Introduction to inflation

Lecture 1: Background

The strongest motivation for primordial inflation is the causality problem of the standard Big Bang model, also sometimes called the horizon problem. To understand the causal structure of the standard Big Bang model it is useful to consider its Penrose diagram.

Penrose diagrams are also sometimes called conformal diagrams, because they show the causal structure of the infinite spacetime by a conformal mapping to a diagram of finite size preserving the causal property that two light rays only intersect if they intersect in the actual spacetime.

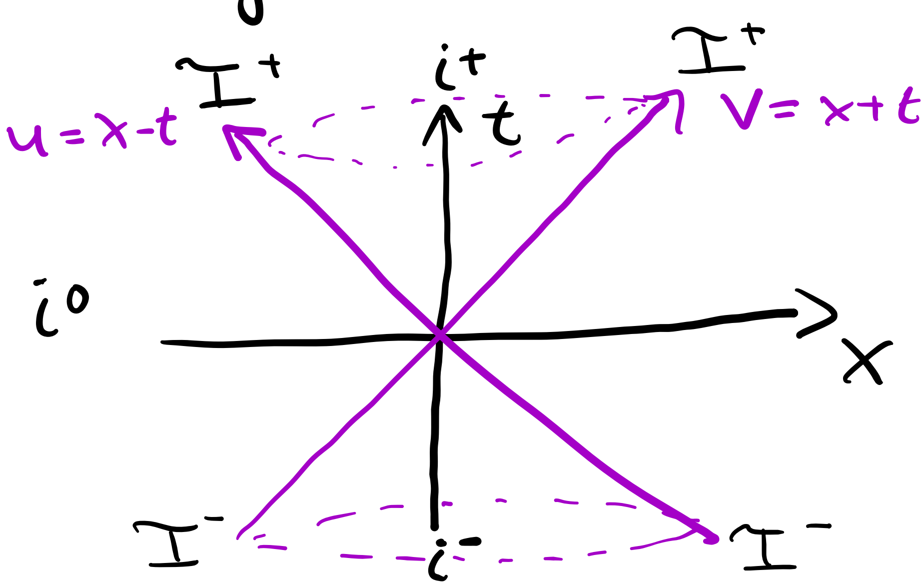
In the standard Big Bang model the universe was radiation dominated early on, so we would like to look at the Penrose diagram of a radiation dominated FRW spacetime to understand the causal structure of the standard Big Bang model.

Penrose diagrams

In order to understand how to draw a Penrose diagram, let's first consider the simplest possible one, the one of Minkowski spacetime. Everyone is familiar with the Minkowski metric ($c \equiv 1$)

$$ds^2 = -dt^2 + d\bar{x}^2 .$$

Now evidently light rays, null geodesics with $ds^2 = 0$, travelling in the x-direction, propagates at 45° angles in the (t,x) plane. So we get the typical light cones



Now, switching to sperical coordinates the Minkowski metric is

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

where $d\Omega^2 = (d\theta^2 + \sin^2\theta\, d\varphi^2)$. Defining the Penrose coordinates

$$\tan(T \pm R) = t \pm r$$

with $-\frac{\pi}{2} < T - R \leq T + R < \frac{\pi}{2}$, the Minkowski metric in those coordinate becomes

$$ds^2 = \frac{1}{4\cos^2(T+R)\cos^2(T-R)}\left(-dT^2 + dR^2 + \sin^2(R)\, d\Omega^2\right)$$

Exercise 1: Demonstrate that this is true.

A conformal rescaling is a rescaling of the metric of the form

$$g_{\mu\nu}(x) \rightarrow \tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x)$$

This is not a change of coordinates, but a change of the actual geometry, which however preserves angles and null geodesics and thus preserves the causal structure.

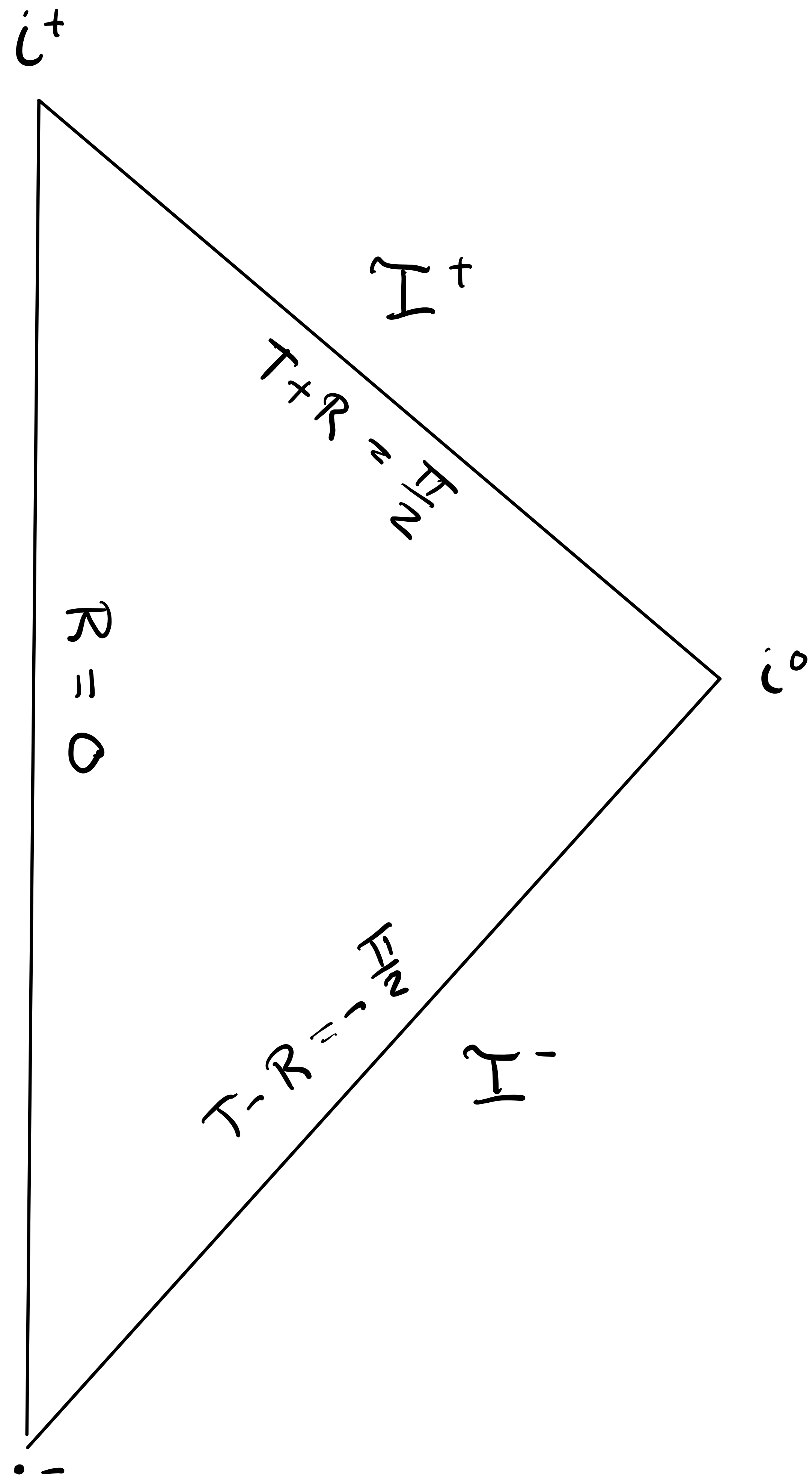
Making a conformal transformation we obtain the (unphysical) metric

$$d\tilde{s}^2 = -dT^2 + dR^2 + \sin^2(R)\,d\Omega^2$$

which is a subsection of the Einstein static universe (spatially closed universe with $a(t) = 1$) restricted to

$$-\frac{\pi}{2} < T - R \leq T + R < \frac{\pi}{2}$$

which is compact. Also notice that $T \in]-\frac{\pi}{2}, \frac{\pi}{2}[$ and $R \geq 0$. Since it is compact, we can draw this entire spacetime in a diagram, which is the Penrose diagram of Minkowski spacetime


i^+
$\mathscr{I}^+$
$T+R=\frac{\pi}{2}$
$R=0$
i^0
$T-R=-\frac{\pi}{2}$
$\mathscr{I}^-$
i^-

Notice that the past and future null infinities $\mathcal{I}^+$, $\mathcal{I}^-$, which are manifolds at $T-R=-\frac{\pi}{2}$, $T+R=\frac{\pi}{2}$, the spatial infinity i^0 at $T=0$, $R=\frac{\pi}{2}$ and past and future time-like infinities, i^+, i^- at $T=-\frac{\pi}{2}$, $R=0$ and $T=+\frac{\pi}{2}$, $R=0$ are not part of Minkowski spacetime.

