NORDITA WINTER SCHOOL, 2024
IN PARTICLE PHYSICS AND COSMOLOGY

AXION COSMOLOGY & ASTROPHYSICS

David Marsh, Stockholm Univeristy

Outline

Lect. 1 & 2: Particle physics preliminaries
Axion cosmology

Lect. 3: Axion cosmology (continued)
Axion astrophysics

Particle physics preliminaries

$$\mathcal{L}_\Theta = \Theta \frac{g_s^2}{32\pi^2} \varepsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta} = \Theta \frac{\alpha_s}{8\pi} \mathrm{tr}(G\tilde{G})$$

Properties:

1) No reason to exclude in renormalisable $\mathcal{L}$.
2) Breaks T and P for generic Θ.
3) Total derivative: $G\tilde{G} = \partial_\mu K^\mu$ — Chern-Simons current
4) Contributions integer quantised: $\mathcal{L}_\Theta = n\Theta$ — Pontryagin index
5) Θ is angular $\in [0, 2\pi[$
6) Θ is experimentally constrained to tiny values...

The chiral anomaly:

$$\mathcal{L}_{QCD} \supset i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - \left(\bar{q}_L M q_R + h.c.\right)$$

The axial rotation $q_L \to e^{-i\alpha} q_L$, $q_R \to e^{+i\alpha} q_R$ is a classical symmetry of $\mathcal{L}$ if $M = 0$ $\left(\bar{q}_L M q_R \to e^{2i\alpha} \bar{q}_L M q_R\right)$.

However, it's anomalous.

$$\int Dq\, D\bar{q} \xrightarrow[\text{by } \alpha]{\text{chiral rotation}} \int Dq\, D\bar{q}\; e^{i\alpha \frac{g_s^2}{32\pi^2} G\tilde{G}}$$

Chiral rotations shuffle Θ between $G\tilde{G}$ and phase of M

Invariant measure of CP-violation:

$$\bar{\Theta} = \Theta_{QCD} - \arg\left(\det\left(Y_u Y_d\right)\right)$$

May expect $\bar{\Theta} \sim \mathcal{O}(1)$, but $\bar{\Theta}$ has physical consequences

1) $\bar{\Theta}$ carries an energy density

$$\mathcal{L}_{\text{chiral}} \supset \Lambda^3_{QCD}\, \mathrm{tr}\left(e^{i\bar{\Theta}} M_q U^\dagger + h.c.\right) \sim \Lambda^3_{QCD}\, m_q \cos\bar{\Theta}$$

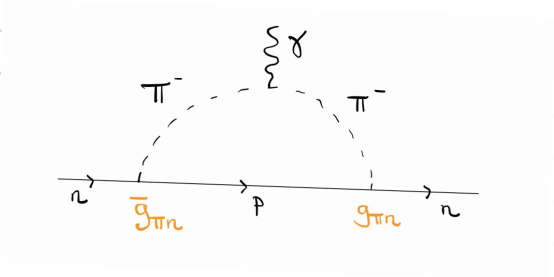
$U = e^{-2i\pi^a\tau^a/f_a}$

Some shift of vac. energy.

2) The "Strong CP Problem"

$$\mathcal{L}_{\pi NN} = \tilde{\pi}^a \bar{\Psi}\left(\underbrace{i\gamma_5 g_{\pi NN}}_{\substack{\text{SM coupling} \\ (\approx 13.4)}} + \underbrace{\bar{g}_{\pi NN}}_{\substack{\text{New coupling} \\ (\approx 0.04\bar{\Theta})}}\right)\tau^a \Psi$$

$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$



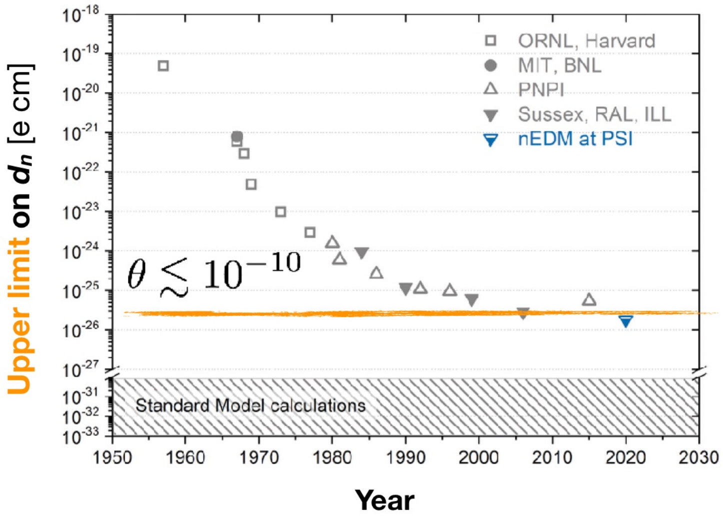
One-loop neutron edm:

$$d_N = \frac{m_n}{4\pi^2}\, g_{\pi nn}\, \bar{g}_{\pi nn} \ln\frac{\Lambda}{m_\pi} \underset{\Lambda = m_n}{=} \left(5.2\times 10^{-16}\ e\cdot cm\right)\bar{\Theta}$$

So, $|d_n|$ naturally $\mathcal{O}(10^{-16}\, e\cdot\text{cm})$.

Experiments: $|d_N| \lesssim 3\times 10^{-26}\, e\cdot\text{cm}$

so that $\bar{\Theta} \lesssim 10^{-10}$.



Perturbatively stable (to at least 7 loops).

No anthropic reason to be small.

Axion resolution: Promote $\bar{\Theta}$ to dynamical field that relaxes dynamically to $\bar{\Theta} = 0$.

$$\mathcal{L}_{new} \supset \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

↑ absorbs $\bar{\Theta}$

The simplest renormalisable theory of the axion (KSVZ)

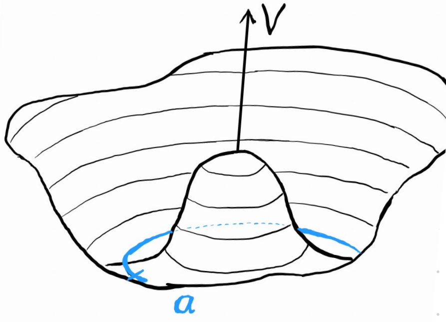
At high energies, introduce Q_L, Q_R in fundamental of $SU(3)$, singlet under $SU(2)$ and $U(1)_Y$, and complex scalar ϕ, a SM singlet.

Postulate a $U(1)_{PQ}$ symmetry:

$$Q_L \to e^{i\alpha/2} Q_L \,, \qquad Q_R \to e^{-i\alpha/2} Q_R \,, \qquad \phi \to e^{i\alpha}\phi \,.$$

Lagrangian:

$$\mathcal{L} \supset -\lambda_\phi \left(|\phi|^2 - \frac{f_a}{2} \right) - \left(y_Q \bar{Q}_L Q_R \phi + h.c. \right)$$



SSB: $\phi = \frac{1}{\sqrt{2}} \left(f_a + \sigma(x) \right) \exp\left(i\, a(x)/f_a \right)$

The axion, $a(x)$, is a massless Nambu-Goldstone boson.

f_a: "axion decay constant"

Integrating out s: $\mathcal{L} \supset \frac{1}{2}\partial_\mu a \partial^\mu a - \left(m_Q \bar{Q}_L Q_R e^{i a/f_a} + h.c.\right)$

Make quark mass independent of a by chiral rotation:

$$Q_L \to e^{i/2\, a/f_a} Q_L \qquad Q_R \to e^{-i/2\, a/f_a} Q_R$$

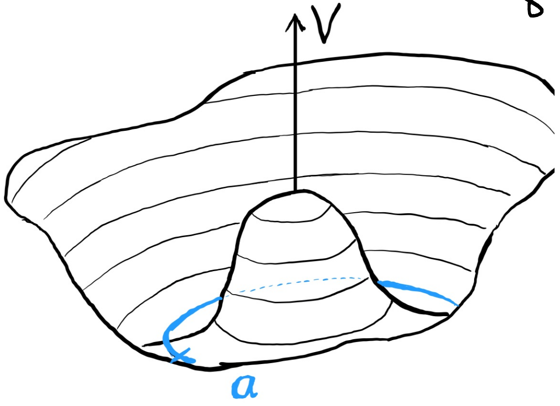
and integrate out Q_L, Q_R, but this transformation is anomalous and, since Q's are charged under $SU(3)$, gives

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G\tilde{G}$$

This promotes $\bar{\Theta}$ to a dynamical field.

Below $\Lambda_{QCD} \approx 200$ MeV, the $G\tilde{G}$ term gives $V(a)$.

