Asymptotic Grand Unification: the $SU(6)$ case

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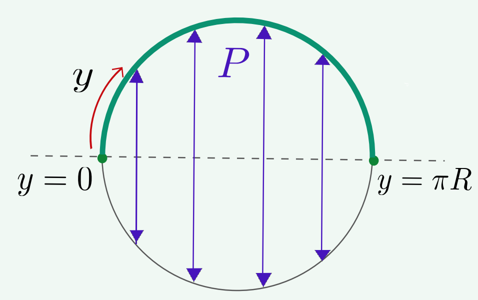
1. Orbifold GUTs - Introduction

- Grand Unified Theories (GUTs) can be formuled in 5 or more space-time dimensions[1]. Gauge symmetry is broken without invoking Higgs fields, but rather using boundary conditions which violate the GUT symmetry.
- Theories are defined on $\mathbb{R}^4 \times K$, where $\mathbb{R}^4$ is the usual 4-dimensional Minkowski space and K defines δ *compact* extra dimensions.

Example: Consider one extra dimension ($\delta = 1$) compactified on the orbifold $K = \mathbb{S}^1/\mathbb{Z}_2$. The inverse radius R^{-1} of the circle sets the scale of compactification.



A given complex field $\Phi(x^\mu, y)$ can be written $\Phi(x^\mu, y) = \Phi_+(x^\mu, y) + i\Phi_-(x^\mu, y)$ and periodicity implies

$$\Phi_+(x^\mu, y) = \sum_{n=0}^{\infty} \Phi_+^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right),$$
$$\Phi_-(x^\mu, y) = \sum_{n=1}^{\infty} \Phi_-^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right).$$

The "four-dimensional" fields $\Phi_\pm^{(n)}$ are called Kaluza-Klein modes and have a mass of n/R.
The Standard Model (SM) fields can be identified with the massless zero modes of of Φ_+ (corresponding to $n = 0$). For energies lower than $1/R$, the heavy Kaluza-Klein towers can be integrated out, resulting in an effective four-dimensional theory with only SM fields as dynamical degrees of freedom. In this way, the orbifold projection has broken the gauge symmetry at low energies.

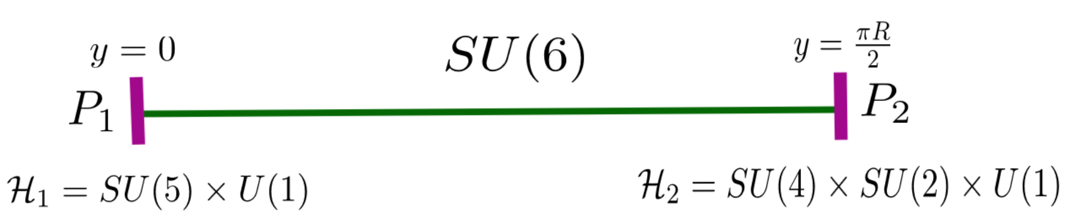
- The intrinsic $\mathbb{Z}_2$ transformation is specified by the parity matrix P acting on the fields: $\Phi(x^\mu, -y) = P\Phi(x^\mu, y)$.
- A model can be fully defined in terms of the gauge group, the parity P and the parity assignments of the fields.
- The Kaluza-Klein towers modify the running of the gauge couplings: instead of the usual logarithmic running, they now exihibit a power law dependence on the energy scale ⇒ they will *asymptotically* flow towards a fixed point.

2. $SU(6)$ GUT in 5 dimensions

- The model[2] is defined on the $\mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold.
- Parities are chosen

$$P_1 = diag(+,+,+,+,+,-)$$
$$P_2 = diag(+,+,+,-,-,+)$$

such that they break $SU(6)$ on the boundaries to



The intersection is $\mathcal{H}_1 \cap \mathcal{H}_2 = G_{\rm SM} \times U(1)_{\rm X}$ and the remnant low energy 4D theory is therefore the SM.

- Field content: gauge bosons live in the adjoint $\mathbf{35}$, while for fermions and scalars there are multiple embedding possibilities.

Example: Fermionic $\mathbf{15}$ and $\overline{\mathbf{15}}$ (Ψ_{15} and $\Psi_{\overline{15}}$) representations as well as a scalar $\mathbf{15}$ (Φ_{15}) and $\mathbf{6}$ (Φ_6) representations.

3. One loop effective potential

- A scalar field is identified with the fifth-dimensional component A_5 of the gauge field. This will generate masses for bulk fields through the Gauge-Higgs mechanics (GHU).
- Gauge symmetry forbids a tree-level potential for A_5 ⇒ the potential will be generated at one loop.
- This A_5 contains a SM-like doublet H_5, and if we assume the potential induces a VEV for it, we can parametrize it[3]

$$\langle H_5 \rangle = \sqrt{2}\begin{pmatrix} 0 \\ \frac{\alpha}{Rg} \end{pmatrix}$$

where g is the the 5D gauge coupling and α is a dimensionless parameter.

Full one loop potential computation

Step 1: Assume a generic bulk field and compute its GHU generated mass

$$m_n^2 = \frac{(n+\beta)^2}{R^2}$$

where β is proportional to α and depends on the representation of the field.

Step 2: Write the contribution to the effective potential after summing over all Kaluza-Klein states[3,4]

$$V_{\rm eff}(\beta) = \frac{\mp 1}{32\pi^2} \frac{1}{(\pi R)^4} \mathcal{F}(\beta)$$

where

$$\mathcal{F}(\beta) = \frac{3}{2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n \beta)}{n^5}.$$

Step 3: Add all contributions coming from bulk fields

$$V_{\rm eff}(\beta) = V_{\rm eff}^{\rm gauge}(\beta) + V_{\rm eff}^{\rm fermionic}(\beta) + V_{\rm eff}^{\rm scalar}(\beta).$$

- Once $V_{\rm eff}(\beta)$ is computed, it can be checked whether it posseses a global minimum and where this minimum lies. This ensures that the VEV of H_5 can be generated for spontaneous symmetry breaking.

4. Preliminary results and outlook

- For the embedding mentioned in the example, and *ignoring bulk masses of gauge bosons*, we find the effective potential at one loop

$$V_{\rm eff}(\alpha) = -\frac{3}{32\pi^2}\frac{1}{(\pi R)^4}\Bigg(5\mathcal{F}^+(\alpha/\sqrt{2}) + 2\mathcal{F}^+(\alpha) + \mathcal{F}^+(\sqrt{2}\alpha/\sqrt{3}) + $$
$$+ 9\mathcal{F}^-(\alpha/\sqrt{2}) - \frac{32}{3}\mathcal{F}^+(\alpha/2) - \frac{32}{3}\mathcal{F}^-(\alpha/2)\Bigg).$$

- However, bulk masses for the gauge bosons are being generated by the kinetic term of Φ_6 ⇒ potential will be modified.
- The modification arises due to the fact that $\mathcal{F}(\beta)$ changes in the presence of fields with a bulk mass, whose contribution will be exponentially suppressed.
- **Next step in the analysis**: compute the full one loop potential, including the contribution of fields which acquire a bulk mass. Later on, perform the matching with the low energy 4D theory and check whether it can fully accommodate the SM.
- The potential is an *important tool*: it gives information about symmetry breaking features, properties of scalar fields and interactions.
- Study the theoretical framework of the model and its phenomenological implications.

References

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