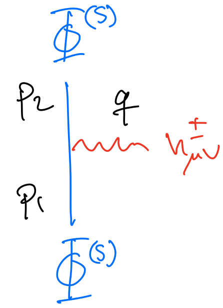
Lecture 3

Recap: 3pt amplitudes for rotating BHs :

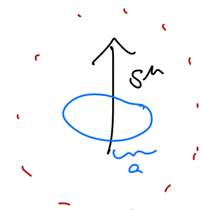


QFT: Classical:

+ helicity: $$(\varepsilon_3^+\cdot p_1)^2 \frac{\langle \mathbf{1}\,\mathbf{2}\rangle^{2s}}{m^{2s}} \xrightarrow[s\to\infty]{\hbar\to 0} (\varepsilon_3^+\cdot p_1)^2 e^{a\cdot q}$$

- helicity: $$(\varepsilon_3^-\cdot p_1)^2 \frac{[\mathbf{1}\,\mathbf{2}]^{2s}}{m^{2s}} \xrightarrow[s\to\infty]{\hbar\to 0} (\varepsilon_3^-\cdot p_1)^2 e^{-a\cdot q}$$

Spin vector : $S^\mu = m\, a^\mu$ ↖ ring radius vector



What are the 3pt ampl's useful for?

- 3pt are not physical, momenta complex
- 3pt is input for physical 4pt calc.

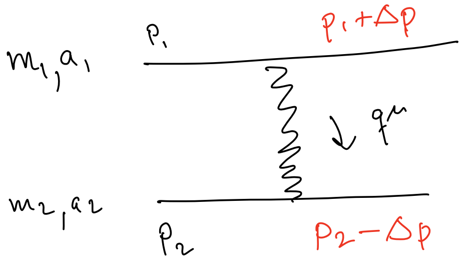
⟹ Gravitational potential → bound systems

⟹ 2 → 2 scattering, angle, impulse

⟹ Compton amplitude

- Effective Lagrangian can be inferred

2→2 Scattering and Impulse @ leading G



$$a^\mu = a_1^\mu + a_2^\mu$$

$$= -8\pi G \frac{m_1^2 m_2^2}{q^2}\left(e^{2w} e^{-q\cdot a} + e^{-2w} e^{q\cdot a}\right)$$

w = rapidity : $\quad e^w = \frac{p_1\cdot \varepsilon_q^-}{m_1}\,\frac{\varepsilon_q^+\cdot p_2}{m_2}$

$$= \underbrace{\frac{p_1\cdot p_2}{m_1 m_2}}_{\gamma = \cosh w} + \underbrace{\frac{\epsilon(p_1 p_2 q r)}{m_1 m_2\, q\cdot r}}_{\sinh w}$$

Levi-Civita contraction

KMOC : Impulse $\Delta p^\mu \sim \langle q^\mu \rangle$

$$\Delta p^\mu = \frac{i}{4}\int \frac{d^4 q}{(2\pi)^4}\,\delta(q\cdot p_1)\,\delta(q\cdot p_2)\, q^\mu\, M_{2\to 2}\, e^{-iq\cdot b}$$

$$= -\frac{2 m_1 m_2 G}{\sinh w}\,\mathrm{Re}\left[\frac{b_\perp^\mu \cosh 2w + 2i\,\epsilon^\mu\left(\frac{p_1}{m_1}\frac{p_2}{m_2} b_\perp\right)\cosh w}{b_\perp^2}\right]$$

where $\quad b_\perp = \Pi_{p_1 p_2}\cdot(b + ia)$

↖ projector perp. to p_1 & p_2

What effective theories (EFTs) give Schwarzschild and Kerr 3pt ampl's ?

minimally coupled scalar ~ schwarzschild BH

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\,\partial_\mu\phi\,\partial^\mu\phi \approx \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\,(\eta^{\mu\nu} - \kappa h^{\mu\nu}) + \frac{1}{2}\eta^{\mu\nu}\cancel{\mathrm{tr}\,h} \to 0$$

$$M(1^0, 2^0, 3) = -\kappa(p_1\cdot\varepsilon_3)(p_2\cdot\varepsilon_3) = \kappa(p_1\cdot\varepsilon_3)^2$$

electron in GR ~ spin dipole moment of Kerr

$$\nabla_\mu\psi = \frac{1}{8}\omega_\mu{}^{ab}[\gamma_a,\gamma_b]\psi$$

$$\mathcal{L} = \sqrt{-g}\,\bar\psi(i\not\nabla - m)\psi$$

$$= \bar\psi\, i\not\partial\,\psi - \frac{i}{2}\underbrace{\bar\psi\,\not h\cdot\partial\,\psi}_{\parallel} + \cancel{\frac{i}{8}\bar\psi\gamma^\mu\omega_\mu{}^{ab}[\gamma_a,\gamma_b]\psi} \to 0 \text{ onshell} \qquad \omega_\mu{}^{ab} \sim \varepsilon_\mu p^{[a}\varepsilon^{b]}$$

$$\boxed{\text{note: } \not\nabla = \gamma^a e_a{}^\mu\partial_\mu \sim \not\partial + \not h\cdot\partial + \cdots}$$

$$\mathcal{M}(1^{1/2} 2^{1/2} 3) = \frac{i}{2}\,\varepsilon_3\cdot p_1\; u_2\,\not\varepsilon_3\, v_1$$

$$= \underbrace{A(1^0 2^0 3)}_{\substack{\text{scalar QED} \\ \parallel \\ \text{scalar QCD}}}\;\underbrace{A(1^{1/2} 2^{1/2} 3)}_{\substack{\text{QED} \\ \parallel \\ \text{QCD}}}$$

$$= \frac{i}{2}\frac{(\varepsilon\cdot p_1)^2}{m^2}\begin{cases}\langle 12\rangle^2 & +\text{ helicity}\\ [12]^2 & -\text{ helicity}\end{cases}$$