Now let's turn to FRW spacetime. Let's for simplicity ignore spatial curvature

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right)$$

Introducing conformal time

$$a\,d\tau = dt$$

$$\Rightarrow ds^2 = a^2\left(-d\tau^2 + dr^2 + r^2 d\Omega^2\right)$$

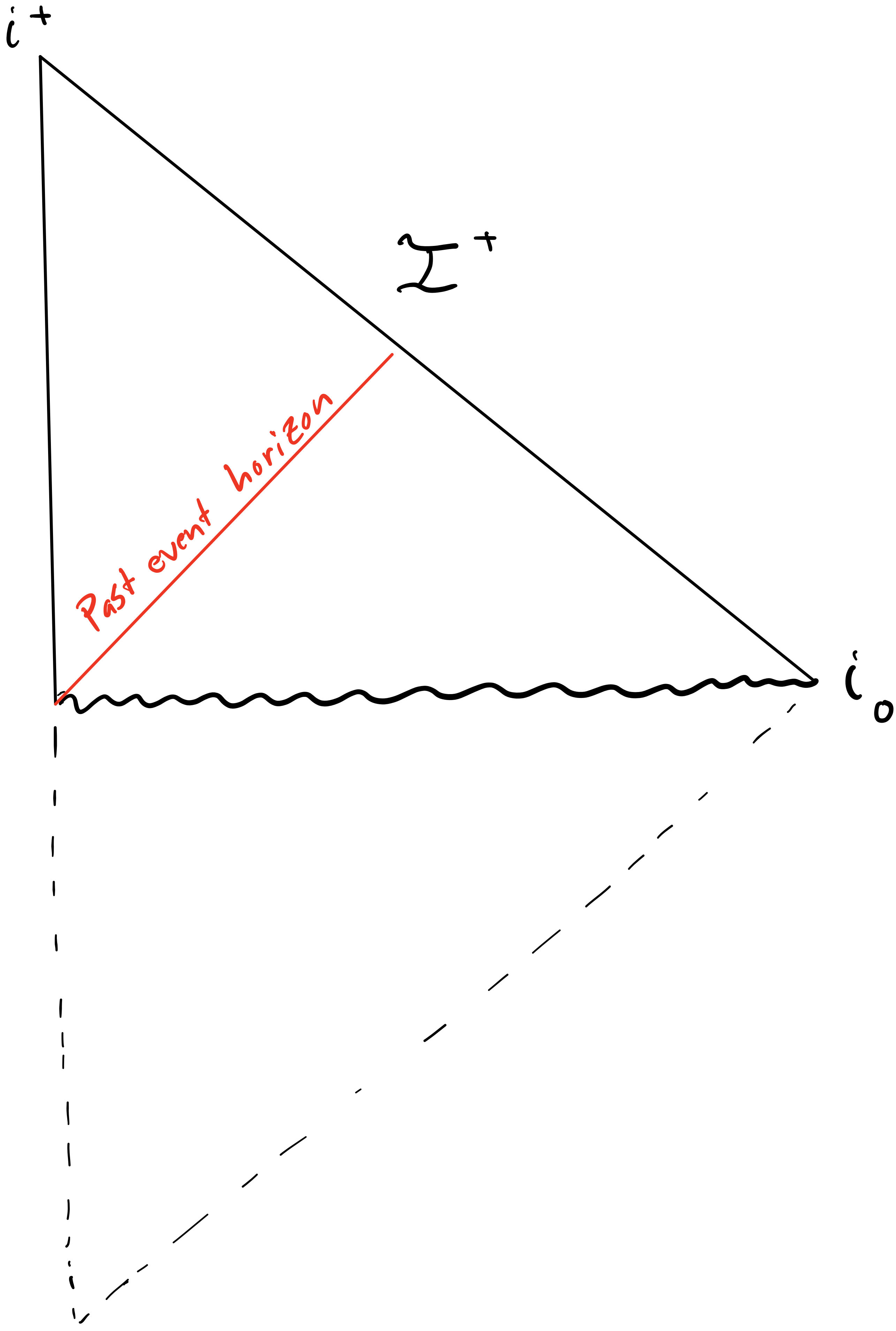
and after a conformal rescaling, $ds^2 \to d\tilde{s}^2$, we obtain the Minkowski metric in conformal coordinates

$$d\tilde{s}^2 = -d\tau^2 + dr^2 + r^2 d\Omega^2$$

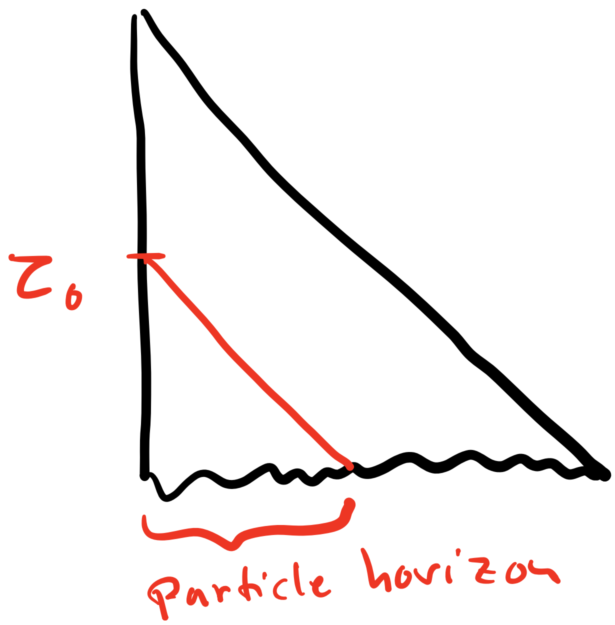
Thus, a flat FRW universe is conformal to Minkowski and therefore has the same Penrose diagram apart from one crucial point. In a radiation (or matter) dominated universe there is a Big Bang singularity at finite time in the past.

Thus FRW with ordinary matter (with $p \geq 0$) is conformal to Minkowski spacetime with the difference that $t > 0$ (in Minkowski $-\infty < t < \infty$) and so the lower part of the Penrose diagram is cutoff by the singularity.

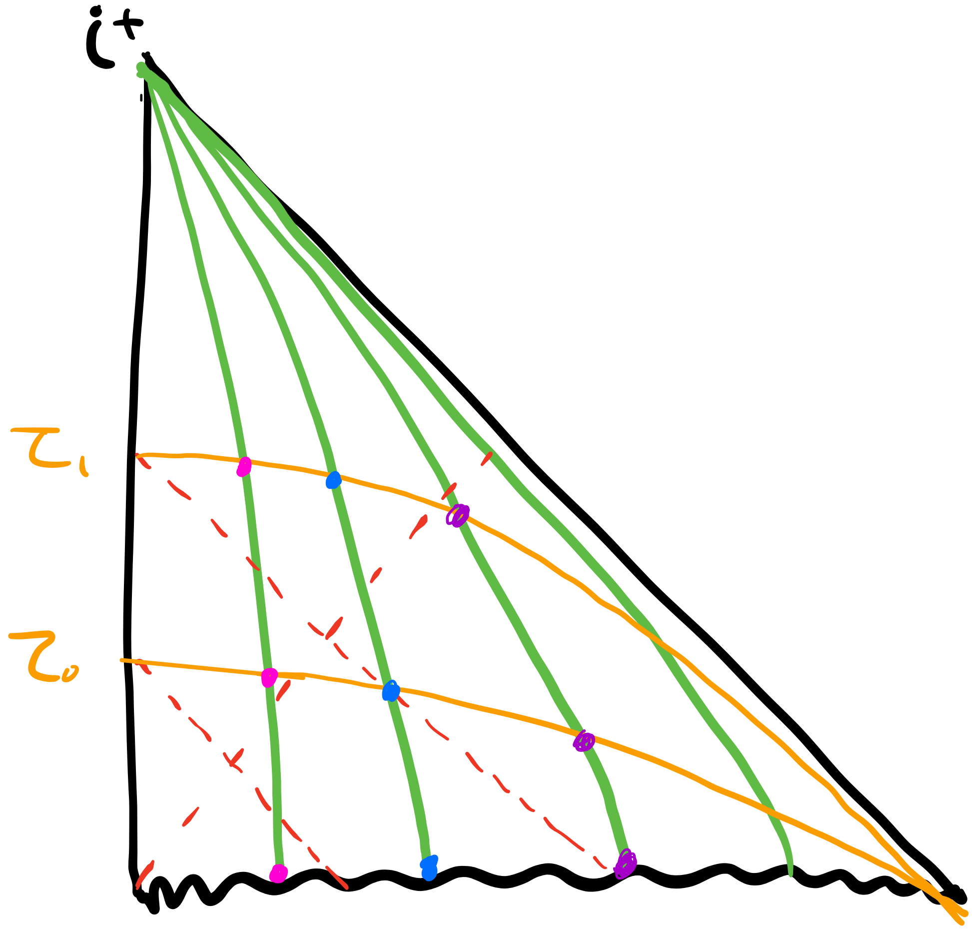
Penrose diagram of FRW
with $P \geq 0$



Clearly we cannot influence any spacetime events until they enter the past-event horizon. Similarly nothing can influence us unless inside our past light cone

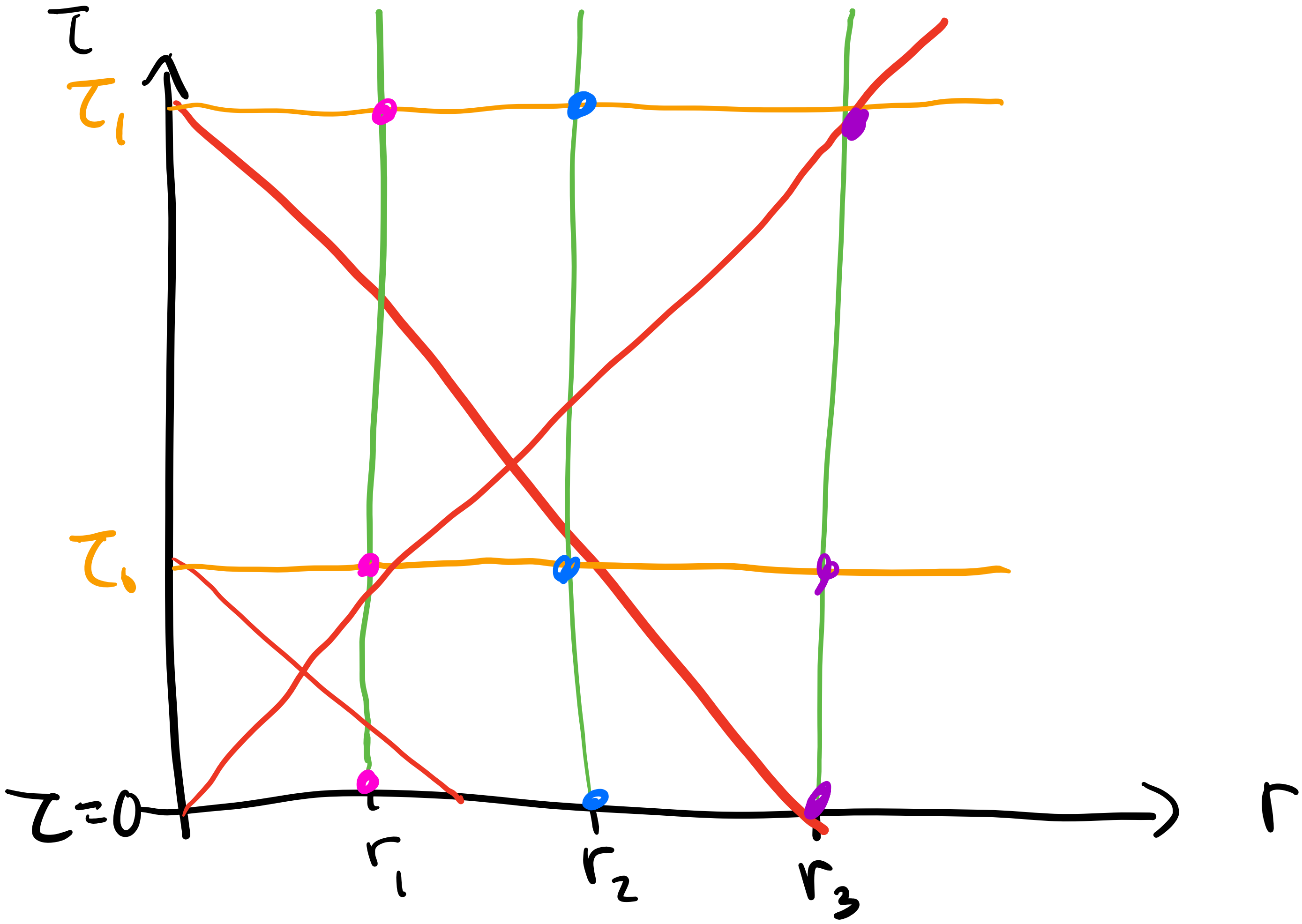


While null geodesics goes at 45° angles, comoving observers are those at rest in FRW coordinates. They have to end up at future time-like infinity, but can start out at any r. So lines of constant radial coordinates looks like this (green lines)



Similarly slices of constant τ has to end on spatial infinity (orange lines).

