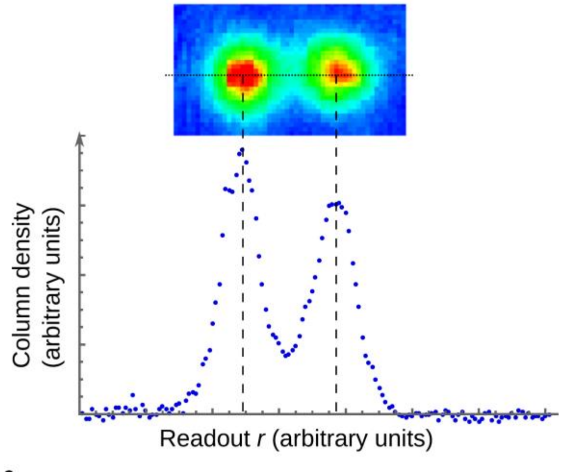
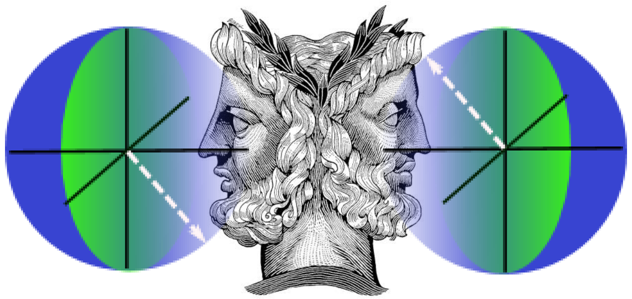
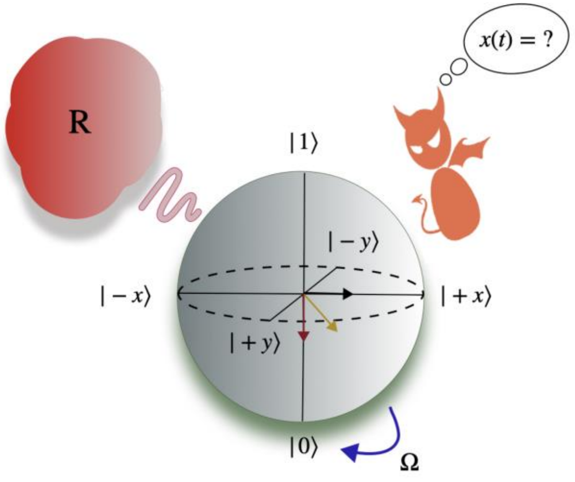






Stochastic Thermodynamics of Quantum Measurements: Engines, Refrigerators, and Clocks

Sreenath K. Manikandan
WINQ fellow, Nordita KTH and Stockholm University

Based on:
Physical Review A, 99(2), 022117.,
Nature communications 12, no. 1 (2021): 1-7., Physical Review E, 105(4), 044137, arXiv:2207.07909 (2022)
Physical Review A 106, no. 4 (2022): 042221., and Physical Review A, 107(2), 023516

The work of SKM is supported by the Wallenberg Initiative on Networks and Quantum Information (WINQ). Nordita is partially supported by Nordforsk.

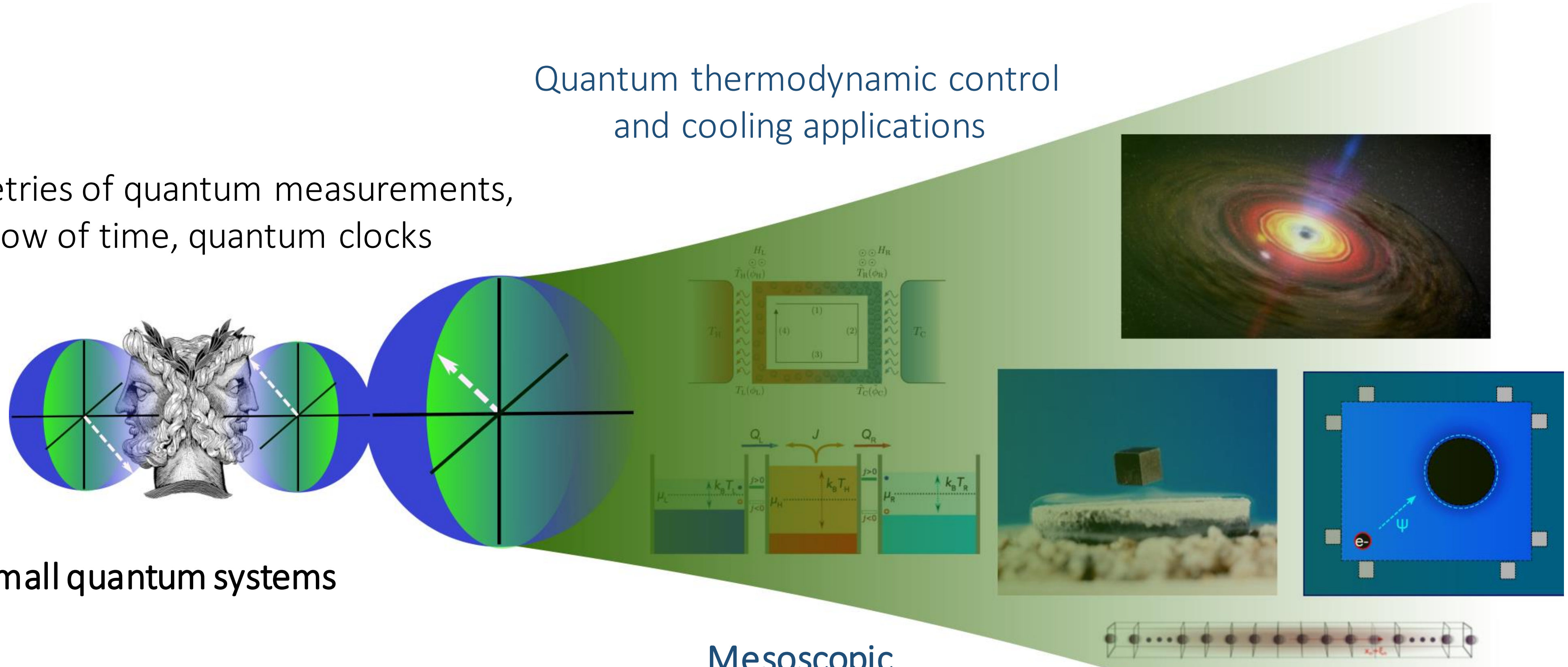
Janus Image Credits: Pinterest.com

Overview of my research

Table-top quantum analogies, and tests of gravity in the quantum regime

Quantum thermodynamic control and cooling applications

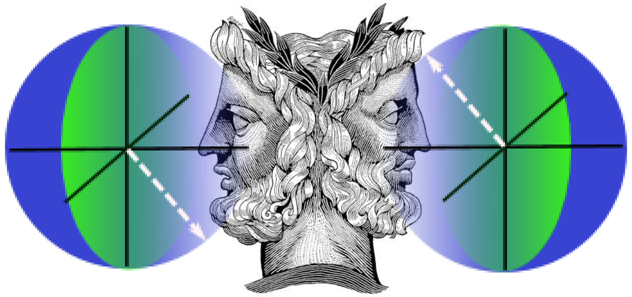
Time-symmetries of quantum measurements, the arrow of time, quantum clocks



Small quantum systems

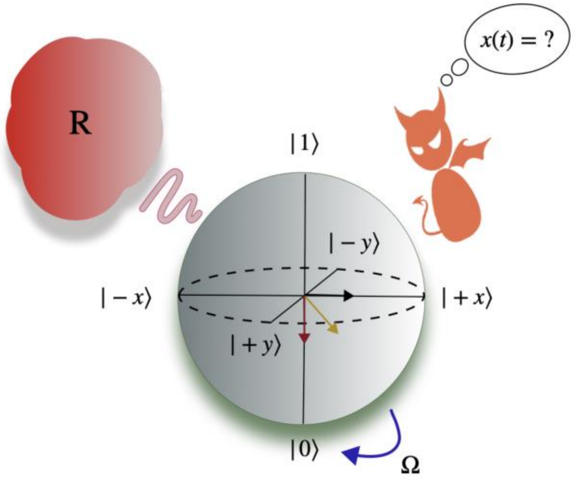
Mesoscopic quantum systems

Large quantum systems

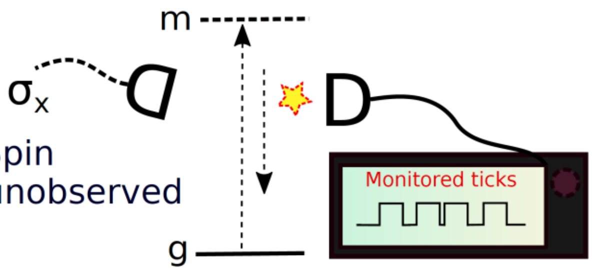


Outline for today's talk

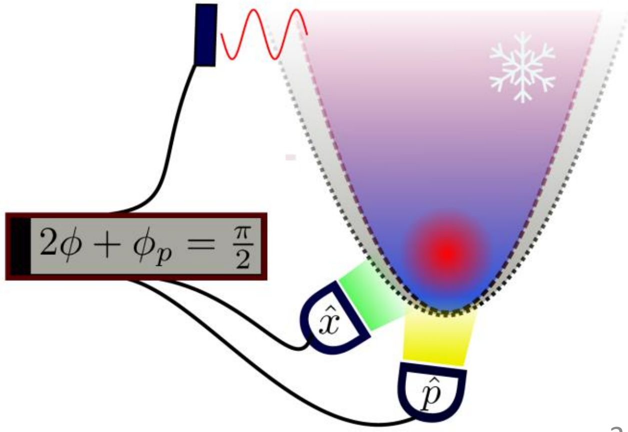
- Introduction
- Arrow of time, fluctuation theorems for quantum measurements
- Thermodynamic device applications of measurement driven quantum systems:



The arrow of time perceived by a quantum Maxwell's demon

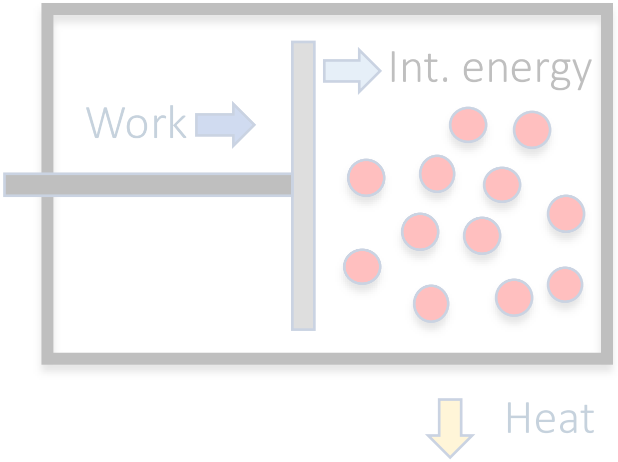


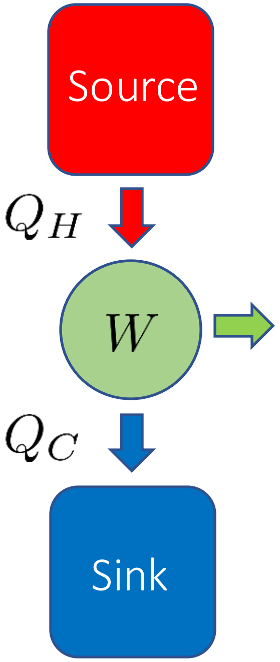
Autonomous quantum clocks



Optimal quantum parametric feedback cooling

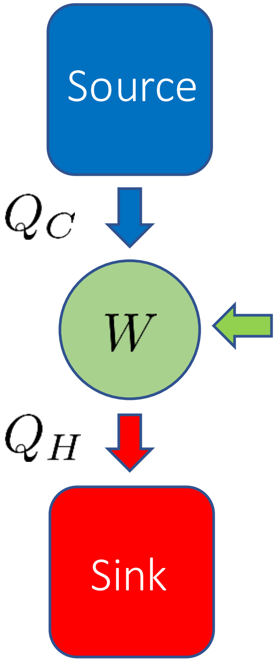
Engines and refrigerators, and clocks





$$\eta = \frac{W}{Q_H}$$

Efficiency of the engine



$$\mathrm{COP} = \frac{Q_C}{W}$$

Coefficient of Performance of the refrigerator

Image credit: https://www.sciencenews.org/article/clock-time-accuracy-entropy-disorder

- Erker, Paul, et al. "Autonomous quantum clocks: does thermodynamics limit our ability to measure time?." *Physical Review X* 7.3 (2017): 031022.
- Milburn, G. J. "The thermodynamics of clocks." *Contemporary Physics* 61.2 (2020): 69-95.

From classical to the quantum regime of thermodynamics

Quantum systems in contact with thermal reservoirs, and **measurement apparatuses**

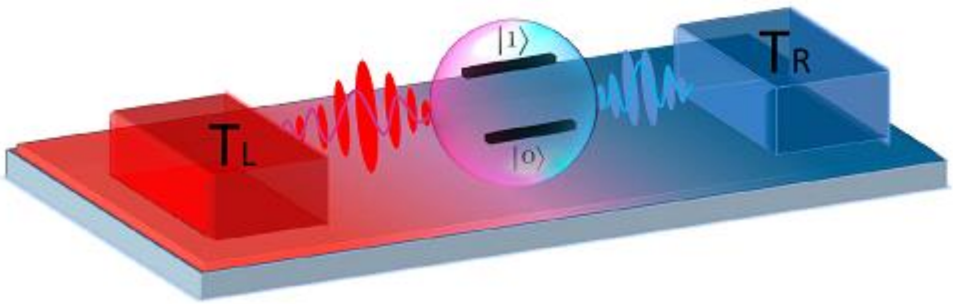
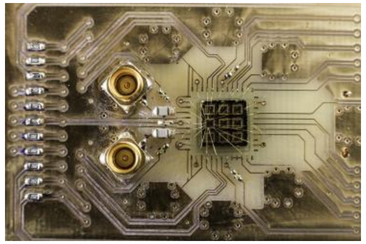


Image credits: http://web.mit.edu/jianshucaogroup/quther2015.html



Pngitem.com

- Vinjanampathy, Sai, and Janet Anders. "Quantum thermodynamics." *Contemporary Physics* 57.4 (2016): 545-579.
- Elouard, Cyril, et al. "The role of quantum measurement in stochastic thermodynamics." npj Quantum Information 3.1 (2017): 9.

Reversal of motion (time reversal)

Classical mechanics:

$$\frac{\partial q_i}{\partial t} = \frac{\partial \mathcal{H}}{\partial p_i}, \qquad \frac{\partial p_i}{\partial t} = -\frac{\partial \mathcal{H}}{\partial q_i},$$

Reversal of motion (time reversal)

Classical mechanics:

$$\frac{\partial q_i}{\partial t} = \frac{\partial \mathcal{H}}{\partial p_i}, \qquad \frac{\partial p_i}{\partial t} = -\frac{\partial \mathcal{H}}{\partial q_i},$$

Quantum mechanics:

$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{\mathcal{H}}\Psi,$$

$$\Theta = \hat{U}\mathrm{K} \qquad \Theta\mathbf{x}\Theta^{-1} = \mathbf{x} \qquad \Theta\mathbf{p}\Theta^{-1} = -\mathbf{p} \qquad \Theta\mathbf{J}\Theta^{-1} = -\mathbf{J} \qquad \Theta\mathbf{S}\Theta^{-1} = -\mathbf{S}$$

Reversal of motion (time reversal)

Classical mechanics:

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Quantum measurement:

$$\hat{A} = \sum_n a_n |a_n\rangle\langle a_n|$$

$$|\Psi\rangle = \sum_n \alpha_n |a_n\rangle$$

$\vdots$

a_2

a_3 $\qquad p_3 = |\alpha_3|^2$

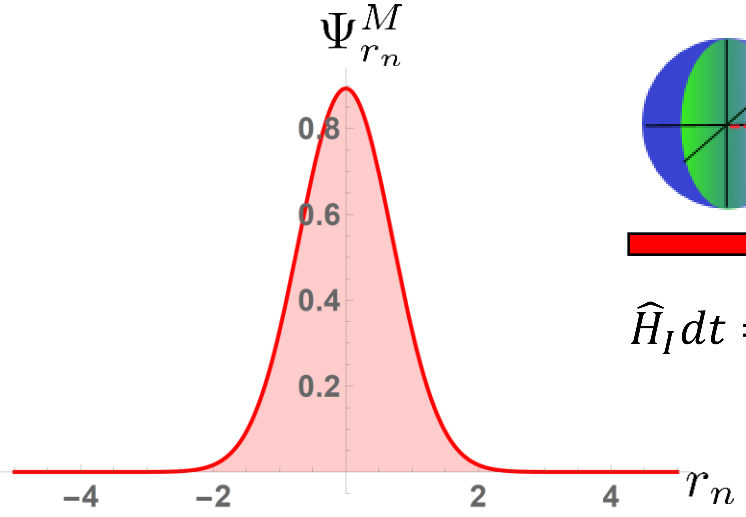
a_4

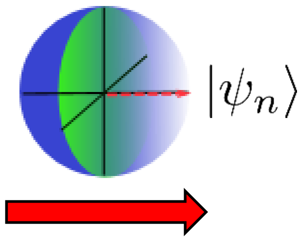
$\vdots$

$$|\Psi'\rangle = |a_n\rangle\langle a_n||\Psi\rangle \propto |a_n\rangle.$$

Sakurai, Jun John, and Eugene D. Commins. "Modern quantum mechanics, revised edition." (1995): 93-95.

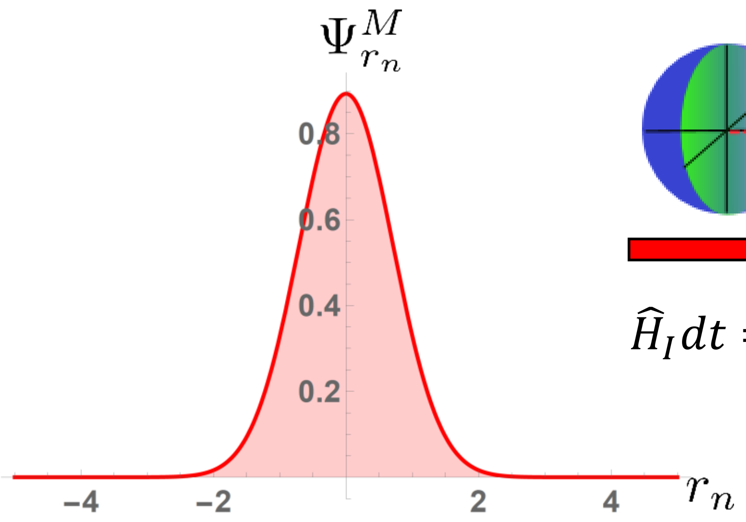
Quantum weak measurement of $\hat{\sigma}_z$

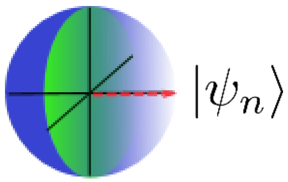




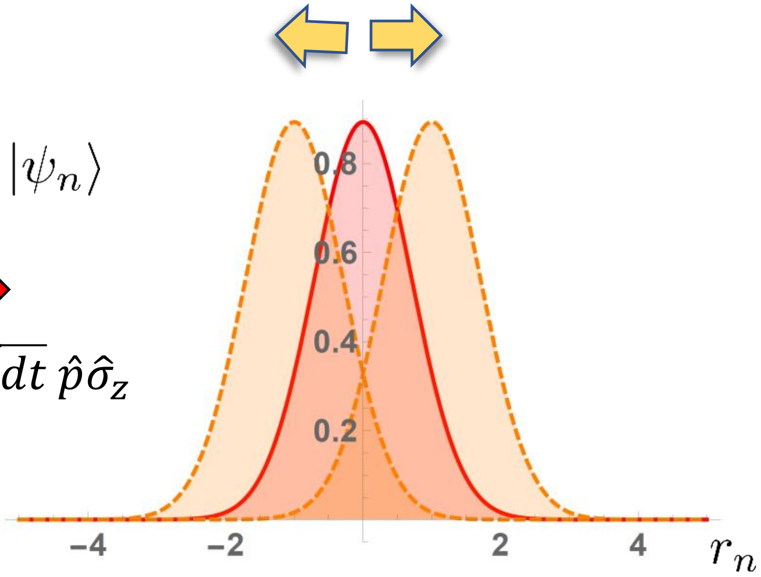
$$\widehat{H}_I dt = \sqrt{dt}\,\hat{p}\hat{\sigma}_z$$

