NORDITA Winter School 2024
in Particle Physics and Cosmology

Neutrino physics I: Neutrino Oscillations

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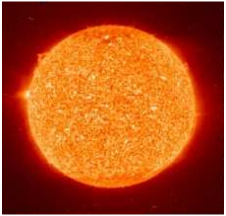
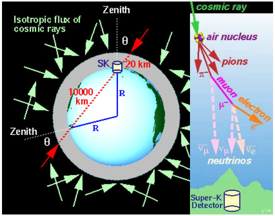
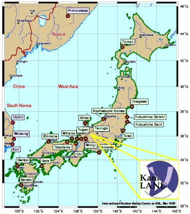
HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

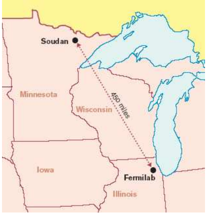
Stockholm, 15-17 Jan 2024

Neutrinos oscillate...









... and have mass ⇒ physics beyond the Standard Model

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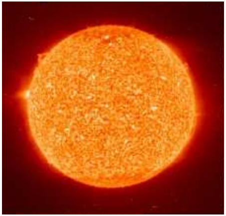
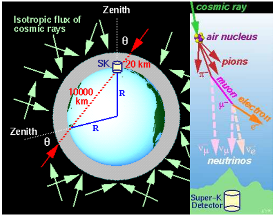
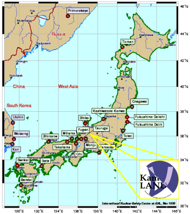
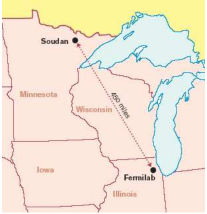







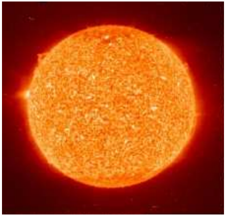
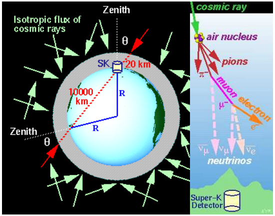
... and have mass ⇒ physics beyond the Standard Model

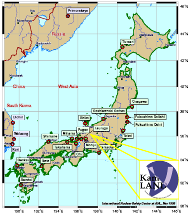
- Part I: Neutrino Oscillations
- Part II: Neutrino mass - Dirac versus Majorana
- Part III: Neutrinos and physics beyond the Standard Model

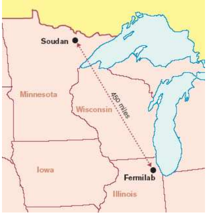
Literature

- Phenomenology:
  C. Giunti, C.W. Kim: Fundamentals of Neutrino Physics and Astrophysics
- Theory aspects:
  R.N. Mohapatra, P.B. Pal, Massive Neutrinos In Physics And Astrophysics (1998, World Scientific Publishing)
- more literature during the lectures

Neutrinos oscillate...





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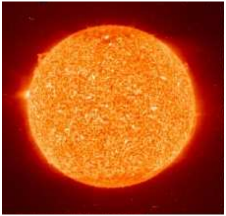

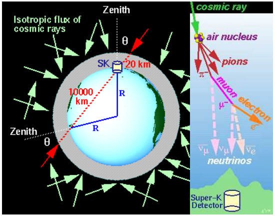

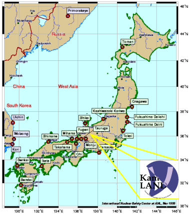

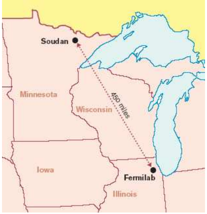

... and have mass $\Rightarrow$ physics beyond the Standard Model

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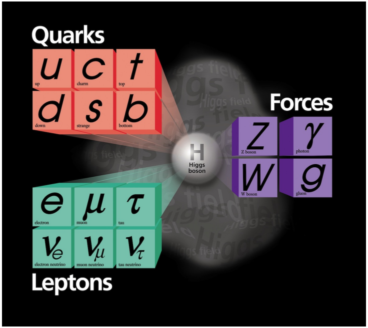
Outline - Neutrino Physics I

Outline

The Standard Model



Fermions in the Standard Model come in three generations (“Flavours”)

Neutrinos are the “partners” of the charged leptons

more precisely: they form a doublet under the SU(2) gauge symmetry

Flavour neutrinos

A neutrino of flavour α is defined by the charged current interaction with the corresponding charged lepton:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L}\gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

for example

$$\pi^+ \to \mu^+ \nu_\mu$$

the muon neutrino ν_μ comes together with the charged muon μ^+

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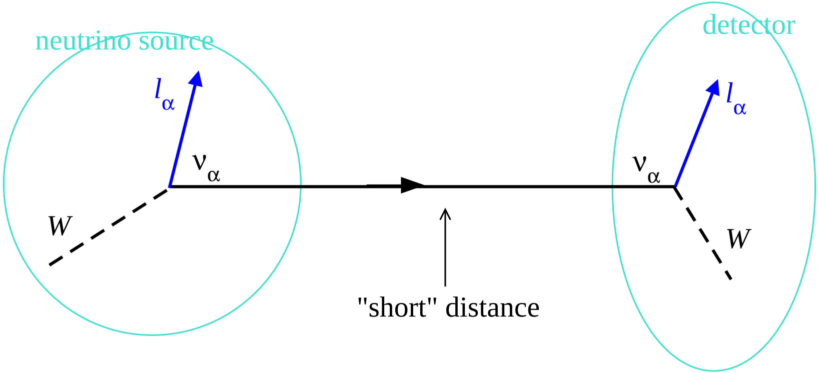

Let's give mass to the neutrinos

Majorana mass term:

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}^T C^{-1} \mathcal{M}_{\alpha\beta} \nu_{\beta L} + \mathrm{h.c.}$$

