023]

$$-\frac{6}{5}f_{\rm NL}(k_1, k_2, k_3) = \sum_{I,J=\mathcal{R},\mathcal{C}} f_{\rm NL}^{IJ} \frac{\tilde{\mathcal{P}}^I(k_1)\tilde{\mathcal{P}}^J(k_2) + (\vec{k} \text{ cyclic perms})}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (\vec{k} \text{ cyclic perms})}$$

$$f_{\rm NL}^{\mathcal{R}\mathcal{R}}, f_{\rm NL}^{\mathcal{R}\mathcal{C}}, f_{\rm NL}^{\mathcal{C}\mathcal{R}}, f_{\rm NL}^{\mathcal{C}\mathcal{C}}$$

$$\tilde{\mathcal{P}}^{\mathcal{R}}(k) = P_{\mathcal{R}}(k) , \ \tilde{\mathcal{P}}^{\mathcal{C}}(k) = C_{\mathcal{R}\mathcal{S}}(k)$$

Rapid-turn:  $\eta_{\perp}\gg 1$ 

[OI, D. Marsh, G. Salinas, 2023]

$$-\frac{6}{5}f_{\rm NL}(k_1, k_2, k_3) = \sum_{I,J=\mathcal{R},\mathcal{C}} f_{\rm NL}^{IJ} \frac{\tilde{\mathcal{P}}^I(k_1)\tilde{\mathcal{P}}^J(k_2) + (\vec{k} \text{ cyclic perms})}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (\vec{k} \text{ cyclic perms})}$$

$$f_{\rm NL}^{\mathcal{R}\mathcal{R}}, f_{\rm NL}^{\mathcal{R}\mathcal{C}}, f_{\rm NL}^{\mathcal{C}\mathcal{R}}, f_{\rm NL}^{\mathcal{C}\mathcal{C}}$$

$$\tilde{\mathcal{P}}^{\mathcal{R}}(k) = P_{\mathcal{R}}(k) , \ \tilde{\mathcal{P}}^{\mathcal{C}}(k) = C_{\mathcal{R}\mathcal{S}}(k)$$

Shape functions!

Rapid-turn:  $\eta_{\perp} \gg 1$ 

[OI, D. Marsh, G. Salinas, 2023]

Assuming the scale-invariant power spectrum, it reduces to:

$$f_{
m NL} \supset \eta_{\perp *} \, I_4 + ilde{M}_{\perp \perp *} \, I_5 + ilde{M}_{\perp \parallel *} \, I_6$$

$$I_i = I_i \left( T_{\mathcal{RS}}, \mathcal{C}_{\mathcal{RS}}, P_{\mathcal{S}}/P_{\mathcal{R}} \right)$$

New model-independent potentially large contributions to the non-Gaussianity parameter!

### **Example: Angular inflation model**

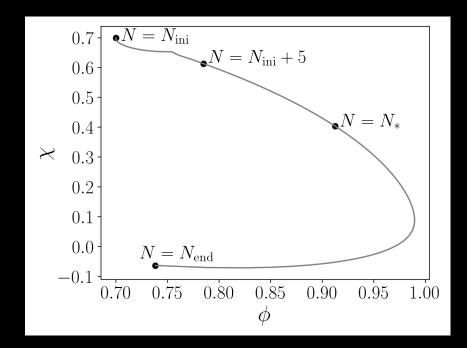
[P. Christodoulidis, D. Roest, E. I. Sfakianakis, 2019]

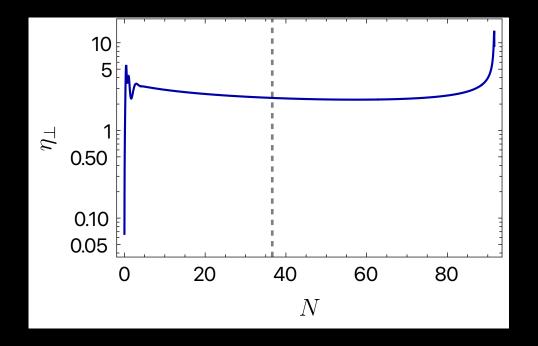
$$V(\phi, \chi) = \frac{\tilde{\alpha}}{2} \left( m_{\phi}^2 \phi^2 + m_{\chi}^2 \chi^2 \right)$$

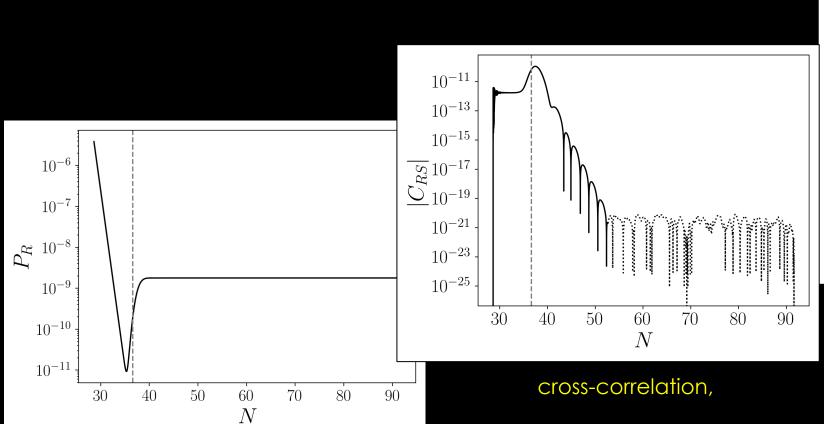
$$G_{ab} = \frac{6\tilde{\alpha}}{\left(1 - \phi^2 - \chi^2\right)^2} \delta_{ab}$$

