Lecture 3

Beyond Linear perturbation theory

So far we have only considered the equation of motion of the perturbations, like δ, to linear order which we derrived from their quadratic action. This is equivalent to saying that we have only treated the free field theory of the perturbation and ignored interactions. Going to higher order in perturbation theory means including interactions.

When quantizing the free field theory, we obtain Gaussian quantum fluctuation, which when streched to large scale with high occupation number becomes Gassian random variables. Gassian fluctuations are completely characterized by their two-point function which we already calculated. But going to higher order in perturbation theory and including interaction, we will find deviations from Gaussianity, characterized by a non-vanish 3-point correlation function and non-vanishing connected 4-point function. We can also have loop corrections to the two-point function. We will now discuss these issues in turn, starting with non-Gaussianity

Non-Gaussianity

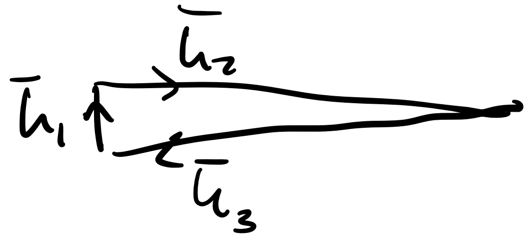
Let's start by considering the 3-point function for the curvature perturbation

$$\langle \zeta_{\bar{k}_1} \zeta_{\bar{k}_2} \zeta_{\bar{k}_3} \rangle \equiv (2\pi)^3 \delta(\sum_a \bar{k}_a) B_\zeta(\bar{k}_1, \bar{k}_2, \bar{k}_3)$$

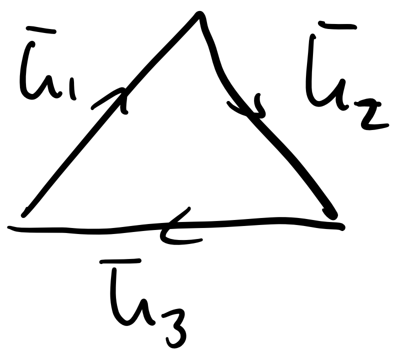
where we have introduced the bispectrum B_ζ, which is a function of the triangle formed by the three momenta, $\bar{k}_1, \bar{k}_2, \bar{k}_3$, due to momentum conservation.

There are three extreme shapes, which are typically used as templates and which embodies different limits of the underlying physics.

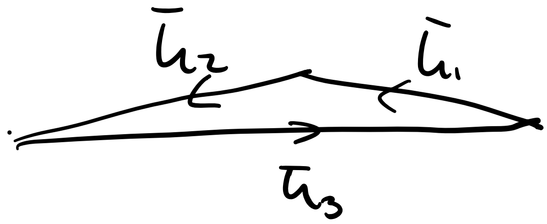
squeezed (local): $|\bar{k}_1| \ll |\bar{k}_2|, |\bar{k}_3|$



Equilateral: $|\bar{k}_1| \simeq |\bar{k}_2| \simeq |\bar{k}_3|$



Folded: $k_3 = 2k_1 = 2k_2$



Single field models of inflation with a standard kinetic term, and the standard curvaton model both have a bispectrum which are peaked around the local squeezed limit, while some higher derivative theories, like DBI inflation, are peaked in the equilateral shape, while f.ex. modifications of initial vacuum could lead to effect ~~ma~~maximizing the bispectrum in the folded shape.

Since the simpler "standard" models of inflation and the curvaton model have a bispectrum that is maximal in the local shape, we are going to focuss on local non-Gaussianity here.

Local non-Gaussianity

As a parametrization of the strength of non-Gaussianity, one usually introduces the dimensionless non-linearity parameter, which in general can depend on momenta

$$B_\zeta = \frac{6}{5} f_{NL} \sum_{a<b} P_\zeta(k_a) P_\zeta(k_b)$$

where the powerspectrum

$$P_\zeta(k) = \frac{1}{4\pi^2} \frac{1}{k^3} \mathcal{P}_\zeta(k)$$

so $\langle \zeta_{\bar{k}_1} \zeta_{\bar{k}_2} \rangle = (2\pi)^3 \delta(\bar{k}_1 + \bar{k}_2) P_\zeta(k_1)$

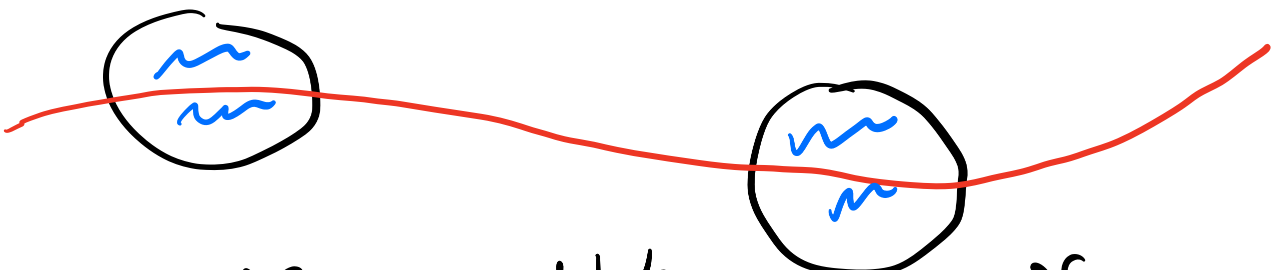
The local shape of non-Gaussianity correspond to the case where f_{NL} is independent of momenta in which case B_ζ can be obtained from the simple ansatz

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}^{local} (\zeta_g^2 - \langle \zeta_g^2 \rangle)$$

where ζ_g is the Gaussian

linear perturbation.