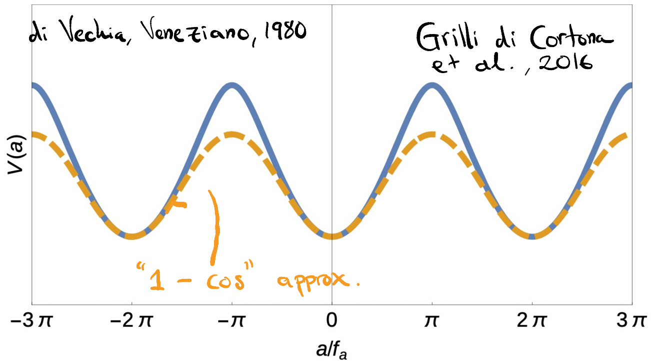
The effective potential can be calculated in chiral pert. theory,



$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2} \sin^2\left(\frac{a}{f_a}\right)}$$

$$m_a^2 = \frac{m_\pi^2 f_\pi^2}{f_a^2} \frac{m_u m_d}{2(m_u+m_d)^2} = 5.6\,\mu eV \left(\frac{10^{12}\,GeV}{f_a}\right)^2 = \frac{\chi_{QCD}(T=0)}{f_a^2}$$

The axion mixes with π^0, gets "universal" coupling to photons:



$$C_{a\gamma} \frac{a}{f_a} \frac{\alpha}{8\pi} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$C_{a\gamma} = \underbrace{C_{a\gamma}^{UV}}_{\substack{\text{KSVZ} \\ \text{"DFSZ"}}} \underbrace{- 1.97}_{\substack{\text{From IR} \\ \pi \text{ mixing}}}$$

$$= \begin{cases} 0 \\ 8/3 \end{cases}$$

More general models can have couplings to SM matter: $\sim \frac{1}{f_a} \partial_\mu a \bar{\Psi} \gamma^\mu \gamma^5 \Psi$

Axion-like particles

Pseudo-Nambu Goldstone bosons independent from the QCD axion.

Arise, e.g., from extra-dimensional theories with p-form gauge potentials: $a = \int_{\Sigma_p} A_p$

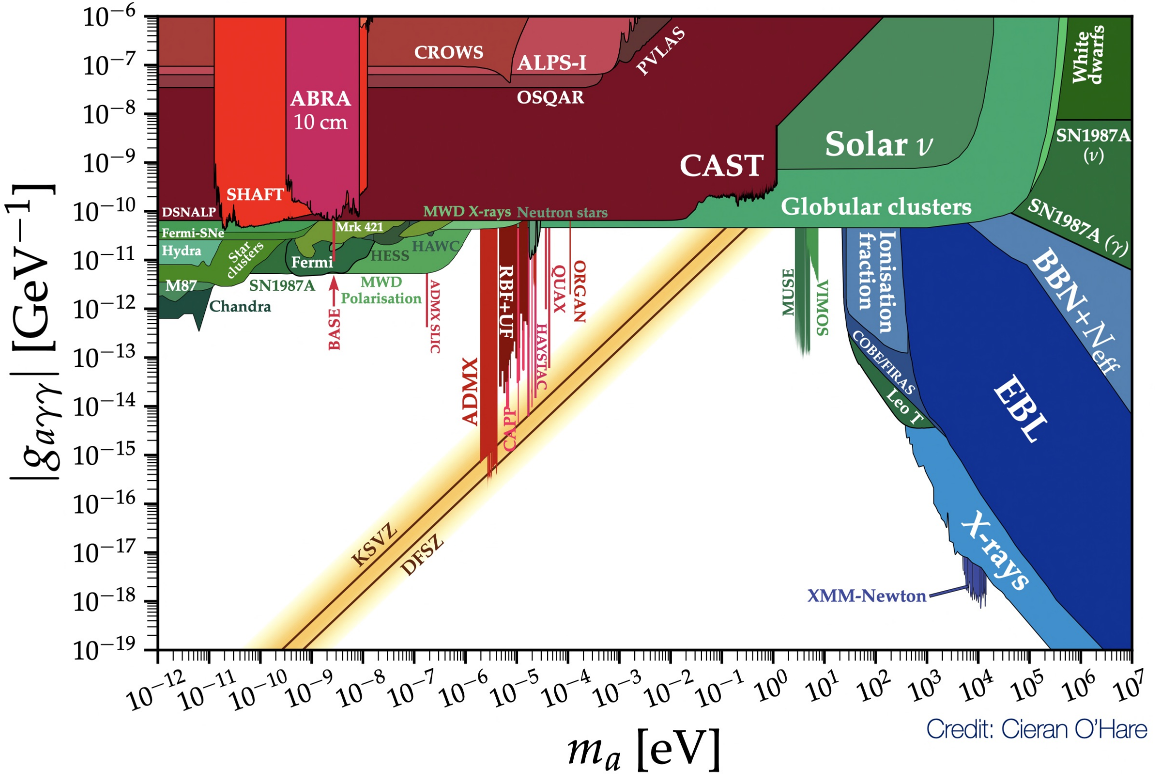
Gauge symmetry $A_p \to A_p + d\omega_{p-1}$ ensures shift symmetry of a.

Relevant EFT:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu a \partial^\mu a - V(a) + \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{af}\partial_\mu a \bar{\Psi}\gamma^\mu\gamma^5\Psi$$

$$V(a) = \Lambda^4\left(1 - \cos\left(a/f_a\right)\right) \quad \text{(typically)}$$

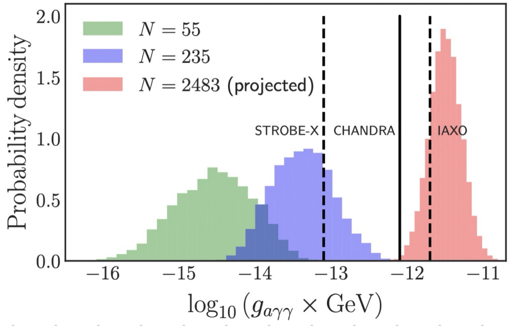
Axion and ALP parameter space (schematically)



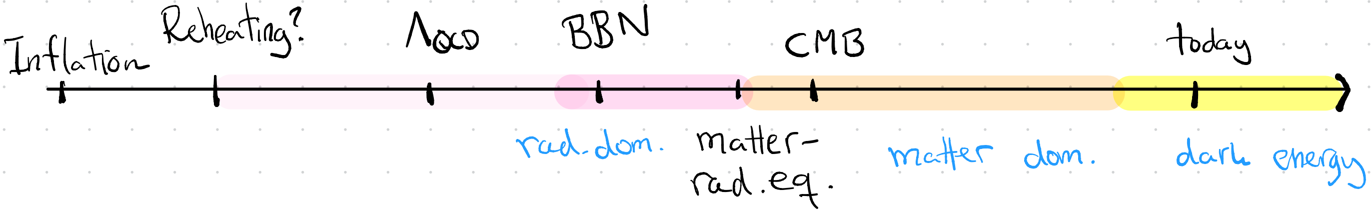
Credit: Cieran O'Hare

String theory expectation?

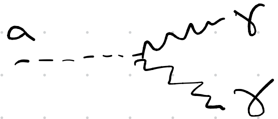
Halverson, Long, Nelson, Salinas



Axion Cosmology



Axions are long-lived. Typically, the dominant decay channel is $a \to \gamma\gamma$:

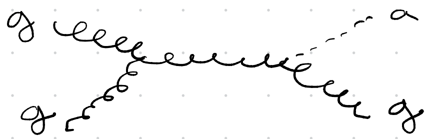


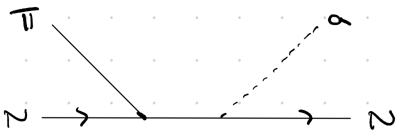
$$\tau_a = \frac{64\pi}{g_{a\gamma}^2 m_a^3} \approx 1.3 \times 10^{25} \sec \left(\frac{10^{-10}\,\mathrm{GeV}^{-1}}{g_{a\gamma}}\right)^2 \left(\frac{\mathrm{eV}}{m_a}\right)^3 \qquad \text{cf.} \quad \tau_{\text{univ.}} \sim 10^{17} \sec.$$

Cosmologically long-lived, if produced.

