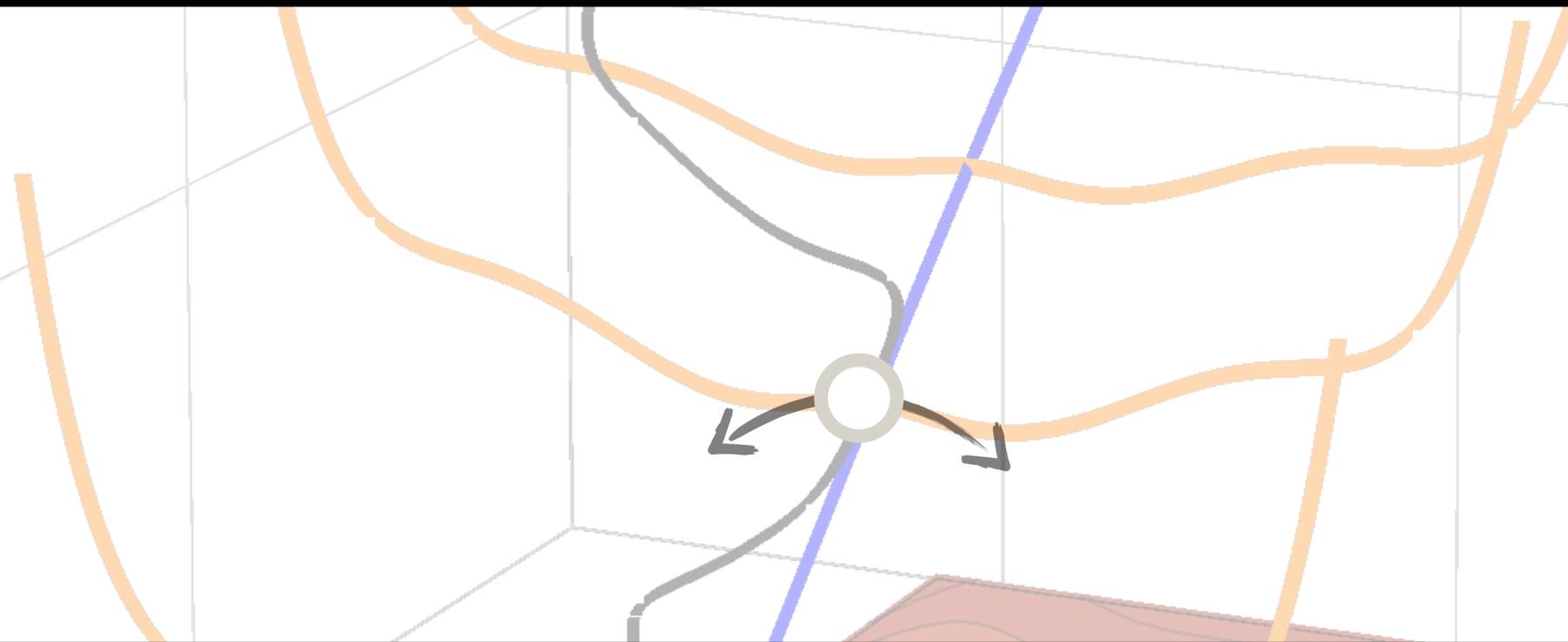


# PBHs and GWs in Higgs- $R^2$ Inflation



Dhong Yeon Cheong (Yonsei U. / CERN)

with Kazunori Kohri, Sung Mook Lee, Seong Chan Park

[DYC, S.M. Lee, S.C. Park, JCAP 01 \(2021\) 032 \(arXiv : 1912.12032\)](#)

[DYC, K. Kohri, S.C. Park, JCAP 10 \(2022\) 015 \(arXiv : 2205.14813\)](#)

# Outline

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Higgs- $R^2$  inflation + SM Higgs Running

# Outline

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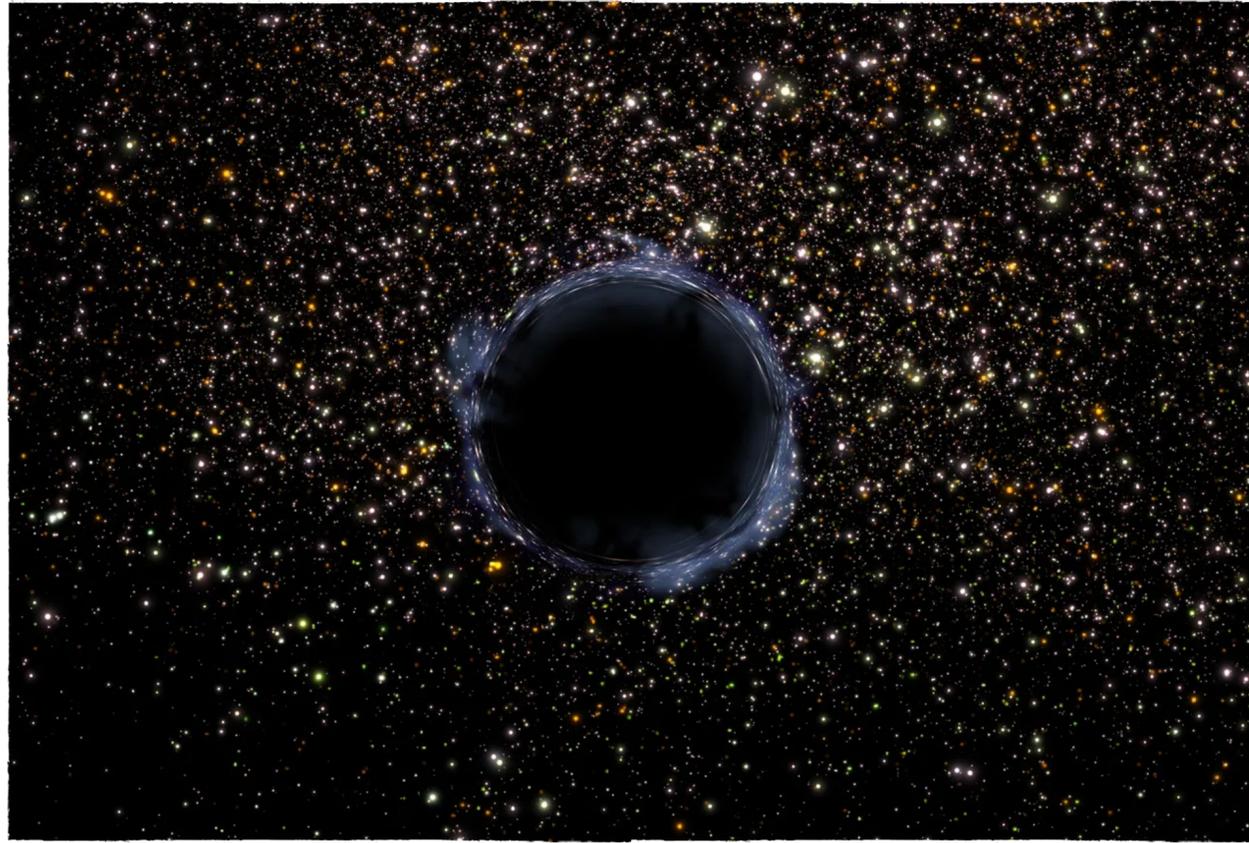
Higgs- $R^2$  inflation + SM Higgs Running



Observational Consequences!

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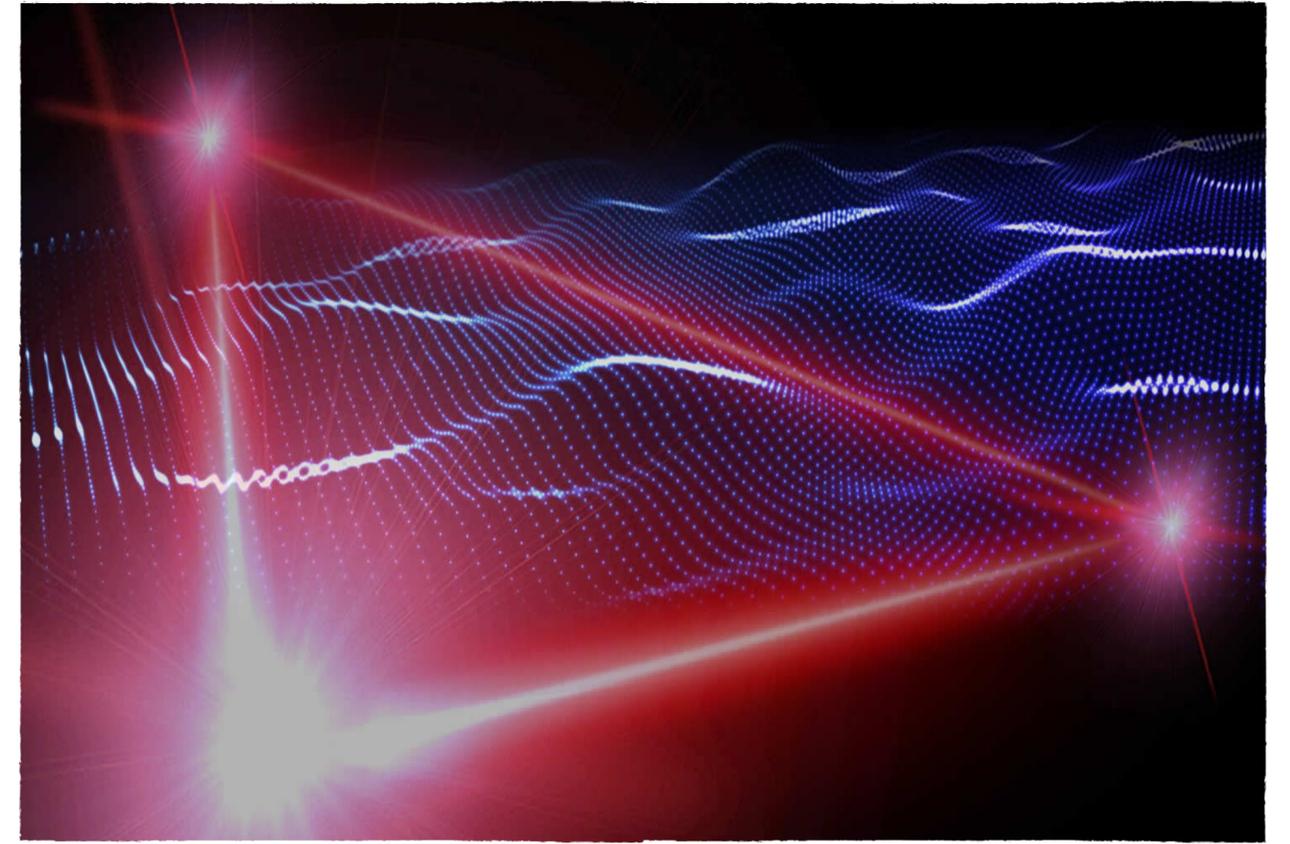
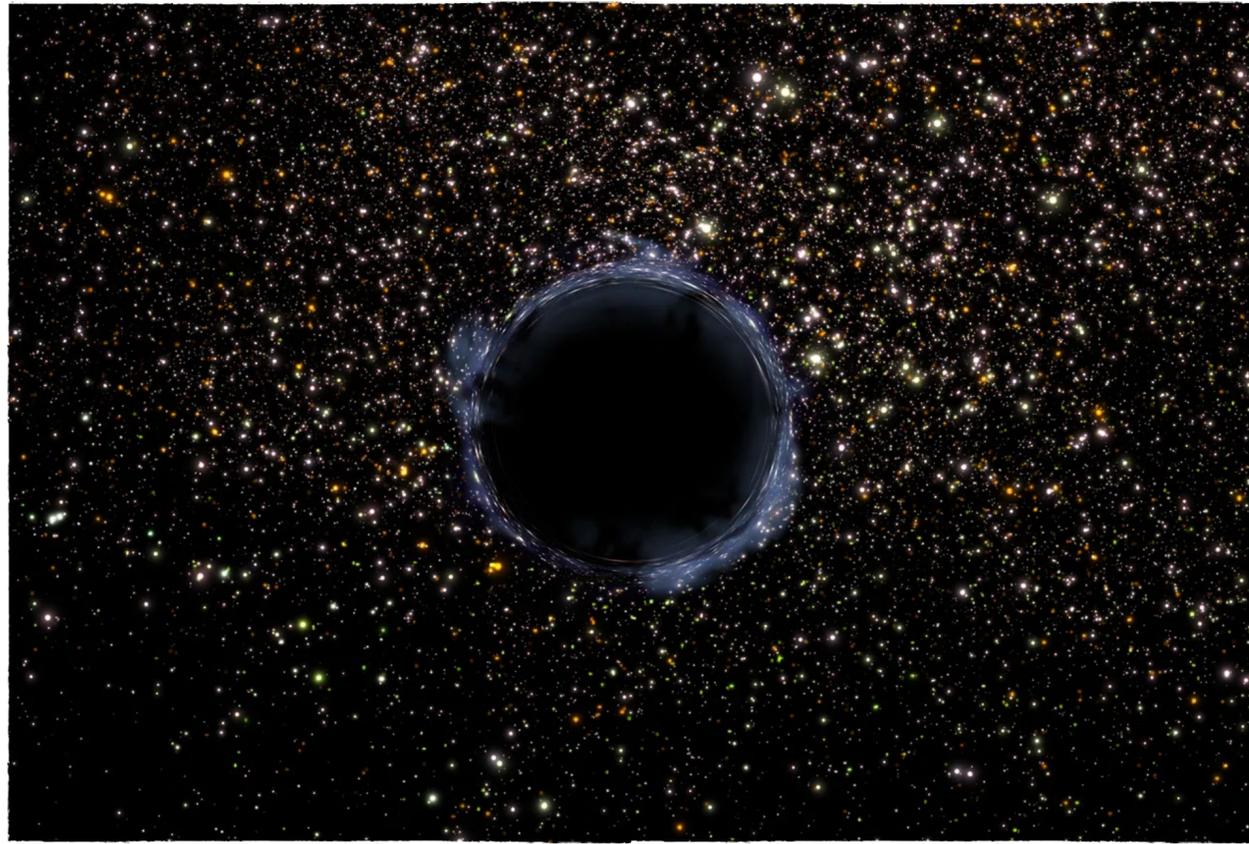
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Higgs- $R^2$  inflation + SM Higgs Running

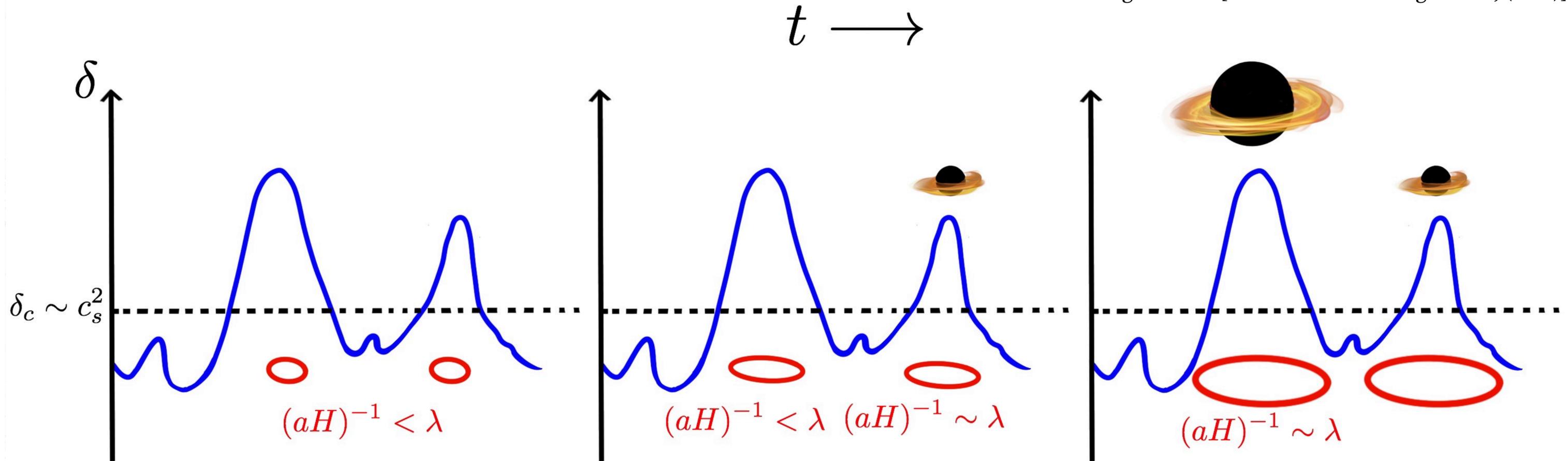


Observational Consequences!

# Primordial Black Holes

PBHs form through the collapse of large overdensities

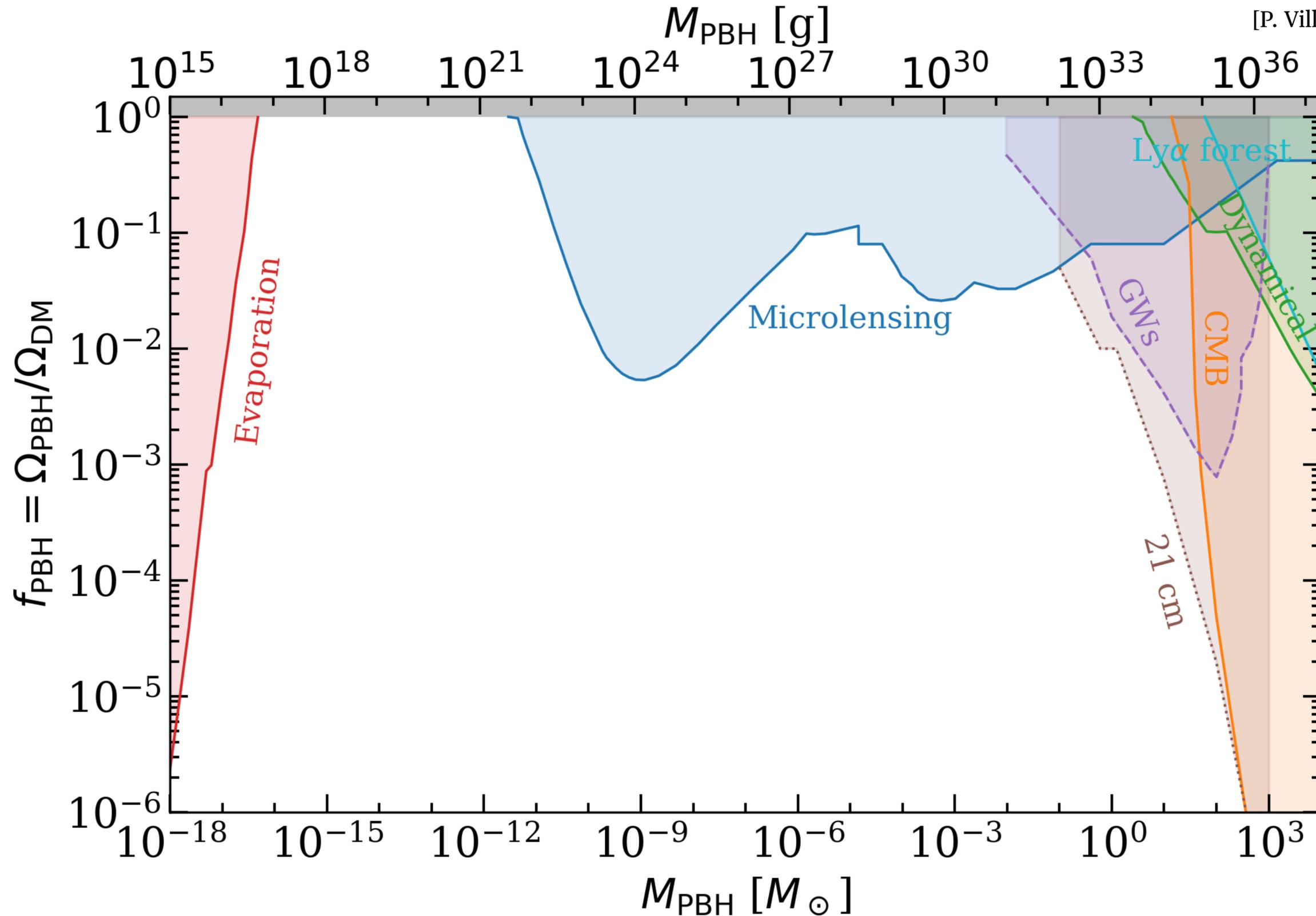
Figure from [P. Villanueva-Domingo *et. al.*, (2021)]



$$\delta_c \sim \mathcal{O}(1) \quad \longleftrightarrow \quad \mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(10^{-2})$$

# Primordial Black Holes - Constraints

[P. Villanueva-Domingo *et. al.*, (2021)]



# Second Order Gravitational Waves

PBH production can be associated with *stochastic gravitational waves!*

[J.R. Espinosa, D. Racco, A. Riotto, (2018)]  
[K. Kohri, T. Terada, (2018)]...

$$\Omega_{\text{GW}}(\eta_0, k) = c_g \frac{\Omega_{r,0}}{6} \int_0^\infty dv \int_{|1-v|}^{1+v} du \left( \frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right)^2 \overline{\mathcal{I}^2(v, u)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

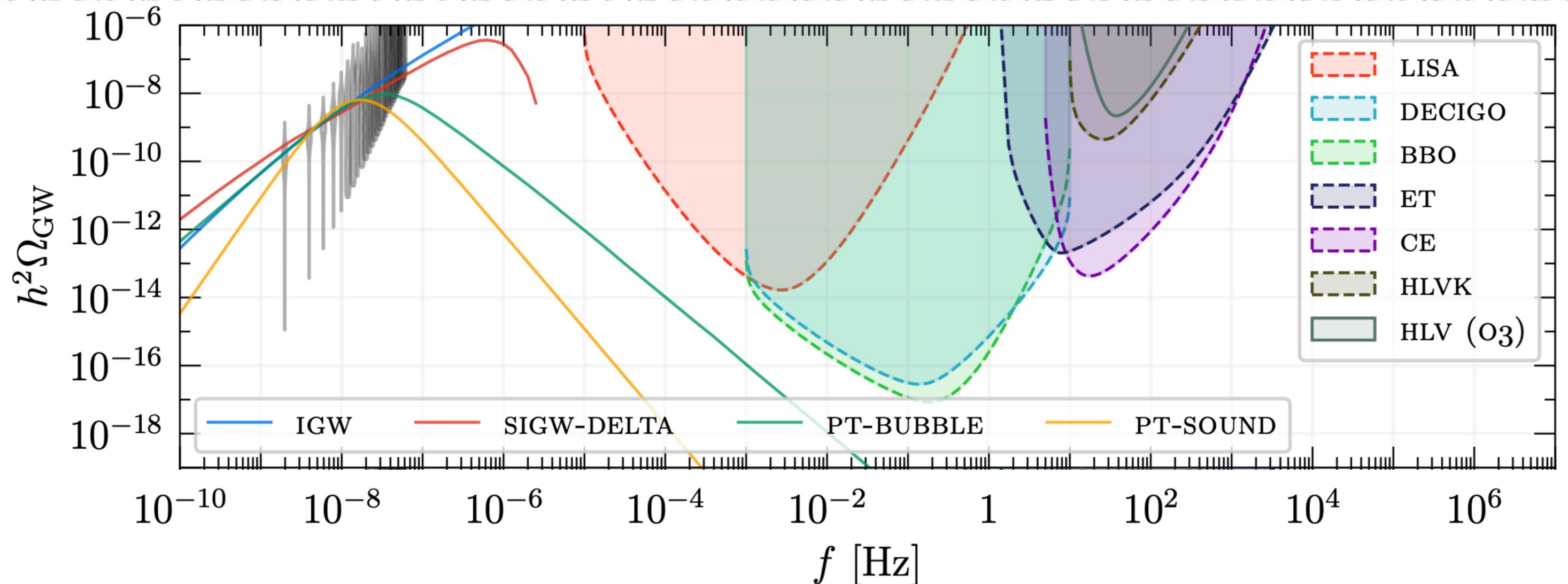


Figure from [NANOGrav 15-year New-Physics Signals, 2306.16219]



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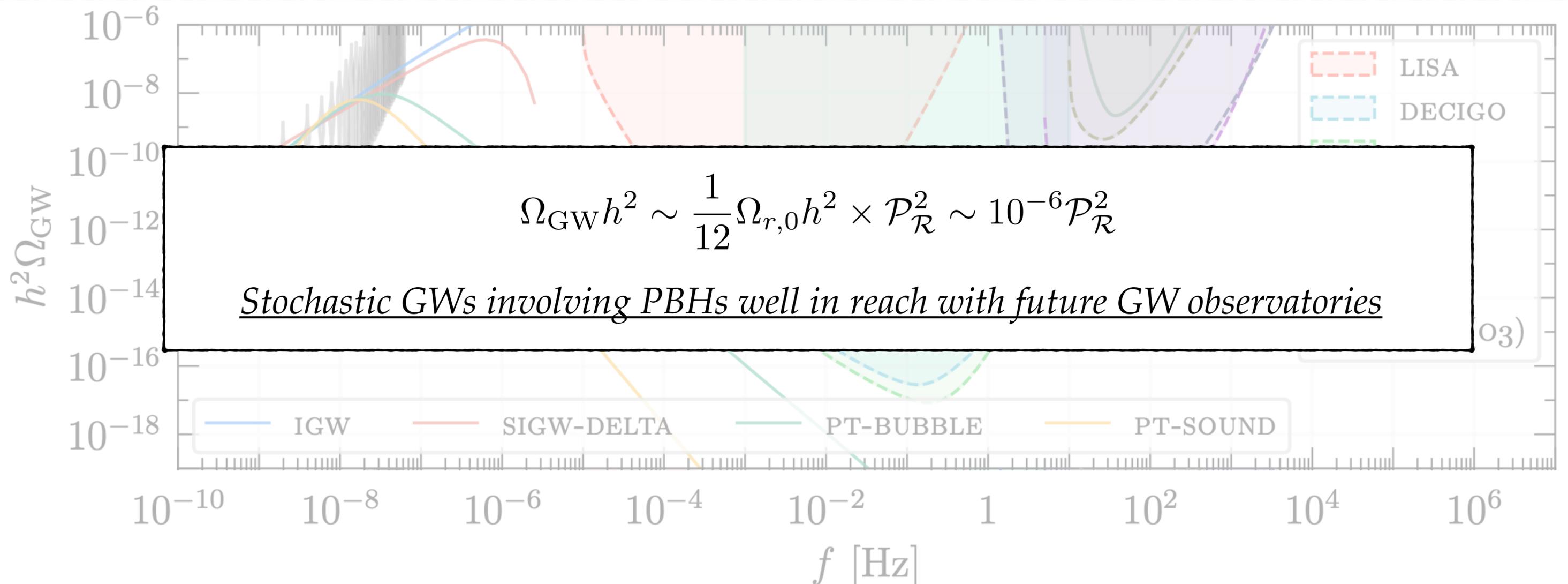


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# Higgs- $R^2$ Inflation

Question) What can be our inflaton?  $\rightarrow$  SM Higgs?