By going back to FRW coordinates in conformal time we can see that a comoving observer (radial coordinate point) which is inside our particle horizon is also inside our past event horizon, they are the same.



We can calculate the particle horizon, $L_{p.h.}$, at τ_0 as the past lightcone at τ_0 or t_0 equivalently. That is how far light can have propagated from $t=0$ to t_0 using for a null geodesic

$dt = a\,dr$

$$L_{p.h.} = a(t_0)\int_0^{r_0} dr = a(t_0)\int_0^{t_0} \frac{1}{a(t)}\,dt$$

$$= a(t_0)\int_0^{a_0} \frac{1}{a\dot{a}}\, da$$

$$= a(t_0)\int_0^{a_0} \frac{1}{a^2 H}\, da$$

In a radiation dominated universe

$\rho_r = 3H^2 M_p^2 \propto \frac{1}{a^4} \Rightarrow H = H_0\left(\frac{a_0}{a}\right)^2$

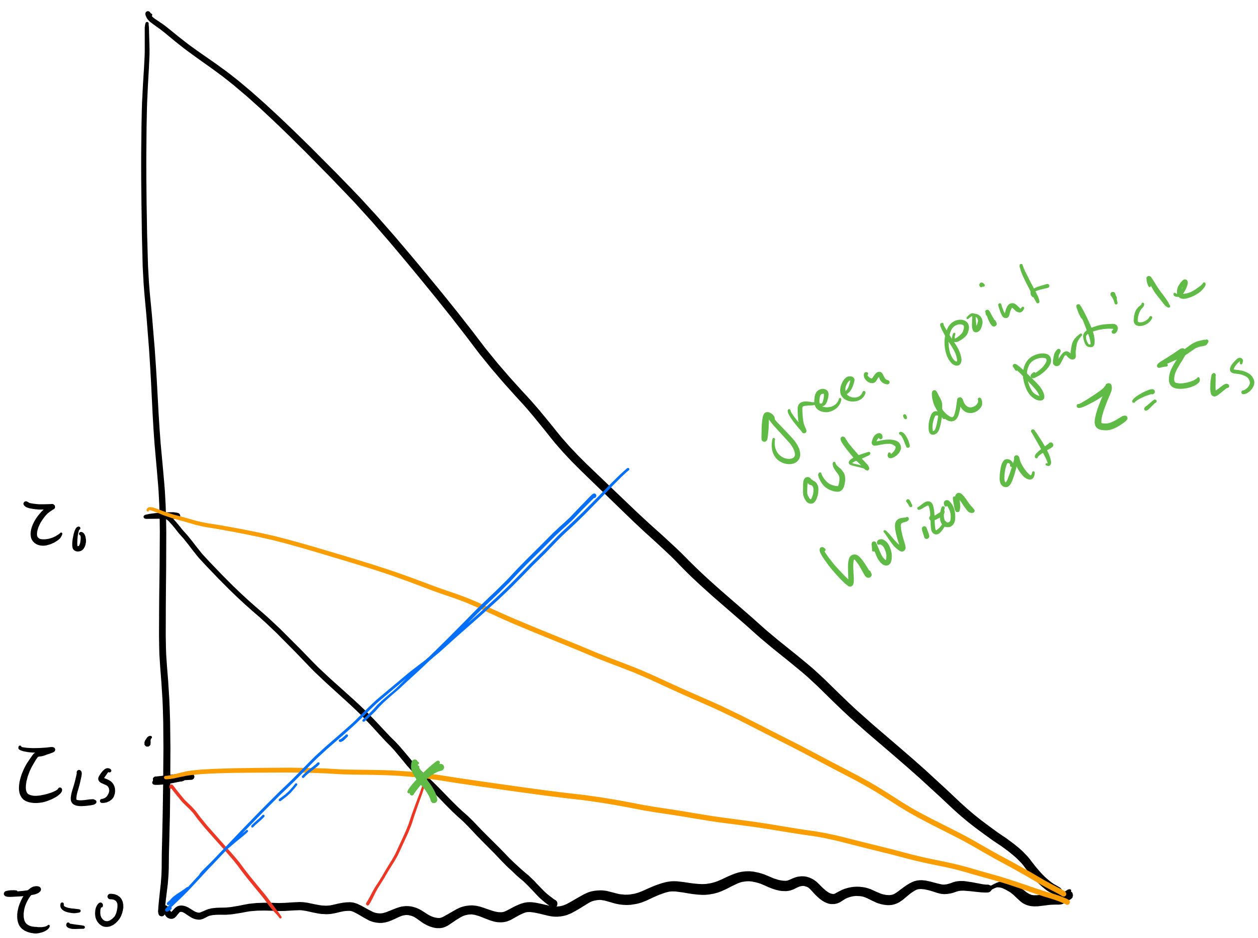
$$\Rightarrow L_{p.h.} = a(t_0)\int_0^{a_0} \frac{1}{H_0 a_0^2}\, da = \frac{1}{H_0}$$

So $1/H_0$ is the age and the size of the observable universe.

Causality problem

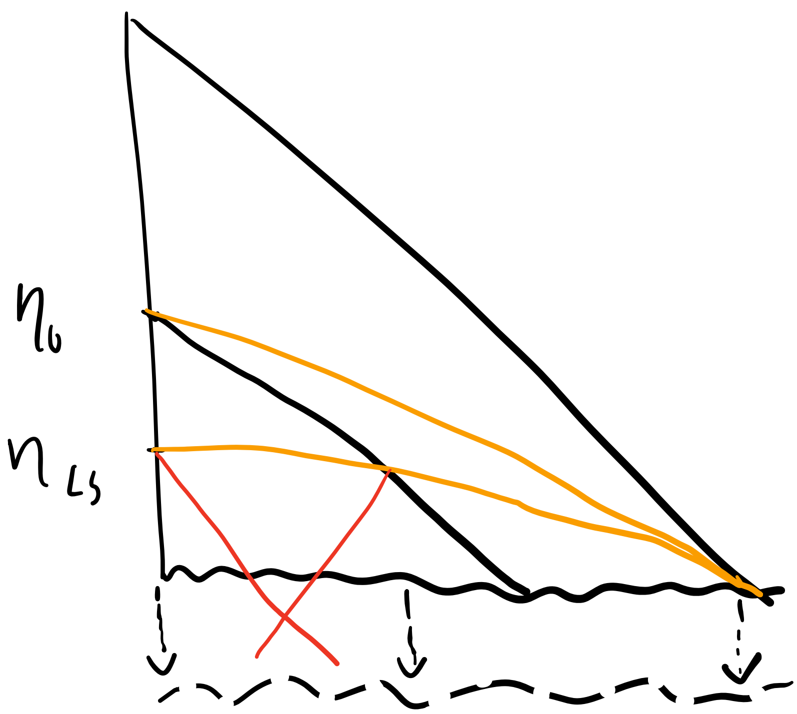
The CMB we observe was created at decoupling of photons ($\sim$ recombination) 380.000 years after Big Bang. We observe the CMB temperature to be the same to pretty high precision all over the sky

$$T_{CMB} = 2.7260 \pm 0.0013\,K$$



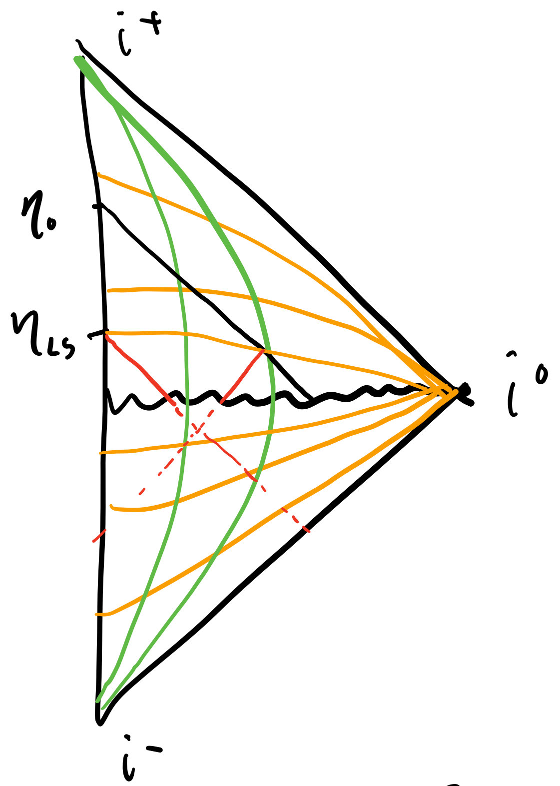
We see that different points at a slize of const. τ_{LS} (or t_{LS}) have past light cones that doesn't overlap and yet at τ_0 we observe them to have same temp. to high precision. This is an apparant paradox — how can they have agreed to have the same temp. if they never could have communicated

$\Rightarrow$ causality problem!



Imagine we could move the singularity back in time or remove it, then problem would be solved!

In particular completing it with a vacuum like solution looking like Minkowski would work



However starting before Big Bang with zero density Minkowski vacuum and then jumping to Planckian radiation density just at the Big Bang is not very physical.

Instead take a vacuum solution with large vacuum energy, i.e. de Sitter space-time.

We see from the vacuum Einstein eq.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = -8\pi G T_{\mu\nu} \underset{\text{in vac. with } T_{\mu\nu}=0}{=} 0$$

we can move the Λ term to right hand side and think of it as a const. energy density with negative pressure

$$P_\Lambda = -\rho_\Lambda \qquad (T^\mu{}_\nu = \mathrm{diag.}(-\rho, P, P, P))$$

Using the first Friedmann eq.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho \propto \text{const}$$

$\Rightarrow \quad a \propto e^{Ht} \qquad$ with $H \propto$ const

The second Friedmann eq.