Quantum weak measurement of $\hat{\sigma}_z$

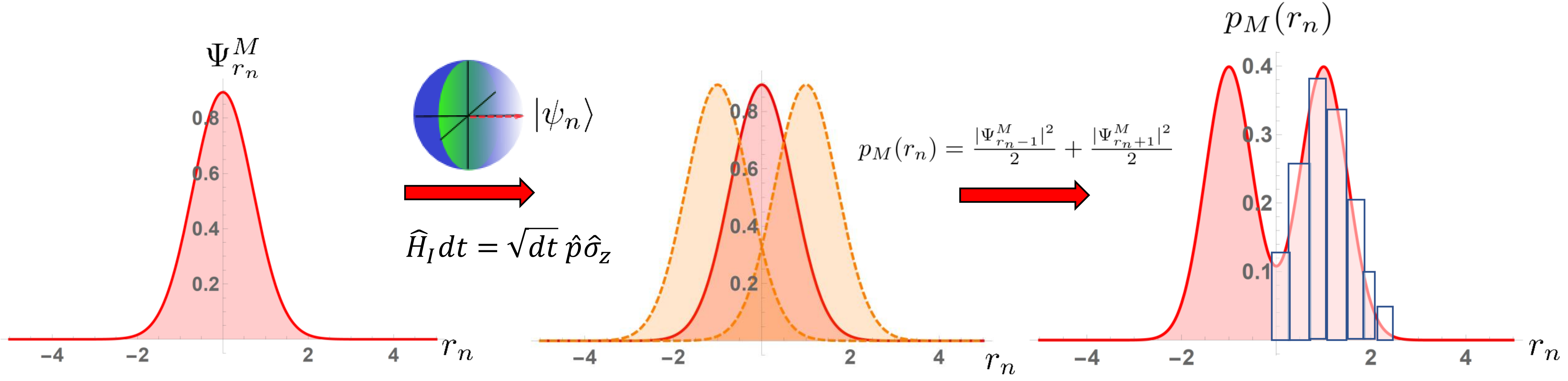




$$\hat{H}_I dt = \sqrt{dt}\,\hat{p}\hat{\sigma}_z$$



Quantum weak measurement of $\hat{\sigma}_z$



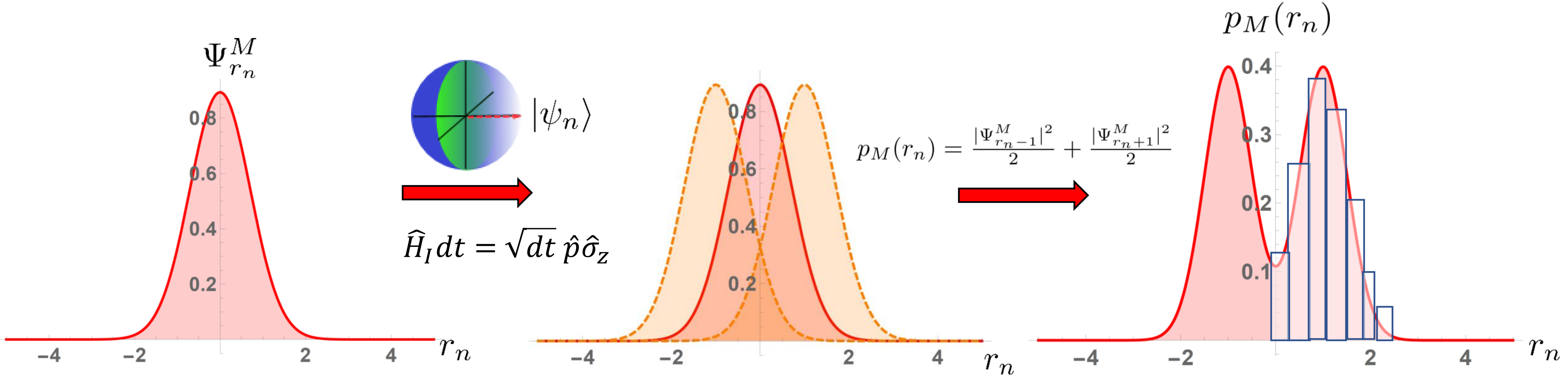
$$|\psi_{n+1}\rangle \propto M_z(r_n)|\psi_n\rangle, \quad \text{where,}$$

$$M_z(r_n) = (dt/2\pi\tau)^{1/4} e^{-(dt/4\tau)(r_n-\sigma_z)^2}.$$

Satisfies the completeness requirement:

$$\int M_z(r_n)^2 dr_n = \hat{1}$$

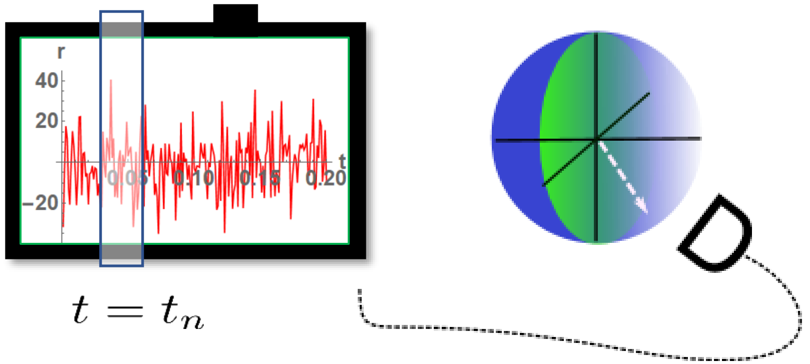
Quantum weak measurement of $\hat{\sigma}_z$



$$\hat{H}_I dt = \sqrt{dt}\,\hat{p}\hat{\sigma}_z$$

$$p_M(r_n) = \frac{|\Psi^M_{r_n-1}|^2}{2} + \frac{|\Psi^M_{r_n+1}|^2}{2}$$

$$r_n \approx \langle \hat{\sigma}_z \rangle_n + \sqrt{\tau}\,\zeta_n$$



$t = t_n$

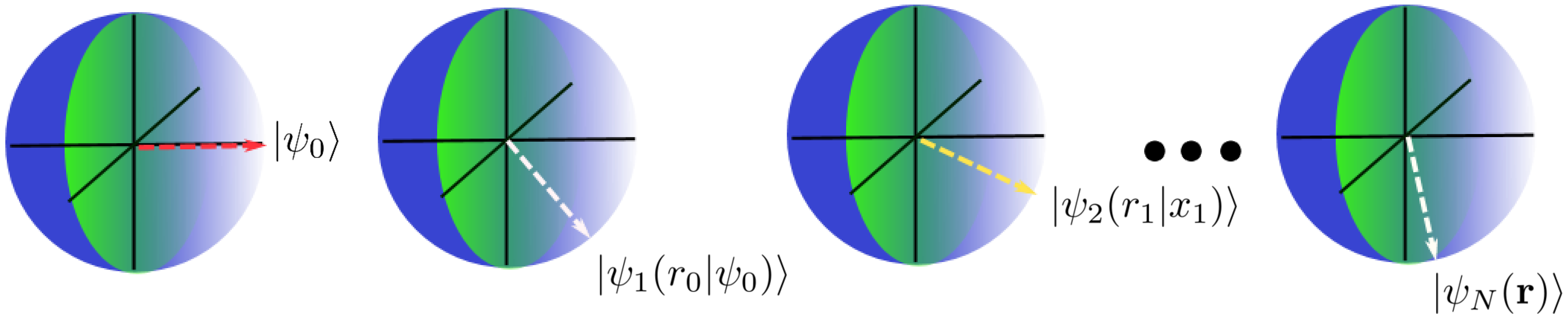
$$|\psi_{n+1}\rangle \propto M_z(r_n)|\psi_n\rangle, \quad \text{where,}$$

$$M_z(r_n) = (dt/2\pi\tau)^{1/4} e^{-(dt/4\tau)(r_n-\sigma_z)^2}.$$

Satisfies the completeness requirement:

$$\int M_z(r_n)^2 dr_n = \hat{1}$$

Quantum trajectory: A single realization



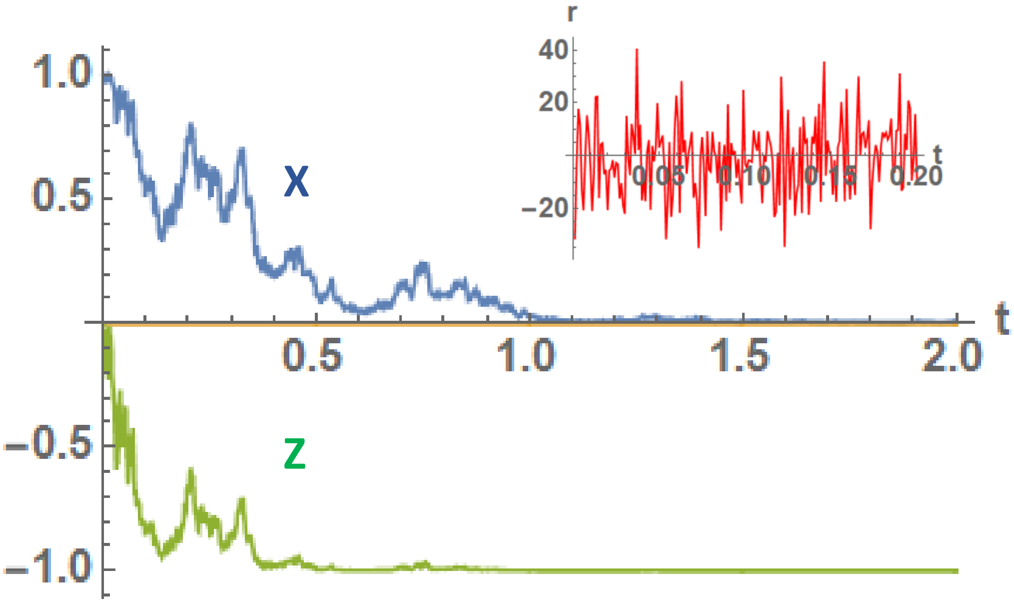
A quantum trajectory is the sequence:

$$\Gamma_{|\psi_0,\mathbf{r}} \equiv \{\psi_0,\ \psi_1(r_0|\psi_0),\ \psi_2(r_1|\psi_1)\ \dots\ \psi_N(\mathbf{r})\},\ \psi_N(\mathbf{r}) = \psi_N[r_{(N-1)}|\psi_{(N-1)}]$$

The sequence is realized with probability: $P_F(\Gamma_{|\psi_0,\mathbf{r}}) \equiv P_F(\mathbf{r}|\psi_0) = \|\overleftarrow{\prod_n} M(r_n)|\psi_0\rangle\|^2$