Considering dim-4 operators,  $R^2$  present!

$$S_J = \int d^4x \sqrt{-g_J} \left[ \frac{M_P^2}{2} \left( R_J + \frac{\xi h^2}{M_P^2} R_J + \frac{R_J^2}{6M^2} \right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu h \nabla_\nu h - \frac{\lambda}{4} h^4 \right]$$

non-minimal coupling

scalar d.o.f

Higgs

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Higgs

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{s}{M_P}} \left\{ \frac{3}{4} M_P^2 M^2 \left( e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda_{\text{eff}}(\mu)}{4} h^4 \right\}$$

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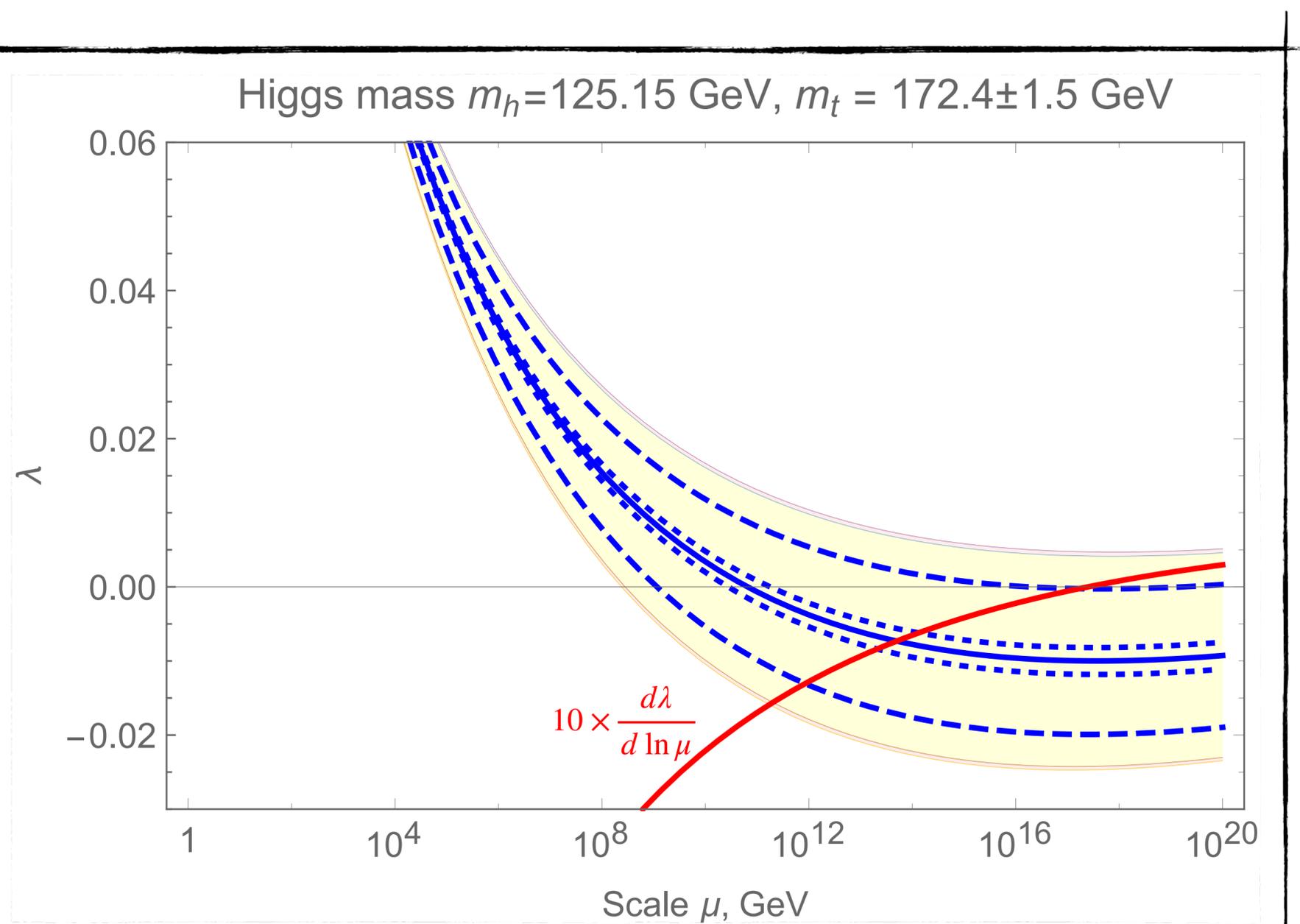
Higgs

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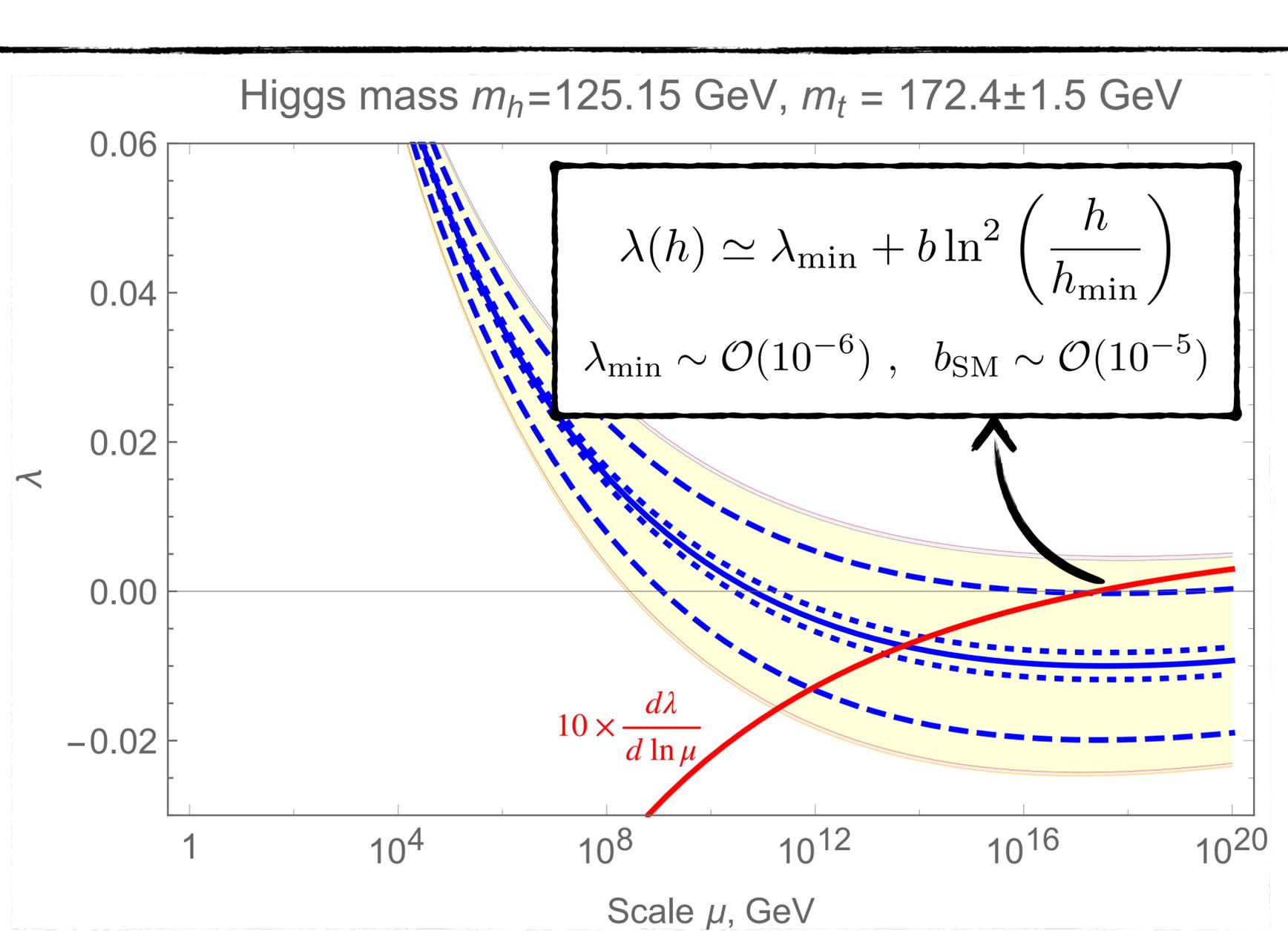
(1) Near-inflection point

(2) Tachyonic hill

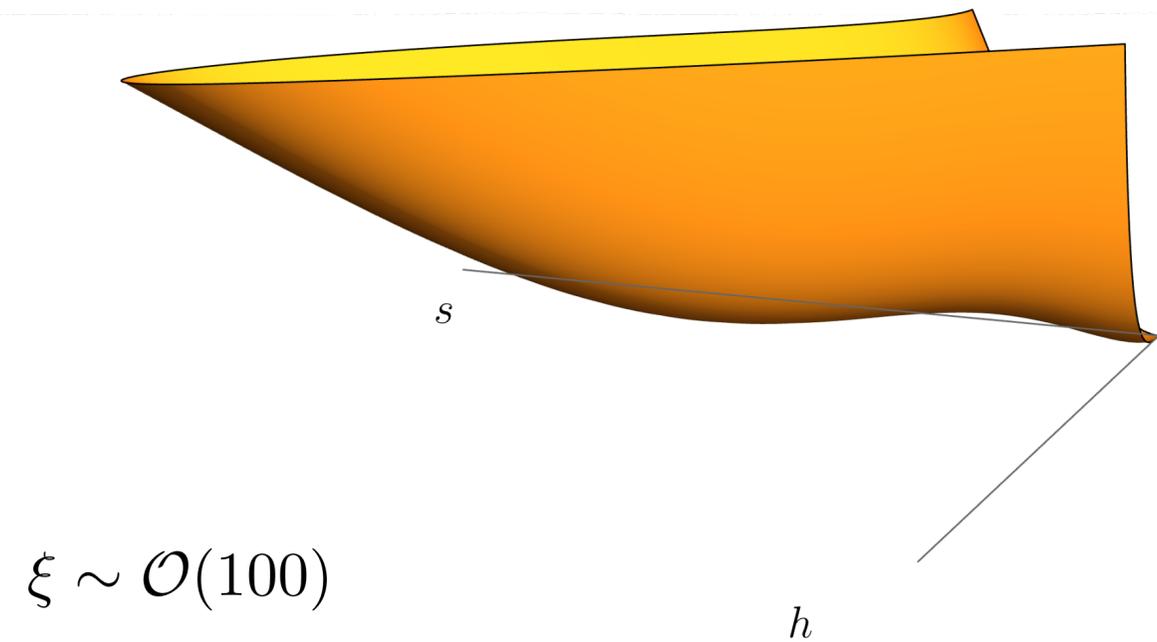
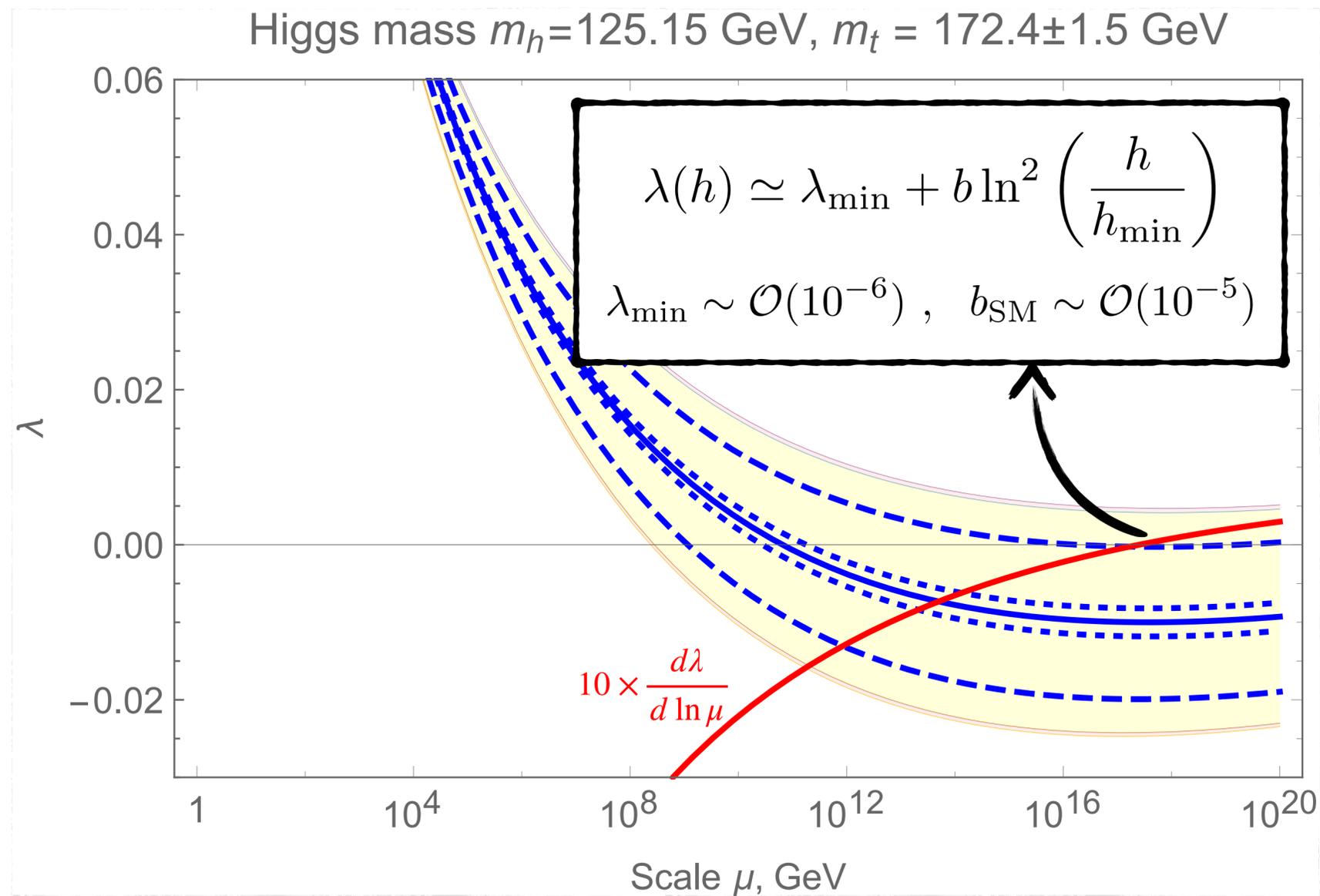
# Higgs- $R^2$ Inflation, SM Higgs Running



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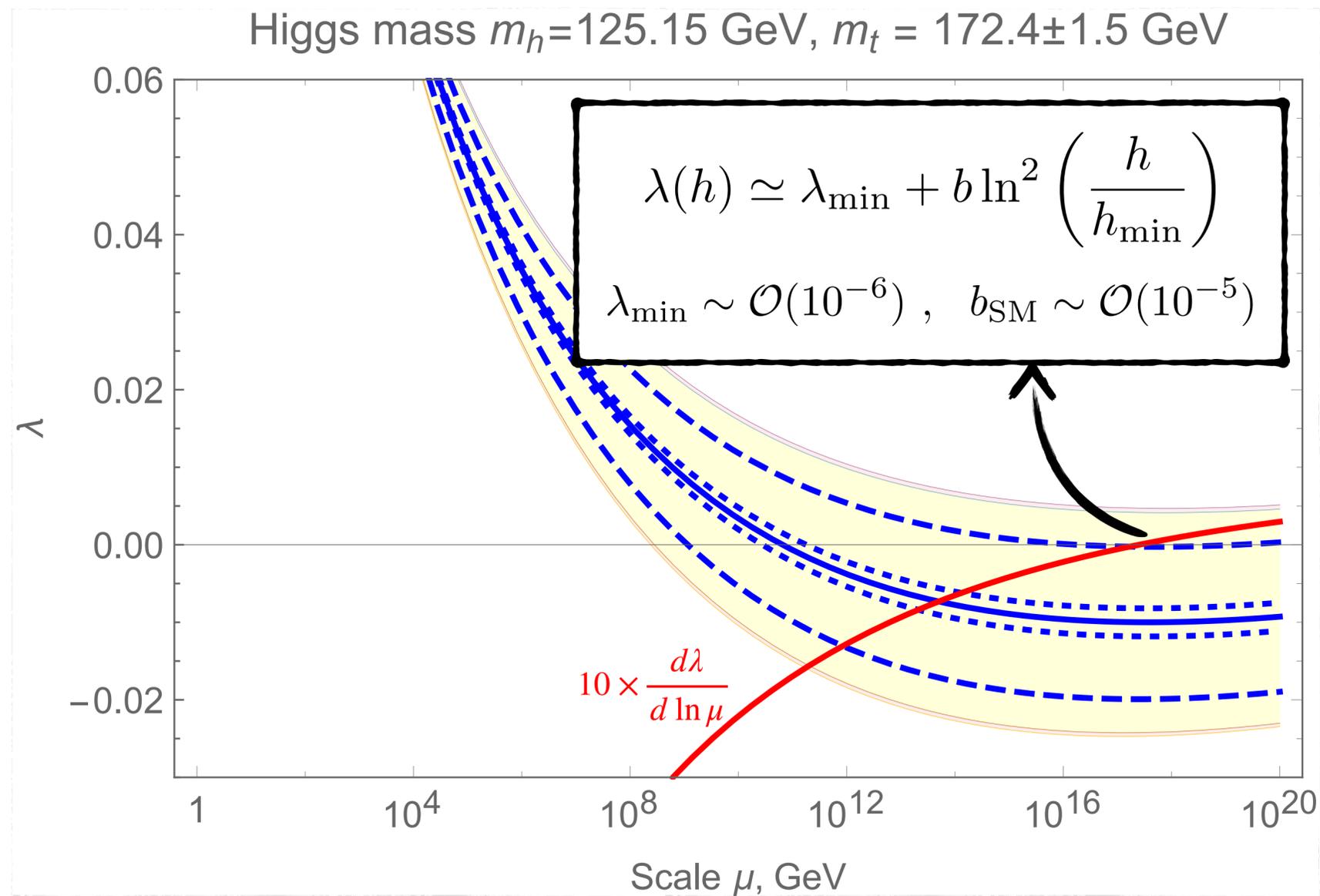


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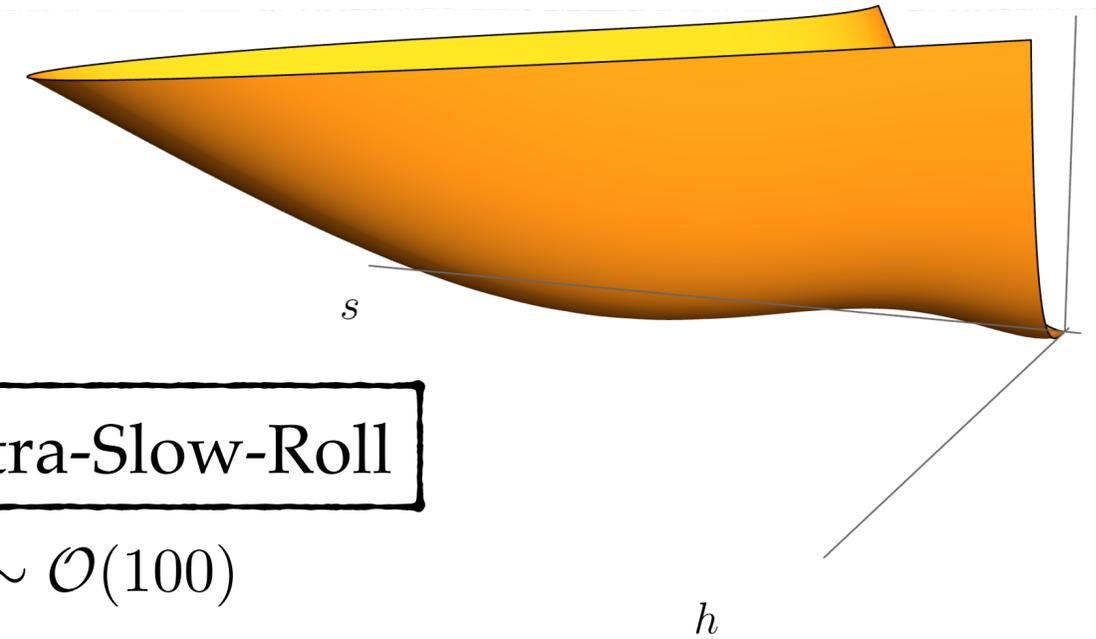


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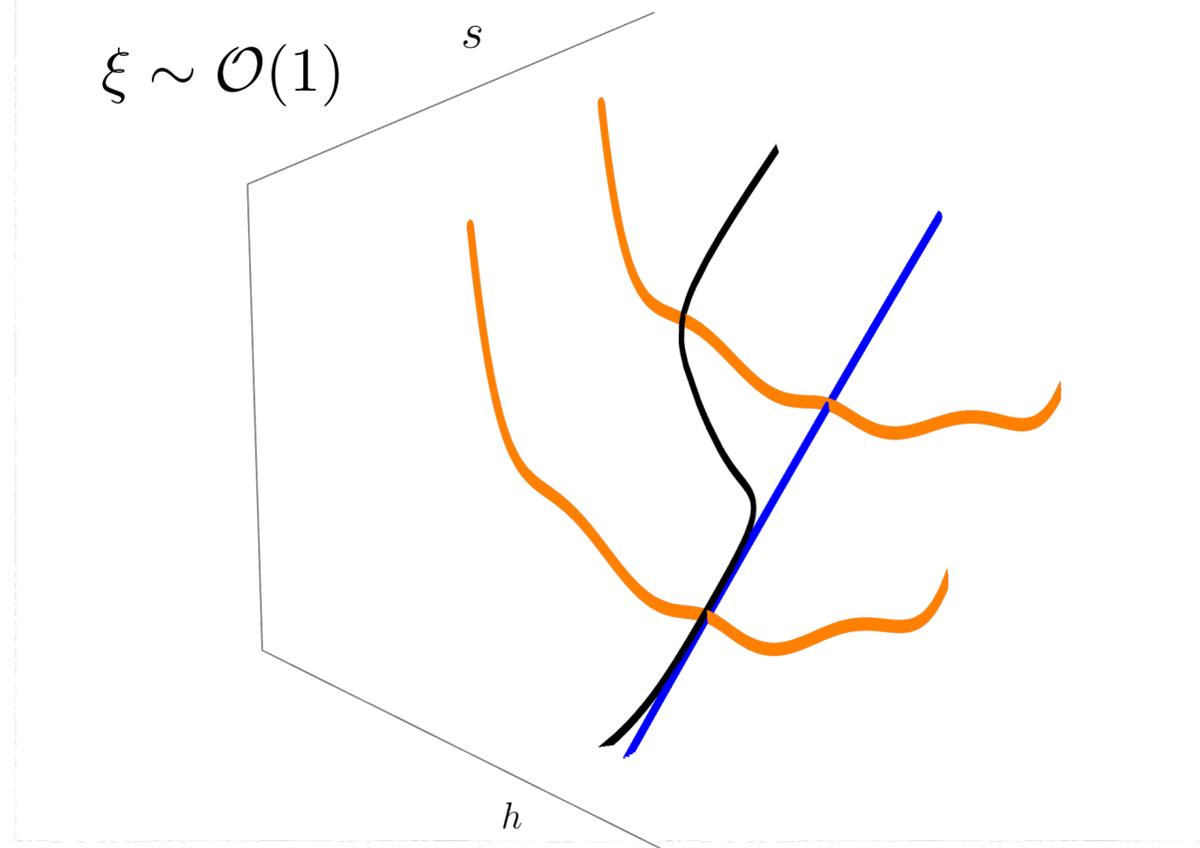
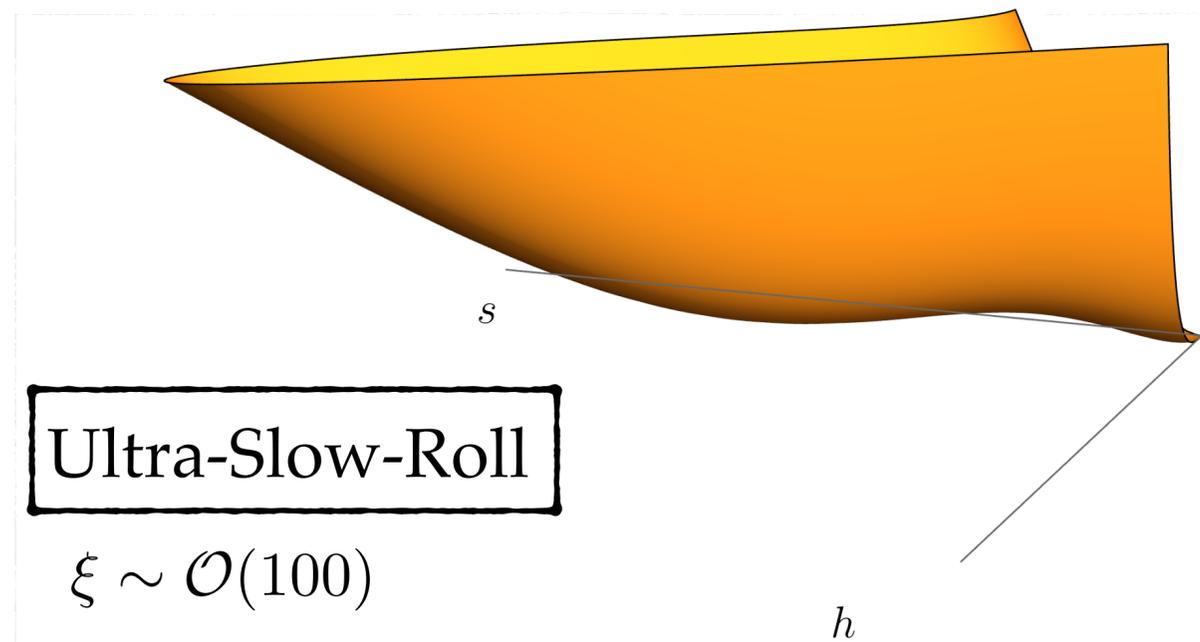
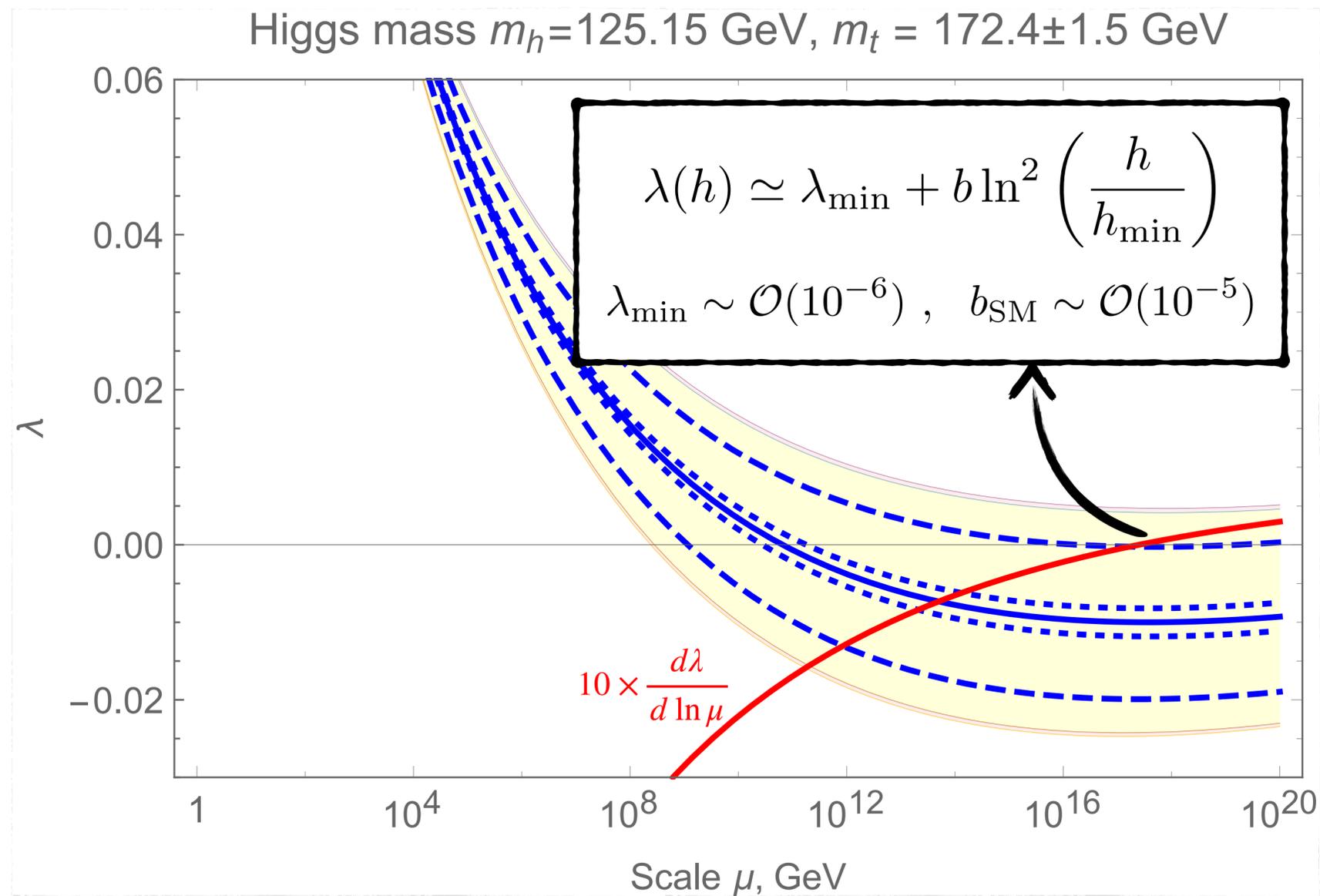