In single field inflation f_{NL}^{local} is given by the Maldacena Consistency relation. If we consider $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$ in the squeezed limit $k_1 \ll k_2, k_3$, we can think of the long wavelength mode as locally rescaling the background for the short wavelength modes



since, if on the scale of $\bar{k}_2$ and $\bar{k}_3$, $\zeta(\bar{k}_1)$ looks as a constant ζ_B, we see from

$$ds^2 = -dt^2 + a^2 e^{2\zeta_B} d\bar{x}^2$$

that it can locally be absorbed in the background

$$d\bar{x} \to d\tilde{\bar{x}} = e^{\zeta_B} d\bar{x}$$

which corresponds to

$$k \to \tilde{k} = e^{-\zeta_B} k$$

We can ~~then~~ Taylor expand the local two-point function the leading order in the long wavelength mode

$$\langle \zeta(x_2)\zeta(x_3)\rangle_{\zeta_B} = \langle \zeta(x_2)\zeta(x_3)\rangle + \zeta_B(x_1)\frac{\partial}{\partial \zeta_B}\langle \zeta(\bar{x}_2)\zeta(\bar{x}_3)\rangle\Big|_0$$

$$+ \ldots$$

Since ζ_B is almost constant on the length scale $|\bar{x}_2 - \bar{x}_3|$, we can choose x_1 freely between x_2 and x_3, but take for simplicity $\bar{x}_1 = (\bar{x}_2 + \bar{x}_3)/2$

Now, going to Fourier space, we have

$$\langle \delta_{u_2}\delta_{u_3}\rangle_B = \int d^3x_2 \int d^3x_3\, e^{-i\bar{x}_2\cdot\bar{u}_2} e^{-i\bar{x}_3\cdot\bar{u}_3} \times \langle \delta(x_2)\,\delta(x_3)\rangle_B$$

$$= \langle \delta_{u_2}\delta_{u_3}\rangle_0$$

$$+ \int d^3x_2 \int d^3x_3\, e^{-i\bar{x}_2\cdot\bar{u}_2} e^{-i\bar{x}_3\cdot\bar{u}_3} \times \delta_B(x_1)\frac{\partial}{\partial\delta_B}\left[\iint \frac{d^3\tilde{q}_2}{(2\pi)^3}\frac{d^3\tilde{q}_3}{(2\pi)^3} e^{i\bar{\tilde{x}}_2\cdot\bar{\tilde{q}}_2} e^{i\bar{\tilde{x}}_3\cdot\bar{\tilde{q}}_3} \times \langle \delta_{\tilde{q}_2}\delta_{\tilde{q}_3}\rangle_B\right]\Bigg|_{\delta_B=0}$$

$$= \langle \delta_{u_2}\delta_{u_3}\rangle_0$$

$$+ \int d^3x_2 \int d^3x_3\, e^{-i\bar{x}_2\cdot\bar{u}_2} e^{-i\bar{x}_3\cdot\bar{u}_3} \int\frac{d^3q_B}{(2\pi)^3} e^{i\bar{x}_1\cdot\bar{q}_B}$$

$$\times \delta_B(q_B)\frac{\partial}{\partial\delta_B}\left[\iint \frac{d^3q_2}{(2\pi)^3}e^{-3\delta_B}\frac{d^3q_3}{(2\pi)^3}e^{-3\delta_B} e^{i\bar{x}_2\cdot\bar{q}_2} e^{i\bar{x}_3\cdot\bar{q}_3} \times \langle \delta(q_2e^{-\delta_B})\,\delta(q_3e^{-\delta_B})\rangle\right]\Bigg|_{\delta_B=0}$$

$$= \langle \zeta_{k_2} \zeta_{k_3} \rangle_0$$

$$+ \int d^3x_2 \int d^3x_3 \int \frac{d^3q_B}{(2\pi)^3}\, e^{-i\bar{x}_2\cdot(\bar{k}_2 - \frac{1}{2}\bar{q}_B)}\, e^{-i\bar{x}_3\cdot(\bar{k}_3 - \frac{1}{2}\bar{q}_B)}$$

$$\times\, \zeta_B(q_B)\frac{\partial}{\partial \zeta_B}\left[\int\!\!\int \frac{d^3q_2}{(2\pi)^3} e^{-3\zeta_B} \frac{d^3q_3}{(2\pi)^3} e^{-3\zeta_B}\, e^{i\bar{x}_2\cdot\bar{q}_2}\, e^{i\bar{x}_3\cdot\bar{q}_3}\right.$$

$$\left.\times \left\langle \zeta(q_2 e^{-\zeta_B})\, \zeta(q_3 e^{-\zeta_B}) \right\rangle\right|_{\zeta_B=0}$$

$$= \langle \zeta_{k_2} \zeta_{k_3} \rangle_0$$

$$+ \int \frac{d^3q_B}{(2\pi)^3} \int \frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}$$

$$\times (2\pi)^3 \delta^3\left(\bar{k}_2 - \tfrac{1}{2}\bar{q}_B - \bar{q}_2\right)(2\pi)^3 \delta^3\left(\bar{k}_3 - \tfrac{1}{2}\bar{q}_B - \bar{q}_3\right)$$

$$\times\, \zeta_B(q_B)\frac{\partial}{\partial \zeta_B}\left[e^{-6\zeta_B}\left\langle \zeta(q_2 e^{-\zeta_B})\, \zeta(q_3 e^{-\zeta_B}) \right\rangle\right]\Big|_{\zeta_B=0}$$

$$= \langle \zeta_{k_2} \zeta_{k_3} \rangle_0$$

$$+ \int \frac{d^3q_B}{(2\pi)^3}\, \zeta_B(q_B)\frac{\partial}{\partial \zeta_B}\left[e^{-6\zeta_B}\left\langle \zeta(q_2 e^{-\zeta_B})\, \zeta(q_3 e^{-\zeta_B}) \right\rangle\right]\Big|_{\zeta_B=0}$$