Quantum measurement reversibility

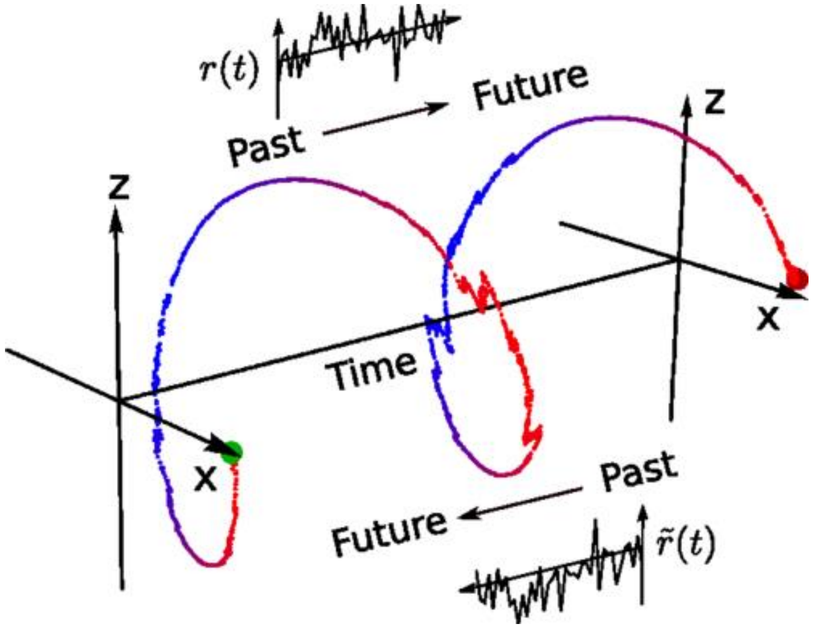
Andrew N Jordan, *Nature* **502**, 177 (2013)



$$M_z(r_n) = (dt/2\pi\tau)^{1/4} e^{-(dt/4\tau)(r_n-\sigma_z)^2}$$

Reversibility at each step: $M_z(-r_n)M_z(r_n) \propto \hat{1}$

Time-symmetry of dynamical equations



$$dt \leftrightarrow \Delta t$$

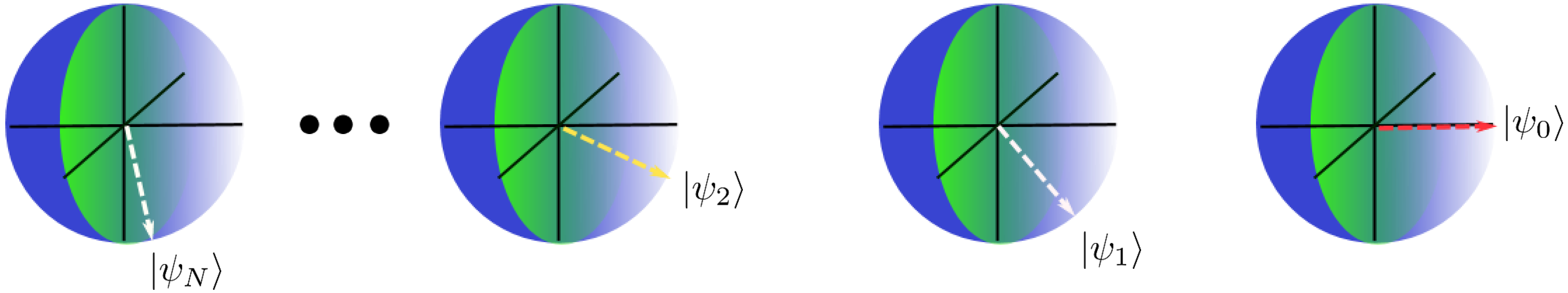
$$\frac{dx}{dt} = \Omega z - \frac{xzr}{\tau}, \qquad \frac{dy}{dt} = -\frac{yzr}{\tau}, \qquad \frac{dz}{dt} = -\Omega x + \frac{(1-z^2)r}{\tau}.$$

Dressel, Justin, et al. "Arrow of time for continuous quantum measurement." *Physical review letters* 119.22 (2017): 220507.

Reversal of arbitrary rank-two measurements

Inverse of a generalized rank-2 qubit measurement:

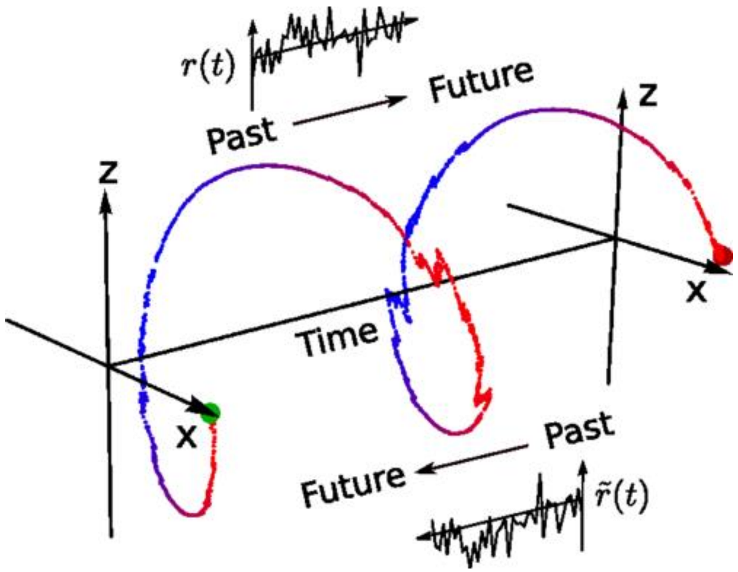
$$\tilde{M}(r_n) = \Theta^{-1} M(r_n)^\dagger \Theta, \quad \text{since} \quad \tilde{M}(r_n) M(r_n) \propto \hat{1}. \qquad \Theta = -i\hat{\sigma}_y \mathrm{K}$$

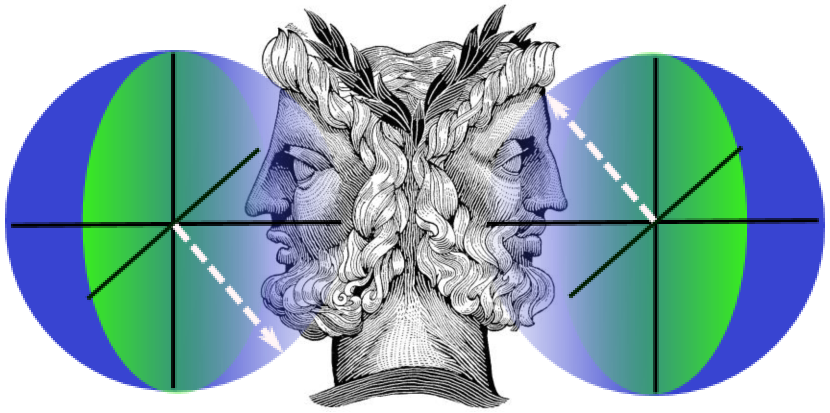


$$P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}] \equiv P_B(\tilde{\mathbf{r}}|\psi_N) = \|\overleftarrow{\prod_n} \tilde{M}(\tilde{r}_n)|\psi_N\rangle\|^2.$$

❖ Sreenath K. Manikandan, and Andrew N. Jordan. "Time reversal symmetry of generalized quantum measurements with past and future boundary conditions." Quantum studies: mathematics and foundations 6.2 (2019): 241-268.

The quantum measurement arrow of time





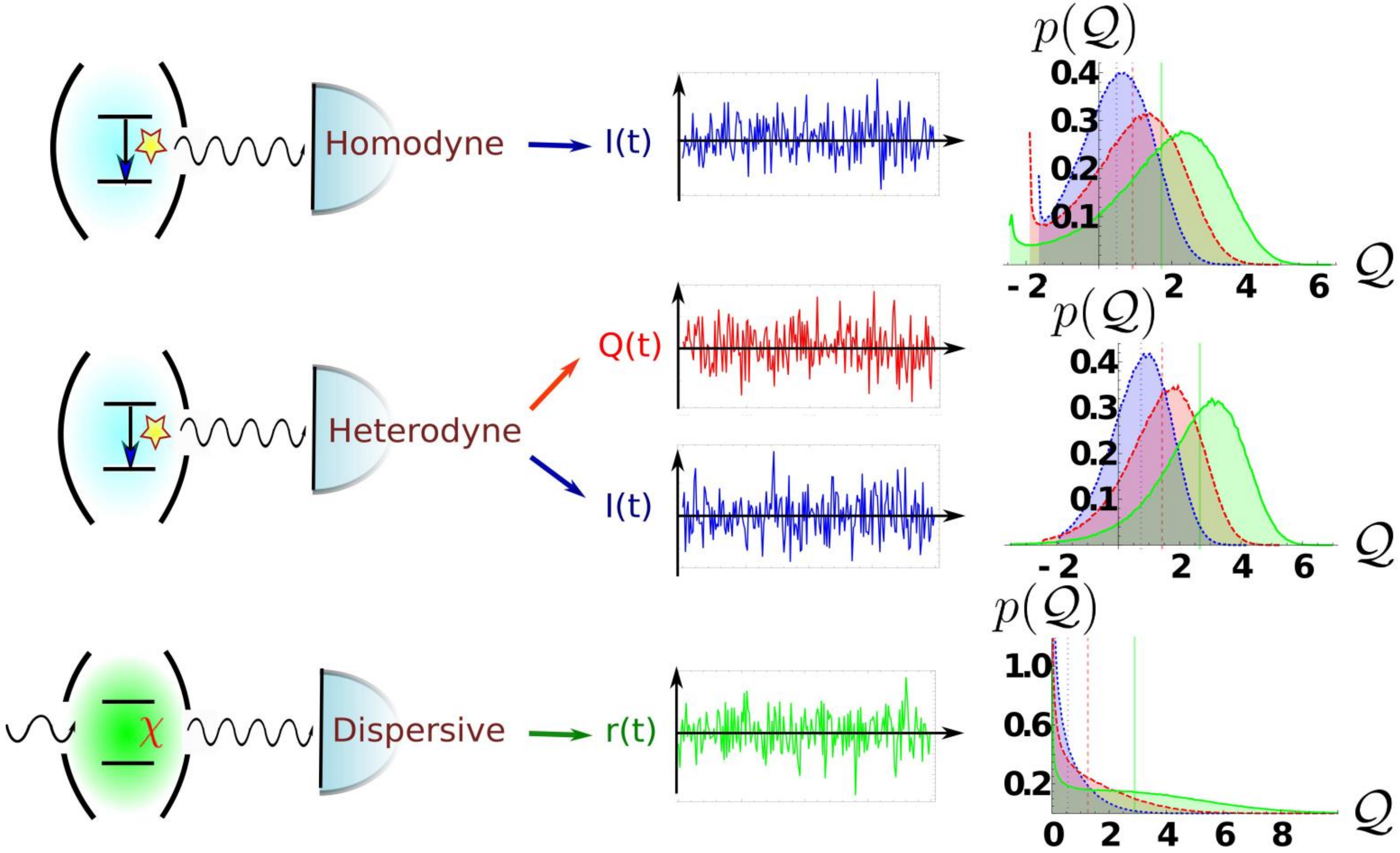
Janus Image Credits: Pinterest.com

The Arrow of Time Measure (using Bayes theorem):

$$\mathcal{Q}(\Gamma_{|\psi_0,\mathbf{r}}) = \log\left\{P_F[\Gamma_{|\psi_0,\mathbf{r}}]/P_B[\tilde{\Gamma}_{|\psi_N(\mathbf{r}),\tilde{\mathbf{r}}}]\right\}.$$

Dressel, Justin, et al. "Arrow of time for continuous quantum measurement." *Physical review letters* 119.22 (2017): 220507.