Thermal production [dim. analysis]

'hot Big Bang' plasma

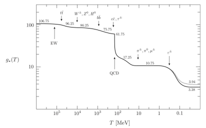




Leading order: $\mathcal{M} \sim \frac{1}{f_a}$ and $\sigma \sim \frac{1}{f_a^2}$

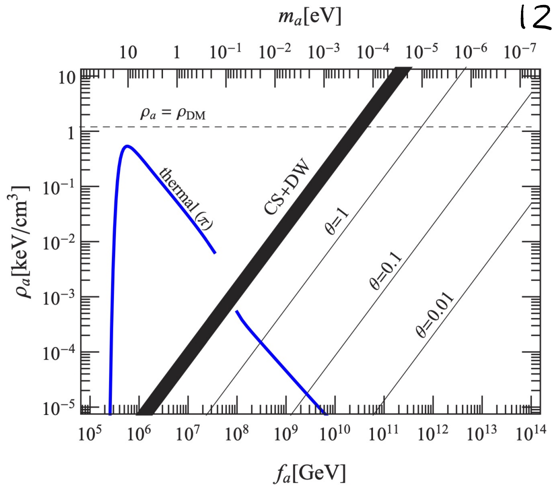
$$\Gamma = \langle n_{plasma} \sigma v \rangle \sim T^3 / f_a^2$$

In thermal equilibrium when $\Gamma \gg H \sim \sqrt{g_*} \frac{T^2}{M_{Pl}}$



Decoupling: $T_d^3 / f_a^2 \approx \sqrt{g_*}\, T_d^2 / M_{Pl}$

$$T_d \approx \sqrt{g_*} \frac{f_a^2}{M_{Pl}} \approx 10^6 \text{ GeV} \left(\frac{f_a}{10^{12}\text{ GeV}}\right)^2$$



Axions are relativistic at decoupling, become non-rel. when $H < m_a$

$$n_a(t_0) = n_a(t_d)\left(\frac{R(t_d)}{R(t_0)}\right)^3$$

scale factor ratio

Thermal axion dark matter is <u>hot</u>, behaves much like ν's. Also $\rho_{a,\,thermal} < \rho_{dm}$.

Dimensional analysis works for typical momenta $\sim T$, but not for low-energy modes as $M \to p_a$ as $p_a \to 0$ for Goldstone bosons (Adler's zero). Axion zero modes created by different mechanism.

Axion misalignment

Simplest case, assume PQ symmetry was broken before inflation. Initial θ takes a single, random value across the observable universe.

$$\ddot{a} + 3H\dot{a} - \frac{1}{R^2}\nabla^2 a + \frac{\partial V}{\partial a} = 0$$

negligible

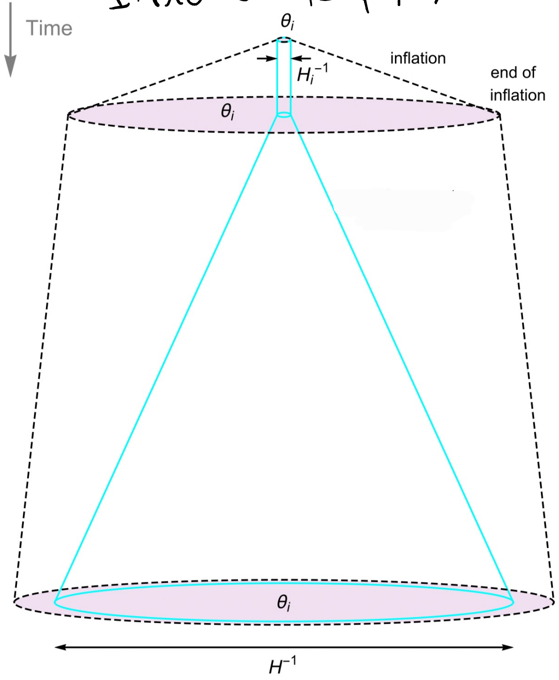
The axion effective potential depends on T:

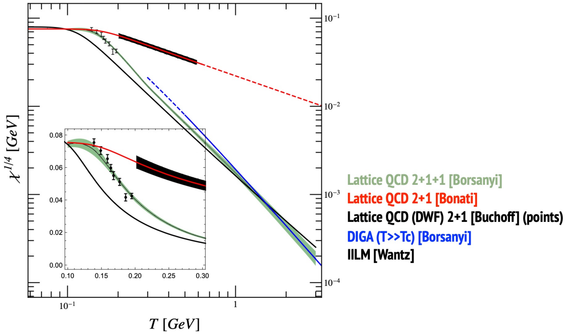
$$m_a^2 = \chi(T)/f_a^2$$

Parametrise: $m_a^2 \sim \frac{1}{T^n} \sim R^n$

For ALPs, typical to assume m_a indep. of T.

IAXO white paper, 2013

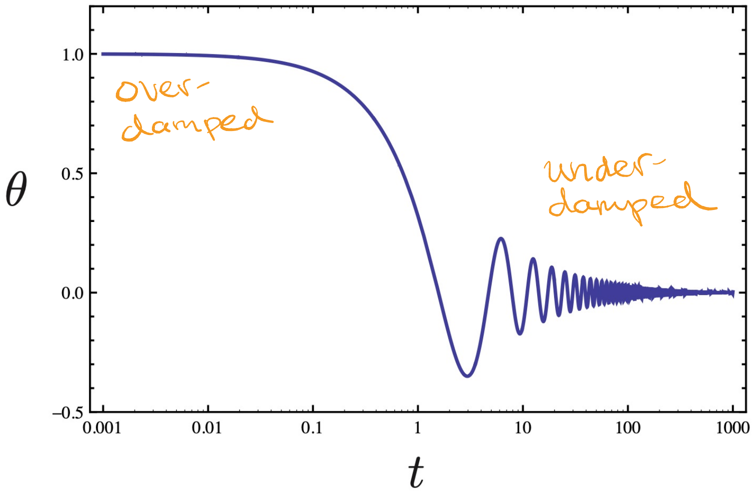




Sufficiently early, $H \gg m_a$,

$$\ddot{a} + 3H\dot{a} = 0 \qquad \text{(over damped)}$$

$$a = \begin{cases} a_i & \text{(constant)} \\ \dot{a} \sim \frac{1}{R^3} & \text{(transient, ignore)} \end{cases}$$



Later:

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)\, a = 0$$

WKB Ansatz: $a(t) = A(t)\, e^{-i\int^t dt' m_a(t')}$

$$\dot{A} + A\left(\frac{3}{2}\frac{\dot{R}}{R} + \frac{\dot{m}_a}{2m_a}\right) = 0 \qquad \text{solved by} \qquad A(t) = \frac{A_0}{R^{3/2}\, m_a^{1/2}}$$

Energy density: $\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2}m_a^2 a^2 = m_a^2\, a^2(t)$ $\qquad \langle \rho_a \rangle = m_a^2 \frac{A_0^2}{2R^3 m_a}$

Non-rel. "dust": $\langle n_a \rangle = \frac{\rho_a}{m_a} \sim R^{-3}$ $\qquad \langle p_a \rangle = \langle \frac{1}{2}\dot{a}^2 - \frac{1}{2}m_a^2 a^2 \rangle = 0.$

Energy density depends on A_0. Rough estimate by match at $3H(t_{osc}) = m_a(t_{osc})$: $A_0^2 = R^3(t_{osc})\, m_a(t_{osc})\, \theta_i^2$ where $\theta_i = \frac{a_i}{f_a}$

$\theta_f < 10^{-10} \Rightarrow \frac{a_f}{f_a} < 10^{-10}$

Energy density today:

$$\rho_a(t_0) = m_a(t_0)\, \frac{1}{2} f_a^2\, m_a(t_{osc})\, \theta_i^2 \left(\frac{R(t_{osc})}{R(t_0)}\right)^3 = m_a(t_0) \frac{\langle \rho_a(t_{osc})\rangle}{m_a(t_{osc})} \left(\frac{R(t_{osc})}{R(t_0)}\right)^3$$

$$= m_a(t_0)\; n_a(t_{osc}) \left(\frac{R(t_{osc})}{R(t_0)}\right)^3$$

Is it enough to account for dark matter?

$\frac{\rho_{dm}(t_0)}{\rho_{crit}} = \Omega_{dm} h^2 = 0.12$.

To evaluate explicitly:

1) $s(T_{osc})\, R^3(T_{osc}) = s(T_0)\, R^3(T_0)$ entropy conservation (in radiation)

2) $m_a(t_{osc}) \approx 3H(t_{osc})$

$$m_a(T) = \frac{\Lambda^2}{f_a}\left(\frac{\Lambda}{T}\right)^{n/2} \qquad \text{for } T > \Lambda = \chi_{QCD}^{1/4}(T \lesssim 100\,\text{MeV}) \simeq 75\ \text{MeV} \quad \text{(rough approx.)}$$

$$H(T) = K\sqrt{g_*(T)}\,\frac{T^2}{M_{Pl}} \qquad T_{osc} = \Lambda\cdot\left(\frac{M_{Pl}}{f_a}\,\frac{1}{3K\sqrt{g_*(t_{osc})}}\right)^{\frac{1}{2+n/2}}$$