Proca field in GR (massive photon) ~ spin quadrupole of Kerr

$$\mathcal{L} = -\sqrt{-g}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m^2 A_\mu A^\mu\right), \quad F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}, \quad \nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma_{\mu\nu}^\rho A_\rho$$

linear order: $\Gamma_{\mu\nu}^{\rho} \sim p_{(\mu} h_{\nu)\rho} - \frac{1}{2}p_\rho h_{\mu\nu}, \quad (F_{\mu\nu})^2 \sim 2F_{\mu\nu}F_{\rho\sigma}\eta^{\mu\rho}h_{\nu\sigma}$

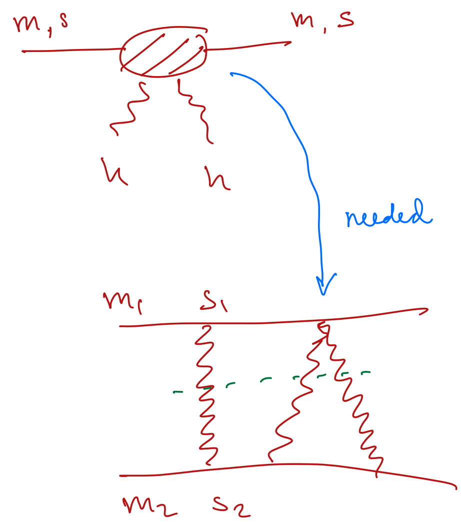
$$M(1^1, 2^1, 3) = \varepsilon_3\cdot p_1\left(\varepsilon_1\cdot\varepsilon_2\,(p_1 - p_2)\cdot\varepsilon_3 + \text{cyclic}\right)$$

$$= \underbrace{A(1^0, 2^0, 3)}_{\text{scalar QED}}\;\underbrace{A(1^1, 2^1, 3)}_{\text{SSB YM = Electro-weak Wwg}}$$

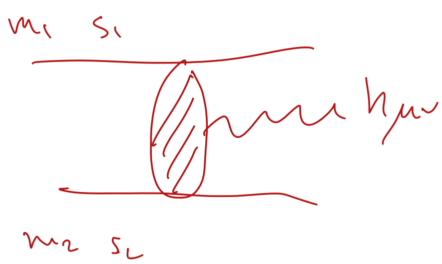
Beyond leading order in G (higher PM)

Compton scattering :

Conservative 2→2

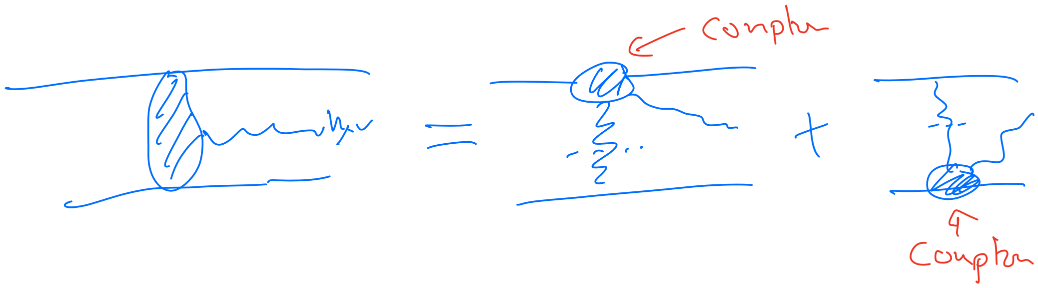


Radiation



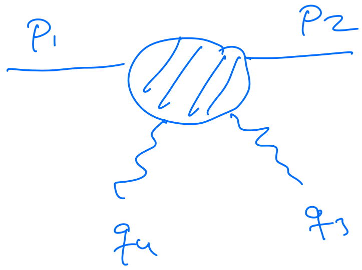
LIGO/Virgo
KAGRA

Note : in limit of large separation $b \gg r_s$



Thus, we need Compton ampl ⟹ NLO predictions

Compton amplitude (at order G)



classical limit

$|q_3|,|q_4| \ll |p_1|,|p_2|$

$s \to \infty$

Quantum Compton known for $s \leq 2$ [AHH]

$$M_4(1^s, 2^s, 3^+, 4^+) = \frac{\langle \mathbf{1}\mathbf{2}\rangle^{2s}[34]^4}{m^{2s-4}\, s\,(t-m^2)(u-m^2)}$$

$$M_4(1^s, 2^s, 3^-, 4^+) = \langle 3|p_1|4]^{4-2s}\, \frac{\left([14]\langle 23\rangle + [24]\langle 13\rangle\right)^{2s}}{s\,(t-m^2)(u-m^2)}$$

spurious pole for $s>2$

$$\xrightarrow[s\to\infty]{\hbar\to 0} M_4^{(s=0)}\, e^{a\cdot(q_4-q_3) + \frac{\langle 3|a|4]}{\langle 3|p_1|4]} p_1\cdot(q_4-q_3)}$$

problem at order a^4

What EFTs?

EFT	$s=1/2$	$s=1$	$s=3/2$	$s=2$	$s\geq 5/2$
Kerr	Majorana	Proca	Rarita-Schwinger	Kaluza-Klein grav.	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	higher-spin (HS)	HS