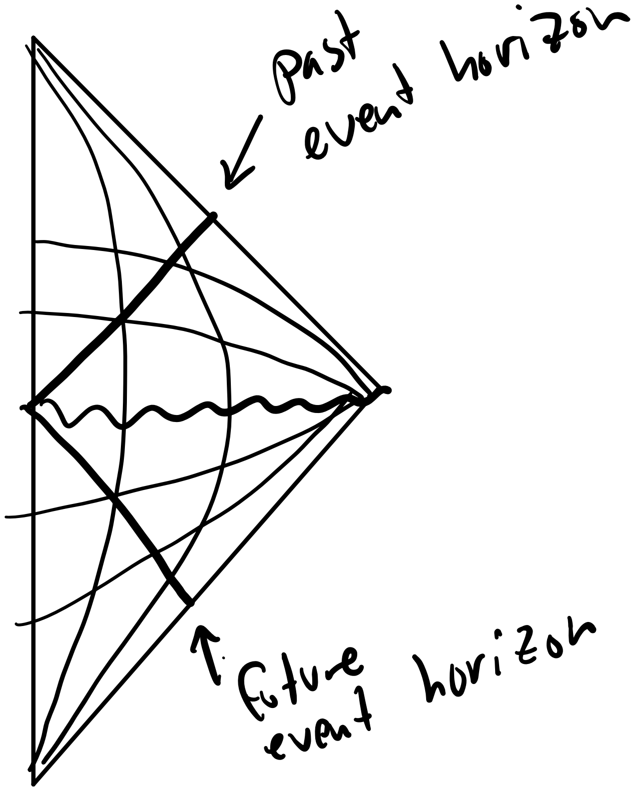
$$\frac{\ddot{a}}{a} = -4\pi G \left(p + \frac{1}{3} \rho \right)$$

then imply $p = -\rho$

Thus we have an FRW metric with exponential expanding scale factor, wich we know is conformal to Minkowski; but the energy density can be high, and it can precede the rad. dom. phase

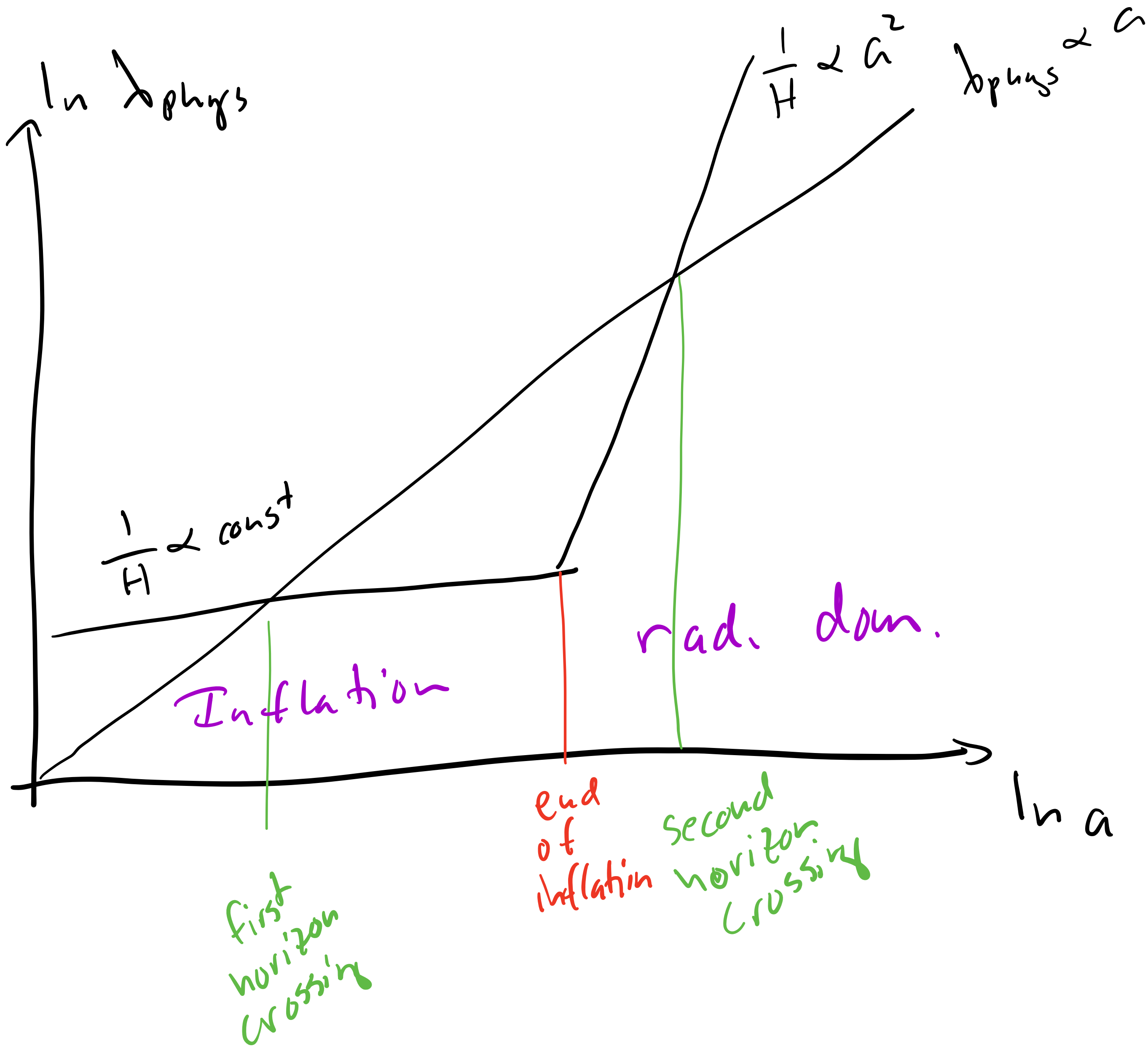
Also we see from $a \propto e^{Ht}$ that $a \to 0$ only when $t \to -\infty$ so singularity indeed moved infinitely back in t.

Having de Sitter FRW from $-\infty < t < 0$ and rad. dom FRW from $0 < t < \infty$ we then get a solution to the causality problem and the Penrose diagram looks like



So an early de Sitter like era solve the causality problem because in the past everything crossed back inside the event horizon and came in causal contact.

Another way to view it, is in a diagram like this



Inflation

As illustrated in the figure above, the horizon problem requires a period initially where the physical scales λ evolves faster than the horizon, so all of the observable universe could be in causal contact in the past ($\lambda \propto a$, $H = \dot{a}/a$)

$$\Rightarrow \frac{d}{dt}\left(\frac{\lambda}{|H^{-1}|}\right) = \frac{d}{dt}\left(a\left|\frac{\dot{a}}{a}\right|\right) = \frac{d}{dt}|\dot{a}| > 0$$

$\Rightarrow$ $\dot{a} > 0$ and $\ddot{a} > 0$

or $\dot{a} < 0$ and $\ddot{a} < 0$

$\Rightarrow$ we need a period of accelerated expansion (Inflation) or a period of accelerated contraction (Pre-big bang, Ekpyrotic)

Assume we have a period of approximately de Sitter like expansion with an almost constant energy density, as mention above we have $a \propto e^{Ht}$; i.e. exponential expansion. It is convenient to take the log when discussing the amount of expansion and measure the duration of inflation in e-folds

$$N = \ln\left(\frac{a(t_R)}{a(t_i)}\right)$$

where t_R is the time of reheating at the end of inflation

To solve the causality/horizon problem the largest observable scale today (the present horizon scale H_0^{-1}) must have inflated from a value $\lambda_{H_0}(t_i)$ smaller than the horizon during inflation

$$\lambda_{H_0}(t_i) = H_0^{-1}\left(\frac{a(t_R)}{a(t_0)}\right)\left(\frac{a(t_i)}{a(t_R)}\right) = H_0^{-1}\left(\frac{T_0}{T_R}\right)e^{-N}$$
$$\lesssim H_I^{-1}$$

where we used that after inflation we have that the temperature, T, drops with the expansion as $T \propto 1/a$.

$$\Rightarrow \quad N \gtrsim \ln\left(\frac{T_0}{H_0}\right) - \ln\left(\frac{T_R}{H_I}\right)$$
$$\sim 67 - \ln\left(\frac{T_R}{H_I}\right)$$
$$\Rightarrow \quad N \gtrsim 70$$

Flatness problem

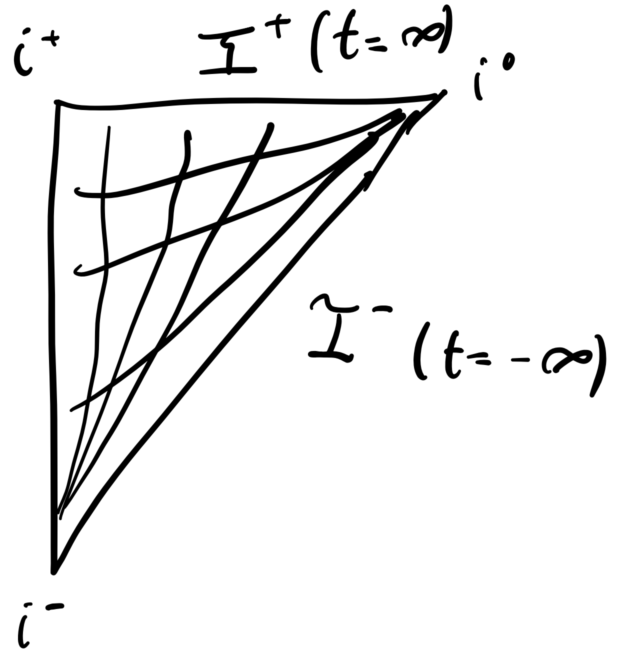
Inflation can also explain why the universe is observed to be spatially flat to high precision, as any initial curvature will be inflated away

$$\Omega - 1 = \frac{k}{(aH)^2} \qquad H \sim \text{const.} \Rightarrow \Omega - 1 \simeq e^{-2N}\frac{k}{H^2}$$

All this together makes it a good assumption that there were a period of quasi de Sitter expansion in the early universe.

de Sitter interlude

We saw that there is a set of coordinates where de Sitter looks like the lower part of Minkowski



$a = e^{Ht}$

$$a\,d\tau \equiv dt \Rightarrow \quad \tau = \int \frac{1}{a} dt = \int e^{-Ht} dt = \frac{-1}{aH}$$

$$\Rightarrow \quad \tau \to 0 \quad \text{for} \quad t \to \infty \quad (a \to \infty)$$

$$\tau \to -\infty \quad \text{for} \quad t \to -\infty \quad (a \to 0)$$

However, just like the Schwartzschild metric is not geodesic complete, as in-falling observers are crossing the horizon in finite time, also these coordinates of de Sitter are not geodesic complete. In fact they only cover half of de Sitter.

