Stockholm University

Knut and Alice Wallenberg Foundation

Quantum superpositions of spacetimes and black holes

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Quantum signatures of black hole mass superpositions

PRL **129** 181301 (2022)

OKC@ 15, Stockholm 18 October 2023

Composite quantum systems in curved space time – foundations and applications
Wallenberg Academy Fellow

Since February 2023 @ Stockholm Uni

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Germain Tobar
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Dr Joshua Foo*
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* co-sup with Tim Ralph

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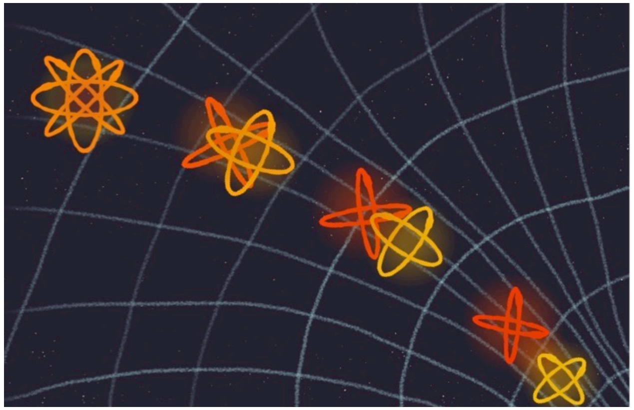
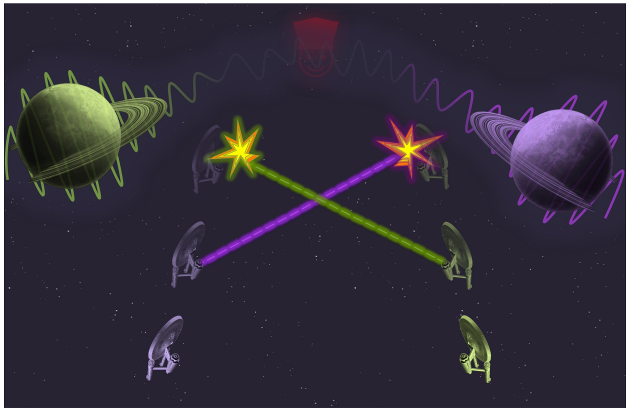
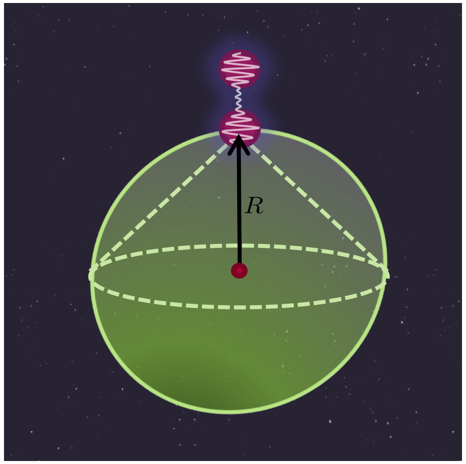
Theory and tests of QM+GR



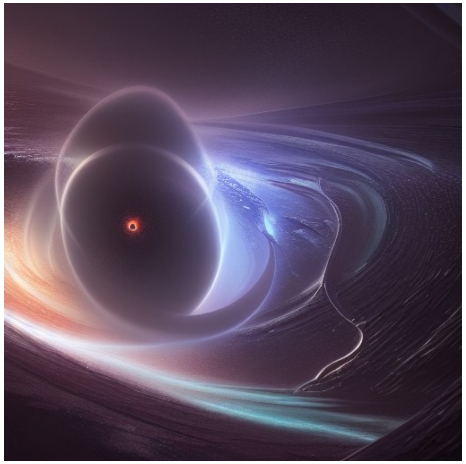
Quantum causal order



Tests of alternative theories
(collapse models, classical-channel gravity)



QFT in classical & quantum curved spacetimes, link to quantum thermo



Quantum superpositions of spacetime?