$\mathcal{M}$: symmetric mass matrix

In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

any complex symmetric matrix $\mathcal{M}$ can be diagonalised by a unitary matrix

$$U_\nu^T \mathcal{M} U_\nu = m \,, \qquad m: \text{ diagonal, } m_i \geq 0$$

Lepton mixing

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \bar\nu_{iL} U^*_{\alpha i} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

$$\mathcal{L}_{\rm M} = -\frac{1}{2} \sum_{i=1}^{3} \nu_{iL}^T C^{-1} \nu_{iL} m_i^\nu - \sum_{\alpha=e,\mu,\tau} \bar\ell_{\alpha R} \ell_{\alpha L} m_\alpha^\ell + \text{h.c.}$$

Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\rm PMNS}$$

Lepton mixing

- Flavour neutrinos ν_α are superpositions of massive neutrinos ν_i:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i \qquad (\alpha = e, \mu, \tau)$$

- mismatch between mass and interaction basis
- Example for two neutrinos:

$$\begin{aligned} \nu_e &= \ \ \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \\ \nu_\mu &= -\sin\theta\, \nu_1 + \cos\theta\, \nu_2 \end{aligned}$$

- The same phenomenon happens also for quarks (CKM matrix)

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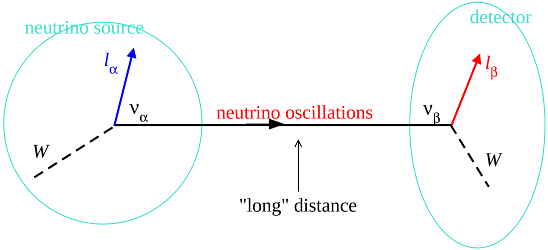
Outline

Neutrino oscillations



$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle \qquad e^{-i(E_i t - p_i x)} \qquad |\nu_\beta\rangle = U^*_{\beta i}|\nu_i\rangle$$

oscillation amplitude:

$$\begin{aligned}\mathcal{A}_{\nu_\alpha\to\nu_\beta} &= \langle\nu_\beta| \quad \text{propagation} \quad |\nu_\alpha\rangle \\ &= \sum_{i,j} U_{\beta j}\langle\nu_j| \quad e^{-i(E_i t - p_i x)} \quad |\nu_i\rangle U^*_{\alpha i} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)}\end{aligned}$$

Neutrino oscillations in vacuum

oscillation amplitude:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)} \quad \to \quad P_{\nu_\alpha \to \nu_\beta} = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta} \right|^2$$

need to calculate phase differences:

$$\phi_{ji} = (E_j - E_i)t - (p_j - p_i)x \quad \text{with} \quad E_i^2 = p_i^2 + m_i^2$$

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after some hand waving:

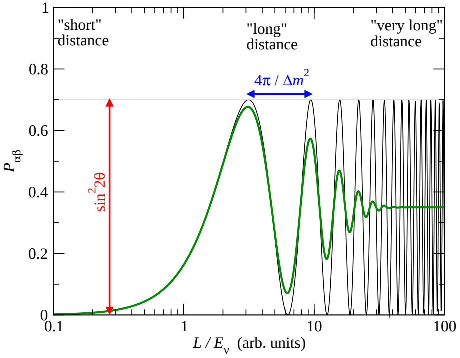
$$\phi_{ji} \approx \frac{\Delta m^2_{ji} L}{2\, E} \quad \text{with} \quad \Delta m^2_{ji} \equiv m_j^2 - m_i^2$$

2-neutrino oscillations

Two-flavour limit:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad P = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

oscillations are sensitive to mass differences (not absolute masses)



$$\frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2[\mathrm{eV}^2]\, L[\mathrm{km}]}{E_\nu[\mathrm{GeV}]}$$

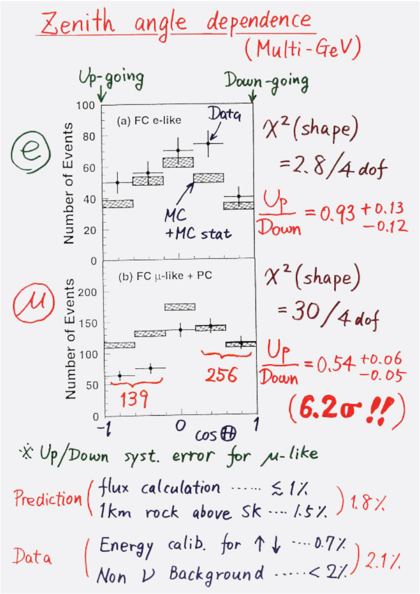
Neutrinos oscillate!

1998: SuperKamiokande atmospheric neutrinos

- zenith-angle dependent deficit of multi-GeV μ-like events
- consistent with $\nu_\mu \to \nu_\tau$ oscillations with

$\Delta m^2 \simeq 2.5 \times 10^{-3}$ eV^2
$\sin^2 2\theta \simeq 1$

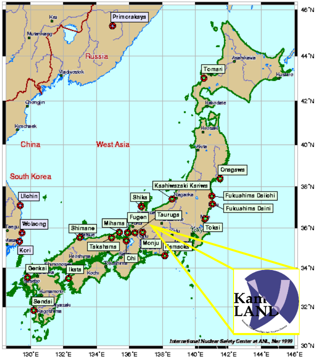
Nobel prize 2015
Takaaki Kajita



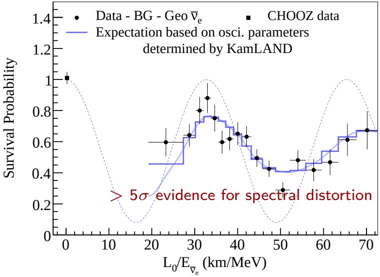
Neutrinos oscillate!

$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

KamLAND $\bar{\nu}_e \to \bar{\nu}_e$



$\langle L \rangle \sim 180$ km



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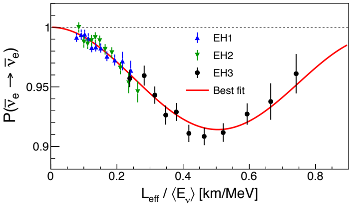
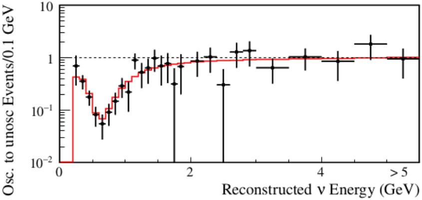
DayaBay, 2015

$\bar{\nu}_e \to \bar{\nu}_e$, $\langle L \rangle \sim 2$ km



T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L \rangle \sim 295$ km



the naive approach to calculate the oscillation probability is problematic at least for the following reasons:

- production and detection regions are localised in space $\rightarrow$ inconsistent with plane wave ansatz for neutrino propagation $\propto e^{-i(E_i t - p_i x)}$
- plane waves correspond to states with exact energy/momentum $\rightarrow$ neutrino mass states are distinguishable particles $\rightarrow$ why is the sum in the amplitude coherent (inside modulus)?