The original singularity theorem of Hawking and Penrose assumes the strong energy condition

$$\rho + 3p \geq 0$$

which is violated during inflation, and so do not apply to inflation. Borde-Guth-Vilenkin however showed that also inflation is not geodesically complete.

For null geodesics one cannot use proper time to parametrize their curve, so one need to use an affine parameter, λ. One can show that the geodesic equation is satisfied for null-geodesics in FRW space times if

$$d\lambda \propto a\,dt$$

<u>Exercise 2:</u> Show that this is true.

Normalizing the affine parameter by choosing

$$d\lambda = \frac{a(t)}{a(t_f)} dt \quad \Rightarrow \quad \left.\frac{d\lambda}{dt}\right|_{t=t_f} = 1$$

Multiplying by $H = \frac{\dot{a}}{a}$ and integrating

$$\int_{\lambda(t_i)}^{\lambda(t_f)} H(\lambda)\, d\lambda = \int_{a(t_i)}^{a(t_f)} \frac{da}{\cancel{dt}} \frac{1}{\cancel{a}} \frac{\cancel{a}}{a_f} \cancel{dt}$$

$$= \int_{a(t_i)}^{a(t_f)} \frac{1}{a(t_f)} da \leq 1$$

Now define the averaged H_{av}

$$H_{av} = \frac{1}{\lambda(t_f) - \lambda(t_i)} \int_{\lambda(t_i)}^{\lambda(t_f)} H(\lambda)\, d\lambda \leq \frac{1}{\lambda(t_f) - \lambda(t_i)}$$

$$\Rightarrow \quad \lambda(t_f) - \lambda(t_i) < \frac{1}{H_{av}}$$

So any backward null-geodesic in spacetime with $H_{av} > 0$ must have a finite affine length. One can show this also hold for time-like geodesics. Thus de Sitter in the FRW coordinates are only past eternal for observers at rest in co-moving coordinates, while other observers will see a universe that has only excisted a finite time. This is because the FRW coordinates of de Sitter, also called the Poincaré patch or flat slicing, only covers half of the de Sitter spacetime.

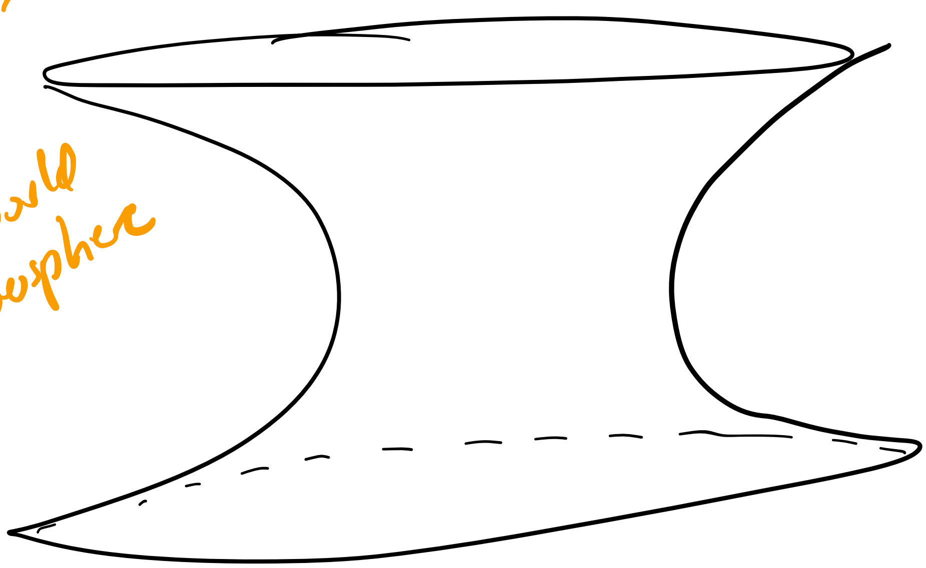
The simplest way to obtain de Sitter spacetime is to realize it as a hypersurface in a 5-d Minkowski space describing a hyper-boloid

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2 \qquad \left[\Lambda = \frac{3}{\ell^2}\right]$$

in a flat five-dimensional space $\mathbb{R}^5$ with metric

$$ds^2 = -dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2$$

had this been a "+" then we would have a hyposphere instead



coordinates

we can now introduce coordinates on the hyper-boloid by

$$\ell \sinh(\hat{t}/\ell) = X_0 \qquad \ell \cosh(\hat{t}/\ell) \cos\chi = X_1$$

$$l \cosh(\hat{t}/l) \sinh\chi \cos\theta = X_2$$

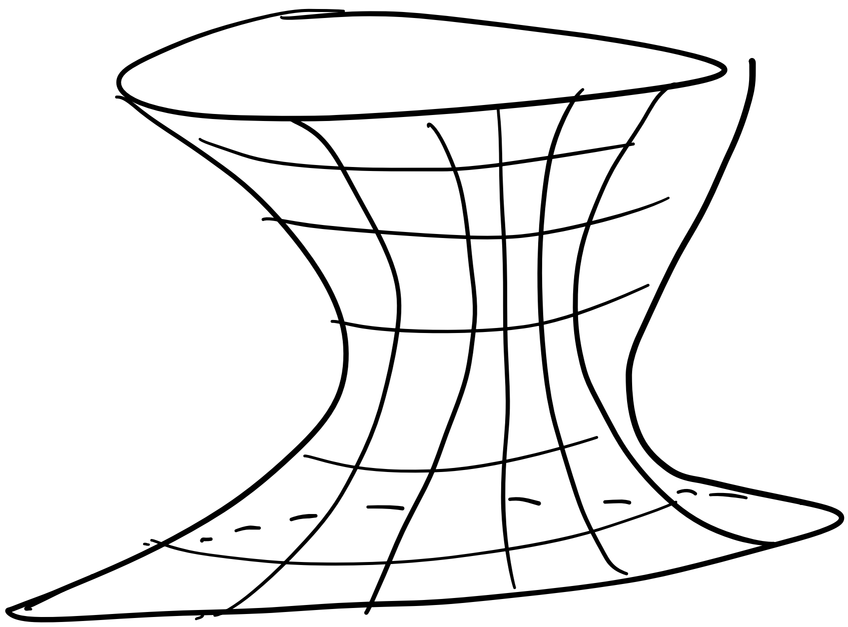
$$l \cosh(\hat{t}/l) \sinh\chi \sin\theta \cos\varphi = X_3$$

$$l \cosh(\hat{t}/l) \sinh\chi \sin\theta \sin\varphi = X_4$$

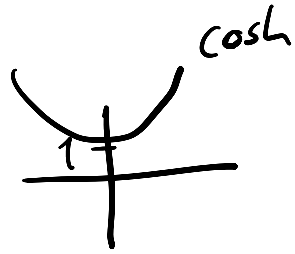
$$\Rightarrow ds^2 = -d\hat{t}^2 + l^2 \cosh^2(\hat{t}/l)[d\chi^2 + \sinh^2\chi \overbrace{(d\theta^2 + \sinh^2\theta \, d\varphi^2)}^{d\Omega^2}]$$

Singularities at $\chi = 0$, $\chi = \pi$ and $\theta = 0$, $\theta = \pi$ are simple coordinate singularities occuring in polar coordinates, but apart from that these coordinates cover the whole space

$-\infty < \hat{t} < \infty$, $0 \leq \chi \leq \pi$, $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$



To obtain the Penrose diagram and understand the causal structure of de Sitter, we can define a new time coordinate



$$\cosh \hat{t}/\ell = \frac{1}{\cos \hat{T}}$$

so

$$-\pi/2 < \hat{T} < \pi/2$$

$$\Rightarrow \quad ds^2 = \frac{\ell^2}{\cos^2(\hat{T})}\left(-d\hat{T}^2 + \sin^2\chi \, d\Omega^2\right)$$

which is conformal to

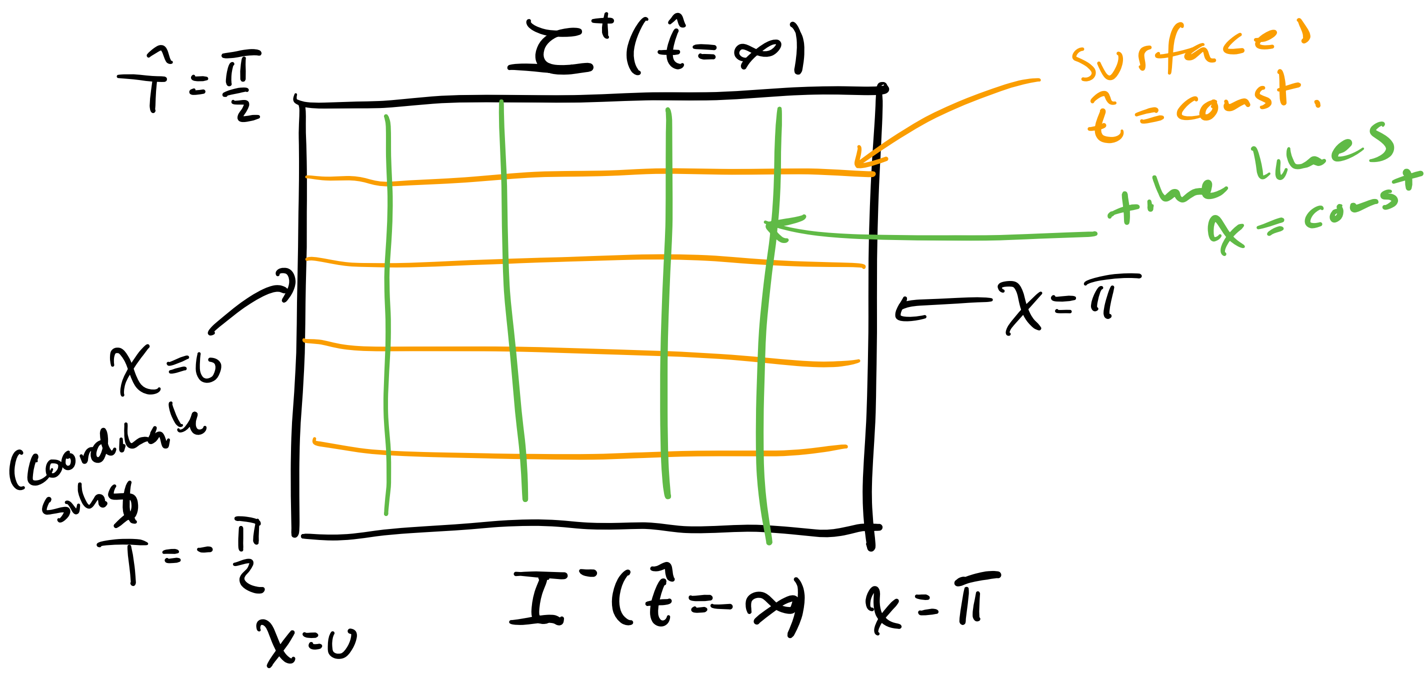
$$d\tilde{s}^2 = -d\hat{T}^2 + \sin^2\chi \, d\Omega^2$$