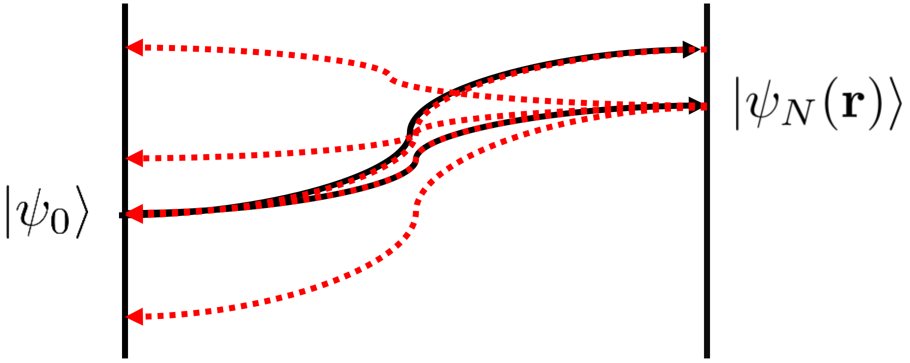
The probability distribution of arrow of Time

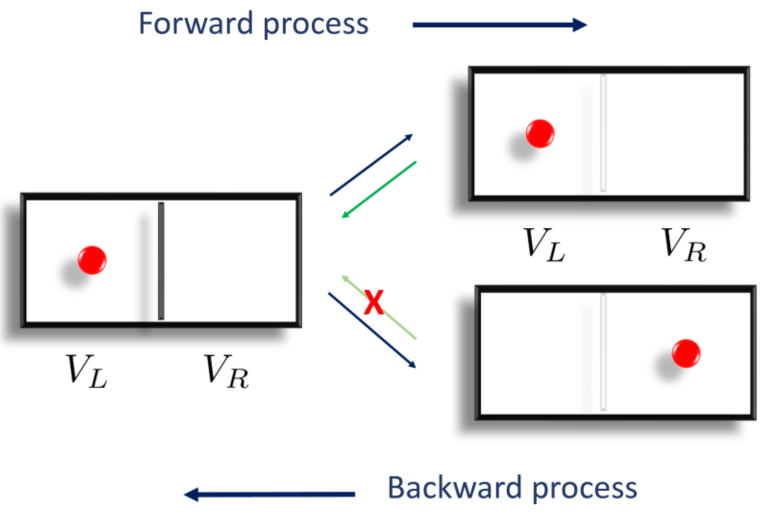


We set $\gamma^{-1} = \tau$. $T = 0.5\tau,\ \tau,\ 2\tau$. All starting from $x = 1$.

Sreenath K. Manikandan, Cyril Elouard, and Andrew N. Jordan. *Physical Review A* 99.2 (2019): 022117.

Absolute irreversibility



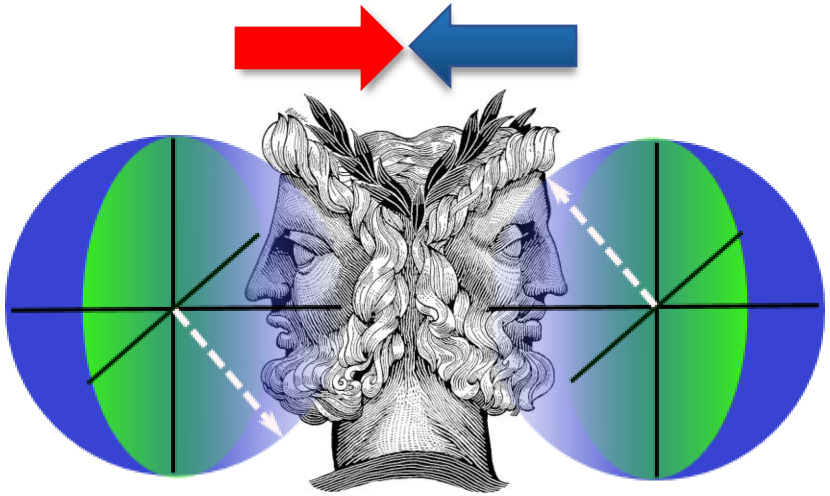


Murashita, Yûto, Ken Funo, and Masahito Ueda. "Nonequilibrium equalities in absolutely irreversible processes." *Physical Review E* 90.4 (2014): 042110.

$$\langle e^{-\mathcal{Q}(\Gamma)}\rangle = 1-\mu, \qquad \mu = 1-\int D\tilde{\Gamma} P_B(\tilde{\Gamma}).$$

Using $e^{-\langle \mathcal{Q}\rangle} \leq \langle e^{-Q}\rangle$, we find $\langle \mathcal{Q}\rangle \geq -\log(1-\mu)$,

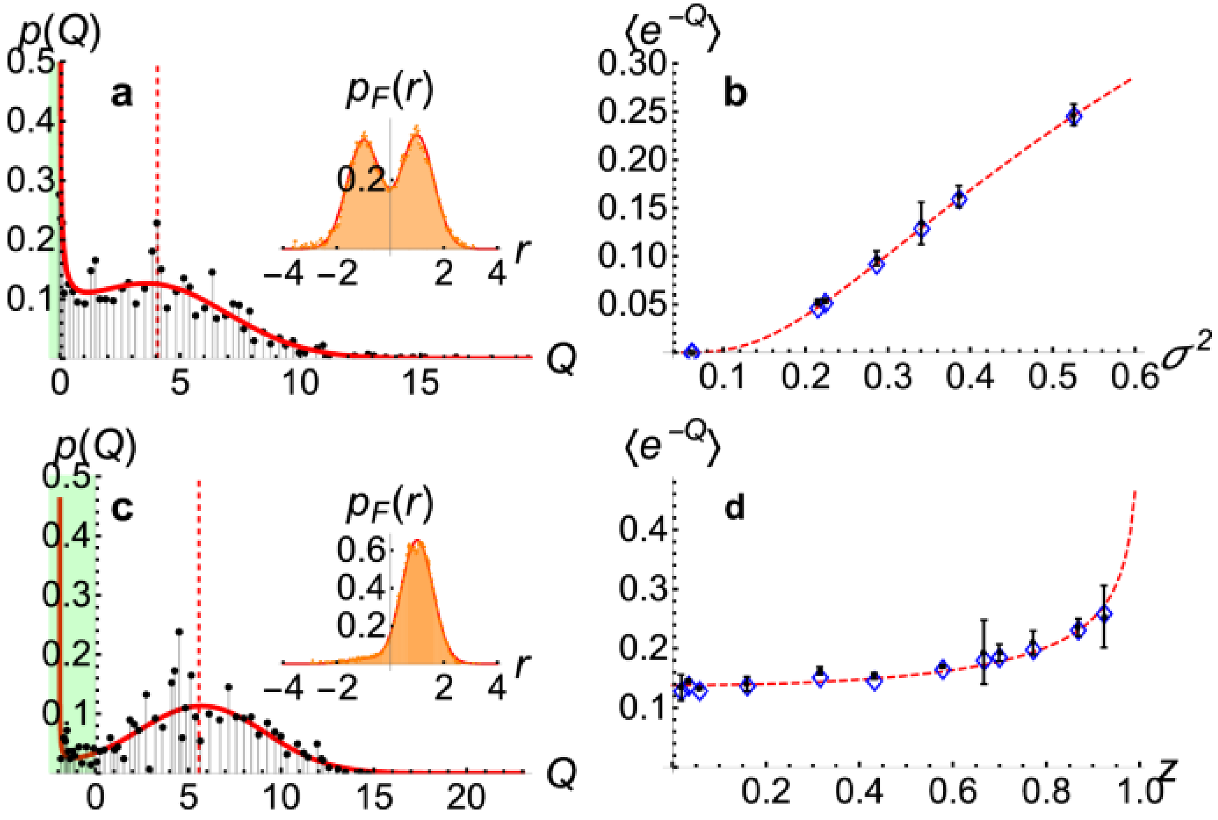
$$\langle e^{-\mathcal{Q}(\Gamma)} \rangle = 1 - \mu, \qquad \mu = \int D\mathbf{r} \frac{|\langle \bar{\psi}_0|\mathcal{M}^\dagger(\mathbf{r})\mathcal{M}(\mathbf{r})|\psi_0\rangle|^2}{\langle \psi_0|\mathcal{M}^\dagger(\mathbf{r})\mathcal{M}(\mathbf{r})|\psi_0\rangle}.$$

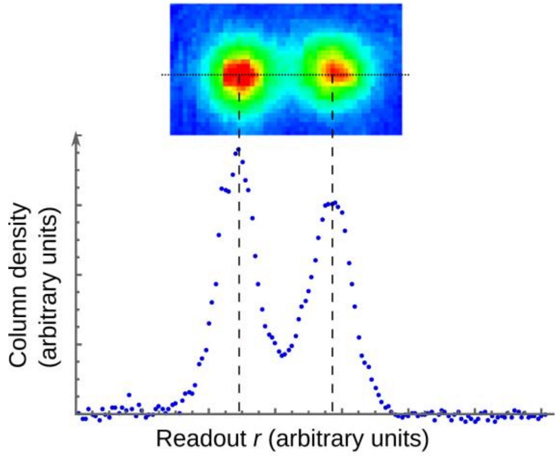


- **A complementary description is contained in μ**
- **Reflects the many to one mapping aspect of the quantum measurement problem.**