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- production and detection regions are localised in space $\to$ inconsistent with plane wave ansatz for neutrino propagation $\propto e^{-i(E_i t - p_i x)}$
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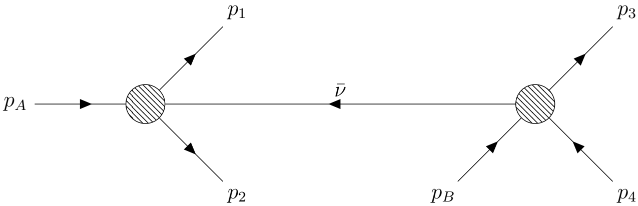
Two approaches:

- assume wave-packets for neutrinos
- QFT approach, neutrino as internal line, wave-packets for external particles

relation of the two approaches e.g., Akhmedov, Kopp, JHEP (2010) [1001.4815]

QFT approach to neutrino oscillations

joint process of neutrino production and detection



early papers:

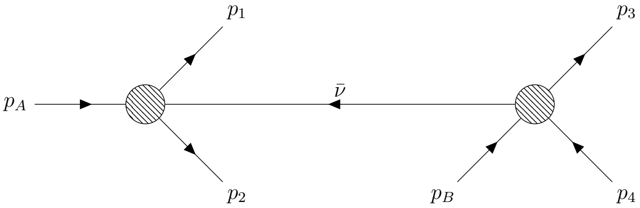
Rich,1993; Giunti,Kim,Lee,Lee,1993; Grimus,Stockinger,1996; Kiers,Weiss,1998

review paper:

M. Beuthe, Oscillations of Neutrinos and Mesons in Quantum Field Theory, Phys. Rept. 375 (2003) 105 [hep-ph/0109119]

QFT approach to neutrino oscillations

joint process of neutrino production and detection



- neutrino corresponds to internal line, unobservable
- "standard" Feynman rules to calculate amplitude $\mathcal{A}$ of the whole process
- take into account that production and detection vertices are macroscopically separated in space and time
- coherence properties determined by localization (or momentum spread) of initial and final state particles

The oscillation amplitude Krueger, TS, 2303.15524

$$i\mathcal{A}_{\alpha\beta} \propto \sum_j U_{\alpha j} U^*_{\beta j} \int \frac{d^4 p}{(2\pi)^4} i\tilde{\mathcal{M}}_P \frac{\not{p} - m_j}{p^2 - m_j^2 + i\epsilon} i\tilde{\mathcal{M}}_D \, e^{-ip(x_D - x_P)}$$

$$\times \prod_{I=P,D} \frac{\pi^2}{\sigma_{pI}^3 \sigma_{EI}} \exp\left[-\frac{(\mathbf{p} - \mathbf{p}_I)^2}{4\sigma_{pI}^2} - \frac{(p^0 - E_I - \mathbf{v}_I(\mathbf{p} - \mathbf{p}_I))^2}{4\sigma_{EI}^2} \right]$$

effective momentum and energy spreads determined by localization and velocity of all external particles:

$$\sigma_p^2 \equiv \sum_{i,f} \sigma_{i,f}^2 \,, \qquad \sigma_e^2 \equiv \sigma_p^2 (\Sigma - \mathbf{v}^2)$$

and a weighted velocity and velocity-squared:

$$\mathbf{v} \equiv \frac{1}{\sigma_p^2} \sum_{i,f} \sigma_{i,f}^2 \mathbf{v}_{i,f} \,, \qquad \Sigma \equiv \frac{1}{\sigma_p^2} \sum_{i,f} \sigma_{i,f}^2 \mathbf{v}_{i,f}^2 \,, \qquad \mathbf{v_i} \equiv \left. \frac{\partial E_i}{\partial \mathbf{k}_i} \right|_{\mathbf{k}_i = \mathbf{p}_i}$$

The oscillation amplitude-squared

after some algebra (and non-trivial manipulations):

$$
\begin{aligned}
\overline{|\mathcal{A}_{\alpha\beta}|^2} \quad & \propto \exp\left[i \frac{\Delta m^2 L}{2E_0} \right] \\
& \times \exp\left[-\frac{1}{2} \left(\frac{\Delta m^2}{4E_0 \sigma_m} \right)^2 \right] \\
& \times \exp\left[-\frac{1}{2} \left(\frac{\Delta m^2 L \sigma_{\rm en}}{2E_0^2} \right)^2 \right]
\end{aligned}
$$

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standard oscillation phase

The oscillation amplitude-squared

after some algebra (and non-trivial manipulations):

$$\begin{aligned}
\overline{|\mathcal{A}_{\alpha\beta}|^2} &\propto \exp\left[i\frac{\Delta m^2 L}{2E_0}\right] && \text{standard oscillation phase} \\
&\times \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2}{4E_0\sigma_m}\right)^2\right] && \text{localization decoherence } \xi_{\rm loc} \\
&\times \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2 L\sigma_{\rm en}}{2E_0^2}\right)^2\right] && \text{energy decoherence } \xi_{\rm en}
\end{aligned}$$

definitions:

$$\frac{1}{\sigma_m^2} \equiv \sum_{I=P,D}\left(\frac{1}{\sigma_{pI}^2} + \frac{v_I^2}{\sigma_{EI}^2}\right), \qquad \frac{1}{\sigma_{\rm en}^2} \equiv \sum_{I=P,D}\frac{1}{\sigma_{I,\rm eff}^2}$$

$$\frac{1}{\sigma_{I,\rm eff}^2} \equiv \frac{1}{\sigma_{pI}^2} + \frac{(1-v_I)^2}{\sigma_{EI}^2},$$