This is the same metric as the conformal or Penrose coordinates of Minkowski except in this case we don't have

$$-\frac{\pi}{2} < T - R \leq T + R < \frac{\pi}{2}$$

but instead

$$-\frac{\pi}{2} < \hat{T} < \frac{\pi}{2} \quad , \quad 0 \le \chi \le \pi$$



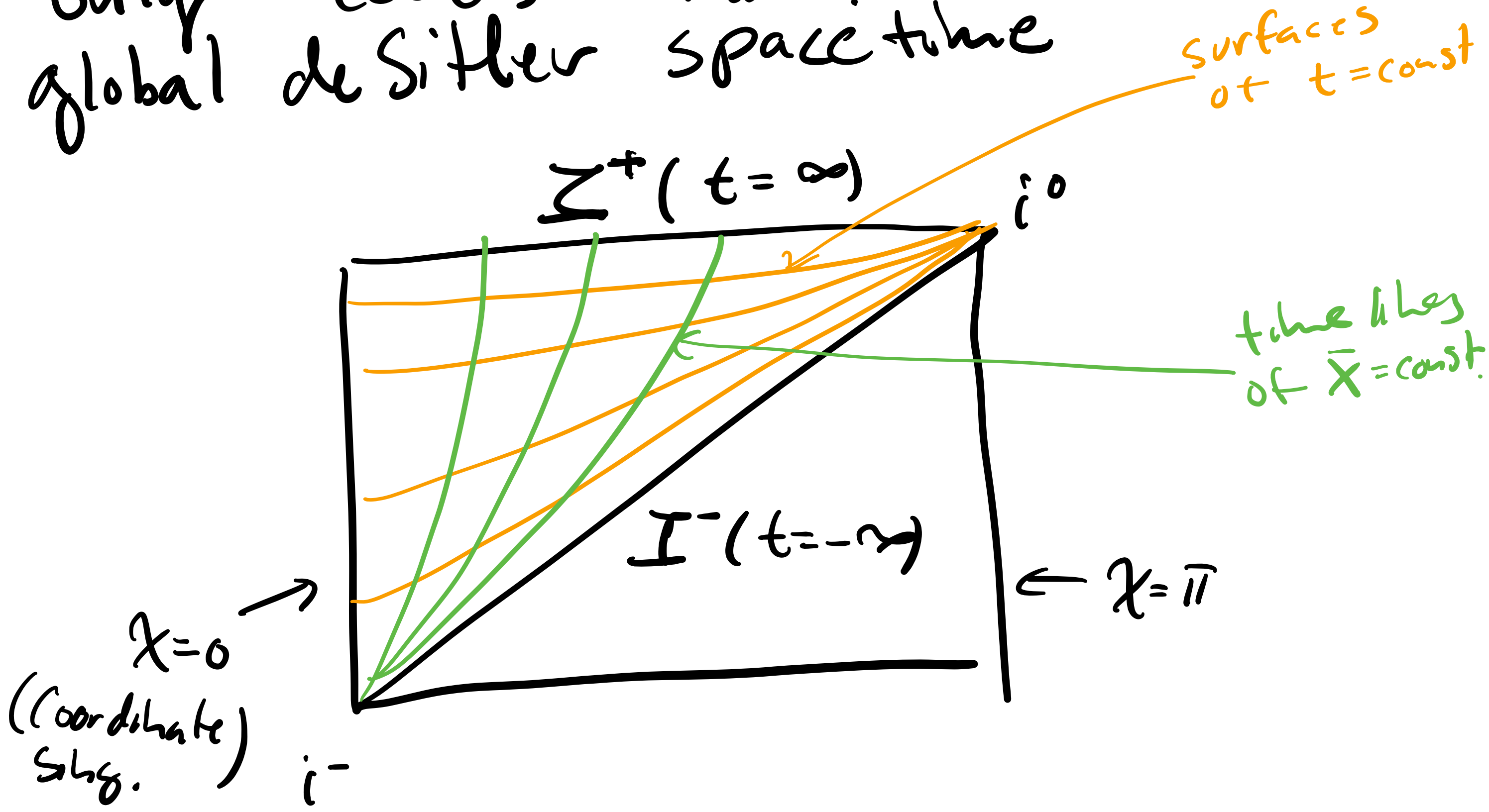
Now clearly the FRW or Planar or Poincaré coordinates of before

$$t = \ell \log \frac{X_0+X_1}{\ell} \qquad x = \frac{\ell X_2}{X_0+X_1}, \quad y = \frac{\ell X_3}{X_0+X_1}, \quad z = \frac{\ell X_4}{X_0+X_1}$$

$$\Rightarrow \quad ds^2 = -dt^2 + \exp(2t/\ell)\, d\vec{x}^{\,2}$$

$$= -dt^2 + e^{2Ht} d\vec{x}^{\,2} \qquad H = \frac{1}{\ell}$$

only covers half of the global de Sitter spacetime



Now it is clear why dS in FRW coordinats is geodesically incomplete.

Note that sometime people discuss observer dependent issues like particle production and disagrees because the compare results in global coord. versus FRW coordinates, however a cosmologically comoving observer is at rest in FRW coord.

Nevertheless, this means that in FRW coord. we do not avoid that a cosmological observer will experience an initial singularity ($\equiv$ geodesic incompleteness)

Of course when pasting dS lower part to FRW upper part to get the inflationary space time we cut the dS at some finite $t=0$ and also do not include the $t \to \infty$ part of dS. So in fact inflation cannot be exact dS but must include some ways of ending inflation at $t=0$.

Models of Inflation (Microphysics)

To end inflation wee need some dynamics playing the role of a clock telling us when to go from dS-like inflation to radiation dominated FRW.

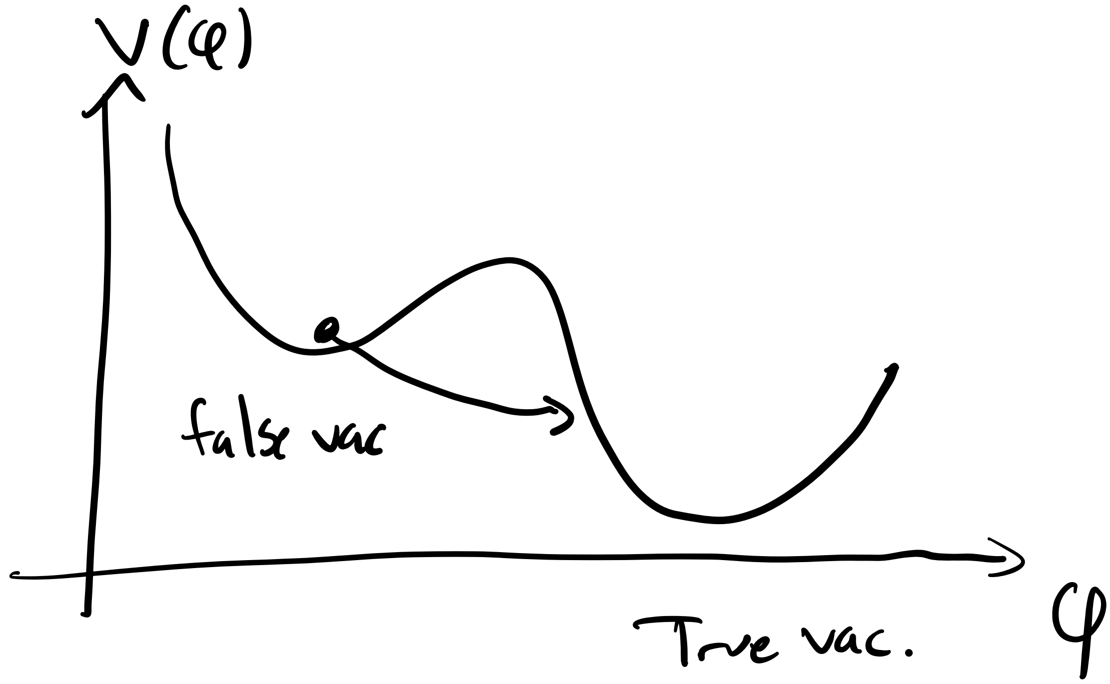
Old Inflation

Global

Pure dS is in thermal equilibrium and therefore quite dead. Nothing happens and the temp. is const.