Janus Image Credits: Pinterest.com

Experiments In the cold atom platform

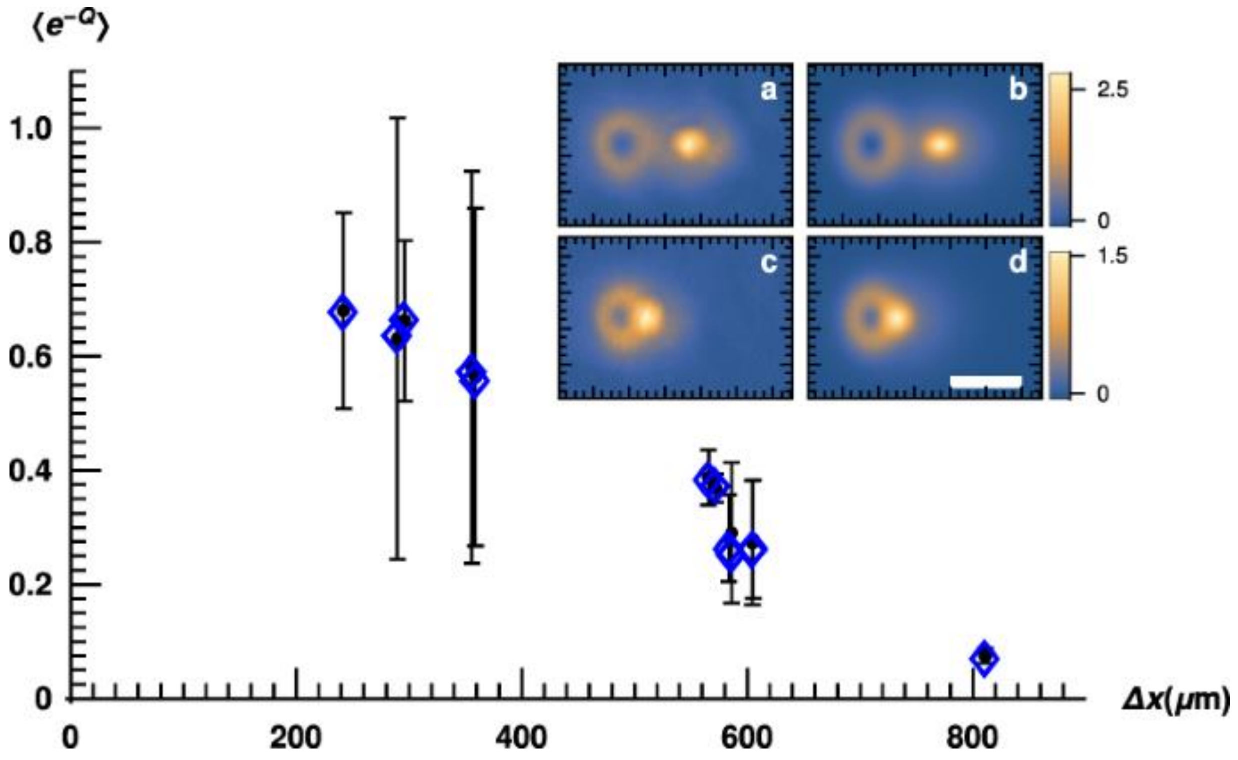




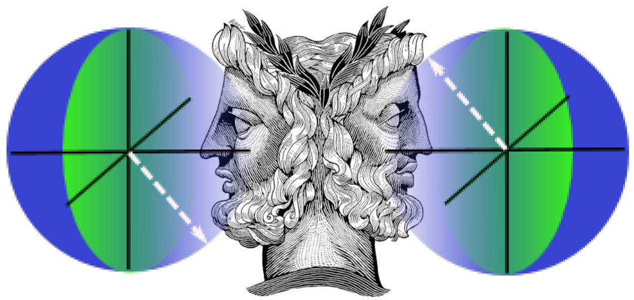
The green shaded region represents realizations where arrow of time was inferred to be negative.

Jayaseelan, M., **Sreenath K. Manikandan**, Jordan, A.N. Bigelow, N.P., Quantum measurement arrow of time and fluctuation relations for measuring spin of ultracold atoms. Nat Commun **12,** 1847 (2021). https://doi.org/10.1038/s41467-021-22094-3

The cold atom platform sheds light on absolute irreversibility of quantum measurements with initial quantum many body interactions.



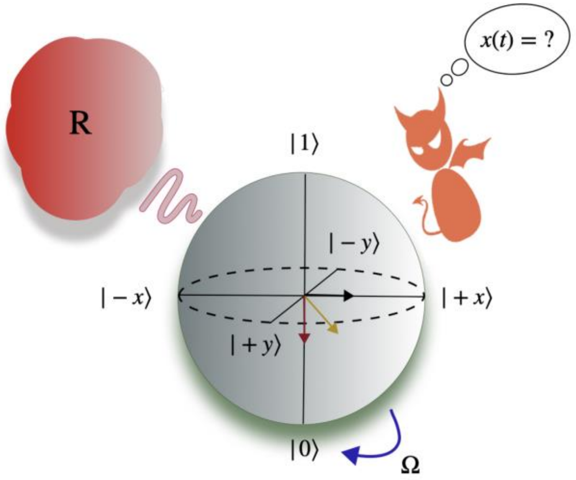
Jayaseelan, M., **Sreenath K. Manikandan**, Jordan, A.N. Bigelow, N.P., Quantum measurement arrow of time and fluctuation relations for measuring spin of ultracold atoms. Nat Commun 12, 1847 (2021). https://doi.org/10.1038/s41467-021-22094-3

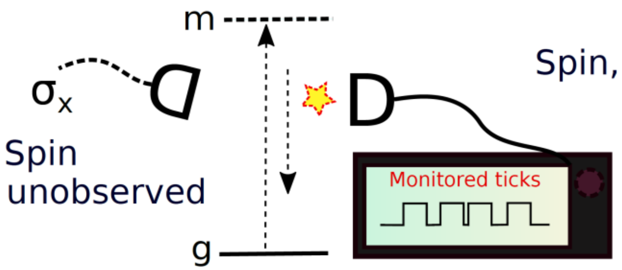


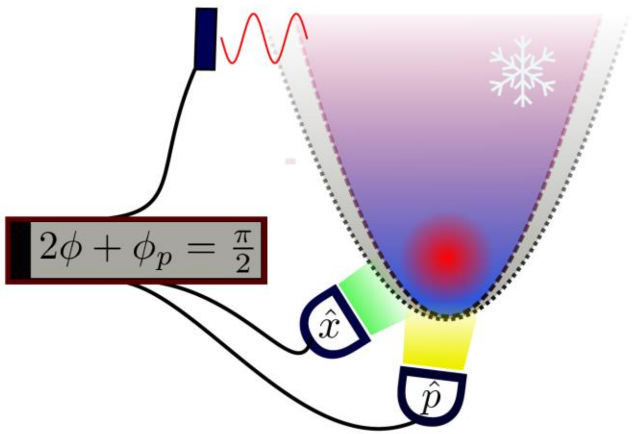
❖ Introduction: quick recap of thermodynamics, the arrow of time problem, fluctuation theorems

❖ Arrow of time, fluctuation theorems for quantum measurements

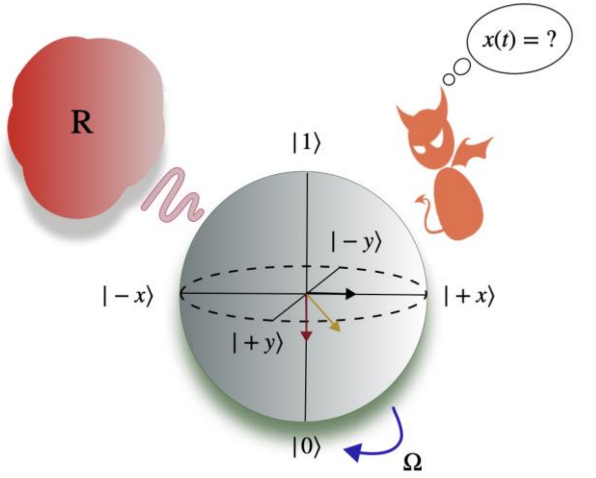
❖ **Thermodynamic device applications of measurement driven quantum systems:**







Example one: a qubit engine

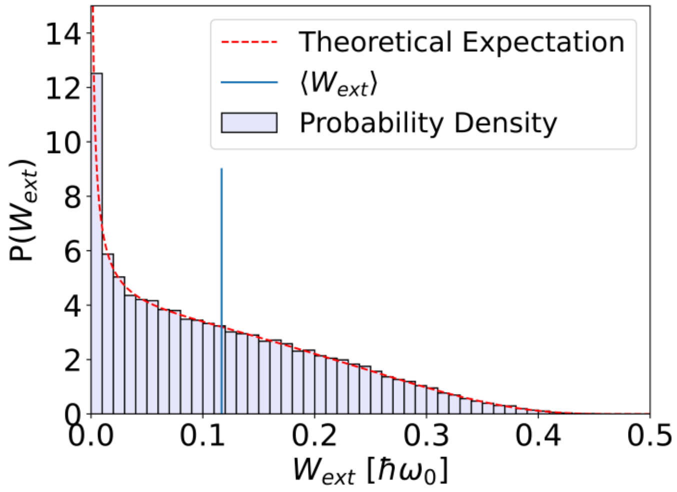


Yanik, K., Bhandari, B., Manikandan, S. K., & Jordan, A. N. (2022). Thermodynamics of quantum measurement and Maxwell's demon's arrow of time. Physical Review A, 106(4), 042221.