(with $q_2 = \bar{k}_2 - \frac{1}{2}\bar{q}_B$, $\bar{q}_3 = \bar{k}_3 - \frac{1}{2}q_B$

$$= \langle \zeta_{k_2} \zeta_{k_3} \rangle_0$$

$$-(n_s - 1)\int \frac{d^3q_B}{(2\pi)^3}\, \zeta_B(q_B) \left\langle \zeta_{k_2 - \frac{1}{2}\bar{q}_B}\, \zeta_{\bar{k}_3 - \frac{1}{2}\bar{q}_B} \right\rangle$$

$$\langle \zeta(\tilde{k}_2)\zeta(\tilde{k}_3)\rangle \propto \delta(\tilde{k}_2+\tilde{k}_3)\frac{1}{\tilde{k}^3}\tilde{k}^{n_s-1}$$

$$= e^{3\zeta}\delta(k_2+k_3)\frac{1}{k^3}e^{3\zeta}k^{n_s-1}e^{(n_s-1)\zeta}$$

$$\propto e^{(6-n_s+1)\zeta}\ \delta(k_2+k_3)\frac{1}{k^3}k^{n_s-1}$$

$$\lim_{k_1\to 0}\langle \zeta_{\bar{k}_1}\zeta_{\bar{k}_2}\zeta_{\bar{k}_3}\rangle$$

$$= -(n_s-1)\int\frac{d^3q_1}{(2\pi)^3}\langle \zeta_{\bar{k}_1}\zeta_{\bar{q}_B}\rangle\langle \zeta_{\bar{k}_2}\zeta_{\bar{k}_3}\rangle$$

$$= -(n_s-1)\langle \zeta_{k_1}\zeta_{-\bar{k}_1}\rangle\langle \zeta_{\bar{k}_2}\zeta_{\bar{k}_3}\rangle$$

$$= -(n_s-1)(2\pi)^3\delta(\bar{k}_2+\bar{k}_3)P_{\zeta_{k_1}}P_{\zeta_{k_2}}$$

$$\Rightarrow \quad f_{NL}^{local} = -\frac{5}{12}(n_s-1)$$

So in single field inflation f_{NL} is pretty small of order the slow-roll parameters. Now lets compare with the curvaton.

Note that Maldacena verified this by calculating the full 3-point function $\langle \zeta_{u_1} \zeta_{u_2} \zeta_{u_3} \rangle$ using in-in formalism in QFT, and then taking the squeezed limit of the full result.

It is straightforward to generalize the Maldacena consistency relation to an n-point function with

$k_1, \dots, k_\ell \gg k_{\ell+1}, \dots, k_n$

$$\Rightarrow P^n(k_1, \dots, k_n) = -(n_\ell - 1) P^\ell(k_1, \dots, k_\ell) \times P^{n-\ell+1}(k_1 + \dots k_\ell, k_{\ell+1}, \dots, k_n)$$

(see Chen, Huang, Shiu, 2008)

or double squeezed limit with two external legs being long wave-length (soft modes)

Curvaton

In the case of the curvaton we found

$$\zeta_\sigma = -\frac{H}{\dot{\rho}_\sigma}\delta\rho_\sigma$$

During matter dom. $\dot{\rho}_\sigma = 3H(\rho_\sigma + P_\sigma) \simeq 3H\rho_\sigma$

$$\Rightarrow \zeta_\sigma \sim \frac{\delta\rho_\sigma}{3\rho_\sigma}$$

Earlier we showed to linear order

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = \frac{m^2\sigma\delta\sigma}{\frac{1}{2}m^2\sigma^2} = 2\frac{\delta\sigma}{\sigma}$$

where $\delta\sigma$ is Gaussian. However we have going to second order we have

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = \frac{m^2\sigma\delta\sigma + \frac{1}{2}\delta\sigma\delta\sigma}{\frac{1}{2}m^2\sigma^2} = 2\left(\frac{\delta\sigma}{\sigma} + \frac{1}{2}\frac{\delta\sigma^2}{\sigma^2}\right)$$

$$\Rightarrow \zeta_\sigma \sim \frac{2}{3}\frac{\delta\sigma}{\sigma} + \frac{1}{3}\frac{\delta\sigma^2}{\sigma^2} \triangleq \zeta_\sigma + \frac{3}{4}\zeta_\sigma^2$$

Lets say the curvaton contributes a fraction $r = \frac{\rho_\sigma}{\rho}$ when it decays

then $\zeta = r\zeta_\sigma \Rightarrow \zeta = \zeta_g + \frac{3}{4}\frac{1}{r}\zeta_g^2$

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}^{local} \left(\zeta_g^2 - \langle \zeta_g^2 \rangle\right)$$

$$\Rightarrow \quad \frac{3}{5} f_{NL}^{local} = \frac{3}{4}\frac{1}{r} \Rightarrow \boxed{f_{NL}^{local} = \frac{5}{4}\frac{1}{r}}$$

Now this result actually only holds for $r \ll 1$ but for $r = 1$ one finds

$$f_{NL}^{local} = -\frac{5}{4}$$

$$\Rightarrow \quad \boxed{|f_{NL}^{local}| = \frac{5}{4}}$$

which is order one and much larger that what we found in single field slow-roll. It can potentially be measured within next 10 years.

So coming back to our plot

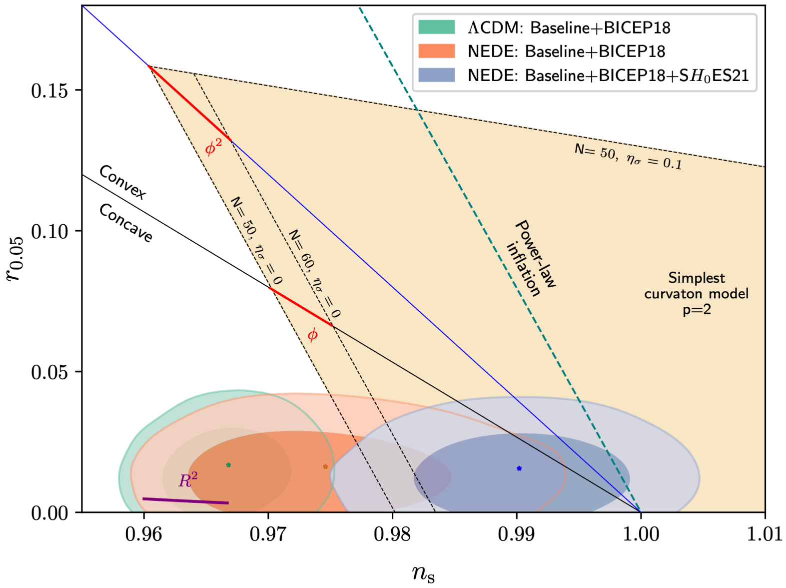


Figure 6. Results for the 68% and 95% C.L. contours relating n_s and r at a pivot scale of $k_* = 0.05$, for the ΛCDM and NEDE models alternating the baseline datasets with SH_0ES while including BICEP18. The small asterisks represent the mean posterior value of the corresponding contours.