Ultra-Slow-Roll

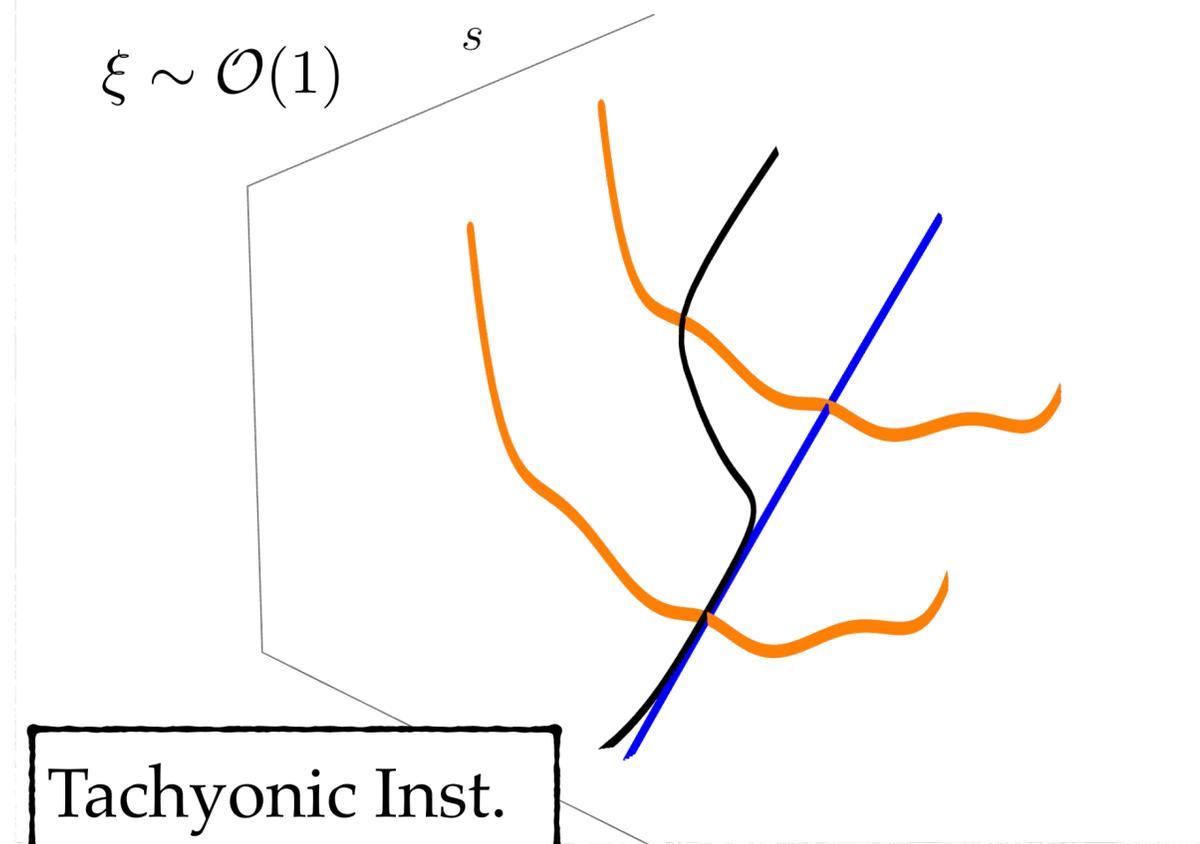
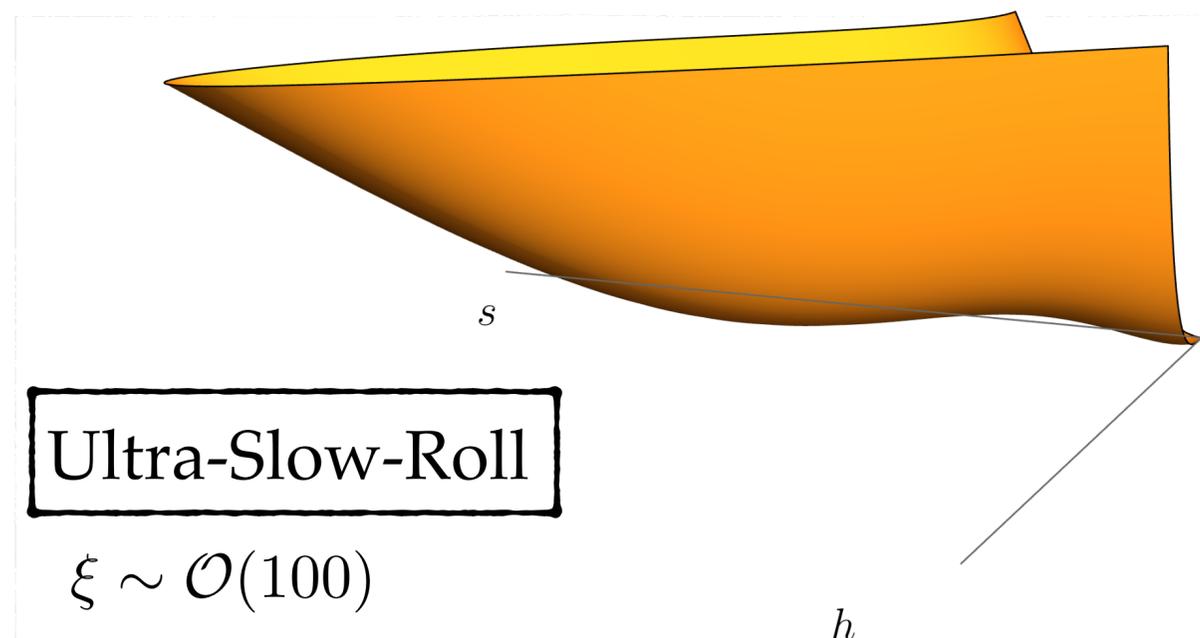
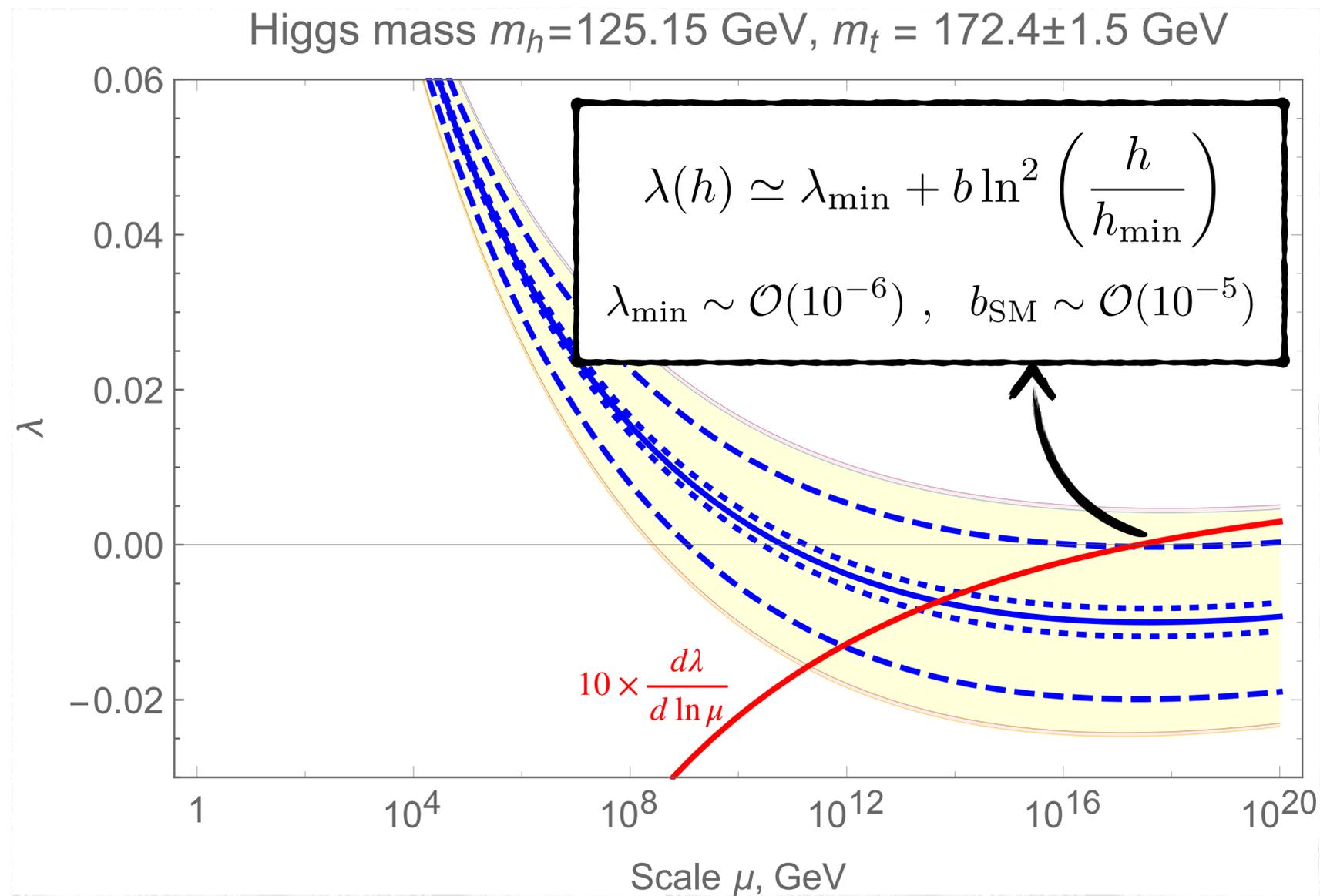
$$\xi \sim \mathcal{O}(100)$$



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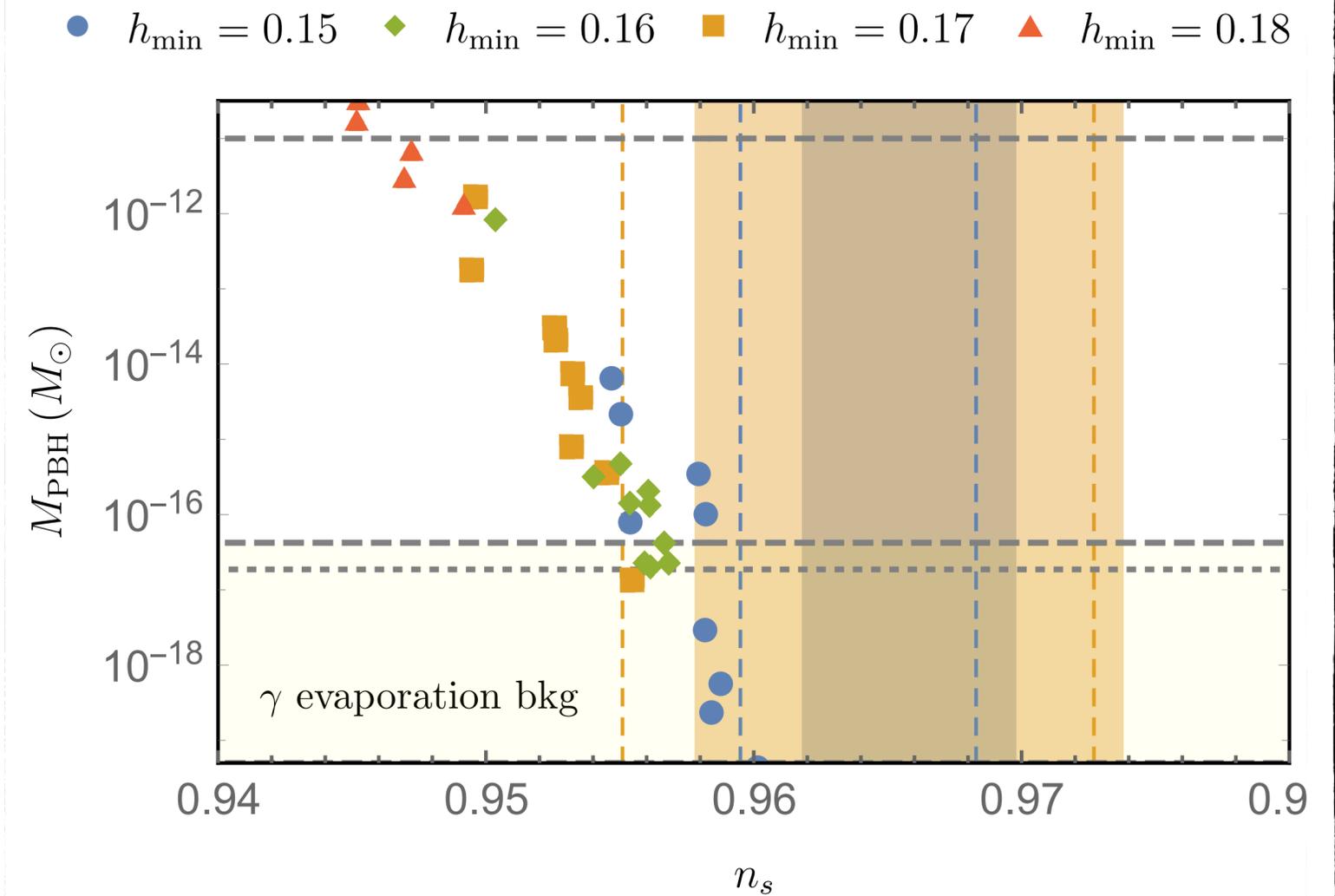
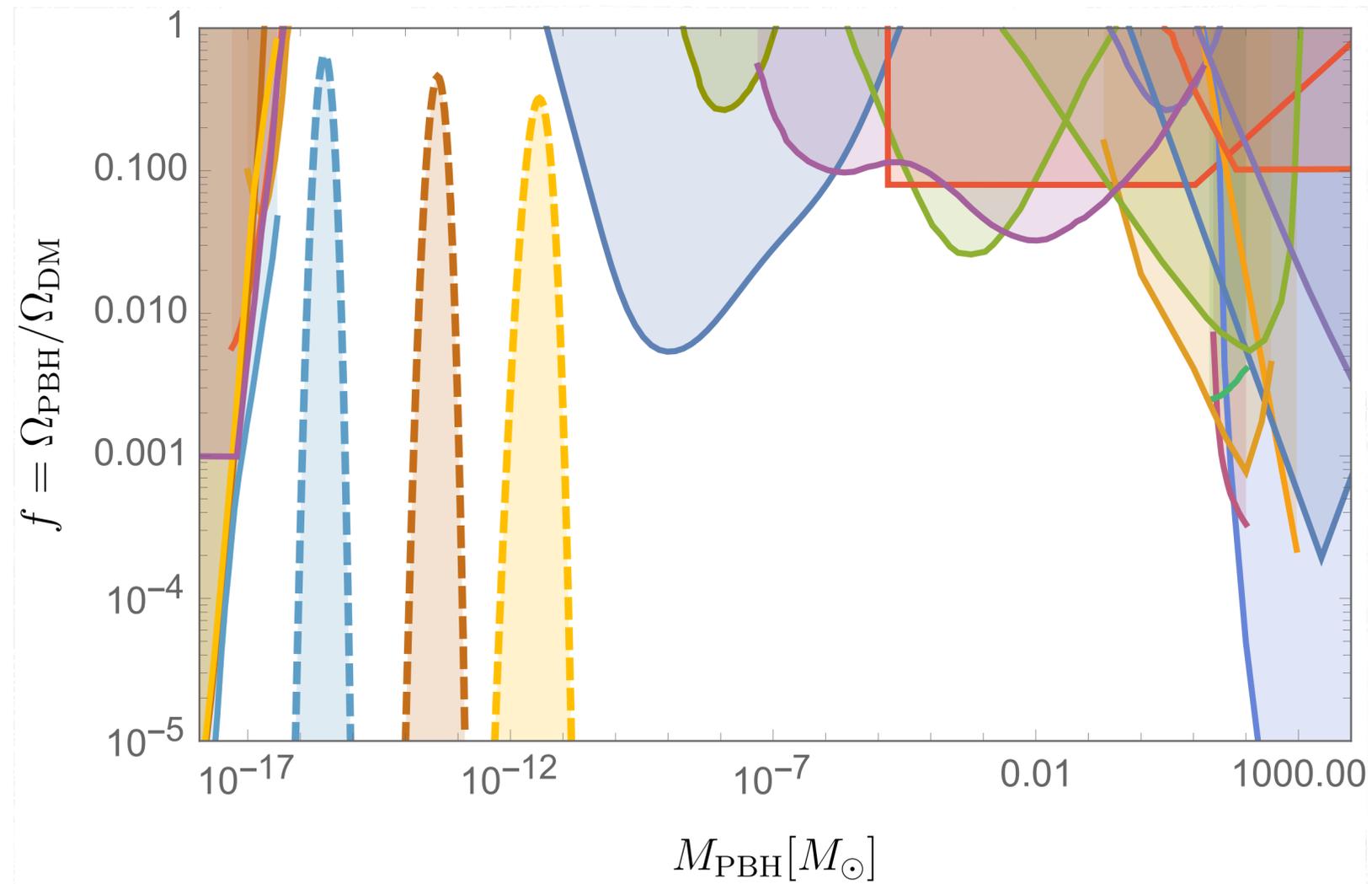


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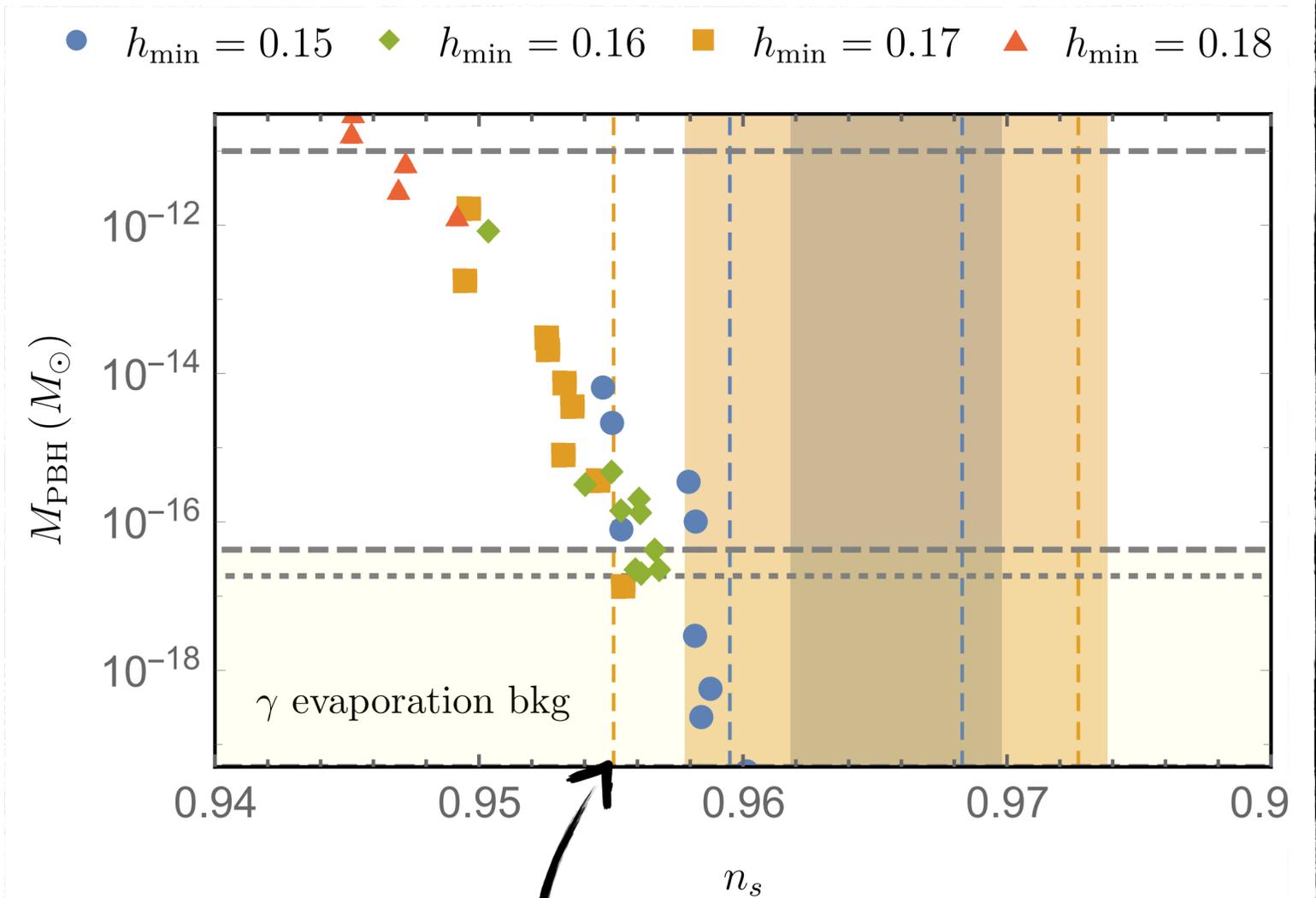
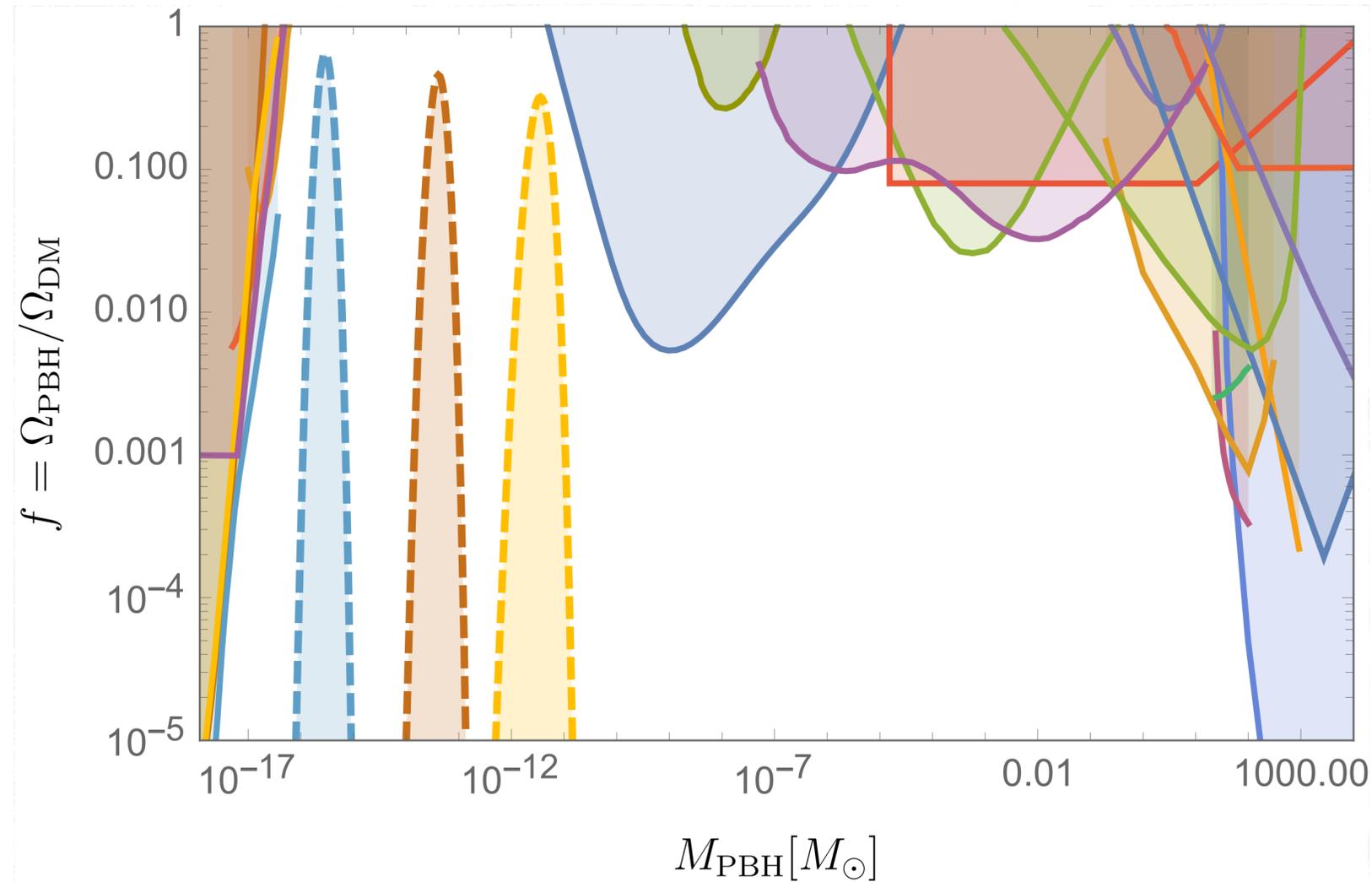
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[DYC, S.M. Lee, S.C. Park, JCAP 01 (2021), 032]