In GR spacetime is a physical system whose states depend on matter-energy DOFs

consider matter configurations sourcing different* classical spacetimes, gA, gB
e.g. a Black Hole with mass MA or MB

QM allows us to assign a state to any physical matter configuration, $|MA\rangle$, $|MB\rangle$

Superposition principle: $\alpha|MA\rangle + \beta|MB\rangle$ is also a physical configuration

→ matter and spacetime described by a superposition of these two configurations

$$|\psi\rangle = \alpha|MA, gA\rangle + \beta|MB, gB\rangle$$

* Not diffeomorphic to each other

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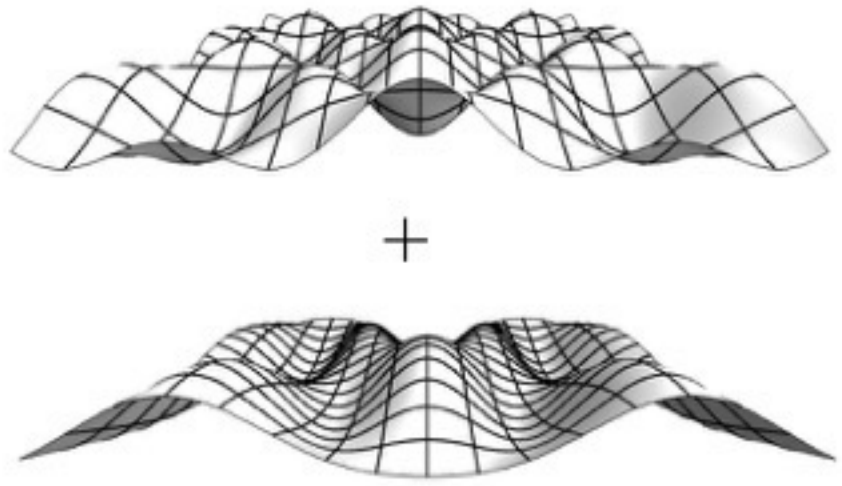
$$|\psi\rangle = \alpha|MA, gA\rangle + \beta|MB, gB\rangle$$

What are physical signatures of $|\psi\rangle$?