Localization decoherence

$$\xi_{\rm loc} = \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2}{4E_\nu\sigma_m}\right)^2\right]$$

energy-momentum uncertainty has to be large enough, such that individual mass states cannot be resolved: $\sigma_m \gg \Delta m^2/E_\nu$

$$\xi_{\rm loc} = \exp\left[-2\pi^2\left(\frac{\delta_{\rm loc}}{L_{\rm osc}}\right)^2\right] \quad \text{with} \quad \sigma_m\delta_{\rm loc} = \frac{1}{2}\,, \quad L_{\rm osc} = 2\pi\frac{2E_\nu}{\Delta m^2}$$

production and detection regions have to be localised much better than the oscillation length: $\delta_{\rm loc} \ll L_{\rm osc}$ (note $\delta^2_{\rm loc} = \delta^2_P + \delta^2_D$)

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Energy decoherence

$$\xi_{\rm en} = \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2 L \sigma_{\rm en}}{2E_\nu^2}\right)^2\right] = \exp\left[-2\pi^2\left(\frac{L}{L_{\rm osc}}\frac{\sigma_{\rm en}}{E_\nu}\right)^2\right]$$

- for experiments at the oscillation maximum ($L \approx L_{\rm osc}$) the neutrino energy needs to be well defined: $\sigma_{\rm en} \ll E_\nu$
- this term can be interpreted as decoherence due to neutrino wave packet separation, identifying $v_j \approx 1 - m_j^2/(2E_\nu^2)$

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Why are oscillations possible?

- $\xi_{\rm loc}$ and $\xi_{\rm en}$ have opposite dependence on spreads: uncertainties have to be large enough that mass states can interfere: $\sigma_m \gg \Delta m^2/E_\nu$
  small enough that intereference is not damped: $\sigma_{\rm en} \ll E_\nu L_{\rm osc}/L$

- assuming $\sigma_m \sim \sigma_{\rm en}$, there are many orders of magnitude available to fulfill both requirements, because

$$\Delta m^2/E_\nu^2 \ll 1 \qquad \text{or} \qquad E_\nu L_{\rm osc} \gg 1$$

Classical averaging

consider averaging of the event rates $R(L, E_\nu) \propto \overline{|\mathcal{A}_{\alpha\beta}|^2}$:

$$\int dL' \, R(L', E_\nu) \frac{1}{\sqrt{2\pi}\delta_{\rm clas}} \exp\left[-\frac{(L'-L)^2}{2\delta_{\rm clas}^2}\right]$$
$$\int dE'_\nu \, R(L, E'_\nu) \frac{1}{\sqrt{2\pi}\sigma_{\rm clas}} \exp\left[-\frac{(E'_\nu-E_\nu)^2}{2\sigma_{\rm clas}^2}\right]$$

same decoherence factors $\xi_{\rm loc}$ and $\xi_{\rm en}$ with (in the Gaussian case)

$$\delta_{\rm loc}^2 \to \delta_{\rm loc}^2 + \delta_{\rm clas}^2 \,, \qquad \sigma_{\rm en}^2 \to \sigma_{\rm en}^2 + \sigma_{\rm clas}^2$$

Classical averaging

- quantum mechanical and classical decoherence have the same effect and are indistinguishable phenomenologically
  Kiers, Nussinov, Weiss, 1996; Stodolsky, 1998; Ohlsson, 2001
- classical averaging due to experimental reasons: size of production region, finite detector resolutions (in space and energy),...
- fundamental averaging effects due to experimental configuration and physics principles: phase space integrals of unobserved particles, Doppler broadening,...

Decoherence parameters - numerical example

estimates for reactor oscillation experiments Krueger, TS, 2303.15524

$$-\ln \xi_{\rm loc} = \frac{1}{2}\left(\frac{\Delta m^2}{4E_\nu \sigma_m}\right)^2 \approx 1.3\times 10^{-19}\left(\frac{\Delta m^2}{1\,{\rm eV}^2}\right)^2\left(\frac{1\,{\rm MeV}}{E_\nu}\right)^2\left(\frac{500\,{\rm eV}}{\sigma_m}\right)^2$$

$$-\ln \xi_{\rm en} = 2\pi^2\left(\frac{L}{L_{\rm osc}}\frac{\sigma_{\rm en}}{E_\nu}\right)^2 \approx 4.9\times 10^{-12}\left(\frac{L}{L_{\rm osc}}\right)^2\left(\frac{1\,{\rm MeV}}{E_\nu}\right)^2\left(\frac{\sigma_{\rm en}}{0.5\,{\rm eV}}\right)^2$$

$\Rightarrow$ QM decoherence (incl. localization and "wave packet separation") is irrelevant for all practical purposes

decoherence effects completely dominated by classical averaging
(e.g., typical energy resolution in reactor exps: $\sigma_{\rm clas} \simeq 0.1$ MeV)

The matter effect

When neutrinos pass through matter the SM interactions with the particles in the background induce an effective potential for the neutrinos

Effective 4-point interaction Hamiltonian

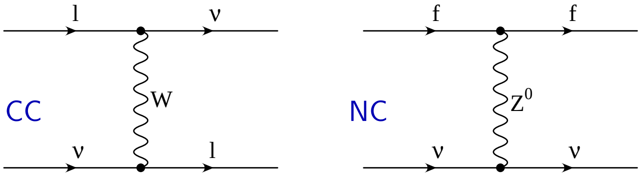
$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}}\,\bar\nu_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha \underbrace{\sum_f \bar f\gamma^\mu(g_V^{\alpha,f} - g_A^{\alpha,f}\gamma_5)f}_{J^\mu_{\rm mat}}$$

coherent forward scattering amplitude leads to an "index of refraction"
$\rightarrow$ proportional to G_F! (not G_F^2)

L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* D **20**, 2634 (1979)

Effective matter potential

$$V_{\mathrm{mat}} = \sqrt{2} G_F \,\mathrm{diag}\,(N_e - N_n/2, -N_n/2, -N_n/2)$$



- only ν_e feel CC (there are no μ, τ in normal matter)
- NC is the same for all flavours $\Rightarrow$ potential proportional to identiy has no effect on the evolution
- NC has no effect for 3-flavour active neutrinos, but is important in the presence of sterile neutrinos