Time-continuous and weak Gaussian measurements

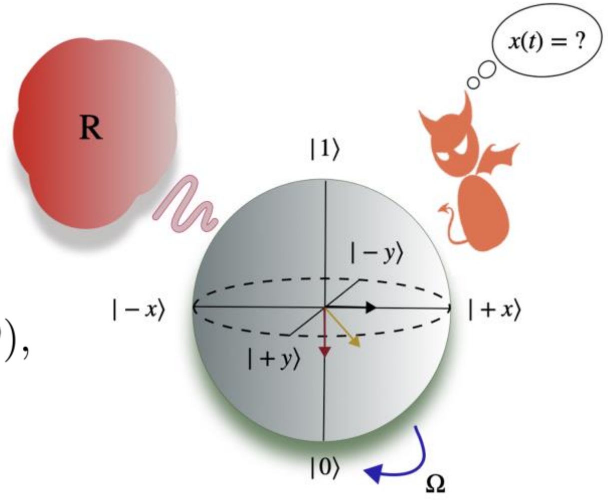
❖ Exact finite time distribution of Q (for $x_0 = 0$):

$$P(Q) = \sqrt{\frac{\tau}{2\pi\mathcal{T}}}\frac{e^Q}{\sqrt{e^Q - 1}}\, e^{\left(-\frac{\mathcal{T}}{2\tau} - \frac{\tau}{2\mathcal{T}}\left[\cosh^{-1}(e^{Q/2})\right]^2\right)}$$



$$P(W_{\text{ext}}) = -\frac{4e^Q}{\hbar\omega_0}\frac{1}{z_0 e^{Q/2} + \frac{z_0^2 - 1}{\sqrt{1 + (z_0^2 - 1)e^{-Q}}}}P(Q),$$

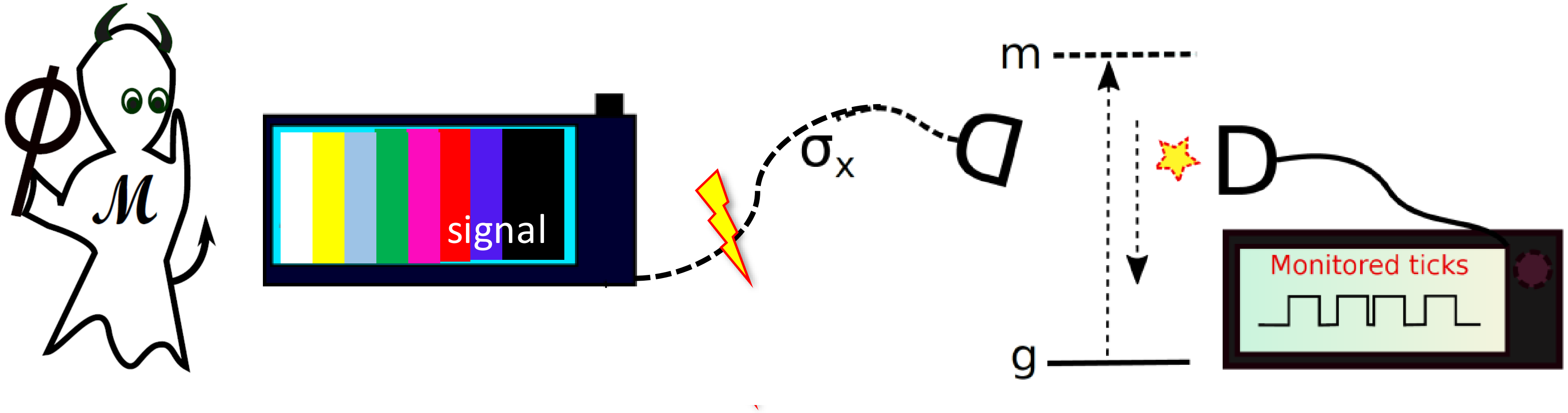
$$e^{-Q} = \left[\frac{2W_{\text{ext}}z_0}{\hbar\omega_0} + \sqrt{1 + \frac{4W_{\text{ext}}^2}{\hbar^2\omega_0^2}(z_0^2 - 1)}\right]^2.$$



Dressel, Justin, et al. "Arrow of time for continuous quantum measurement." *Physical review letters* 119.22 (2017): 220507

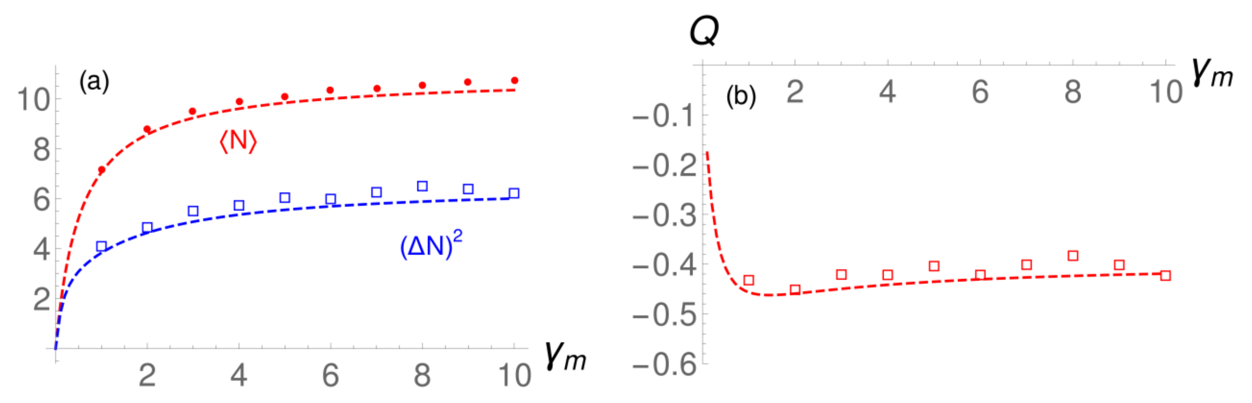
Yanik, K., Bhandari, B., Manikandan, S. K., & Jordan, A. N. (2022). Thermodynamics of quantum measurement and Maxwell's demon's arrow of time. Physical Review A, 106(4), 042221 (Editors' suggestion)

Example two: an autonomous quantum clock



Manikandan, Sreenath K. "Autonomous quantum clocks using athermal resources." arXiv preprint arXiv:2207.07909 (2022).

Statistics of ticks: Large deviation principle



Mean and variance of ticks

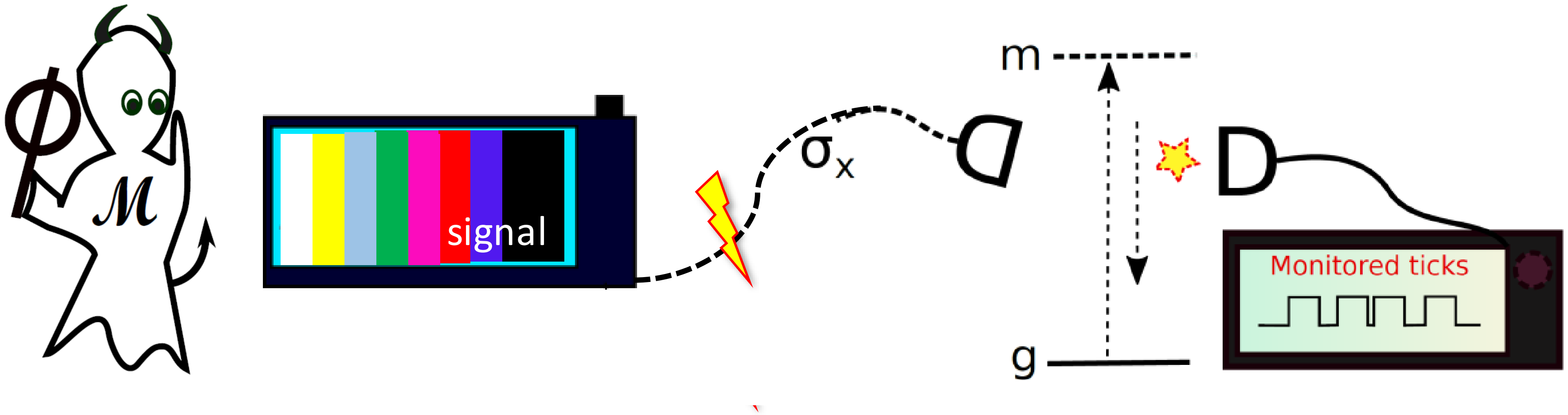
Mandel's Q parameter

Tilted Linbladian operator:

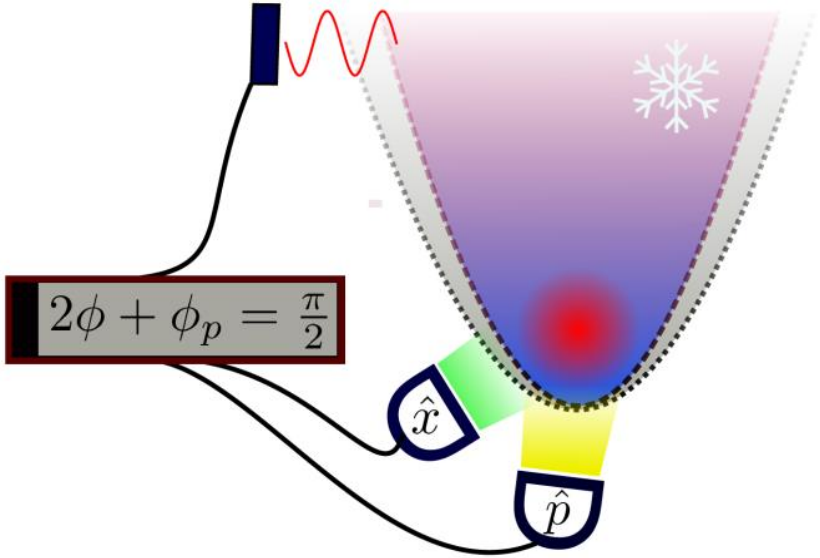
$$\mathcal{W}(s)[\rho] = -i[H,\rho] - \gamma_m[\sigma_x,[\sigma_x,\rho]] + \gamma_w e^{-s}\sigma_-\rho\sigma_+ - (\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)/2.$$

Accuracy of the clock:

$$\frac{\delta t_w}{t_w} = \sqrt{(1+Q)}/\sqrt{\gamma_{\rm tick} t_w}$$



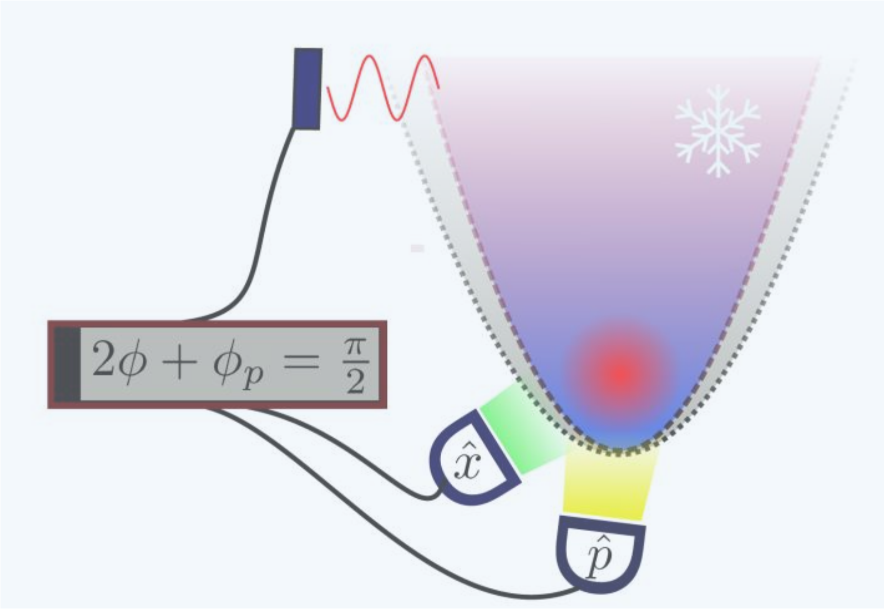
Example three: parametric quantum feedback cooling