$$K = \frac{\pi}{3\sqrt{10}}$$

3) Evaluate $n_a(t_0)$

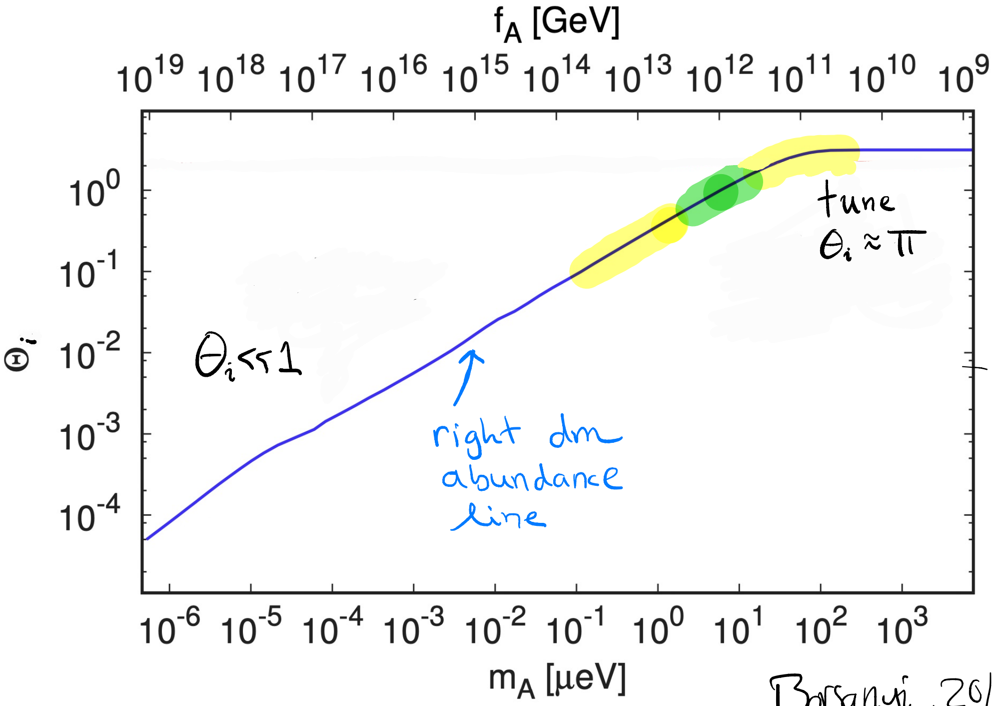
$$n_a(t_{osc})\left(\frac{R(t_{osc})}{R(t_0)}\right)^3 = \frac{n_a(t_{osc})}{s(t_{osc})}\,s(t_0) = \frac{\frac{1}{2}f_a^2\, m_a(t_{osc})\left(\frac{a_i}{f_a}\right)^2}{g_{*s}(T_{osc})\,T_{osc}^3}\; g_{*s}(T_0)\,T_0^3$$

Final result (numerical eval.)

$$\Omega_a h^2 = 0.12\, \Theta_i^2 \left(\frac{f_a}{9\times 10^{11}\,\mathrm{GeV}}\right)^{1.165}$$

$$= 0.12\, \Theta_i^2 \left(\frac{6\,\mu\mathrm{eV}}{m_a}\right)^{1.165}$$

Parameters to explain dark matter



Borsanyi, 2016

Two more points

1) Perturbations suppressed on scales smaller than the axion Jeans length,

$$\lambda_J = 50\, h^{-1/2} \left(\frac{10^{-5}\ \mathrm{eV}}{m_a} \right)^{1/2} \mathrm{AU}.$$

Has been used indirectly to rule out canonical models of "fuzzy dark matter" with $m_a \lesssim 10^{-20}$ eV.

2) Axion will fluctuate during inflation — source of "non-adiabatic" isocurvature mode. $P_{iso} = \frac{H_I^2}{\pi^2 f_a^2 \Theta_i^2} \lesssim 0.04 P_\zeta$

$$f_a \gtrsim 8.2 \cdot 10^{12}\ \mathrm{GeV} \left(\frac{r}{0.04} \right)^{1/2} \quad \text{where } r = A_t / A_s .$$

Presently, $r < 0.04$ by BICEP/PLANCK.

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Recap:

The strong CP problem: why is $\bar{\Theta} \lesssim 10^{-10}$.

The QCD axion: $\bar{\Theta} \mapsto a(x)$, with $\langle a \rangle = 0$.

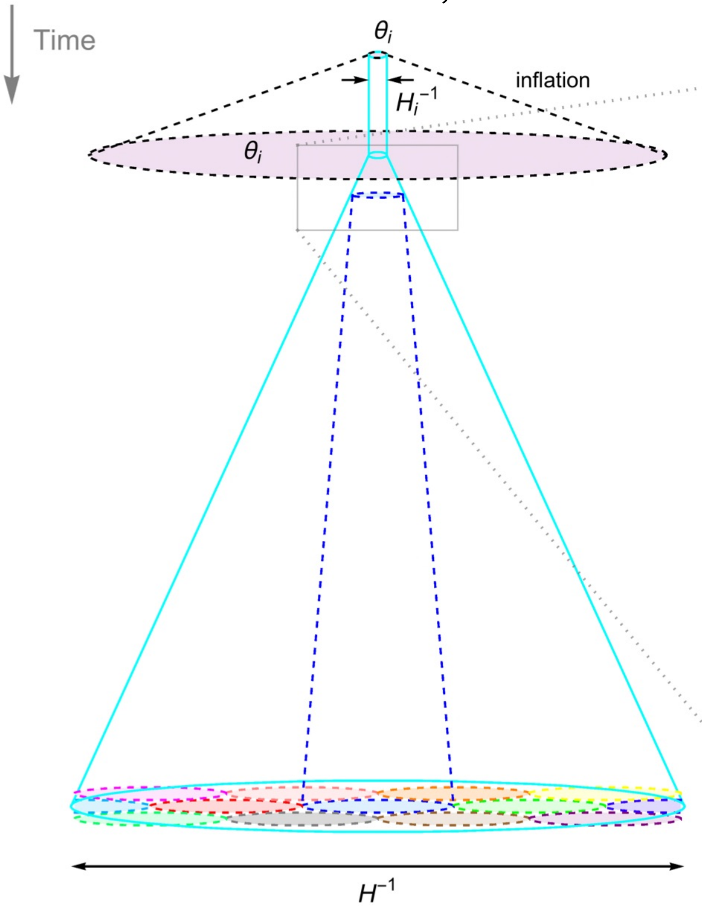
ALPs generalizes idea, not related to strong CP.

Thermally produced QCD axions give hot d.m. contrib.

Axion misalignment can account for dark matter if $f_a \sim 10^{12}$ GeV and $\Theta \sim \mathcal{O}(1)$ if PQ breaking happened before inflation.

The Post-Inflationary Scenario

IAXO White Paper, 2019



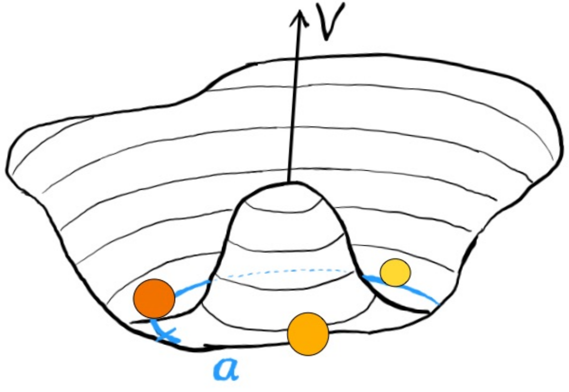
If PQ symmetry broken after inflation, different patches of size $H^{-1}(t_{PQ})$ take different values of θ.

Our present universe contains many different patches with different values of θ_i.

Misalignment contribution $\propto \langle \theta_i^2 \rangle = \frac{\pi^2}{3}$.

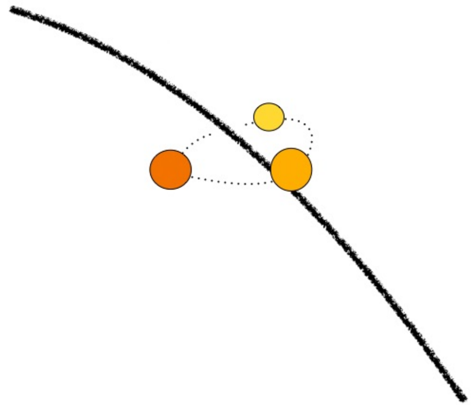
However, there are now additional contributions.

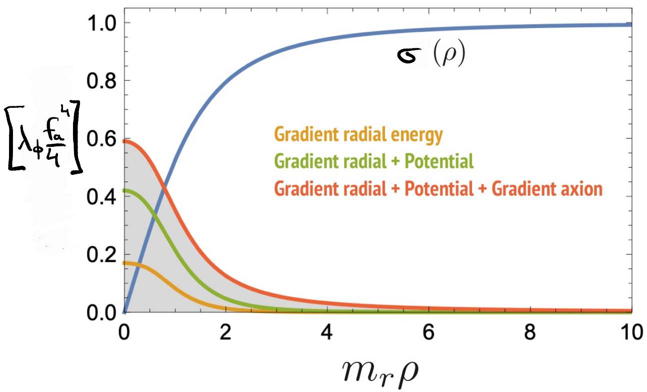
At PQ symmetry breaking, series of patches winding the axion field are formed (Kibble mechanism).



This leads to 'global cosmic strings'.

At the centre of the string, the radial mode sits at the top of the potential:





Strings relevant, as we will see, between t_{PQ} and t_{osc}.