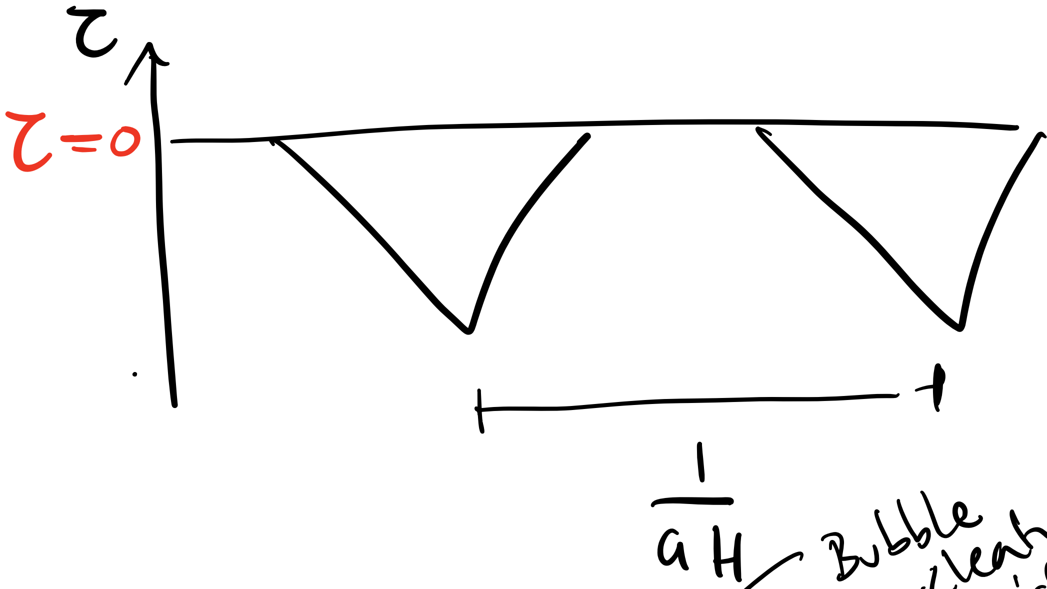
$$T_{dS} = \frac{H}{2\pi}$$

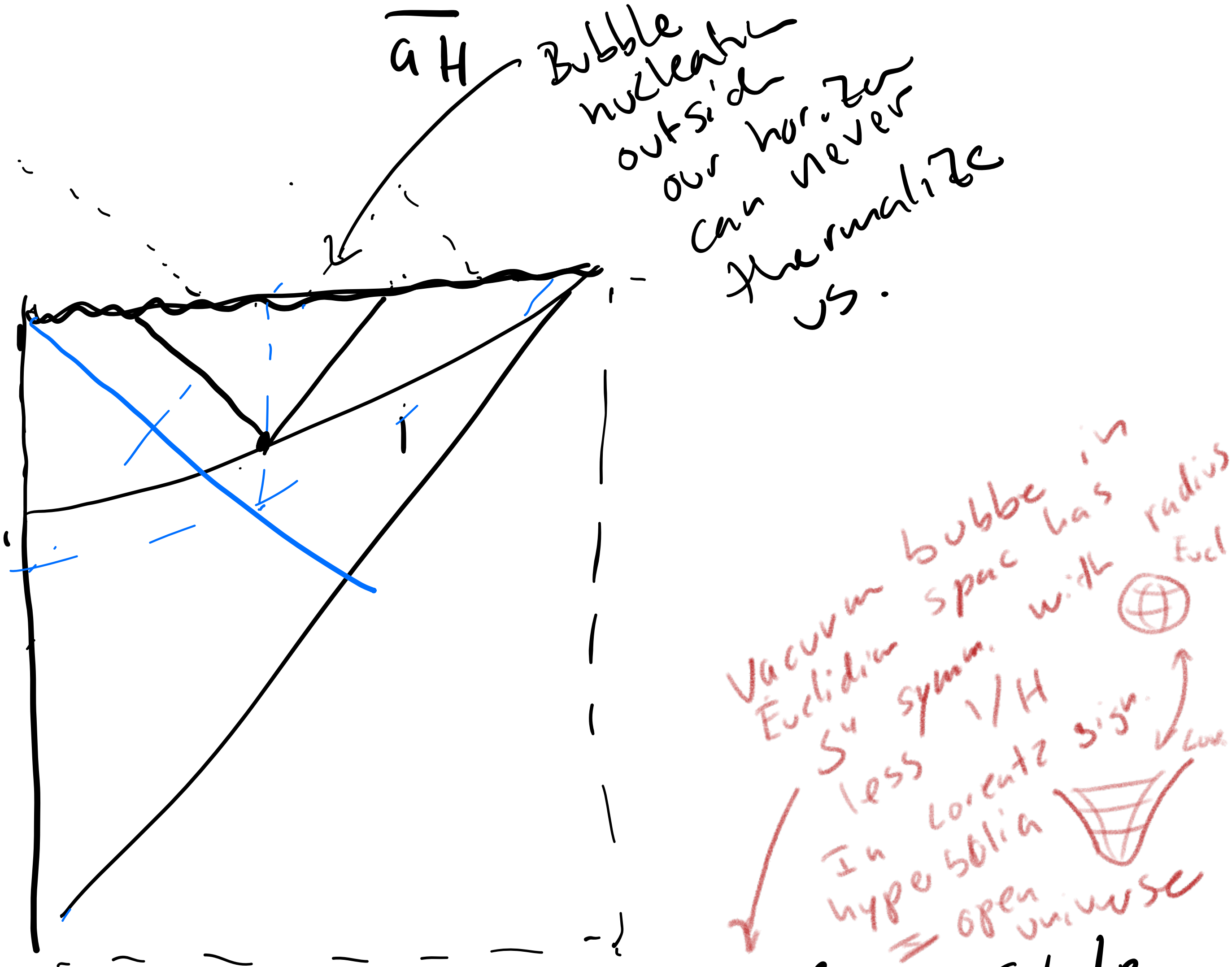
However imagine that there is also some raditation with a changing temp. Note, the radiation may still be in local thermal equilibrium such that the local temp. determines the state of the field at each point of spacetime as if it was in a thermal bath. Then inspired by our understanding of particle physics and f. ex. the EW phase transition, it is natural to think of some scalar field, with a potential which develops a new "true" vacuum below some critical temperature T_c.



Initially stuck in the false minimum the potential energy of the field is like cosmological constant, and if radiation density is small enough the universe will be dominated by a cc. Once the temp. drops below T_c and the second minimum appears, the field will tunnel and the false vac. energy converted into radiation in a first order phase transition.

The phase transition completes, by bubbles of true vacuum nucleating and expanding by the speed of light, only if the bubbles meet and perculate. For that to happen, one needs more than one bubble per event horizon since a bubble can never grow larger than this. However in that case the nucleation rate, Γ, is so high that one never achieves enough inflation. In the other case of one bubble per horizon volume or less, the PT never completes.



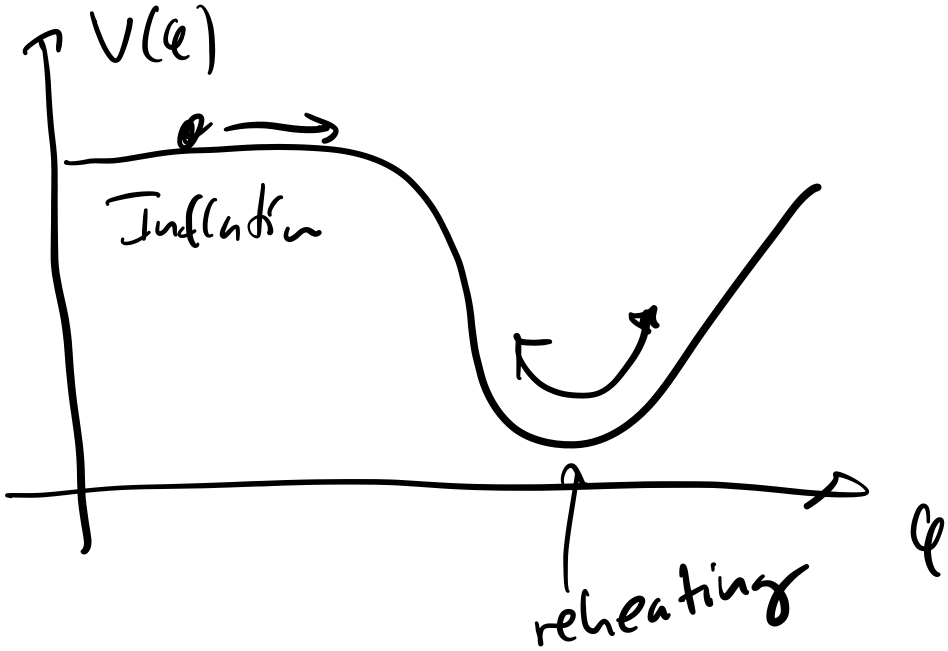


Also we can not live inside a single bubble as that is an open universe with curvature radius = initial size of bubble i.e. less than $1/H$ ⇒ curvature dominated

⇒ Gracefull exit problem!

Slow-roll inflation

The idea is to have a slow roll over P.T. instead



Assume that in addition to gravity we have a scalar field called the inflaton, such that the total action is

$$S = S_{grav} + S_\varphi$$

with

$$S_{grav} = \frac{1}{2}\int d^4x \sqrt{-g}\, R$$

$$S_\varphi = \int d^4x \sqrt{-g}\, \mathcal{L}_\varphi$$

$$= -\int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi + V(\varphi)\right]$$

The field equation of motion is

$$\frac{\delta S}{\delta \varphi} = 0 \Rightarrow \ddot{\varphi} + 3H\dot{\varphi} - \frac{(\partial\varphi)^2}{a^2} + V'(\varphi) = 0$$

From the definition of $T_{\mu\nu}$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\varphi}{\delta g^{\mu\nu}}$$

Jacobi's formula $\delta g = g\, g^{\mu\nu} \delta g_{\mu\nu}$

and using

$$\delta\sqrt{-g} = -\frac{1}{2}\frac{1}{\sqrt{-g}}\delta g = \frac{1}{2}\sqrt{-g}\, g^{\mu\nu}\delta g_{\mu\nu}$$

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$$

$$\Rightarrow \quad T^{\mu\nu} = \partial^\mu\varphi\,\partial^\nu\varphi + g^{\mu\nu}\mathcal{L}_\varphi$$

lowering indices and assuming the ideal fluid form

$$T_{\mu\nu} = \begin{bmatrix} \rho & & & \\ & a^2 p & & \\ & & a^2 p & \\ & & & a^2 p \end{bmatrix}$$

$$\Rightarrow \qquad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\frac{1}{a^2}(\partial_i \varphi)^2 + V(\varphi)$$

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{6}\frac{1}{a^2}(\partial_i \varphi)^2 - V(\varphi)$$

Now assuming the scalar field is homogeneous we have $\partial_i \varphi \to 0$ and from the first Friedmann eq. we find

$$H^2 = \frac{8\pi G_N}{3}\rho = \frac{8\pi G_N}{3}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]$$

For

$$V(\varphi) \gg \dot{\varphi}^2$$

we have quasi-de Sitter expansion with

$$P \simeq -\rho$$

The slow-roll approximation is therefore to assume $\dot{\varphi}$ small and $\ddot{\varphi}$ small to keep $\dot{\varphi}$ small long enough to solve the causality problem, i.e. $\ddot{\varphi} < 3H\dot{\varphi}$

$$\Rightarrow \quad H^2 \simeq \frac{8\pi G_N}{3} V(\varphi) \quad , \quad 3H\dot{\varphi} = -V(\varphi)$$

from first Fridmann eq. and the equation of motion These are called the slow-roll equations.