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$H = \underbrace{U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger}_{\text{vaccum}}$$

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$N_e(x)$: electron density along the neutrino path

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

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$N_e(x)$: electron density along the neutrino path

for non-constant matter: $H(t) \rightarrow$ time-dependent Schrödinger eq.
"MSW resonance" Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)

Neutrino oscillations in constant matter

diagonalize the Hamiltonian in matter:

$$
\begin{aligned}
H^{\nu}_{\mathrm{mat}} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \mathrm{diag}(\sqrt{2} G_F N_e, 0, 0) \\
&= U_m \mathrm{diag}\left(\lambda_1, \lambda_2, \lambda_3\right) U_m^\dagger
\end{aligned}
$$

Same expression for oscillation probability, but replace "vacuum" parameters by "matter" parameters

2-neutrino oscillations in constant matter

Two-flavour case:

$$P_{\rm mat} = \sin^2 2\theta_{\rm mat} \, \sin^2 \frac{\Delta m^2_{\rm mat} L}{4E}$$

with

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

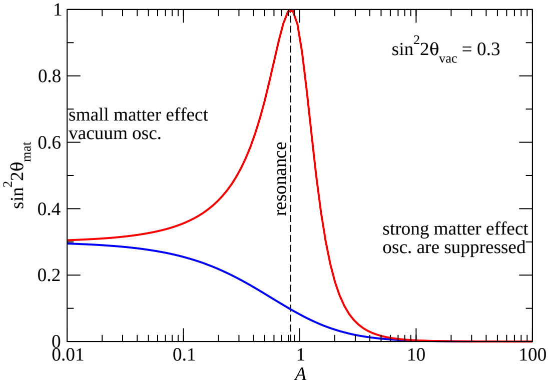
$$\Delta m^2_{\rm mat} = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A \equiv \frac{2EV}{\Delta m^2}$$

2-neutrino oscillations in constant matter

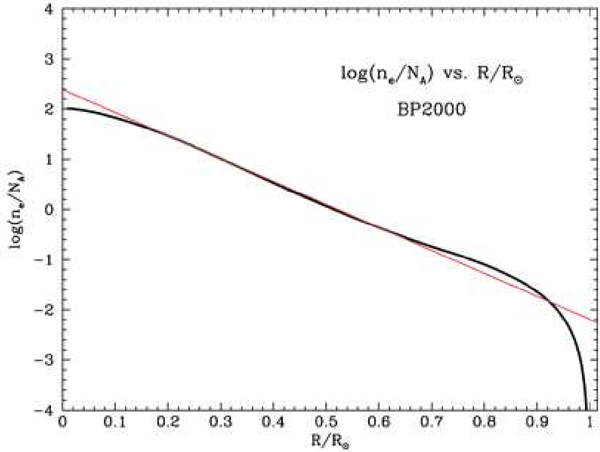
$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A \equiv \frac{2EV}{\Delta m^2}$$

resonance for $\cos 2\theta = A$: "MSW resonance"
Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)



Varying matter density: example solar neutrinos

The electron density in the sun:



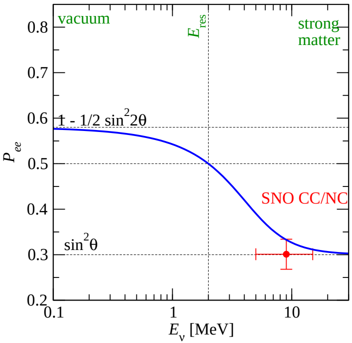
Solar neutrinos and the Sudbury Neutrino Observatory

2002: SNO: CC to NC ratio of solar neutrino flux

CC: $\nu_e + d \to p + p + e^-$
NC: $\nu_x + d \to p + n + \nu_x$

- evidence for $\nu_e \to \nu_\mu, \nu_\tau$ conversion
- MSW effect inside the sun adiabatic conversion through resonance

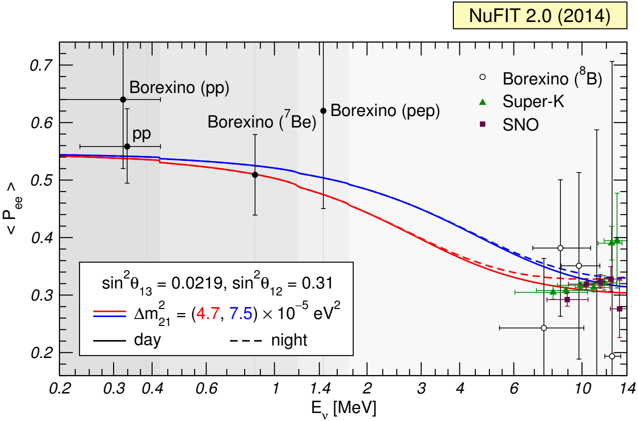
Nobel prize 2015
Art McDonald



$$P_{ee} = \frac{\phi_e}{\phi_e + \phi_\mu + \phi_\tau} = \frac{\phi_{CC}}{\phi_{NC}}$$

Evidence for LMA-MSW

solar neutrino experiments Homestake, SAGE+GNO, Super-K, SNO, Borexino



- $\sin^2\theta < 0.5$ is strong evidence for MSW conversion
- for energies above resonance: $P_{ee} \approx \sin^2\theta$ → best determination of θ_{12}

Outline

3-flavour neutrino parameters

- 3 masses: Δm^2_{21}, Δm^2_{31}, m_0
- 3 mixing angles: θ_{12}, θ_{13}, θ_{23}
- 3 phases: 1 Dirac (δ), 2 Majorana (α_1, α_2)

neutrino oscillations
absolute mass observables
lepton-number violation (neutrinoless double-beta decay)

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\Delta m^2_{31} \qquad \qquad \qquad \qquad \Delta m^2_{21}$$

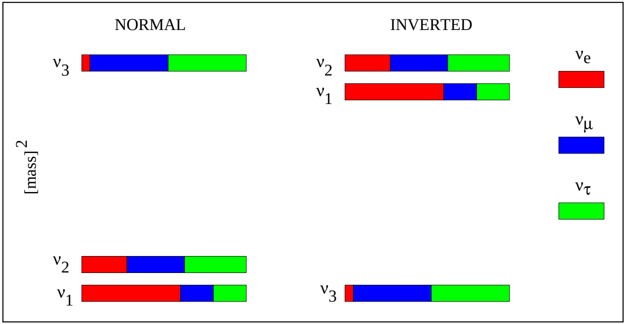
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm+LBL(dis)   react+LBL(app)   solar+KamLAND