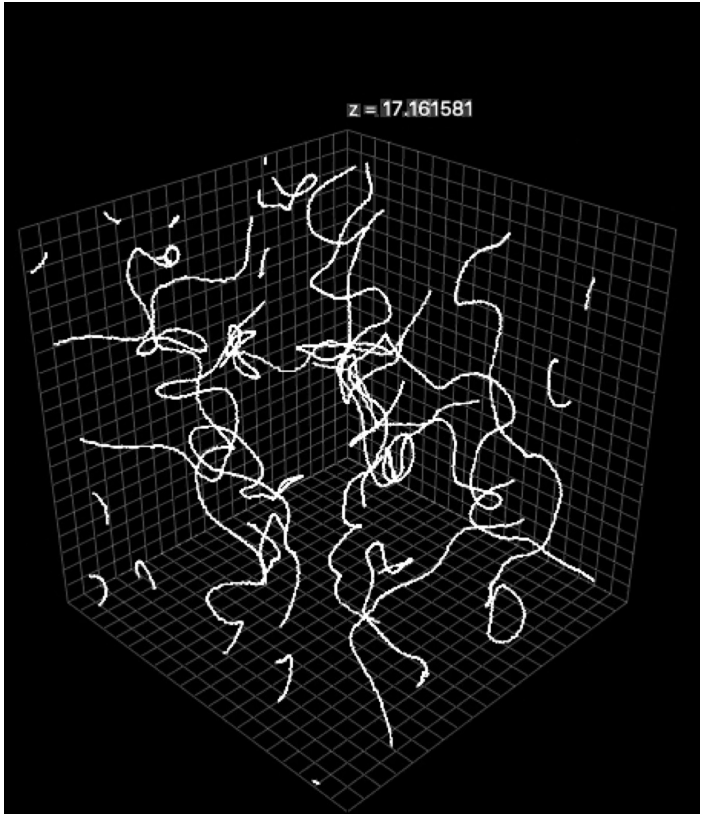
Properties

1) very narrow, $\mathcal{O}(f_a^{-1})$
2) high tension,
$$\mu \sim \frac{f_a^2}{2}\left(2.6 + \pi \ln \frac{m_r \xi}{1.55}\right)$$
3) evolution shortens strings by straightening, pinching off, intersecting.

Approach a scaling solution

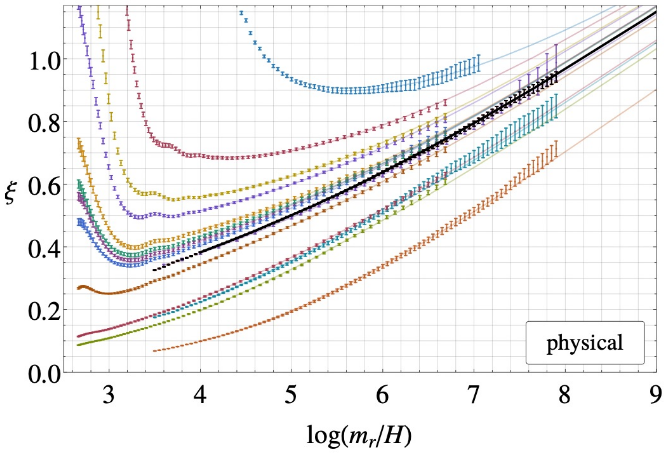
$$\xi \sim \frac{\text{string length}}{\text{causal vol.}}\left(\frac{H^{-1}}{(H^{-1})^3}\right)^{-1} \approx 1.5$$

Recent controversy — is ξ constant or log-enhanced?



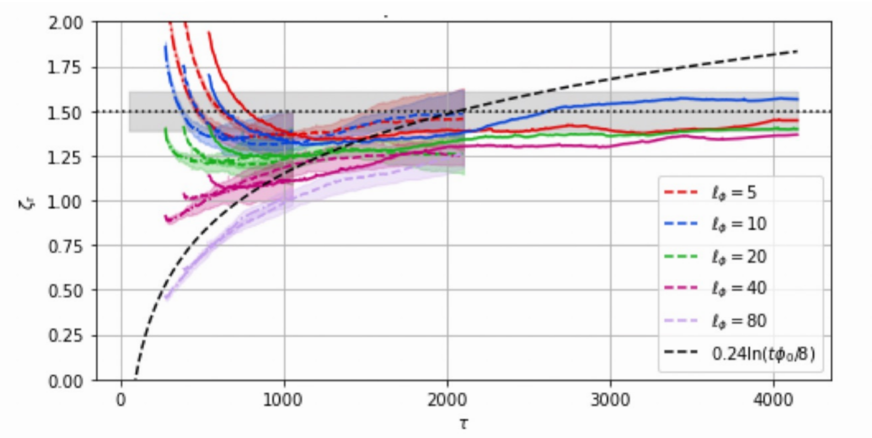
Small changes in time can have big effect over time
(most interested in $\log(m_r/H) \approx 60-70$)

" $\xi = 15$ at $\log\left(\frac{m_r}{H}\right) \approx 60$ "



Gorghetto et al. 2021

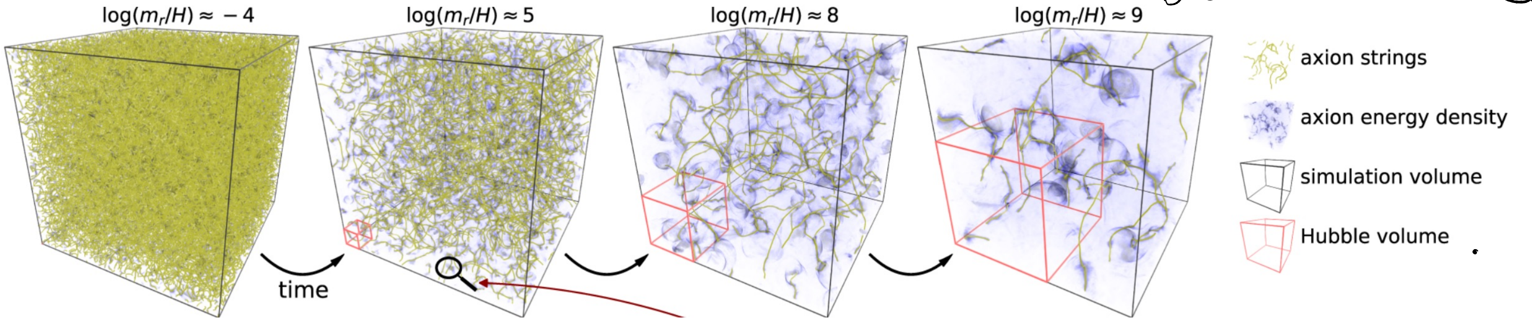
" slow approach to scaling sol. "



Hindmarsh, Urrestilla et al.
(preliminary, private comm.)

String network radiates axions

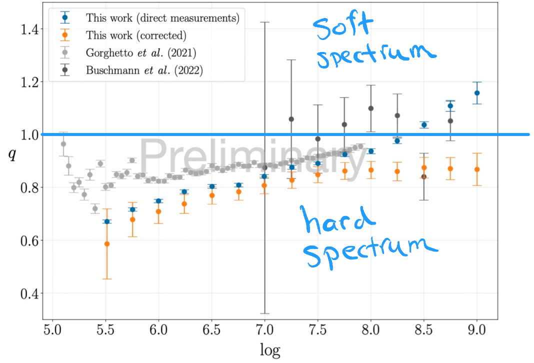
Buschmann et al. 2022



DM abundance depends on n_a, not only ρ_a; the spectrum matters.

Currently, no consensus on slope of spectrum:

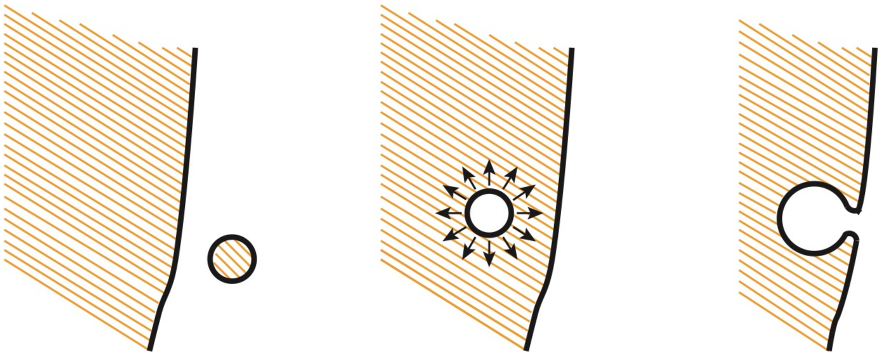
$$\text{“flux”} \sim k^{-q}$$



Saikawa et al.

Once $H \approx m_a$, the $U(1)$ symmetry breaking is felt by the string, and, at most points $\theta(x) \to 0$. This leads to the transient formation of domain walls where $\theta \approx \pi$. Domain walls are attached to strings, and make string/wall system collapse.

In variants of the theory with $N_{DW} > 1$, domain walls are a priori stable, and problematic.