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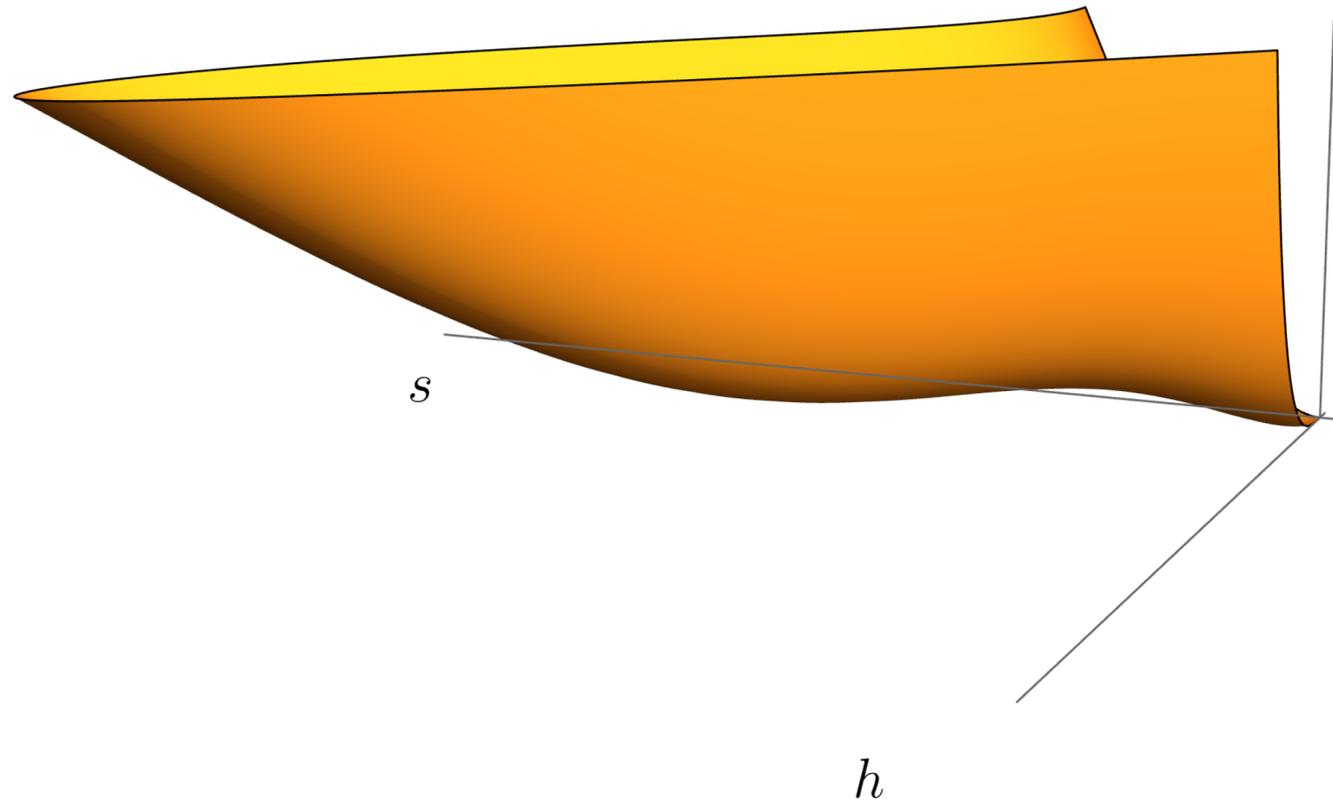
Tension with CMB!

Sensitive to higher order terms e.g.  $\sim R^3$

# Higgs- $R^2$ Inflation / USR vs Tachyonic Instability?

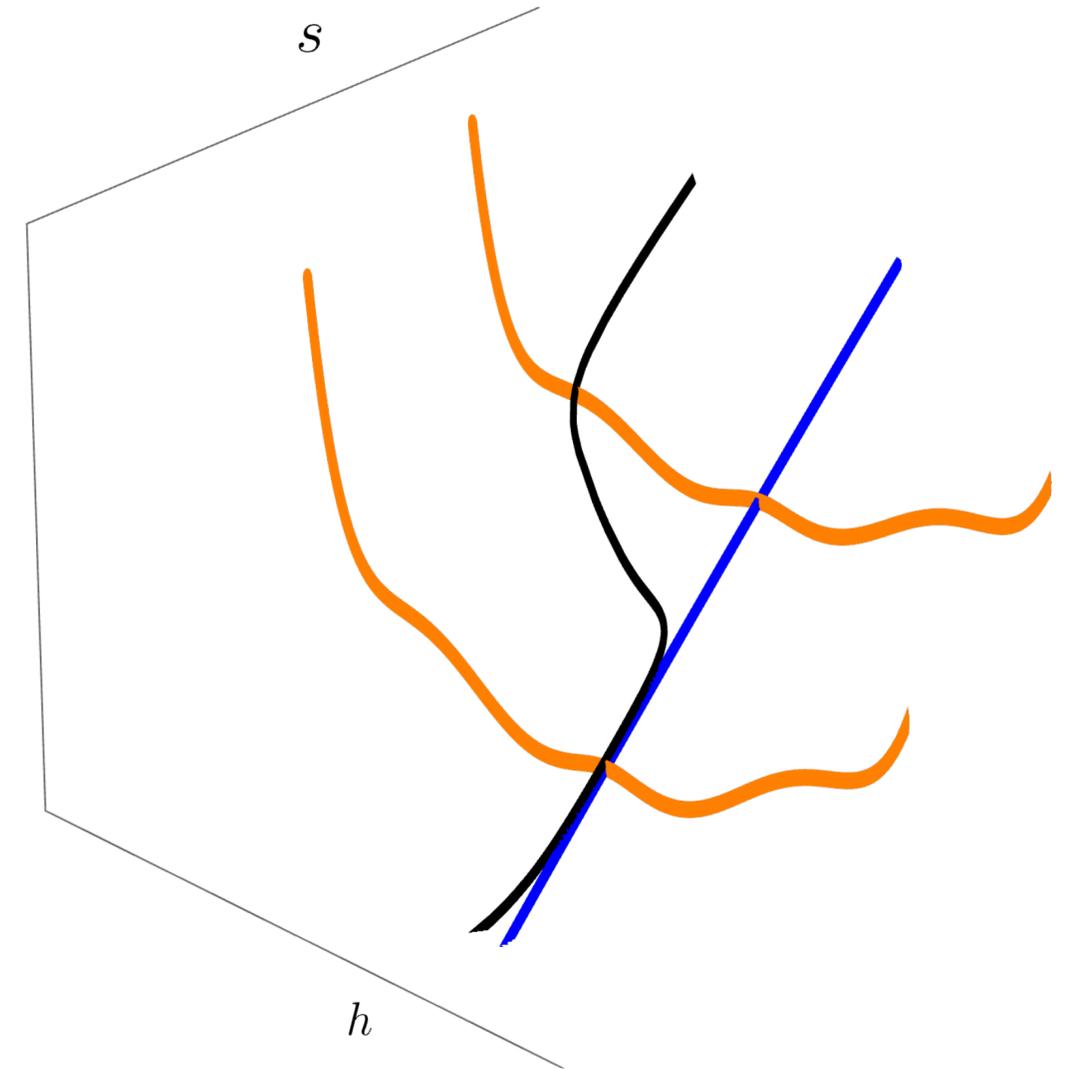
Ultra-Slow-Roll

$$\xi \sim \mathcal{O}(100)$$



Tachyonic Inst.

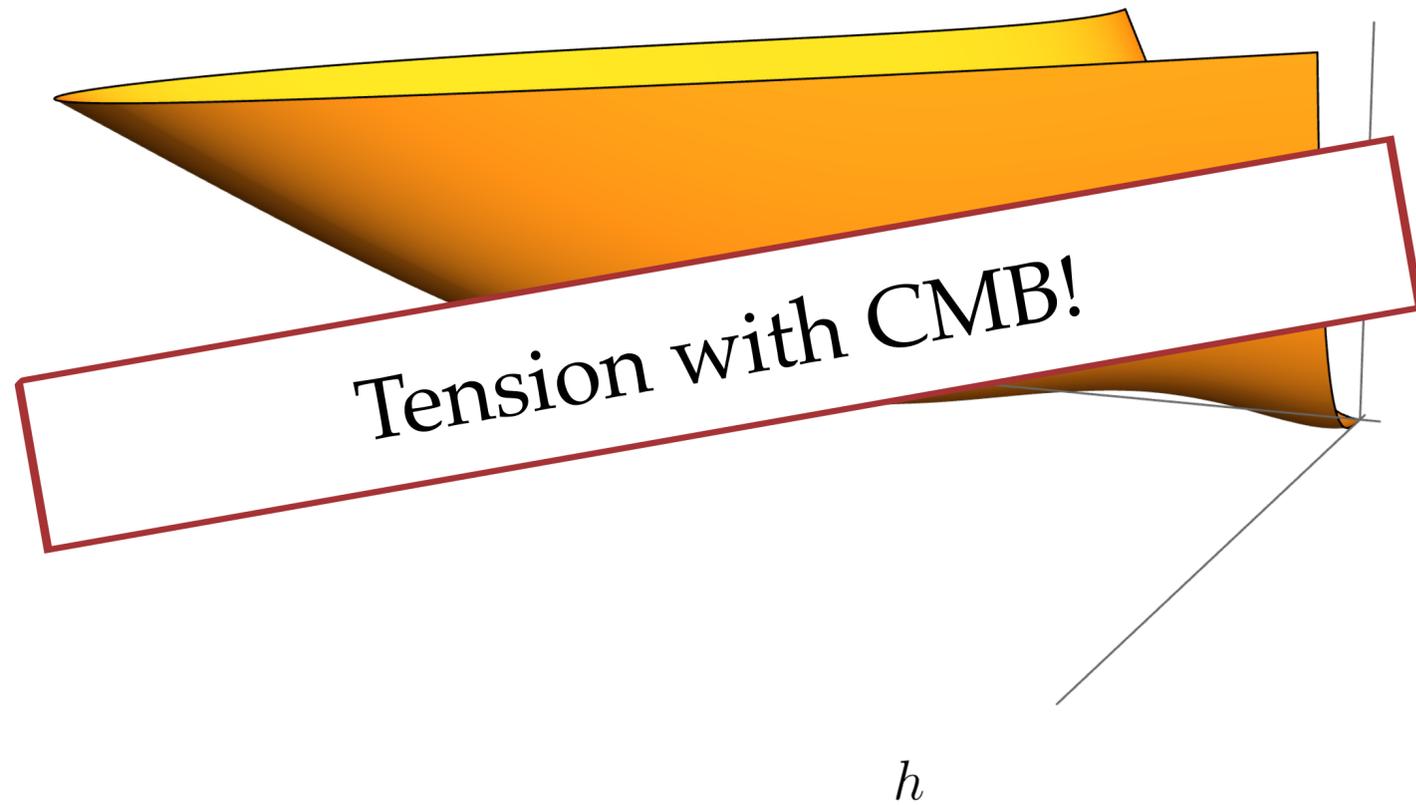
$$\xi \sim \mathcal{O}(1)$$



# Higgs- $R^2$ Inflation / USR vs Tachyonic Instability?

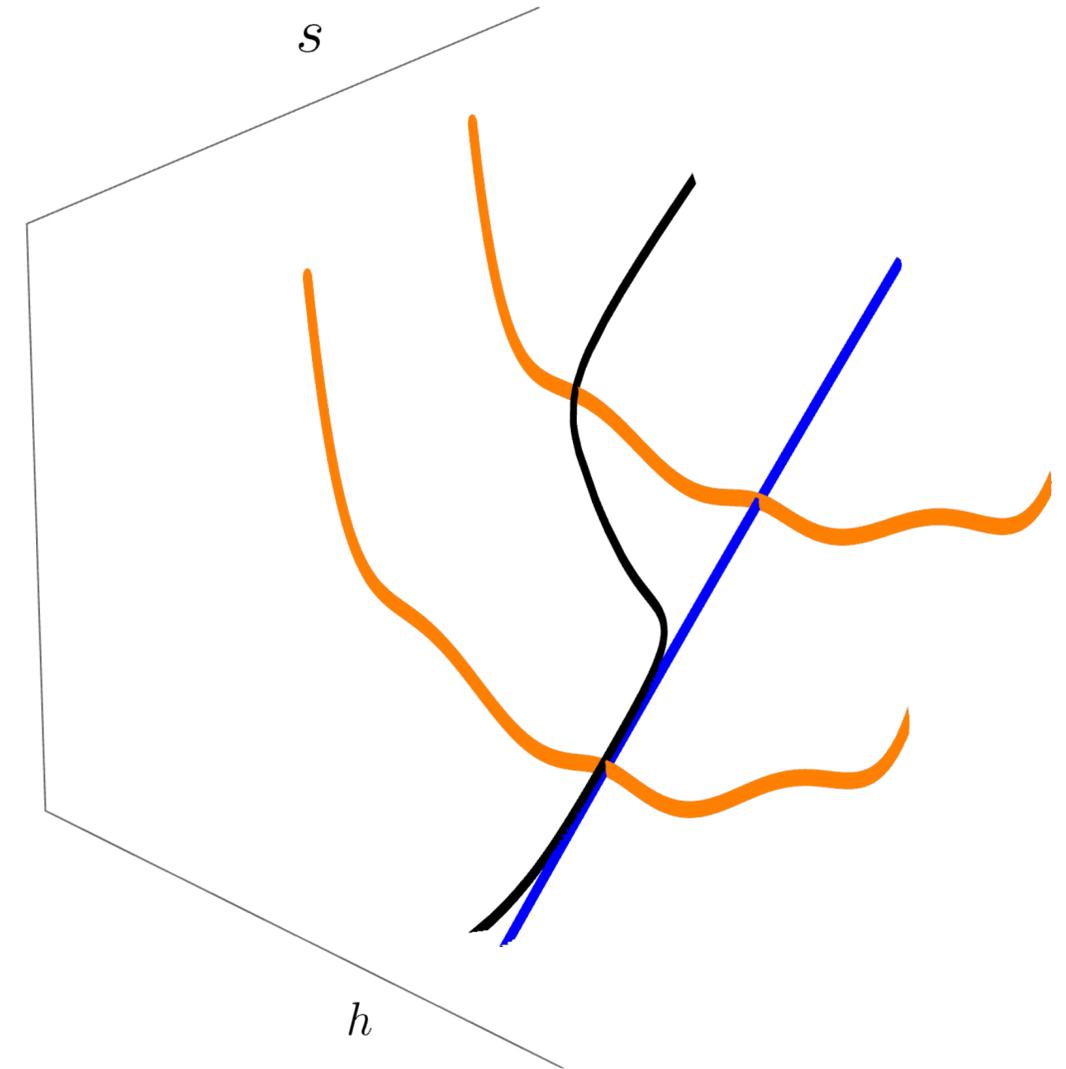
Ultra-Slow-Roll

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Tachyonic Inst.

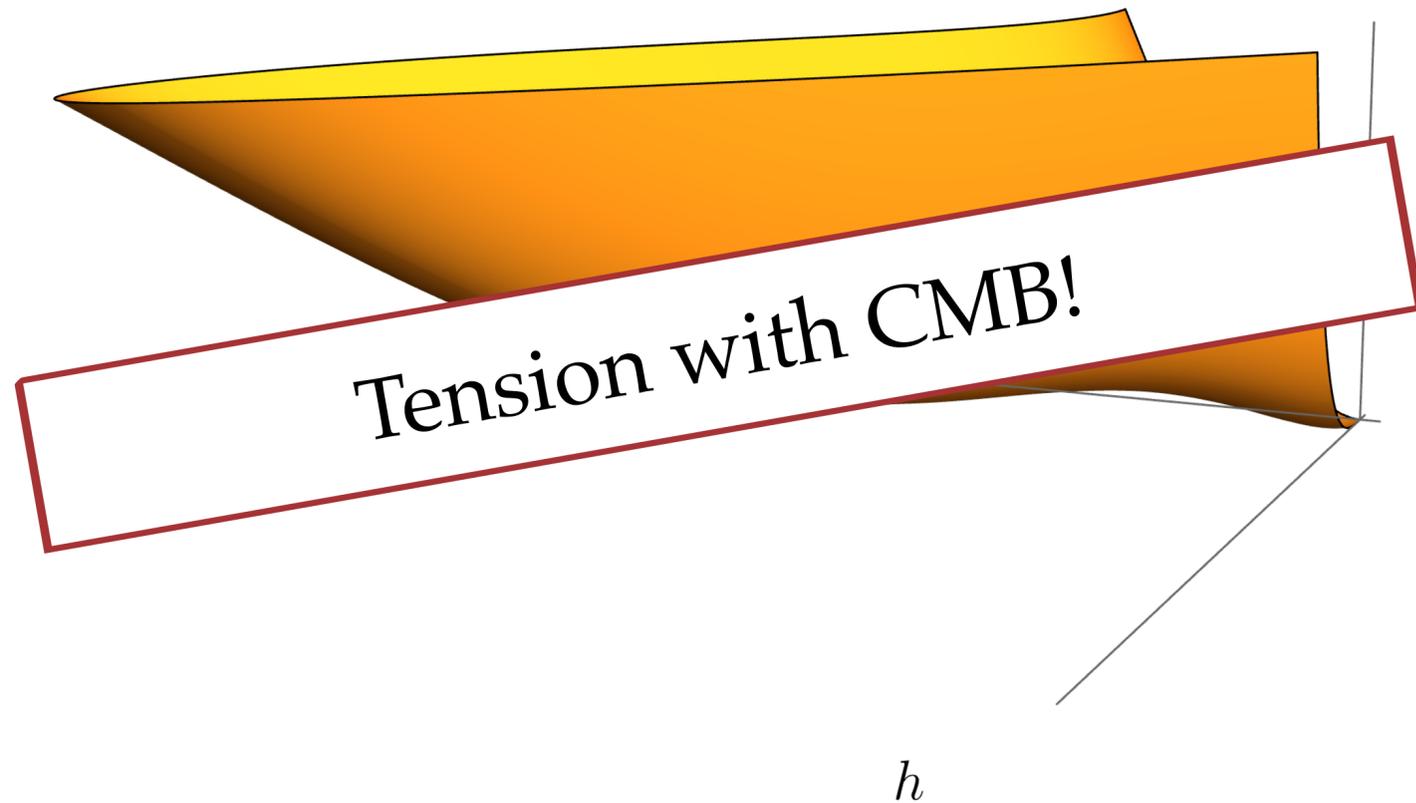
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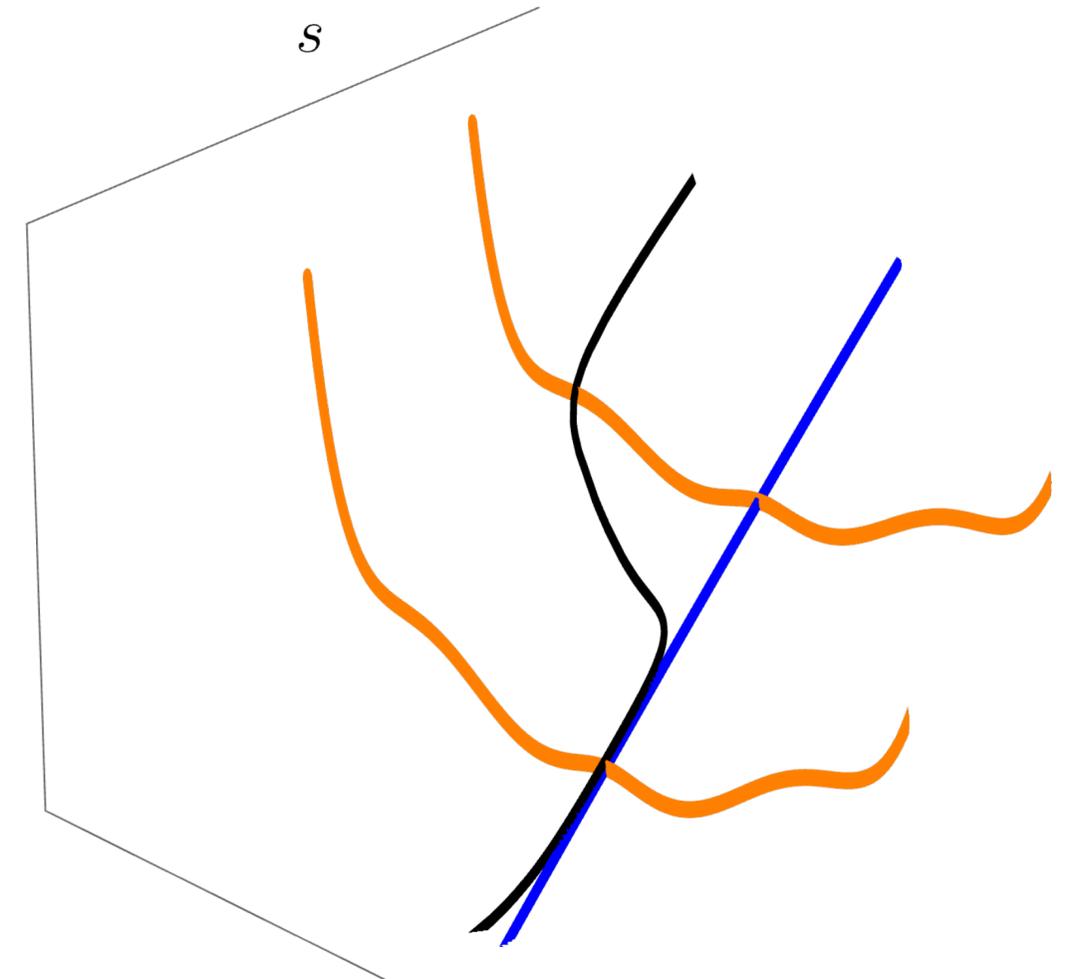
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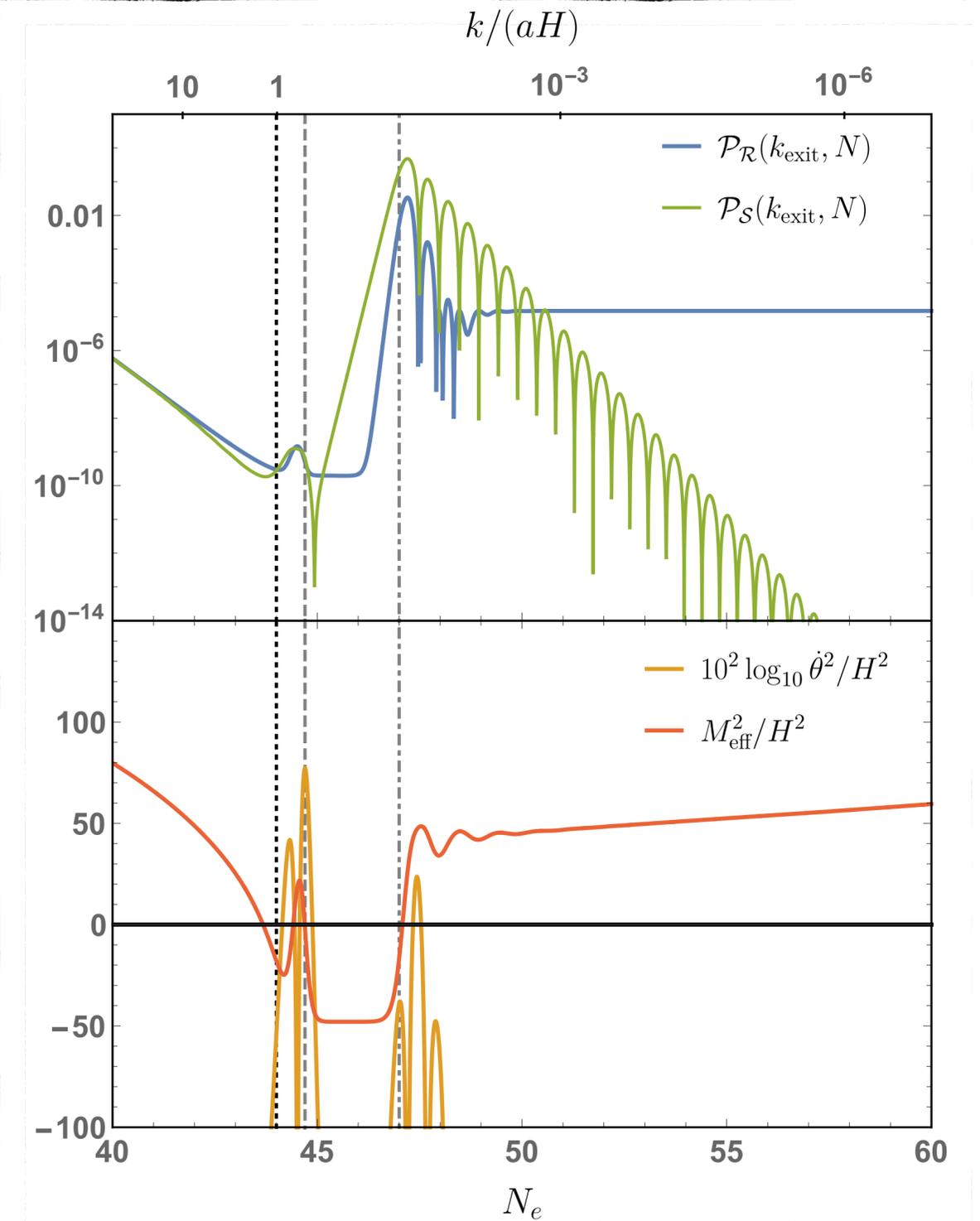
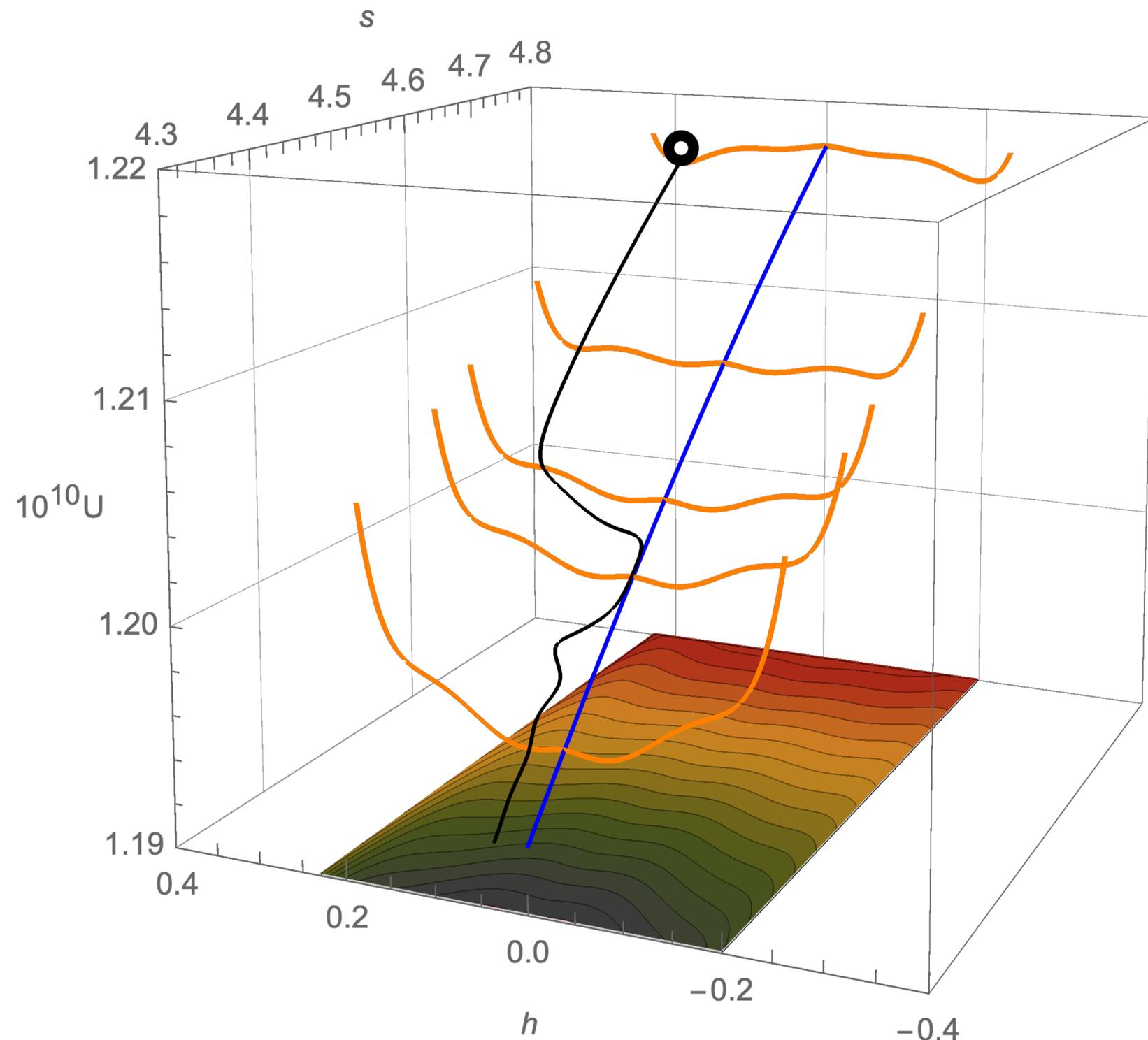


