Hawking radiation for detectors in superposition of locations outside a black hole

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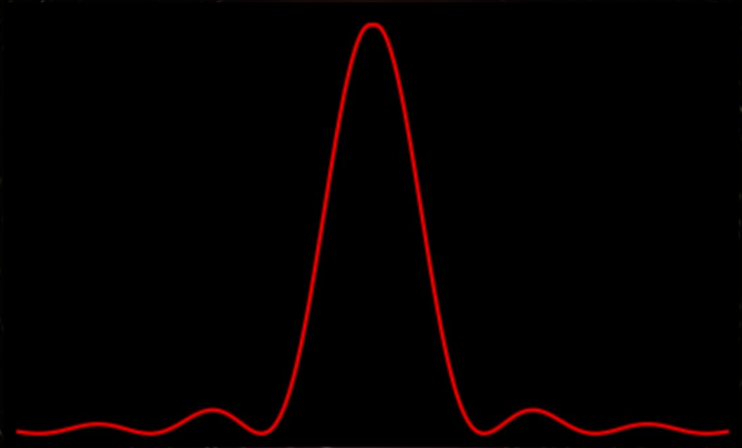
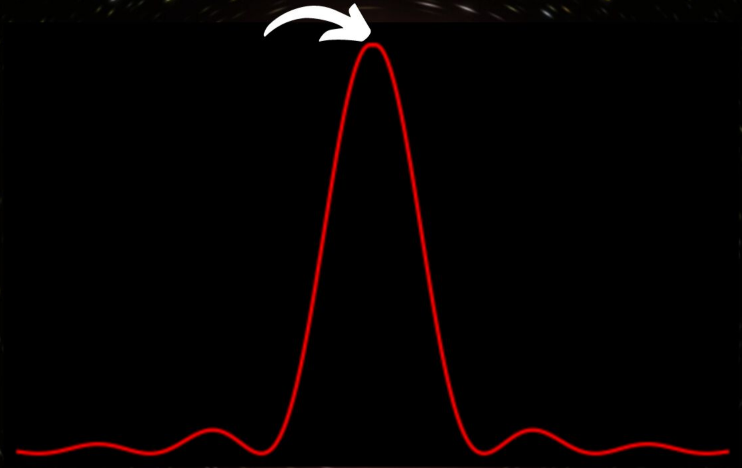
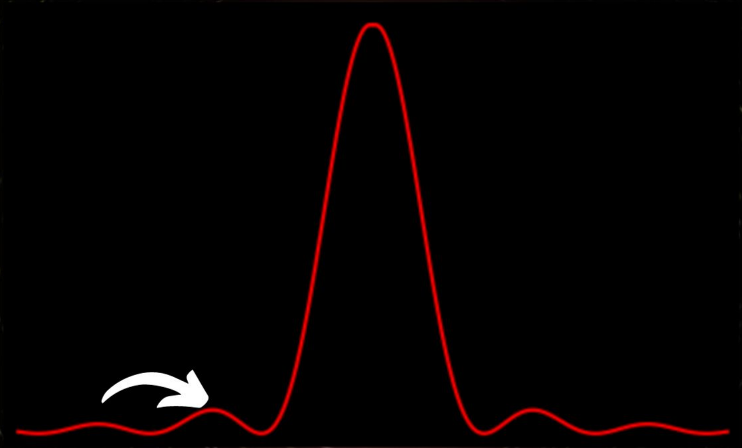
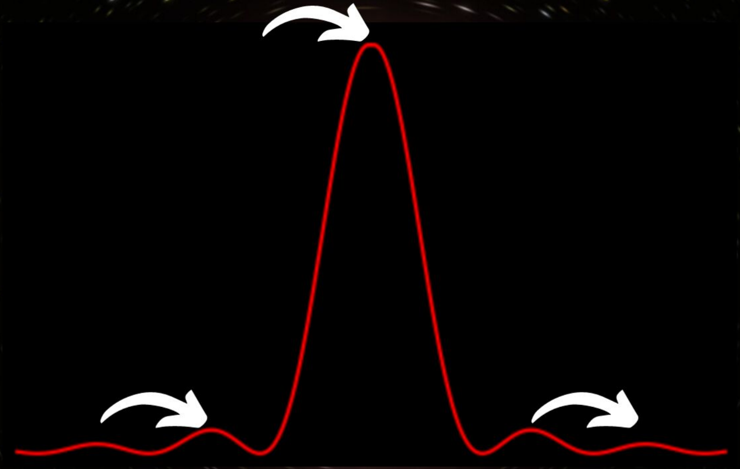
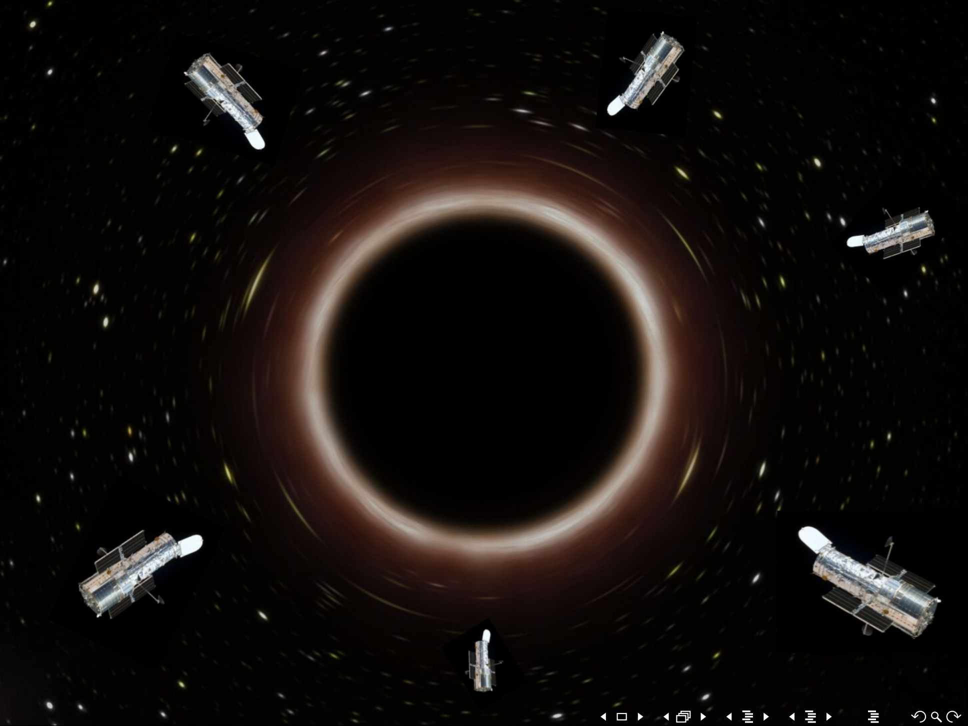
RQI circuit, November 2023