(a) (b) (c)

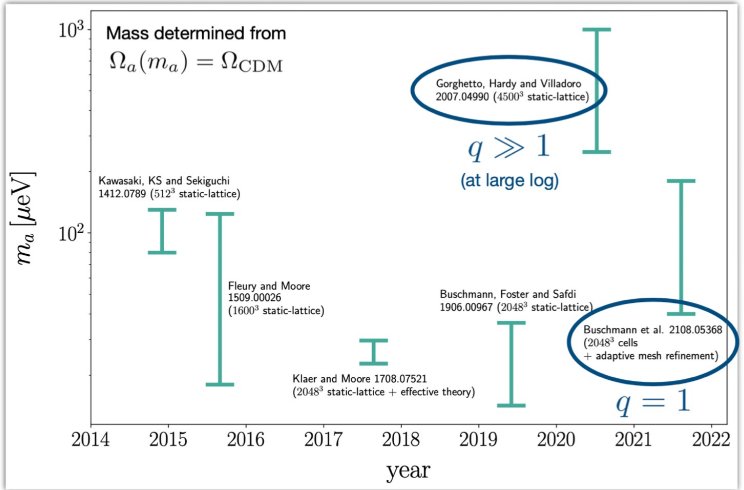
Kim, 2014

Current estimates

$$\frac{\rho_{a,\,\text{string}}}{\rho_{a,\,\text{misalign}}} \sim \mathcal{O}(0.1 - 200)$$

In the post-inflationary scenario, a single value of m_a gives the correct dark matter abundance for the QCD axion, but what is it?

Saikawa '23



$$m_a \begin{cases} > 20\ \mu\text{eV} & \text{Hiramatsu} \\ = 26.2 \pm 3.4\ \mu\text{eV} & \text{Klaer, Moore} \\ = 25.2 \pm 11\ \mu\text{eV} & \text{Buschmann et al.} \\ = 115 \pm 25\ \mu\text{eV} & \text{Kawasaki et al.} \\ \sim 40 - 180\ \mu\text{eV} & \text{Buschmann et al} \end{cases}$$

Axion clumps

Miniclusters: $\delta\rho_a/\rho_a$ has $\mathcal{O}(1)$ inhomogeneities on scales $L_{osc} \sim \frac{1}{H(t_{osc})R(t_{osc})}$. At around matter-rad. equality, these collapse into miniclusters, decoupling the axions within from the Hubble flow.

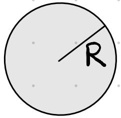
Expected properties:

$$R_{MC} \sim 10^9 \text{ km} \left(\frac{10^{12}\text{ GeV}}{f_a}\right)^{1/6}$$

$$M_{MC} \sim 10^{-12} M_\odot \left(\frac{f_a}{10^{12}\text{ GeV}}\right)^{7/3}$$

[astroid-sized]

Axion stars: stable solutions of the field equations coupled to gravity.



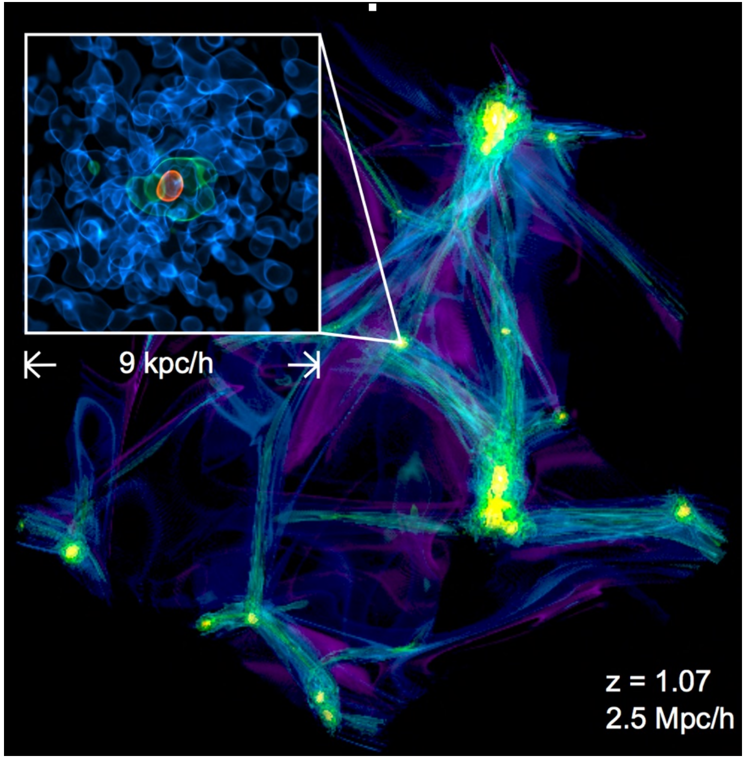
$$U \sim -\frac{GM}{R} + \int d^3\underline{x}\, \frac{1}{2} (\underline{\nabla} a)^2$$

Gravitational self-attraction balanced by gradient "pressure".

$$M_* \lesssim \frac{f_a M_{pl}}{m_a} \sim 10^{-13}\, M_\odot \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^2$$

$$R_* \gtrsim \frac{1}{m_a}\left(\frac{M_{pl}}{f_a}\right) \sim 4000 \text{ km}$$

$$v_{esc,*} \sim \left(f_a/M_{pl}\right) \sim \mathcal{O}(m/s)$$



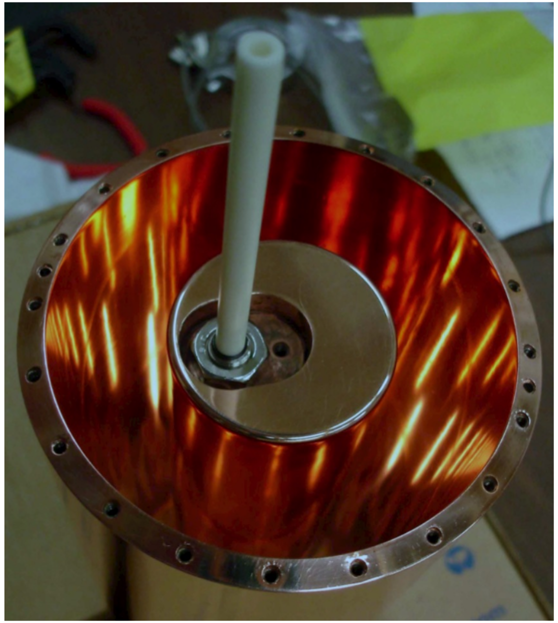
Veltmaat et al.

Interesting opportunities for astrophysics.

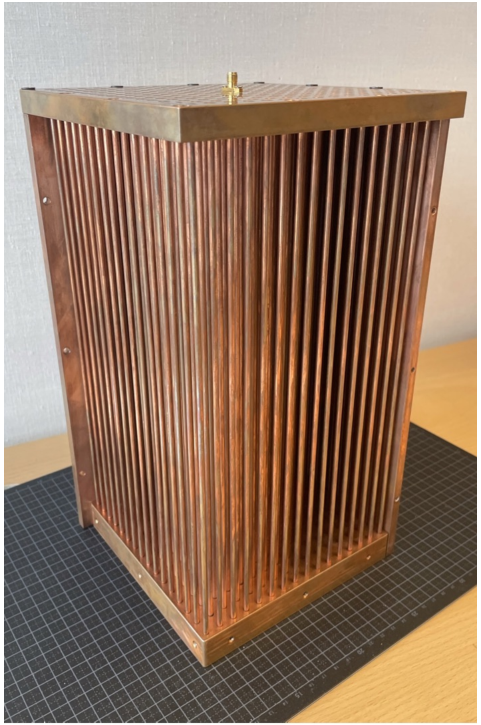
Present-day abundance in MW not very well known.

If most of the dark matter is in clumps, it can complicate things for direct detection experiments.