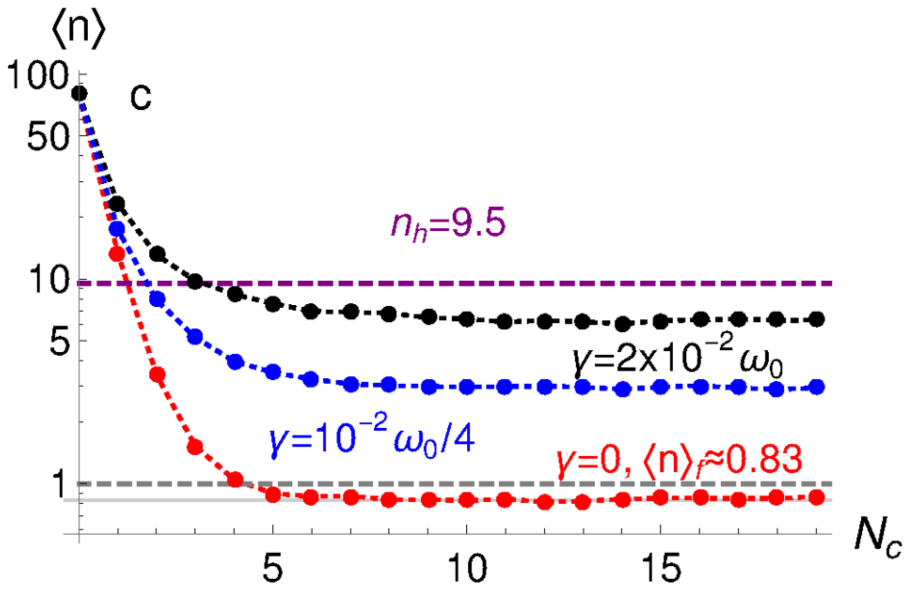


Qvarfort, S

Manikandan, S. K., & Qvarfort, S. (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.

The Quantum Noise Limit To Cooling





In the absence of any dissipation, the oscillator can be cooled down to $\boldsymbol{n_f = 0.83}$. The noise added by measurements plays a crucial role.

$$n_{min}(\xi_{j-1}) = \int d^2\xi_j \, Q[\xi_j, |\xi_{j-1}|, r_{sq}(\xi_{j-1})] \, n_{min}(\xi_j),$$

$$n_{min}(\xi_j) = (\sqrt{1 + 4|\xi_j|^2} - 1)/2.$$

Manikandan, S. K., & Qvarfort, S. (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.

Conclusions and outlook

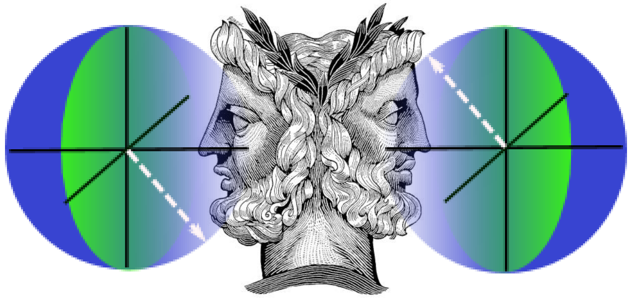
- The thermodynamics of quantum measurement driven devices is a fascinating field where principles of quantum mechanics and stochastic thermodynamics meet. Measurements can be thought of an additional resource in the problem.

- This also allows to address fundamental questions, such as the emergence of an arrow of time in the quantum regime (time-reversal symmetry breaking), and the nature of clocks (time-translational symmetry breaking) in the quantum regime.

- Experiments can be done in superconducting quantum circuits, or ultracold atoms:

  - Harrington, P. M., Tan, D., Naghiloo, M., & Murch, K. W. (2019). Characterizing a statistical arrow of time in quantum measurement dynamics. Physical Review Letters, 123(2), 020502.

  - Jayaseelan, M., K. Manikandan, S., Jordan, A. N., & Bigelow, N. P. (2021). Quantum measurement arrow of time and fluctuation relations for measuring spin of ultracold atoms. Nature communications, 12(1), 1847.



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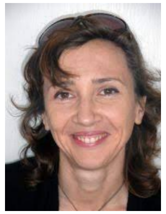
M Jayaseelan


N P Bigelow


A Korotkov


A N Jordan

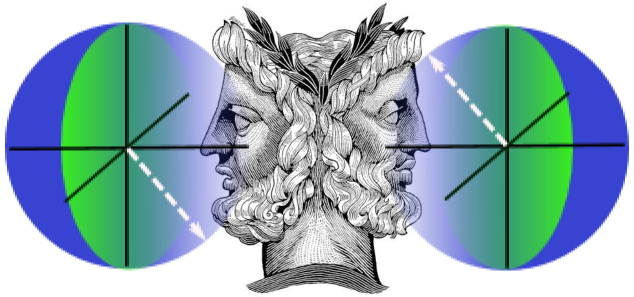

A Auffèves


KW Murch


S Qvarfort


S Krishnamurthy


P Nayak



Thank you for your time :)

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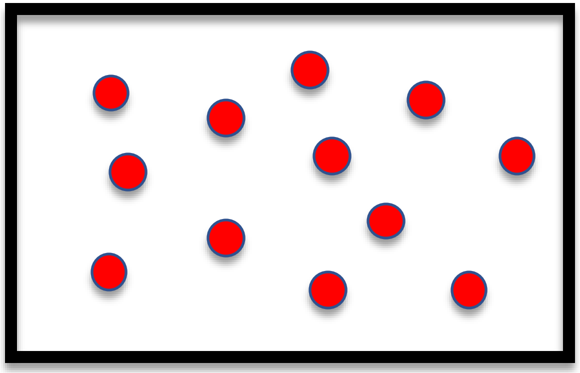
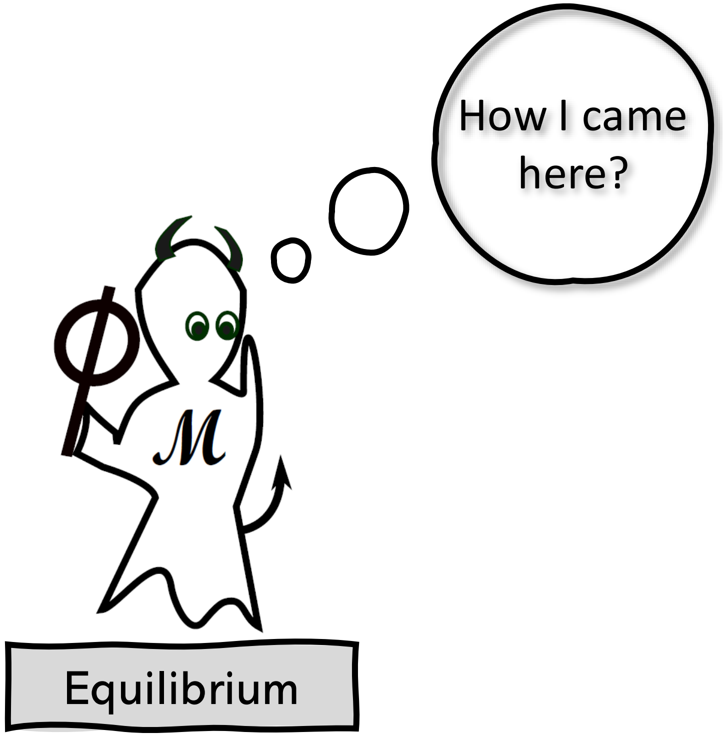
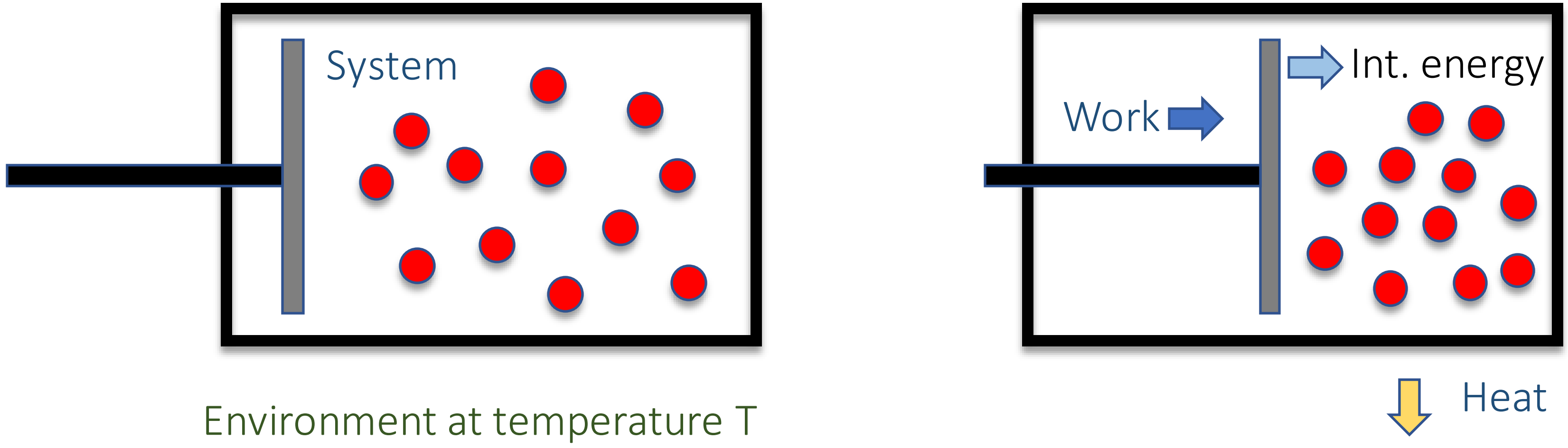
Principles of thermodynamics





Equilibrium states, characterized by very few parameters, U, V, N ... are the ones which maximizes entropy.

Processes in thermodynamics



Work: W Internal energy: E Heat: Q

The first law: $đW = dE - đQ$

The second law: $dS_{\mathrm{tot}} = dS_{\mathrm{sys}} + dS_{\mathrm{env}} \geq 0$

For systems in equilibrium at initial and final stages, $đW \geq dE - TdS_{\mathrm{sys}} \equiv dF.$