- In plot on the previous slide, we used the relations for the simplest curvaton model

$$V(\phi,\sigma) = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2$$

which implies

$$n_s = 1 - \frac{1}{1+R}\frac{8}{4N+2} + \frac{R}{1+R}\left[-2\epsilon + 2\eta_\sigma\right] \qquad f_{\rm NL} = \left(\frac{R}{1+R}\right)^2\left[\frac{5}{3} - \frac{5}{4r_{\rm dec}} + \frac{5}{6}r_{\rm dec}\right]$$

$$r = \frac{16\epsilon}{1+R}$$

$$R = \frac{\mathcal{P}_{\zeta_{\rm curvaton}}}{\mathcal{P}_{\zeta_{\rm inflaton}}}$$

Inflation dominates

$R \to 0$

$r \to 16\epsilon$

$f_{\rm NL} \to 0$

Curvaton dominates

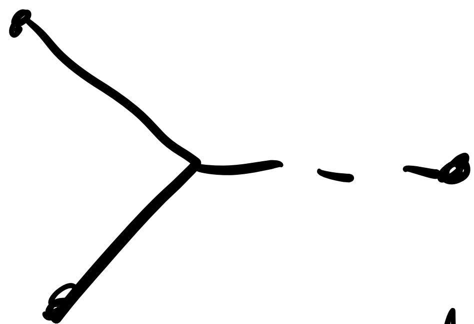
$R \to \infty$

$r \to 0$

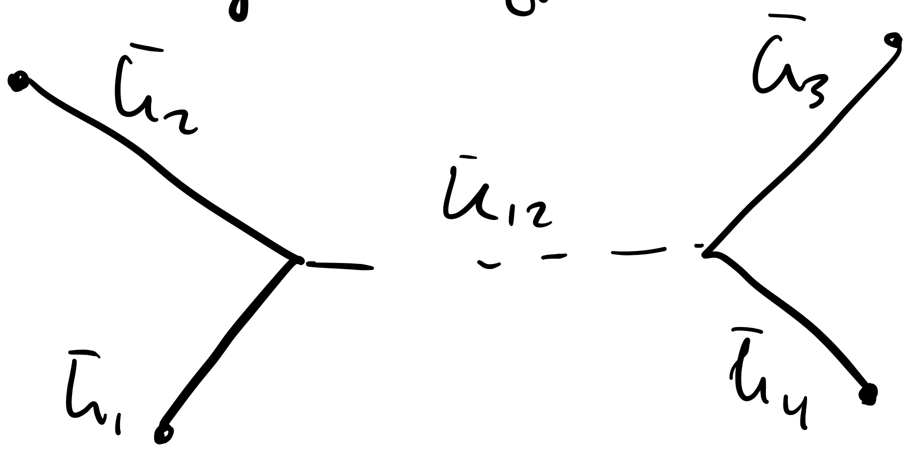
$|f_{\rm NL}| \to 5/4 \qquad (r_{\rm dec} = 1)$

Exchange consistency relation

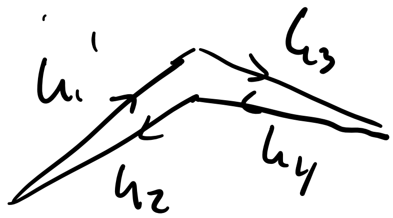
One can think of the Maldacena consistency relation for the 3-point function to be pictorically of the form



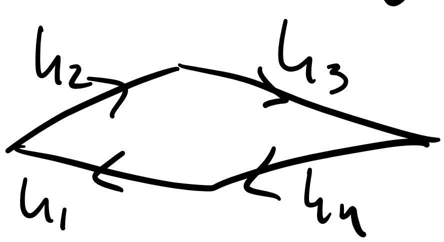
where the dashed line is a long mode and the solid lines are short wavelength modes. Now it was first understood by Seery, Sloth and Vernizzi (SSV) that the four-point function in the counter-colinear, also called collapsed shape satisfy a new consistensy-relation for the exchange diagram contribution



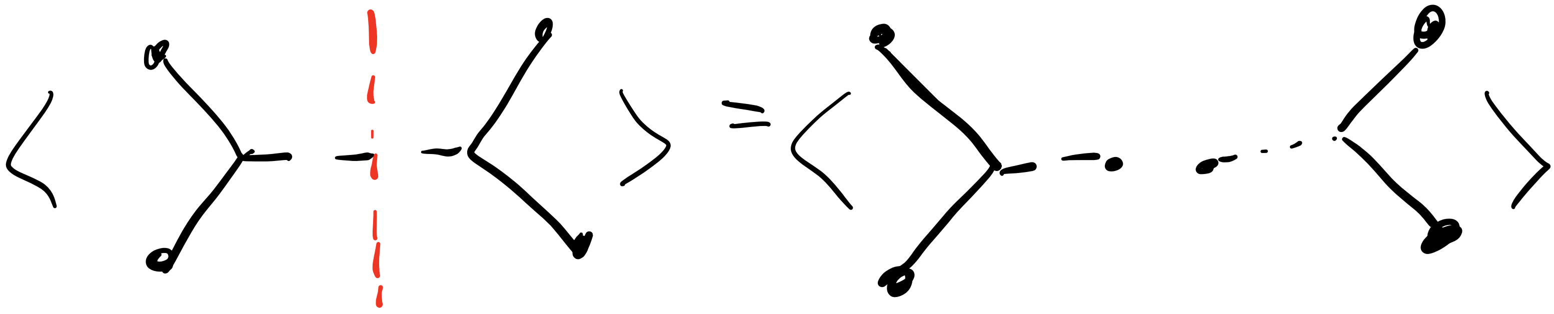
with $\bar{k}_{12} = \bar{k}_1 + \bar{k}_2$ for momentum conservation reasons and we assume $k_{12} \ll k_1 \approx k_2, k_3 \approx k_4$ like a folded kite