Multi-field effect / Exponential Growth



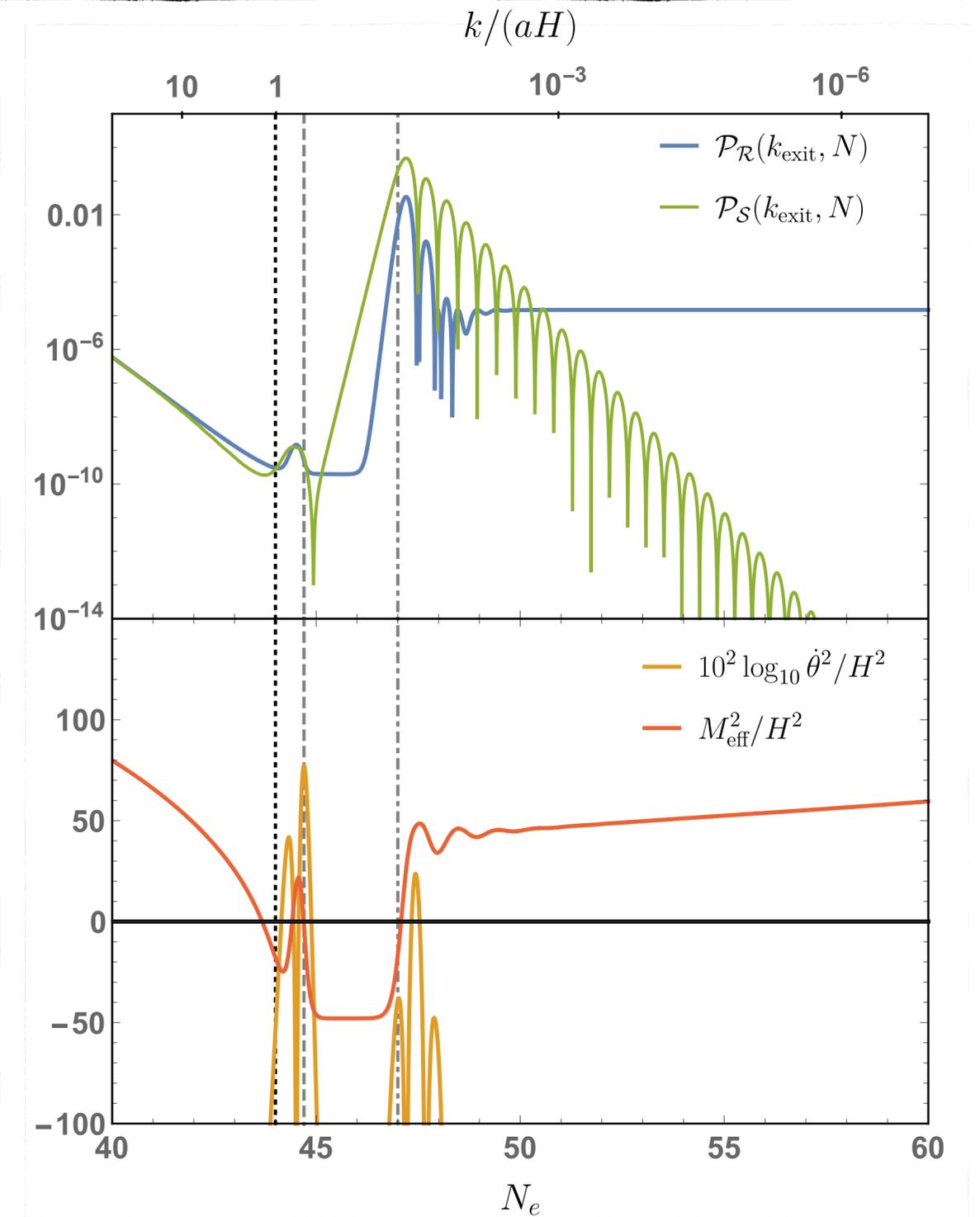
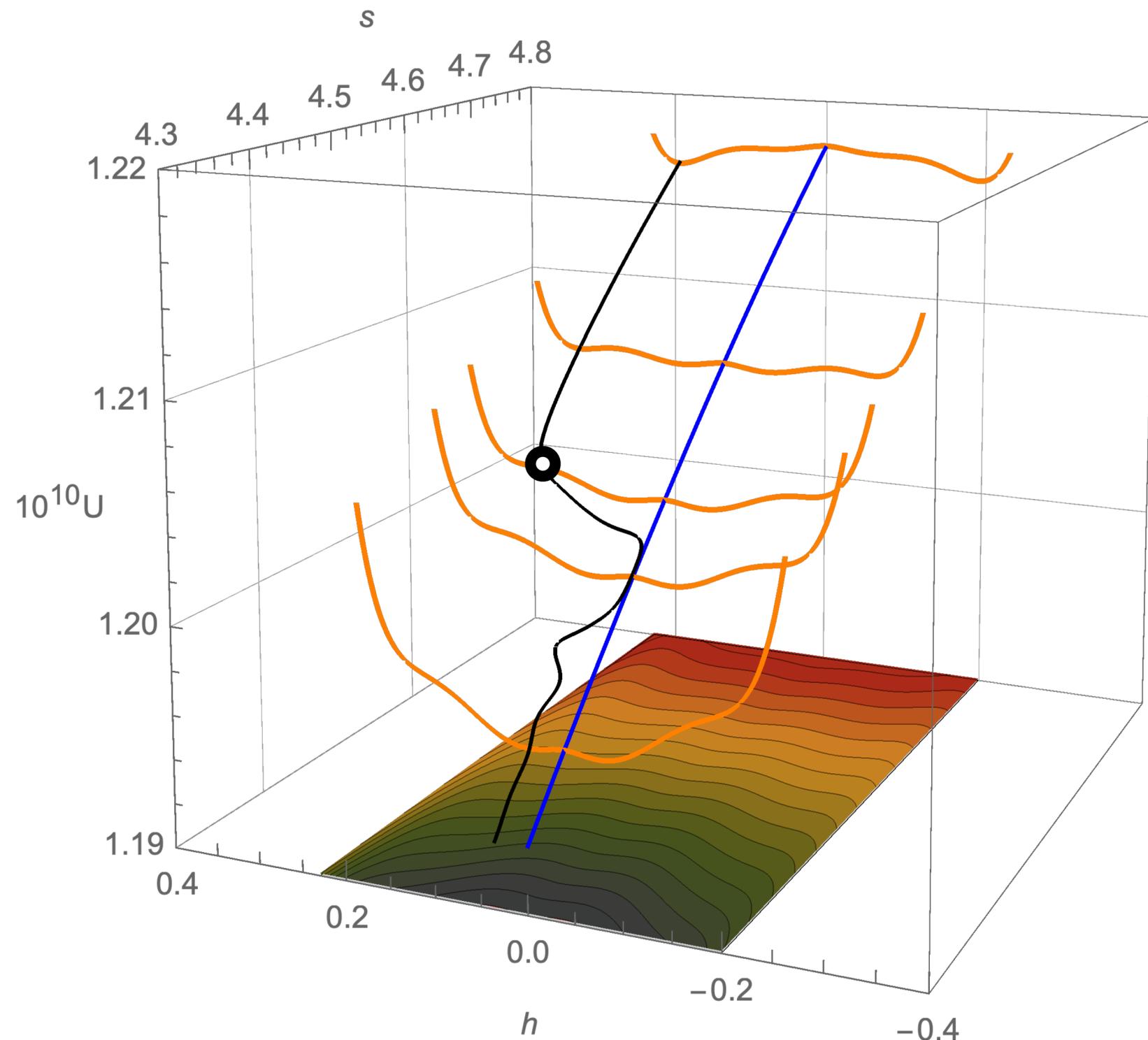
# Higgs- $R^2$ Inflation, Tachyonic Instability

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



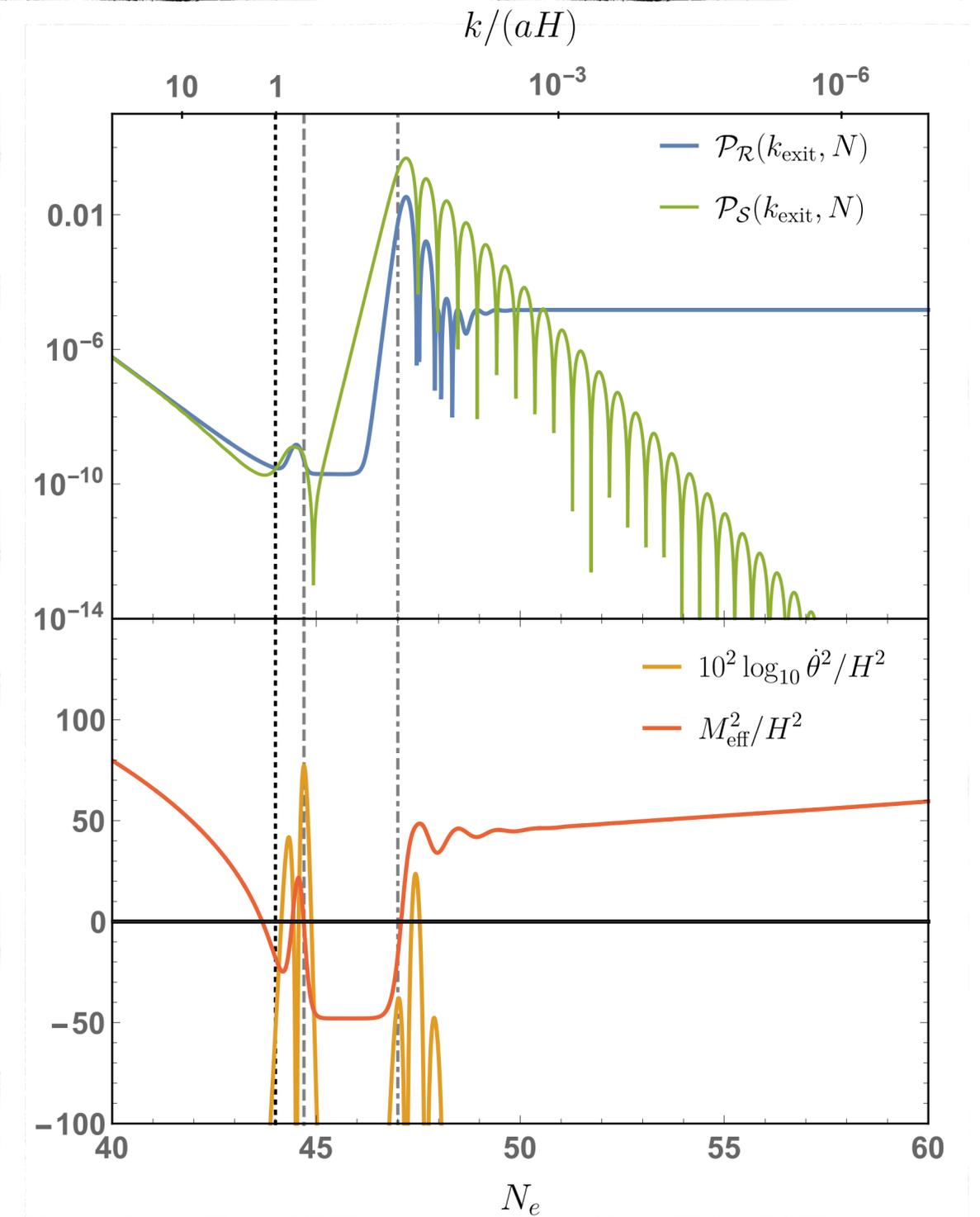
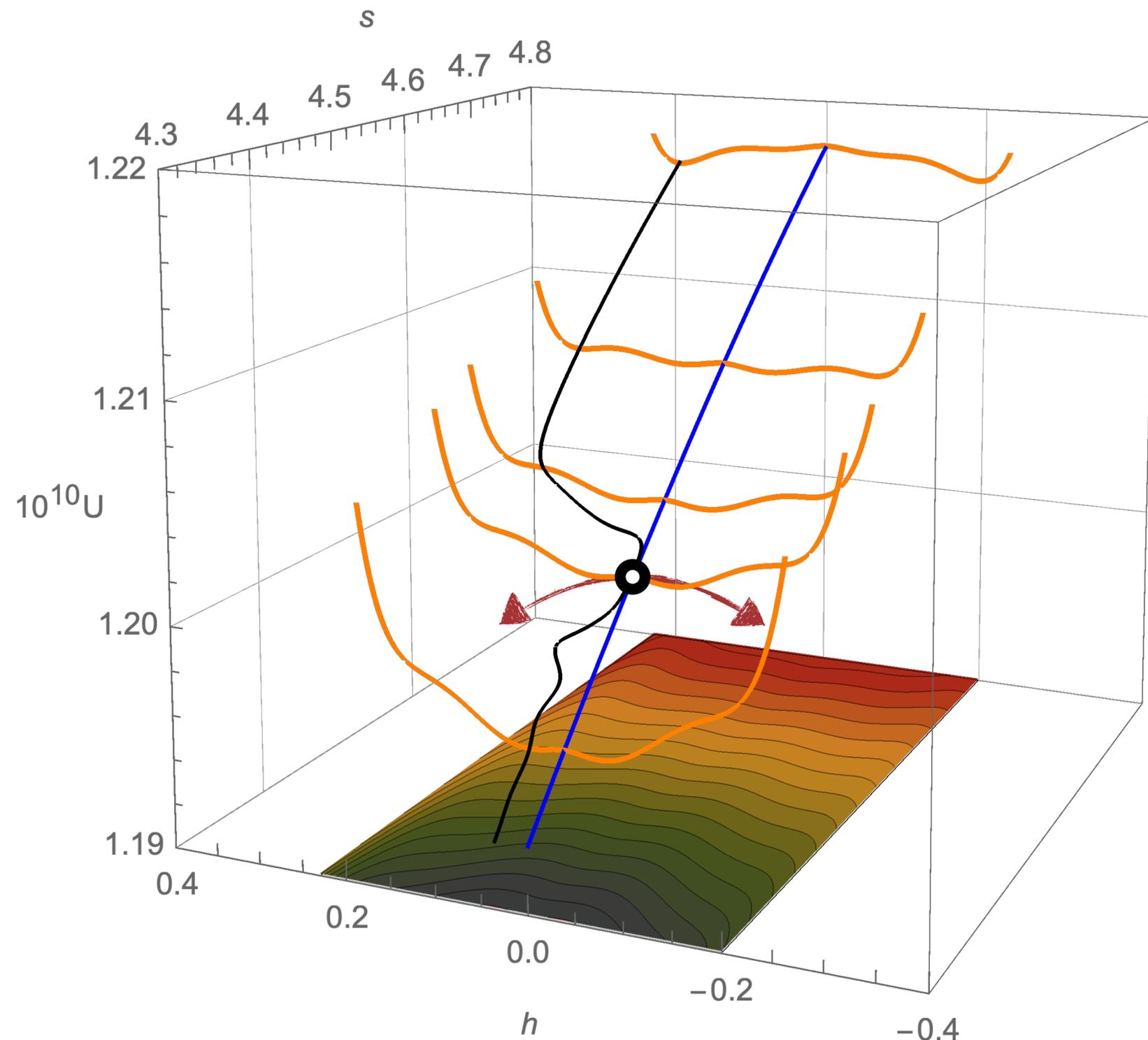
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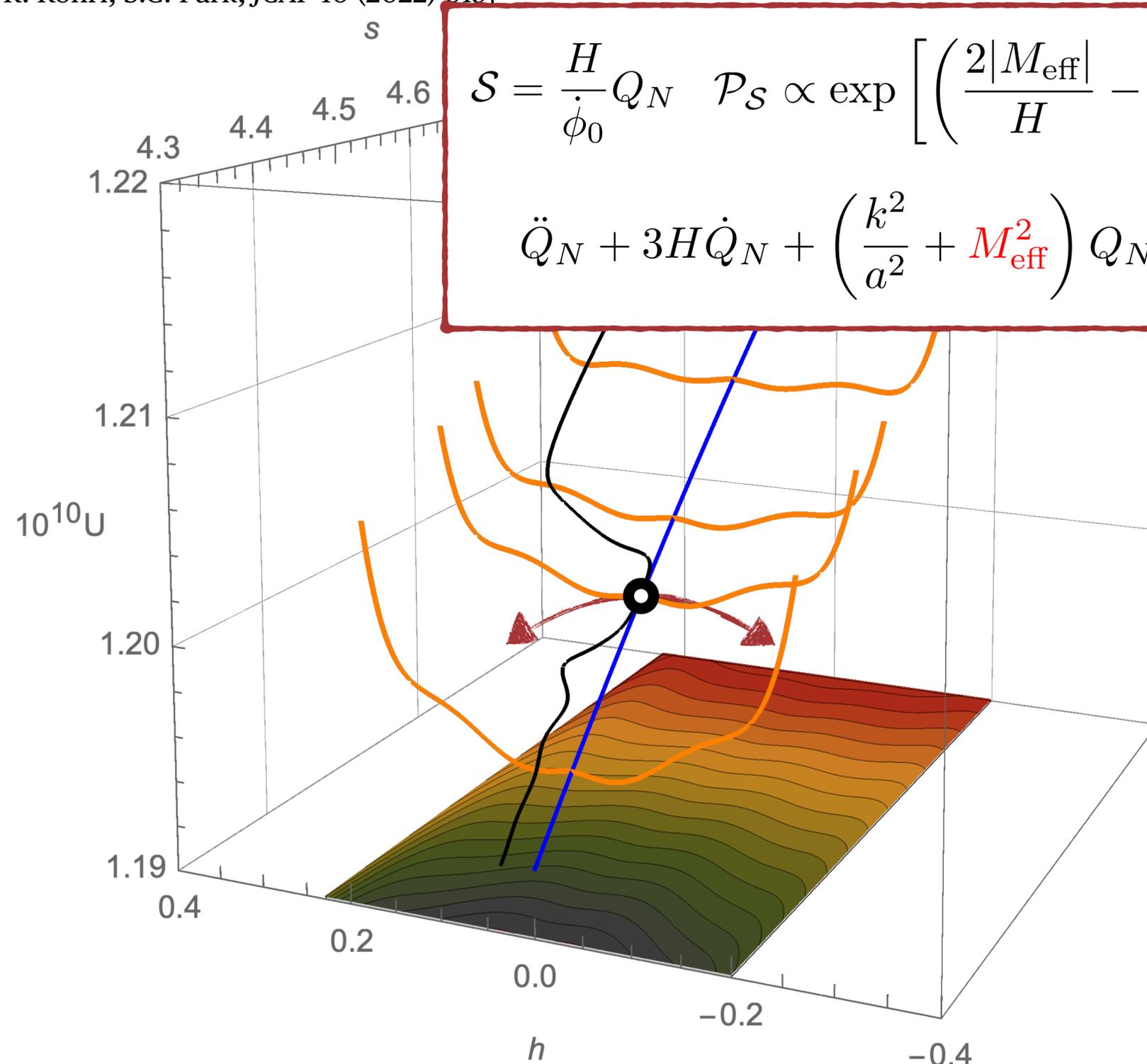
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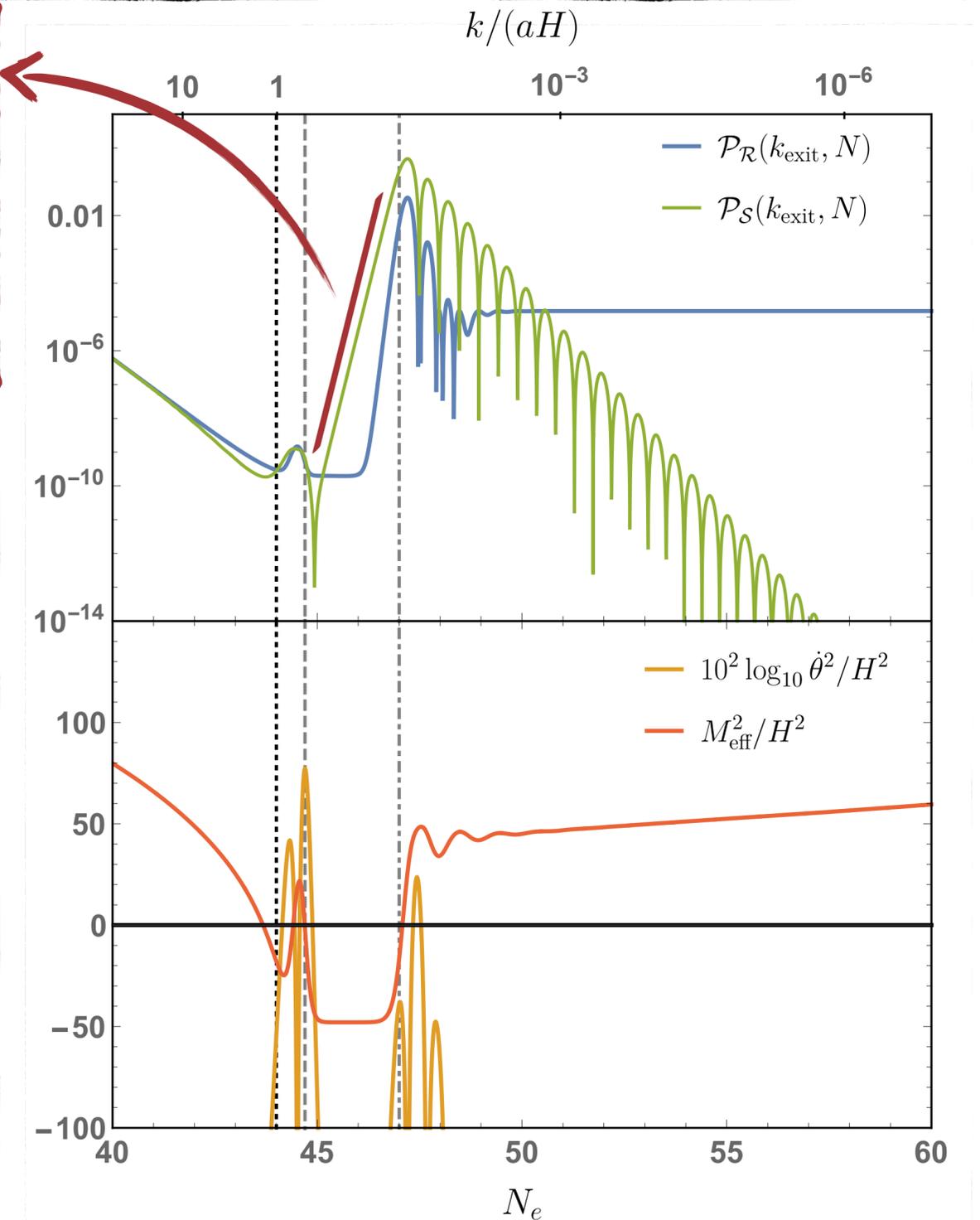
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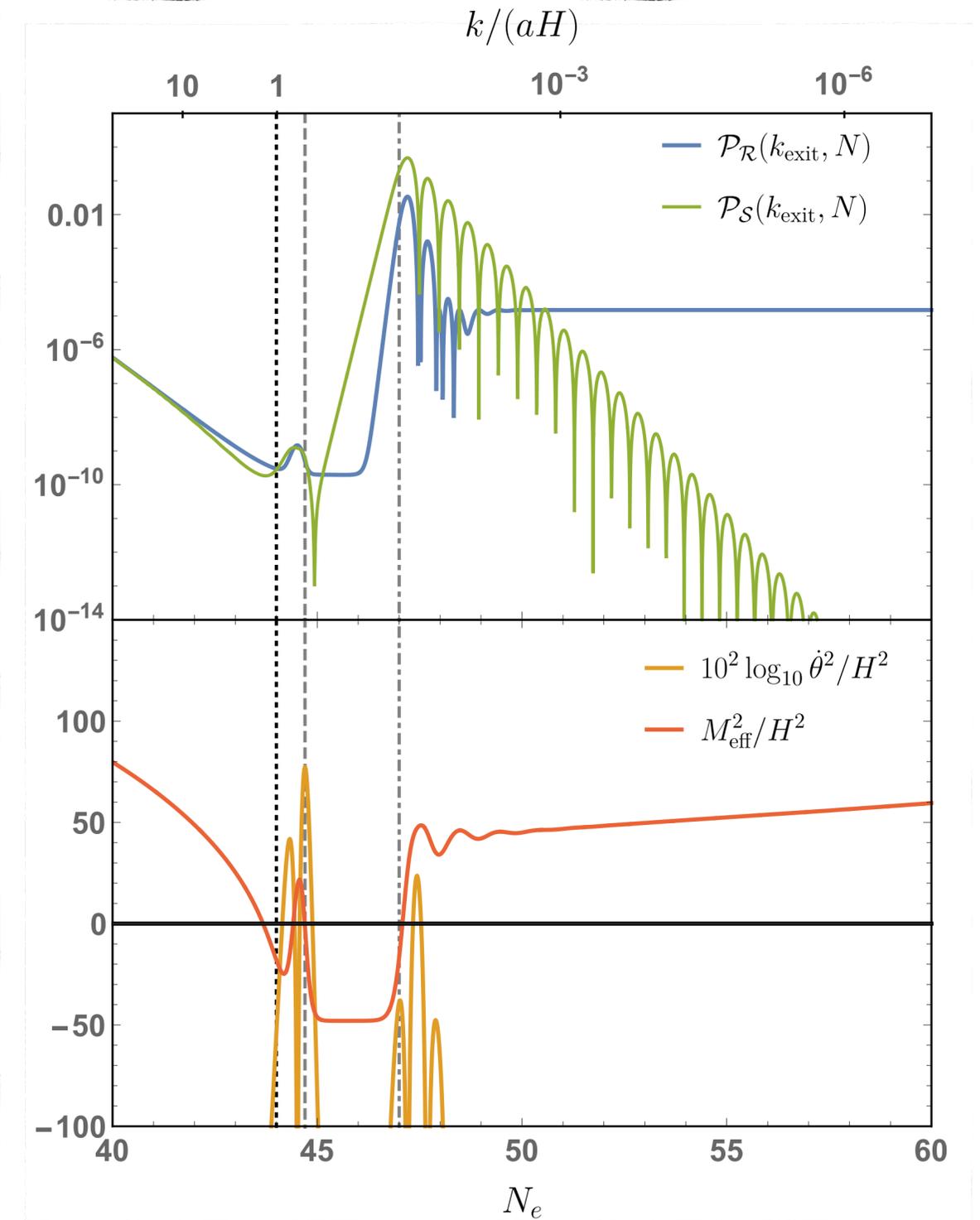
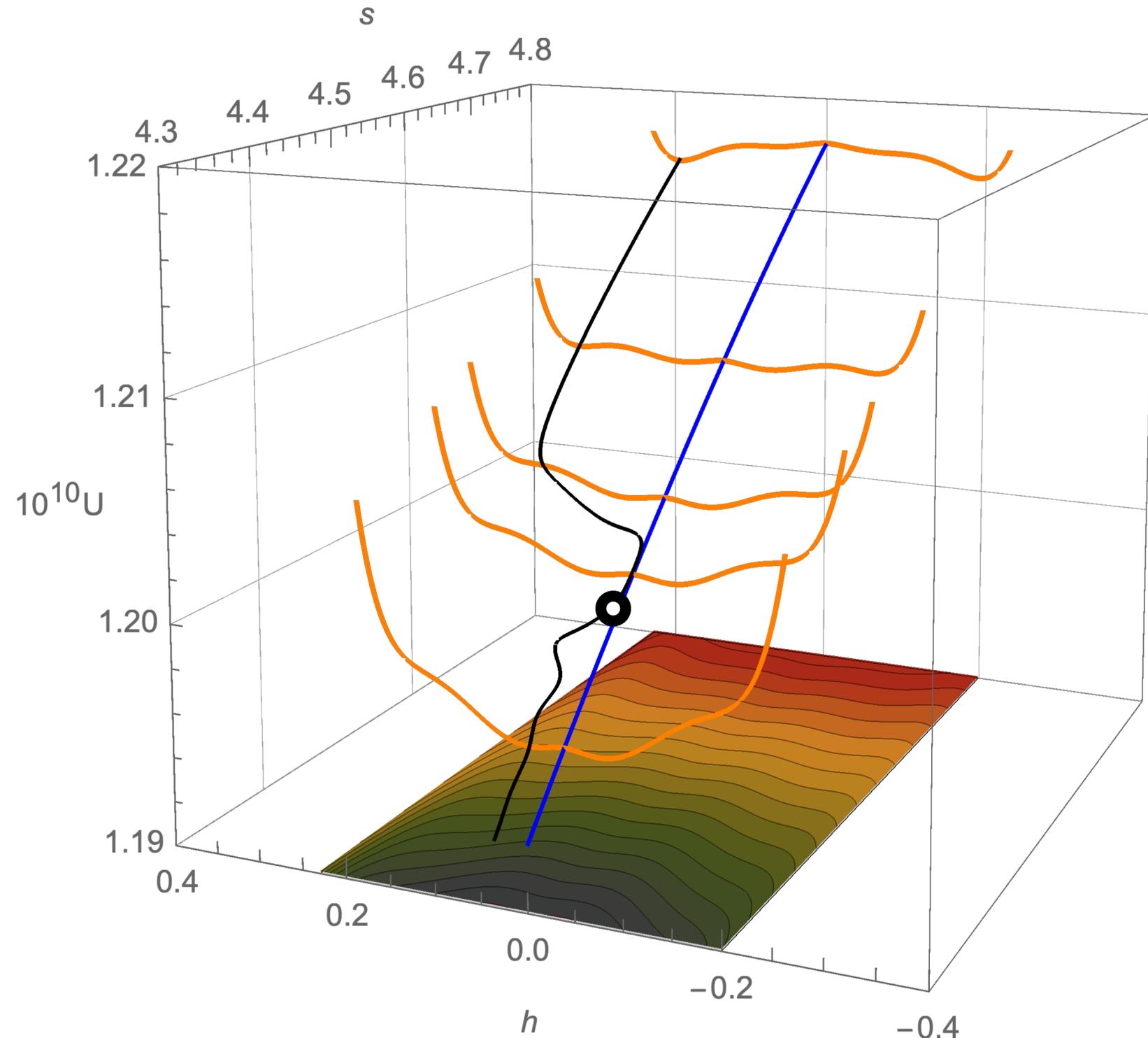
$$S = \frac{H}{\dot{\phi}_0} Q_N \quad \mathcal{P}_S \propto \exp \left[ \left( \frac{2|M_{\text{eff}}|}{H} - 3 \right) N_e \right]$$