Axion dark matter experiments

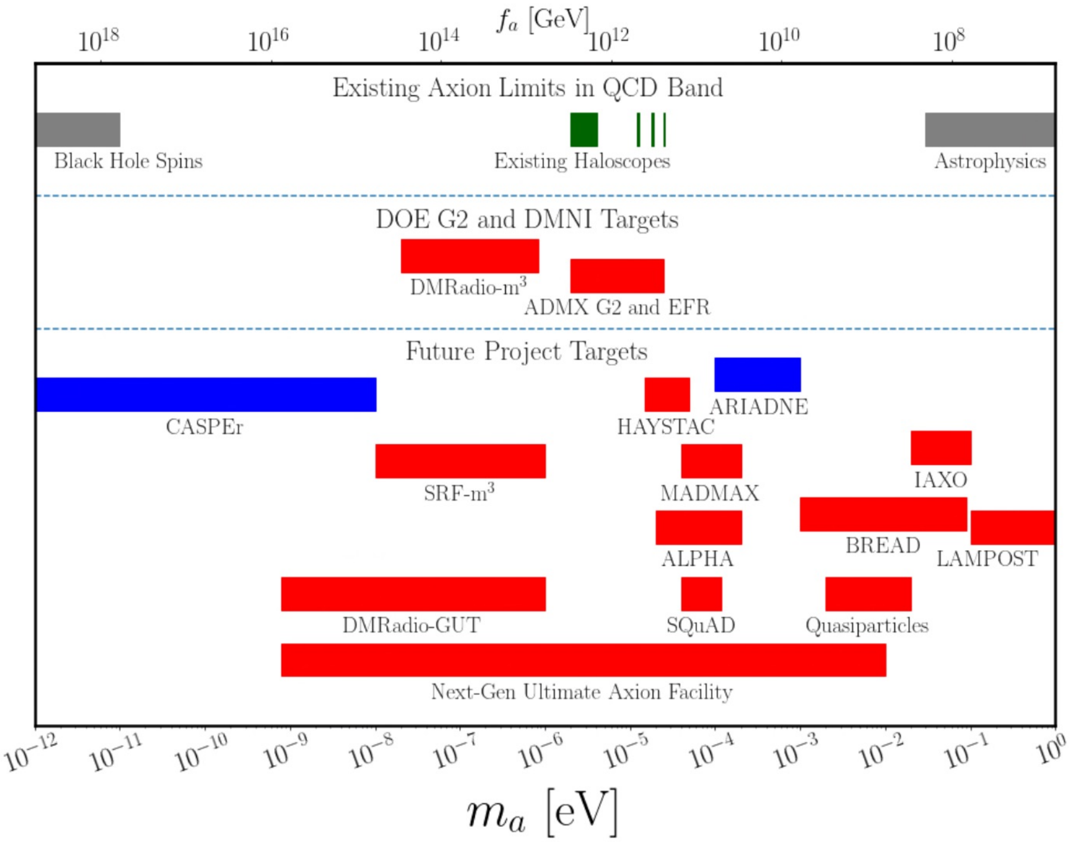


ADMX
(U. Washington)



ALPHA
(Stockholm U.)

A flurry of activity



Axion dark radiation, cf. "Cosmic axion background"

Relativistic axion-like particles would contribute to the effective # neutrinos

$$N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_{d.r.}}{\rho_{CMB}}$$

Thermal origin: $\Delta N_{eff} = c \left(\frac{g_*(t_\nu)}{g_*(t_{dr})}\right)^{1/3}$ $\quad c = \begin{cases} 1 & \text{Weyl} \\ 2 & \text{Dirac} \\ 4/7 & \text{scalar} \end{cases}$

Particle decay: $\Delta N_{eff} = \frac{43}{7}\left(\frac{g_*(t_\nu)}{g_*(t_{dr})}\right)^{1/3} \frac{B_{dr}}{1 - B_{dr}}$

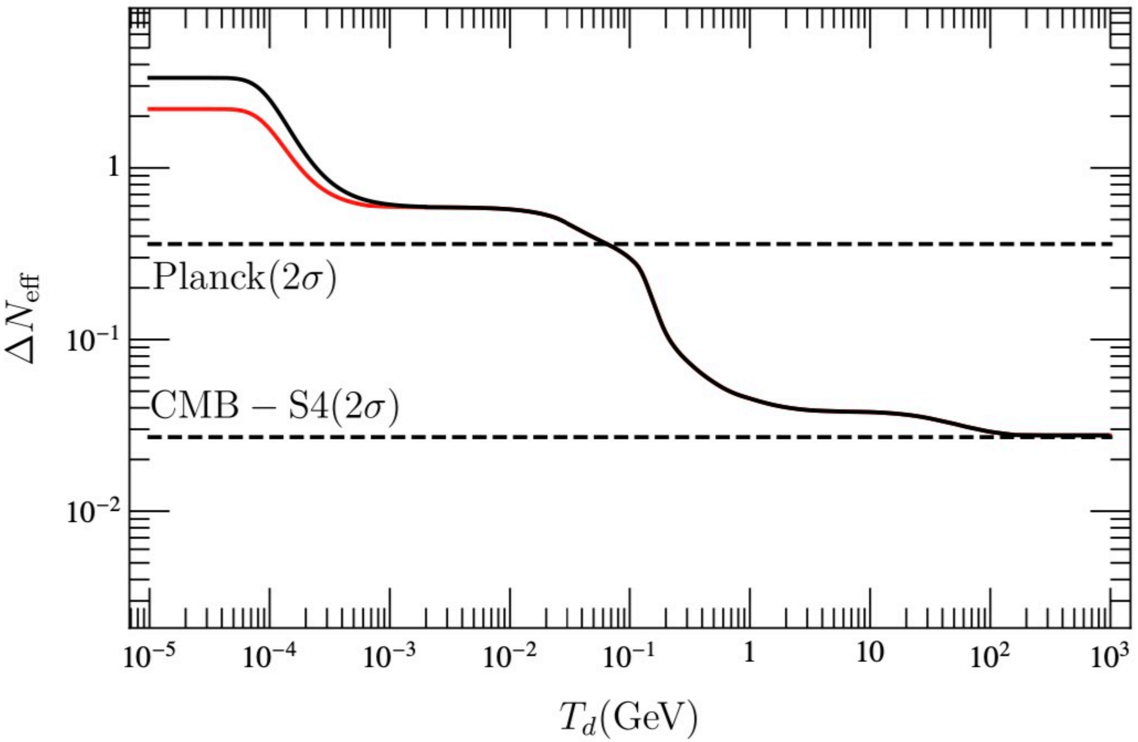
$\Phi \to \begin{cases} h, \gamma, Z, \psi, & \text{[thermalises]} \\ a & \text{[non-thermal]} \end{cases}$

branching ratio

$E_a = m_\Phi/2$ $\qquad T_{reh} = m_\Phi^{3/2}/\Lambda^{1/2}$

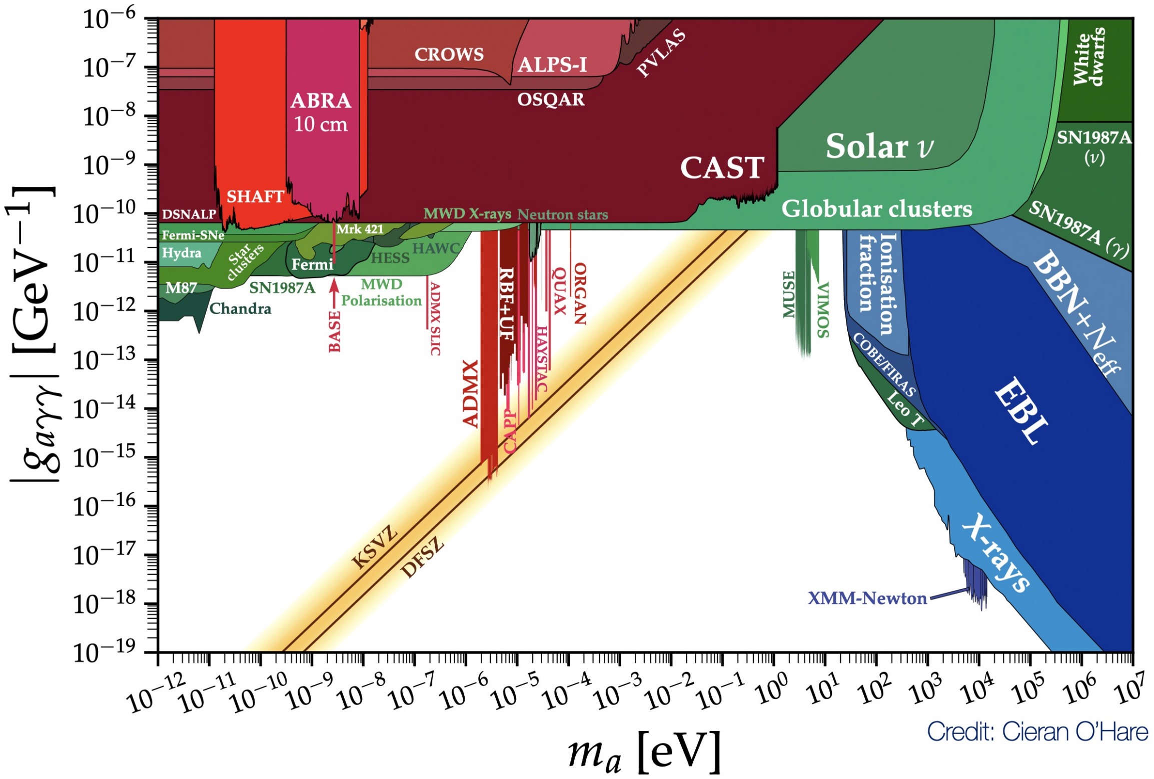
axions can be more energetic than T.

IAXO White Paper, 2019



thermally produced

Axion astrophysics



First signals from axions or ALPs may well come from the sky.

Types of searches

Particles from the sun

Lifetime of stars

White dwarf luminosity fnc.