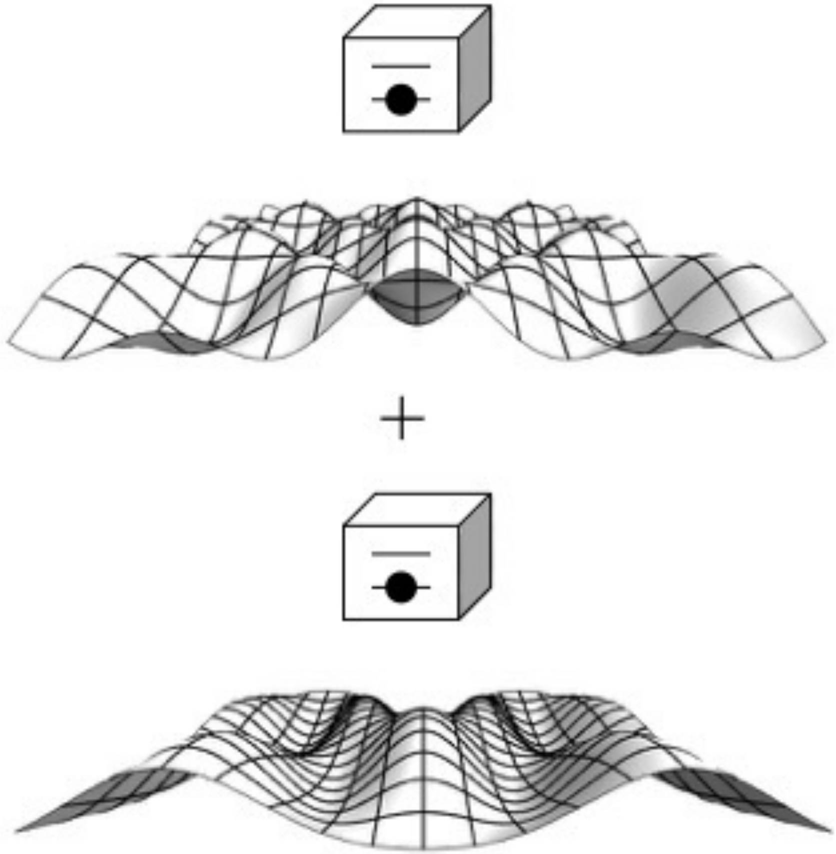
* Not diffeomorphic to each other

Operational approach



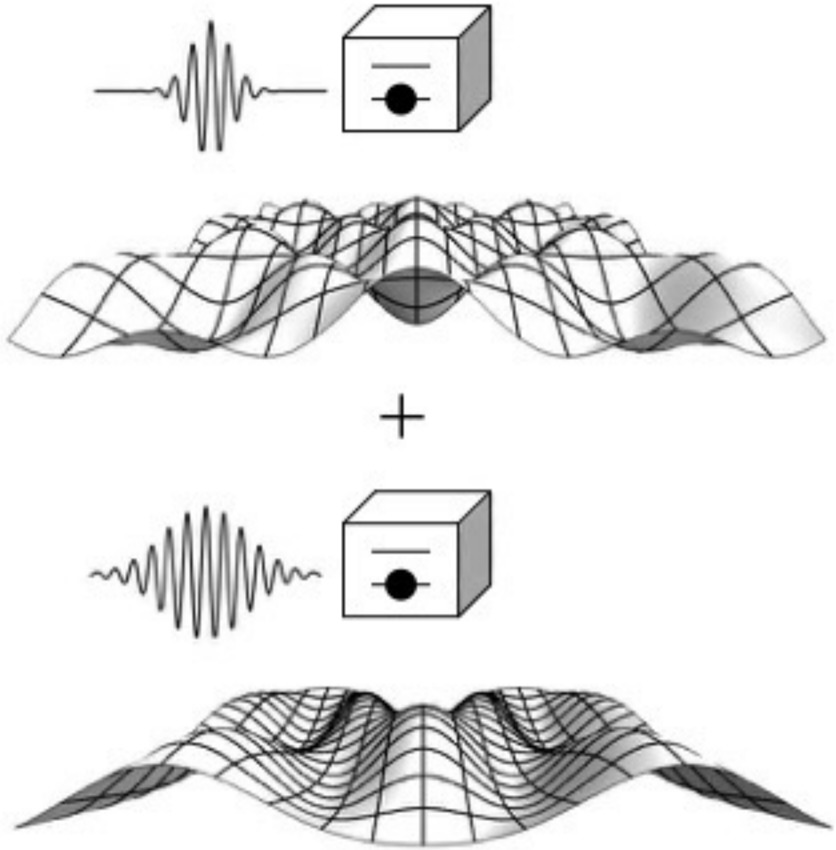
How to image spacetime superpostion?

Operational approach



Measurements with another system that couples to spacetime

Operational approach



Here – indirect imaging: measure quantum fields in the (quantum) spacetime

quantum field detectors

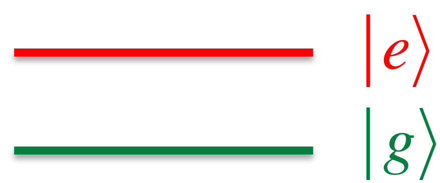
Example: atoms – respond to EM field (to photons)

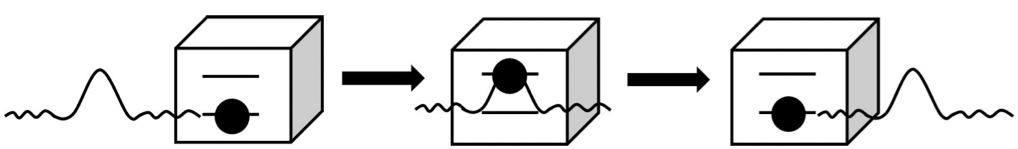
Absorb a photon → internal energy jumps up a level

Emit a photon → internal energy drops down a level

Toy-model of an atom: Unruh—DeWitt (UDW) detector

Energy gap Ω





like a quantum dot ☺

B.S. DeWitt in *General Relativity: An Einstein Centenary Survey* (CUP 1980)

Unruh-DeWitt model

Characterised by detector-field interaction (in the interaction picture)

$$\hat{\mathcal{H}}_i(\tau) = \lambda\,\eta(\tau)\,\sigma(\tau)\,\hat{\Phi}(\mathrm{x}_i(\tau))$$

Coupling constant

"switching function"

field

$$|e\rangle\langle g|e^{i\Omega\tau} + |g\rangle\langle e|e^{-i\Omega\tau}$$

Unruh-DeWitt model

Characterised by detector-field interaction (in the interaction picture)

$$\hat{\mathcal{H}}_i(\tau) = \lambda\eta(\tau)\sigma(\tau)\hat{\Phi}(\mathrm{x}(\tau))$$