3-flavour effects are suppressed: $\Delta m^2_{21} \ll \Delta m^2_{31}$ and $\theta_{13} \ll 1$ ($U_{e3} = s_{13} e^{-i\delta}$)

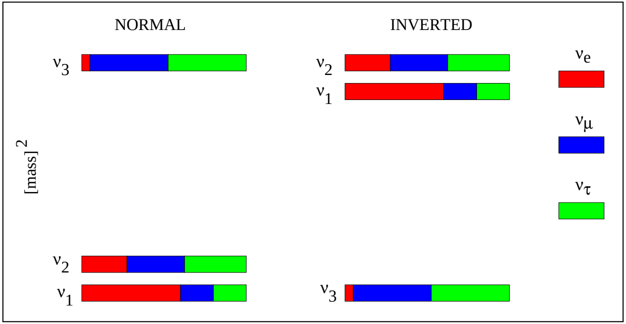
$\Rightarrow$ dominant oscillations are well described by effective two-flavour oscillations
$\Rightarrow$ present data is already sensitive to sub-leading effects
$\Rightarrow$ CP-violation is suppressed by θ_{13} and $\Delta m^2_{21}/\Delta m^2_{31}$

What we know – masses



- The two mass-squared differences are separated roughly by a factor 30: $\Delta m^2_{21} \approx 7 \times 10^{-5}\mathrm{eV}^2\,, \quad |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx 2.4 \times 10^{-3}\mathrm{eV}^2$
- at least two neutrinos are massive

Physical interpretation of mixing angles



$\sin\theta_{13} = |U_{e3}|$ (ν_e component in ν_3) = (ν_3 component in ν_e)

$\tan\theta_{12} = \frac{|U_{e2}|}{|U_{e1}|}$ ratio of ν_2 and ν_1 component in ν_e

$\tan\theta_{23} = \frac{|U_{\mu 3}|}{|U_{\tau 3}|}$ ratio of ν_μ and ν_τ component in ν_3

What we know – mixing

- approx. equal mixing of ν_μ and ν_τ in all mass states:
  $\theta_{23} \approx 45^\circ$ (with significant uncertainty)
- there is one mass state ("ν_1") which is dominantely ν_e ($\theta_{12} \approx 30^\circ$), and it is the lighter of the two states of the doublet with the small splitting (MSW in sun)
- there is a small ν_e component in the mass state ν_3: $\theta_{13} \approx 9^\circ$
  we do not know whether this mass state is the heaviest (normal ordering) or the lightest (inverted ordering)

Complementarity of global oscillation data

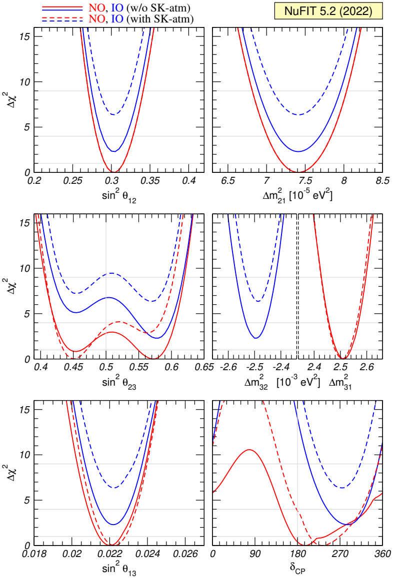
param	experiment	comment
θ_{12}	SNO, SuperK, (KamLAND)	resonant matter effect in the Sun
θ_{23}	SuperK, T2K, NOvA	ν_μ disappearance atmospheric (accelerator) neutrinos
θ_{13}	DayaBay, RENO, D-Chooz (T2K, NOvA)	$\bar{\nu}_e$ disappearance reactor experiments @ ~ 1 km
Δm^2_{21}	KamLAND, (SNO, SuperK)	$\bar{\nu}_e$ disappearance reactor @ ~ 180 km (spectrum)
$\lvert\Delta m^2_{31}\rvert$	MINOS, T2K, NOvA, DayaBay	ν_μ and $\bar{\nu}_e$ disapp (spectrum)
δ	T2K, NOvA + DayaBay	very weak sensitivity combination of $(\nu_\mu \to \nu_e) + \bar{\nu}_e$ disap

- global data fits nicely with the 3 neutrinos from the SM
- a few “anomalies” at 2-3 σ: LSND, MiniBooNE, reactor anomaly, no LMA MSW up-turn of solar neutrino spectrum – SOLVED 2020 (!)

Global 3-flavour fit

- NuFit collaboration: www.nu-fit.org
  with M.C. Gonzalez-Garcia, M. Maltoni, et al.
- latest paper:
  Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, 2007.14792
- latest version: 5.2 (as of Nov 2022)
- provides updated global fit results
  tables & figures, χ^2 data for download

Global 3-flavour fit



- robust determination (relat. precision at 3σ):

$$\theta_{12}\,(14\%)\quad,\quad\theta_{13}\,(9\%)$$
$$\Delta m^2_{21}\,(16\%)\,,\quad|\Delta m^2_{3\ell}|\,(6.7\%)$$

- broad allowed range for $\theta_{23}\,(27\%)$, non-significant indications for non-maximality/octant
- ambiguity in sign of $\Delta m^2_{3\ell}$ $\rightarrow$ mass ordering
- values of $\delta_{\rm CP} \simeq 90^\circ$ disfavoured

Open questions in the three flavour framework

- Determination of δ_{CP} $\rightarrow$ leptonic CP violation
- Determination of the neutrino mass ordering (normal versus inverted)

CP violation in neutrino oscillations

Leptonic CP violation will manifest itself in a difference of the vacuum oscillation probabilities for neutrinos and anti-neutrinos
Cabibbo, 1977; Bilenky, Hosek, Petcov, 1980, Barger, Whisnant, Phillips, 1980

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = -16\, J_{\alpha\beta} \sin \frac{\Delta m^2_{21} L}{4E_\nu} \sin \frac{\Delta m^2_{32} L}{4E_\nu} \sin \frac{\Delta m^2_{31} L}{4E_\nu}\,,$$

where

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J\,,$$

with $+(-)$ for (anti-)cyclic permutation of the indices e, μ, τ.