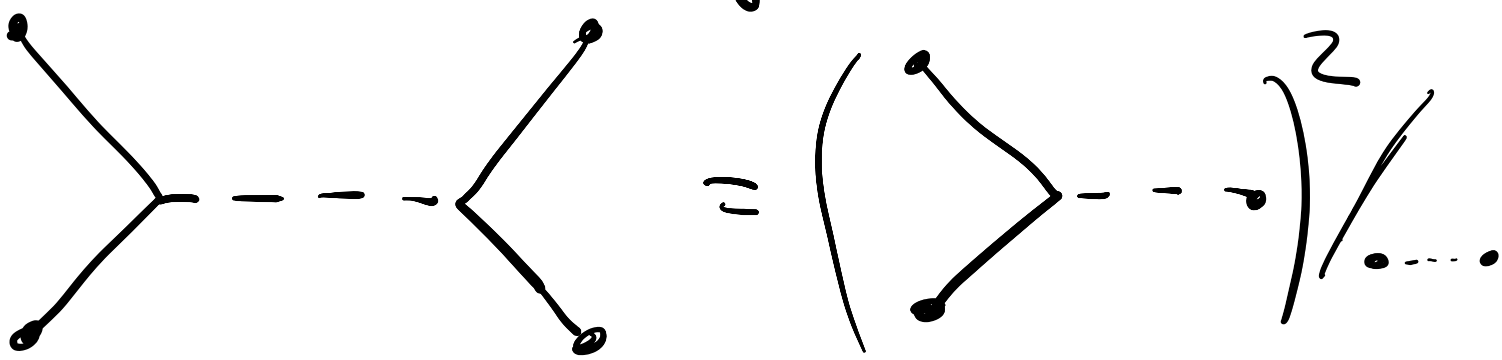
or a parallelogram



The SSV consistency relation for the 4-point exchange diagram in the counter-collinear limit tells us that the diagram can be cut



so that it satisfy a relation



in the following sense:

Remember we had in the presence of a long/soft mode ζ_B

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} = \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \zeta_B \frac{\partial}{\partial \zeta_B} \langle \zeta_{k_1} \zeta_{k_2} \rangle \Big|_0 + \dots$$

$$= \langle \zeta_{k_1} \zeta_{k_2} \rangle_0$$

$$- (n_s - 1) \int \frac{d^3 q_B}{(2\pi)^3} \zeta_B(q_B) \langle \zeta_{k_1 - \frac{1}{2} q_B} \zeta_{k_2 - \frac{1}{2} q_B} \rangle$$

So now we can obtain the semiclassical contribution to the four-point function from the correlation of a pair of two point functions along the long mode

$$\lim_{k_{12}\to 0} \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = \left\langle \langle \zeta_{k_1}\zeta_{k_2}\rangle_B \langle \zeta_{k_1}\zeta_{k_2}\rangle_B \right\rangle$$

$$\Downarrow \quad = (n_s-1)^2 \langle \zeta_{k_1}\zeta_{k_2}\rangle \langle \zeta_{k_{12}}\zeta_{k_{12}}\rangle \langle \zeta_{k_3}\zeta_{k_4}\rangle$$

This is the SSV consistency relation which can also be written in terms of the trispectrum

$$\lim_{k_{12}\to 0} T(k_1,k_2,k_3,k_4) = (n_s-1)^2 P_\zeta(k_1) P_\zeta(k_3) P_\zeta(k_{12})$$

$$= 4\tau_{NL}^{local} P_\zeta(k_1) P_\zeta(k_3) P_\zeta(k_{12})$$

where $\tau_{NL}^{local} = \left(\frac{6}{5} f_{NL}^{local}\right)^2$

with the trispectrum, T, defined as

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = (2\pi)^3 \delta\left(\sum_a \bar{k}_a\right) T_\zeta(k_1,k_2,k_3,k_4)$$

Of course the SSV consistency relation can be generalized to include exchange of graviton [SSV + GS] and in other ways.

Note that the full trispectrum was calculated in [SSV, SSZ] and the SSV consistency relation was verified by full in-in calculation.

Semi-classical consistency relations loops and IR effects

At higher order in perturbation theory we can also have loop corrections to f.ex. the two-point correlation function $\langle \zeta_{k_1} \zeta_{k_2} \rangle$. This has been a hot topic of discussions because of apparent IR divergencies that has to be dealt with correctly, but may also teach us about the global nature of inflationary spacetimes.

It was first shown by Giddings & Sloth (GS) that one can similarly use semi-classical relations (soft theorems) to extract the IR contribution from the loops.

The basic insight of GS is that if one goes to one order higher in the background expansion around the Long mode

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} = \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \zeta_B \frac{\partial}{\partial \zeta_B} \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} \Big|_0$$

$$+ \frac{1}{2} \zeta_B^2 \frac{\partial^2}{\partial \zeta_B^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} \Big|_0 + \cdots$$

and then take and average over the long modes

$$\langle \langle \zeta_{k_1} \zeta_{k_2} \rangle \rangle_B = \frac{1}{2} \langle \zeta_B^2 \rangle \frac{\partial^2}{\partial \zeta_B^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} \Big|_0 + \cdots$$

using from earlier

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} = e^{-6\zeta_B} \langle \zeta(e^{-\zeta_B} k_1) \, \zeta(e^{-\zeta_B} k_2) \rangle$$

we obtain the GS consistency relation for IR 1-loop contributions

$$\langle \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\zeta_B} \rangle = \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \left(\tfrac{1}{2}(n_s-1)^2 + \alpha_s \right) \langle \zeta_{k_1} \zeta_{k_2} \rangle \times \langle \zeta_B^2(x) \rangle_*$$

where

$$\langle \zeta_B^2(x) \rangle_* \simeq \int_{a_i H}^{a_* H} \frac{dq}{q} \frac{1}{2\epsilon} \frac{H^2}{(2\pi)^2}$$

and $\alpha_s = dn_s/d\ln(k)$ is the running of the spectrum.

In principle the integral is IR divergent, but the IR cutoff should be the largest relevant scale, and UV cutoff $(a_* H)$ is when the short modes cross horizon $a_* H \simeq k_1 \sim k_2$.