Time evolution

$$\hat{U} = \mathcal{T} e^{-i\int d\tau \frac{dt}{d\tau}\hat{\mathcal{H}}_i}$$ / perturbatively in λ

Excitation probability (g → e) given some initial field state

$$|\psi_f\rangle = \hat{U}|g\rangle|0\rangle \rightarrow \mathrm{Tr}_{\mathrm{Field}}\{|\psi_f\rangle\langle\psi_f|\} \rightarrow \begin{bmatrix} 1-\mathcal{P}_E & 0 \\ 0 & \mathcal{P}_E \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$$\mathcal{P}_E = \lambda^2 \int d\tau \int d\tau' \eta(\tau)\eta(\tau') e^{-i\Omega(\tau-\tau')} \underline{\langle 0|\hat{\Phi}(x(\tau)\hat{\Phi}(x(\tau'))|0\rangle}$$

Two-point/Wightman function $W(x,x')$

Unruh-DeWitt model

Accelerated detectors thermalize even if field in vacuum

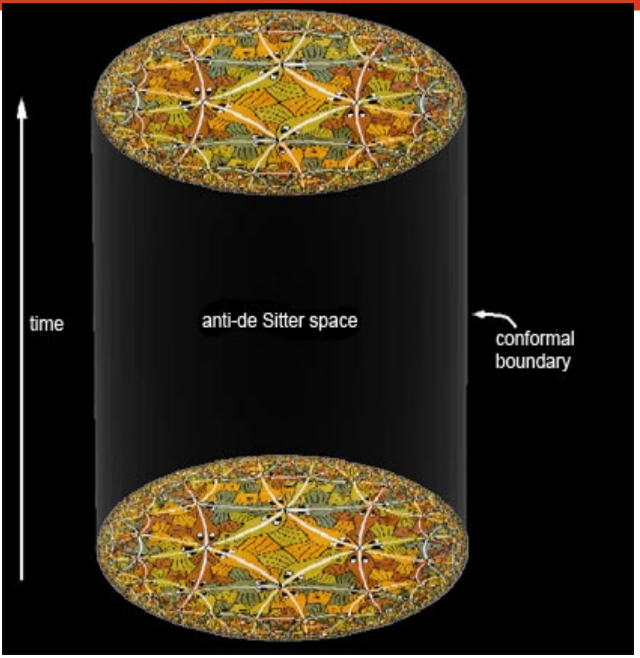
Unruh effect/temperature:

$$T = \frac{a}{2\pi}\left(\frac{\hbar}{k_B c}\right)$$

Constant proper acceleration

S.A. Fulling *PRD* **7** 2850 (1973)
P.C.W. Davies *J Phys A* **8** 609 (1975)
W. G. Unruh *PRD* **14** 3251 (1976)

UDW detectors in superposition of spacetimes



Graphic based on: J.Maldacena
The Illusion of Gravity,
Scientific American 293 (5): 56–63 (2005)

Anti de Sitter Spacetime
max symmetric Lorentzian manifold
negative scalar curvature

$ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + dX_2^2$ covering space metric

$-l^2 = -T_1^2 - T_2^2 + X_1^2 + X_2^2$ hyperbolic constraint surface

$$T_1 = l\sqrt{\frac{r^2}{l^2}}\cosh\phi \qquad X_1 = l\sqrt{\frac{r^2}{l^2}}\sinh\phi$$

$$T_2 = l\sqrt{\frac{r^2}{l^2} - 1}\sinh\frac{t}{l} \qquad X_2 = l\sqrt{\frac{r^2}{l^2} - 1}\cosh\frac{t}{l}$$

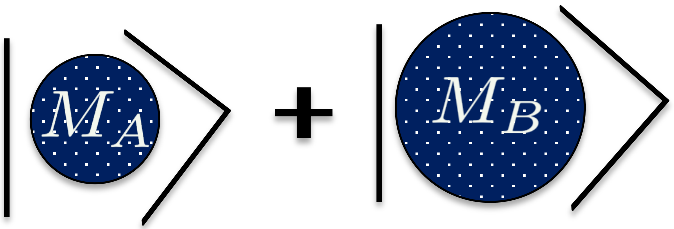
$$ds^2 = -\left(\frac{r^2}{l^2} - 1\right)dt^2 + \left(\frac{r^2}{l^2} - 1\right)dr^2 + r^2 d\phi^2 \qquad \Gamma : \phi \to \phi + 2\pi\sqrt{M}$$