- Schwarzschild metric:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

- Degrees of freedom:
  - Internal energy levels: $\{|0\rangle_{\mathsf{D}}, |E_1\rangle_{\mathsf{D}}, |E_2\rangle_{\mathsf{D}}, \dots\}$
  - Static trajectories: $\{|1\rangle_{\mathsf{T}}, |2\rangle_{\mathsf{T}}, \dots\}$
  - States of the field: $\{|\Omega\rangle_{\mathsf{F}}, \dots\}$
- Interaction:

$$\hat{H}_{\mathsf{I}}(\tau) = -\varepsilon\chi(\tau)\hat{m}(\tau)\hat{\phi}(\hat{x}(\tau))$$

- Initial state:

$$|\Psi_i\rangle = |0\rangle_{\mathsf{D}}\,|\Omega\rangle_{\mathsf{F}}\sum_n A_n\,|n\rangle_{\mathsf{T}}$$

The final state of the detector (energy + trajectory):

$$\begin{aligned}\rho_{\mathsf{DT}} \approx &\left(\sum_{n,m} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}}\right) \left|0\right\rangle \left\langle 0\right|_{\mathsf{D}} \\ &+ \frac{\varepsilon^2 T}{2\pi} \sum_n \left|A_n\right|^2 \left|n\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_i \left|\zeta_i\right|^2 \sigma_{in} \frac{E_i}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_i\right\rangle \left\langle E_i\right|_{\mathsf{D}} \\ &+ \frac{\varepsilon^2 T}{2\pi} \sum_{\substack{n,m \\ n\neq m}} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_{\substack{i,j \\ i\neq j}}^{\text{cond}} \zeta_i^* \zeta_j \Lambda_{nm}^{ij} \sqrt{\sigma_{in}\sigma_{jm}} \frac{\sqrt{E_i E_j}}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_j\right\rangle \left\langle E_i\right|_{\mathsf{D}}\end{aligned}$$