This can be generalized to higher order by going to higher order in the taylor expansion

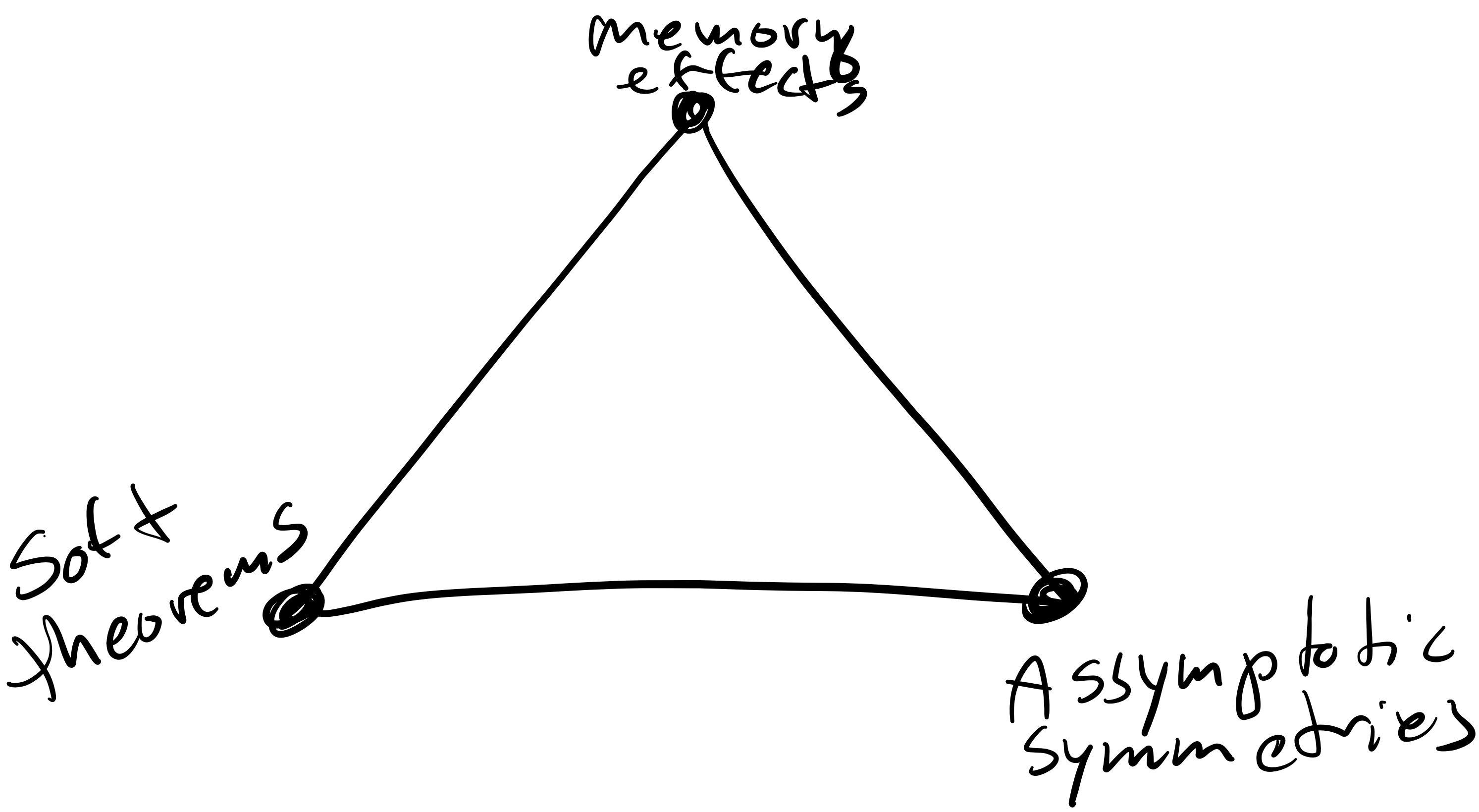
The infrared triangle

The infrared triangle was put forward in the context of Black Hole physics by Strominger as an outcome of attempts to understand the black hole information paradox in his work with Perry and Hawking. However,

in work of Ferreira, Sandora & Sloth it was argued that a similar relation exist for inflationary space-times.

The infrared triangle relates the semiclassical consistency relation, also sometimes called soft theorems, to gravitational memory effects and assymptotic symmetries

Infrared triangle of dS



[see Ferreira, Sandora, Sloth 2016 & 2017 and Anninos, Ng & Strominger 2010]

Assymptotic symmetries

Since graviton freezes and becomes constant on superhorizon scales, it can locally be viewed as just a rescaling of the local coordinate equalling a large gauge transformation.

However this is not true when comparing different patches along the variation of the long mode. The presence of the long mode at superhorizon (≡ assymptotic infinity) spontaneously break the assymptotic symmetry of spatial diffeomorphisms, and the long (soft) mode can be viewed as the Goldstone mode of the spontaneoulsy broken assymptotic symmetry.

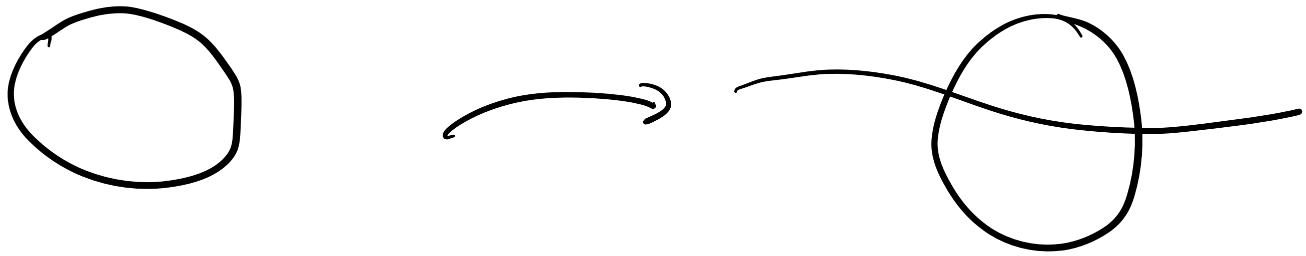
Focussing on tensor mode (a similar story holds for ζ) in the transverse and traceless gauge

$$ds^2 = -dt^2 + a^2 [e^{\gamma}]_{ij} dx^i dx^j$$

where we choose the exponentiated parametrization, with

$$[e^{\gamma}]^i{}_i = \partial^j [e^{\gamma}]_{ij} = 0 .$$

Since modes freezes and becomes constant on superhorizon, the addition of a super-horizon soft mode is a large gauge transformation (gauge transformation not falling off at infinity) corresponding to a spatial diff. on $\mathbb{R}^3$ (the assymptotic symm. of dS).