J: leptonic analogue to the Jarlskog-invariant in the quark sector
Jarlskog, 1985

CP violation

Jarlskog-invariant:

$$J = |\text{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2})| = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta \equiv J^{\max} \sin\delta$$

neutrino oscillation data:

$$J^{\max} = 0.0332 \pm 0.0008 (\pm 0.0019) \qquad 1\sigma\ (3\sigma) \qquad \text{nu-fit 5.0}$$

in the quark sector:

$$J_{\rm CKM} = (3.18 \pm 0.15) \times 10^{-5} \qquad \text{PDG}$$

CP violation

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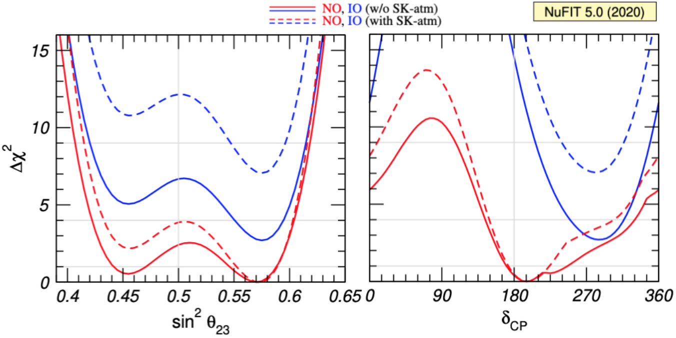
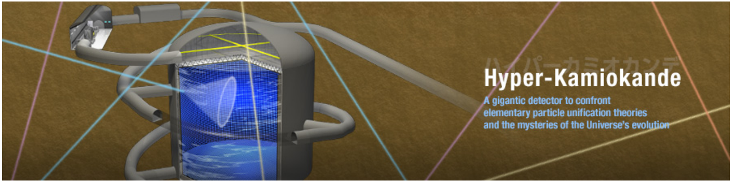
Status of δ_{CP}

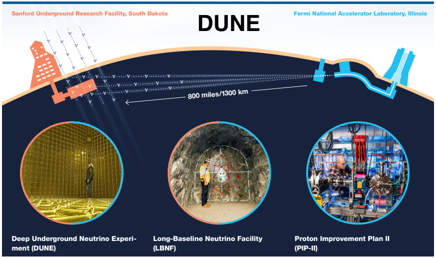


- some indications on the allowed range of δ_{CP} due to the interplay of reactor (Daya Bay) and accelerator (T2K, NOvA) neutrino experiments
- values of $\delta_{\mathrm{CP}} \simeq 90^\circ$ disfavoured
- no significant indication of CPV (yet)

T2K: J-PARC → HyperK (285 km, WC detector)



DUNE: Fermilab → Homestake (1300 km, LAr detectors)



oscillation science goals: determine mass ordering and CP phase

Determining the mass ordering

- Looking for the matter effect in transitions involving Δm^2_{31}
  - long-baseline accelerator experiments NOvA, DUNE
  - atmospheric neutrino experiments IceCube, ORCA, HyperK

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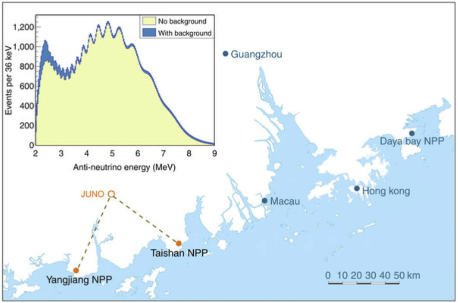
  - reactor experiment at 60 km JUNO



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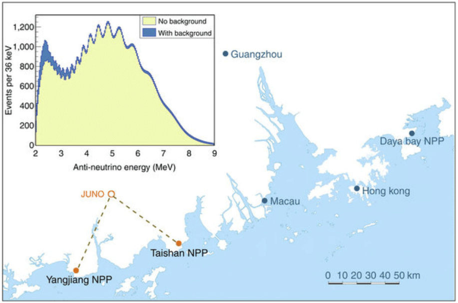

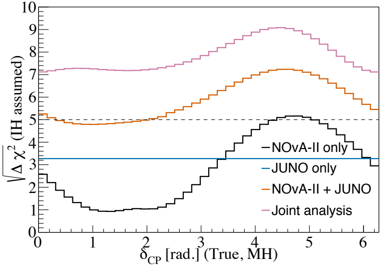

Cao et al., 2009.08585

Outline

Summary

- global data on neutrino oscillations is (mostly) consistent with 3-flavour oscillations
- at least two neutrinos are massive
- typical mass scales

$$\sqrt{\Delta m^2_{21}} \sim 0.0086\,\text{eV}$$
$$\sqrt{\Delta m^2_{31}} \sim 0.05\,\text{eV}$$

  are much smaller than all other fermion masses
- all three mixing angles are measured with reasonable precision
- lepton mixing is VERY different from quark mixing

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The SM flavour puzzle

Lepton mixing:

$$\theta_{12} \approx 33^\circ$$
$$\theta_{23} \approx 45^\circ$$
$$\theta_{13} \approx 9^\circ$$

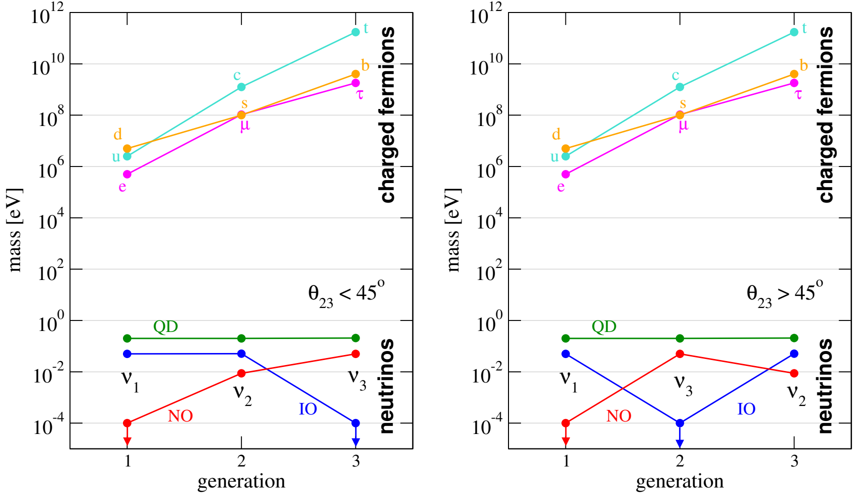
$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Quark mixing:

$$\theta_{12} \approx 13^\circ$$
$$\theta_{23} \approx 2^\circ$$
$$\theta_{13} \approx 0.2^\circ$$