The final state of the detector (energy + trajectory):

$$\rho_{\mathsf{DT}} \approx \left(\sum_{n,m} A_n^* A_m \left| m \right\rangle \left\langle n \right|_{\mathsf{T}} \right) \left| 0 \right\rangle \left\langle 0 \right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_n |A_n|^2 \left| n \right\rangle \left\langle n \right|_{\mathsf{T}} \sum_i |\zeta_i|^2 \sigma_{in} \frac{E_i}{e^{q_{in}/T_{\mathsf{H}}} - 1} \left| E_i \right\rangle \left\langle E_i \right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_{\substack{n,m \\ n \neq m}} A_n^* A_m \left| m \right\rangle \left\langle n \right|_{\mathsf{T}} \sum_{\substack{i,j \\ i \neq j}}^{\text{cond}} \zeta_i^* \zeta_j \Lambda_{nm}^{ij} \sqrt{\sigma_{in} \sigma_{jm}} \frac{\sqrt{E_i E_j}}{e^{q_{in}/T_{\mathsf{H}}} - 1} \left| E_j \right\rangle \left\langle E_i \right|_{\mathsf{D}}$$

The final state of the detector (energy + trajectory):

$$\rho_{\mathsf{DT}} \approx \left(\sum_{n,m} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}} \right) \left|0\right\rangle \left\langle 0\right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_n |A_n|^2 \left|n\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_i |\zeta_i|^2 \sigma_{in} \frac{E_i}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_i\right\rangle \left\langle E_i\right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_{\substack{n,m \\ n \neq m}} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_{\substack{i,j \\ i \neq j}}^{\mathsf{cond}} \zeta_i^* \zeta_j \Lambda_{nm}^{ij} \sqrt{\sigma_{in}\sigma_{jm}} \frac{\sqrt{E_i E_j}}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_j\right\rangle \left\langle E_i\right|_{\mathsf{D}}$$

The final state of the detector (energy + trajectory):

$$
\begin{aligned}
\rho_{\mathsf{DT}} \approx & \left(\sum_{n,m} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}}\right) \left|0\right\rangle \left\langle 0\right|_{\mathsf{D}} \\
& + \frac{\varepsilon^2 T}{2\pi} \sum_{n} \left|A_n\right|^2 \left|n\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_{i} \left|\zeta_i\right|^2 \sigma_{in} \frac{E_i}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_i\right\rangle \left\langle E_i\right|_{\mathsf{D}} \\
& + \frac{\varepsilon^2 T}{2\pi} \sum_{\substack{n,m \\ n\neq m}} A_n^* A_m \left|m\right\rangle \left\langle n\right|_{\mathsf{T}} \sum_{\substack{i,j \\ i\neq j}}^{\mathsf{cond}} \zeta_i^* \zeta_j \Lambda_{nm}^{ij} \sqrt{\sigma_{in}\sigma_{jm}} \frac{\sqrt{E_i E_j}}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left|E_j\right\rangle \left\langle E_i\right|_{\mathsf{D}}
\end{aligned}
$$

The final state of the detector (energy + trajectory):

$$\rho_{\mathsf{DT}} \approx \left(\sum_{n,m} A_n^* A_m \left| m \right\rangle \left\langle n \right|_{\mathsf{T}} \right) \left| 0 \right\rangle \left\langle 0 \right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_n \left| A_n \right|^2 \left| n \right\rangle \left\langle n \right|_{\mathsf{T}} \sum_i \left| \zeta_i \right|^2 \sigma_{in} \frac{E_i}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left| E_i \right\rangle \left\langle E_i \right|_{\mathsf{D}}$$

$$+ \frac{\varepsilon^2 T}{2\pi} \sum_{\substack{n,m \\ n \neq m}} A_n^* A_m \left| m \right\rangle \left\langle n \right|_{\mathsf{T}} \sum_{\substack{i,j \\ i \neq j}}^{\text{cond}} \zeta_i^* \zeta_j \Lambda_{nm}^{ij} \sqrt{\sigma_{in} \sigma_{jm}} \frac{\sqrt{E_i E_j}}{\mathrm{e}^{q_{in}/T_{\mathsf{H}}} - 1} \left| E_j \right\rangle \left\langle E_i \right|_{\mathsf{D}}$$