$$x^i \longrightarrow [e^{\gamma_L/2}]^{ij} x_j$$

$$|0\rangle \longrightarrow |0'\rangle = e^{iQ}|0\rangle$$

From the definition of the Nöther charge related to a variation, $\delta\gamma_{ij}$, of the canonical variable γ_{ij}, if the variation is a symmetry transformation, we have (Π = canonical conjugate momentum)

$$Q = \frac{1}{2}\int d^3x\, \Pi^{ij}\, \delta\gamma_{ij} + \text{h.c.}$$

Demanding that the field variation corresponds to a large gauge transformation, in the form of a spatial diffeomorphism

$$\mathcal{L}_\xi [e^\gamma]_{ij} = \delta [e^\gamma]_{ij}$$

gives to first order

$$\delta \gamma_{ij} = \xi_a \partial^a \gamma_{ij}$$

If the large gauge transformation equals adding a long wavelength soft graviton mode then we must have

$$\xi_i = -\frac{1}{2} \gamma_{L\,ij} x^j$$

and we obtain ($\Pi_{ij} = \frac{1}{4} a^3 \dot{\gamma}_{ij}$)

$$Q = -\frac{a^3}{16} \int d^3x \, \dot{\gamma}_{ij} \gamma^L_{ab} x^b \partial^a \gamma_{ij} + h.c.$$

($M_P \equiv 1$)

If the assymptotic symmetry is spontaneously broken then the charge associated with it will act

non-trivially on the vacuum

$$e^{iQ}|0\rangle = |0'\rangle \neq |0\rangle$$

To se that this really changes the vacuum $|0\rangle$ into $|0'\rangle$ which now include a soft mode γ_L consider the 3-point graviton amplitude in the squeezed limit

$$\langle \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3} \rangle$$

where $q_1 \ll q_2, q_3$

This correlation function is zero at tree-level, so

$$\langle 0 | \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3} | 0 \rangle = 0$$

at tree level.

Now let's consider the same correlation function when a soft mode is added

$$\langle 0' | \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3} | 0' \rangle$$

$$= \langle 0 | e^{-iQ} \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3} e^{iQ} | 0 \rangle$$

$$= \langle 0 | \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3} | 0 \rangle - i \langle 0 | [Q, \gamma_{q_1}^{s_1} \gamma_{q_2}^{s_2} \gamma_{q_3}^{s_3}] | 0 \rangle + \mathcal{O}(Q^2)$$

$\vdots$

$$= \frac{3-n_t}{2} \frac{\epsilon_{ij} k_i k_j}{k^2} \langle \gamma_{k_1} \gamma_{-k_1} \rangle \langle \gamma_{k_2} \gamma_{k_3} \rangle$$

In agreement with Maldacena 2002 computed with Maldacena consistency relation (soft theorem).

$\Rightarrow$ Soft theorems $\longleftrightarrow$ assymptotic symmetries!

Exercise 4: Connect the dots above (top of page).

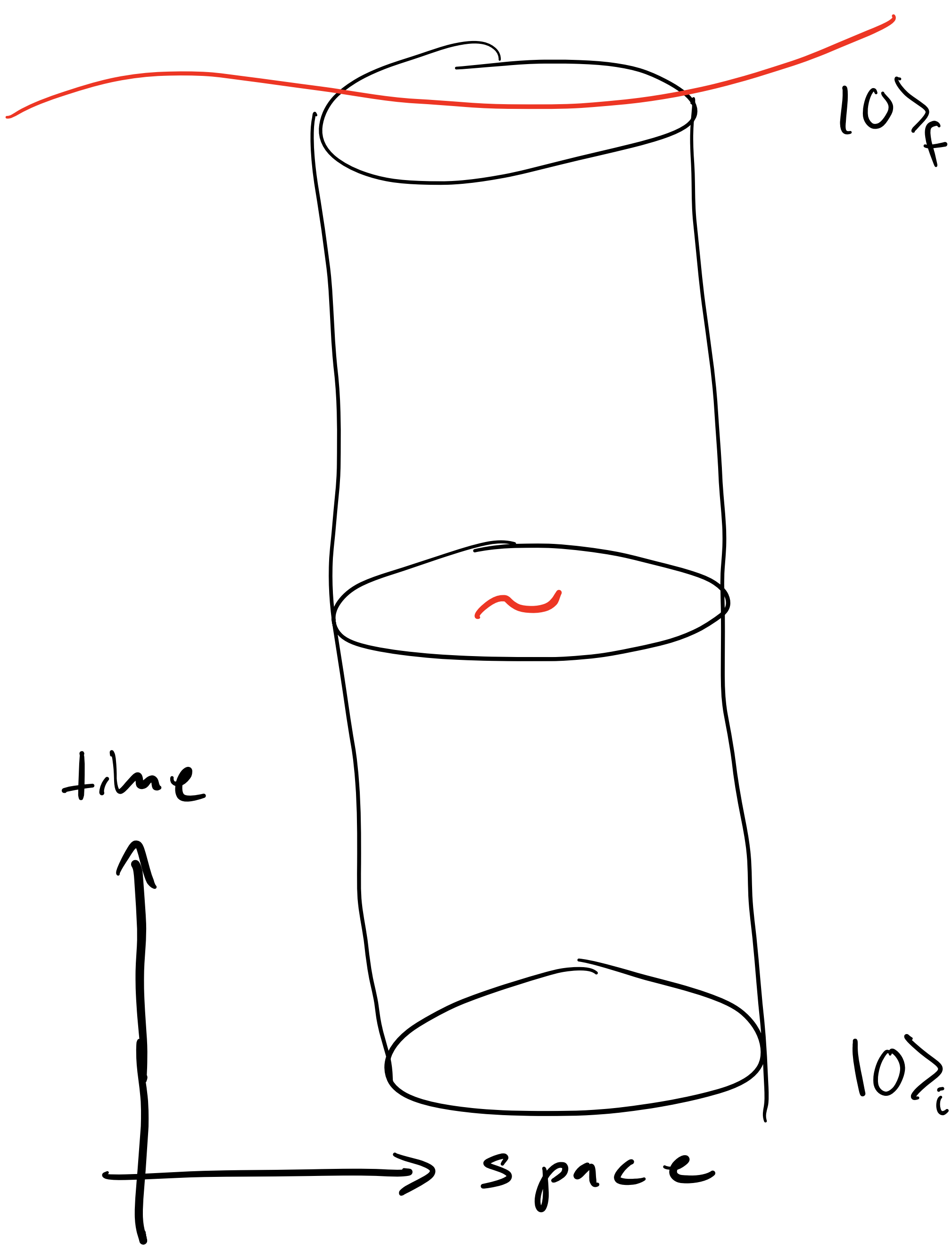
[hint: have a look at arXiv: 1609.06318 sect. 4.1]