To keep control of the slow-roll approximation, we introduce the slow-roll parameters

$$\epsilon = 4\pi G_N \frac{\dot{\varphi}^2}{H^2} \quad , \quad \eta = \frac{1}{8\pi G_N}\left(\frac{V''}{V}\right)$$

where $\eta - \epsilon = -\dfrac{\ddot{\varphi}}{H\dot{\varphi}}$

The slow-roll approximation is then equivalent to requiring

$$\epsilon << 1 \, , \qquad |\eta| << 1$$

In particular, inflation $\Leftrightarrow \epsilon < 1$

The constraint on the number of e-folds for solving causality and flatness problem becomes a constraint on the potential

$$N \equiv \ln\left(\frac{a(t_f)}{a(t_i)}\right) = \int_{t_i}^{t_f} H\,dt$$

$$\simeq 8\pi G_N \int_{\varphi_f}^{\varphi_i} \frac{V}{V_\varphi}\,d\varphi \gtrsim 60$$

where we used $\dot{\varphi} = -\frac{V_\varphi}{3H}$

$$dt = \frac{dt}{d\varphi}d\varphi \Rightarrow H\,dt = \frac{H}{\dot{\varphi}}d\varphi \Rightarrow H\,dt = \frac{3H^2}{V_\varphi} = \frac{V}{V_\varphi}$$

While the details of inflationary model building is a huge topic, we will mention four classes of <u>inflation models</u>

1. Large field models
2. small field models
3. Hybrid models
4. Curvaton models

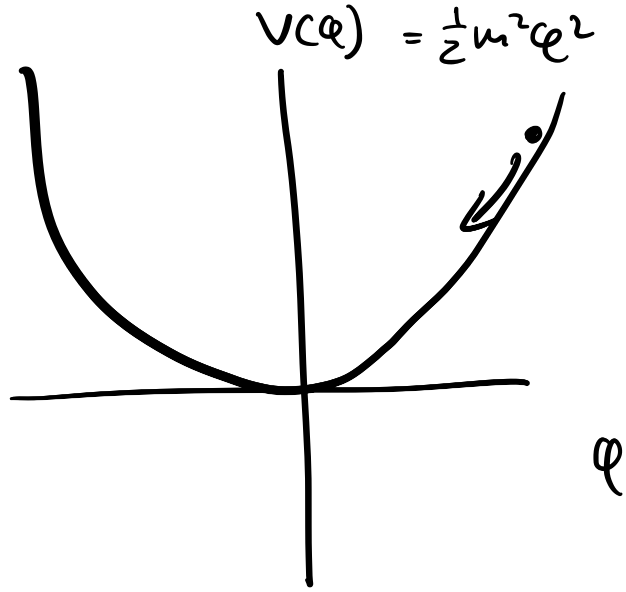
If we first focuss on single field models, a single field inflation potential can be described by two energy scales, the hight of the potential, Λ, and its width, μ, related to the field excursion of the inflaton field $\Delta\varphi$ during inflation

$$V(\varphi) = \Lambda^4 f\left(\frac{\varphi}{\mu}\right)$$

Large field models

In large field models $\Delta\varphi \gg M_p$ so one of the energy scales describing the potential is Planckian $\mu \gtrsim M_p$. This can be achieved if the inflaton field starts high-up in a monomial type potential

$$V(\varphi) = \Lambda^4 \left(\frac{\varphi}{\mu}\right)^n, \quad n \geq 1$$



$\varphi_f << \varphi$

$$\epsilon = \frac{n^2 M_p^2}{2\varphi^2}, \quad \eta = \frac{n(n-1) M_p^2}{\varphi^2}$$

$$N = -\int_{\varphi}^{\varphi_f} \frac{V}{V'} \frac{d\varphi}{M_p} = -\frac{1}{n M_p} \int_{\varphi}^{\varphi_f} \varphi \, d\varphi \simeq \frac{\varphi^2}{2n M_p^2}$$

$$\Rightarrow \quad \epsilon = \frac{n}{4N}, \quad \eta = \frac{n-1}{2N}$$

$$\Rightarrow \quad \epsilon \sim \eta \sim \frac{1}{N}$$

and $\epsilon << 1 \Rightarrow \varphi >> M_p$

As long as $\rho \simeq m^2 \varphi^2 << M_p^4$ $(n=2)$
we don't need quantum gravity
and the model is safe as

a QFT, so this just require $\varphi \ll \frac{M_p^2}{m}$ which can be larger than M_p if $m \ll M_p$.

However some symmetry needs to protect the potential from getting corrected by an infinite tower of higher dim. operators of the form induced by graviton loops

$$\mathcal{L} \supset \frac{\varphi^n}{M_p^{n-4}}$$

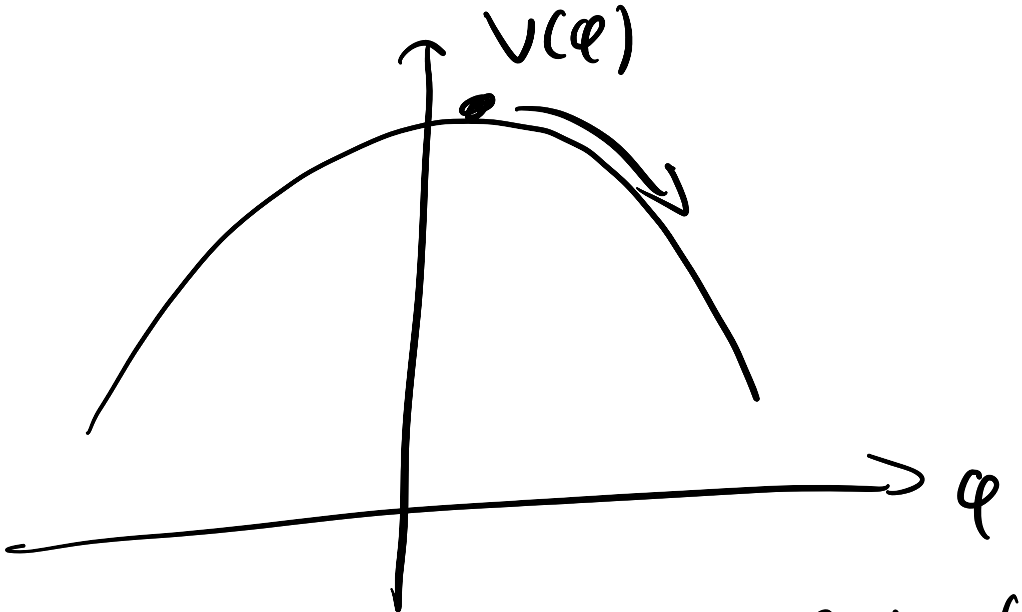
that becomes important for $\varphi \gg M_p$

One way is to use spontaneously broken shift-symmetry and letting the inflaton be the pNGB associated with it.

prime examples are chaotic infl. and axion monodromy infl.

Small field models

Here the potential is inverted so we start at small field value and roll away



One can think of this a effectively a tachyonic mass

$$V'' < 0 \Rightarrow \eta < 0$$

This type of potential is inspired by spontaneous symm. in particle phys

If one taylor expand the function $f\left(\frac{\varphi}{\mu}\right)$ for $\varphi \ll \mu$ one get a potential of the form

$$V(\varphi) = \Lambda^4 \left[1 - \left(\frac{\varphi}{\mu}\right)^n + \ldots \right] \;, \quad n \geq 2$$

Obviously this breaks down when $\varphi \sim \mu$, which is where inflation ends, $\varphi_f \sim \mu$.

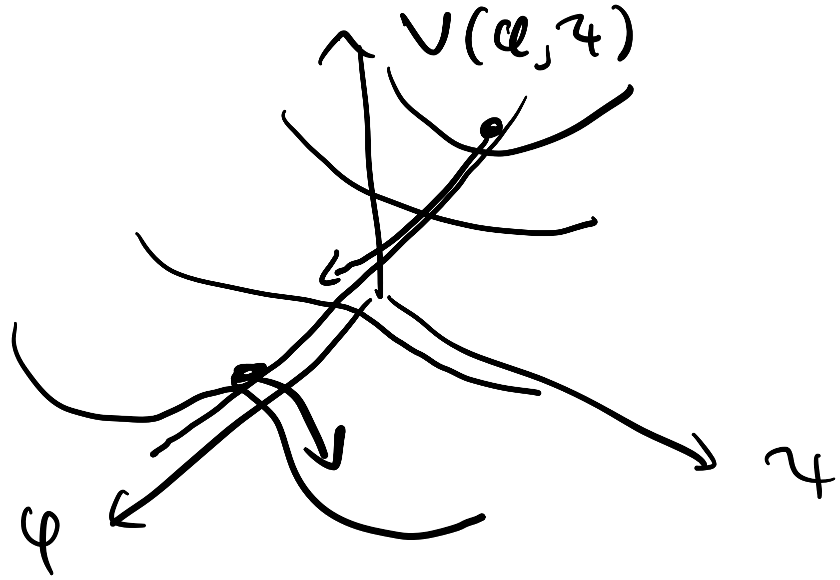
Since φ is small and to leading order $V(\varphi) \approx \Lambda^4$, one typically has $\epsilon \simeq 0 << 1$

Examples are new infl and natural infl. In natural infl. the pot. is the $\cos(\varphi/\mu)$ of a p Nambu-Goldstone-Boson (pNGB) field which expanded could give a potential of the form above with $n=2$.

Hybrid models

Hybrid inflation has been much discussed in the context of Supersymmetry and supergravity. It is effectively a single field model, but where the end of inflation is triggered by a second field, the waterfall field, with a typical potential of the form

$$V(\varphi,\psi) = \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\tilde{\lambda}\varphi^2\psi^2 + \frac{1}{4}\lambda(M^2-\psi^2)^2$$



One could also consider variants where the PT at the end is first order.

Taylor expanding the potential above for small $\varphi \ll \mu$, one gets effectively

$$V(\varphi) = \Lambda^4\left[1 + \left(\frac{\varphi}{\mu}\right)^n + \dots\right], \quad n \geq 2$$

so typically one has $\eta = 2\left(\frac{M_p}{\mu}\right)^2 > 0 \Rightarrow \mu > M_p$

and $\epsilon = \left(\frac{\varphi}{\mu}\right)^2 \eta \ll \eta$

Curvaton

The models above are constrained by observations, since the fluctuations of the inflaton creates the CMB perturbations in those models. Since CMB perturbations are close to scale invariant, this implies that the inflation potential also needs to be close to scale inv., i.e. close to de Sitter with $\epsilon, \eta \ll 1$. We say the potential has to be very flat. This is what f.ex. rules out the Pre-big bang scenario and the old Ekpyrotic scenario in absence of the curvaton.

The curvaton is another light field that remains frozen during inflation and because of being almost massless during inflation, it aqcuires a close to flat spectrum. After inflation the curvaton dominates the universe and decay into radiation, so all the CMB perturbations are created by the curvaton instead of by the inflaton.

In the curvaton scenario, the inflaton potential therefore be much steeper. Also pre-big bang and the new Ekpyrotic scenario requires a curvaton to be compatible with observations.

In fact the curvaton was first introduced in 2001, by Enquist and Sloth, exactly to explain how pre-big bang could lead to a flat spectrum in agreement with CMB observations. It was shortly after named the "Curvaton" by Lyth and Wands.

The same is true for Ekpyrotic scenario. The new Ekpyrotic scenario, introduced later, was therefore also called "pre-big bang with a curvaton heart" by Andrei Linde.