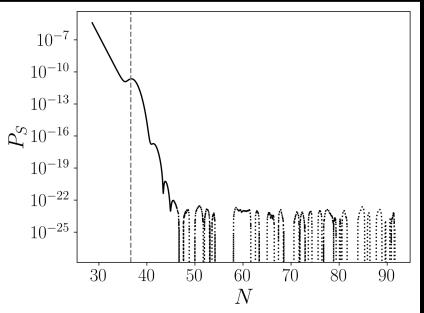
$$V(\phi, \chi) = \frac{\tilde{\alpha}}{2} \left( m_{\phi}^2 \phi^2 + m_{\chi}^2 \chi^2 \right)$$

$$G_{ab} = \frac{6\tilde{\alpha}}{\left(1 - \phi^2 - \chi^2\right)^2} \delta_{ab}$$





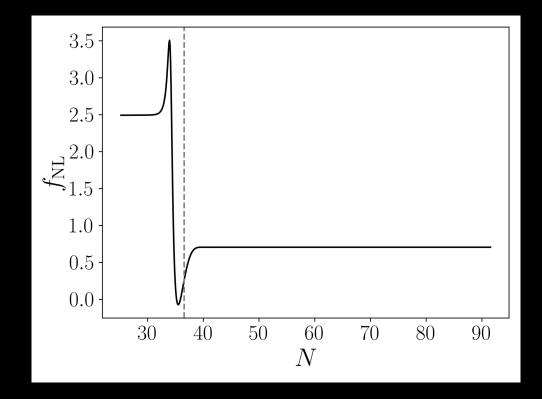




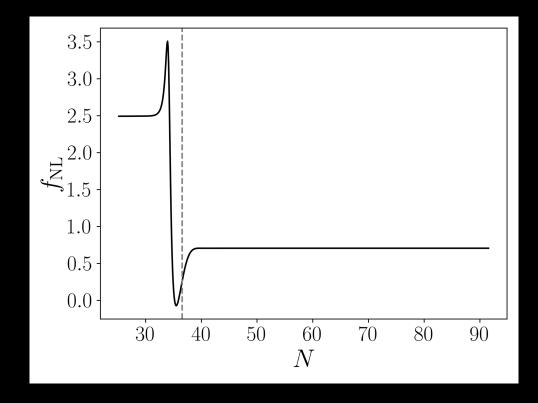
isocurvature perturbations.

Power spectrum of curvature perturbations,

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$$f_{\rm NL} = -\frac{5}{6} (0.006 I_{1*} + 1.89 I_{2*} + 0.004 I_{3*} - 2.35 I_{4*} - 0.015 I_{5*} + 2.3 I_{6*})$$



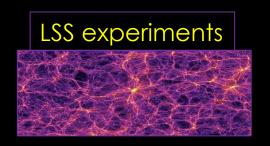
$$f_{\rm NL} = -\frac{5}{6} (0.006 I_{1*} + 1.89 I_{2*} + 0.004 I_{3*} - 2.35 I_{4*} - 0.015 I_{5*} + 2.3 I_{6*})$$

$$f_{\rm NL} = 0.93 \simeq \mathcal{O}\left(1\right)$$

### Conclusions

- 1. Extended the  $\delta N$  -formalism to rapid-turn inflation.
- 2. Identified new model-independent potentially large contributions to the non-Gaussianity parameter.
- 3. Detection of Non-Gaussianity  $f_{
  m NL}^{
  m loc} \simeq \mathcal{O}\left(1
  ight)$  signals:
- New particles inflation with more than one field, curved field-space, steep potentials, UV competions...
- OR non-inflationary perturbations?





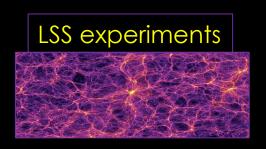
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#### Conclusions

# Thank you!

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## Back up slides

The effective mass of isocurvature perturbation:

$$\mu^2 = e_{\perp}^a e_{\perp}^b \nabla_a \nabla_b V + \epsilon H^2 \mathbb{R} + 3H^2 \eta_{\perp}^2,$$

[D.H. Lyth and Y. Rodriguez, (2005)]

$$\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = N_a N_b N_c \left\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \delta \phi^c_{*\vec{k}_3} \right\rangle$$
$$+ \frac{1}{2} N_a N_b N_{cd} \left\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \left( \delta \phi^c_{*} * \delta \phi^d_{*} \right)_{\vec{k}_3} \right\rangle + (\vec{k} \text{ cyclic perms}) + \cdots$$

$$-\frac{6}{5}f_{\rm NL}\approx -\frac{6}{5}f_{\rm NL}^{(3)}-\frac{6}{5}f_{\rm NL}^{(4)}$$
 horizon crossing super-horizon contribution evolution

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## The end