Neutron star magnetospheres

Supernovae
ν duration,
γ ray burst

Black hole superradiance

AGN, Quasars, GRB
distorted spectra
anomalous transparency

Axion-photon conversion (relativistic)

Consider an ALP of mass m_a coupled to photons as

$$\mathcal{L} \supset \frac{1}{2}\left(\partial_\mu a\, \partial^\mu a - m_a^2 a^2\right) - \frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \text{in Minkowski space.}$$

<u>EoM</u>

$$\left(\Box + m_a^2\right) a = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\nabla_\nu F^{\nu\mu} = J^\mu_{ext} + g_{a\gamma} \nabla_\nu \left(a \tilde{F}^{\nu\mu}\right)$$

Assume the existence of a backround magnetic field, $\underline{B}_0$, and expand fluctuations in gauge potential and axion to leading order.

This gives:

$$\begin{cases} (\Box + m_a^2)\, a = -g_{a\gamma}\, \dot{\underline{A}} \cdot \underline{B}_0 \\ (\Box + \omega_{pl}^2)\, \underline{A} = g_{a\gamma}\, \dot{a}\, \underline{B}_0 \end{cases}$$

↑ finite density photon mass $\omega_{pl}^2 = \frac{4\pi\alpha\, n_e}{m_e}$

Specialise to wave travelling in z-direction w/ $\omega \gg m_a, \omega_{pl}$.

Ansatz: $A_i = \tilde{A}_i(z)\, e^{-i\omega t}$ for $i = x, y$

$a = \tilde{a}(z)\, e^{-i\omega t}$

EoM:

$$\begin{cases} (\omega^2 + \partial_z^2 - m_a^2)\, a = i\omega\, g_{a\gamma} (\tilde{A}_x B_x + \tilde{A}_y B_y) \\ (\omega^2 + \partial_z^2 - \omega_{pl}^2)\, \tilde{A}_j = -i\omega\, g_{a\gamma}\, a\, B_j \end{cases} \qquad j = x, y$$

"Rotating wave approx."

$$(\omega^2 + \partial_z^2) = \underbrace{(\omega + i\partial_z)}_{= \omega + k \,\approx\, 2\omega \text{ for relativistic particles}}(\omega - i\partial_z) \approx 2\omega(\omega - i\partial_z)$$

$$\begin{cases} \left(-i\partial_z - \frac{m_a^2}{2\omega}\right) a = \frac{i}{2} g_{a\gamma} \left(\tilde{A}_x B_x + \tilde{A}_y B_y\right) \\ \left(-i\partial_z - \frac{\omega_{pl}^2}{2\omega}\right) \tilde{A}_j = -\frac{i}{2} g_{a\gamma}\, a B_j \end{cases}$$

Schrödinger eqn: $i\partial_z \Psi = (H_0 + H_I)\Psi$

$$\Psi = \begin{pmatrix} \tilde{A}_x \\ \tilde{A}_y \\ a \end{pmatrix} \quad H_0 = \omega \mathbb{1} + \begin{pmatrix} \Delta_\gamma & & \\ & \Delta_\gamma & \\ & & \Delta_a \end{pmatrix} \quad H_I = \begin{pmatrix} 0 & 0 & \Delta_x \\ 0 & 0 & \Delta_y \\ \Delta_x & \Delta_y & 0 \end{pmatrix}$$

$$\Delta_\gamma = -\frac{\omega_{pl}^2}{2\omega} \qquad \Delta_a = -\frac{m_a^2}{2\omega} \qquad \Delta_i = g_{a\gamma} B_i / 2$$

For general parameters, the Schr. eqn. must be solved numerically. However, for small $g_{a\gamma}$, QM perturbation theory applies.

Interaction picture: $H_{int} = U_0^\dagger H_I U_0$

$$\Psi_{int} = U_0^\dagger \Psi$$

$$U_0 = e^{-i\int^z dz' H_0(z')}$$

$$\Psi_{int}(z) = \sum_{n=0}^{\infty} (-i)^n \int^z dz_1' \int^{z_1} dz_2' \ldots \int^{z_{n-1}} dz_n' H_{int}(z_1') \ldots H_{int}(z_n')$$

$$\simeq \Psi_{int}(0) - i \int^z dz' H_{int}(z') \Psi_{int}(0)$$

Transition amplitudes

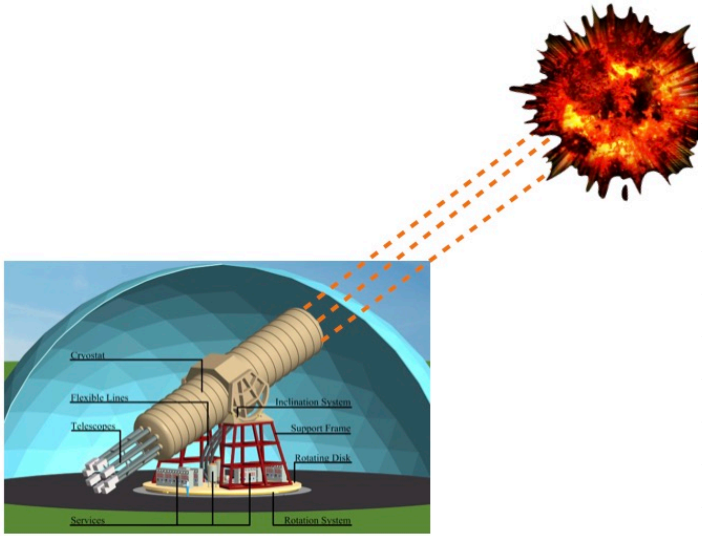
Assume $\Psi_i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, i.e. an x-polarised photon.

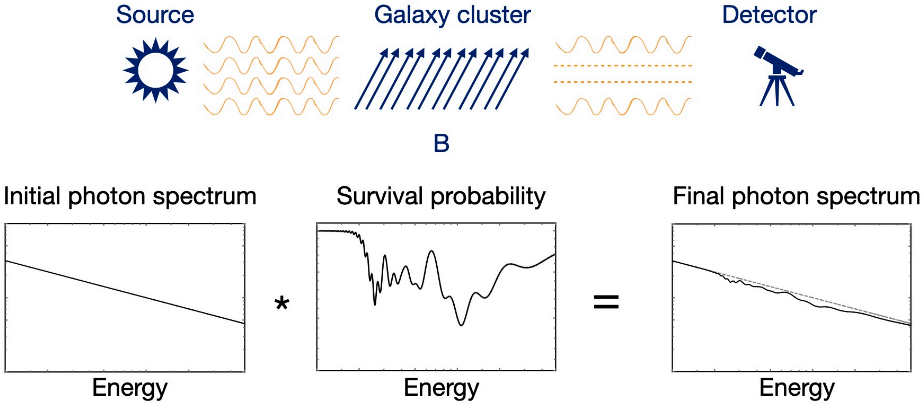
$$A_{\gamma_x \to a}(z) = (0,0,1)\cdot\Psi(z) = -i\int^{z} dz'\,\Delta_x(z')\, e^{i\int^{z'} dz''(\Delta_a - \Delta_\gamma)}$$

Conversion probability: $P_{\gamma_x \to a}(z) = \left| \int^{z} dz'\,\Delta_x(z')\, e^{+i\int^{z'} dz''(\Delta_a - \Delta_\gamma)} \right|^2$

Measures fraction of EM flux transferred into axions.

Applications include:





$$P_{a\gamma} \approx 2\times10^{-19}\left(\frac{g_{a\gamma}}{10^{-10}\,\mathrm{GeV}^{-1}}\,\frac{B}{10\,\mathrm{T}}\,\frac{L}{10\,\mathrm{m}}\right)^2$$

$P_{a\gamma} \sim \mathcal{O}(1)$ achievable for $g_{a\gamma} \sim 10^{-12}\,\mathrm{GeV}^{-1}$. Data allows for few percent fluctuations.

Summary

Axion physics is a scientifically rich and diverse area.

Approaching a 'golden age'?

Much progress and new ideas from junior people.

Scientific discovery may be very rapid.

Exercises on axion cosmology

1) Show that, in a $U(1)$ gauge theory the Θ-term violates CP, P and T given that

$$P: \ E(\underline{x},t) \mapsto -E(-\underline{x},t) \ ; \ \underline{B}(\underline{x},t) \mapsto B(-\underline{x},t)$$

$$T: \ \underline{E}(\underline{x},t) \mapsto \underline{E}(\underline{x},-t) \ ; \ B(\underline{x},t) \mapsto -B(\underline{x},-t)$$

$$C: \ \underline{E}(\underline{x},t) \mapsto -\underline{E}(\underline{x},t) \ ; \ \underline{B}(\underline{x},t) \mapsto -\underline{B}(\underline{x},t)$$

2) Determine the minimal mass of ALP dark matter. Calculate the corresponding de Broglie wavelength for $v \sim 10^{-3}$.

3) Derive an analytical formula for $\Omega_a h^2$.
Choose relevant parameter values and evaluate.

4) Derive the axion photon conversion probability for a constant magnetic field of size L.