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & {\scriptstyle \epsilon} \\ \epsilon & 1 & {\scriptstyle \epsilon} \\ {\scriptstyle \epsilon} & {\scriptstyle \epsilon} & 1 \end{pmatrix}$$

The SM flavour puzzle



Summary

open questions for oscillation experiments:

- identify neutrino mass ordering
- establish leptonic CP violation
- precision measurments (e.g., $\theta_{23} \approx 45^\circ$?)
- over-constrain 3-flavour oscillations (search for non-standard properties, sterile neutrinos, exotic neutrino interactions,...)

questions which cannot be addressed by oscillations:

- absolute neutrino mass scale
- Dirac or Majorana nature

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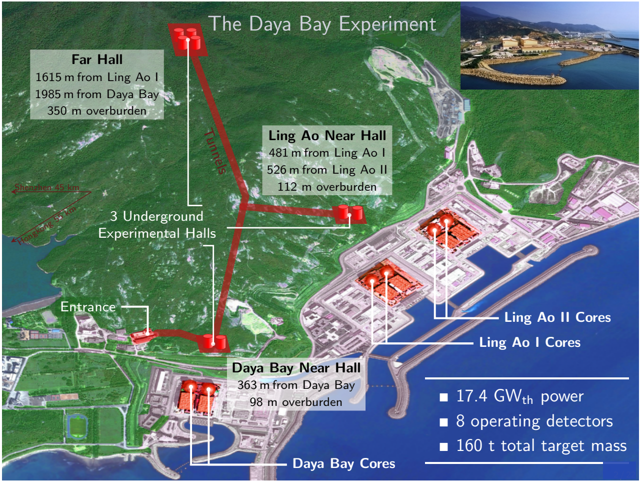
Supplementary slides

Daya Bay reactor experiment

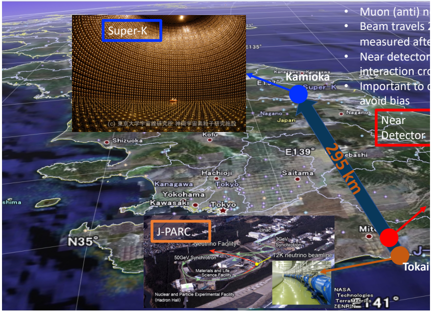
- $\bar{\nu}_e \to \bar{\nu}_e$ disappearance

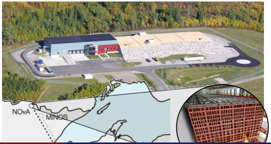


T2K and NOvA accelerator experiments

- $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ disappearance
- $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance

The T2K Experiment





The NOvA Experiment

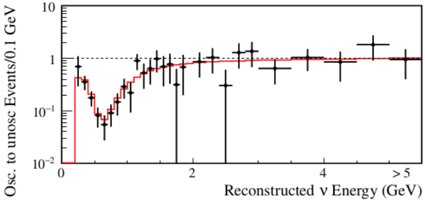
- Long-baseline neutrino oscillation experiment
- NuMI beam: ν_μ or $\bar{\nu}_\mu$
- 2 functionally identical, tracking calorimeter detectors
  - Near: 300 T underground

Disappearance due to Δm^2_{31}

$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4} \frac{L}{E_\nu} \right)$$

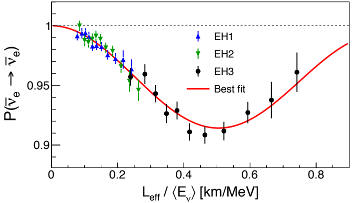
T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L \rangle \sim 295$ km



DayaBay, 2015

$\bar{\nu}_e \to \bar{\nu}_e$, $\langle L \rangle \sim 2$ km



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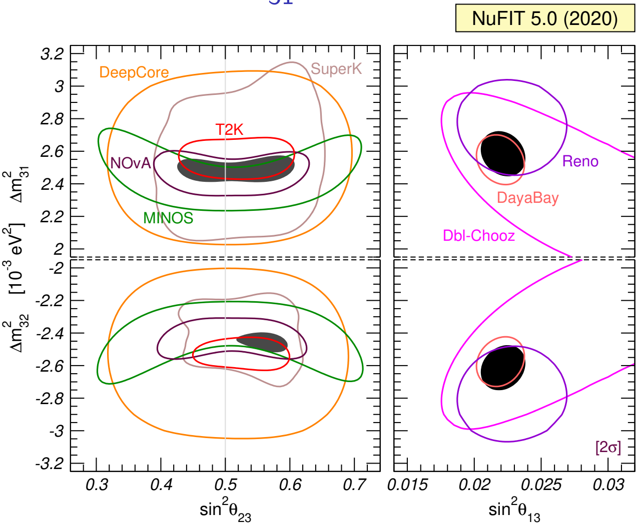

Complementarity between beam and reactor experiments

- $\nu_\mu \to \nu_e$ appearance probability (T2K, NOvA):

$$P_{\mu e} \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2(1-A)\Delta}{(1-A)^2} + \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\Delta + \delta_{\rm CP})$$

with $$\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu}, \quad \hat{\alpha} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\theta_{12}, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}}$$

- ν_e survival probability (reactor experiments, e.g. Daya Bay)

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \Delta$$

Latest restults from T2K and NOvA

$$P_{\mu e} \approx \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2(1-A)\Delta}{(1-A)^2} + \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A}\frac{\sin A\Delta}{A} \cos(\Delta + \delta_{\rm CP})$$