Banados-Teitelboim-Zanelli (BTZ) black hole → event horizon at $r_H = \sqrt{M}\, l$

BTZ *The Black hole in three-dimensional space-time PRL*, **69** (13): 1849–51 (2008)

UDW detectors in superposition of spacetimes

Superpose mass of the BTZ black hole



Initial state of mass, field, internal state of detector

$$|\psi(t_i)\rangle = \frac{1}{\sqrt{2}}(|M_A\rangle + |M_B\rangle)|0_{\mathrm{AdS}}\rangle|g\rangle$$

Interaction

$$\hat{H}_{\mathrm{int.}} = \lambda \sum_{i=A,B} \eta(\tau_i)\hat{\sigma}(\tau_i)\hat{\phi}(\mathbf{X}_i) \otimes |M_i\rangle\langle M_i|$$

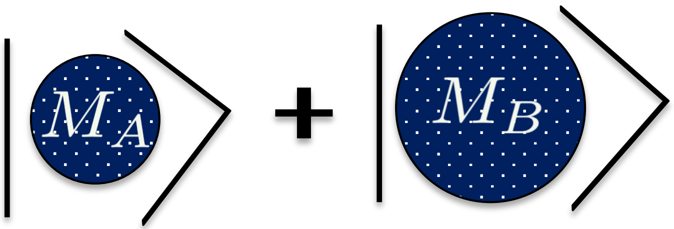
Automorphic field

$$\hat{\phi}(\mathbf{X}_i) = \frac{1}{\sqrt{\sum_n \eta^{2n}}} \sum_n \eta^n \hat{\psi}(\Gamma_i^n \mathbf{X})$$

Isometry $\Gamma_i^n \mathbf{X} : (t, r, \phi) \mapsto (t, r, \phi + 2\pi n \sqrt{M_i})$

UDW detectors in superposition of spacetimes

Superpose mass of the BTZ black hole



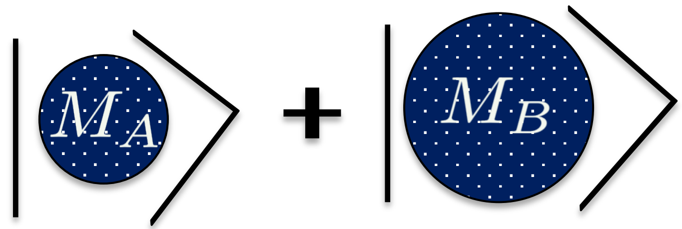
- Detector at fixed r = R
- Time evolution defined with respect to a faraway clock

  (at large r proper time diff. between a clock in spacetime with M_A , M_B is negligible)
- Prepare and measure the mass in $|\pm\rangle = (|M_A\rangle \pm |M_B\rangle)/\sqrt{2}$

$$P_E^{(\pm)} \propto \int_{-t_f}^{t_f} d\tau \int_{-t_f}^{t_f} d\tau' \eta(\tau)\eta(\tau') e^{-i\Omega(\tau-\tau')} \Big(\underbrace{W(x_A, x'_A)}_{P_A} + \underbrace{W(x_B, x'_B)}_{P_B} \pm 2\cos(\Delta E \Delta t) \underbrace{W(x_A, x'_B)}_{L_{AB}} \Big)$$