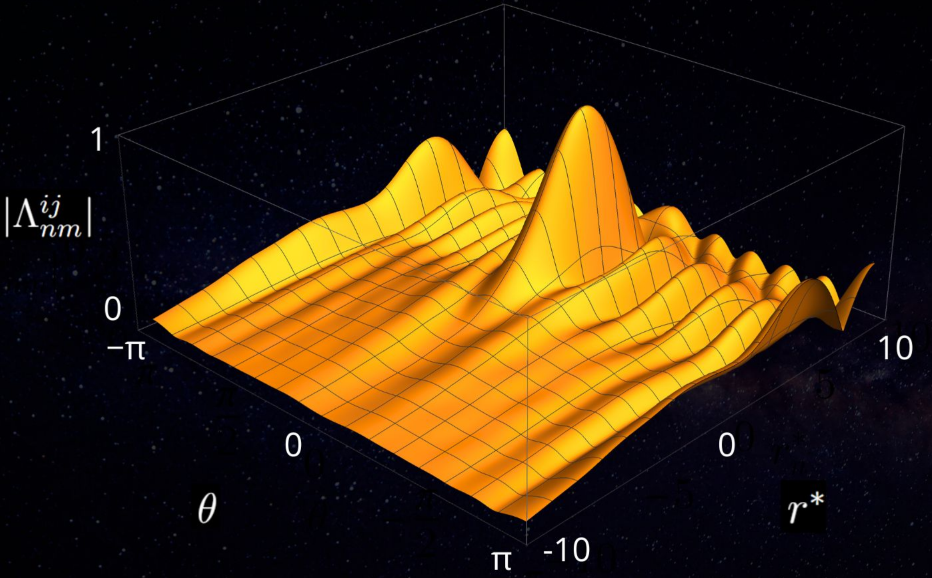
$$\Lambda^{ij}_{nm} = \frac{\langle E_i, n | E_j, m\rangle_{\mathsf{F}}}{\sqrt{\langle E_i, n | E_i, n\rangle_{\mathsf{F}} \langle E_j, m | E_j, m\rangle_{\mathsf{F}}}}$$

- $|E_i, n\rangle_{\mathsf{F}}$ — state of the field after the excitation on the trajectory n to the energy E_i
- $\Lambda^{ij}_{nm} = 0$ unless

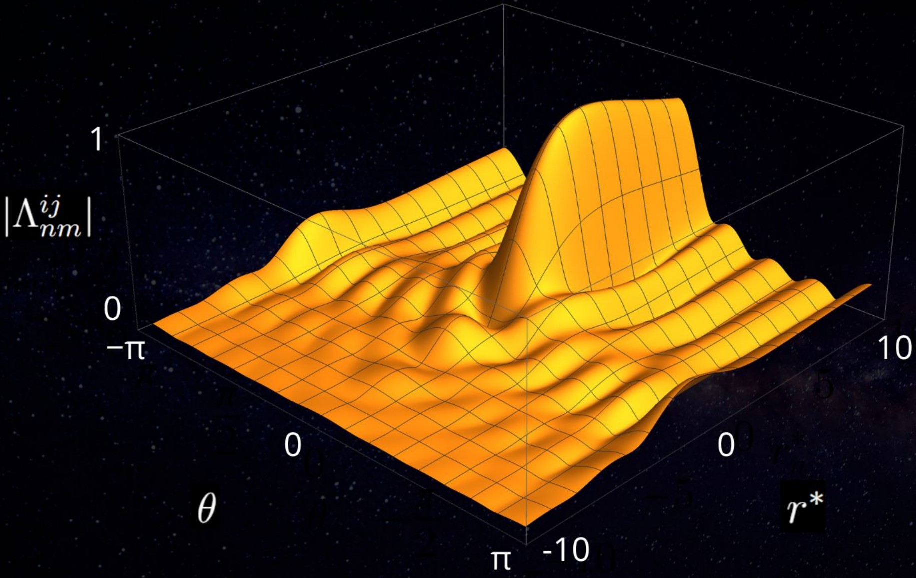
$$E_i/\alpha_n = E_j/\alpha_m$$

  - α_n — gravitational time dilation factor on the trajectory n
  - The energy of excitation as perceived by any static observer must be the same on trajectories n and m

Hartle-Hawking state



Unruh state



To consider:

- Other kinds of trajectories
- Other spacetimes
- Quantum reference frames