$$\ddot{Q}_N + 3H\dot{Q}_N + \left( \frac{k^2}{a^2} + M_{\text{eff}}^2 \right) Q_N \simeq 0$$



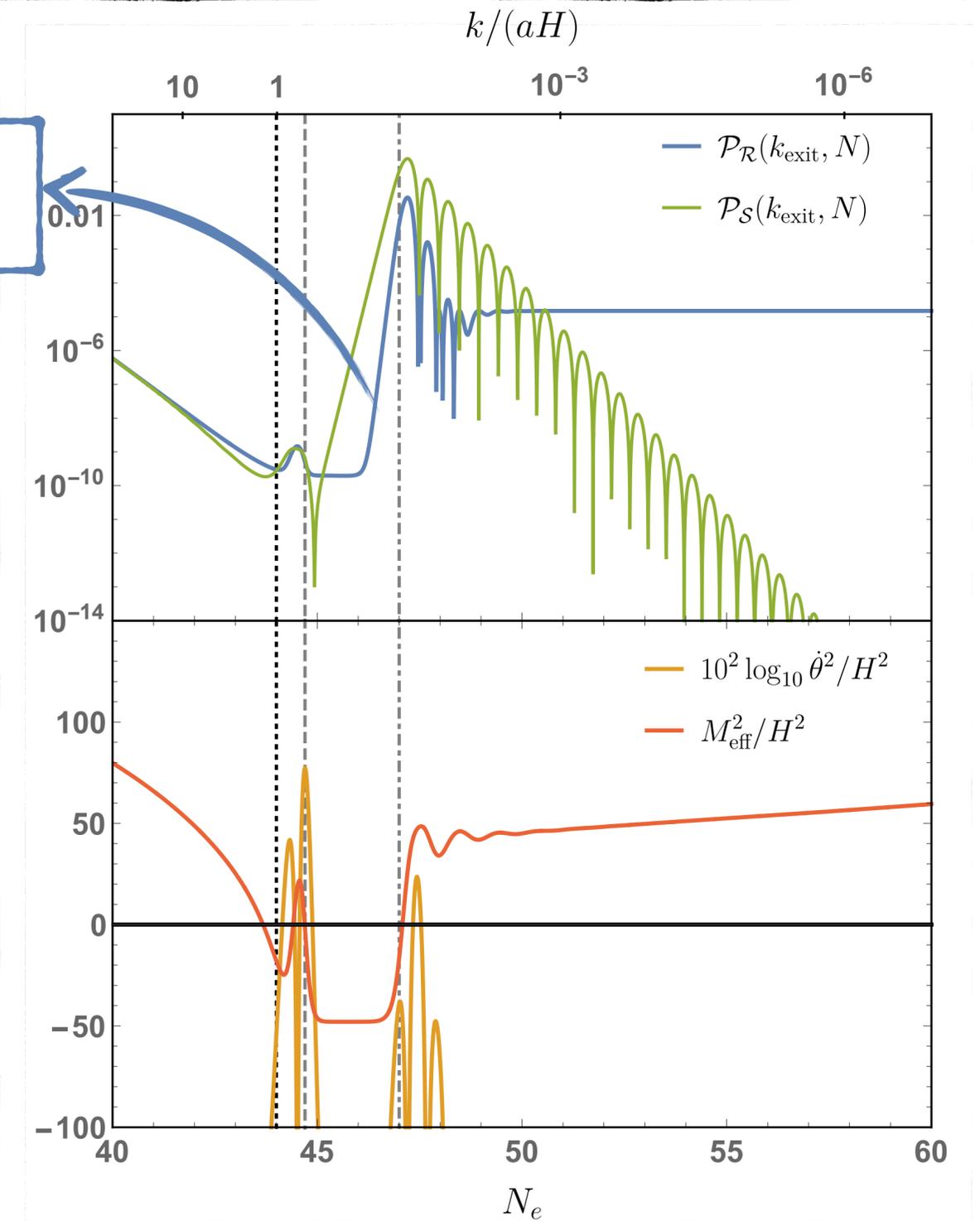
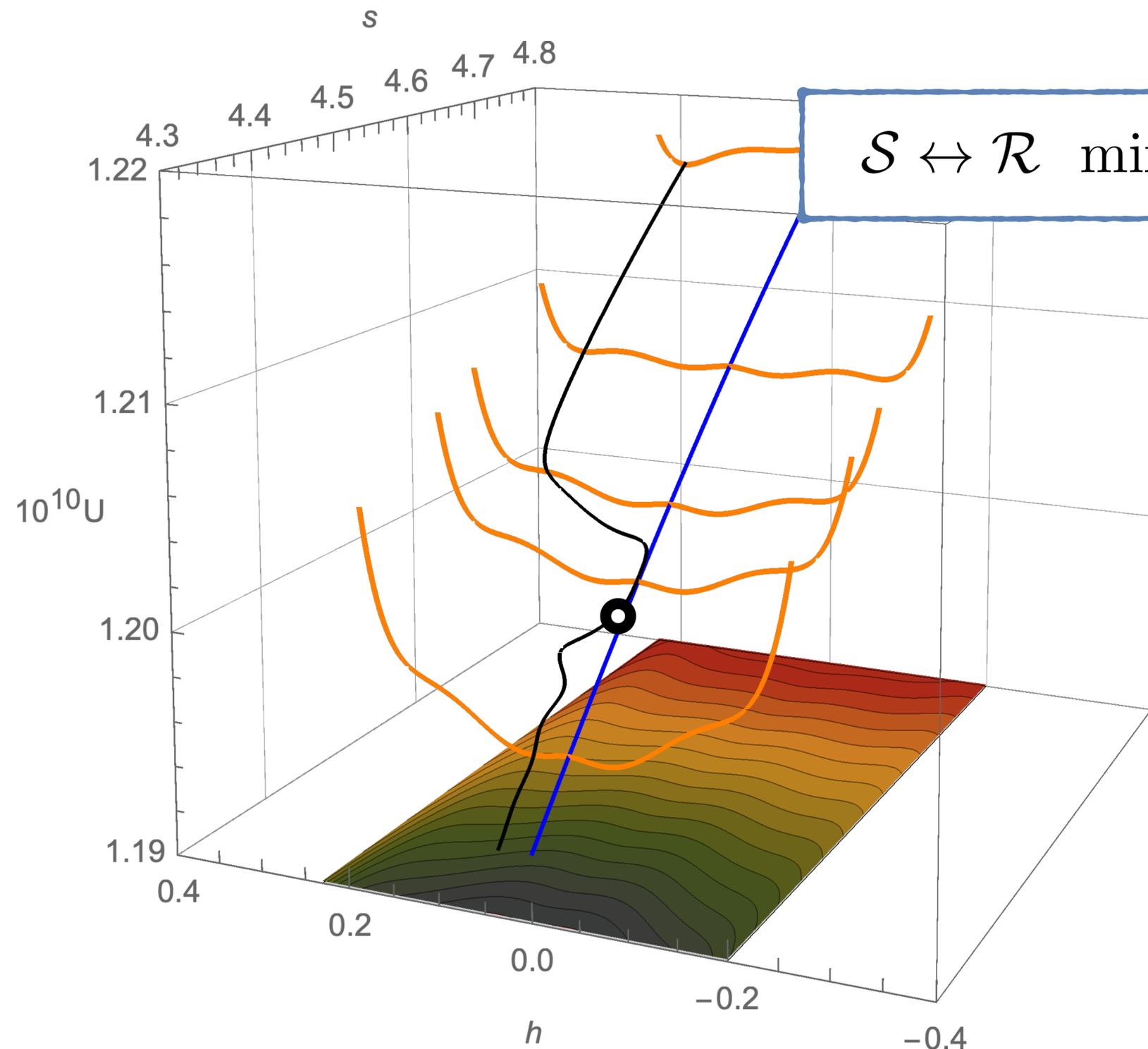
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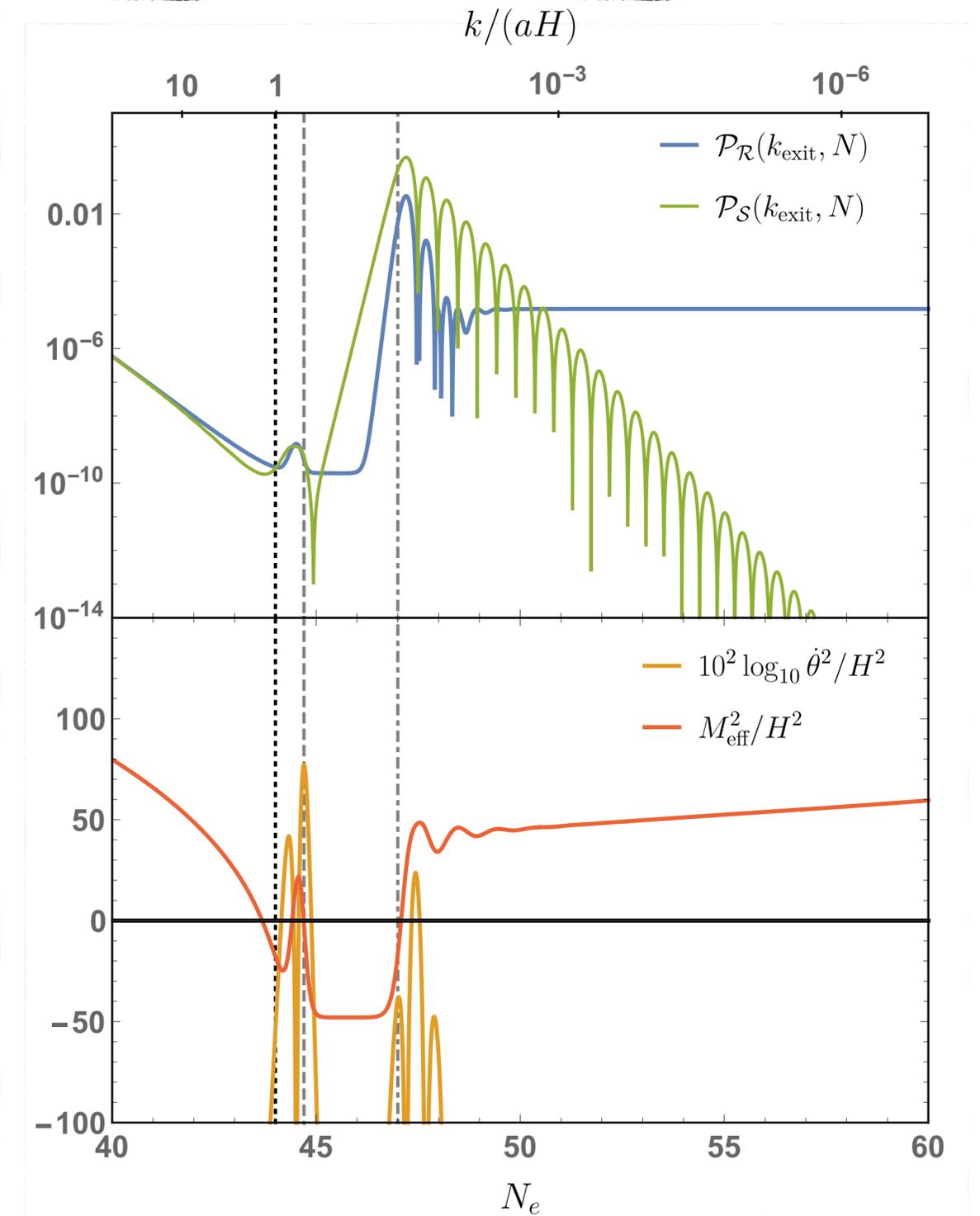
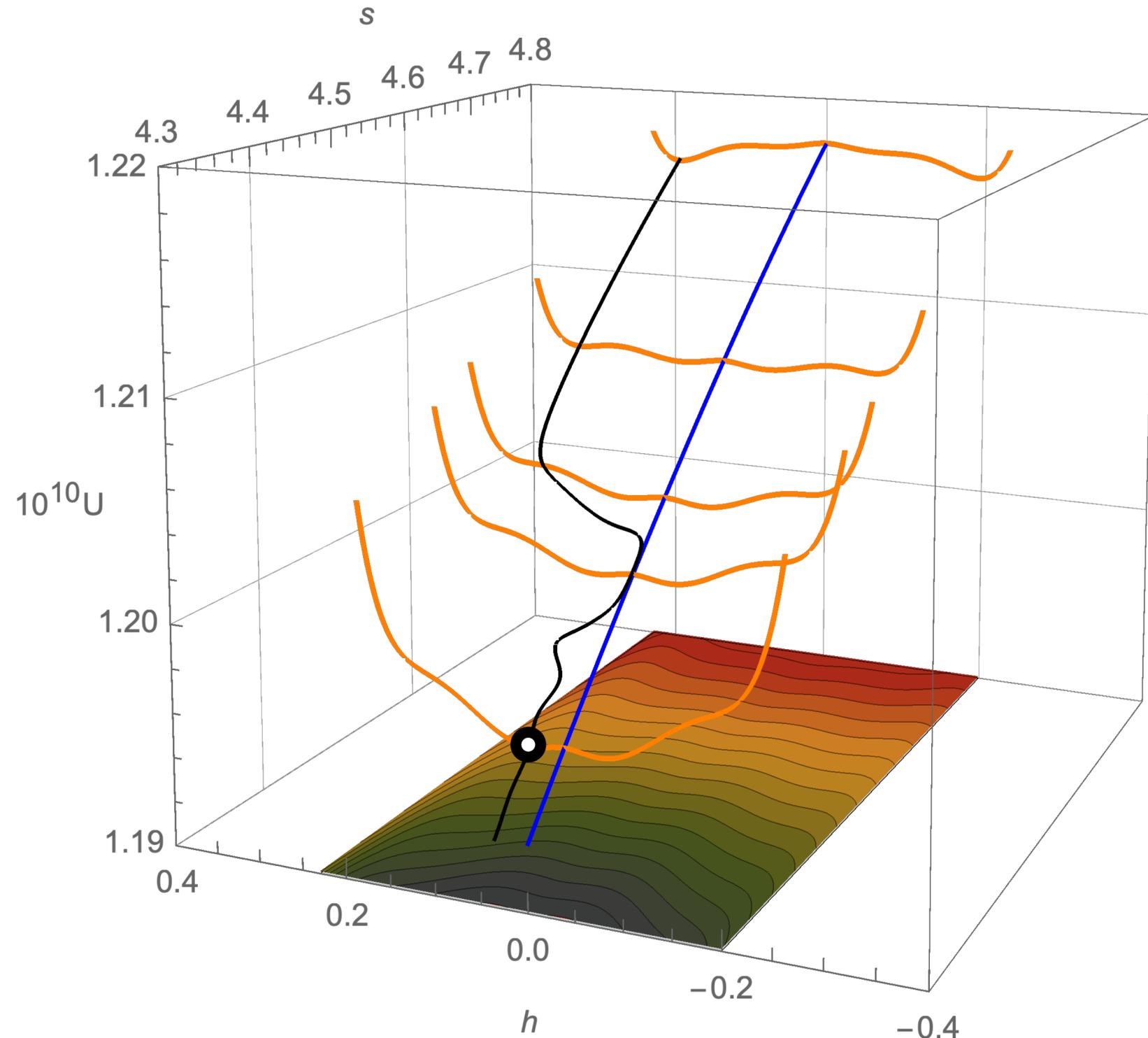
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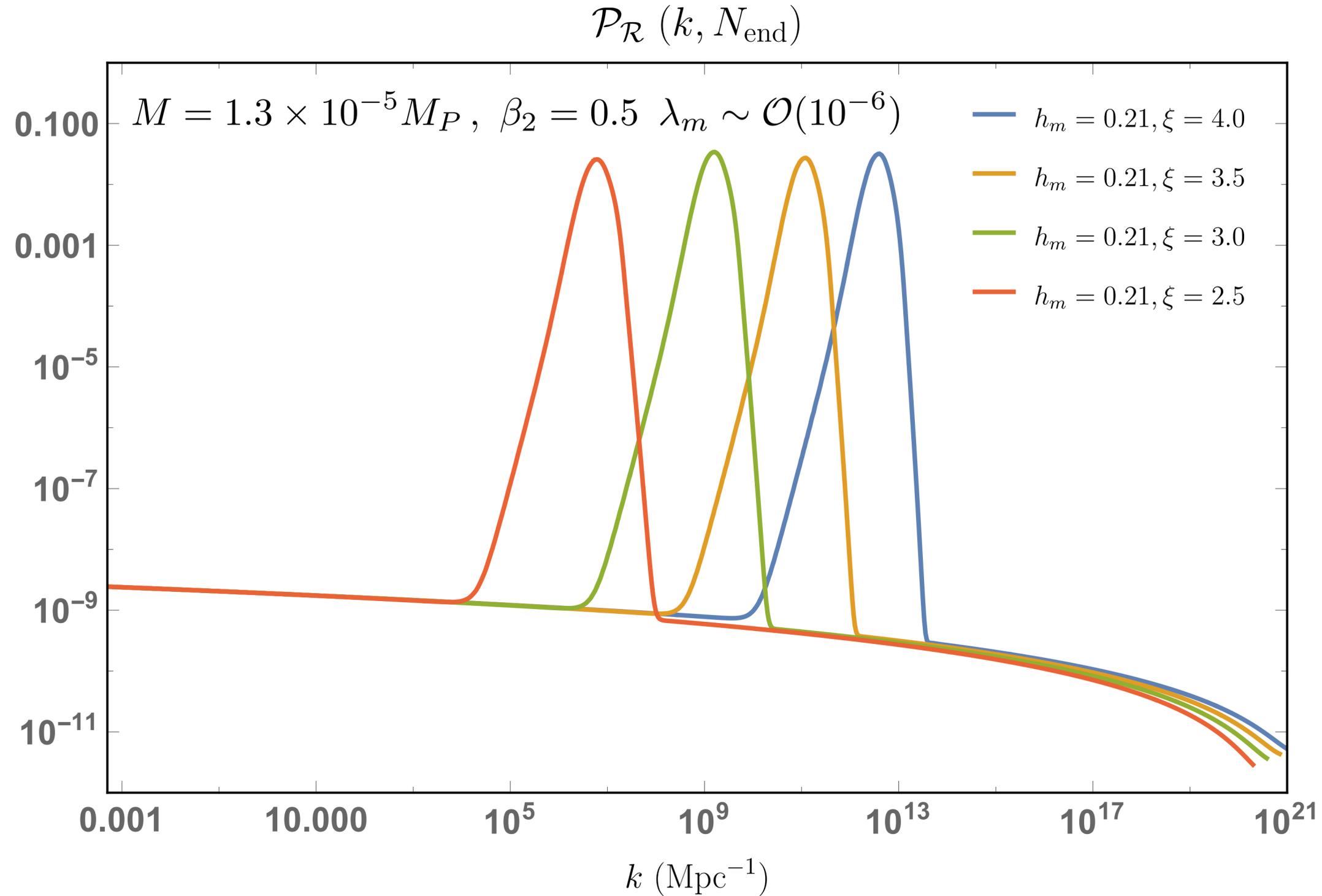
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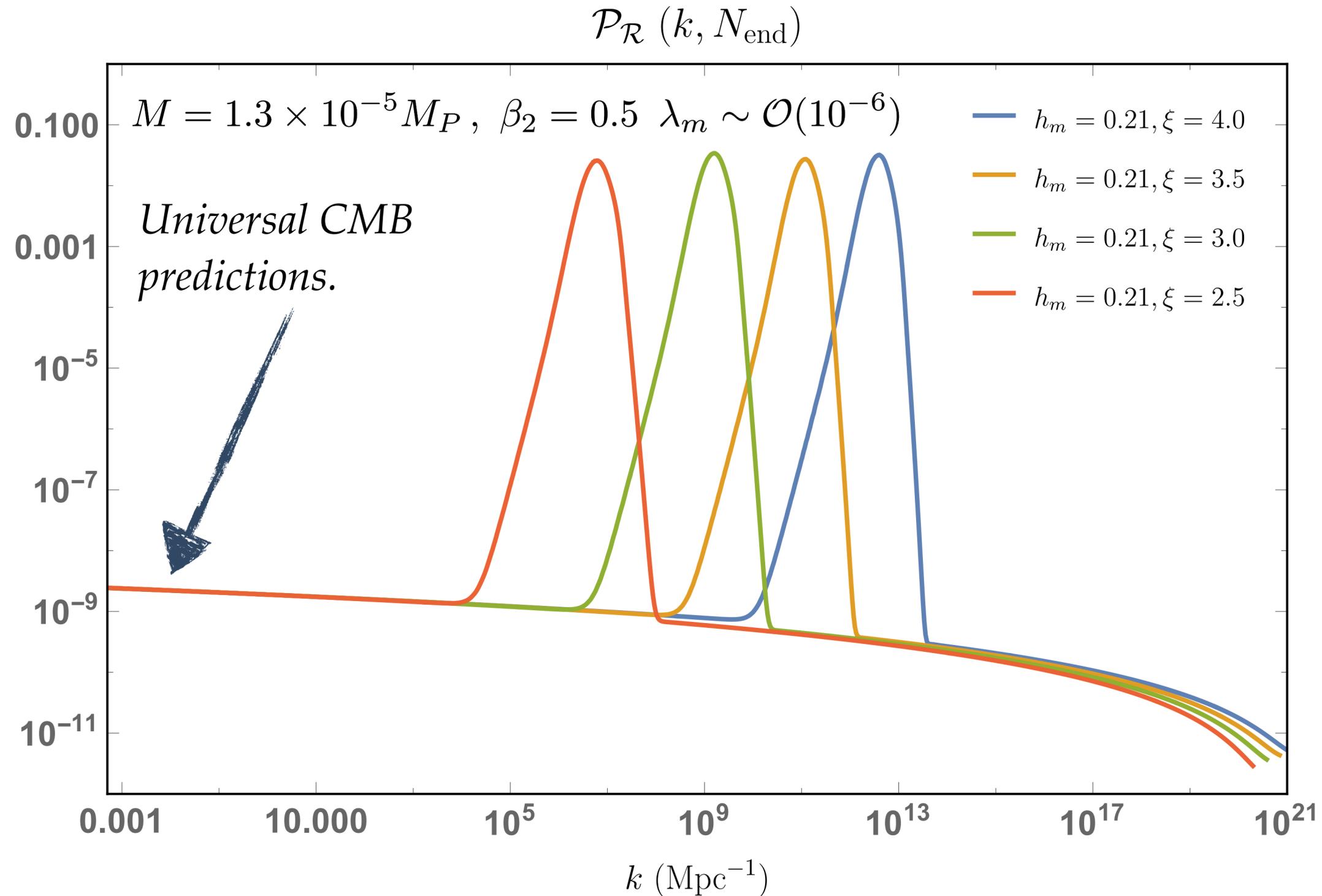
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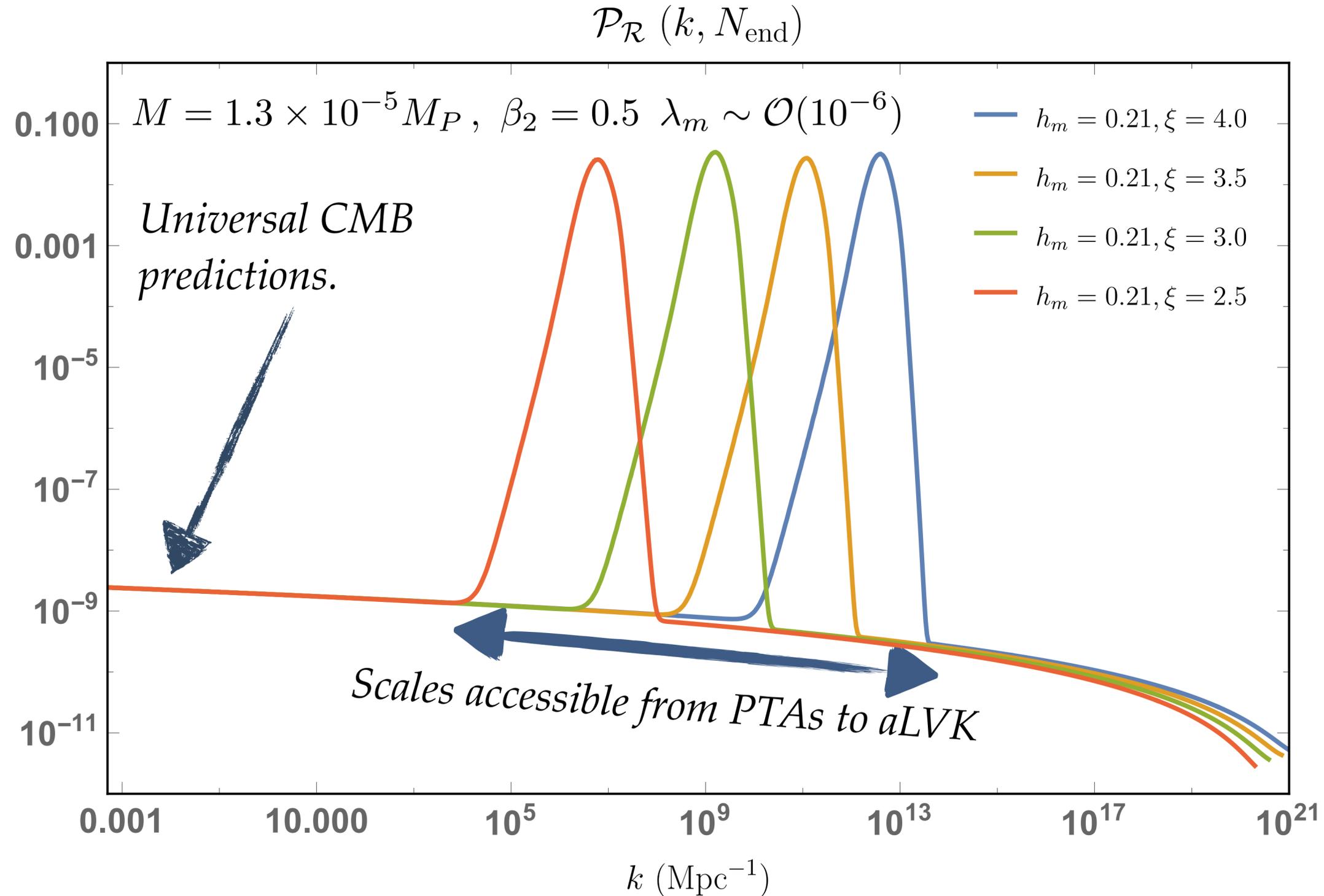
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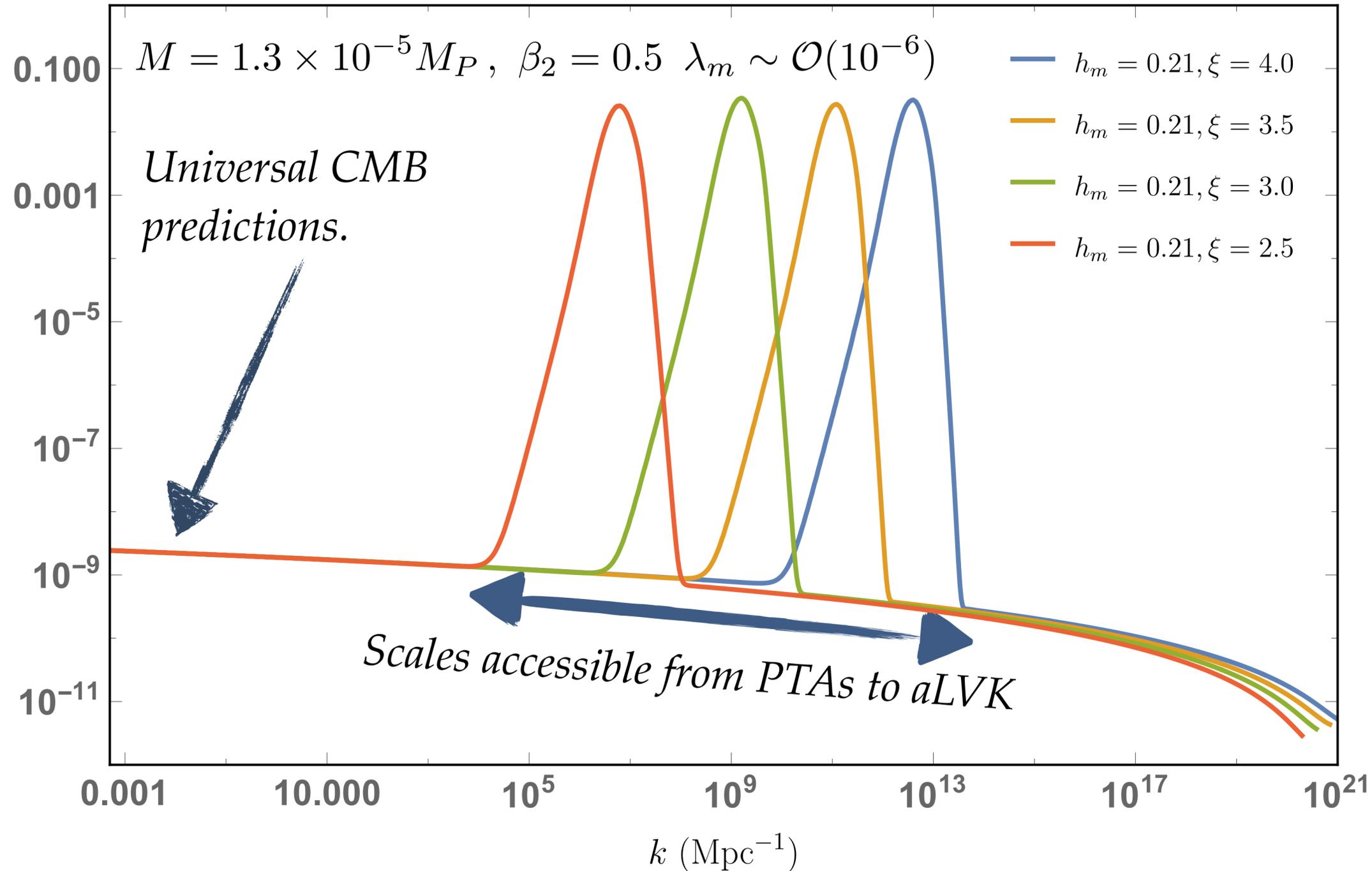
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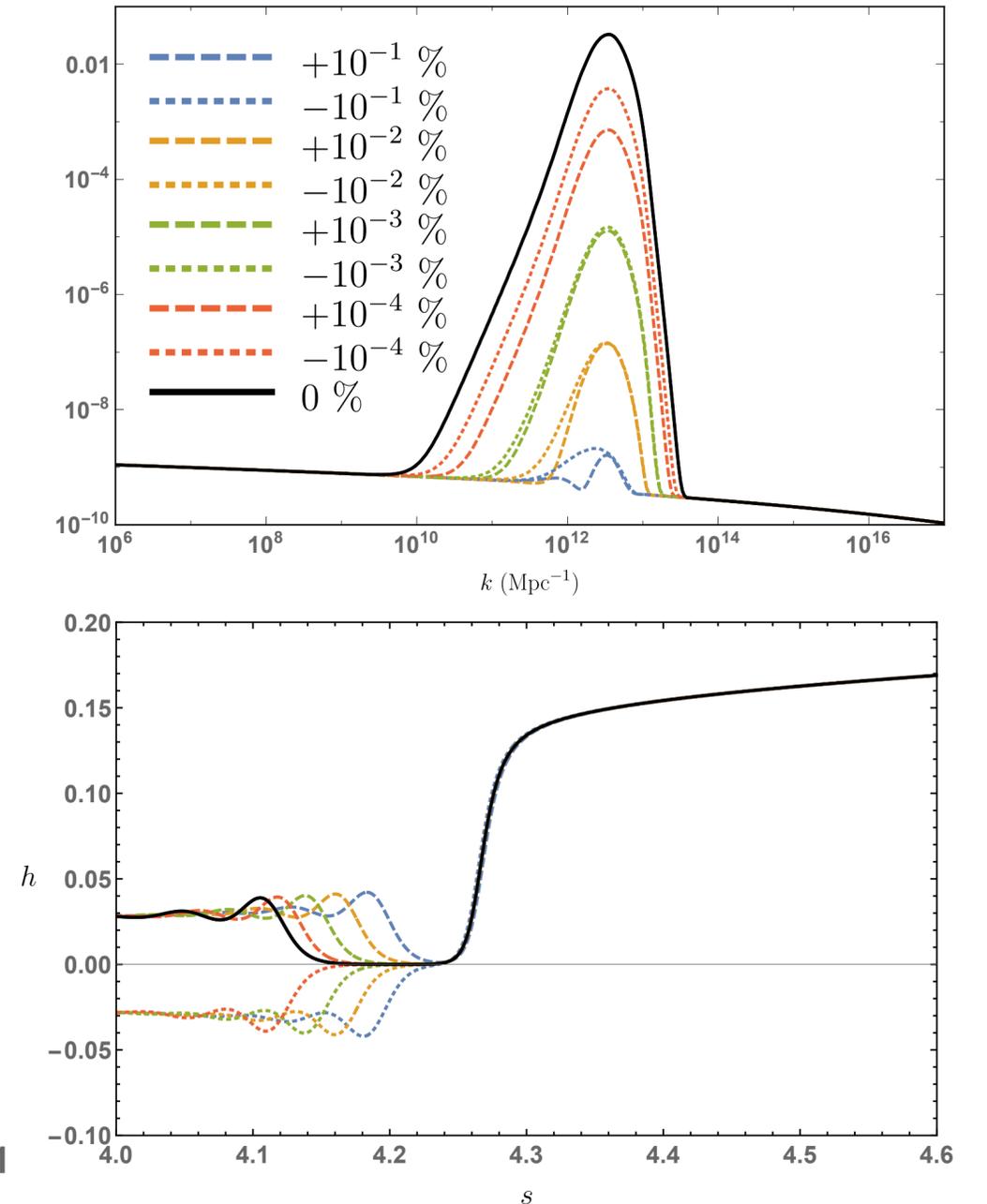