UDW detectors in superposition of spacetimes

Superpose mass of the BTZ black hole



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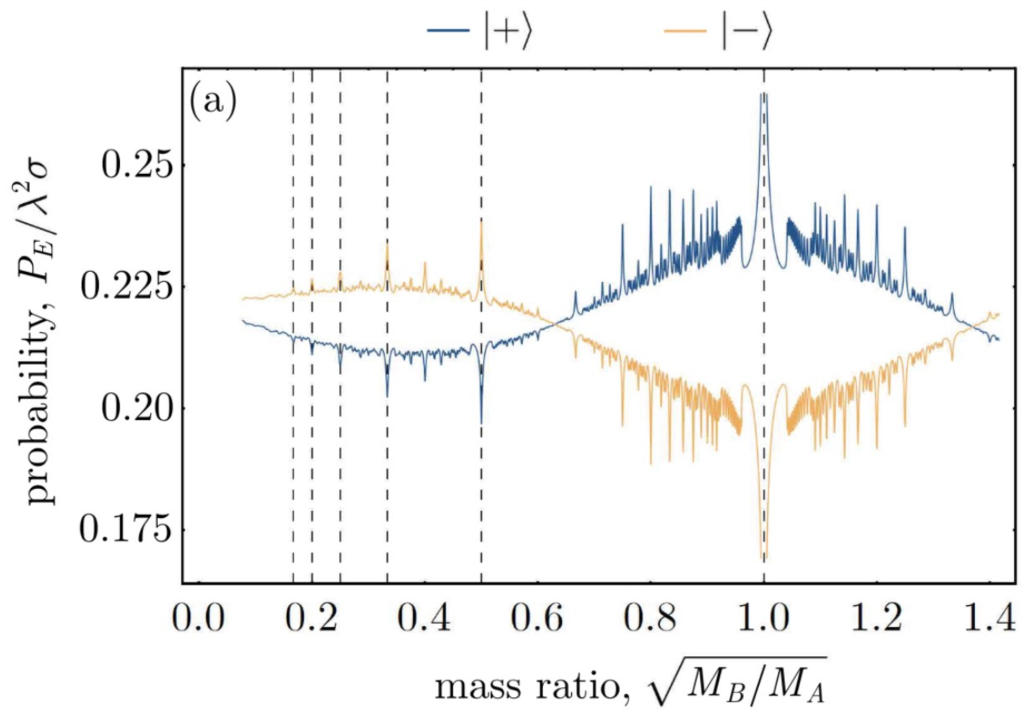
$$L_{AB} : \sum_{n,m} f\left(m\sqrt{M_A} - n\sqrt{M_B}\right)$$