Gravitational memory

To complete the infrared triang of de Sitter, let's briefly consider gravitational memory.

we will introduce the concept of a patient observer as one that carries gravitational memory.

One example of a patitient observer, connecting to the other two corners of the triangle, is an obser which records the initial state before the soft mode is created, and is around long enoug to compare with the final state after the soft mode has left the horizon.



Such an observer will see

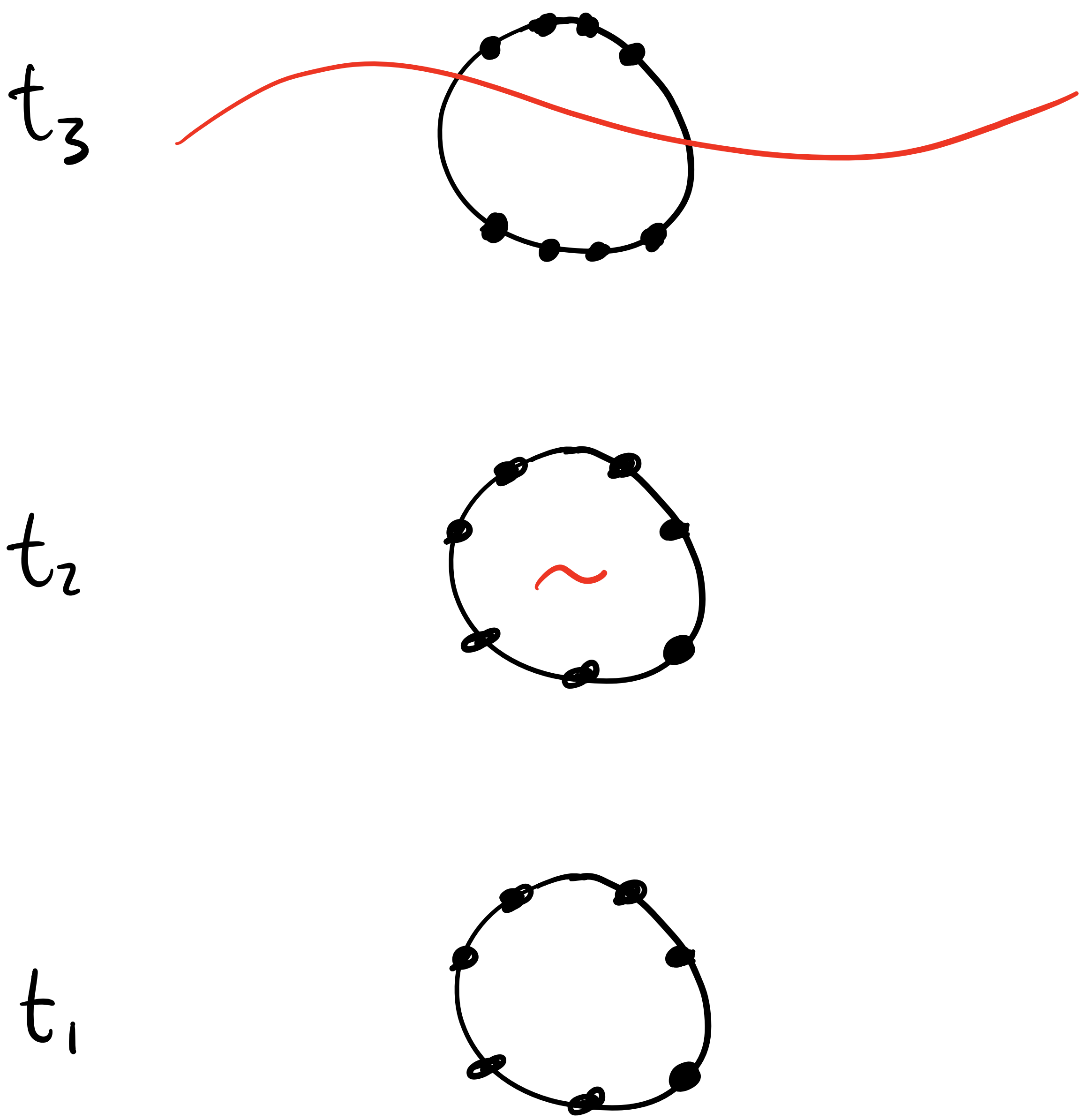
$${}_i\langle 0 | 0 \rangle_f \neq 1$$

since

$$|0\rangle_f = e^{iQ} |0\rangle_i .$$

A patient observer could be a circular array of satellites very carefully bound together in the radial direction, but not preventing them from feeling shear effects when a long mode is added, the spatial distance between the satellites changes by

$$ds^2 = a^2\delta_{ij}dx^i dx^j \to ds'^2 = a^2(e^{\gamma_L})_{ij}dx^i dx^j$$



This thought experiment connects gravitational memory effects to soft theorems and assymtotic symmetries.

[For details and Challenges see Ferreira, Sandora, Sloth, 2016 & 2017]