# Higgs- $R^2$ Inflation, Tachyonic Instability

$\mathcal{P}_{\mathcal{R}}(k, N_{\text{end}})$



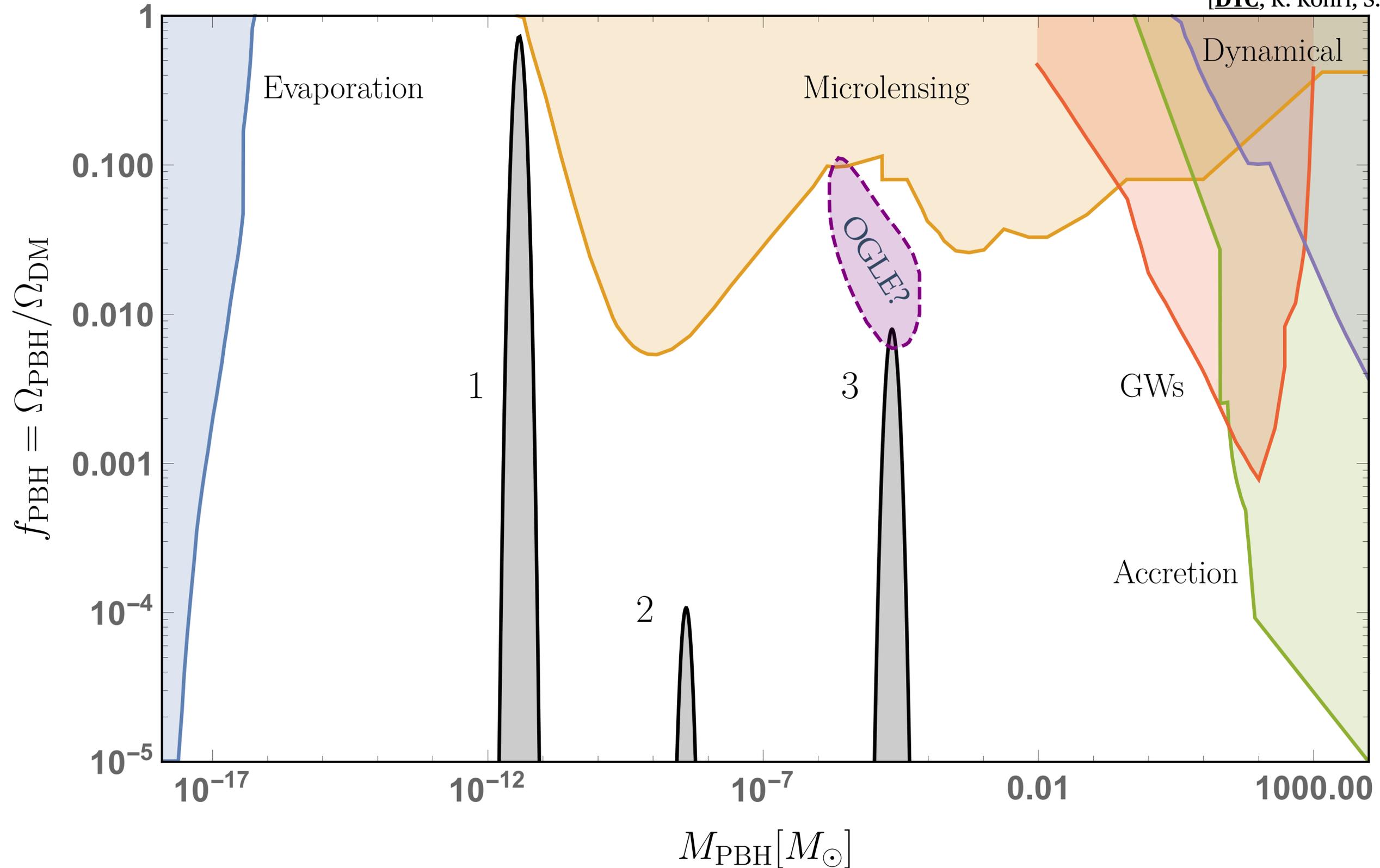
[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



$$\frac{\delta \lambda_m}{\lambda_m} \equiv \frac{\lambda_m^{\text{dev}} - \lambda_m}{\lambda_m} \sim 10^{-4} \%$$

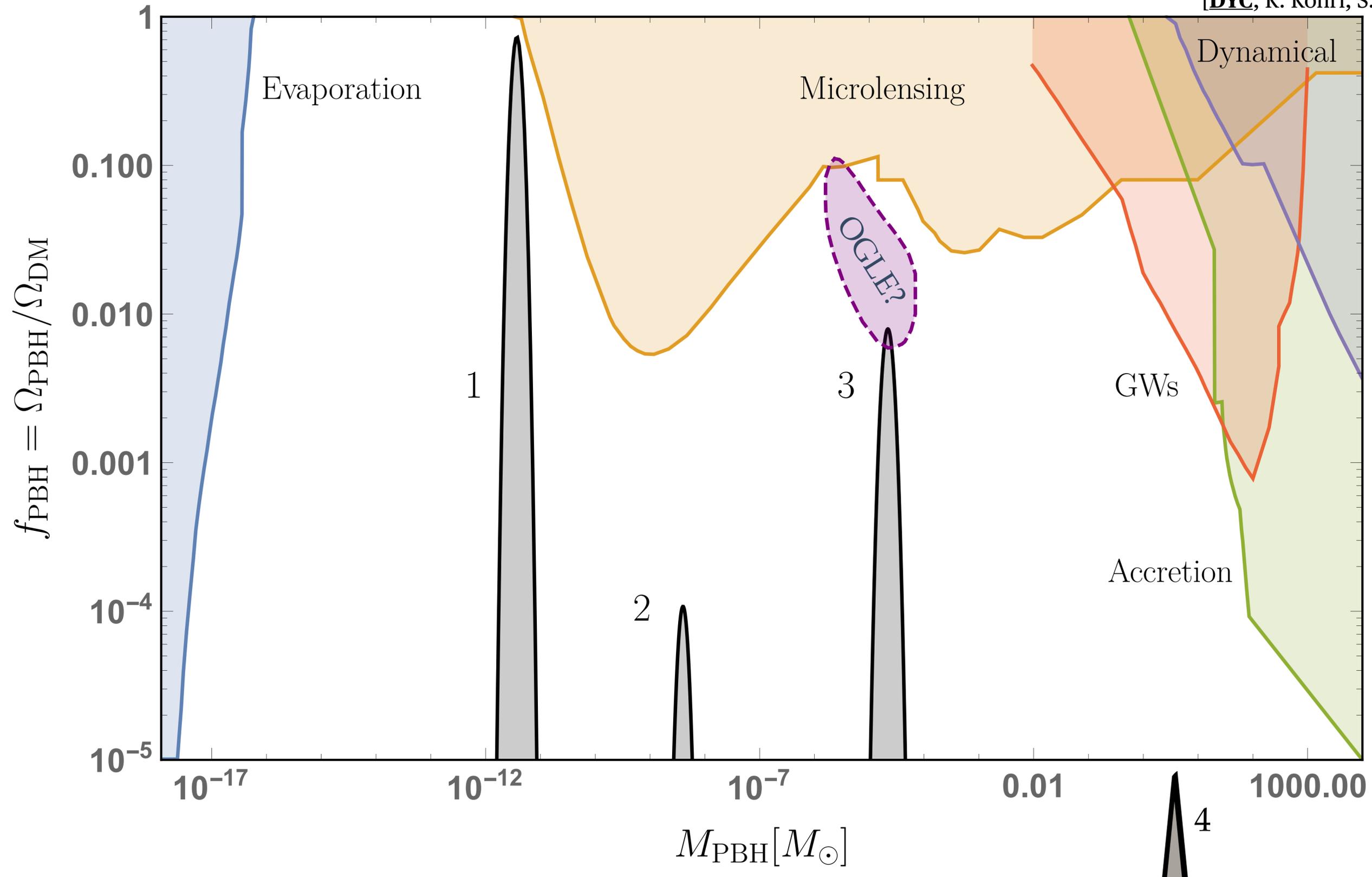
# Phenomena — Primordial Black Holes

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



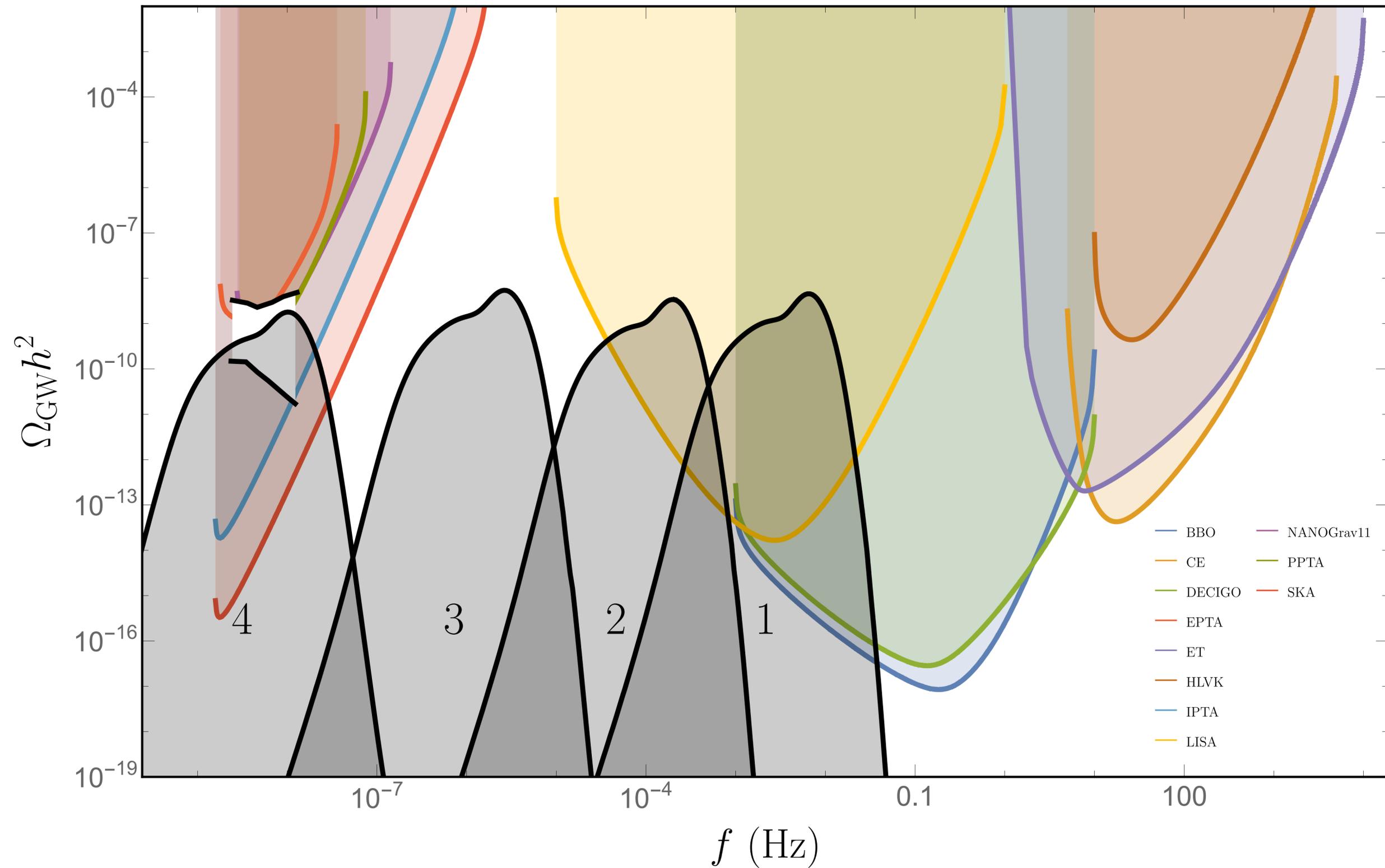
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[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