with local maxima at $m\sqrt{M_A} - n\sqrt{M_B}$ = 0

UDW detectors in superposition of spacetimes

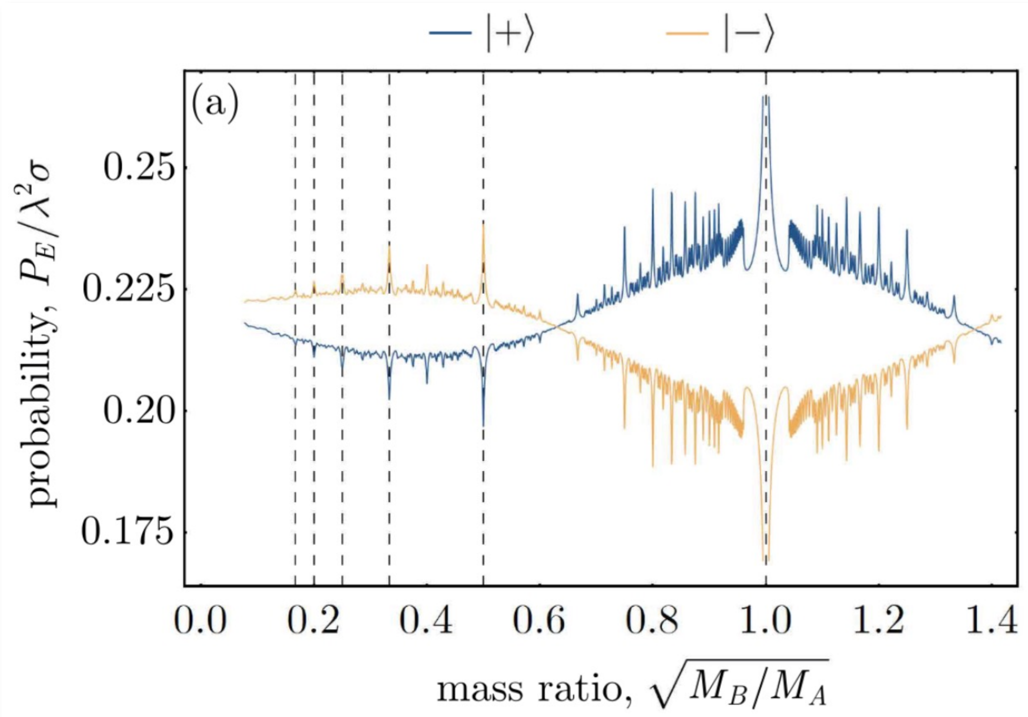


Dashed lines:
$\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \ldots 8\}$

Resonant peaks at $r_{H_B}/r_{H_A} = \sqrt{M_B/M_A} = n/m$

Foo, Arabaci, MZ, Mann PRL **129** (2022) 181301

J.Bekenstein Conjecture



Dashed lines:
$\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \ldots 8\}$

Jacob Bekenstein conjecture: Area of black hole $A \propto n$

BTZ black hole (2+1 dim) $A = 2\pi\, r_H$

$$r_H = \sqrt{M}l = n\hbar \xrightarrow{\text{superpose}} r_{H_B}/r_{H_A} = \sqrt{M_B/M_A} = n/m \in \mathbb{Q}$$

Bekenstein, *Lettere al Nuovo Cimento* **11**, pages 467–470 (1974)
Bekenstein, Mukhanov *Phys.Lett. B* **360** 7-12 (1995)

Conclusions

- Construction and characterization of ‘superposed spacetime’
  - mass-superposed black hole
  - Operational description via Wightman function
  - Generalizable to other spacetimes and rotating BTZ
- Particle detector response
  - Peaks at rational values of superposed horizon ratio
  - Consistent with Bekenstein’ Conjecture

Foo, Mann, MZ
CQG **38** (2021) 115010
Arabaci, Foo, MZ, Mann
PRD **107** 045014 (2023)

Open questions

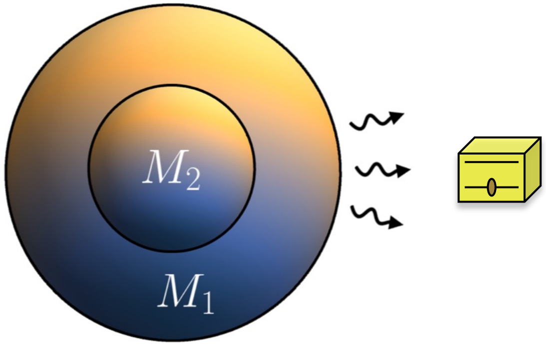
- Construct BH spacetimes with mass superpositions within a Quantum Gravity framework
- Are there any BH superselection rules?
- Apply to Schwarzschild/Kerr mass superpositions

Thank You

UDW detectors in superposition of spacetimes

$$P_G^{(\pm)} = \frac{1}{2}\Big(1 \pm \cos(\Delta E \Delta t)\Big)\Big[1 - \frac{\lambda^2}{2}\Big(P_A + P_B\Big)\Big]$$

$$P_E^{(\pm)} = \frac{\lambda^2}{4}\Big(P_A + P_B \pm 2\cos(\Delta E \Delta t) L_{AB}\Big)$$



Probabilities oscillate in time
due to different energies (masses) of the black hole

$$\frac{P_D}{\sigma} = \frac{\sqrt{\pi} H_0(0)}{8} - \frac{i}{8\sqrt{\pi}} \mathrm{PV} \int_{-t_f/2l}^{t_f/2l} \frac{dz\, X_0(2lz) H_0(2lz)}{\sinh(z)}$$
$$+ \frac{1}{4\sqrt{2\pi}\sum_n \eta^{2n}} \sum_{n \neq m} \mathrm{Re} \int_0^{t_f/l} \frac{dz\, X_0(lz) H_0(lz)}{\sqrt{\beta_{nm} - \cosh(z)}}$$

$$\beta_{nm} = \frac{1}{\gamma_D^2}\left[\frac{R_D^2 \cosh(2\pi(n-m)\sqrt{M_D})}{M_D l^2} - 1\right]$$

$$\frac{L_{AB}}{\sigma} = \frac{Y_0}{\sum_n \eta^{2n}} \sum_{n,m} \mathrm{Re} \int_0^{t_f/l} \frac{dz\, Z_0(lz) Q_0(lz)}{\sqrt{\alpha_{nm} - \cosh(z)}}.$$

$$\alpha_{nm} = \frac{1}{\gamma_A \gamma_B}\left[\frac{R_D^2 \cosh(2\pi(m\sqrt{M_A} - n\sqrt{M_B}))}{\sqrt{M_A M_B} l^2} - 1\right]$$

P_E sensitive to particular ratios of the horizons/mass

$$r_{H_B}/r_{H_A} = \sqrt{M_B/M_A} = n/m$$