# Phenomena — Second order GWs

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



# Summary

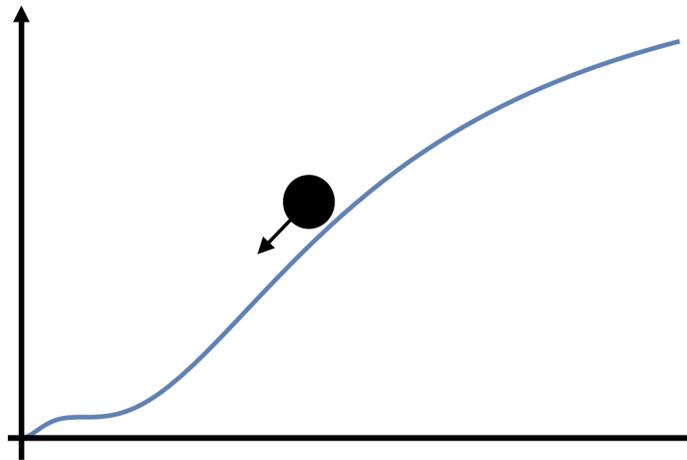
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Higgs- $R^2$  inflation + SM Higgs Running

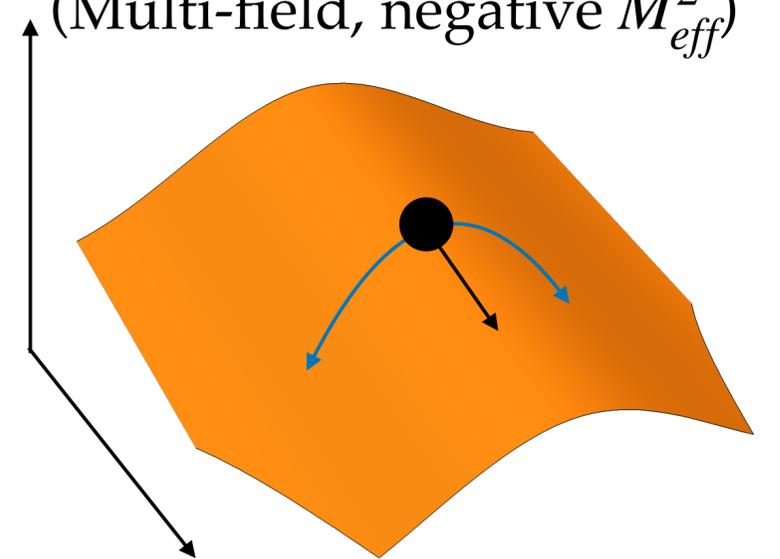
# Summary

## Higgs- $R^2$ inflation + SM Higgs Running

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(Effective single field, USR)



2. Tachyonic instability  
(Multi-field, negative  $M_{eff}^2$ )

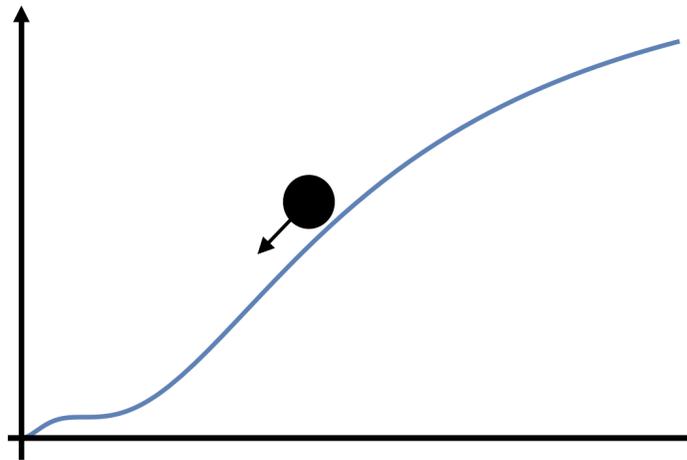




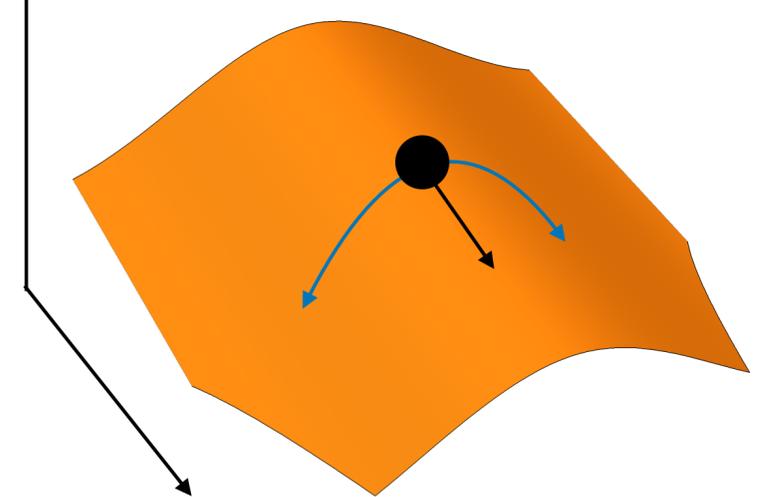
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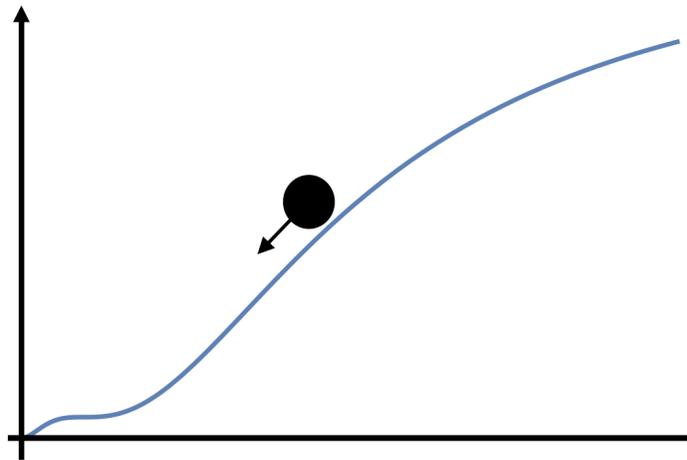
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*PBH & SGWB Production - SM consistency!*

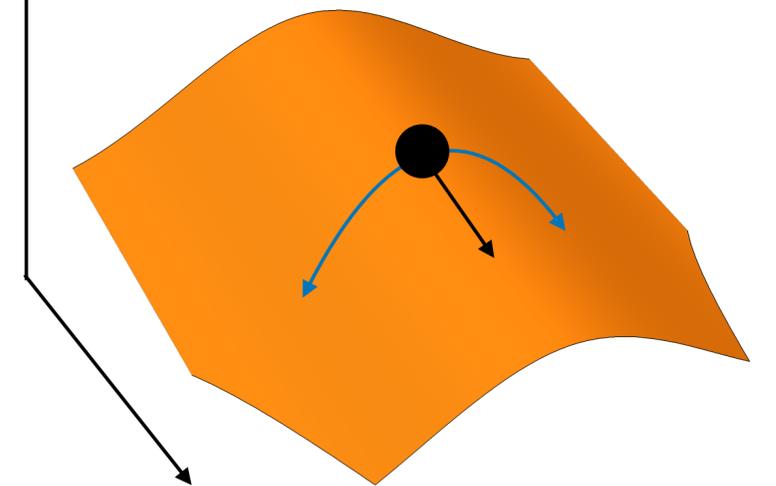
Higgs- $R^2$  inflation + SM Higgs Running

1. Near Inflection Point  
(Effective single field, USR)



*Possible Signatures of the  
Higgs in the Early Universe!*

2. Tachyonic instability  
(Multi-field, negative  $M_{eff}^2$ )



*PBH & SGWB Production - SM consistency!*

Backup slides.

# Introduction - Primordial Black Holes

Then, how large should the density perturbations / curvature perturbations be?

$$\beta \sim \frac{\sigma}{\sqrt{2\pi}\delta_c} e^{-\delta_c^2/(2\sigma^2)} \sim e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}}$$

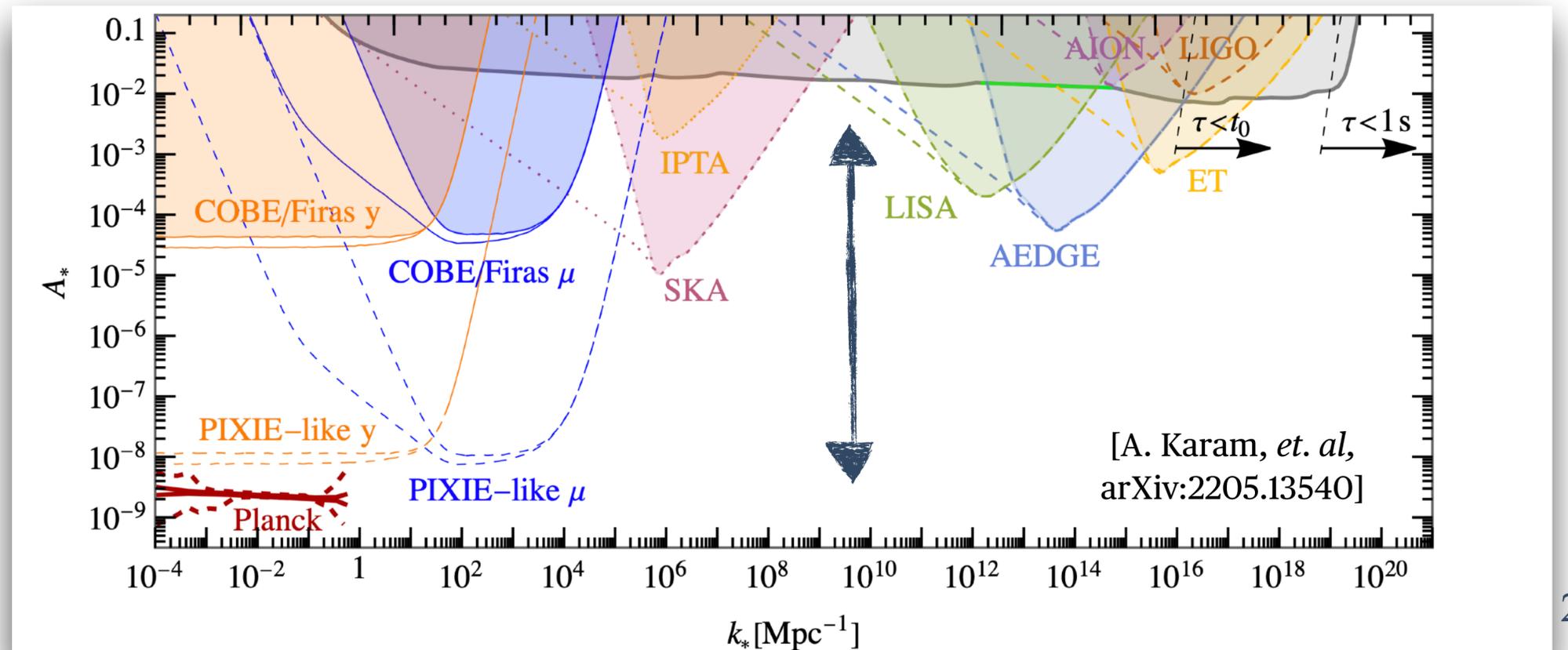
Naive!  $\sigma^2 \sim \mathcal{P}_{\mathcal{R}}$  Exponential dependence

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{DM}}} = \left( \frac{a_{\text{eq}}}{a_{\text{form}}} \right) \beta(M)$$

Take  $f_{\text{PBH}} \sim \mathcal{O}(1) \rightarrow \beta(M) \sim 10^{-8}$  then

$$\mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(10^{-2})$$

For solar-mass black holes,  $a_{\text{eq}}/a_{\text{form}} \sim 10^8$  in RD



# Higgs- $R^2$ Inflation, Perturbations

Second order perturbation with  $\phi^a(t, \vec{x}) = \phi_0^a(t) + \delta\phi^a(t, \vec{x})$ ,  $ds^2 = -(1 + 2\psi)dt^2 + a(t)^2(1 - 2\psi)\delta_{ij}dx^i dx^j$ .

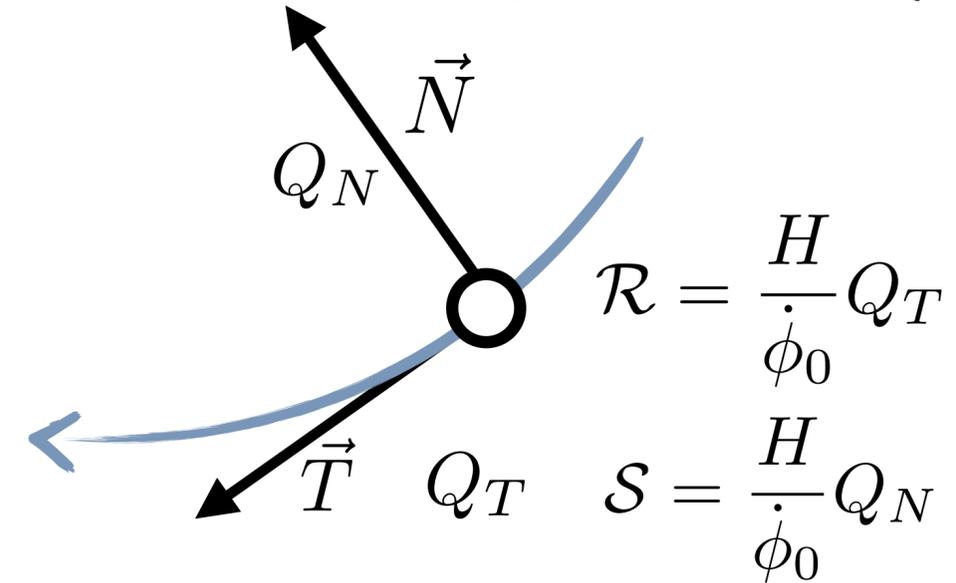
$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel}) H \dot{\mathcal{R}} + \frac{k^2}{a^2} \mathcal{R} = -2 \frac{H^2}{\dot{\phi}_0} \eta_{\perp} \left[ \dot{Q}_N + \left( 3 - \eta_{\parallel} + \frac{\dot{\eta}_{\perp}}{H \eta_{\perp}} \right) H Q_N \right]$$

$$\ddot{Q}_N + 3H \dot{Q}_N + \left( \frac{k^2}{a^2} + M_{\text{eff}}^2 \right) Q_N = 2\dot{\phi}_0 \eta_{\perp} \dot{\mathcal{R}}.$$

[S. Groot Nibelink, B.J.W. van Tent, (2002)] , [S. Cespedes et. al, (2012)]  
[A. Achucarro et. al, (2012)] ...

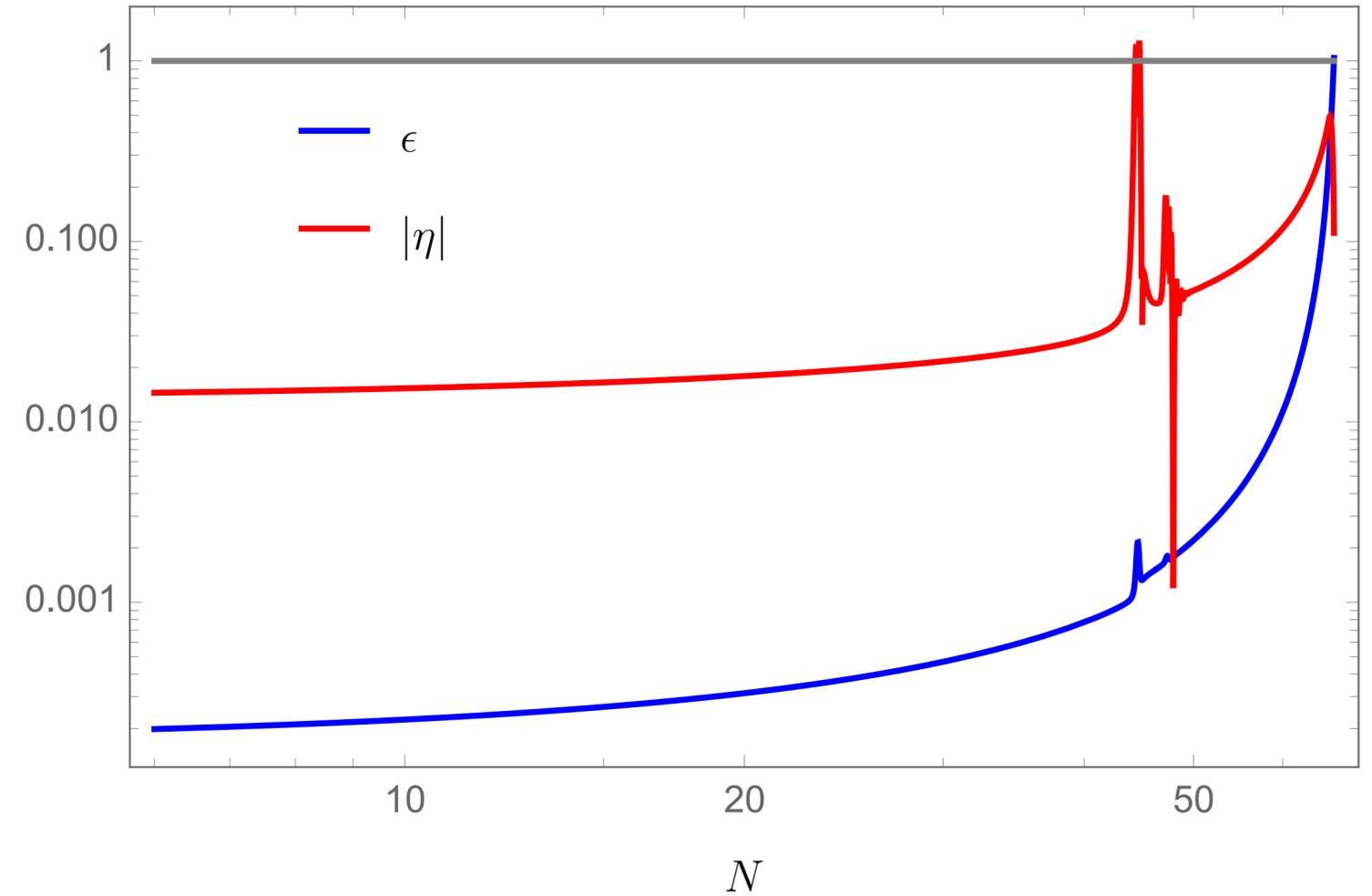
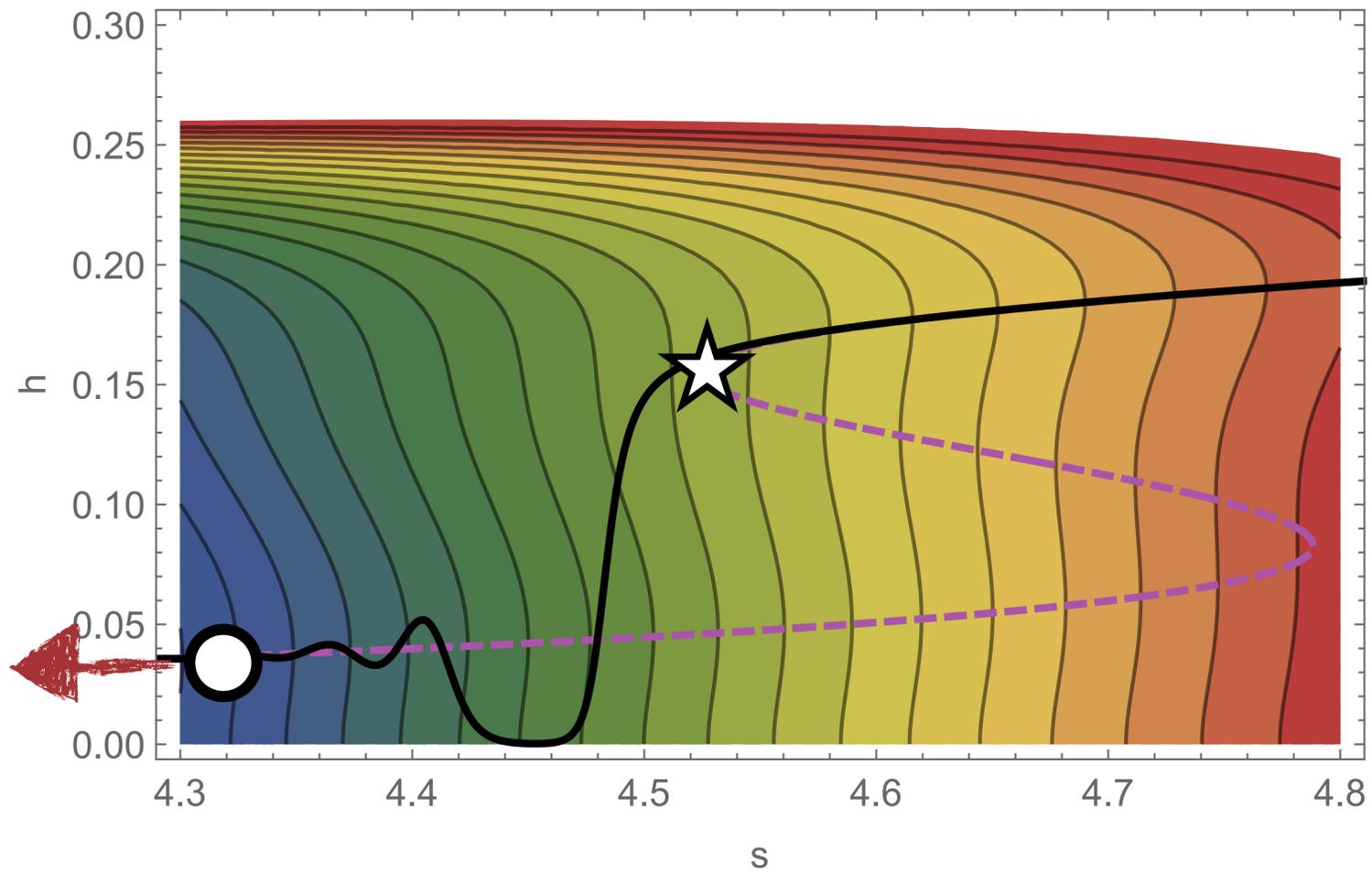
$$\eta_{\parallel} \equiv -\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} \quad \eta_{\perp} \equiv \frac{U_N}{\dot{\phi}_0 H} \quad Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H} \psi \quad \dot{\theta} \equiv H \eta_{\perp}$$

$$M_{\text{eff}}^2 = U_{NN} + H^2 \epsilon \mathbb{R} - \dot{\theta}^2.$$



- $M_{\text{eff}}^2 < 0$  leads to *tachyonic growth of  $Q_N$* , then gets *sourced to  $\mathcal{R}$*  through *turns in the trajectory*.

# Higgs- $R^2$ Inflation, Slow-Roll Parameters



# Higgs- $R^2$ Inflation / Scalaron

$$S_J = \int d^4x \sqrt{-g} \left[ \frac{M_P^2 + \xi h^2}{2} R + \frac{M_P^2}{12m_s^2} R^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{g^2}{4} g^{\mu\nu} W_\mu^+ W_\nu^- (h+v)^2 + \dots - \frac{\lambda}{4} h^4 + \mathcal{L} \right].$$



Auxiliary field  $\chi$

$$S_J = \int d^4x \sqrt{-g} \left[ \frac{M_P^2 + \xi h^2}{2} \chi + \frac{M_P^2}{12m_s^2} \chi^2 + \left( \frac{M_P^2 + \xi h^2}{2} + \frac{M_P^2}{6\mu^2} \chi \right) (R - \chi) - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{g^2}{4} g^{\mu\nu} W_\mu^+ W_\nu^- (h+v)^2 + \dots - \frac{\lambda}{4} h^4 + \mathcal{L} \right]$$

$$\Omega^2 \equiv 1 + \xi \frac{h^2}{M_P^2} + \frac{\chi}{3m_s^2} \equiv e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}}$$

Weyl



$$g_{\mu\nu}^E = \Omega^2 g_{\mu\nu}, \quad g_E = \Omega^8 g.$$

Notice “non-canonical field space metric”

Multi-field /  $(h, R) \rightarrow (h, s)$  system.

$$S = \int d^4x \sqrt{-g_E} \left[ \frac{M_P^2}{2} R_E - \frac{1}{2} (\partial_\mu s)^2 - \frac{1}{2} \Omega^{-2} (\partial_\mu h)^2 - V(h, s) + \dots \right]$$

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{s}{M_P}} \left\{ \frac{3}{4} M_P^2 M^2 \left( e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda}{4} h^4 \right\}$$

$(s, h) \simeq (0, 0)$

$$\simeq \frac{\lambda}{4} h^4 + \frac{3\xi^2 M^2}{4M_P^2} h^4 + \frac{1}{2} M^2 s^2 + \dots - \frac{\lambda}{\sqrt{6} M_P} s h^4 - \frac{M^2}{6\sqrt{6} M_P^3} s^5 + \left( \frac{\lambda}{3M_P^2} + \frac{\xi^2 M^2}{M_P^4} \right) h^4 s^2 + \dots$$

$$\Lambda \sim \mathcal{O} \left( \frac{M_P^2}{\xi^2 M^2} \right) M_P > M_P \quad \text{Theory unitarized through the scalaron!}_{27}$$